Introduction to Solid State Physics

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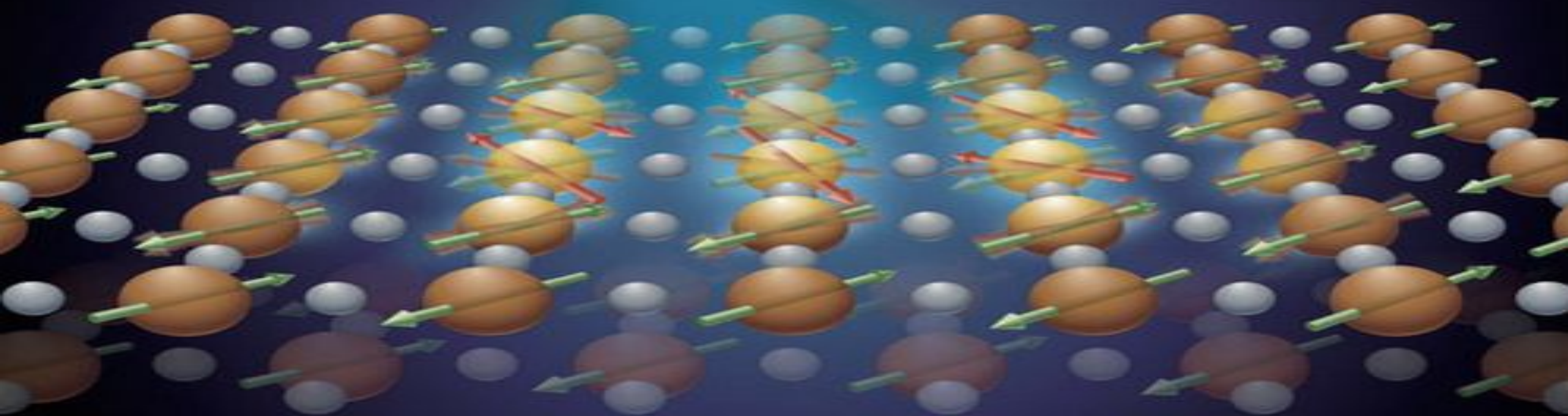
Faculté des Sciences de Tunis, Université Tunis El Manar



Outline

Lecture I: Introduction to Solid State Physics

- Brief story…
- Solid state physics in daily life
- Basics of Solid State Physics

Lecture II: Electronic band structure and electronic transport

- Electronic band structure: Tight binding approach
- Applications to graphene: Dirac electrons

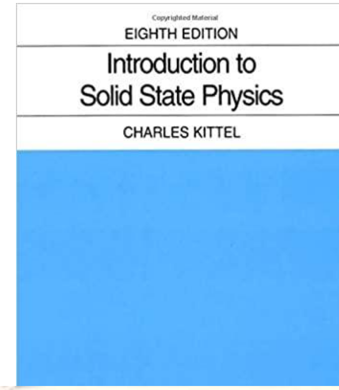
Lecture III: Introduction to Topological materials

- Introduction to topology in Physics
- Quantum Hall effect
- Haldane model

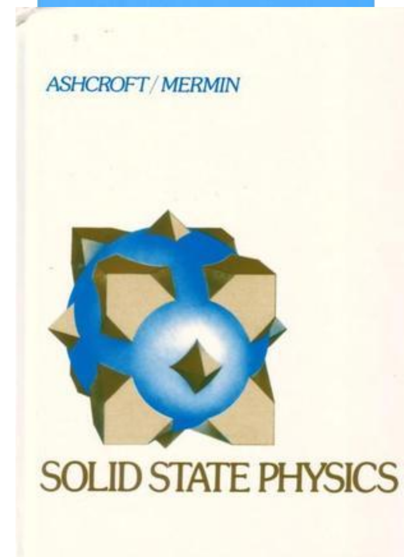
**It's an online lecture, but…stay focused…
there will be Quizzes and Assignments!**

References

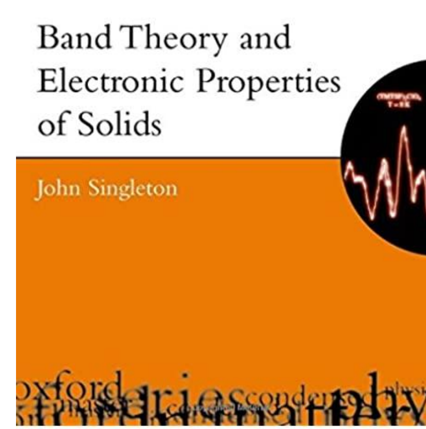
Introduction to Solid State Physics, Charles Kittel



Solid State Physics Neil Ashcroft and N. Mermin



Band Theory and Electronic Properties of Solids, John Singleton



Outline

Lecture I: Introduction to Solid State Physics

- **A Brief story…**
- **Solid state physics in daily life**
- **Basics of Solid State Physics**

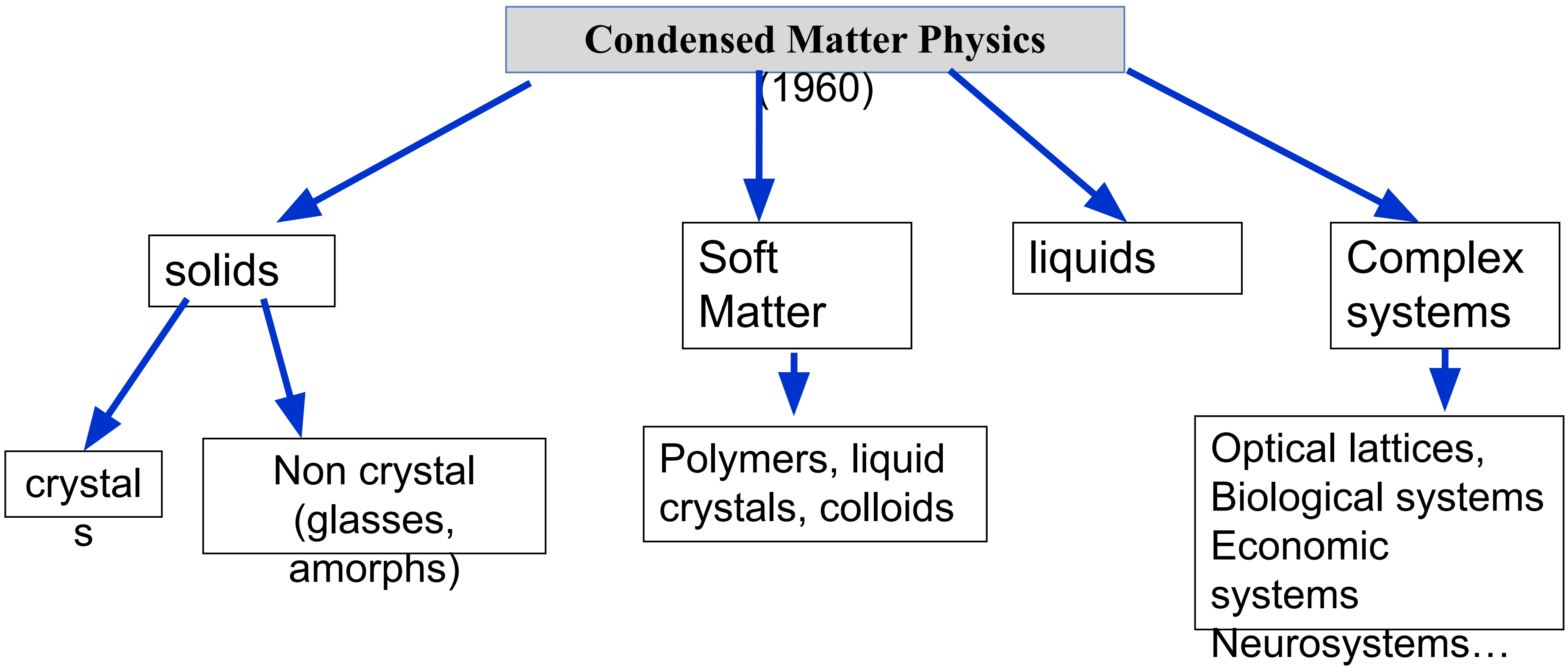
Lecture II: Electronic band structure and electronic transport

- **Tight binding approach**
- **Applications to graphene: Dirac electrons**

Lecture III: Introduction to Topological materials

- **Introduction to topology in Physics**
- **Quantum Hall effect**
- **Haldane model**

What is solid state Physics?

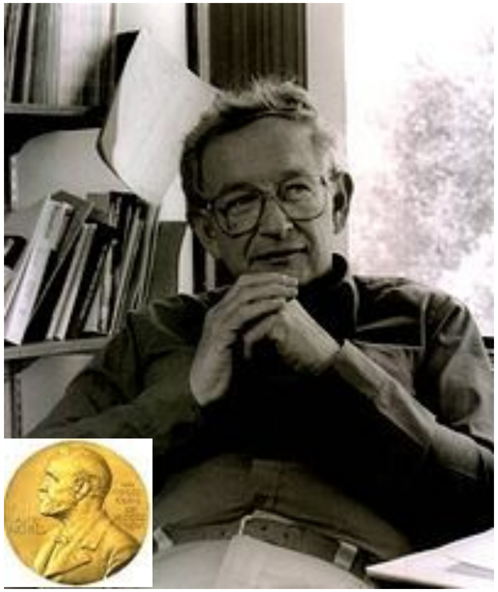


What is condensed Matter Physics?

"*More is different!*"

P.W. Anderson

$1+1 \neq 2$



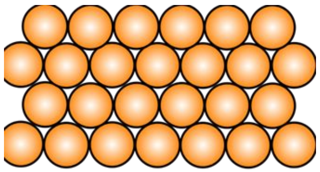
What is Condensed Matter Physics?

Condensed matter physics deals with systems of large number of interacting particles (constituents)

Explores macroscopic behavior of matter based on its microscopic properties

What is solid state Physics?

Crystalline solids



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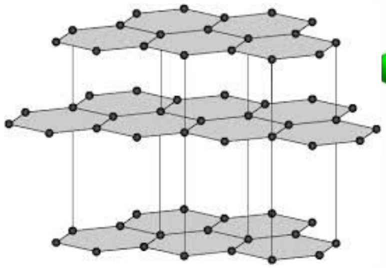

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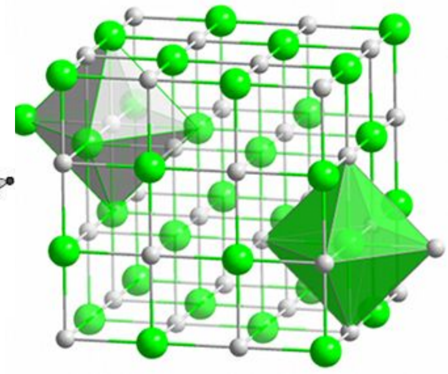
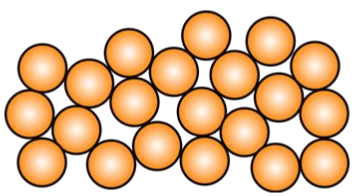
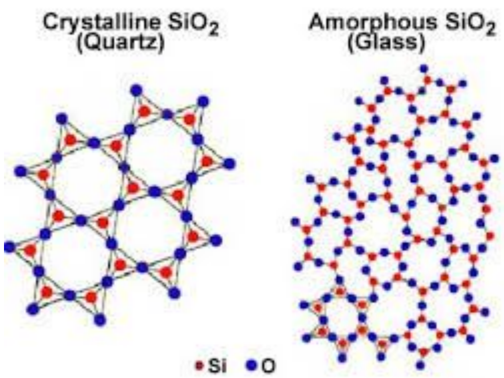
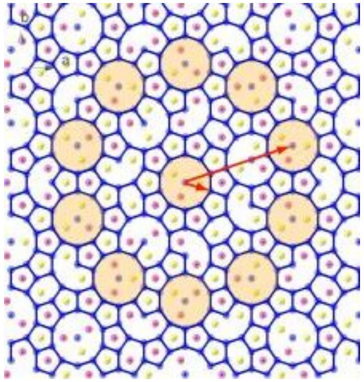


Na
Cl

Non crystalline solids







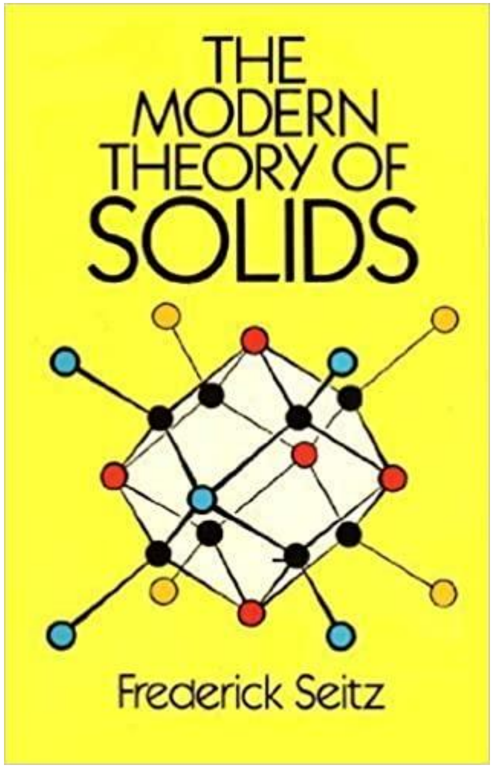
Quasi-crystals:
Ordred at large scale but are not periodic

What is solid state Physics?

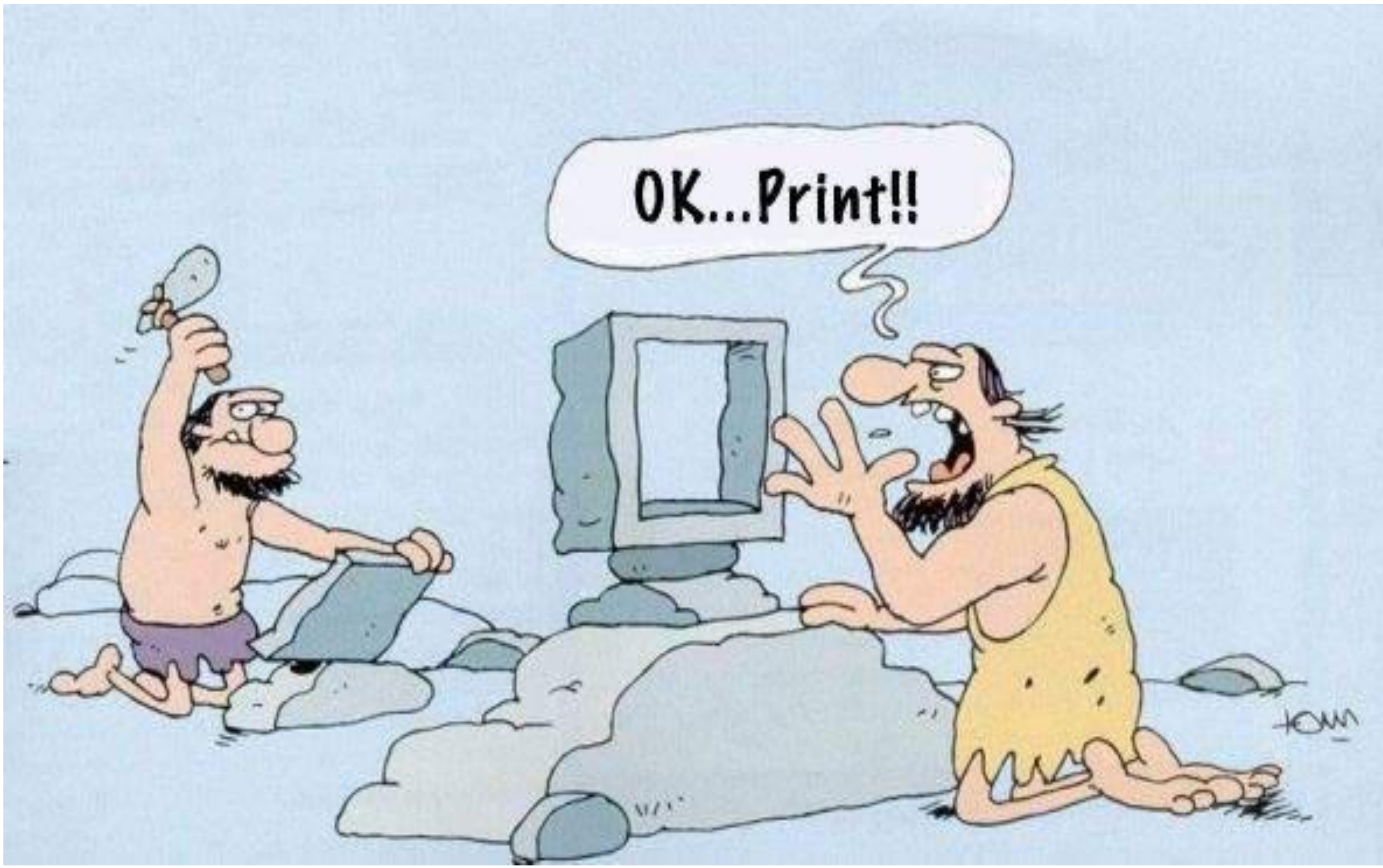
- Study the microscopic properties (electrical, thermal, magnetic…) of solids
- Solid: system with a large number of particles (atoms, molecules, ions) in strong interaction (contrary to a gas)

Birth of Solid state Physics:

publication of Fredrick Seitz book
Modern Theory of Solids (1940)



Solid state and Materials Physics: a Brief story…



Brief story…

More than 2.6 millions years ago

a Brief story…

The stone age

More than 2.6 millions years ago in Africa

Flint
(contains crystals of quartz SiO2)

Knapping process of a flint

a Brief story…

The stone age

a Brief story…

The bronze age

3000 to 1000 BC

Bronze: alloy of copper.

