Introduction to Solid State Physics

Sonia Haddad

Laboratoire de Physique de la Matière Condensée
Faculté des Sciences de Tunis, Université Tunis El Manar
Outline

Lecture I: Introduction to Solid State Physics

• Brief story…
• Solid state physics in daily life
• Basics of Solid State Physics

Lecture II: Electronic band structure and electronic transport

• Electronic band structure: Tight binding approach
• Applications to graphene: Dirac electrons

Lecture III: Introduction to Topological materials

• Introduction to topology in Physics
• Integer Quantum Hall effect
• Haldane model
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• A Brief story…
• Solid state physics in daily life
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Lecture III: Introduction to Topological materials

• Introduction to topology in Physics
• Quantum Hall effect
• Haldane model
In 1961, George Gamow remarked that

“only number theory and topology had no application to physics”

Gamow was mistaken!

**Topology on top**

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10k Accesses | 4 Citations | 18 Altmetric | Metrics

Topology has journeyed from the purely mathematical arena to feature throughout physics.
Lord Kelvin: **Vortex theory of the atom** (period 1870-1890)

*Atoms are knotted vortices in the ether*  
(postulated medium for the propagation of light) *(Wikipedia)*

**Vortex**: region in a fluid in which the flow revolves around an axis line

*Carbon atom according to the Vortex theory*
The Konigsberg bridge puzzle (1736) in mathematics: find a path to walk through the city of Konigsberg (Germany) that crosses each of the seven bridges once and only once.

Leonhard Euler

Euler work:

- replace each land by "node" and each bridge by an "edge" graphs
- There is no possible path
- topology or architecture of the graph is important and not the details of the geography.
- The problem is the same for any city having the same topological invariants (number of lands and bridges)
- Introduced foundations of graph theory and the mathematical idea of topology (an important area of mathematics with the work of Poincaré around 1890).
Topology (Greek words τόπος, 'place, location', and λόγος, 'study') branch of mathematics deals with the properties of a geometric object that are preserved under continuous deformations, such as stretching, twisting, without closing holes, opening holes, tearing, gluing.

Topological invariant: genus number of holes
To a Topologist, this is a Doughnut
Dirac in 1931, introduced magnetic monopole is a hypothetical massive particle that is an isolated magnet with only one magnetic pole.

Magnetic monopole may solve one of the physics puzzle: the quantization of the electric charge.

Dirac “I am inclined now to believe that monopoles do not exist.” (1981)
In the 1960s and early 1970s

Solitons as topological defects cosmology, particle physics, supercondutors… objects with localised energy and stability

Example Skyrmion (1962) : topologically protected vortex like spin structures
Aharonov-Bohm effect

- described in 1949 by Ehrenberg and Siday
- Reformulated by Aharonov-Bohm in 1959

- Interference pattern of two electron beams modified by the presence of solenoid carrying a current ($B = 0$ outside the solenoid)

- Coupling electron wave function and the vector potential $A$

- The wave function acquires an extra phase known as geometrical or topological phase

$$\varphi = \frac{q}{\hbar} \int_P A \cdot dx$$

$B = 0 \quad A \neq 0$
In the 1970s, Berezinski, Thouless et Kosterlitz introduced the topological phase transition taking place without symmetry-breaking (contrary to Landau transitions).
Key example of topological phase: Integer Quantum Hall effect (IQHE)

- Discovered by K. von Klitzing in 1980
- 2D electron gas cooled down (few K) under strong magnetic field (few Tesla)
- Quantification of the Hall conductivity
- Quantification robust against disorder
- Theoretical interpretation of IQHE: 1982, Thouless, Kohmoto, Nightingal et Nijs (TKNN)
- Conductivity related to a topological invariant: Chern number
- Defined from the geometrical phase of the electronic wavefunction called Berry phase (Michael Berry)
Bloch theorem:

\[ \psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})e^{i\vec{k}.\vec{r}} \]

\[ u_{\vec{k}}(\vec{r} + \vec{R}) = u_{\vec{k}}(\vec{r}) \text{ cellular function} \]

