

# Consistent description for $^{11}\text{Li}$ scattering from light to heavy targets

Manuela Rodríguez-Gallardo  
José Antonio Lay    Antonio Moro  
Joaquín Gómez-Camacho

Universidad de Sevilla

28 June 2022

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description for  
 $^{11}\text{Li}$  scattering  
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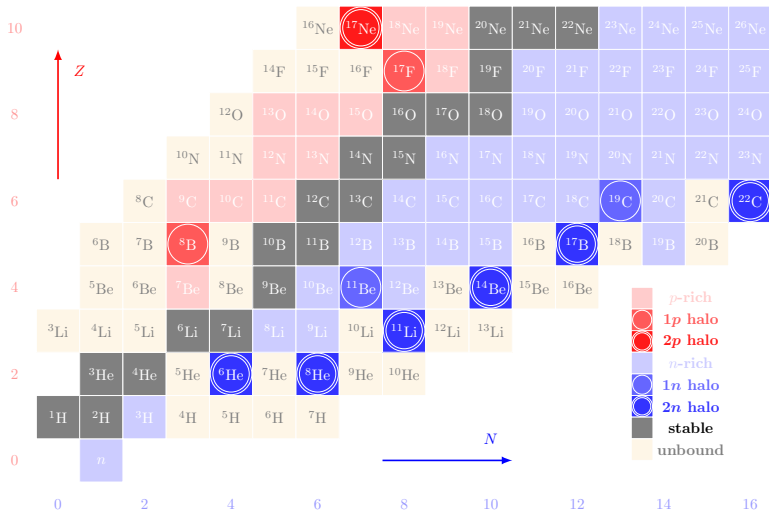
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Gallardo

- Motivation: Borromean nucleus  $^{11}\text{Li}$
- Continuum-Discretized Coupled-Channels (CDCC)
  - ⇒ Discretization methods
- $B(E1)$  using different structure models
- Four-body CDCC applied to:
  - ⇒  $^{11}\text{Li} + ^{208}\text{Pb}$  at 29 MeV
    - ↻ Dipolar resonance at low energies
  - ⇒  $^{11}\text{Li} + ^{64}\text{Zn}$  at 22.5 MeV
  - ⇒  $^{11}\text{Li} + d$  at 55.3 MeV
  - ⇒  $^{11}\text{Li} + p$  at 66 MeV
- $B(E1)$  vs  $B(E1)_{iso}$ ?
- Summary and future work

# Motivation

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

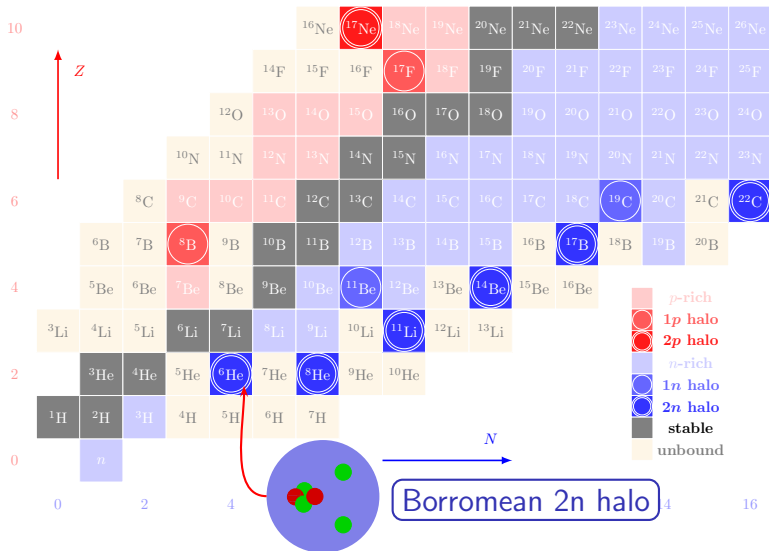
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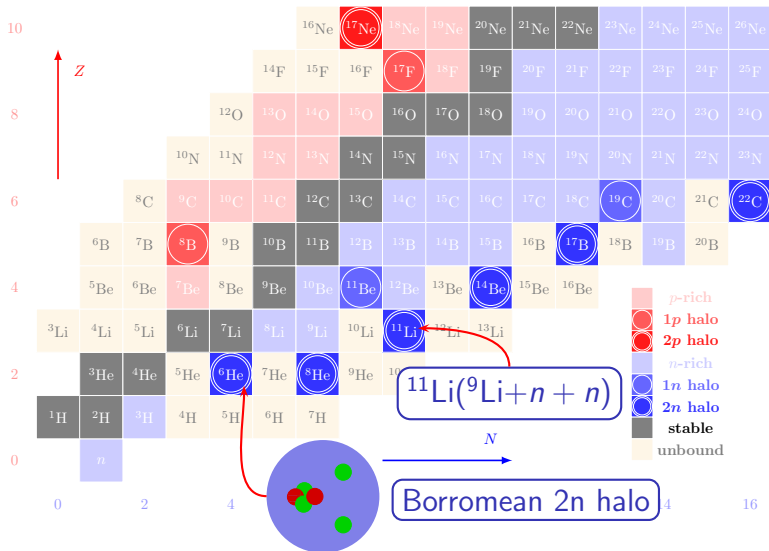
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Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