a Brief story…

The iron age

à 1100 BC

a Brief story…

1400 to 50 BC

1320 BC

4th-century

Lycurgus Cup
Glass+ gold nanoparticles

a Brief story…

Hannibal Barca
Carthaginian general

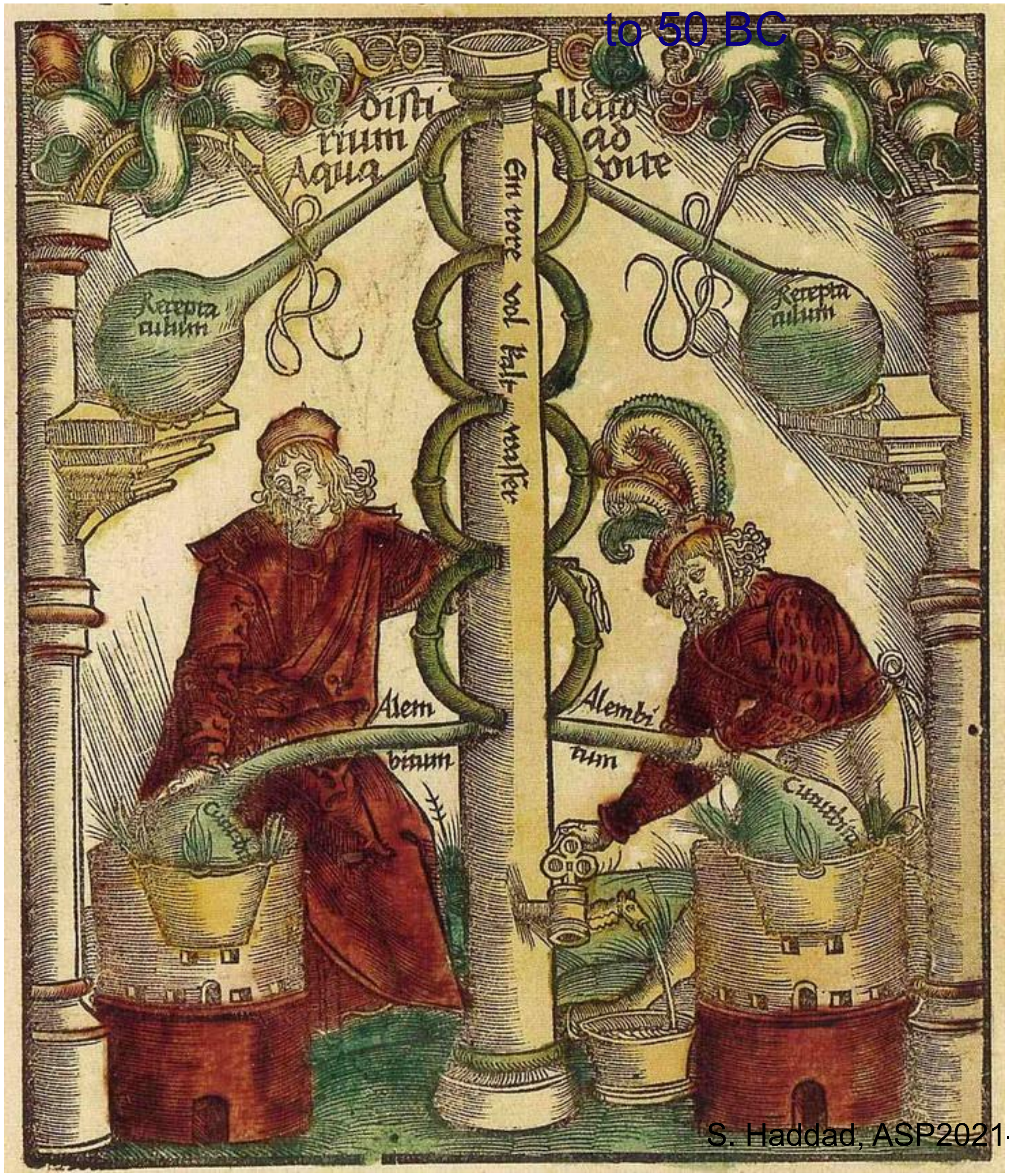
2sd punic war 218 BC

Quiz: Did Hannibal use African or Indian Elephants?

Materials Physics: a Brief story…

Middle age:
500 to 1500

to 50 BC



Use of:
Gold (Au), silver (Ag), iron (Fe), tin (Sn), mercury (Hg), , copper (Cu), lead (Pb), carbon (C), sulfer (S)…

Alchemy:
transform metals in gold !

Materials Physics: Birth of crystallography…

Modern period
1500 to 1800

Discovery of crystallographic order

1781

René Just Haüy

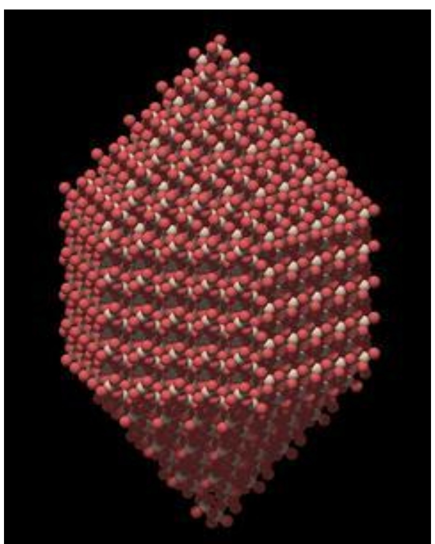
Materials Physics: Birth of crystallography…

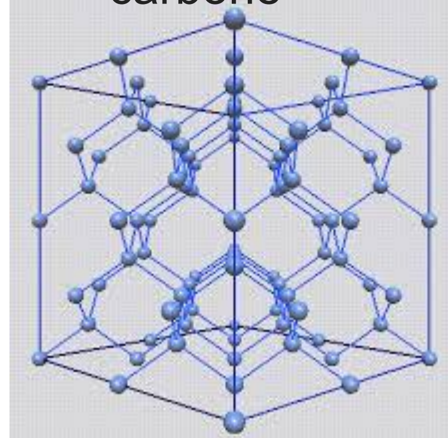
Modern period
1500 to 1800

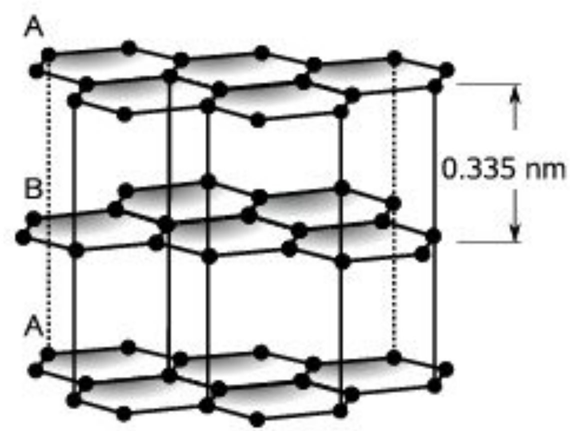
Quartz
SiO_2

Diamon
d
carbone

Graphite
carbone







Materials Physics: Birth of crystallography… 1895

How to observe the crystallographic order ?

X rays

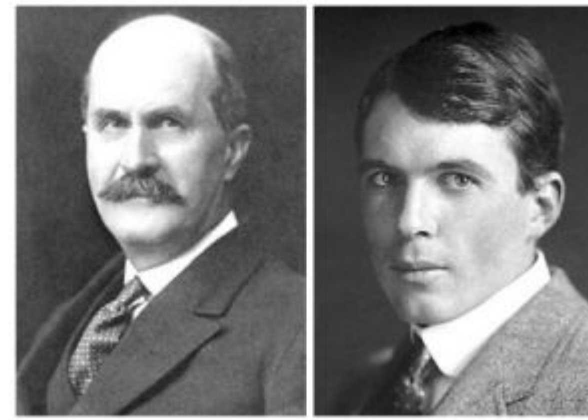
1st Nobel prize in Physics 1901

Materials Physics: Birth of crystallography… 1895

How to observe the crystallographic order ?

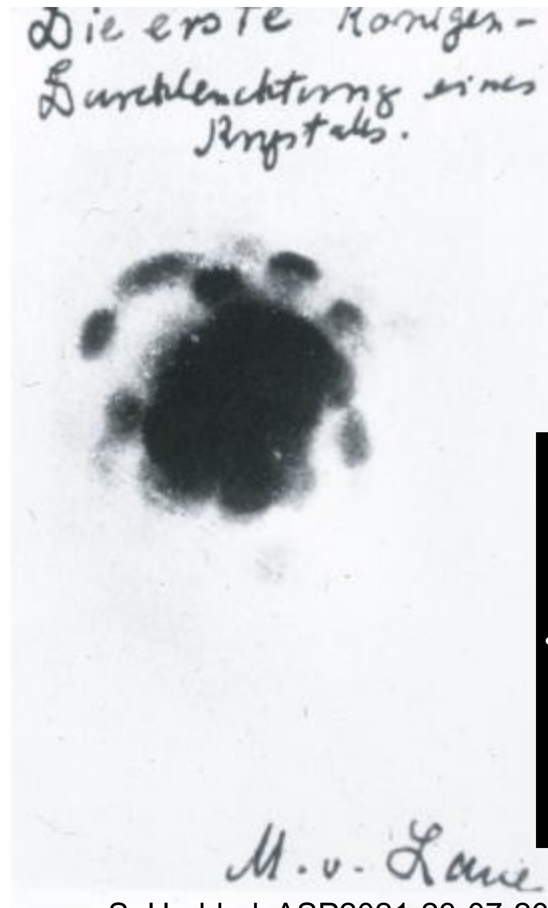
1915

X rays

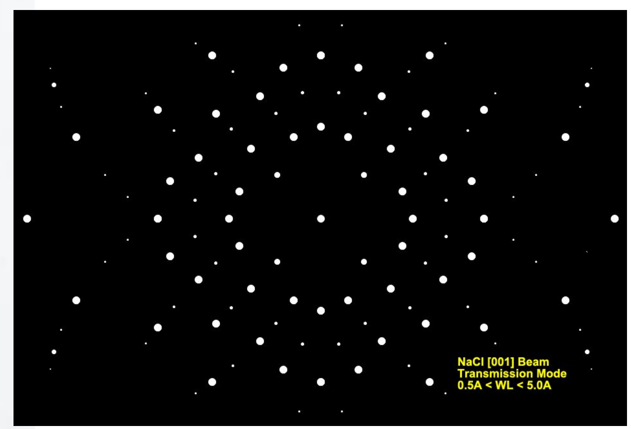


Max von Laue

1914



William Henry Bragg
William Lawrence Bragg



Emergence of Materials Physics

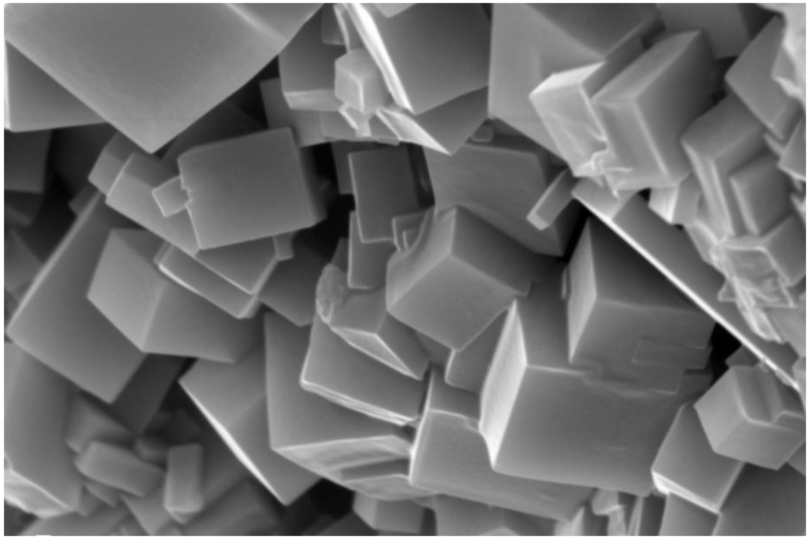
How to observe the crystallographic order ?

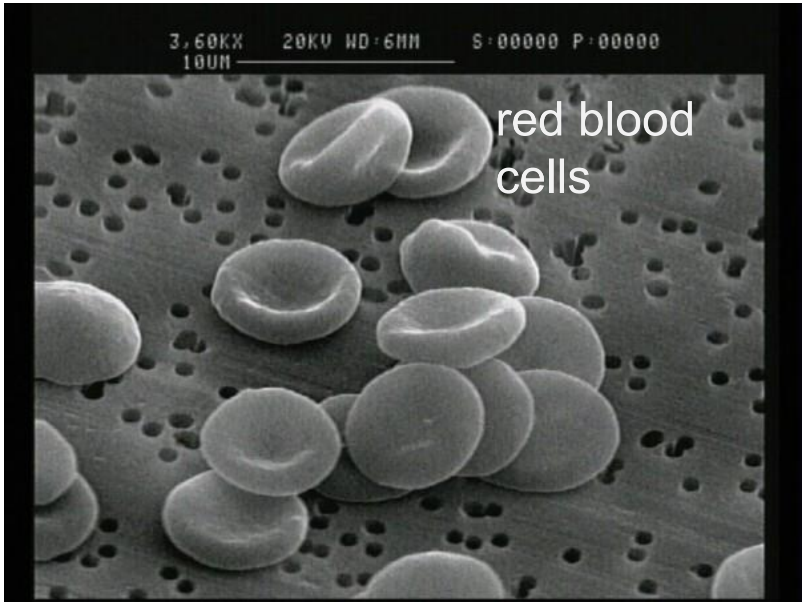
Electron Microscopy (1931)
Max Knoll and Ernst Ruska…

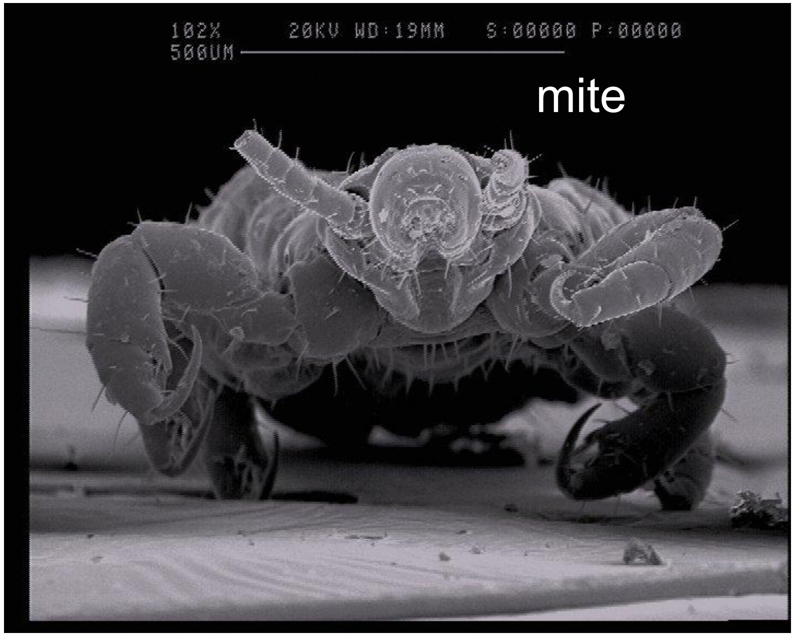
https://www.youtube.com/watch?v=wMLvodIVYNI

Emergence of Materials Physics

Electron Microscopy







Salt crystals

Lecture I: Introduction to solid state Physics

Some milestones

1897: Discovery of the electron by Thomson

Joseph John Thomson

1900: Quanta theory of Max Planck

1900: Drude model: 1^{st} model of metals

Paul Drude

1924: De Broglie duality

Louis de Broglie

1926: Fermi – Dirac statistics

Enrico Fermi

Paul Adrien Maurice Dirac

1926: Sommerfeld model of electrons in metals

Arnold Johannes Sommerfeld

1926: Schrödinger equation

Erwin Schrödinger

1928: Bloch theorem and electronic band theory

Felix Bloch

1948: Invention of transistor

William Bradford Shockley

John Bardee n

Walter Houser Brattain

Emergence of Materials Physics

1956

Fabrication 1st transistor in 1949 based on Germanium

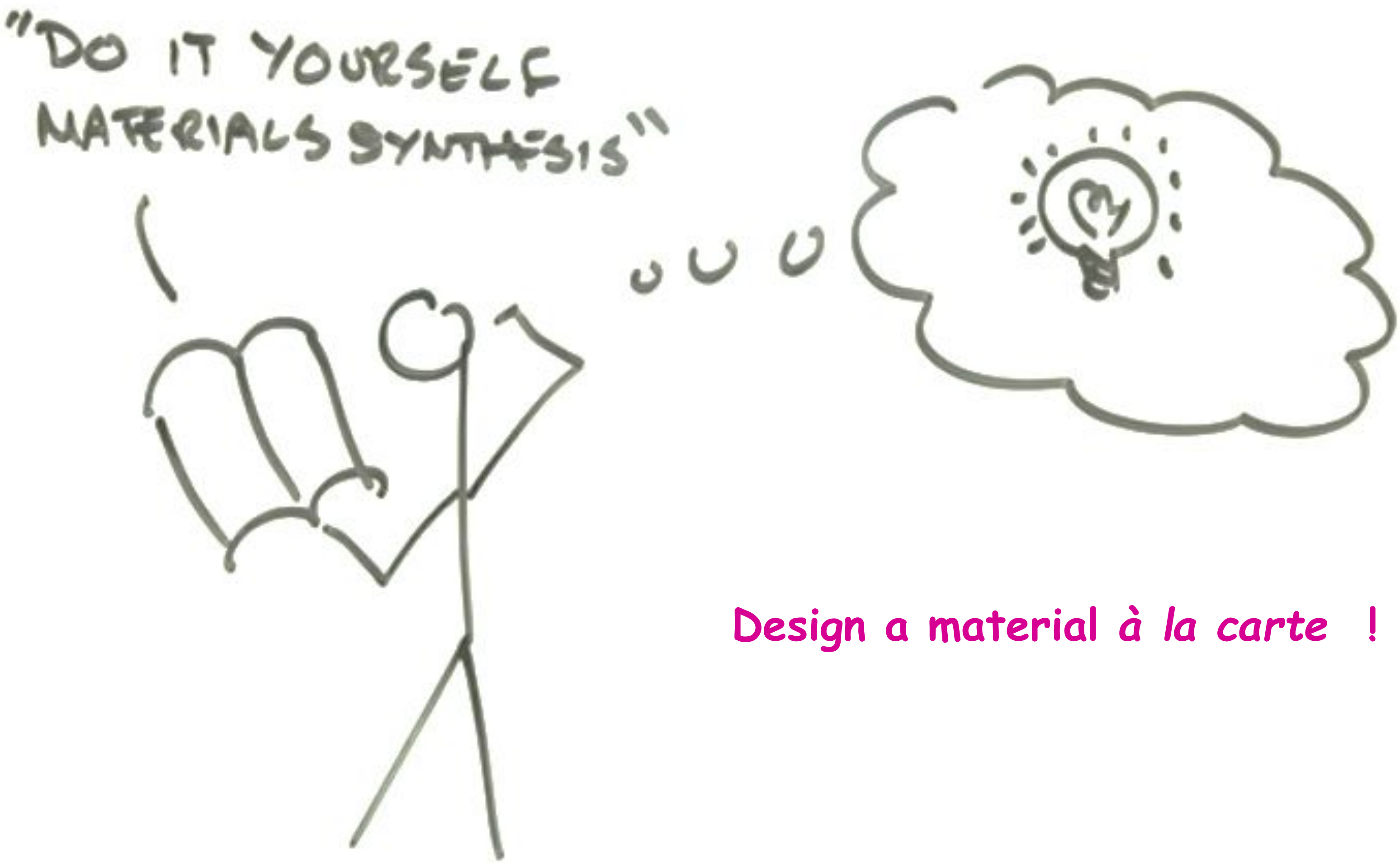
John Bardeen, William Shockley et Walter Brattain

