Trapped-Ion Entangling Gates Robust Against Qubit Frequency Errors

ECCTI 2020

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Mølmer–Sørensen Scheme

- Two-qubit entangling gate
- Laser- or microwave-driven
- Qubits interact via shared motion
- Global radiation
- Difficult to control at scale

Prior work

- Requires additional fields
- Only for hyper-fine qubits
- Perturbative analysis
- Relies on commutation with gate

Our aim

Strong, modulated control with no additional fields

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Energy Levels and Errors

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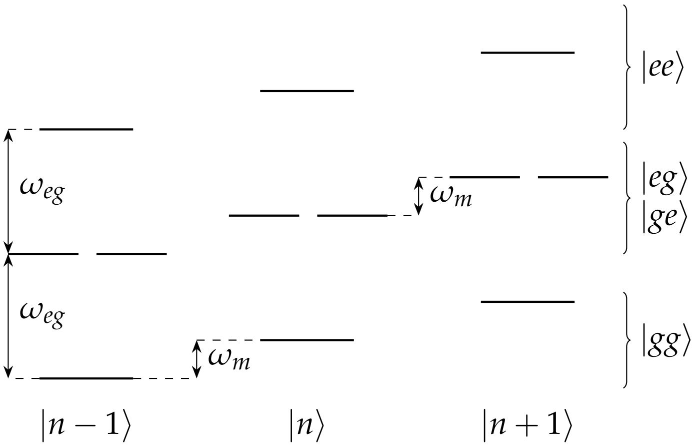

$$\frac{1}{\hbar}\hat{\mathcal{H}}_{\text{sys}} = \frac{1}{2}\left(\omega_{eg}\hat{\sigma}_z^{(1)} + \omega_{eg}\hat{\sigma}_z^{(2)}\right) + \omega_m\,\hat{a}^\dagger\hat{a}$$

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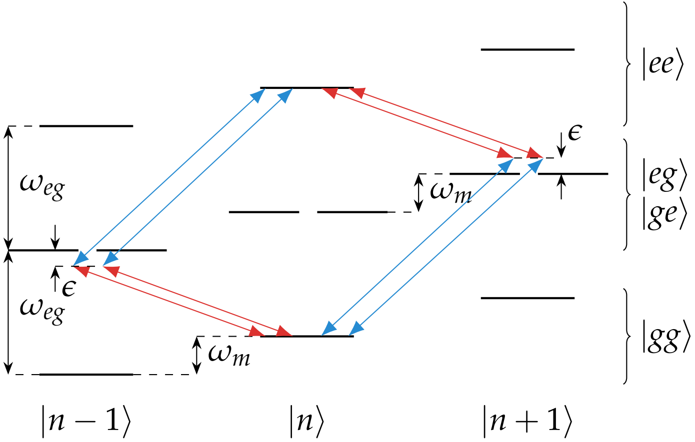

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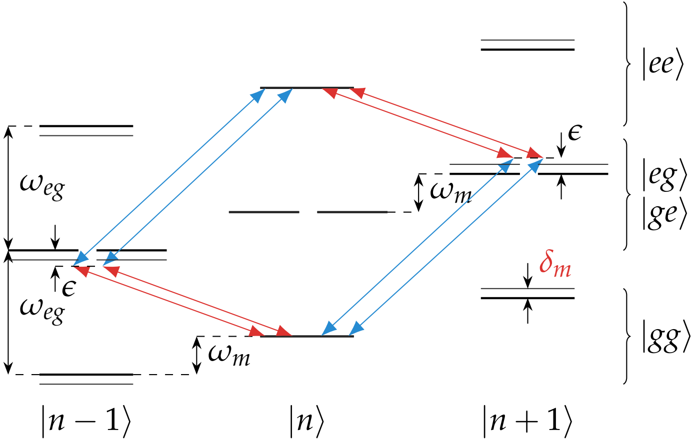
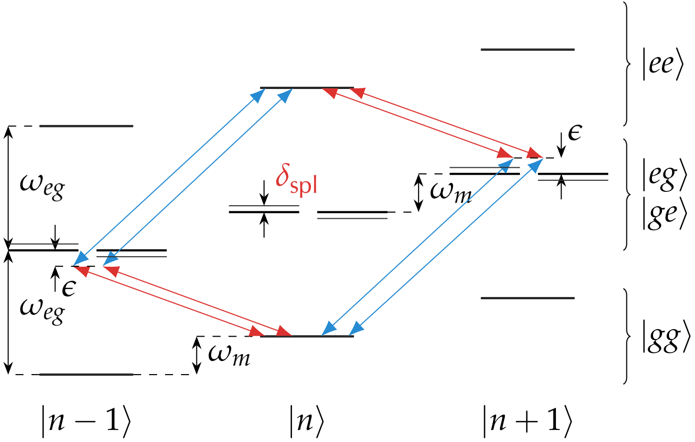


