Introduction to Solid State Physics

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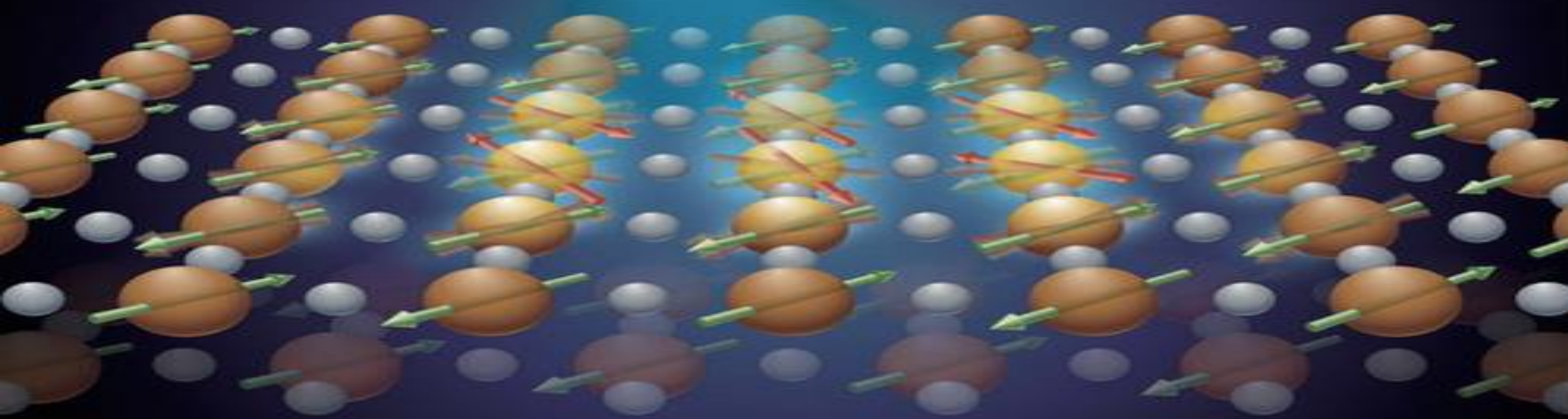

Outline

Lecture I: Introduction to Solid State Physics

- **Brief story…**
- **Solid state physics in daily life**
- **Basics of Solid State Physics**

Lecture II: Electronic band structure and electronic transport

- **Electronic band structure: Tight binding approach**
- **Applications to graphene: Dirac electrons**

Lecture III: Introduction to Topological materials

- **Introduction to topology in Physics**
- **Integer Quantum Hall effect**
- **Haldane model**

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- A Brief story…
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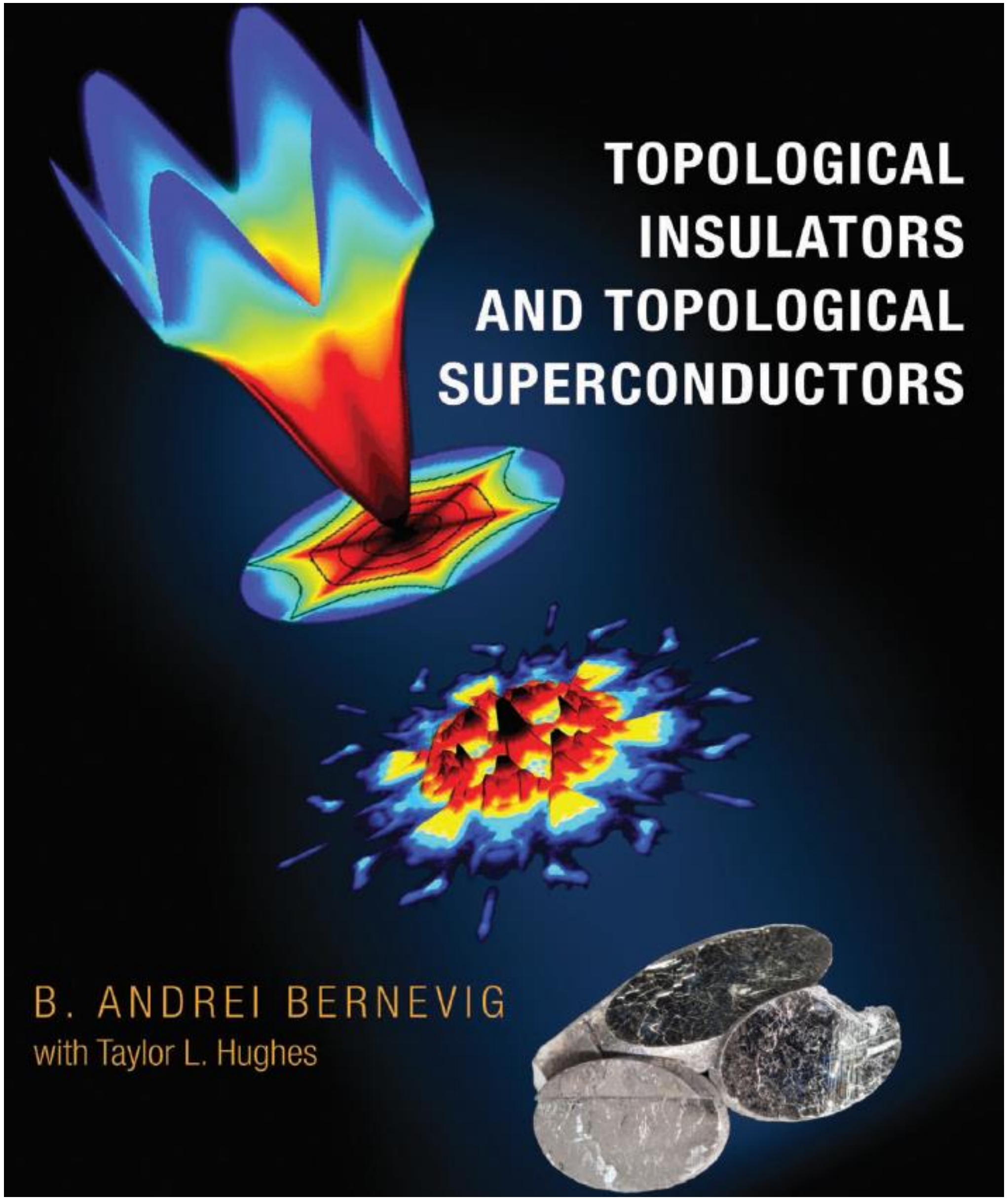
Lecture III: Introduction to Topological materials

- **Introduction to topology in Physics**
- **Quantum Hall effect**
- **Haldane model**

References


TOPOLOGICAL
INSULATORS
AND TOPOLOGICAL
SUPERCONDUCTORS
B. ANDREI BERNEVIG
with Taylor L. Hughes

In 1961, George Gamow remarked that

"***only number theory and topology had no application to physics***"

Gamow was mistaken!

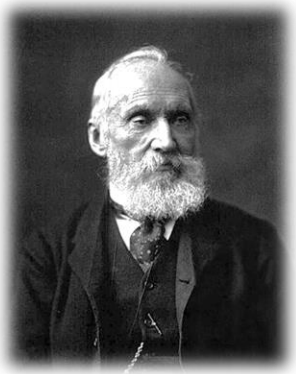
Topology on top

Nature Physics **12**, 615 (2016) | Cite this article

10k Accesses | **4** Citations | **18** Altmetric | Metrics

Topology has journeyed from the purely mathematical arena to feature throughout physics.

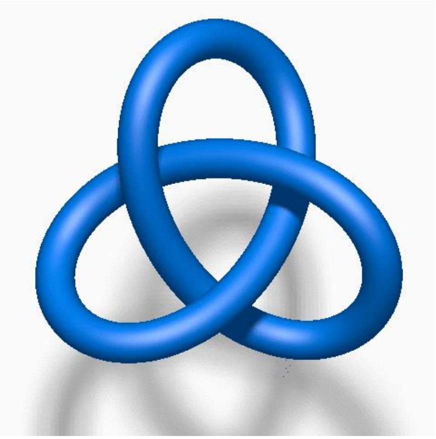
Lord Kelvin: Vortex theory of the atom (period 1870-1890)



William Thomson, Lord Kelvin

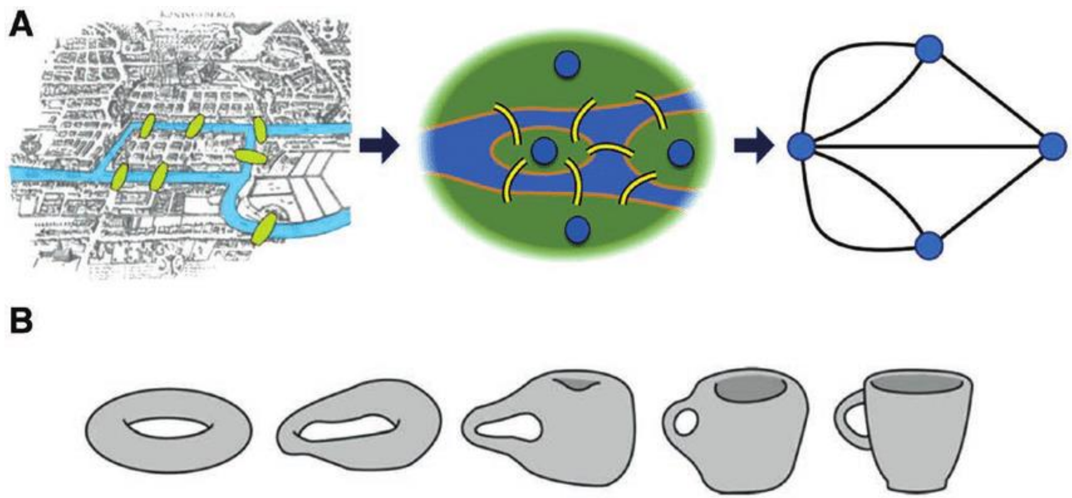
Atoms are knotted vortices in the ether
(postulated medium for the propagation of light) (Wikipedia)

Vortex: region in a fluid in which the flow revolves around an axis line



Carbon atom according to the Vortex theory

The Konigsberg bridge puzzle (1736) in mathematics: find a path to walk through the city of Konigsberg (Germany) that crosses each of the seven bridges once and only once.