Starting of la microelectronics

Emergence of Advanced Materials Physics

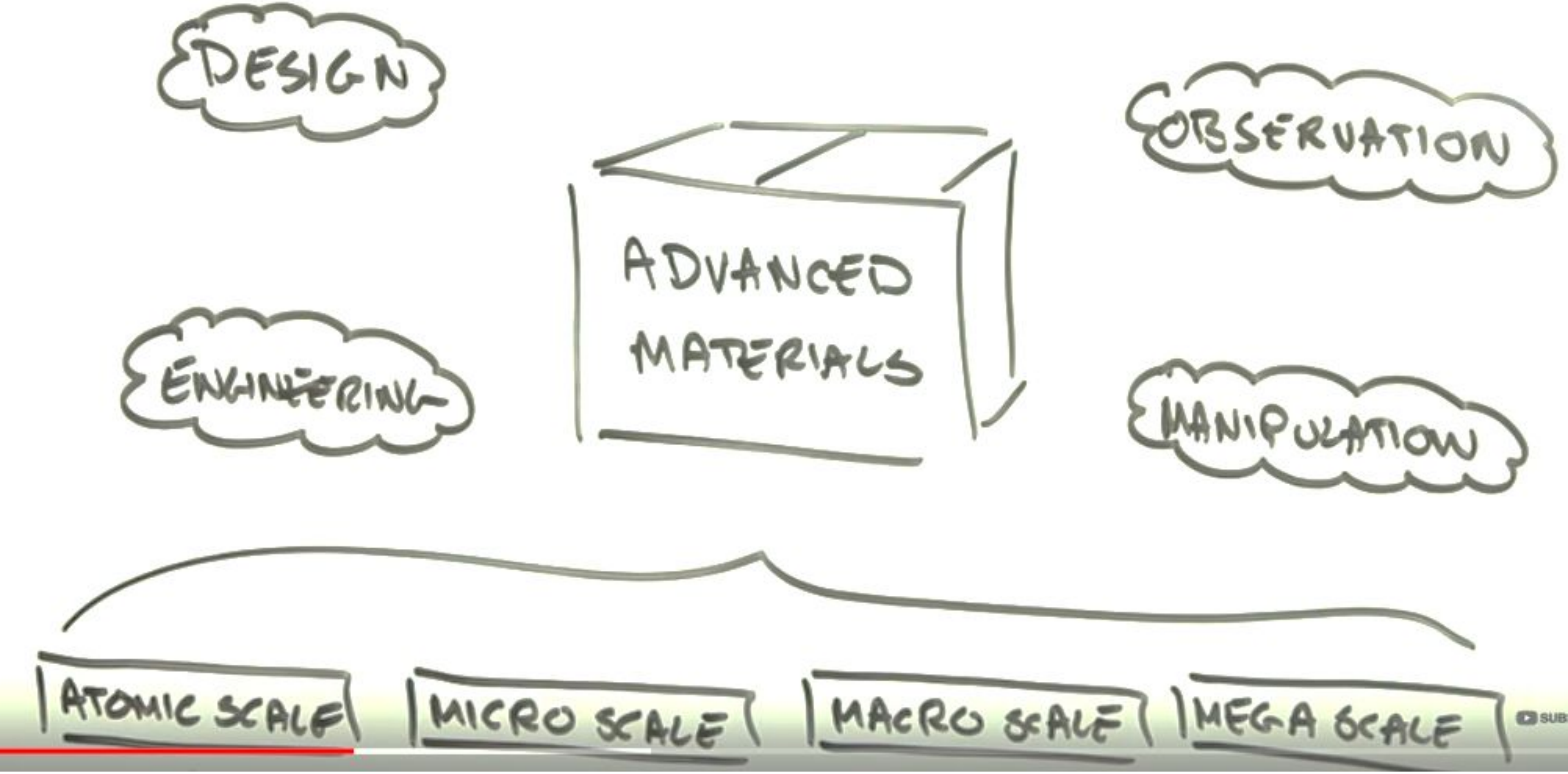
Emergence of Materials Physics

Materials for technology: how to rise the challenge?



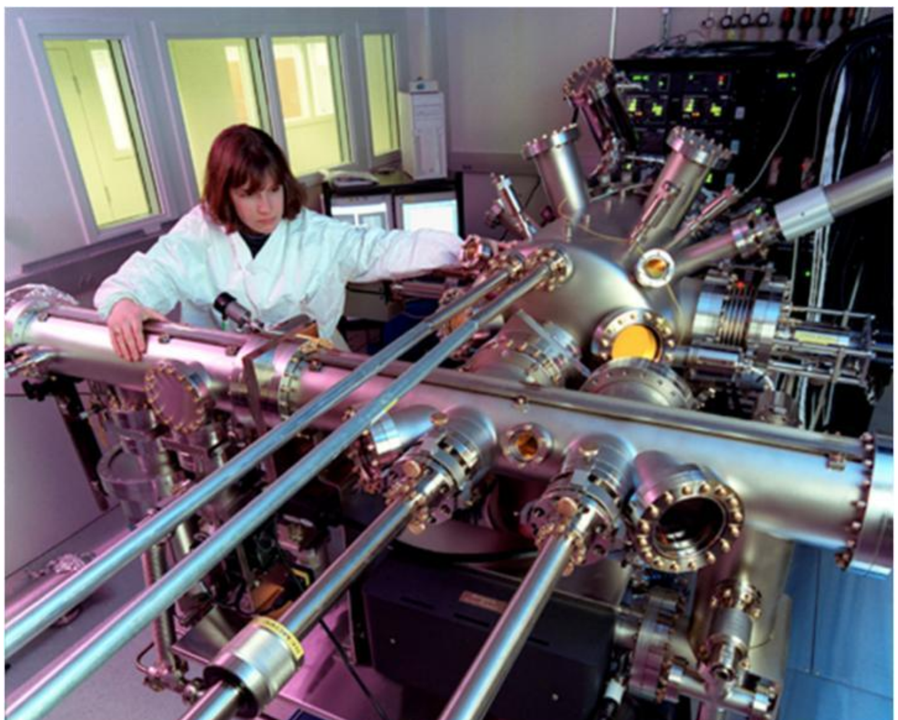
Design a material *à la carte* !

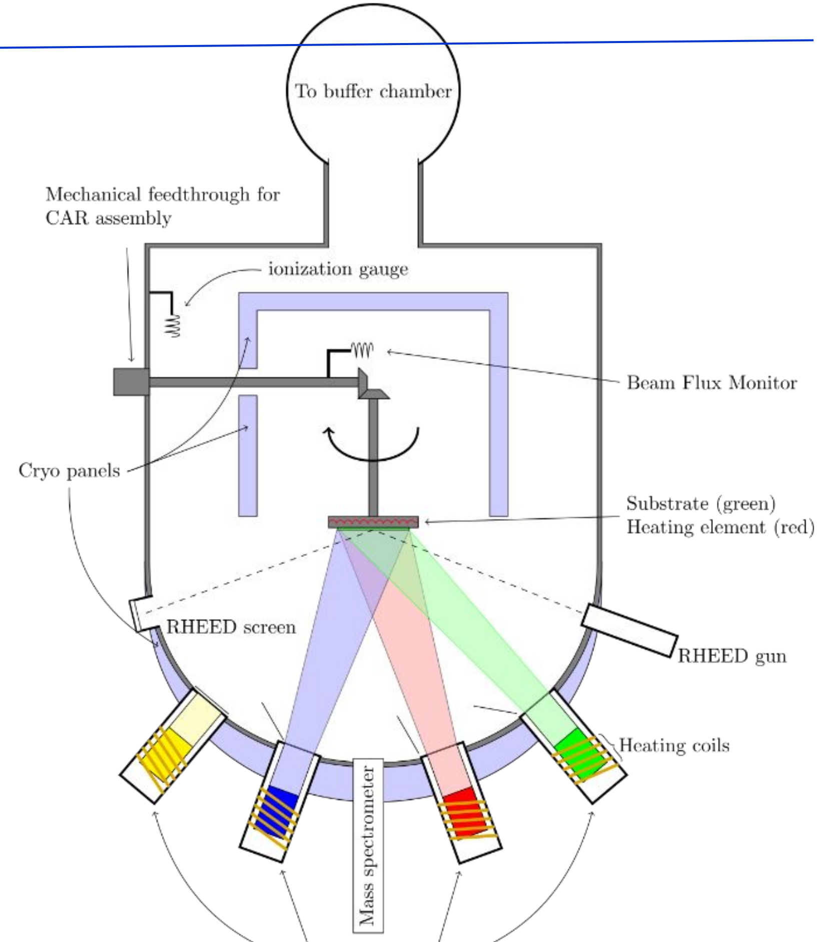
Analytical, Simulations,
Machine learning

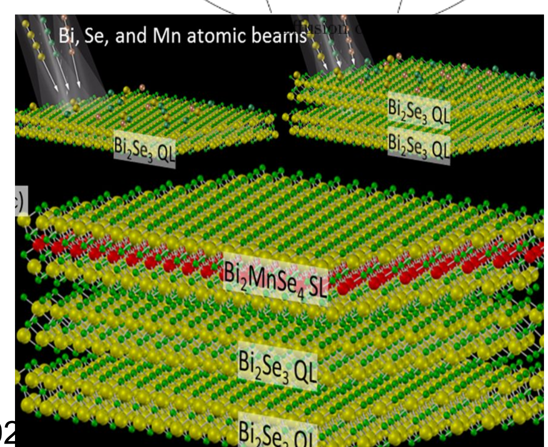


Advanced Materials Physics

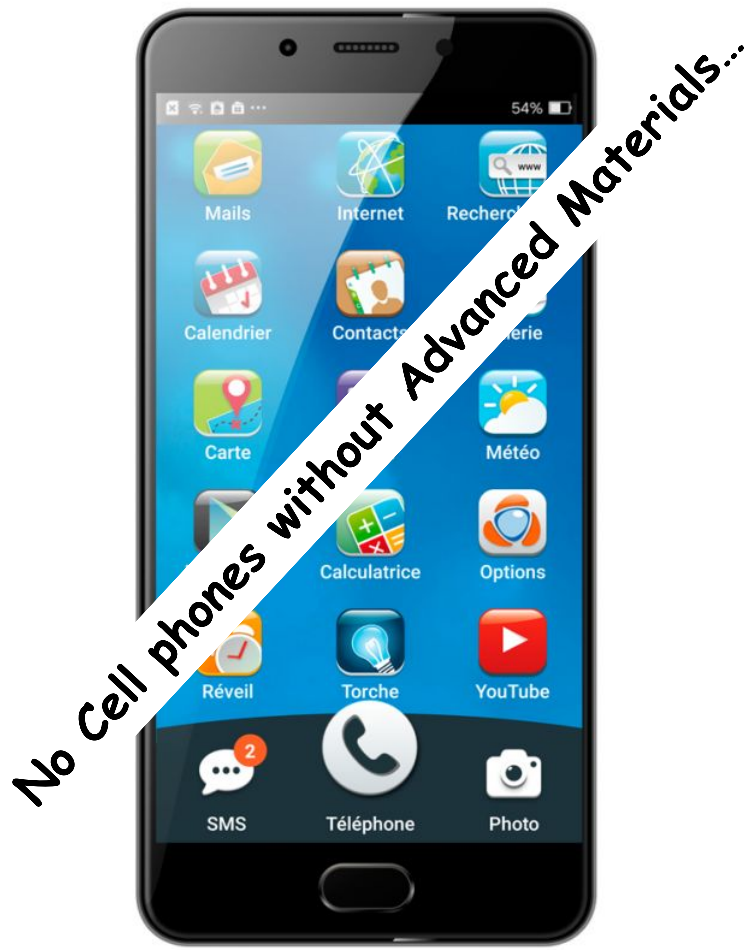
molecular beam epitaxy



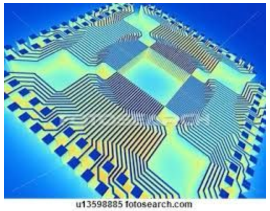




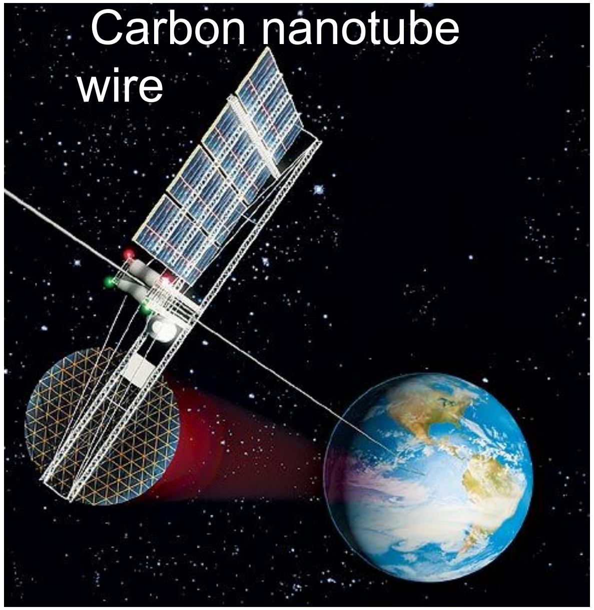
Advanced Materials Physics



Technological applications

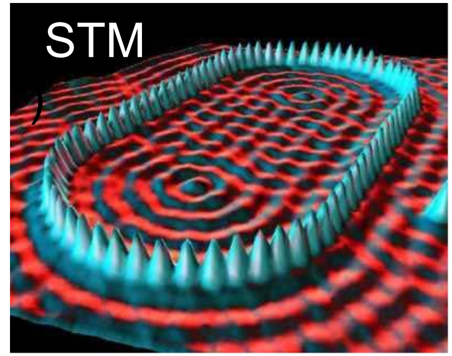


Carbon nanotube wire

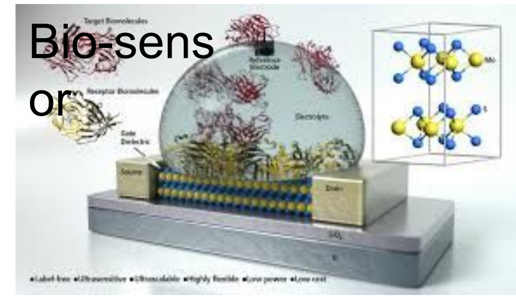


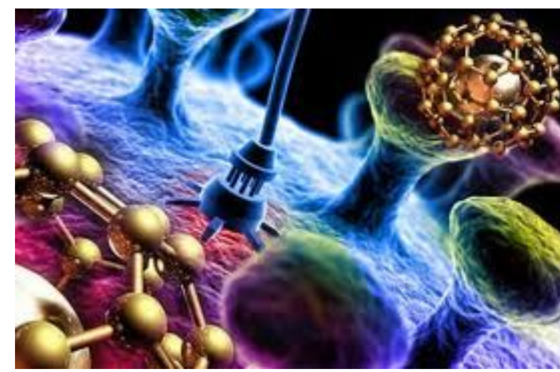
It is not Science-fiction !