$$\frac{1}{\hbar}\hat{\mathcal{H}}_{\text{sys}} = \frac{1}{2}\left(\omega_{eg}\hat{\sigma}_z^{(1)} + \omega_{eg}\hat{\sigma}_z^{(2)}\right) + (\omega_m + \delta_m)\hat{a}^\dagger\hat{a}$$

Energy Levels and Errors



$$\frac{1}{\hbar}\hat{\mathcal{H}}_{sys} = \frac{1}{2}\Big(\big(\omega_{eg} + \delta_{spl}\big)\hat{\sigma}_z^{(1)} + \big(\omega_{eg} - \delta_{spl}\big)\hat{\sigma}_z^{(2)}\Big) + \omega_m\,\hat{a}^\dagger\hat{a}$$

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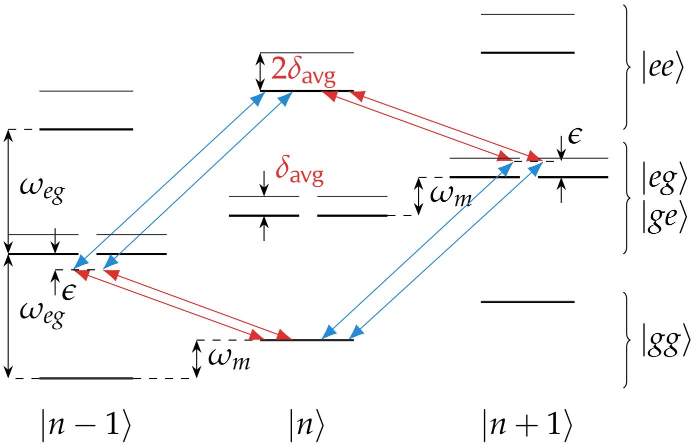

$$\frac{1}{\hbar}\hat{\mathcal{H}}_{\text{sys}} = \frac{1}{2}\Big((\omega_{eg} + \delta_{\text{avg}})\hat{\sigma}_z^{(1)} + (\omega_{eg} + \delta_{\text{avg}})\hat{\sigma}_z^{(2)} \Big) + \omega_m \hat{a}^\dagger \hat{a}$$

Hamiltonians

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What we want

$$\hat{\mathcal{H}}_{\mathrm{MS}} = -\eta f(t)\left(\hat{\sigma}_y^{(1)} + \hat{\sigma}_y^{(2)}\right)\hat{a}^\dagger + \mathrm{H.c.}$$