If \( \vec{k} \) describe a closed path in the Brillouin zone during time

How will be modified the Bloch function?
### Berry phase

**Bloch function**

\[
|u_m(\vec{k}(t))\rangle = e^{i\gamma_m} e^{-i \int_0^T \epsilon_m(t') dt'} |u_m(\vec{k}_0)\rangle
\]

**Classical parallel transport:** by moving the vector along a curve on a sphere without changing it: angle between initial and final position

### Dynamical phase

In quantum mechanics:

- electron wavefunction acquires a **geometrical phase** (Berry phase) if it describes a closed path in the Hilbert space
- Bloch function will acquire a **Berry phase** if \( \vec{k} \) describe a closed path in the Brillouin zone
In general, for a wavefunction depending on a parameter $\vec{R}(t)$

Phase de Berry

$$\gamma_n = i \oint \langle \Psi_n \mid \nabla_\vec{R} \mid \Psi_n \rangle . d\vec{R}$$

Depends on the form of the path described by the wavefunction in the parameter space

**Berryology**

$$\vec{A}_n(\vec{k}) = i \left\langle u_{n,\vec{k}} \mid \nabla_{\vec{k}} \mid u_{n,\vec{k}} \right\rangle$$

Berry connection

$$\vec{\Omega}_n(\vec{k}) = \nabla_{\vec{k}} \wedge \vec{A}_n(\vec{k})$$

Berry curvature

Chern number: Berry flux in the BZ

$$C_n = \frac{1}{2\pi} \oint_{S_f} \vec{\Omega}(\vec{k}).d\vec{k},$$
Analogy between electromagnetism and Berry parameter

**Berry curvature**
\[ \Omega(\vec{\lambda}) \]

**Berry connection**
\[ \langle \psi | i \frac{\partial}{\partial \lambda} | \psi \rangle \]

**Geometric phase**
\[ \oint d\lambda \langle \psi | i \frac{\partial}{\partial \lambda} | \psi \rangle = \iint d^2 \lambda \ \Omega(\vec{\lambda}) \]

**Chern number**
\[ \iint d^2 \lambda \ \Omega(\vec{\lambda}) = \text{integer} \]

**Magnetic field**
\[ B(\vec{r}) \]

**Vector potential**
\[ A(\vec{r}) \]

**Aharonov-Bohm phase**
\[ \oint dr \ A(\vec{r}) = \iint d^2 r \ B(\vec{r}) \]

**Dirac monopole**
\[ \iint d^2 r \ B(\vec{r}) = \text{integer} \ \frac{h}{e} \]
Chern theorem: the integral of the Berry curvature over a closed surface if the space parameters of wavefunctions is quantized

\[ \int_{S_f} \mathbf{\Omega}(\mathbf{R}) d\mathbf{S} = 2\pi m \]

$m$: Chern number

A nonvanishing flux of the Berry curvature through a closed surface is equivalent of having a nonzero magnetic charge: topological object

S. S. Chern

A nonzero Chern number is a signature of the topological aspect of the wavefunction
Integer Quantum Hall Effect

- Important on the fundamental aspects (manifestation of topology)
- Difficult to realize (extreme conditions T and B)

\[ R_{Hall} = \frac{h}{e^2 n} \quad n \text{ integer (Chern number)} \]
In life there is something else besides magnetic field!

Haldane model:

Haldane, PRL 61,2015 (1988)

Seminal paper of Topological Material

Haldane proposed IQHE without magnetic field!