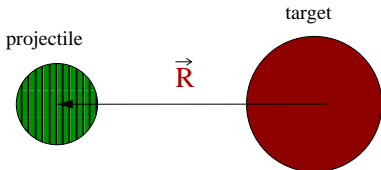
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# CDCC formalism

Consistent  
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 $^{11}\text{Li}$  scattering  
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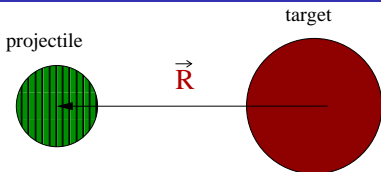
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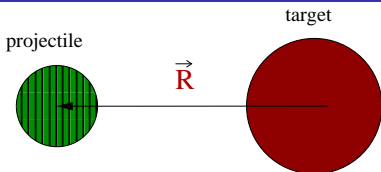


$$\Psi_J^M(\vec{R}, \xi) = \sum \phi_{jn}^\mu(\xi) \langle LM_L j \mu | JM \rangle \frac{i^L}{R} Y_L^{M_L}(\hat{R}) f_{Lnj}^J(R)$$

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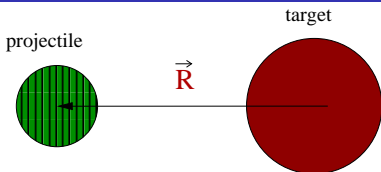
↑  
1, 2, 3...n particles



# CDCC formalism

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$$\Psi_J^M(\vec{R}, \xi) = \sum \phi_{jn}^\mu(\xi) \langle LM_L J \mu | JM \rangle \frac{i^L}{R} Y_L^{M_L}(\hat{R}) f_{Lnj}^J(R)$$

Coupled-channels system

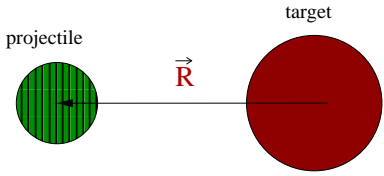
$$\left[ -\frac{\hbar^2}{2m_r} \left( \frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + \varepsilon_{nj} - E \right] f_{Lnj}^J(R) + \sum_{L'n'j'} i^{L'-L} V_{Lnj, L'n'j'}^J(R) f_{L'n'j'}^J(R) = 0$$

1, 2, 3... n particles

# CDCC formalism

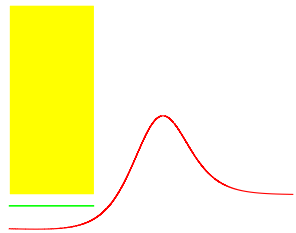
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$$\Psi_J^M(\vec{R}, \xi) = \sum \phi_{jn}^\mu(\xi) \langle LM_L j \mu | JM \rangle \frac{i^L}{R} Y_L^{M_L}(\hat{R}) f_{Lnj}^J(R)$$