Hamiltonians

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What we get

$$\hat{\mathcal{H}}_{\mathrm{MS}} = -\eta f(t) e^{i\delta_m t}\hat{a}^\dagger \cdot \begin{pmatrix} \cos\left((\delta_{\mathrm{avg}} + \delta_{\mathrm{spl}})t\right)\hat{\sigma}_y^{(1)} \\ + \sin\left((\delta_{\mathrm{avg}} + \delta_{\mathrm{spl}})t\right)\hat{\sigma}_x^{(1)} \\ + \cos\left((\delta_{\mathrm{avg}} - \delta_{\mathrm{spl}})t\right)\hat{\sigma}_y^{(2)} \\ + \sin\left((\delta_{\mathrm{avg}} - \delta_{\mathrm{spl}})t\right)\hat{\sigma}_x^{(2)} \end{pmatrix} + \mathrm{H.c.}$$

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Perturbative solutions are not possible

Optimising the Gate

Target dynamics

- $\hat{\mathcal{U}}_{\text{tg}} \left|\chi_k\right\rangle = \left|\psi_k\right\rangle$, e.g. $\hat{\mathcal{U}}_{\text{tg}} \left|gg\right\rangle \propto \left|gg\right\rangle + i \left|ee\right\rangle$
- $\hat{\mathcal{U}}_{\text{MS}}(\vec{\delta})$ should match $\hat{\mathcal{U}}_{\text{tg}}$
- Evolve $\hat{\mathcal{H}}_{\text{MS}}$ to find infidelity I

$$I = 1 - \frac{1}{K} \sum_{k=1}^{K} \left| \langle \psi_k | \hat{\mathcal{U}}_{\text{MS}} | \chi_k \rangle \right|^2$$

Optimisation

- Infidelity I averages all start states

$$E\left[I(\vec{\delta})\right] = \int I(\vec{\delta})\,\mathrm{d}w(\vec{\delta})$$

- Drive gate with several tones

$$f(t) = ce^{it} \to \sum_{k=1}^{n} c_k e^{ik\epsilon t}$$

$$\hat{\mathcal{H}}_{\text{MS}} = -f(t) \cdot \hat{\mathcal{S}}_{x,y}^{(1,2)}(\delta_{\text{avg}}, \delta_{\text{spl}}, \delta_m)\,\hat{a}^\dagger + \text{H.c.}$$

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Numerical Techniques

Target evaluation

- Target is 2D integral

$$\text{FoM} = \iint I(\vec{\delta}) e^{-\frac{\vec{\delta}\cdot\vec{\delta}}{2\sigma^2}} \, \mathrm{d}\vec{\delta}$$

- Use Gauss–Hermite quadrature

$$\text{FoM} \approx \sum_{k=1}^{m} I(\vec{\lambda}_k) w_k$$

- Few evaluations to reach high accuracy degree

Power constraint

$$f(t) = \sum_k c_k e^{ik\epsilon t}$$

$$\max_t |f(t)|^2 = \text{const}$$

- Find zeros of polynomial

$$e^{in\epsilon t} \partial_t |f(t)|^2 = \sum_{j=0}^{2n} \zeta_j e^{ij\epsilon t}$$

- Scale c_k to satisfy constraint

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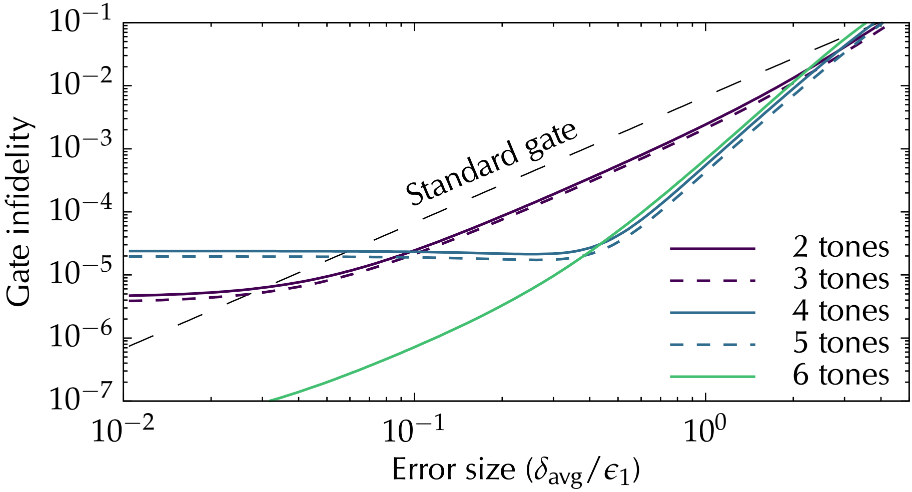
Infidelity Improvement