Anomalous Quantum Hall effect (AQH)
Model ingredients:

- Graphene-like crystal with two types of atoms A and B
- Complex second-neighboring hopping parameters
- Zero total magnetic flux in the unit cell
- Breaks time reversal symmetry without magnetic field

Haldane, PRL 61,2015 (1988)

https://topocondmat.org/w4_haldane/haldane_model.html
Kane and Mele model = 2 copies of Haldane model for spins $\uparrow$, and $\downarrow$

Quantum Spin Hall Effect

- Inversion symmetry breaking not really needed
- Spin orbit term connecting sites in the same sublattice
- Time reversal symmetry preserved
- No external magnetic field

1st realization in quantum well (HgTe) (mercury-telluride)

Birth of topological insulator

https://topocondmat.org/w5_qshe/fermion_parity_pump.html

C. L. Kane, E. J. Mele
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Brief story of topology in Physics

Topological Dirac Cone
featured in Top-10 Nature Physics

Top 10 Physics Discoveries of the last 10 Years

Nobel prize in Physics 2016

F. Duncan M. Haldane  J. Michael Kosterlitz  David J. Thouless

“For theoretical discoveries of topological phase transitions and topological phases of matter”

S. Haddad, ASP2021-26-07-2021-III
Topology and the Bambaloni of Sidi Bou Saïd village (Tunis)
Integer quantum Hall effect

Anomalous Quantum Hall effect (Haldane model)

Quantum Spin Hall effect (Kane-Mele model)

Topological materials

Topological Crystalline Insulators and Topological Superconductors: From Concepts to Materials

Annual Review of Condensed Matter Physics
Vol. 6:361-381 (Volume publication date March 2015)
First published online as a Review in Advance on January 22, 2015
https://doi.org/10.1146/annurev-conmatphys-031214-014501
Classical Hall effect

\[ \rho_{xx} = \frac{E_x}{j_x} \]

\[ \rho_{yx} = -\frac{E_y}{j_x} = -\frac{E_H}{j_x} \]
Classical Hall effect: origin

Drude model

\[
\begin{pmatrix}
1 & \omega_c \tau \\
-\omega_c \tau & 1
\end{pmatrix}
J = \frac{e^2 n \tau}{m} E
\]

\[J = \sigma E\]

Conductivity tensor

\[
\sigma = \frac{\sigma_D}{1 + \omega_B^2 \tau^2}
\begin{pmatrix}
1 & -\omega_c \tau \\
\omega_c \tau & 1
\end{pmatrix}
\]

\[
\sigma_D = \frac{ne^2 \tau}{m}
\]

\[
\omega_c = \frac{qB}{M}
\]

Resistivity tensor

\[
\rho = \sigma^{-1} = \begin{pmatrix}
\rho_{xx} & \rho_{xy} \\
-\rho_{xy} & \rho_{yy}
\end{pmatrix}
\]

\[
\rho = \frac{1}{\sigma_D} \begin{pmatrix}
1 & \omega_B \tau \\
-\omega_B \tau & 1
\end{pmatrix}
\]

Hall resistance

\[
R_{xy} = \frac{V_y}{I_x} = \frac{LE_y}{LJ_x} = \frac{E_y}{J_x} = -\rho_{xy}
\]

Hall coefficient

\[
R_H = -\frac{E_y}{J_x B} = \frac{\rho_{xy}}{B}
\]

\[
R_H = \frac{\omega_c}{B \sigma_D} = \frac{1}{e n_{elc}}
\]
Classical Hall effect: what is for?

- Identification of p or n type of semi-conductor
- Measure of carrier concentrations in semi-conductors

\[ R_H = \frac{\omega_c}{B \sigma_D} = \frac{1}{nq} \]

- Hall effect sensor: detect the presence and magnitude of a magnetic field
Classical Hall effect: what is for?
Classical Hall effect (1879)

Edwin Hall

Integer quantum Hall effect (1980)

von Klitzing

$$R_{\text{Hall}} = R_{xy} = \frac{U_H}{I_x} = \frac{B}{n_s e}$$

$$R_{\text{Hall}} = R_{xy} = \frac{h}{e^2 n} \quad n \text{ integer}$$
Lecture III: Introduction to topological materials

Integer Quantum Hall effect

5.2.1980 BIRTHDAY OF QHE
(at 2 a.m.)

Resistance at B=0
Resistance at B=19.8 T

Hallresistance

von Klitzing data, Séminaire Poincaré 2 (2004) 1-16
Hall resistance $R_{Hall} = \frac{h}{e^2 n}$

$\frac{h}{e^2}$ universal constant

Low field
$R_L = \text{constant}$
$R_{Hall} \alpha B$
(classical HE)

Intermediate field values:
Shubnikov-de Haas Oscillations
$R_L$ oscillations quantiques
$R_{Hall} \alpha B$

strong champ: IQHE
$R_L$ vanishes
$R_{Hall}$ in a plateau
The new kilogram has arrived

On May 20, 2019, a kilogram will still weigh a kilogram, although it will no longer be defined in relation to a material prototype that weighs by definition exactly one kilogram, but instead in relation to the exact value now fixed by the Planck constant (h).