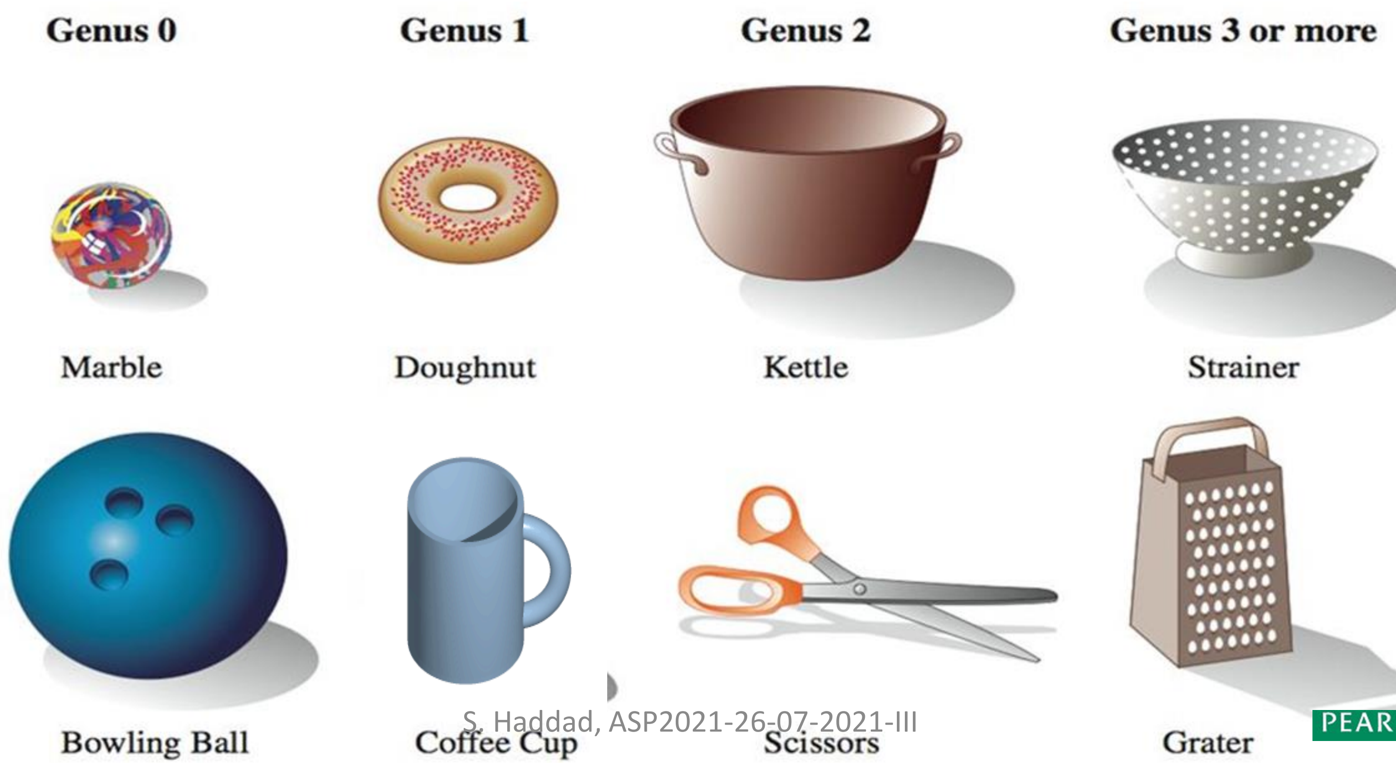
Leonhard Euler

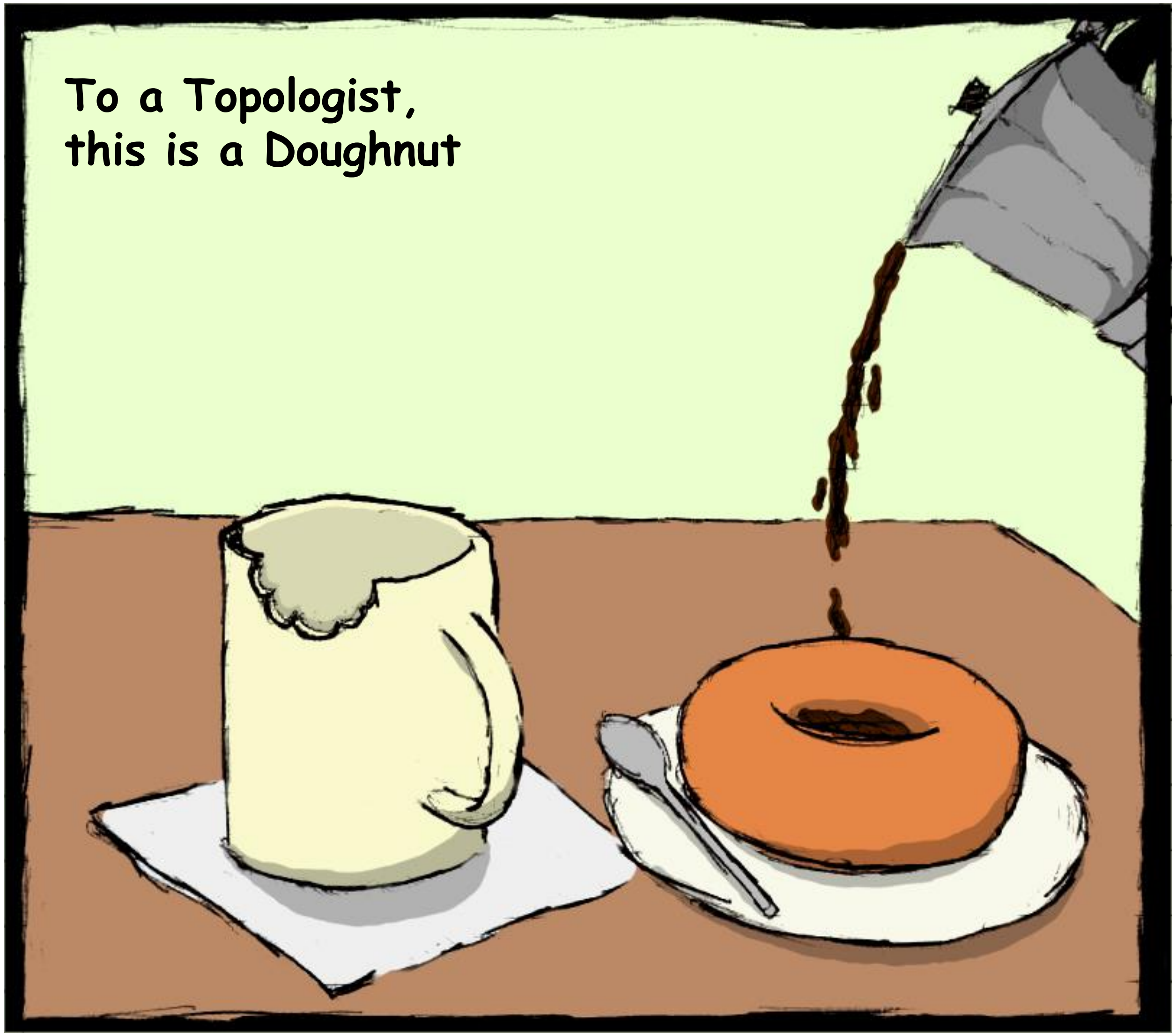
Euler work:

- replace each land by "node" and each bridge by an "edge" ⟹ graphs
- There is no possible path
- *topology* or architecture of the graph is important and not the details of the *geography*.
- The problem is the same for any city having the same topological invariants (number of lands and bridges)
- Introduced foundations of graph theory and the mathematical idea of topology (an important area of mathematics with the work of Poincaré around 1890).

Topology (Greek words τόπος, 'place, location', and λόγος, 'study') branch of mathematics deals with **the properties of a geometric object that are preserved under continuous deformations**, such as stretching, twisting, without closing holes, opening holes, tearing, gluing.

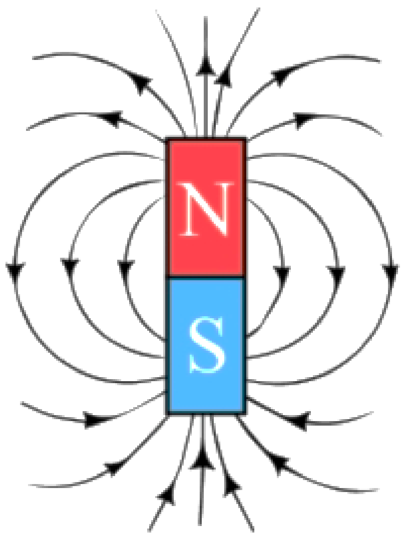
Topological invariant: genus number of holes

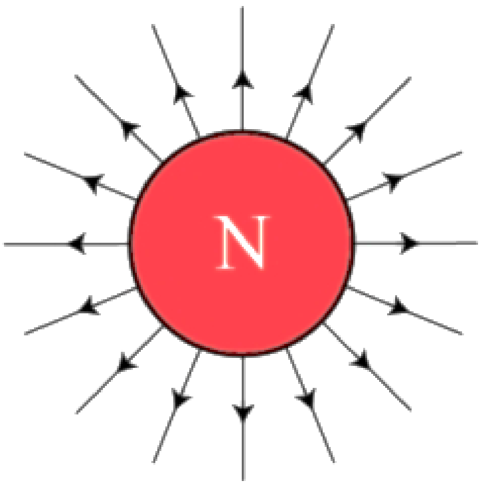


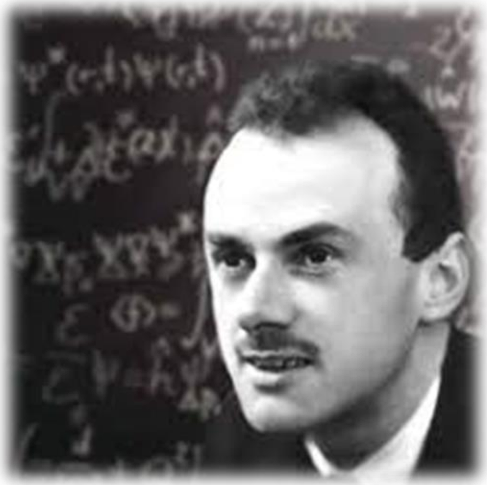

To a Topologist,
this is a Doughnut

- Dirac in 1931, introduced magnetic monopole is a hypothetical massive particle that is an isolated magnet with only one magnetic pole

- Magnetic monopole may solve one of the physics puzzle: the quantization of the electric charge