STM

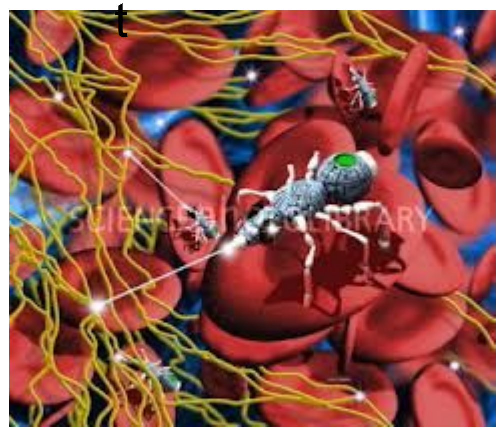


Bio-sensor





Nano-robot



Meta-materials

Quantum Computing

Material System	$\|0\rangle$	$\|1\rangle$
Ion traps		
Defects in solids		
Semiconductor quantum dot		
Superconducting		
Topological nanowire		

https://devopedia.org/quantum-computing

Solid state physics :

- New fundamental concepts (composite fermions, topology…)
- New phenomena: superconductivity, quantum Hall effect, spin Hall effect…
- New class of materials: multiferroics, 2D materials, Dirac electrons systems, topological insulators, Weyl semi-metals
- Playground for fundamental concepts: Higgs mechanism, Majorana fermions, Dirac fermions…

Higgs Bosons in Particle Physics and in Condensed Matter

G. E. Volovik & M. A. Zubkov

Journal of Low Temperature Physics **175**, 486–497 (2014) | Cite this article

Topological materials move from the world of theoretical physics to experimental chemistry

These exotic materials could find applications in electronics, catalysis, and quantum computing

by Neil Savage, special to C&EN

June 23, 2018 | A version of this story appeared in **Volume 96, Issue 26**

International Journal of Modern Physics B | Vol. 30, No. 19, 1630012 (2016)

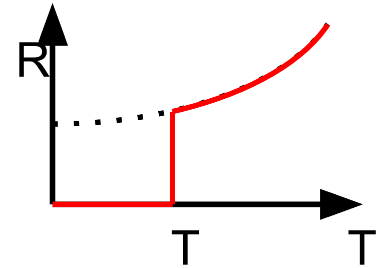
| Review Paper

Majorana fermions in condensed-matter physics

A. J. Leggett

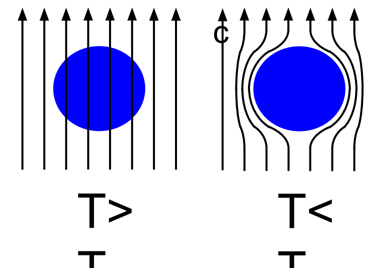
Superconductivity and its applications

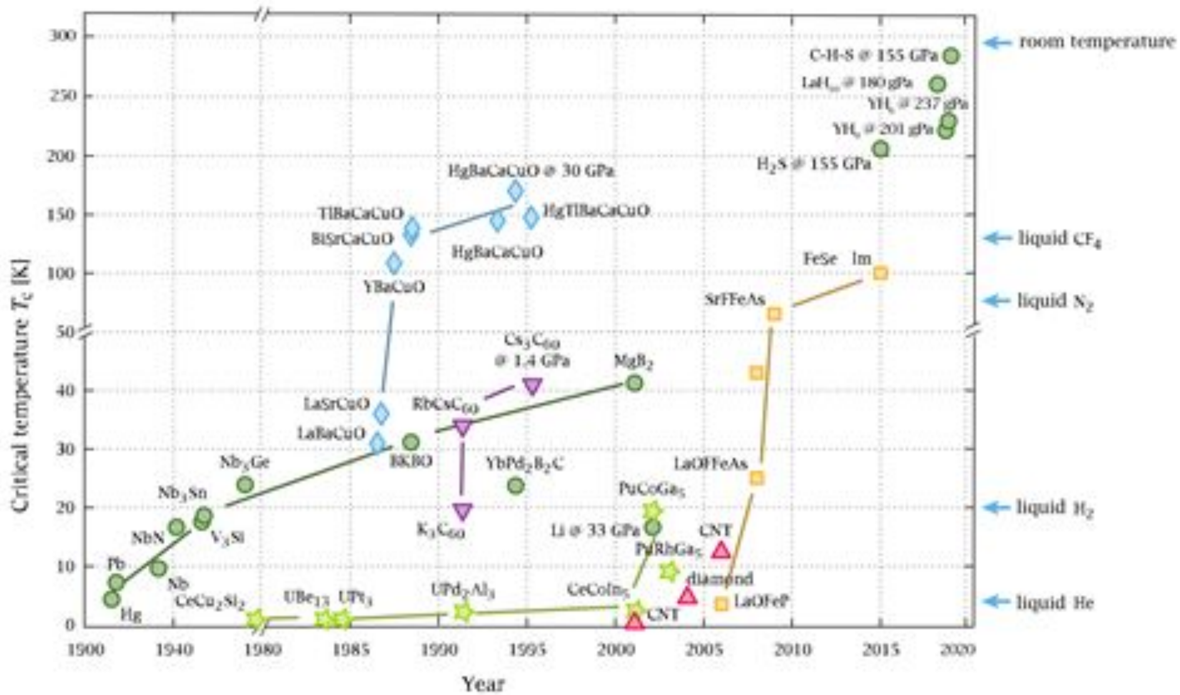
1911 :
H. Kamerlingh Onnes
Electrical resistivity drops to zero at a critical temperature



1933 : W. Meissner et R. Ochsenfeld

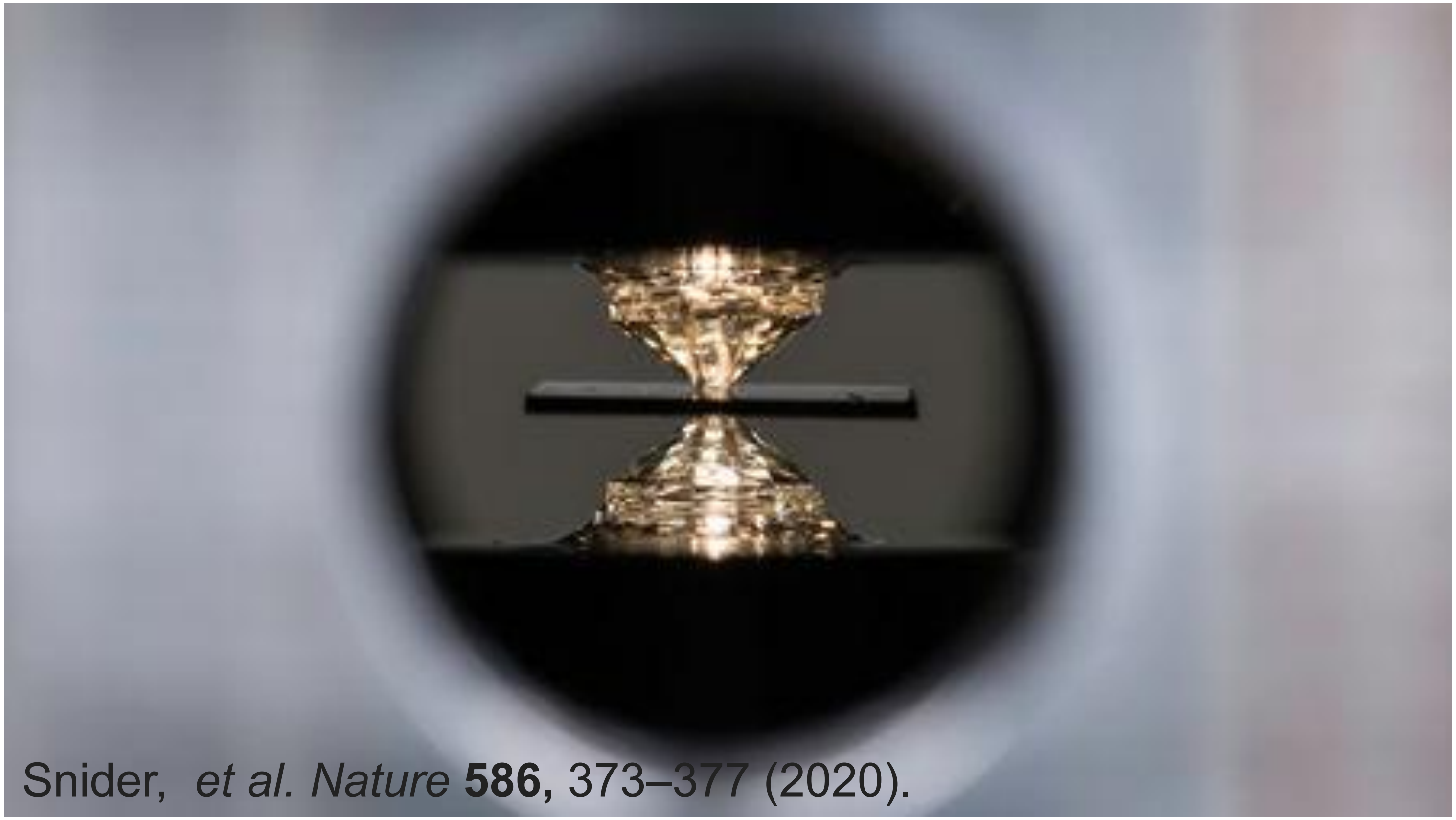
Total diamagnetism below T_c and H_c





After decades, room temperature superconductivity achieved

14 octobre 2020

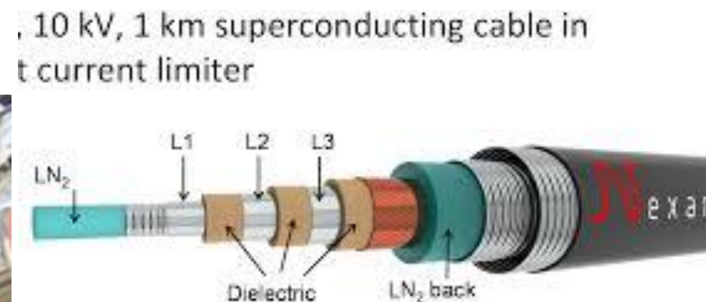


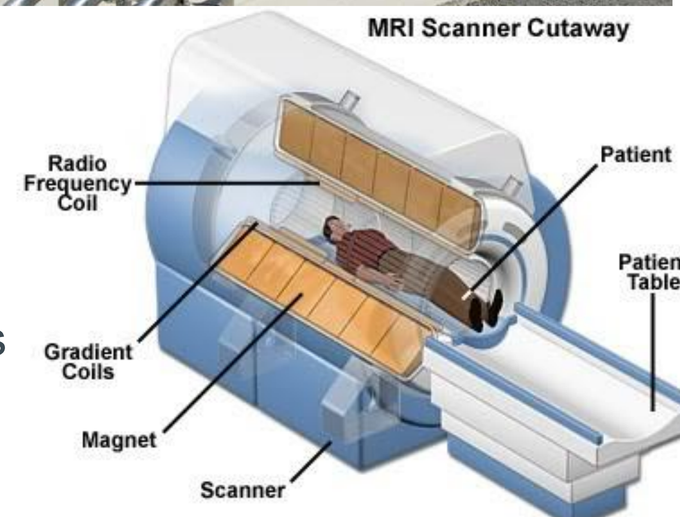
Crushed between two diamonds, a compound of hydrogen, sulfur, and carbon superconducts at room temperature.

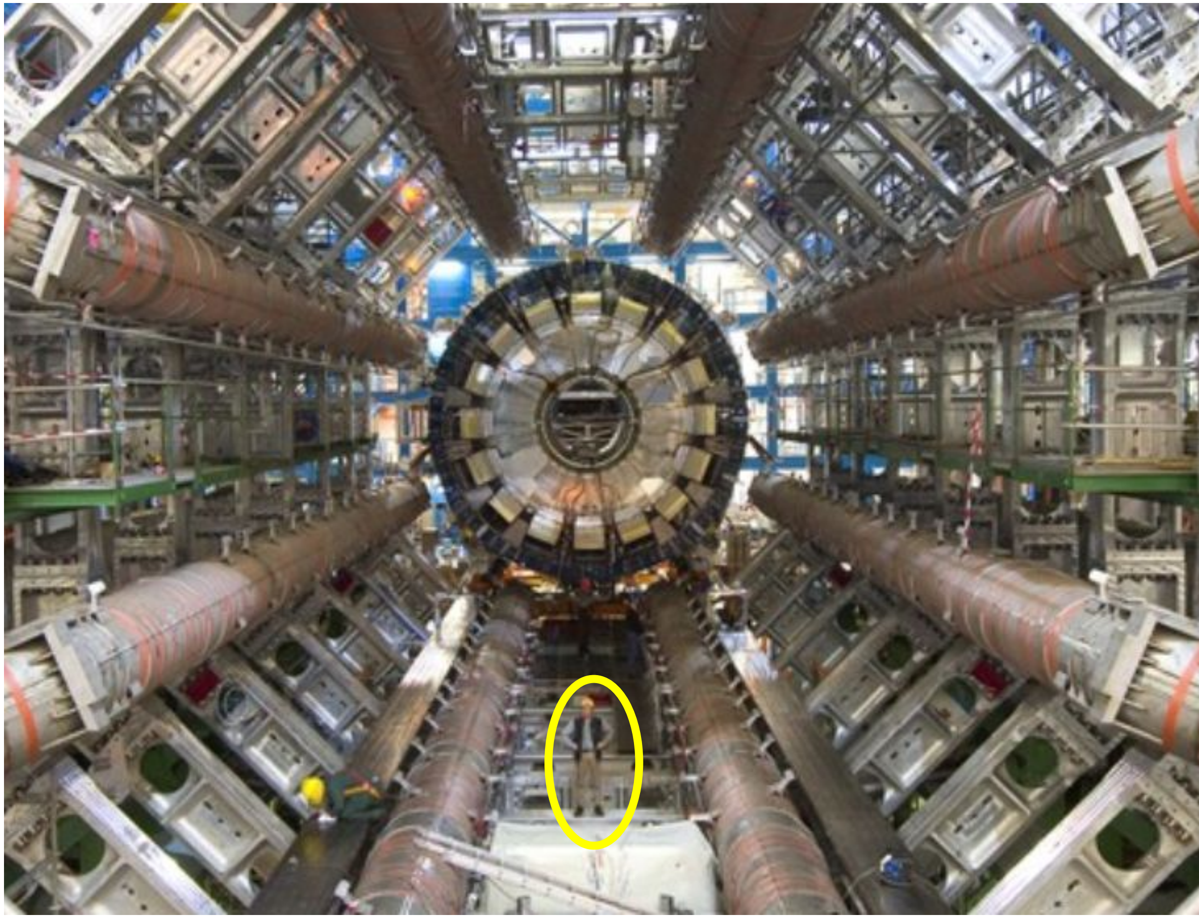
Mechanism of high temperature superconductivity is still under debate!

Advanced Materials Physics

Superconductors







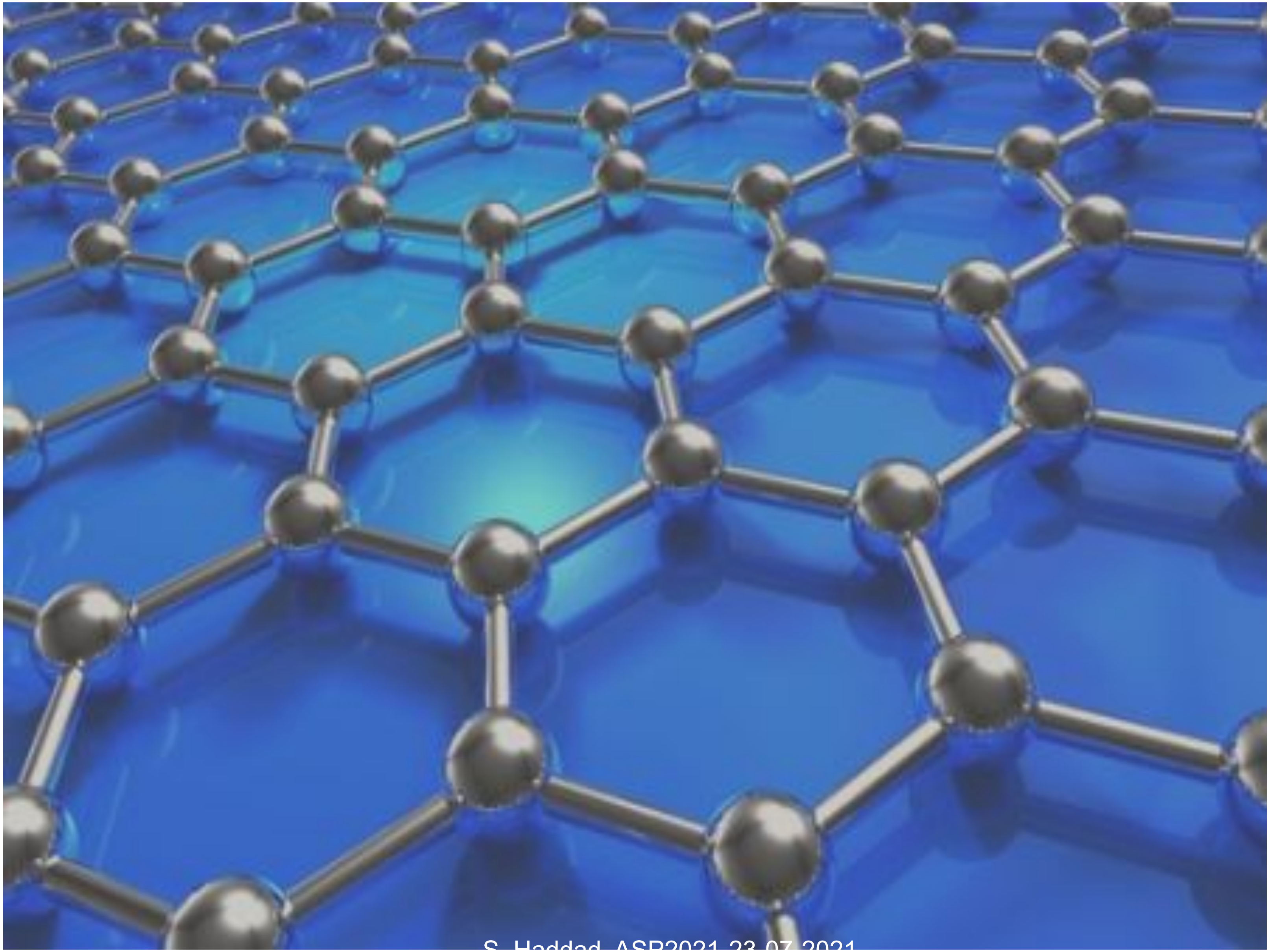
Central view of the ATLAS detector with its eight toroids around the calorimeter (Image: CERN)

Superconductors

Superconductors

Applications…

Graphene, the wonder material

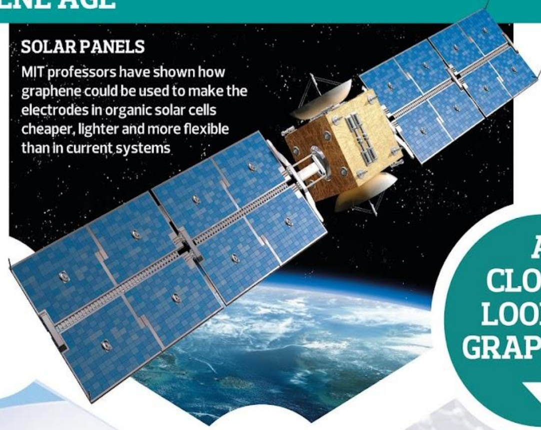


WELCOME TO THE GRAPHENE AGE

George Osborne on a visit to the Manchester University lab of Professors Geim and Novoselov