**PREVIOUS DEFINITION**

 adopted at the first meeting of the CGPM in 1889

"The kilogram is the unit of mass; it is equal to the mass of the international prototype kilogram."

**NEW DEFINITION**

 which should be adopted at the 26th meeting of the CGPM in November 2018

"The kilogram (kg) is the unit of mass of the SI. It is defined by taking the fixed numerical value of the Planck constant (h), and equals 6,626 070 15 × 10⁻³⁴ when it is expressed in J.s, a unit equal to kg m² s⁻¹, the meter and the second being defined in accordance with c and ΔνCs."

---

\[ h \]

Weighs by definition 1 kilogram

\[ h \]

is measured based on the international prototype kilogram (IPK) using a Kibble balance.

\[ h \]

equals exactly 6,62607015 \times 10^{-34} \text{ J.s}

\[ h \]

A standard of the kilogram was realised using \( h \) by means of a Kibble balance.
**IQHE**

$R_L = 0 \ (\rho_{xx} = 0)$ if $R_{\text{Hall}}$ is in plateau ($\rho_{xx} \neq 0$)

\[ \rho_{xx} = 0 \quad ? \quad \text{Perfect conductor?} \]

\[
\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}
\]

\[
\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \quad \sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}
\]

If $\rho_{xy} \neq 0$, then $\rho_{xx} = 0 \Rightarrow \sigma_{xx} = 0$.

Paradox!!!
IQHE

One could have

\[ \rho_{xx} = 0 \implies \sigma_{xx} = 0 \]

\( \rho_{xx} = 0 \) \hspace{1cm} \text{No dissipation (conductor)}

\( \sigma_{xx} = 0 \) \hspace{1cm} \text{No current in the longitudinal direction (insulator)}

In the bulk: insulator (localization of electrons by B and disorder)

At the edges: conducting current (non localized electrons)

EHQE: example of topological insulator
IQHE: Origin of the plateaus

Landau quantization

\[ H = \frac{1}{2m} \left( \vec{P} + e \vec{A} \right)^2 \]

\[ \pi = p + eA \]

\[ [\Pi_x, \Pi_y] = -i \frac{\hbar^2}{l_B^2} = -i\hbar eB \]

\[ l_B = \sqrt{\hbar/|eB|} \simeq 25 \text{nm}/\sqrt{\text{B}[T]} \]

\[ a = \frac{l_B}{\sqrt{2\hbar}} (\Pi_x - i\Pi_y) \]

\[ a^\dagger = \frac{l_B}{\sqrt{2\hbar}} (\Pi_x + i\Pi_y) \]

\[ H = \frac{1}{2m} \pi \cdot \pi = \hbar \omega_c \left( a^\dagger a + \frac{1}{2} \right) \]

\[ a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle \]

\[ a |n\rangle = \sqrt{n} |n-1\rangle \]

\[ E_n = \hbar \omega_c \left( n + \frac{1}{2} \right) \quad n \in \mathbb{N} \]

Degeneracy of \( \Pi \cdot \Pi = \hbar \omega_c \frac{mS}{2\pi \hbar^2} = \frac{BS}{\hbar/e} = \frac{\phi}{\phi_0} = N\phi \)
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Integer Quantum Hall effect

Varying B $\rightarrow$ varying the filling of LL $\nu = \frac{N}{N_\Phi} = \frac{n_c}{n_\Phi} = \frac{\hbar n_c}{eB}$

If LL is filled: fill a higher LL $\Rightarrow R_{\text{Hall}}$ changes the plateau