Amplitude Variation

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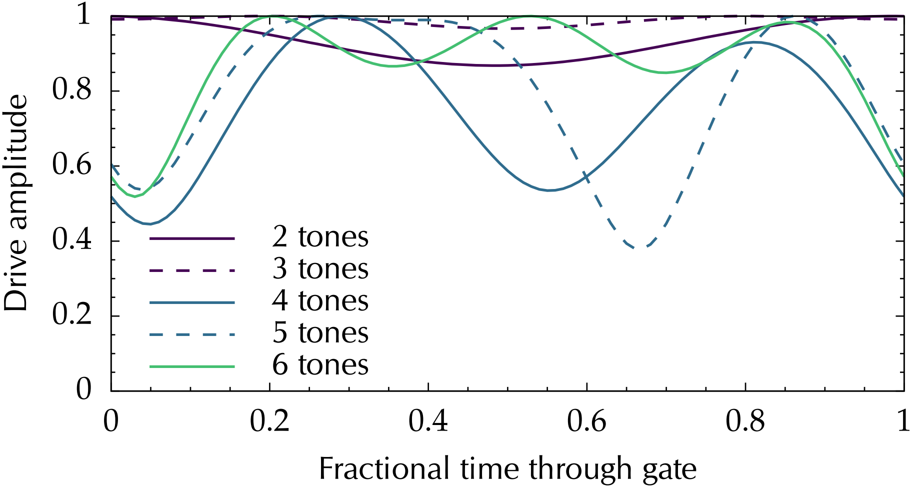
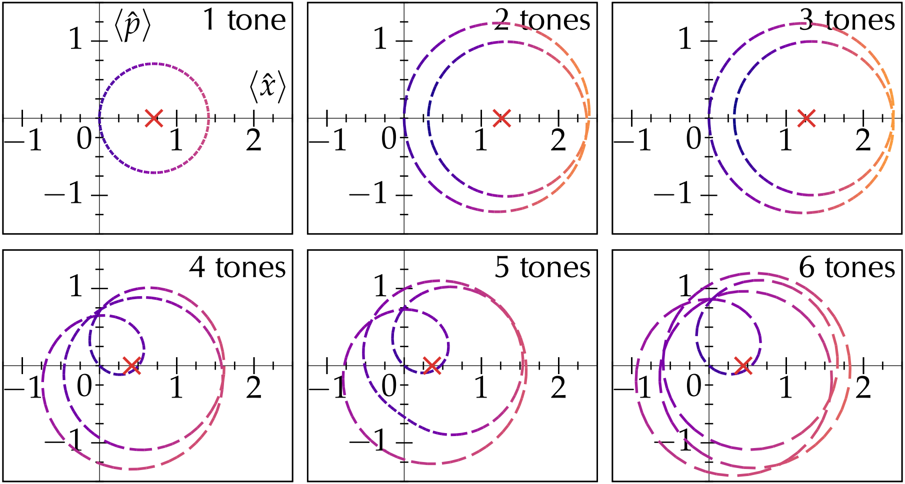


Phase-Space Diagrams



Outlook

Achieved

- Minimised qubit errors with no additional fields
- Unified several errors
- Optimised nonlinear constraints

Further work

- Verify experimentally
- Vary phase to reduce motional heating
- Move outside Lamb–Dicke regime

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Imperial Ion Trapping

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QSE

EPSRC
Engineering and Physical Sciences Research Council

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