SOLAR PANELS

MIT professors have shown how graphene could be used to make the electrodes in organic solar cells cheaper, lighter and more flexible than in current systems



AIR TRAVEL

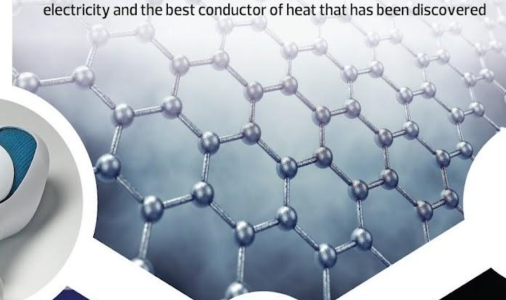
Using graphene would enable aeroplane manufacturers to develop extremely strong yet light components – bringing down weight and therefore reducing fuel costs

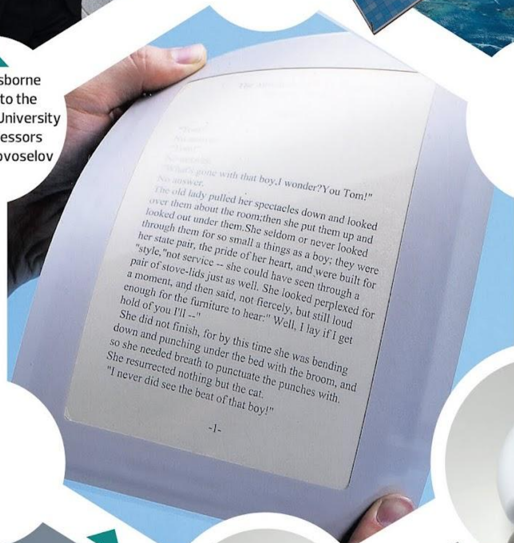
MOBILE PHONES

Nokia is exploring the potential uses of graphene in mobile devices. Aside from smaller, more flexible phones, it may allow built-in solar power and transparent electronics

A CLOSER LOOK AT GRAPHENE

Graphene is a one-atom thick form, or allotrope, of carbon – other allotropes include diamond and graphite. It is often described as an atomic-scale chicken wire constructed of carbon atoms and their bonds. When graphene sheets are stacked, three million sheets would be needed to create a 1mm thickness. It's been claimed that it is the strongest material known to man, that a clingfilm-thick layer could support an elephant. Despite its strength it can be stretched by 20% without being damaged. It is also an excellent conductor of electricity and the best conductor of heat that has been discovered





FLEXIBLE SCREENS

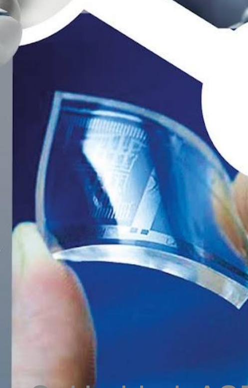
Researchers in South Korea have produced a continuous layer of graphene 63cm wide. This has opened up possibilities in electronics, "You could theoretically roll up your iPhone and stick it behind your ear like a pencil," claims one scientist

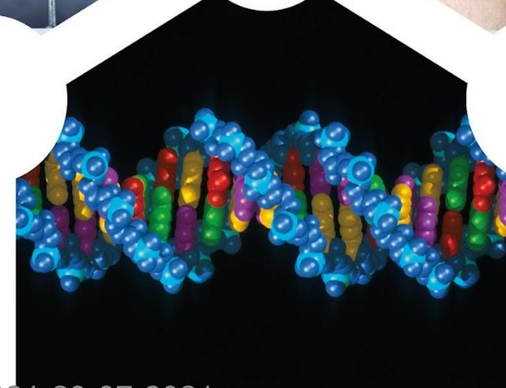
PROSTHETICS

Aside from allowing for the construction of stronger, more flexible and lighter limbs, its conductivity opens up new possibilities for its use in the electrodes used to turn brain signals into movement

COMPUTER CHIPS

Geim and Novoselov have been working on demonstrating how graphene could replace silicon as the key material in electronic circuits. IBM is one of many electronics firms experimenting with graphene conductors





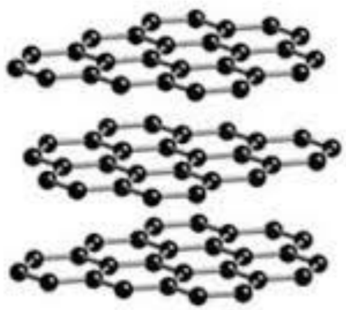
DNA SEQUENCING

Researchers at British firm Oxford Nanopore, building on discoveries made at Harvard, claim that using graphene could reduce the cost and speed up the process of DNA sequencing

GRAPHIC: PETE GUEST

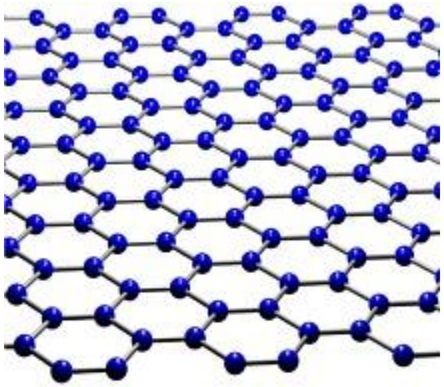
Graphene, the wonder material

Graphite



Geim et Novoselov
Nobel prize of Physics
2010

Exfoliation (scotch)



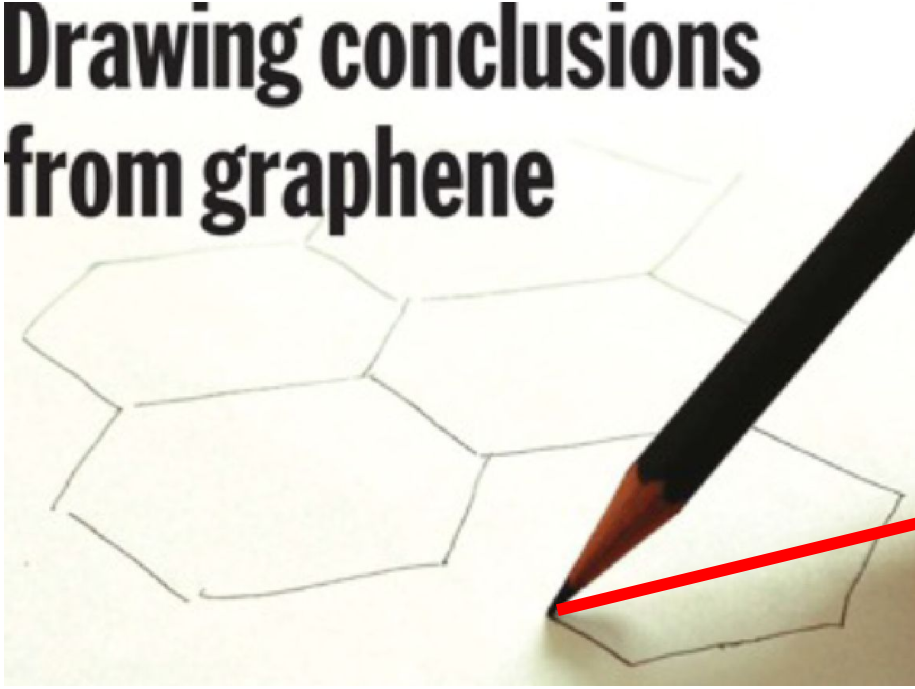
Graphene discovered in 2004

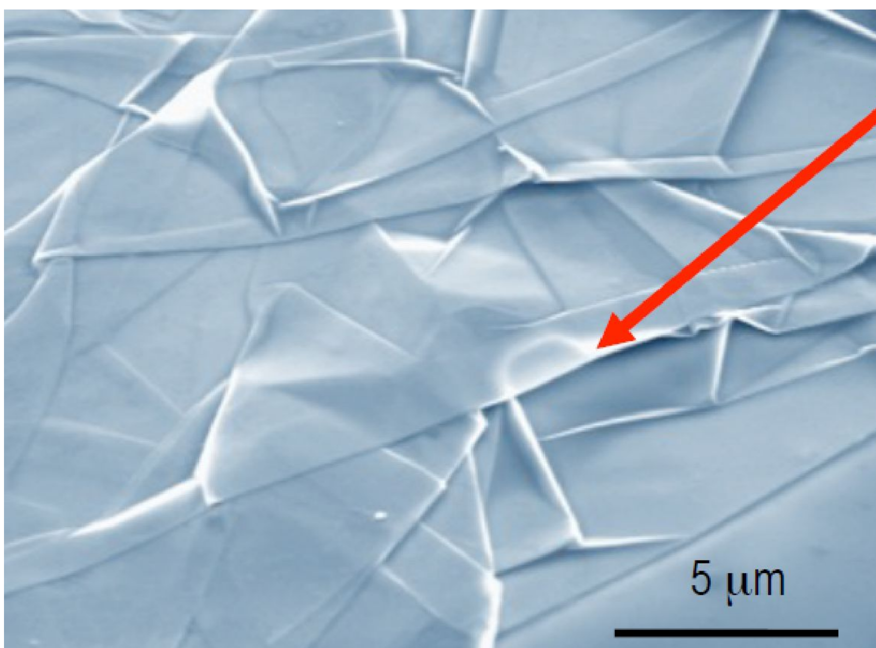
Graphene, the wonder material

Friday scribble yields Nobel Prize

By David Bradley | 1 January 2011

The work by Geim and Novoselov on graphene is equally bizarre and unexpected. Geim explains that the pair discovered how to make graphene in a 'fun Friday afternoon experiment' where they used adhesive tape to peel off the top layer of carbon atoms from a pencil mark on a glass surface and, with repeated folding and peeling of the sticky tape, were able to whittle the deposit down to a layer one single atom thick. Of course, the fine details of this 'cleavage' technique and its implications for science were published in research papers with rather esoteric, yet intriguing, titles such as *Electric Field Effect in Atomically Thin Carbon Films*[1] in the journal *Science* in October 2004 and *Two-dimensional atomic crystals* in *Proceedings of the National Academy of Sciences*[2] in 2005.

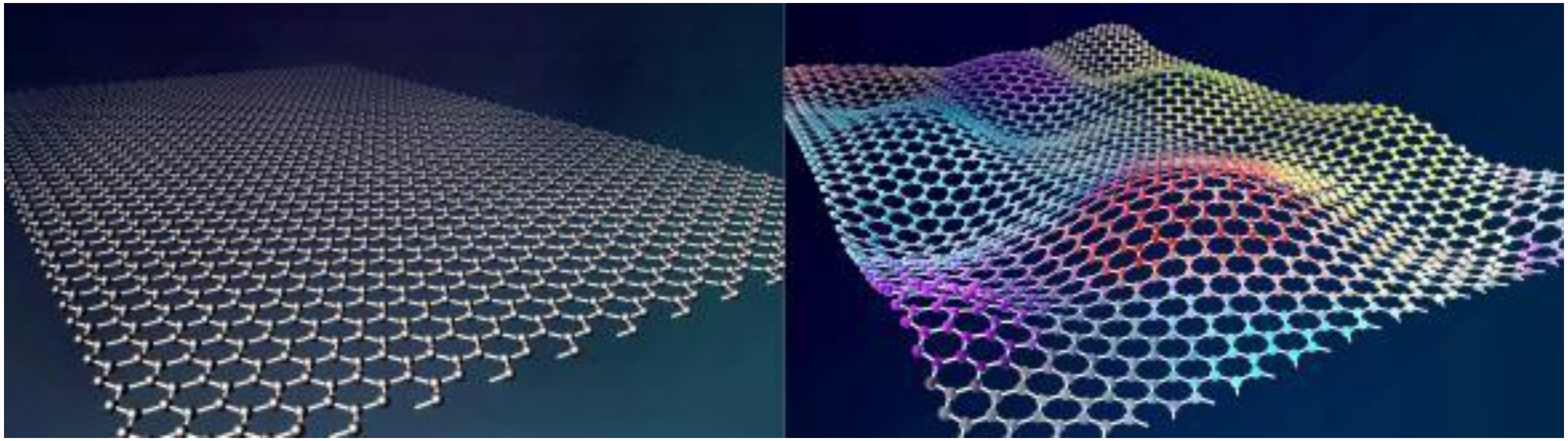




Pencil used in England since 1560 (Angleterre)!

Graphene: 2D or not 2D?

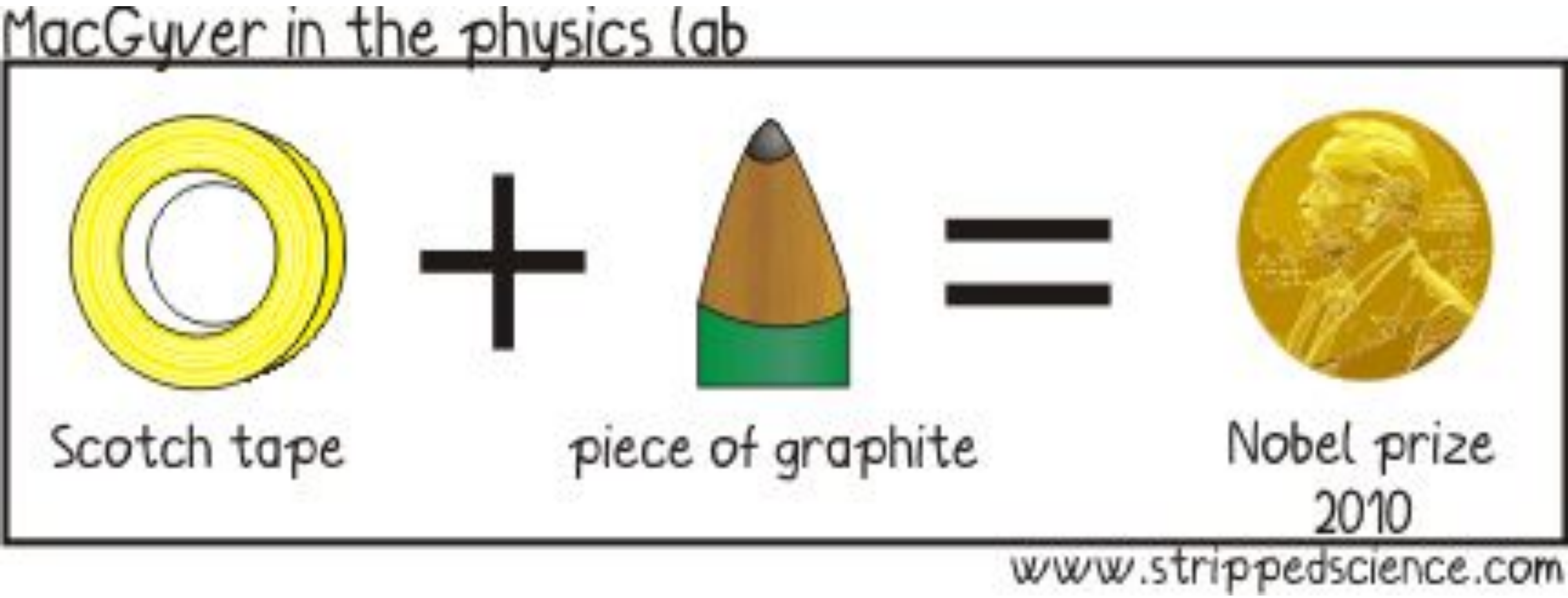
It was long believed that strictly two dimensional (2D) arrangements of atoms would be unstable (Mermin-Wigner theorem)



Ideal 2D graphene

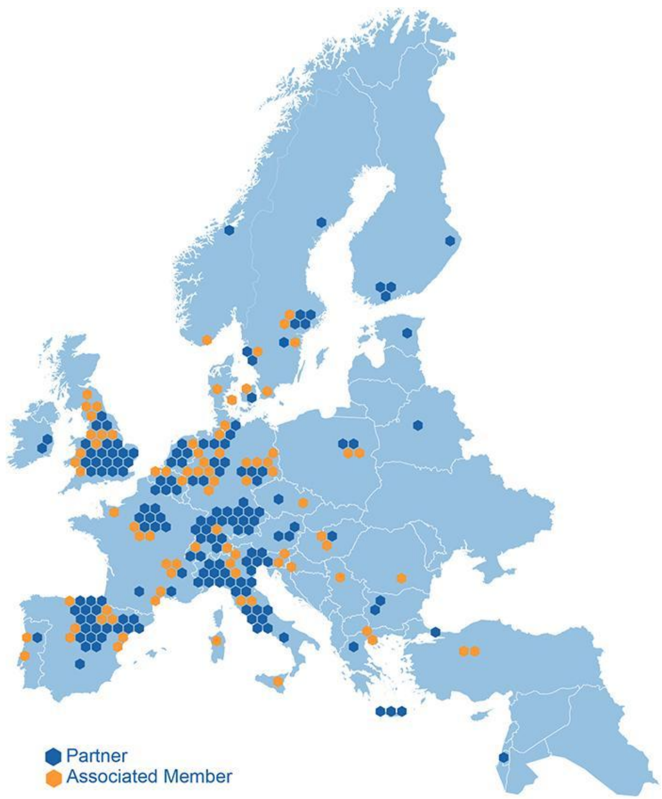
Real graphene: ripples heights are of atomic size
graphene is 2D material

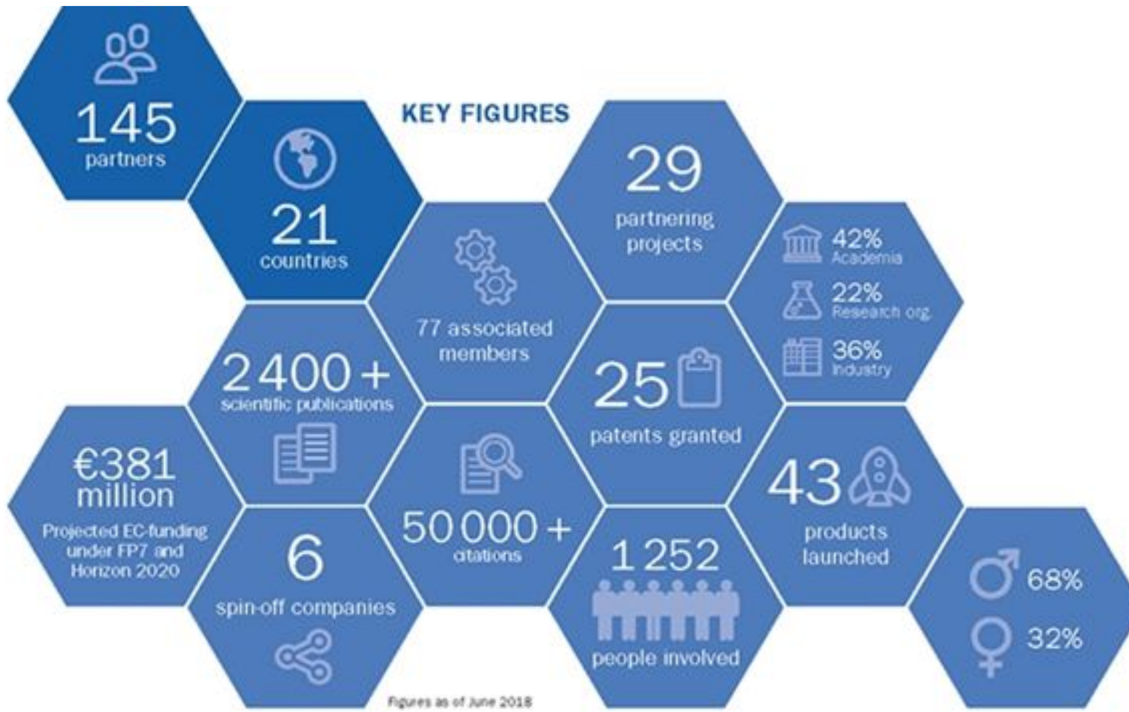
Wonder nanomaterial which may one-day enable faster and tinier computational and electronic devices.