System is of a finite size: Fermi level intercept LL at states appearing at the edges

conducting edge states
• IQHE: edge states

\[ \sigma_{xy} = \left( \frac{e^2}{h} \right) n \ \text{avec} \ n : \text{number of edge states} \]
IQHE importance of disorder

\[ \sigma_{xy} = (e^2 / h) \ n \]

\( \sigma_{xy} \) not sensitive to
  
  - disorder
  
  - to the sample geometry

Topological character of \( \sigma_{xy} \)

\( n \): Chern number (topological invariant)

IQHE is topological

Definition 1: in an ordered topological phase, the response functions are expressed in terms of topological invariants

What is a topological invariant?

topological invariant = quantity which does not change under continuous deformations (genus in Math)

Definition 2: a topological phase is insulating but has surface/edge metallic states if it is close to a trivial (ordinary) phase
Review

(a) Hall (1879)

(b) AHE (1881)

(c) SHE (2004)

(d) QHE (1980) high H

(e) QAHE (2013)

(f) QSHE (2007)

Chang and Li, J. Phys.: Condens. Matter 28 (2016) 123002
"I can never remember which is a stalagmite and which is a stalactite."

S. Haddad, ASP2021-26-07-2021-III
There are not only metal, semi-conductors and insulators in life...
Lecture III: Introduction to topological materials

*Topological band structure*

- **Trivial band insulator**
- **Topological insulator (2005)**
Origin of edge states

At the interface

The topological invariant $n$ passe changes from 0 à 1

Gap should close at some points

Appearance of edge states inside the gap

Topological insulator
**Haldane model**

**Key ingredients:**

- Honeycomb lattice at zero magnetic field with...
- Locally broken Time Reversal Symmetry (TRS) by local magnetic fluxes
- Complex second neighbor hopping parameters

Haldane, PRL 61,2015 (1988)
Haldane model

Haldane hamiltonian

\[ H_{Haldane}(\vec{k}) = H_{grahene}(\vec{k}) + \left[ M - 2t_2 \sin \phi \sum_{i=1}^{3} (\sin \vec{k}.\vec{b}_i) \right] \sigma_z \]

\( M \): mass term breaks inversion symmetry

\( \Phi \): breaks Time Reversal Symmetry (TRS): \( \phi \rightarrow \vec{A} \rightarrow \vec{B}_{local} \)

Total flux: \( \iint \vec{B}.d\vec{s} = 0 \)

\( \sigma_{xy} = (e^2/h) \ C \), where \( C \) Chern number

\[ C = \frac{1}{2} [\text{sign}(m_+) - \text{sign}(m_-)] \]

\( m_{\pm} = M \mp 3t_2 \sqrt{3} \sin \phi \)

Quantum Anomalous Hall effect (QAH)
Haldane model

Haldane phase diagram

\[ M/t_2 \]

\[ 3\sqrt{3} \]

\[ 0 \]

\[ -3\sqrt{3} \]

\[ K \quad C_1 = 0 \]

\[ K' \quad C_1 = -1 \]

\[ K' \quad C_1 = +1 \]

\[ K \quad C_1 = 0 \]

Haldane, PRL 61, 2015 (1988)

Band insulator (trivial)

Semi-metal

Topological insulator
Haldane model

Zigzag graphene nanoribbon

Topological phases  Edge states  Trivial phases  no edge states
Haldane model

Zigzag graphene nanoribbon

**Edge states**

**Edge currents**

\[ E/t \]

\[ k_x d \]

\[ C=+1 \]

\[ |\psi|^2 \]

\[ x \]

\[ y \]

\[ n=N \]

\[ n=4 \]

\[ n=3 \]

\[ n=2 \]

\[ n=1 \]
Topological Quantum Materials from the Viewpoint of Chemistry

Nitesh Kumar*, Satya N. Guin, Kaustuv Manna, Chandra Shekhar, and Claudia Felser*
Lecture III: Introduction to topological materials

Topological band structure

Énergie

Bande de conduction

États de surface

Niveau de Fermi

Bande de valence

Impulsion
Thank you for your attention

Any questions?