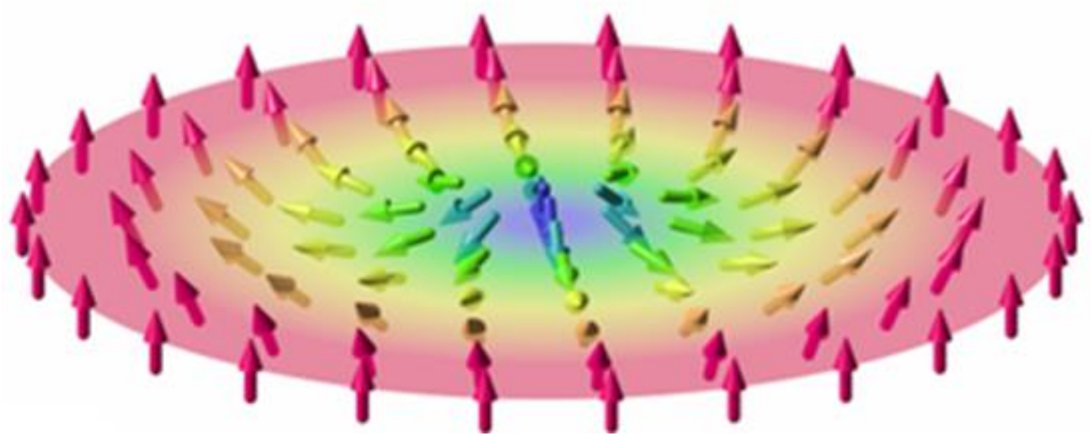


Dirac “I am inclined now to believe that monopoles do not exist.” (1981)

In the 1960s and early 1970s

Solitons as topological defects cosmology, particle physics, supercondutors…
objects with localised energy and stability

Example Skyrmion (1962) : topologically protected vortex like spin structures



Tony Skyrme

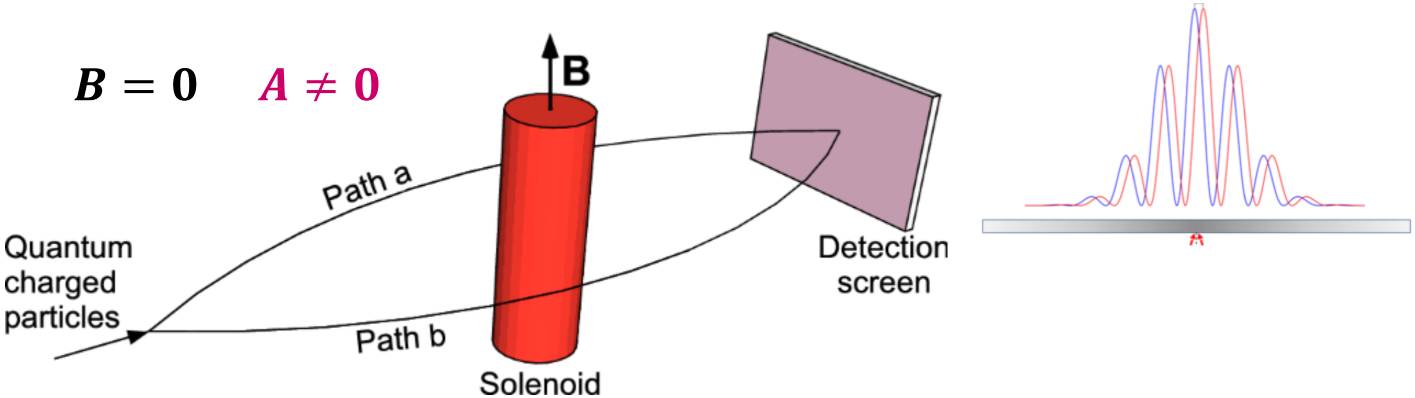
Aharonov-Bohm effect

- described in 1949 by Ehrenberg and Siday
- Reformulated by Aharonov-Bohm in 1959
- Interference pattern of two electron beams modified by the presence of solenoid carrying a current ($\boldsymbol{B} = \boldsymbol{0}$ outside the solenoid)
- Coupling electron wave function and the vector potential $\boldsymbol{A}$
- The wave function acquires an extra phase known as geometrical or topological phase

Yakir Aharonov

$$\varphi = \frac{q}{\hbar}\int_P \mathbf{A}\cdot d\mathbf{x}$$

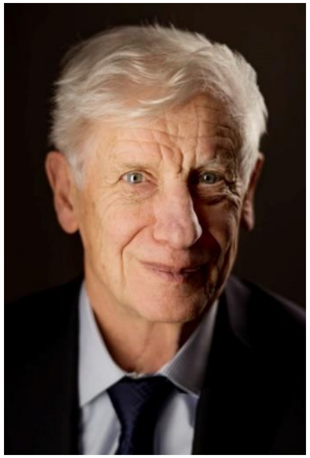
$\boldsymbol{B} = \boldsymbol{0}$ $\quad$ $\boldsymbol{A} \neq \boldsymbol{0}$



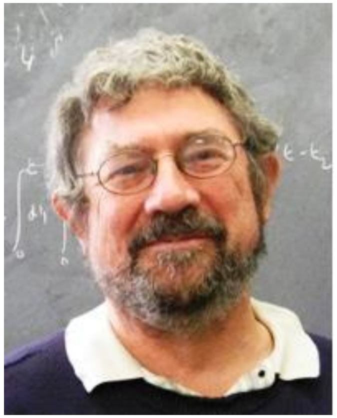
David Bohm

In the 1970s, Berezinski, Thouless et Kosterlitz introduced the topological phase transition taking place without symmetry-breaking (contrary to Landau transitions)

Vadim Berezinskii



David J. Thouless

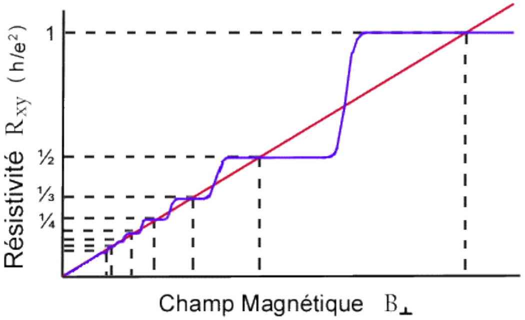


John M. Kosterlitz

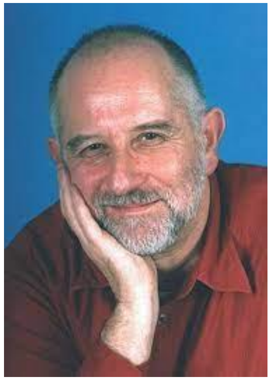
von Klitzing

Key example of topological phase: Integer Quantum Hall effect (IQHE)

- Discovered by K. von Klitzing in 1980
- 2D electron gas cooled down (few K) under strong magnetic filed (few Tesla)
- Quantification of the Hall conductivity
- Quantification robust against disorder

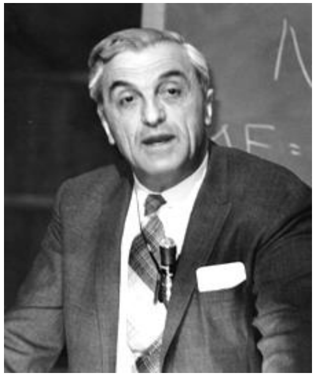


- Theoretical interpretation of IQHE: 1982, Thouless, Kohmoto, Nightingal et Nijs (TKNN)
- Conductivity related to a topological invariant: Chern number
- Defined from the geometrical phase of the electronic wavefunction called Berry phase (Michael Berry)



Michael Berry

Berry phase



Felix Bloch

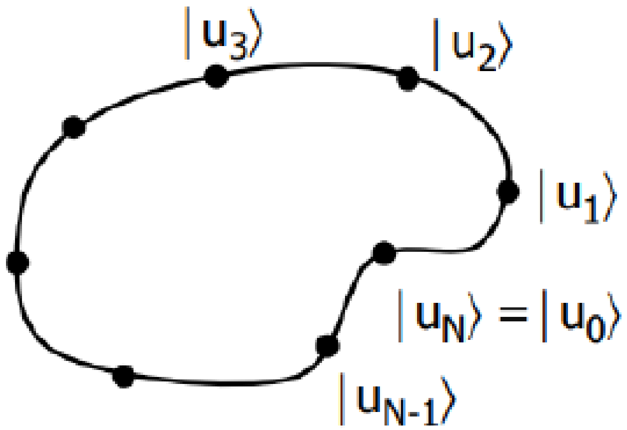
Bloch theorem:

$$\psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r})e^{i\vec{k}.\vec{r}}$$

$$u_{\vec{k}}(\vec{r}+\vec{R}) = u_{\vec{k}}(\vec{r})\text{ cellular function}$$

If $\vec{k}$ describe a closed path in the Brillouin zone during time

How will be modified the Bloch function?

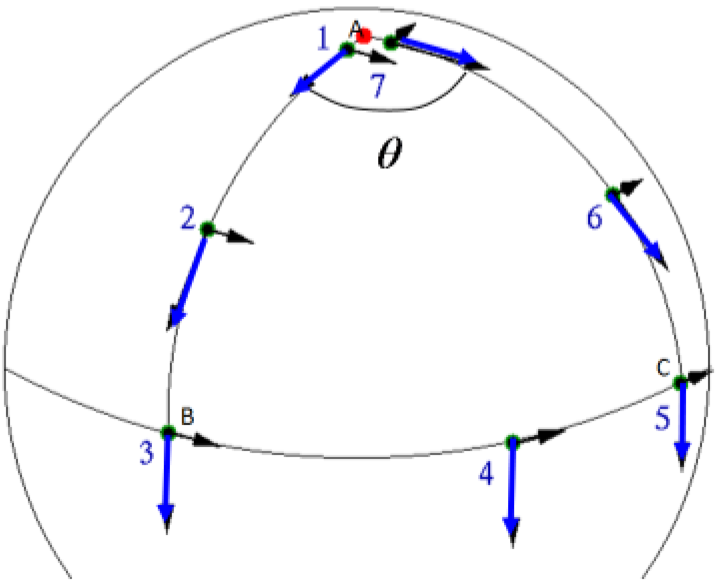


Berry phase Dynamical phase

Bloch function $|u_m(\vec{k}(t))\rangle = e^{i\gamma_m} e^{-i\int_0^T \varepsilon_m(t')dt'} |u_m(\vec{k}_0)\rangle$

Berry *"face"*

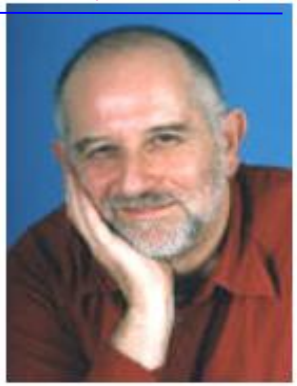
Classical parallel transport: by moving the vector along a curve on a sphere without changing it: angle between initial and final position



In quantum mechanics:

- electron wavefunction acquires a geometrical phase (Berry phase) if it describes a closed path in the Hilbert space
- Bloch function will acquire a Berry phase if $\vec{k}$ describe a closed path in the Brillouin zone

In general, for a wavefunction dependening on a parameter $\vec{R}\,(t)$



Berry *"face"*

Phase de Berry

$$\gamma_n = i\oint \langle \Psi_n | \vec{\nabla}_{\vec{R}} | \Psi_n \rangle . d\vec{R}$$

Depends on the form of the path described by the wavefunction in the parameter space