MacGyver in the physics lab
Scotch tape
piece of graphite
Nobel prize
2010
www.strippedscience.com

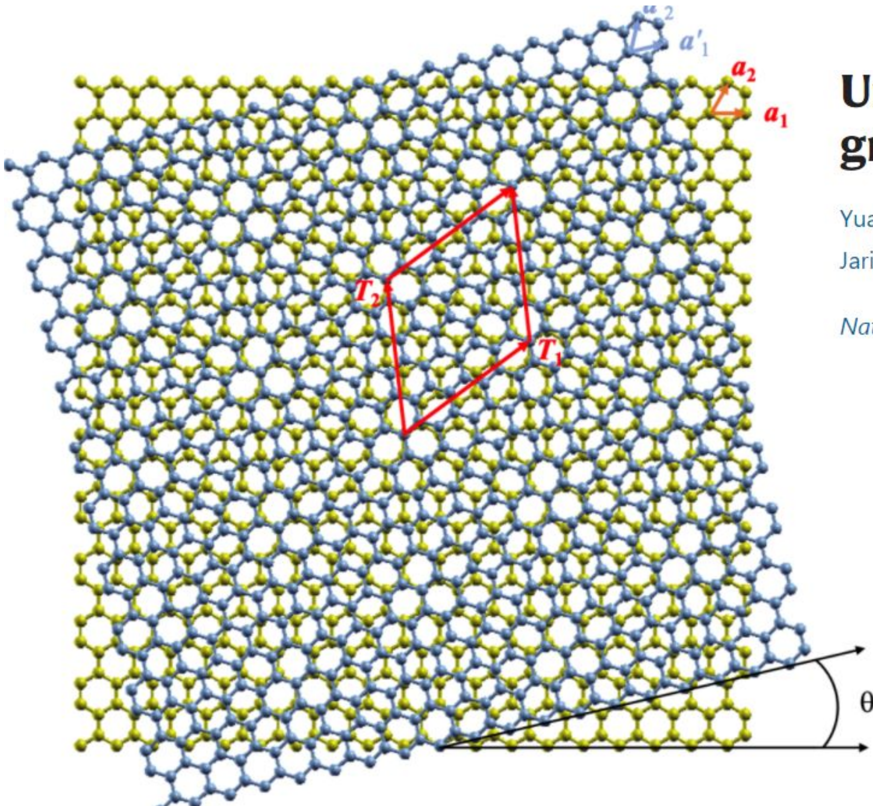

06:59:59
AM

GRAPHENE FLAGSHIP





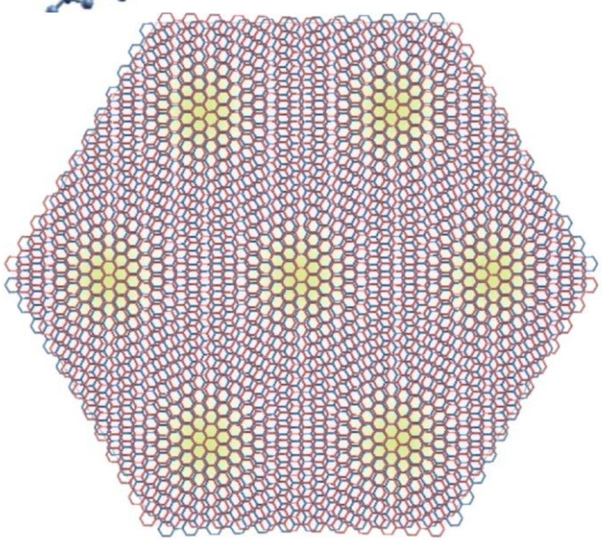
Lecture I: Introduction to solid state Physics

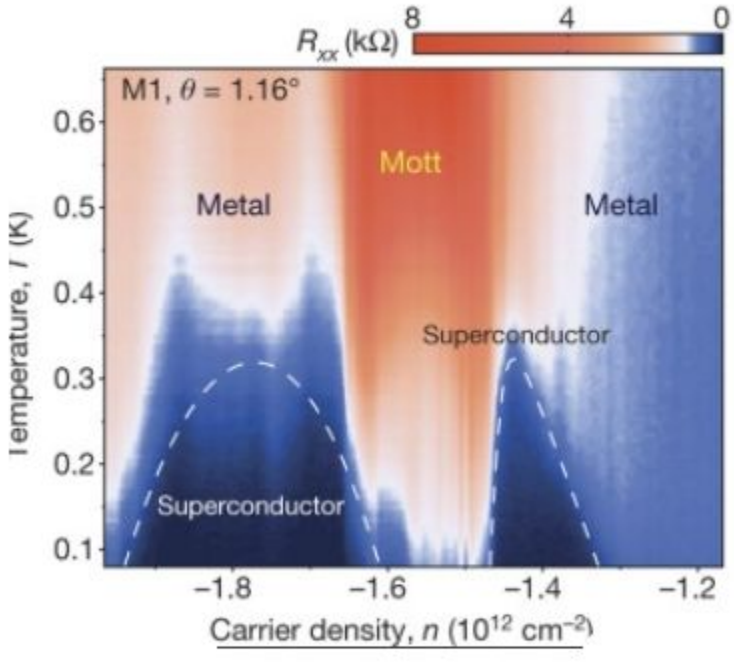


Unconventional superconductivity in magic-angle graphene superlattices

Yuan Cao, Valla Fatemi, Shiang Fang, Kenji Watanabe, Takashi Taniguchi, Efthimios Kaxiras & Pablo Jarillo-Herrero

Nature **556**, 43–50 (2018) | Cite this article





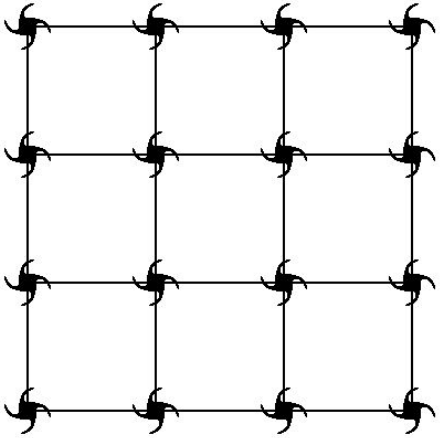
How to model electronic properties in crystals?

Real system

Physics Model

After, A. Posazhennikova

Overview of basic concepts



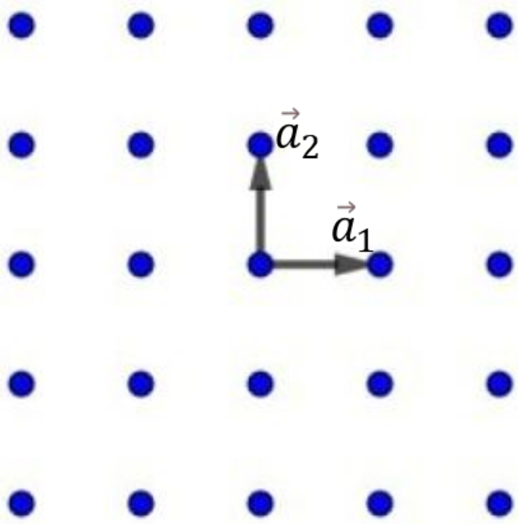
Crystal:

- regular arrangement of a unit structure (atoms, molecules…) in the space.
- Can be described by a lattice

Lattice:

- Set of points arranged periodically, in the space, in the same way as the unit structure
- Each lattice point correspond to a unit structure
- Lattice is purely geometric

Crystal= Lattice + unit structure

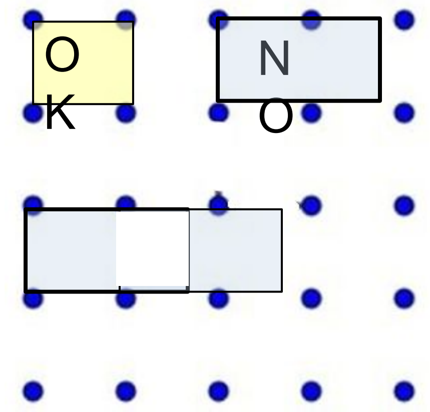


Lattice: described by an origin + basis

Each point of lattice : $\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3, n_i \in \mathbb{Z}$

Unit cell:

- Volume which, under translations, **fills up all the space without any overlapping**

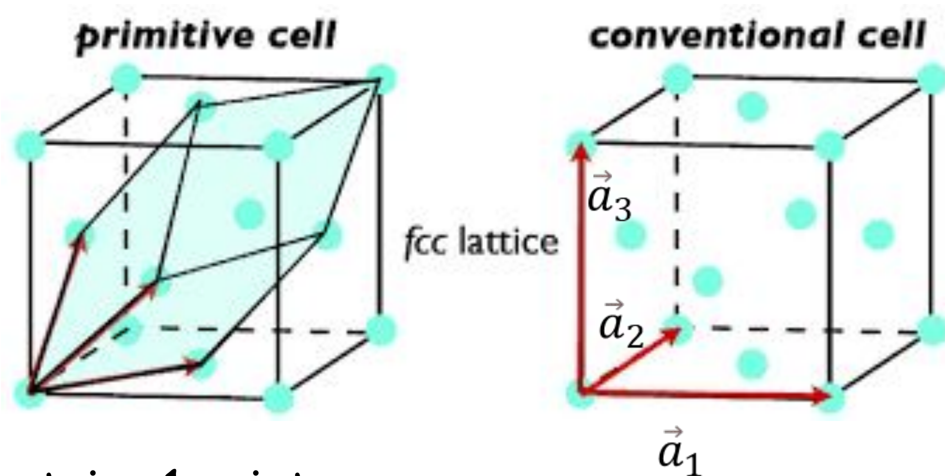


Primitive cell:

- unit cell with the **smallest volume**
- Contains **one lattice point**

Conventional unit cell:

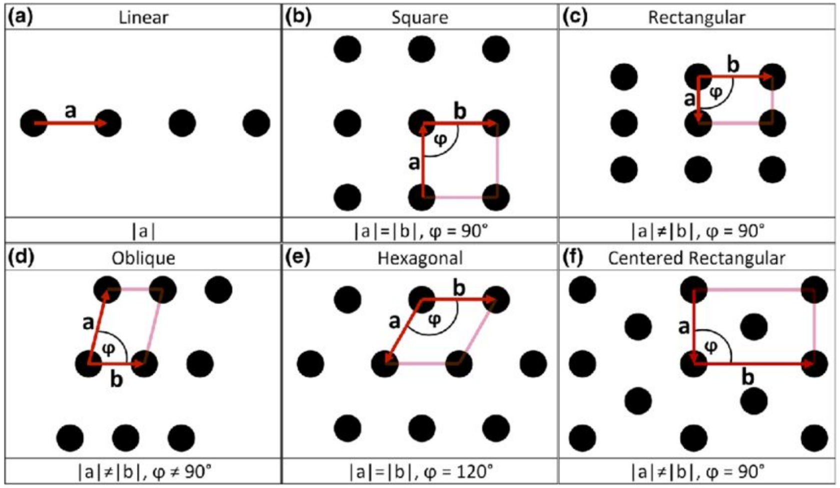
- may contain **more than a lattice point**
- brings out the **symmetry** of the crystal

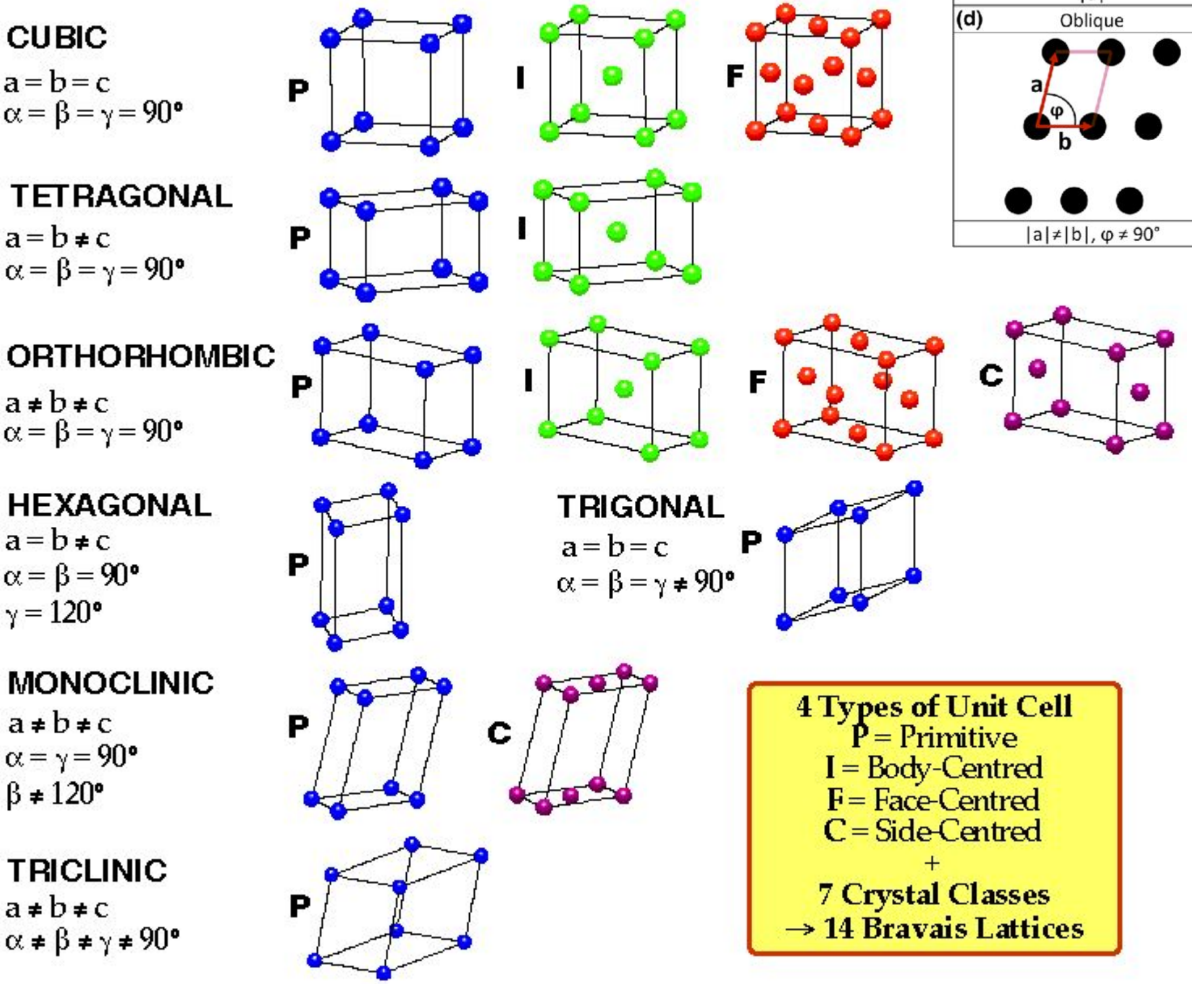


Contains 1 point

Contains 4 points

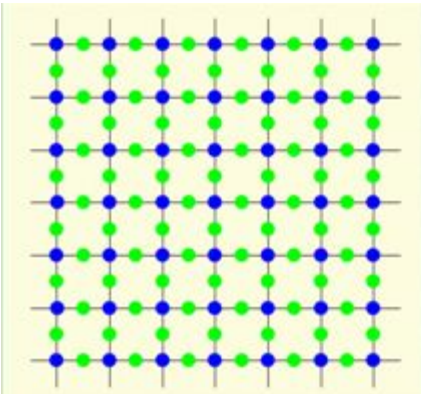
2D: 5 Bravais Lattices
3D: 7 crystalline classes
14 Bravais Lattices



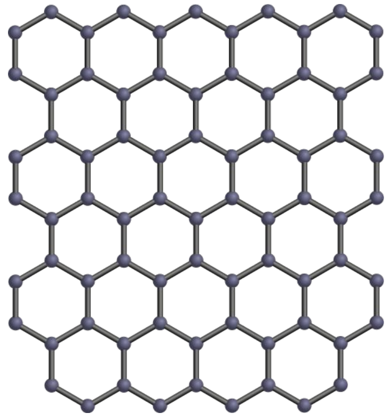


Assignment 1

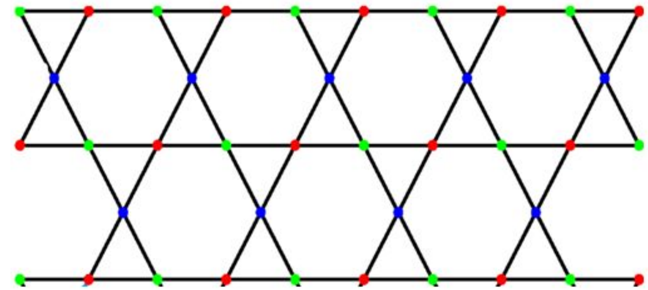
1/ Find the Bravais Lattice of the following structure



Copper oxide plane of cuprates (high temperature superconductors)
Blue dot : copper, green: oxygen

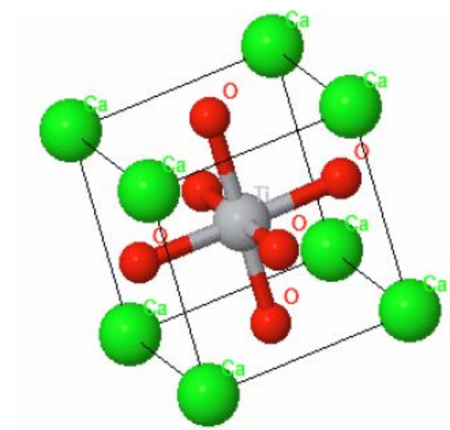


graphene



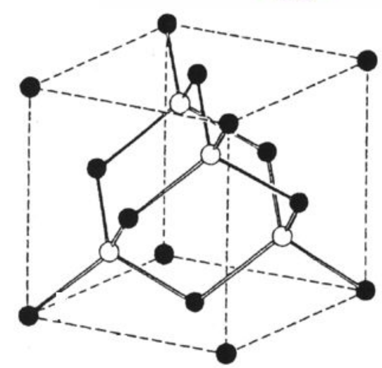
Kagome Lattice

2/ Find the unit structure and the chemical formula of this crystal



3/ The crystal of GaAs has a zinc blende structure. Ga atoms are in the sites (0,0,0) and As atoms As are at sites (1/4,1/4,1/4). The lattice parameter is 5.65 Å

Calculate the density of atoms of Ga et As per cm^3.