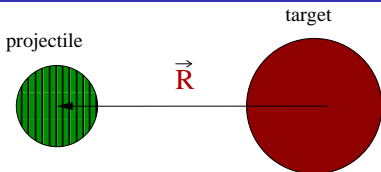
$\phi_n^{j\mu}(\xi)?$



# CDCC formalism

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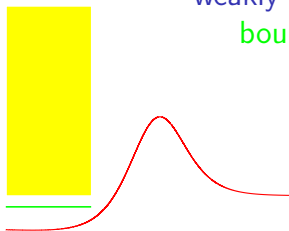
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$$\Psi_J^M(\vec{R}, \xi) = \sum \phi_{jn}^{\mu}(\xi) \langle LM_L j \mu | JM \rangle \frac{i^L}{R} Y_L^{M_L}(\hat{R}) f_{Lnj}^J(R)$$

weakly-bound systems  
bound + continuum

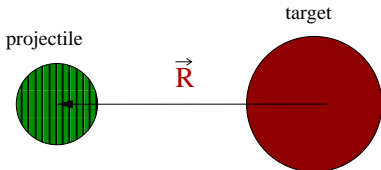
$\phi_n^{j\mu}(\xi)?$



# CDCC formalism

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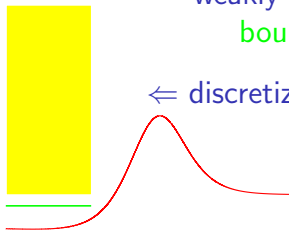
$$\Psi_J^M(\vec{R}, \xi) = \sum \phi_{jn}^{\mu}(\xi) \langle LM_L j \mu | JM \rangle \frac{i^L}{R} Y_L^{M_L}(\hat{R}) f_{Lnj}^J(R)$$

weakly-bound systems

bound + continuum

$\phi_n^{j\mu}(\xi)$ ?

← discretization methods

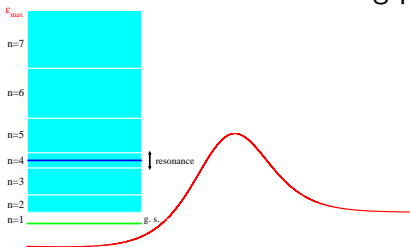


# Discretization methods

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- Pseudo-State (PS)** methods: They consist in diagonalizing the Hamiltonian in a complete discrete basis, truncated at a maximum number of states
  - Transformed Harmonic Oscillator (THO) method
- Binning** procedure: It consists in calculating the true continuum states and making packages of energy

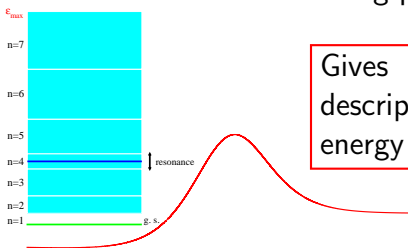


# Discretization methods

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- 1 **Pseudo-State (PS)** methods: They consist in diagonalizing the Hamiltonian in a complete discrete basis, truncated at a maximum number of states
  - ➡ Transformed Harmonic Oscillator (**THO**) method
- 2 **Binning** procedure: It consists in calculating the true continuum states and making packages of energy



Gives a more precise description of the low-energy continuum

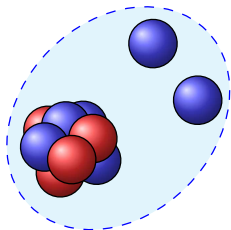
# Application to $^{11}\text{Li}$

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$$\varepsilon_{gs} = -0.369 \text{ MeV}$$

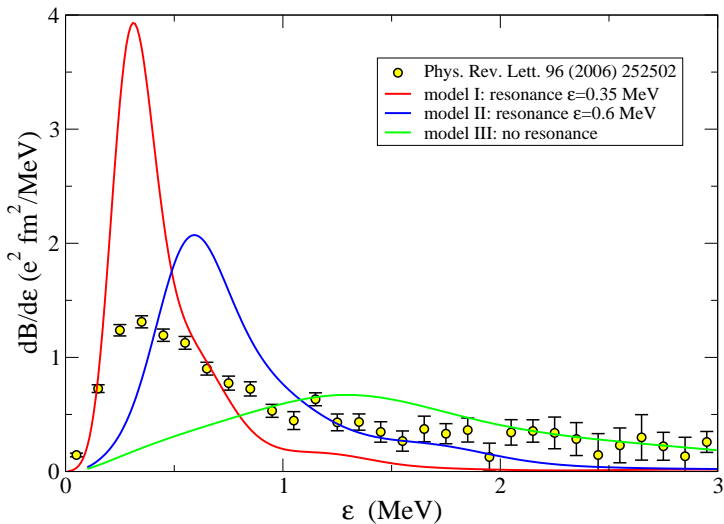


$$\begin{array}{l}
 \text{-----} \quad 1/2^+, 3/2^+, 5/2^+(1^-)? \\
 \text{g.s.} \quad \text{-----} \quad 3/2^-(0^+)
 \end{array}$$