Berryology

$$\vec{A}_n(\vec{k}) = i\langle u_{n,\vec{k}} | \vec{\nabla}_{\vec{k}} | u_{n,\vec{k}} \rangle$$

Berry connection

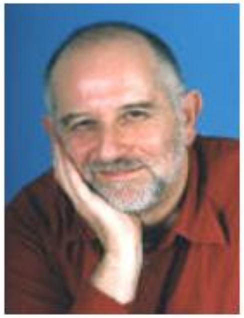
$$\vec{\Omega}_n(\vec{k}) = \vec{\nabla}_{\vec{k}} \wedge \vec{A}_n(\vec{k})$$

Berry curvature

$$C_n = \frac{1}{2\pi}\oint_{S_f} \vec{\Omega}(\vec{k}).\mathrm{d}\vec{k},$$

Chern number: Berry flux in the BZ

Analogy between electromgnetism and Berry parameter



Berry curvature $\Omega(\vec{\lambda})$	Magnetic field $B(\vec{r})$
Berry connection $\langle\psi\vert i\frac{\partial}{\partial\lambda}\vert\psi\rangle$	Vector potential $A(\vec{r})$
Geometric phase $\oint d\lambda\,\langle\psi\vert i\frac{\partial}{\partial\lambda}\vert\psi\rangle=\iint d^2\lambda\;\Omega(\vec{\lambda})$	Aharonov-Bohm phase $\oint dr\,A(\vec{r})=\iint d^2r\,B(\vec{r})$
Chern number $\oiint d^2\lambda\;\Omega(\vec{\lambda})=\text{integer}$	Dirac monopole $\oiint d^2r\;B(\vec{r})=\text{integer}\,h/e$

Chern theorem: the integral of the Berry curvature over a closed surface if the space paramters of wavefunctions is quantized

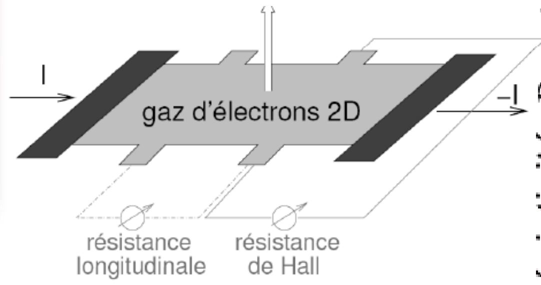
$$\oint_{S_f} \vec{\Omega}(\vec{R})\mathrm{d}\vec{S} = 2\pi m$$

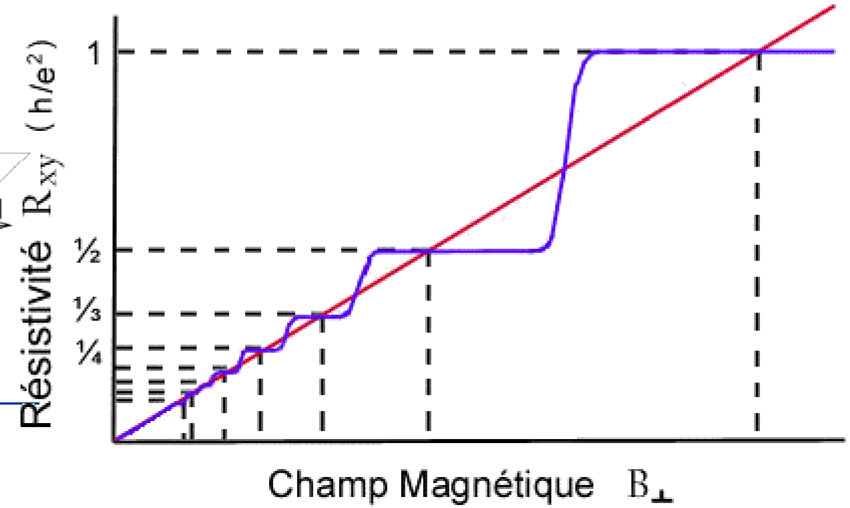
m: Chern number

S. S. Chern

A nonvanishing flux of the Berry curvature through a closed surface is equivalent of having a nonzero magnetic charge: topological object

A nonzero Chern number is a signature of the topological aspect of the wavefunction



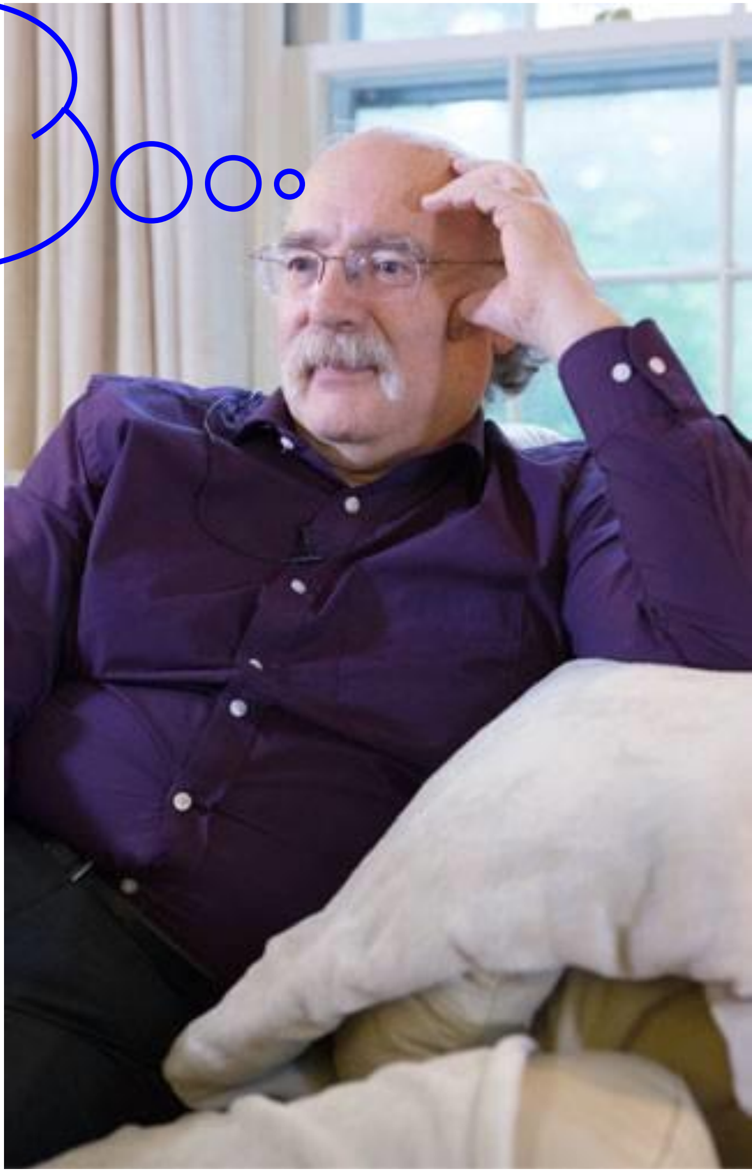


$R_{Hall} = \frac{h}{e^2 n}$ n interger (Chern number)

Integer Quantum Hall Effect

- Important on the fundamental aspects (manifestation of topology)
- Difficult to realize (extreme conditions T and B)

In life there is something else besides magnetic field!



Haldane model:

Haldane, PRL 61,2015 (1988)

Seminal paper of Topological Material

Haldane proposed IQHE without magnetic field!

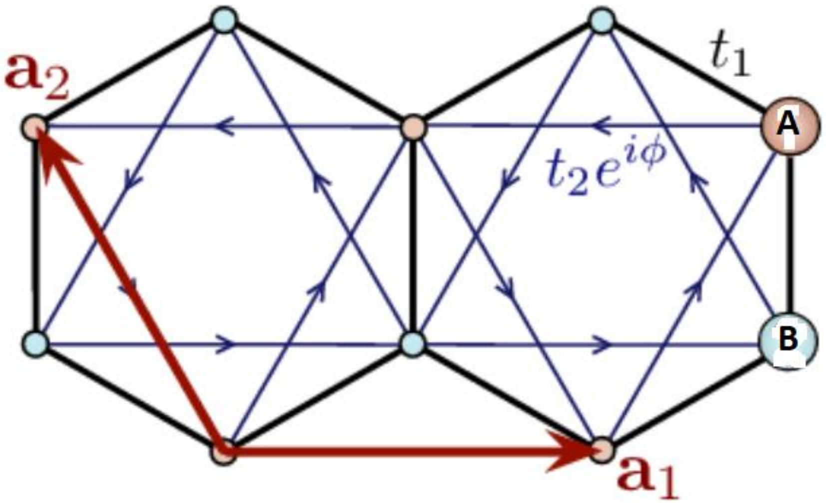
Anomalous Quantum Hall effect (AQH)

Haldane, PRL 61,2015 (1988)

F. D. M. Haldane

Model ingredients:

- Graphene-like crystal with two types of atoms A and B
- Complex second-neighboring hopping parameters
- zero total magnetic flux in the unit cell
- Breaks time reversal symmetry without magnetic field

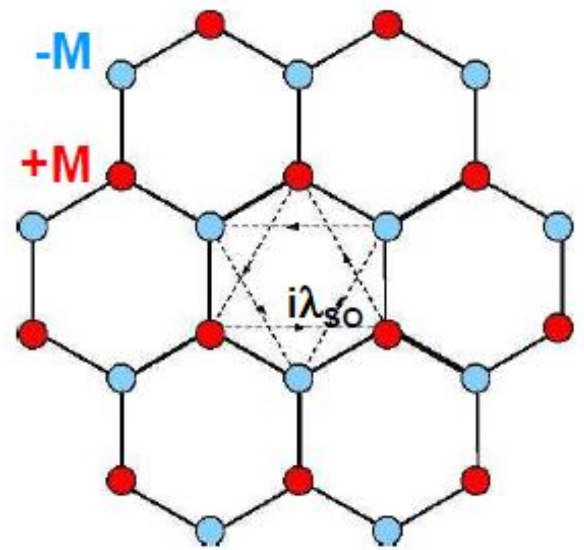


https://topocondmat.org/w4_haldane/haldane_model.html

Kane and Mele model = 2 copies of Haldane model for spins ↑, and ↓

C. L. Kane, E. J. Mele
Phys. Rev. Lett. 95, 146802 (2005)
Phys. Rev. Lett. 95, 226801 (2005)

Quantum Spin Hall Effect



- Inversion symmetry breaking not really needed
- Spin orbit term connecting sites in the same sublattice
- Time reversal symmetry preserved
- No external magnetic filed