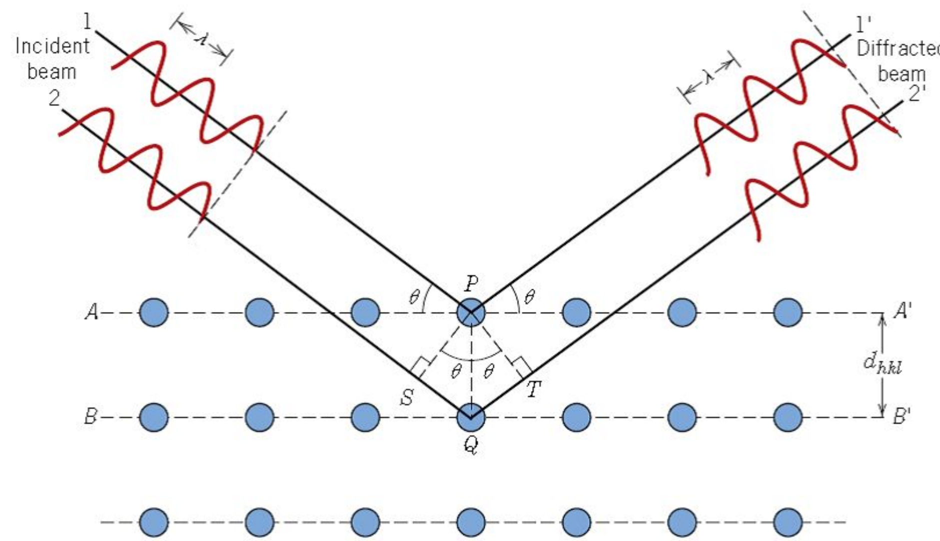
To observe the crystalline order: diffraction of photons, neutrons and electrons

X ray diffraction $\lambda = \frac{hc}{E}, \ \lambda(\text{Å}) = \frac{12.4}{E(eV)}$

Quiz 1: Why X rays ?

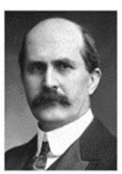
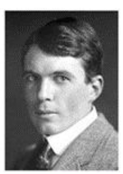
Neutron diffraction $\lambda = \frac{h}{\sqrt{2m_n E}}, \lambda(\text{Å}) = \frac{0.28}{\sqrt{E(eV)}}$

Electron diffraction $\lambda = \frac{h}{\sqrt{2m_e E}}, \ \lambda(\text{Å}) = \frac{12}{\sqrt{E(eV)}}$



Bragg Law

$$n\lambda = 2d\sin\theta \quad (1)$$


Bragg Law ⟺ von Laue Law

$$e^{i\vec{K}.\vec{R}} = 1 \quad (2)$$

Quiz 2: Show the equivalence between (1) and (2)

$$\vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3, n_i \in \mathbb{Z}$$

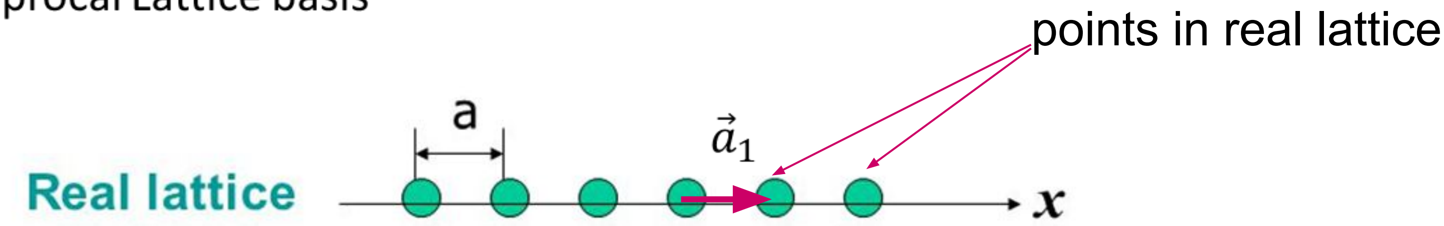
Reciprocal lattice

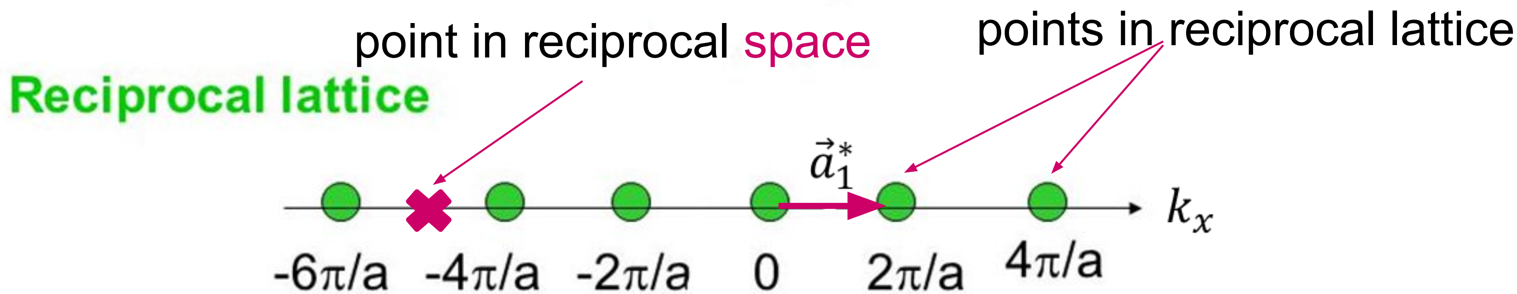
$$e^{i\vec{K}.\vec{R}} = 1 \qquad \vec{R} = n_1\vec{a}_1 + n_2\vec{a}_2 + n_3\vec{a}_3,\ n_i \in \mathbb{Z}$$

$$\vec{K} = m_1\,\vec{a}_1^* + m_2\,\vec{a}_2^* + m_3\,\vec{a}_3^*,\ m_i \in \mathbb{Z}$$ point in the reciprocal Lattice

$$\vec{a}_i^*.\vec{a}_j = 2\pi\,\delta_{ij}$$

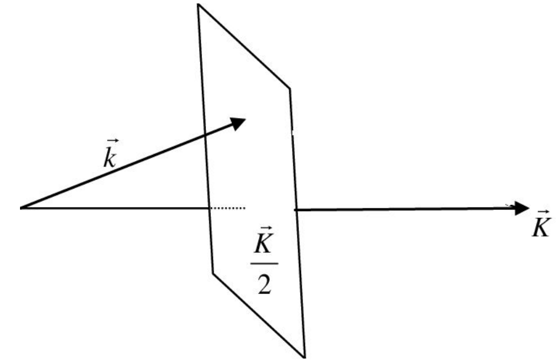
$(\vec{a}_1^*, \vec{a}_2^*, \vec{a}_2^*)$ Reciprocal Lattice basis





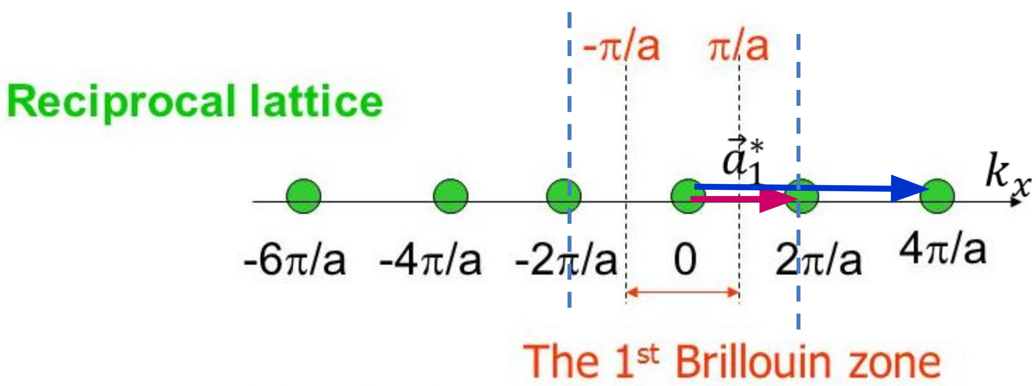
Quiz 3: Write $\vec{K}$ (reciprocal lattice) and $\vec{k}$ (reciprocal space)

Bragg plane: bisecting plane of $\vec{K}$: $|\vec{k}.\vec{K}| = \frac{K^2}{2}$



1^{st} Brillouin zone: set of reciprocal space points which can be reached from the origin without crossing any Bragg plane (0 plane)

n^{th} Brillouin zone: set of reciprocal space points which can be reached from the origin by crossing (n-1) Bragg planes

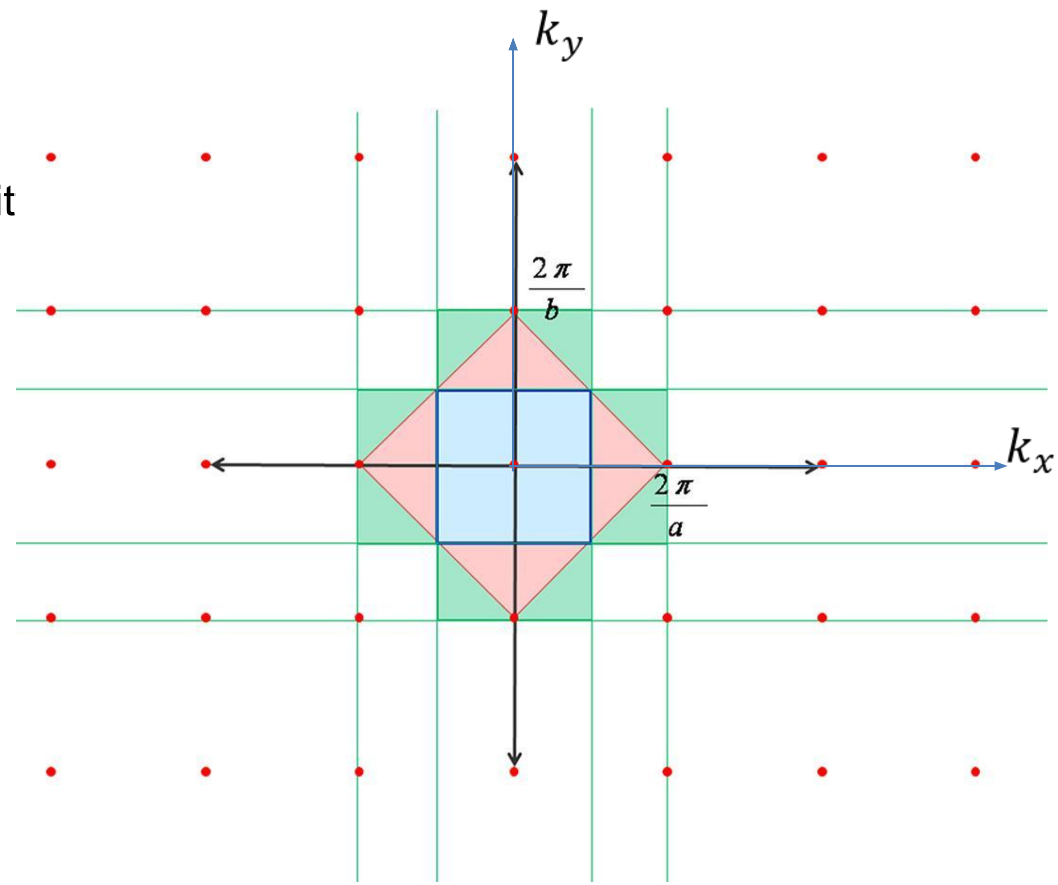


1D: 1st BZ is $k_x \in \left]-\frac{\pi}{a}, \frac{\pi}{a}\right]$

2sd BZ is $k_x \in \left]-\frac{2\pi}{a}, -\frac{\pi}{a}\right] \cup \left]\frac{\pi}{a}, \frac{2\pi}{a}\right]$

BZ of square lattice

- All BZ have the same volume
- Each vector from nth zone can be writ

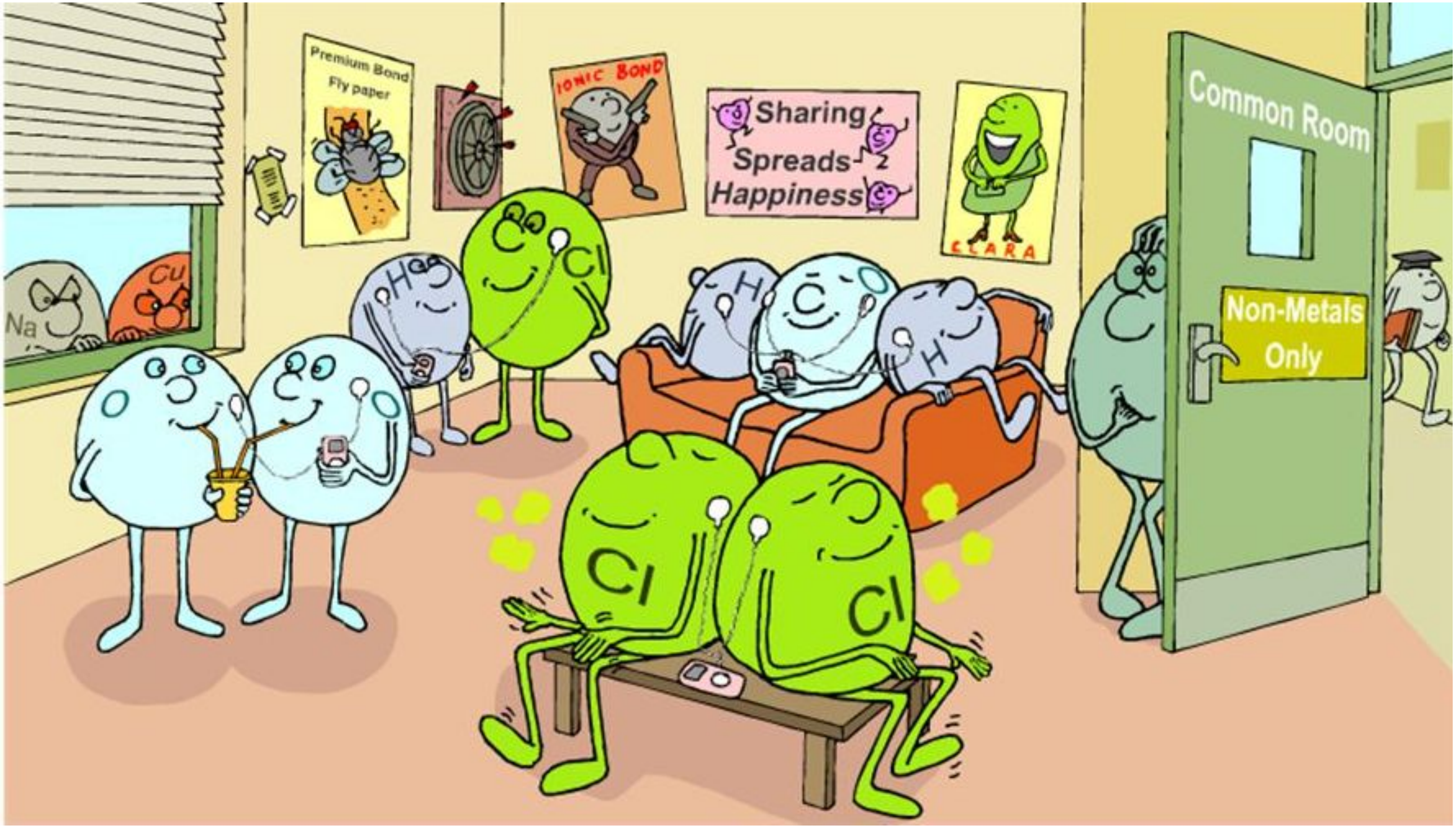


$\vec{k}_n = \vec{k} + \vec{K}, \vec{K} \in RL, \vec{k} \subset 1st\ BZ$

$\vec{k}$ and $\vec{k}_n$ are equivelent

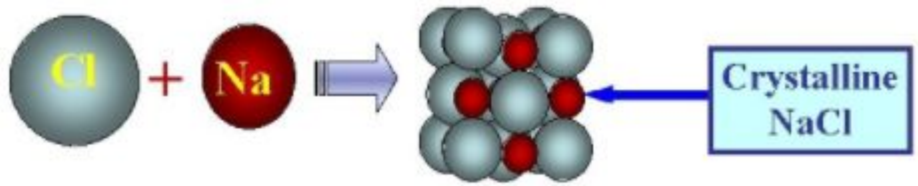
Assignment 2: Draw the 1st and 2sd BZ of triangular lattice

What makes atoms bond to form crystals?