# $B(E1)$ distribution

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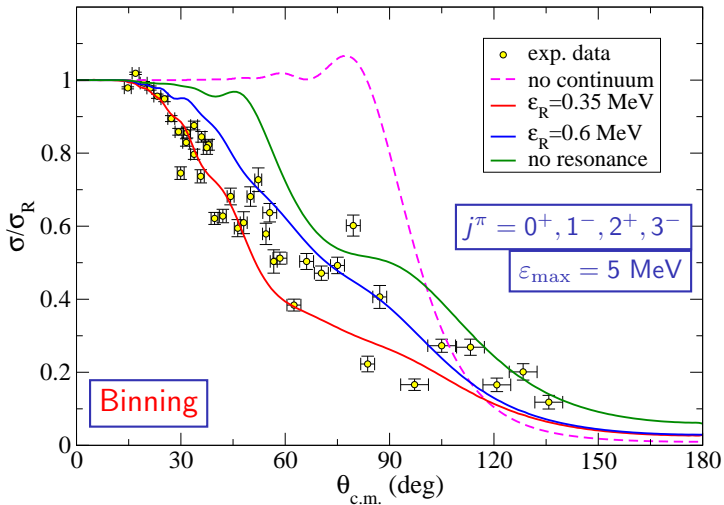




# $^{11}\text{Li} + ^{208}\text{Pb}$ at 29.8 MeV (elastic)

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

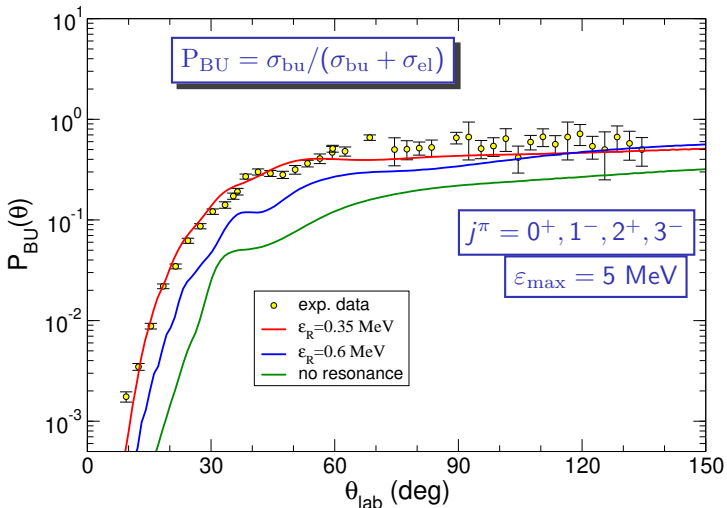
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# $^{11}\text{Li} + ^{208}\text{Pb}$ at 29.8 MeV (breakup)

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

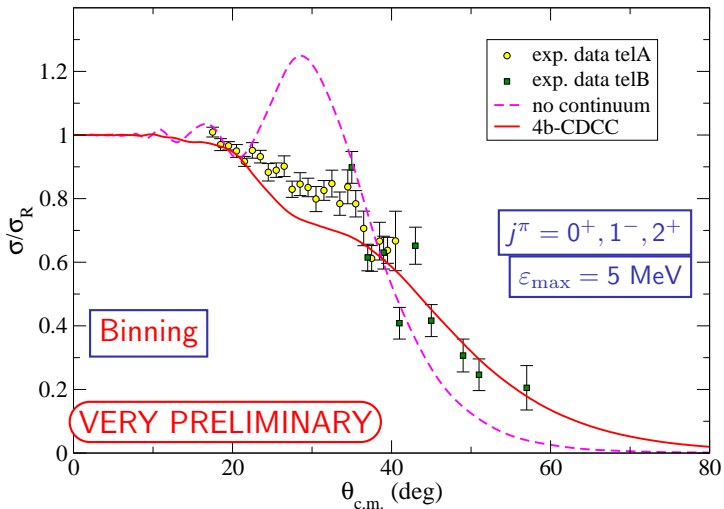
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# $^{11}\text{Li} + ^{64}\text{Zn}$ at 22.5 MeV (elastic)