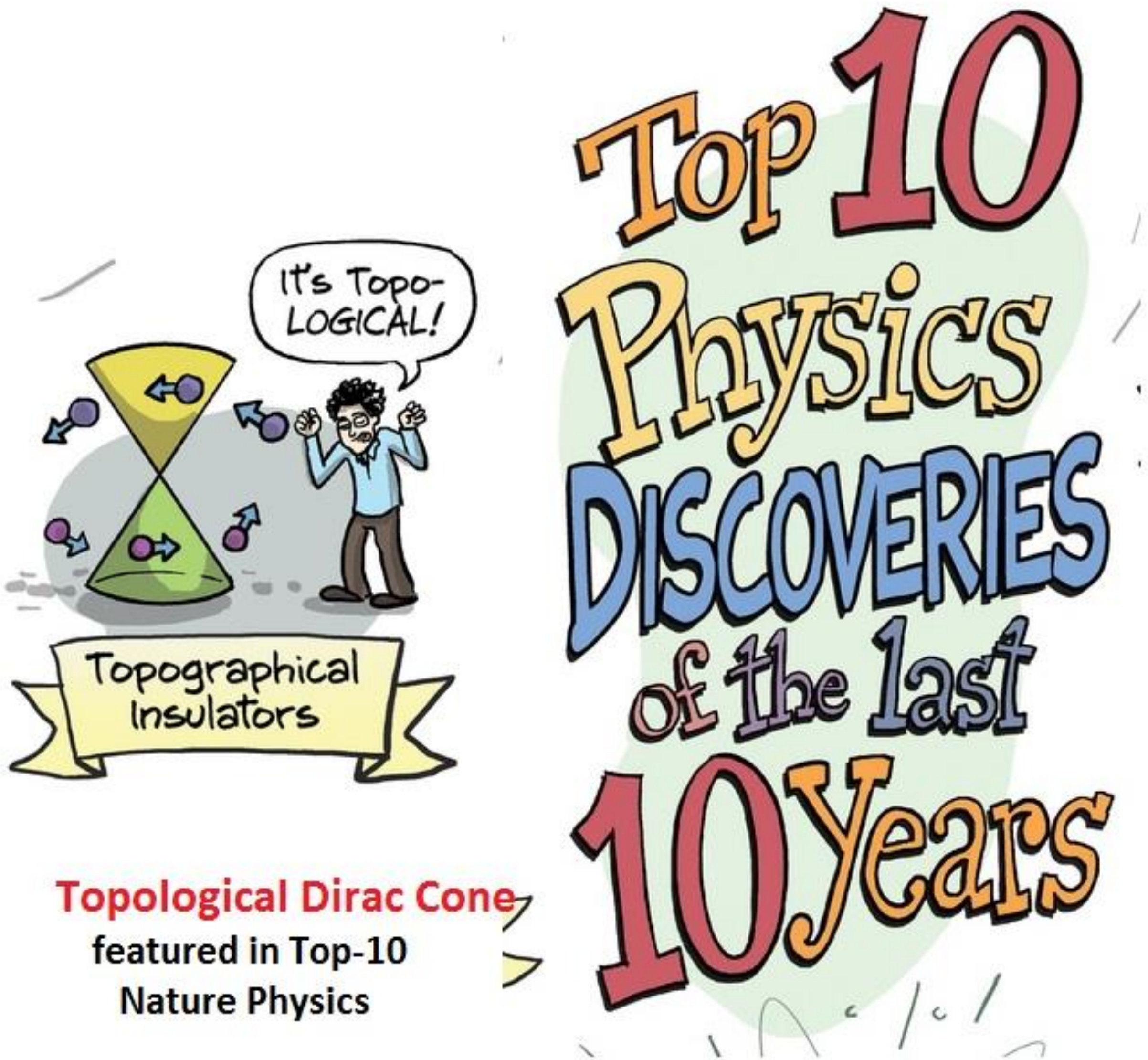
1st realization in quantum well (HgTe) (mercury-telluride)
König et al., Science 318, 766 (2007)

Birth of topological insulator


Charles Kane


John "Gene" Mele

https://topocondmat.org/w5_qshe/fermion_parity_pump.html



Topological Dirac Cone
featured in Top-10
Nature Physics

Nature Physics 11, 799 (2015)

Nobel prize in Physics 2016

F. Duncan M. Haldane — J. Michael Kosterlitz — David J. Thouless

"for theoretical discoveries of topological phase transitions and topological phases of matter"

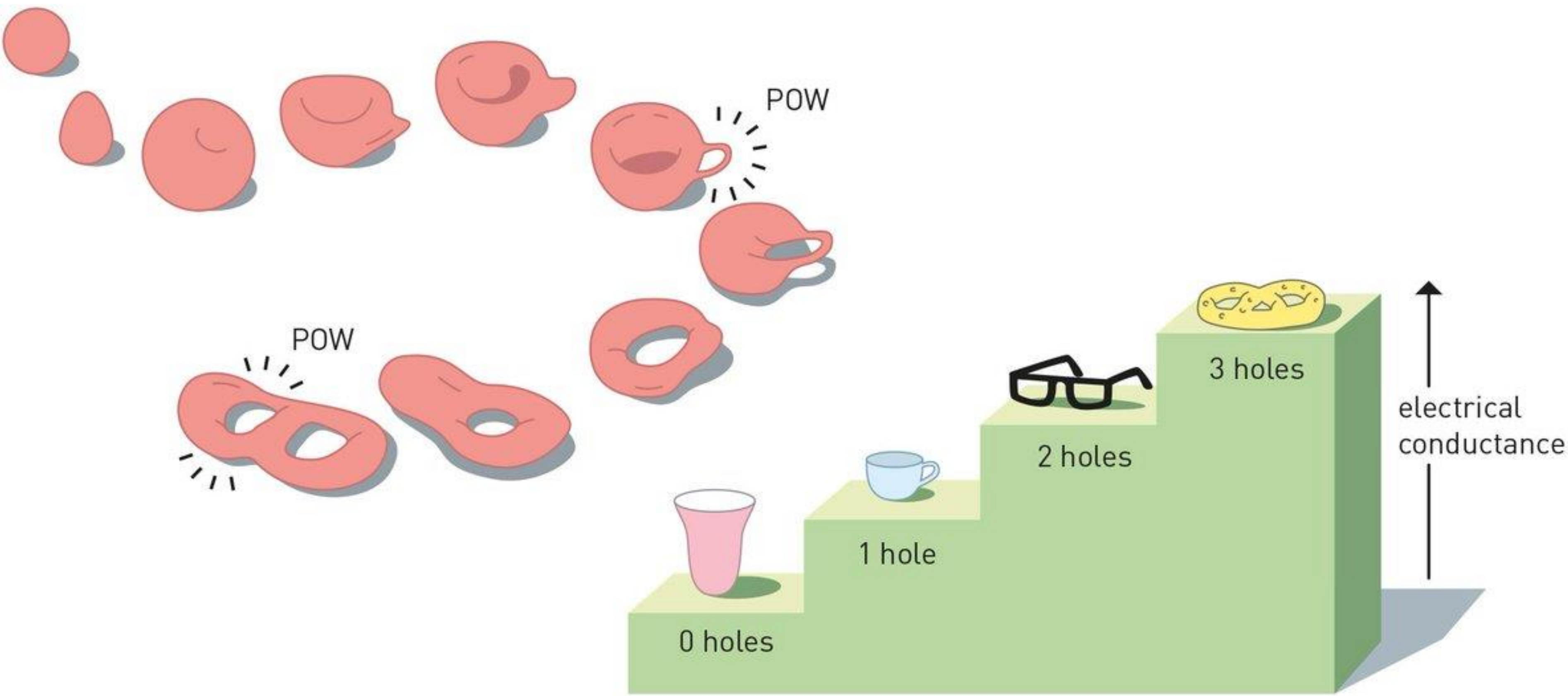
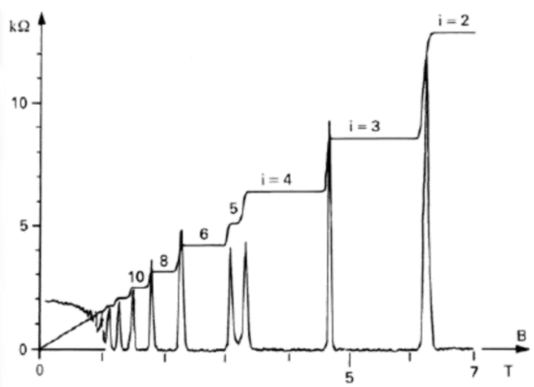


Illustration: ©Johan Jarnestad/The Royal Swedish Academy of Sciences

Topology and the Bambaloni of Sidi Bou Saïd village (Tunis)



Integer quantum Hall effect

Anomalous Quantum Hall effect (Haldane model)

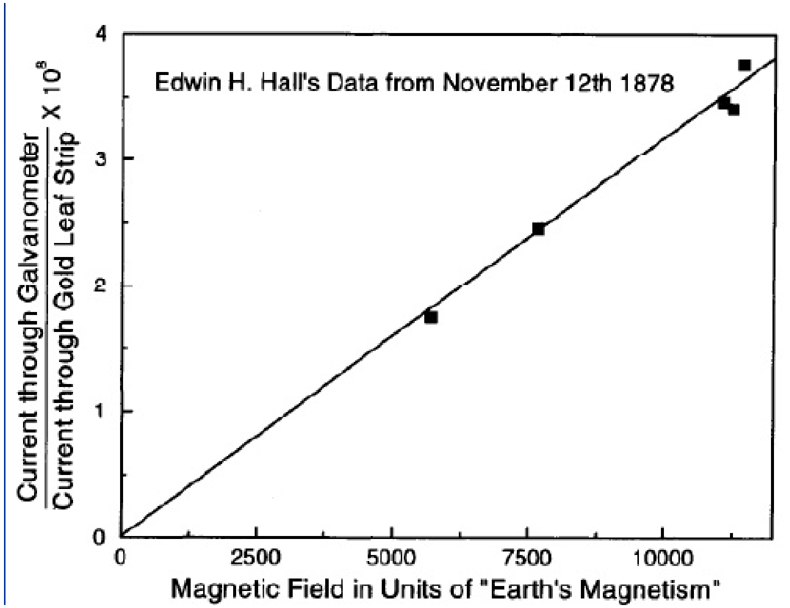
Quantum Spin Hall effect (Kane-Mele model)