Bonding energy

Ionic crystals

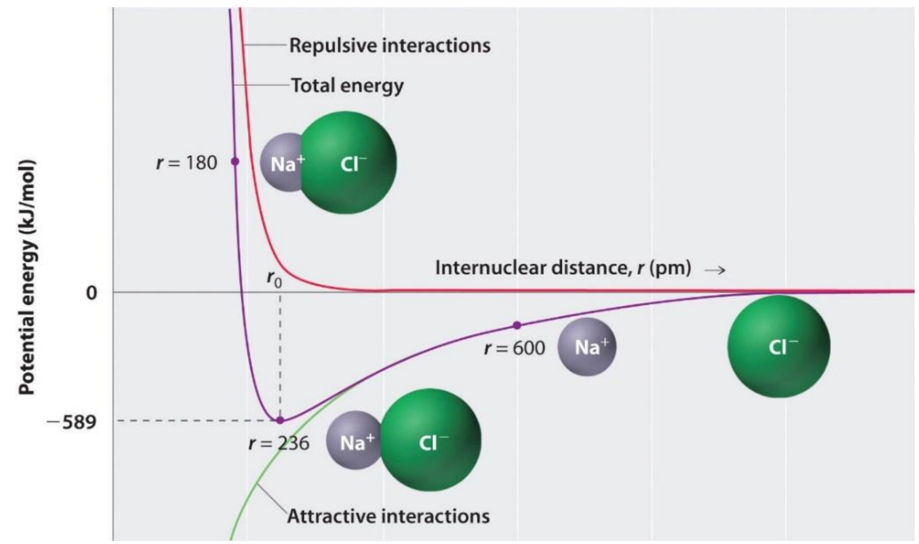


Electrostatic interaction

Total bonding energy of ionic crystal with 2N ions

repulsive attractive

$$U_{total} = NU_i = N\left(z\lambda e^{-\frac{R}{\rho}} - \frac{\alpha q^2}{R}\right)$$

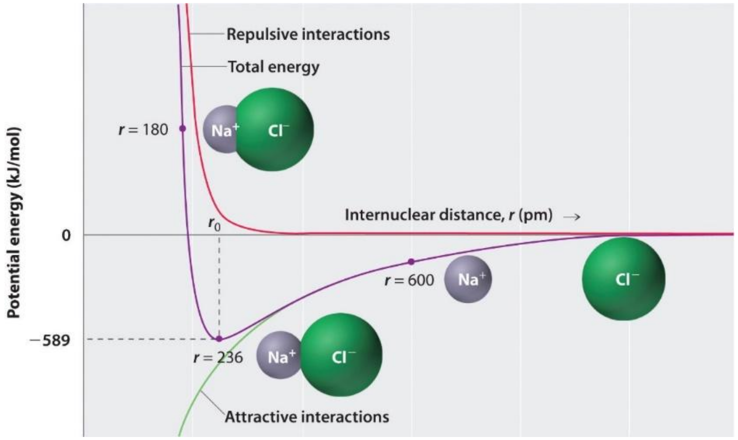


q: ion charge
R : distance between 1st nearest neighbor atoms
z : nb of 1st nearest neighbor ions
α: Madelung constant, depends on the crystal structure
λ and ρ: empirical parameters

Bonding energy

Ionic crystals

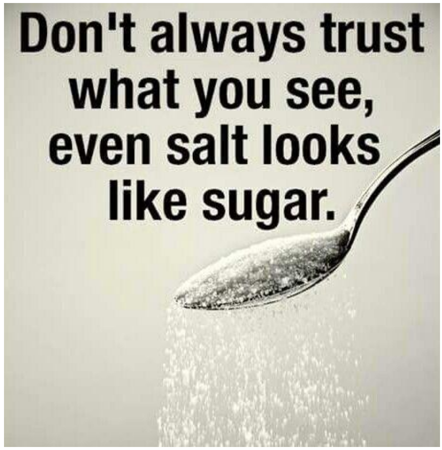
$$U_{total} = NU_i = N\left(z\lambda e^{-\frac{R}{\rho}} - \frac{\alpha q^2}{R}\right)$$



Cohesive energy = crystal energy- energy of free atoms

$$\frac{dU}{dR_{R_o}} = 0 \Rightarrow U_{tot}(R_o)$$

The atomic cohesive energy of NaCl per ion pair ≈ - 6 eV



Quiz 4: how could you distinguish between salt and sugar without tasting?

Bonding energy

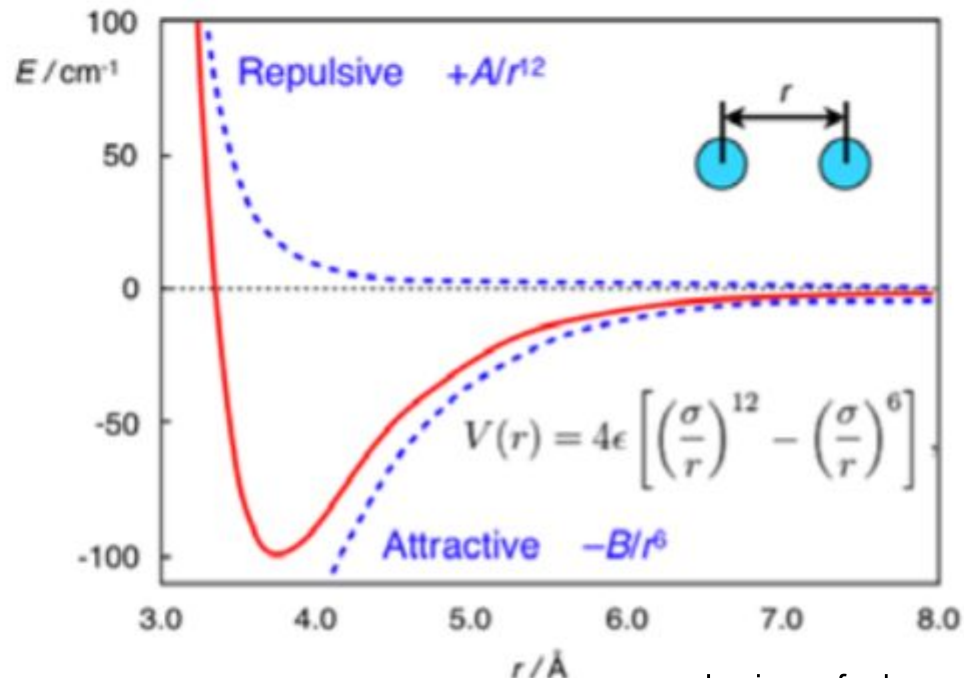
Molecular crystals

noble gas crystals (Ne, Ar, Kr, Xe)

Attractive interaction due to van der Waals forces (force between electric dipoles)

Total bonding energy per atom

$$U(R) = 4\epsilon\left[\left(\frac{\sigma}{R}\right)^{12} - \left(\frac{\sigma}{R}\right)^{6}\right]$$



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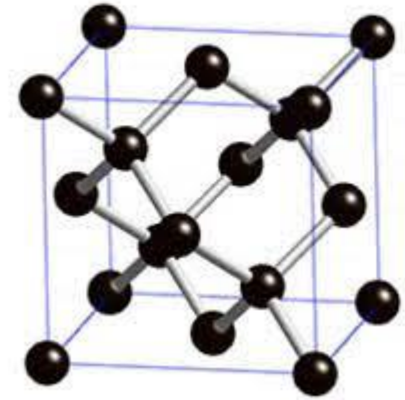
R : distance between 1st nearest neighbo
σ: depends on the crystal structure
ϵ: empirical parameter

Cohesive energy meV

Bonding energy

Covalent crystals



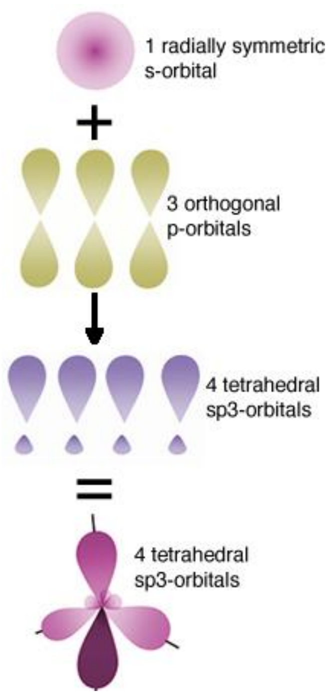
Zinc Blende crystal structure

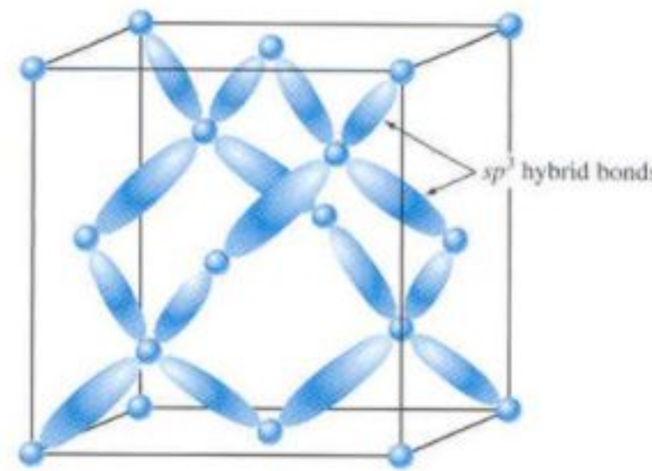
Attractive interaction due to covalent bonding

- Need Quantum Mechanics
- Hybridization and overlap of the electron wave function between atoms
- Exchange energy (spin dependent interaction)

Carbon C: [He] $2s^2 2p^2$

hybridation sp3

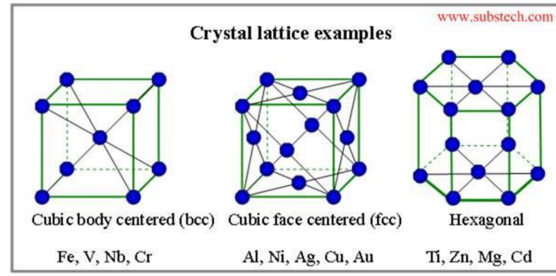


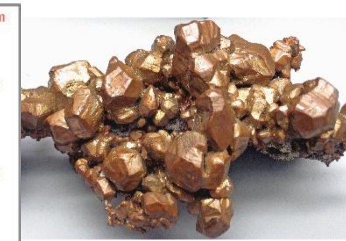


Cohesive energy/atom eV

Bonding energy

Metallic crystals

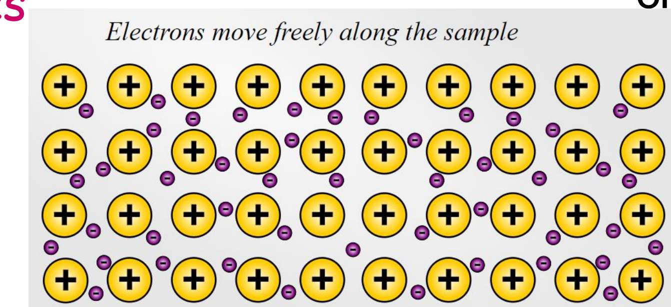




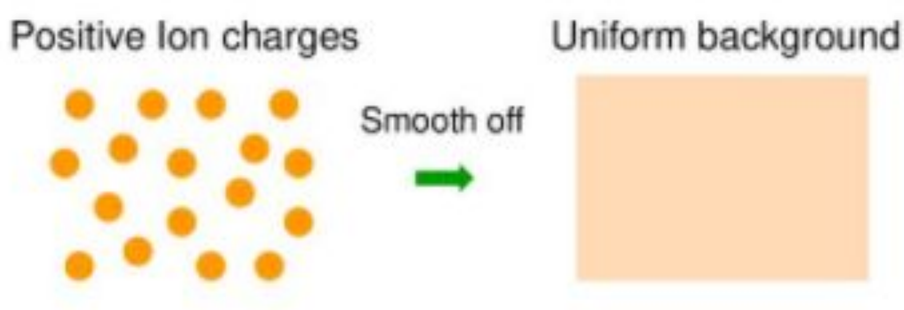
Copper crystal

Attractive interaction due to metallic bonding

- Need Many-body Quantum Mechanics



Jellium model: positive charges are assumed to be uniformly distributed in space and the electron density is a uniform quantity in space.

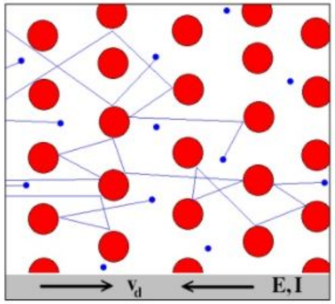


Cohesive energy/atom 0.5 eV

Drude Model (1900):

Paul Karl Ludwig Drude (1863–1906)

- 1st microscopic model to interpret the electronic conductivity of metals
- Conducting electrons described by classical gas (Maxwell-Boltzmann statistics)
- Electrons undergo collisions with positive ions
- Explain the increase of resistivity with temperature ☺
- BUT discrepancies for heat capacity ☹



Arnold Johannes Sommerfeld

Sommerfeld Model (1928):

- Conducting electrons described by quantum gas (Fermi-Dirac statistics)
- Explain the contribution of electrons to the heat capacity of metals ☺
- BUT did not explain why there are insulating, semi-conducting crystals? ☹

What did they miss?

There are symmetries in the crystal!

Translations

What are the consequences of the translational symmetry on the electronic properties?

Electron in crystals: classical or quantum?

Characteristic temperature in metals $T_F \approx 10^4\mathrm{K} \gg T_{ambiant}$ ⇒ Quantum Mechanics

Crystal Hamiltonian:

$$H_{crys} = T_{ions} + T_e + V_{e-ions} + V_{e-e} + V_{ions-ions}$$ Many-body problem! ☹

Approximations

- Born-Oppenheimer: ions fixe compared to electrons
$\langle T_{ions}\rangle \ll \langle T_e\rangle$, $\langle V_{ions-ions}\rangle \approx constant$

- Independent electrons

$H_{crys} = \sum_i H(\vec{r}_i)$, $H(\vec{r}) = \frac{p_i^2}{2m} + V(\vec{r})$, one-body problem ☺

$V(\vec{r}) = V(\vec{r}+\vec{R})$ periodic potential

$H(\vec{r}) = \frac{p_i^2}{2m} + V(\vec{r})$ with $V(\vec{r}) = V(\vec{r}+\vec{R})$ ⟹ $H(\vec{r})$ commute with $T_{\vec{R}}$ (translational operator)

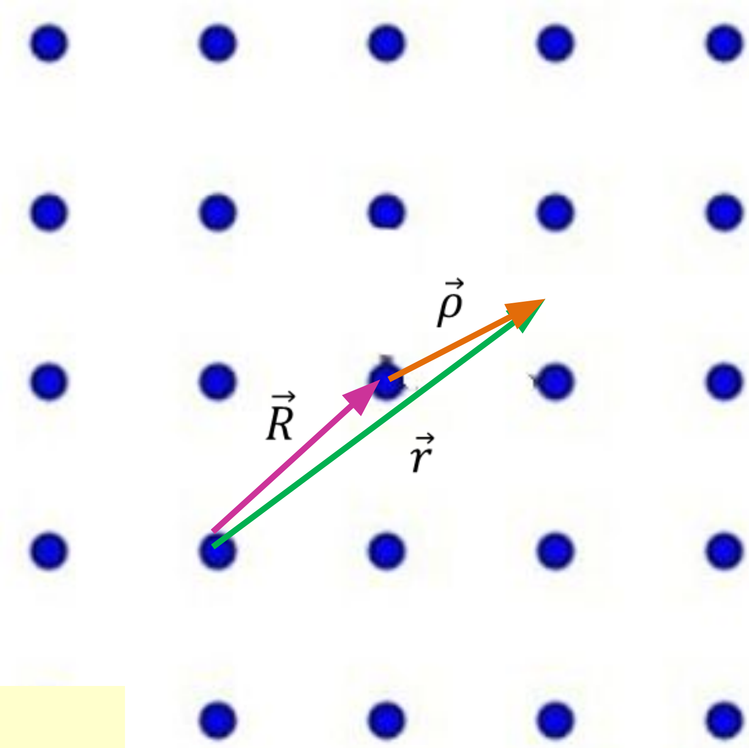
$T_{\vec{R}}$ is unitary ⟹ Eigenfuntions of the form $e^{i\vec{k}.\vec{R}}$, $\vec{k}$ wave vector

Quiz 5: Prove it.

$[T_{\vec{R}}, H(\vec{r})]=0$ ⟹ Eigenfuntions of $H(\vec{r})$ the form $\psi_{\vec{k}}(\vec{\rho},\vec{R}) = f(\vec{\rho})e^{i\vec{k}.\vec{R}}$

$\vec{\rho}$: position of the electron in the unit cell
$f(\vec{\rho})$ defined in te unit cell
$\vec{r}$: position of the electron in the crystal

$$\vec{r} = \vec{\rho} + \vec{R}$$



$$\psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})e^{i\vec{k}.\vec{r}}$$

$u_{\vec{k}}(\vec{r}+\vec{R}) = u_{\vec{k}}(\vec{r})$ cellular function

Felix Bloch

Bloch Theorem

Quiz 6: Prove the Bloch theorem.

Thank you for your attention

Any questions?