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

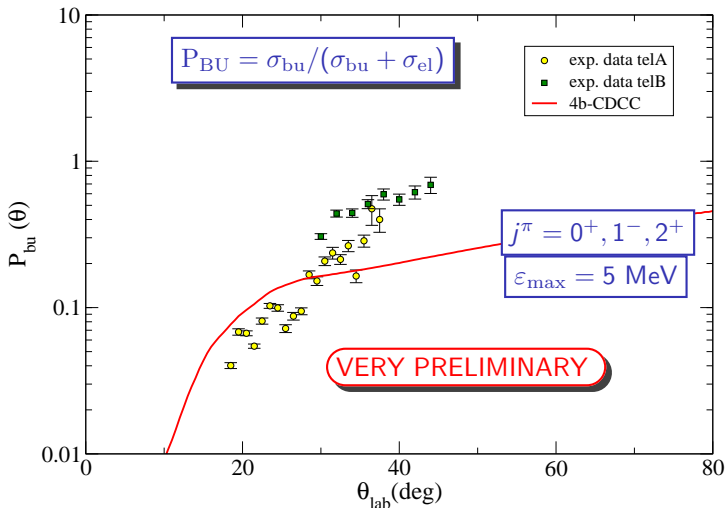
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# $^{11}\text{Li} + ^{64}\text{Zn}$ at 22.5 MeV (breakup)

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☒ S1847 (TRIUMF, Canada): A. di Pietro, J.P. Fernández et al.

# What about light targets?

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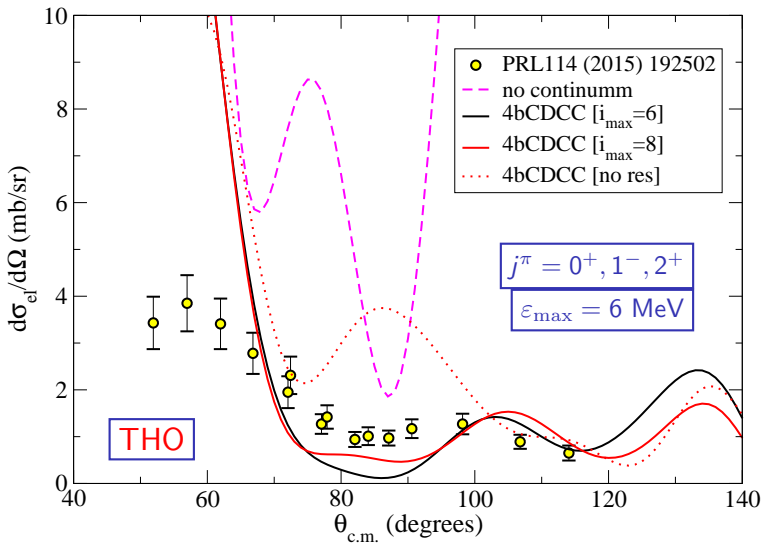
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- $^{11}\text{Li} + ^{208}\text{Pb}, ^{64}\text{Zn}$  at energies around the Coulomb barrier are **Coulomb-dominated** reactions.
- These reactions can be then understood as an **isovectorial** type excitation, being due mainly to the dipolar electric excitation ( $B(E1)$ ).
- However, the reactions on **light targets** like  $p$  and more specially  $d$  are not Coulomb-dominated reactions so the excitations can be understood **more as an isoscalar** excitation.
- What happens if we apply the same  $^{11}\text{Li}$  structure model reproducing the data on  $^{208}\text{Pb}, ^{64}\text{Zn}$  to the reactions on  $p$  and  $d$  published more recently?