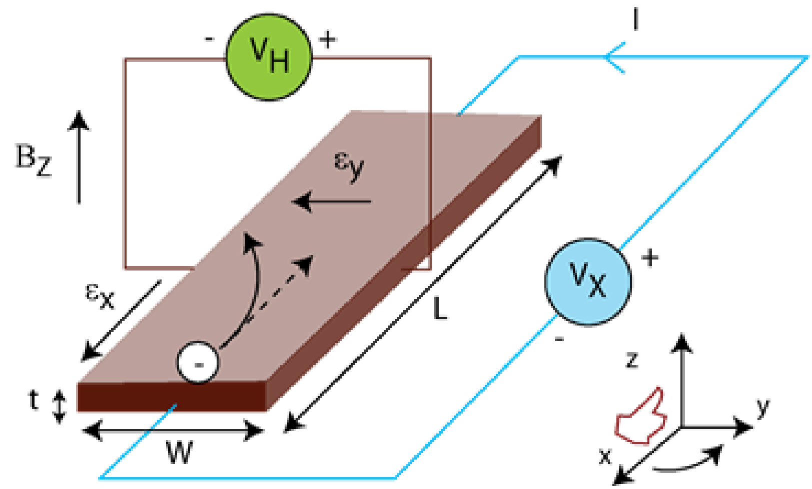
Topological materials

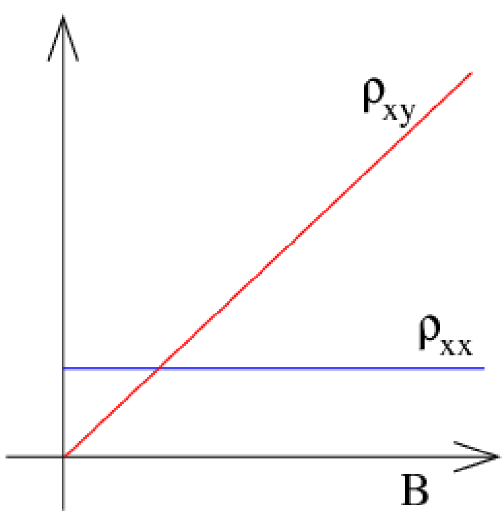
Topological Crystalline Insulators and Topological Superconductors: From Concepts to Materials

Annual Review of Condensed Matter Physics
Vol. 6:361-381 (Volume publication date March 2015)
First published online as a Review in Advance on January 22, 2015
https://doi.org/10.1146/annurev-conmatphys-031214-014501

Classical Hall effect







$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{yx} = -\frac{E_y}{j_x} = -\frac{E_H}{j_x}$$

Classical Hall effect: origin

Drude model

$$\begin{pmatrix} 1 & \omega_c\tau \\ -\omega_c\tau & 1 \end{pmatrix} \mathbf{J} = \frac{e^2 n\tau}{m}\mathbf{E} \qquad \mathbf{J} = \sigma\mathbf{E}$$

Conductivity tensor $\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}$

$$\sigma = \frac{\sigma_D}{1+\omega_B^2\tau^2}\begin{pmatrix} 1 & -\omega_c\tau \\ \omega_c\tau & 1 \end{pmatrix} \qquad \sigma_D = \frac{ne^2\tau}{m} \qquad \boxed{\omega_c = \frac{qB}{M}}$$

Resistivity tensor $\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$ $\qquad \rho = \frac{1}{\sigma_D}\begin{pmatrix} 1 & \omega_B\tau \\ -\omega_B\tau & 1 \end{pmatrix}$

$$\rho_{xx} = \frac{E_x}{j_x}$$

$$\rho_{yx} = -\frac{E_y}{j_x} = -\frac{E_H}{j_x}$$

Hall resistance $R_{xy} = \frac{V_y}{I_x} = \frac{LE_y}{LJ_x} = \frac{E_y}{J_x} = -\rho_{xy}$

Hall coefficient $R_H = -\frac{E_y}{J_x B} = \frac{\rho_{xy}}{B}$ $\qquad \boxed{R_H = \frac{\omega_c}{B\sigma_D} = \frac{1}{e\, n_{elc}}}$

Classical Hall effect: what is for ?

- Identification of p or n type of semi-conductor
- Mesure of carrier concentrations in semi-conductors

$$R_H = \frac{\omega_c}{B\sigma_D} = \frac{1}{nq}$$

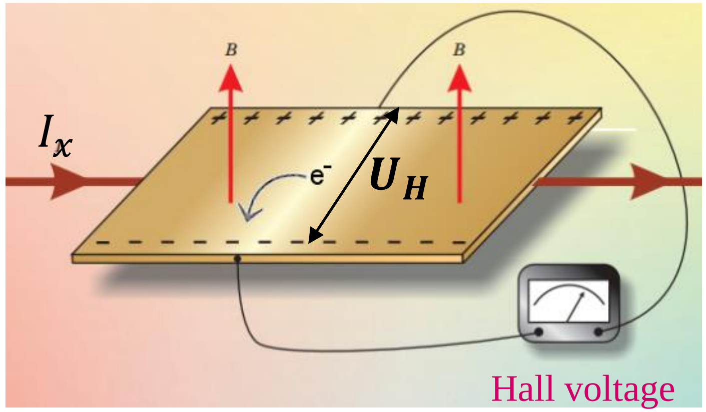
- Hall effect sensor: detect the presence and magnetude of of magnetic field

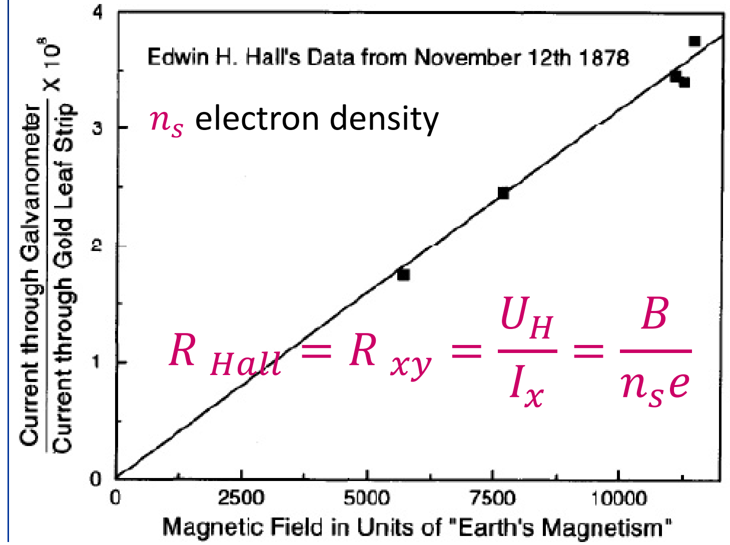
Classical Hall effect: what is for ?

Integer Quantum Hall effect

Classical Hall effect (1879)

Edwin Hall

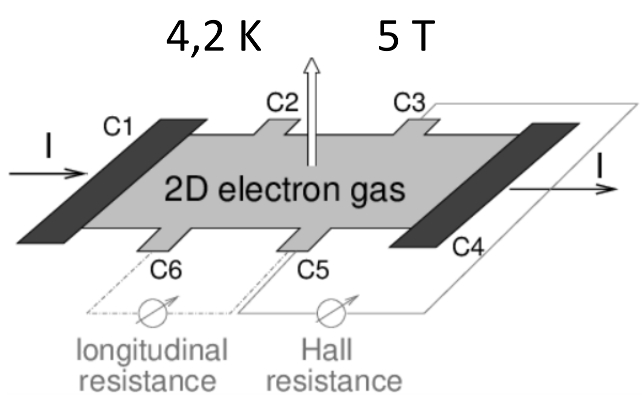


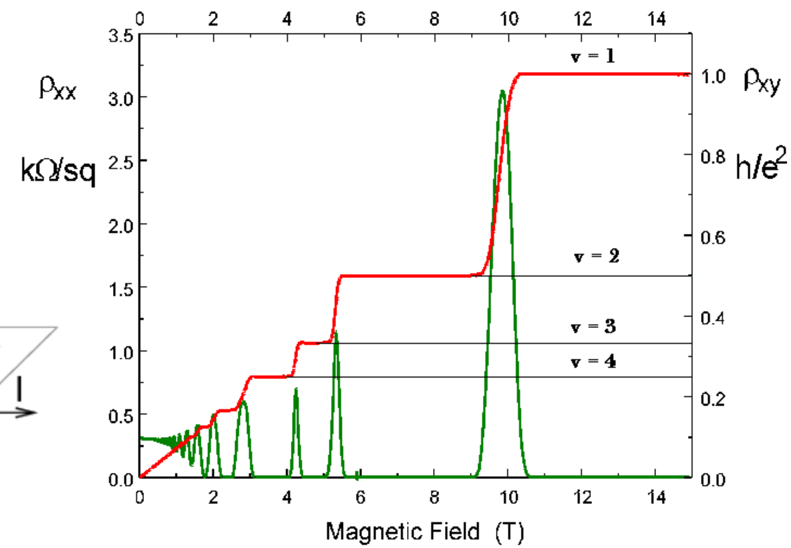


$$R_{Hall} = R_{xy} = \frac{U_H}{I_x} = \frac{B}{n_s e}$$

Integer quantum Hall effect (1980)