# $^{11}\text{Li}+d$ at 55.3 MeV (elastic)

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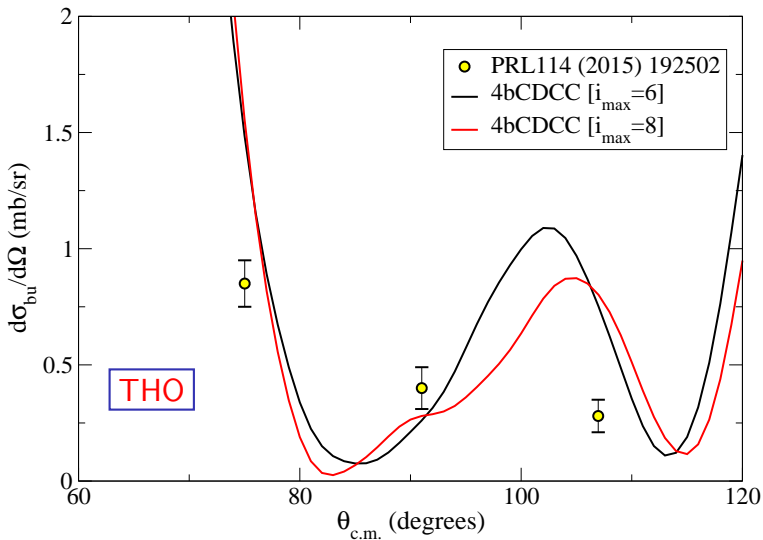
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# $^{11}\text{Li}+d$ at 55.3 MeV (bu to resonance)

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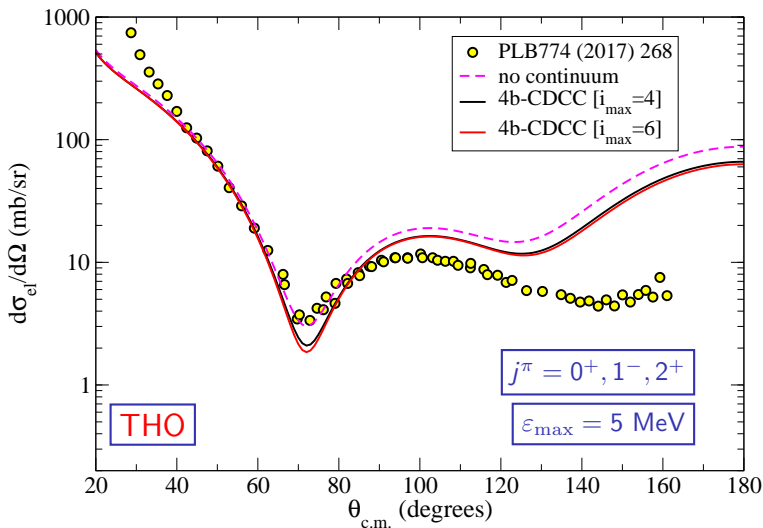
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# $^{11}\text{Li} + p$ at 66 MeV (elastic)

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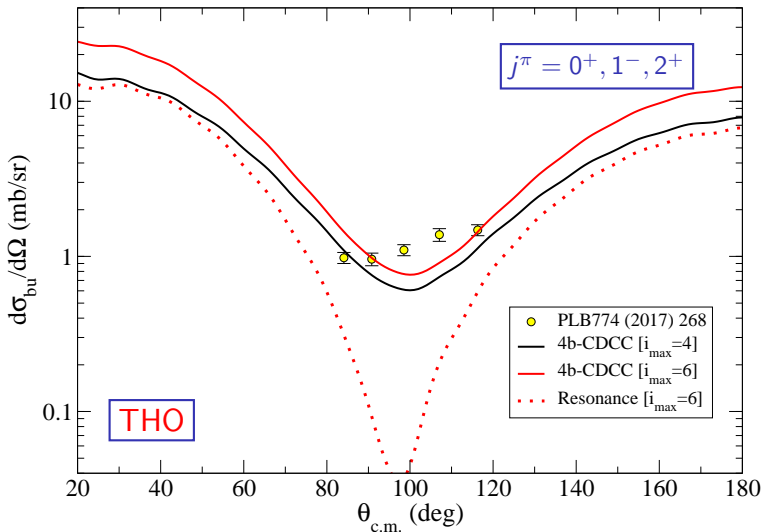