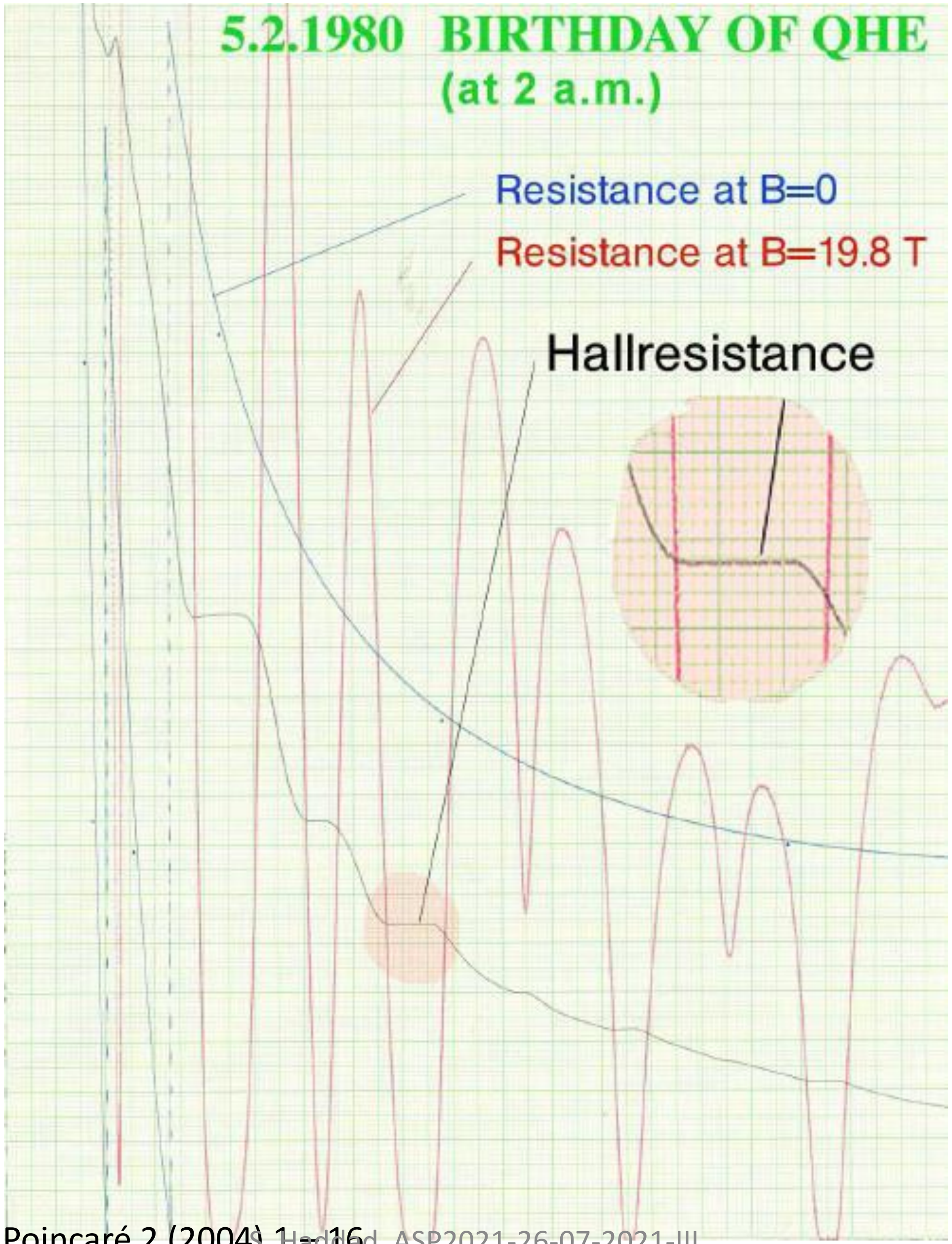
von Klitzing





$$R_{Hall} = R_{xy} = \frac{h}{e^2 n} \qquad n \ integer$$

Integer Quantum Hall effect



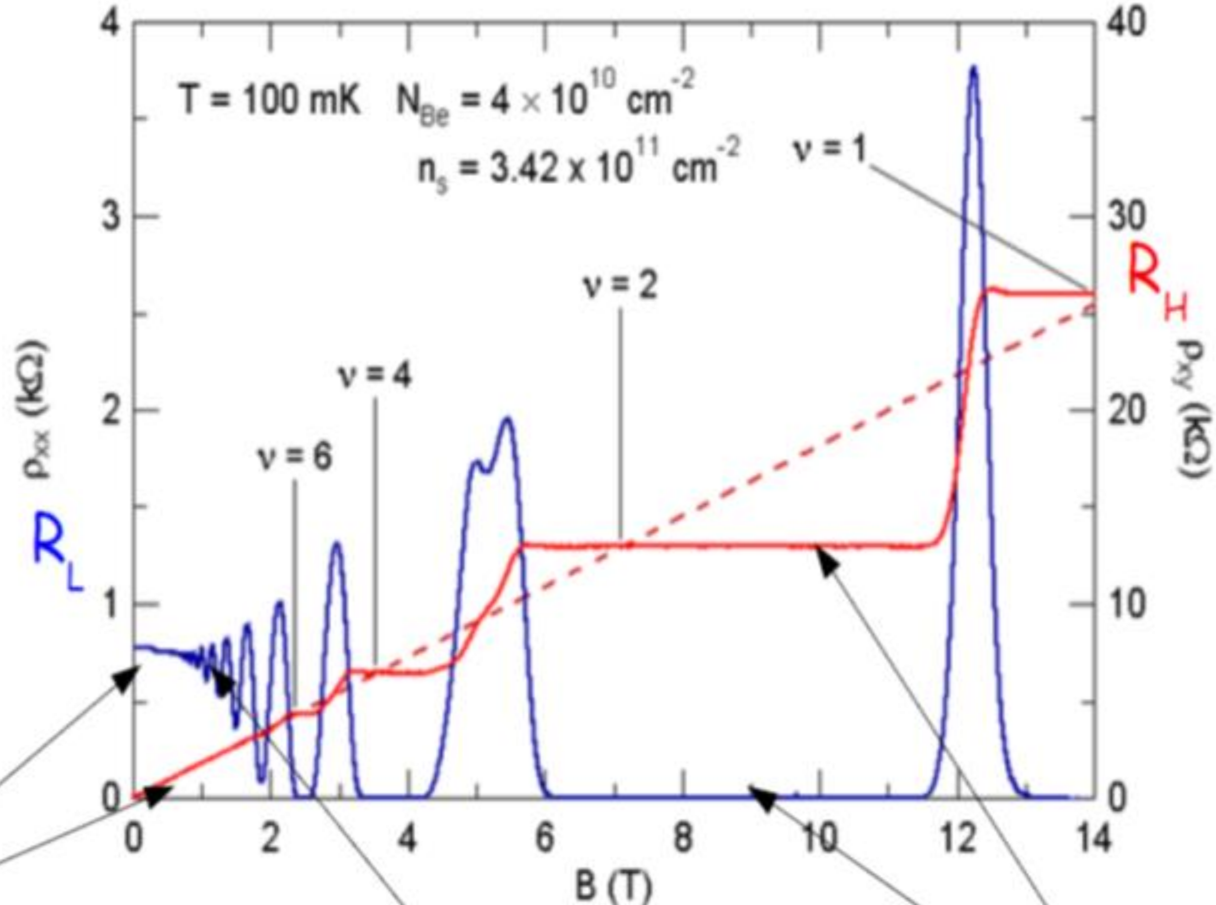
von Klitzing data, Séminaire Poincaré 2 (2004) 1 - 16

Integer Quantum Hall effect

Hall resistance …

$$R_{Hall} = \frac{h}{e^2}\frac{1}{n}$$

$$\frac{h}{e^2}$$ *universal constant*



Low field
R_L = constant
R_{Hall} α B
(classical HE)

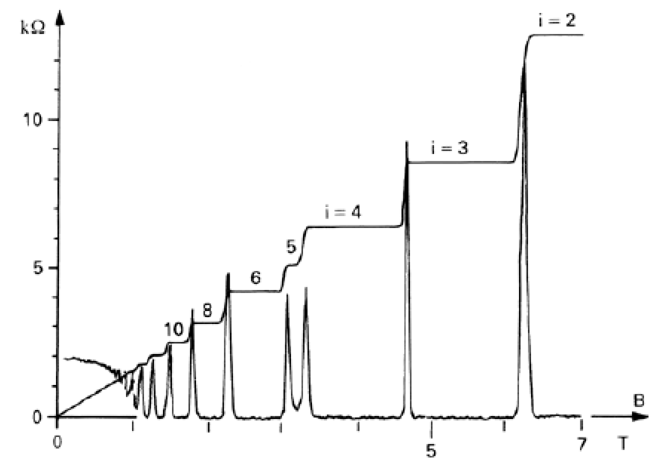
Intermediate field values:
Shubnikov-de Haas Oscillations
R_L oscillations quantiques
R_{Hall} α B

strong champ : IQHE
R_L vanishes
R_{Hall} in a plateau

IQHE What is for?

IQHE

R_L=0 (ρ_{xx}=0) if R_{Hall} is in plateau ($\rho_{xx} \neq 0$)



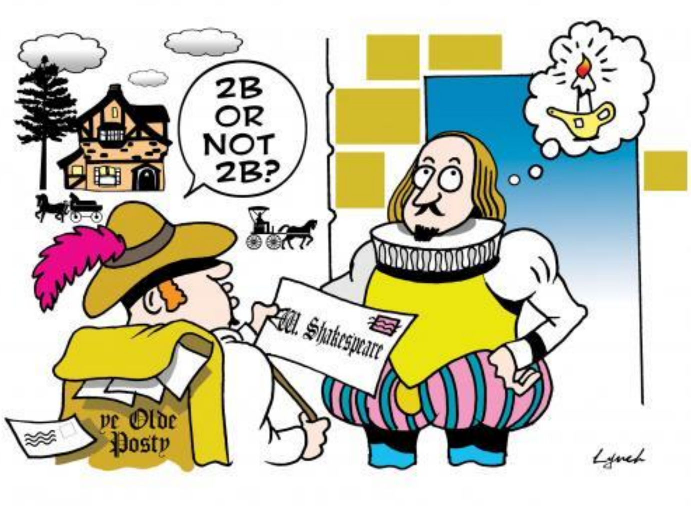
$\rho_{xx} = 0$? → Perfect conductor?

$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$$

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} \qquad \sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}$$

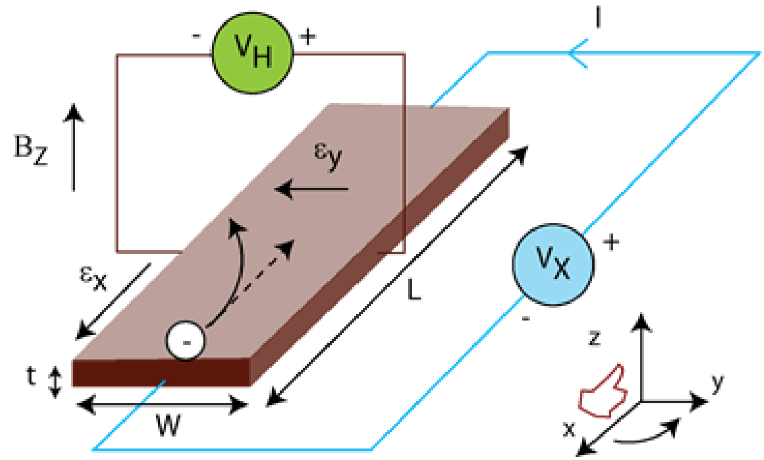
$\rho_{xy} \neq 0,$ $\boxed{\rho_{xx} = 0 \quad \Rightarrow \quad \sigma_{xx} = 0}$

conductor ? insulator ?



Paradox !!!