# $^{11}\text{Li} + p$ at 66 MeV (bu up to 3 MeV)

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

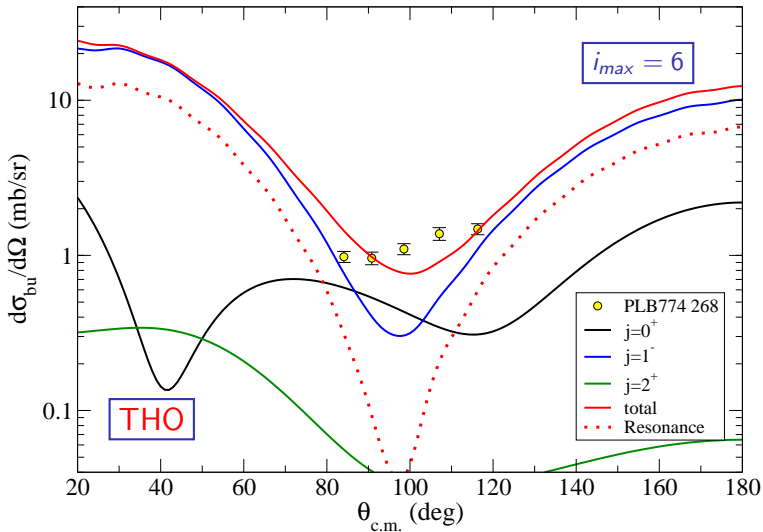
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# $^{11}\text{Li} + p$ at 66 MeV (bu up to 3 MeV)

Consistent description for  $^{11}\text{Li}$  scattering from light to heavy targets

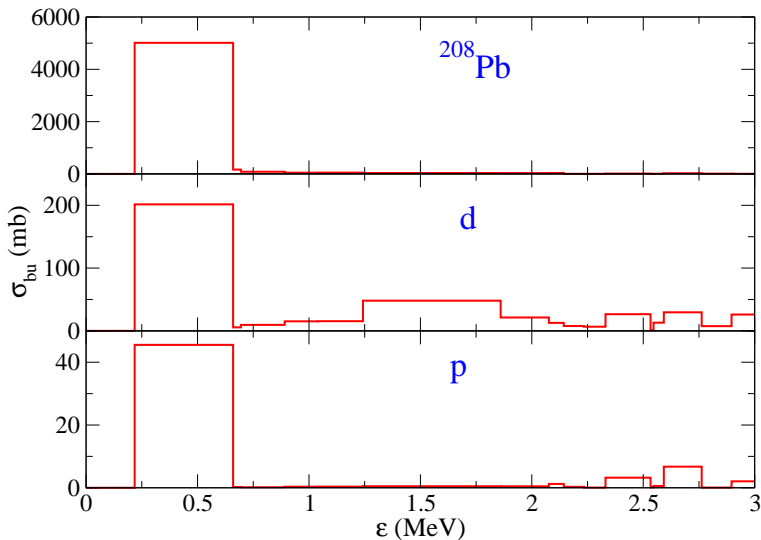
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# Energy distributions states $j = 1^-$

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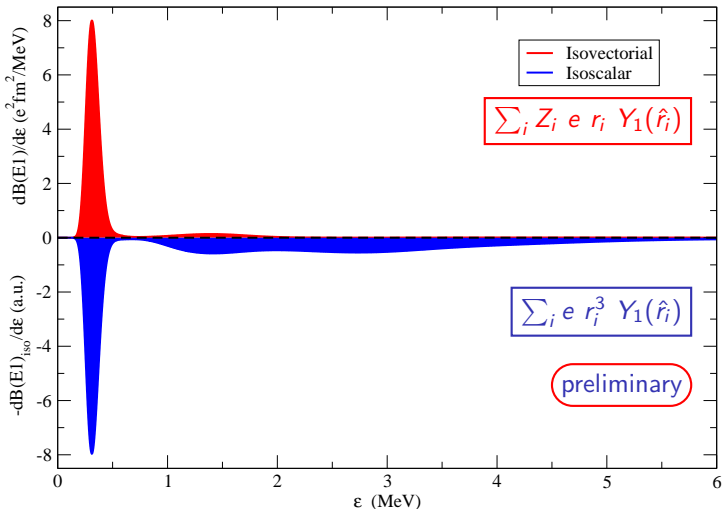
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# $B(E1)$ vs $B(E1)_{iso}$

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☒ Thanks to E. Lanza for discussions on the isoscalar part.

# Summary and conclusions

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- Systematic analysis of existing  $^{11}\text{Li}$  elastic and bu data on light and heavy targets have been performed in terms of the 4-body CDCC formalism.
- The comparison between experimental data (elastic and bu) and 4b-CDCC calculations for  $^{11}\text{Li}+^{208}\text{Pb}$  suggested the existence of a dipolar resonance at about 0.7 MeV over the g.s.
- The same structure model, with a dipolar resonance at low energies, can also explain reasonably well the experimental data for  $^{11}\text{Li}+^{64}\text{Zn}$  at 22.5 MeV,  $^{11}\text{Li}+d$  at 55.2 MeV and  $^{11}\text{Li}+p$  at 66 MeV.
- The comparison (preliminary) between isovectorial and isoscalar  $B(E1)$  distributions reveals a large coupling to the same resonant state.

# Acknowledgments

Consistent  
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PID2020-114687GB-I00



*"Una manera de hacer Europa"*

P20\_01247

# Four-body CDCC (three-body projectiles)

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→ Three-body THO:

Rodríguez-Gallardo et al., PRC 72 (2005) 024007

→ Four-body CDCC with THO method:

Rodríguez-Gallardo et al., PRC 77 (2008) 064609

→ Four-body CDCC with binning procedure:

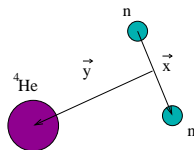
Rodríguez-Gallardo et al., PRC 80 (2009) 051601(R)

→ Three-body Analytical THO (ATHO) method:

Casal et al., PRC 88 (2013) 014327

→ Four-body CDCC with ATHO:

Casal et al., PRC 92 (2015) 054611



# Optical potentials

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- $^9\text{Li} + ^{208}\text{Pb}, ^{64}\text{Zn}$  from adjustment to experimental data from same experiment.
- $^9\text{Li} + p$  from folding with JLM nucleon-nucleon interaction ( $^9\text{Li}$  density as a simple Fermi distribution).
- $^9\text{Li} + d$  as a TELP from  $d$  breakup on  $^9\text{Li}$  with  $n/p + ^9\text{Li}$  optical potential with same folding with JLM.
- $n + ^{208}\text{Pb}, ^{64}\text{Zn}$  Koning & Delaroche.
- $n + p$  effective Gaussian potential.
- $n + d$  as a TELP from previous  $n + p$  interaction.



# Experimental $B(E1)$

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PRL 96, 252502 (2006)

PHYSICAL REVIEW LETTERS

week ending  
30 JUNE 2006

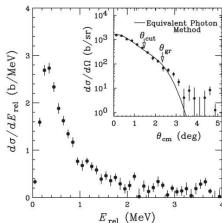


FIG. 2. Breakup cross sections for  $^{11}\text{Li} + \text{Pb}$  at 70 MeV/nucleon as a function of the three-body relative energy for data with  $\theta_{\text{cm}} \leq 5^\circ$ . Inset: Angular distribution of  $^{11}\text{Li}$  (the  $^9\text{Li} + n + n$  c.m.) scattered by the Pb target in the range  $0 \leq E_{\text{rel}} \leq 4$  MeV.  $\theta_{\text{gr}}$  denotes the grazing angle ( $2.34^\circ$ ). The calculation using the equivalent photon method is shown by the solid curve.

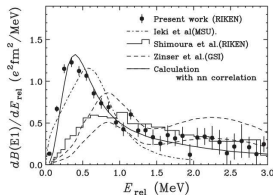


FIG. 3. The  $B(E1)$  distribution obtained in the present work (solid circles) is compared with those from previous measurements [dotted-dashed line [13], solid histogram [14], dashed lines (zone) [15]]. The present data are also compared with the calculation (solid line) [20] which included the full  $n$ - $n$  correlation.

- Indirect measurement: from Coulomb breakup data.
- Based on the Equivalent Photon Model (EPM).