IQHE



One could have

$$\rho_{xx} = 0 \quad \Rightarrow \quad \sigma_{xx} = 0$$

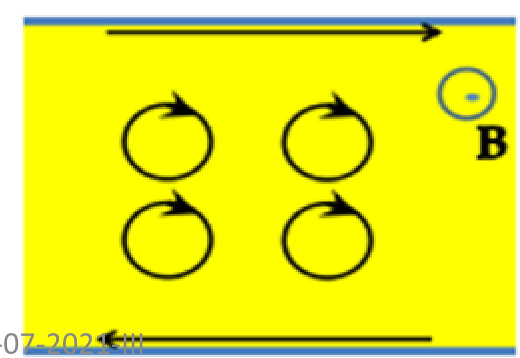
$\rho_{xx} = 0$ ➡ No dissipation (conductor)

$\sigma_{xx} = 0$ ➡ No current in the longitudinal direction (insulator)

In the bulk: insulator (localization of electrons by B and disorder)

At the edges: conducting current (non localized electrons)

EHQE: example of topological insulator



IQHE: Origin of the plateaus

Landau quantization $$H = \frac{1}{2m}\left(\vec{P} + e\,\vec{A}\right)^2 \qquad \boldsymbol{\pi} = \mathbf{p} + e\mathbf{A}$$

$$[\Pi_x, \Pi_y] = -i\frac{\hbar^2}{l_B^2} = -i\hbar eB \qquad l_B = \sqrt{\hbar/|e|B} \simeq 25\text{nm}/\sqrt{B[\text{T}]}$$

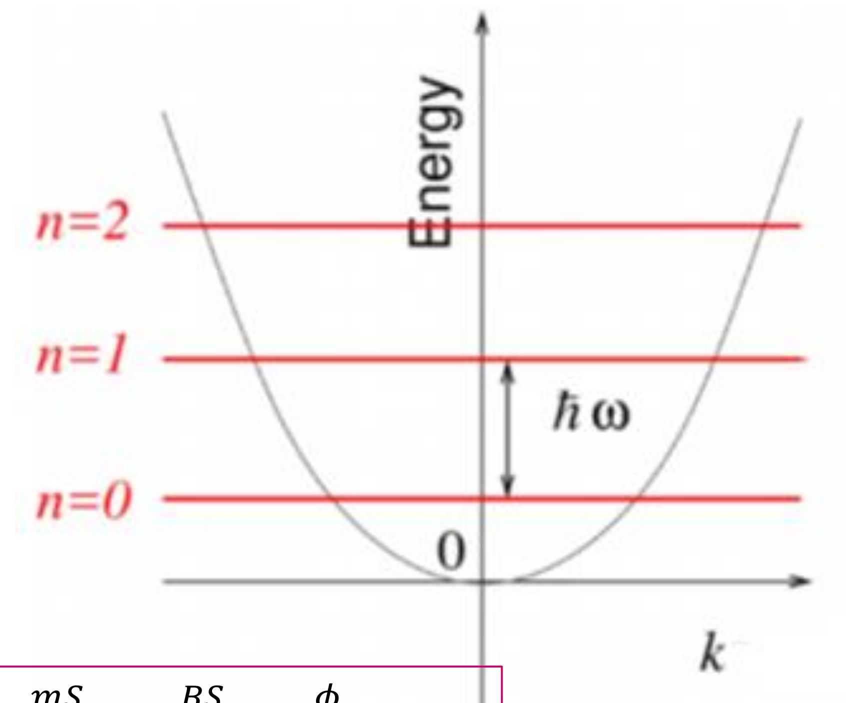
$$a = \frac{l_B}{\sqrt{2}\hbar}(\Pi_x - i\Pi_y) \qquad a^\dagger = \frac{l_B}{\sqrt{2}\hbar}(\Pi_x + i\Pi_y)$$

$$H = \frac{1}{2m}\boldsymbol{\pi}\cdot\boldsymbol{\pi} = \hbar\,\omega_c\left(a^\dagger a + \frac{1}{2}\right)$$

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$a|n\rangle = \sqrt{n}|n-1\rangle$$

$$E_n = \hbar\,\omega_c\left(n + \frac{1}{2}\right) \qquad n \in \mathbf{N}$$



Degeneracy of LL $= \hbar\omega_c\frac{mS}{2\pi\hbar^2} = \frac{BS}{(h/e)} = \frac{\phi}{\phi_0} = N_\phi$

Integer Quantum Hall effect

Varying B ⟹ varying the filling of LL $\nu = \frac{N}{N_\phi} = \frac{n_c}{n_\phi} = \frac{h n_c}{eB}$

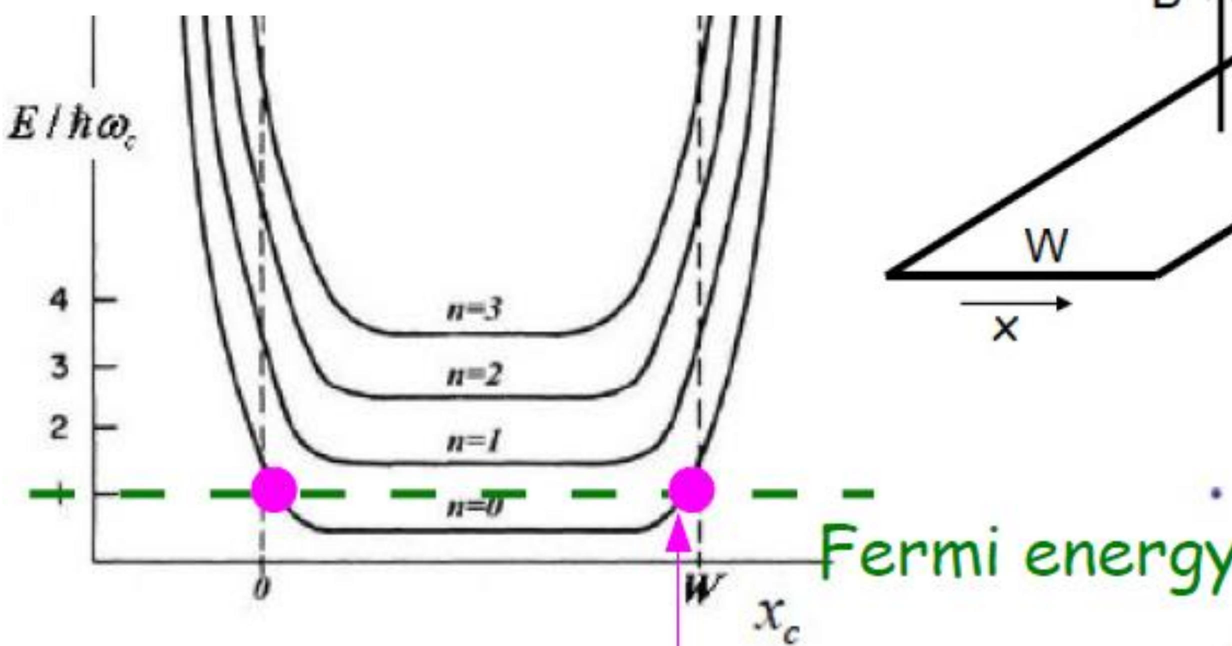
If LL is filled: fill a higher LL ⟹ R_{Hall} changes the plateau

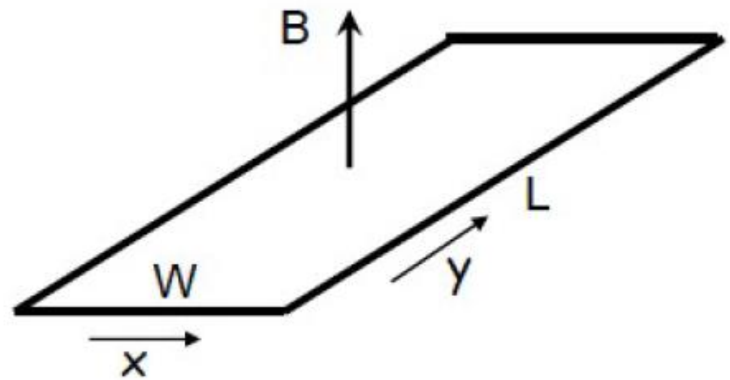
System is of a finite size: Fermi level intercept LL at states appearing at the edges

⇓

conducting edge states

Halperin 1982

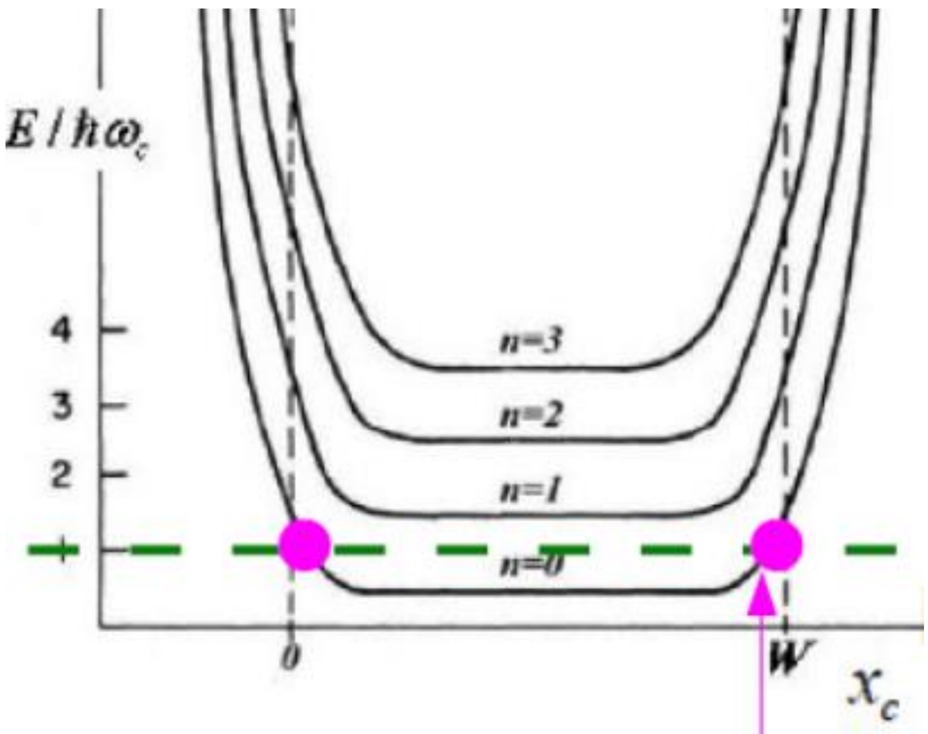


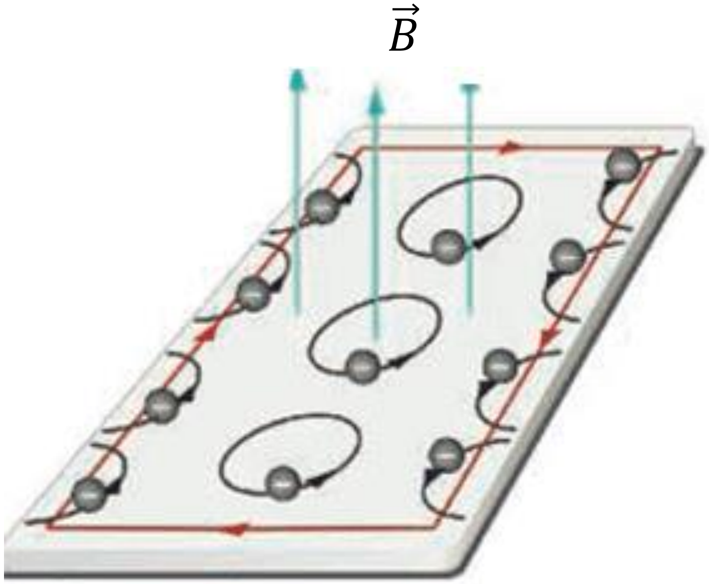


conducting edge states

- IQHE: edge states

σ_{xy}=(e^2/h) n avec n : number of edge states



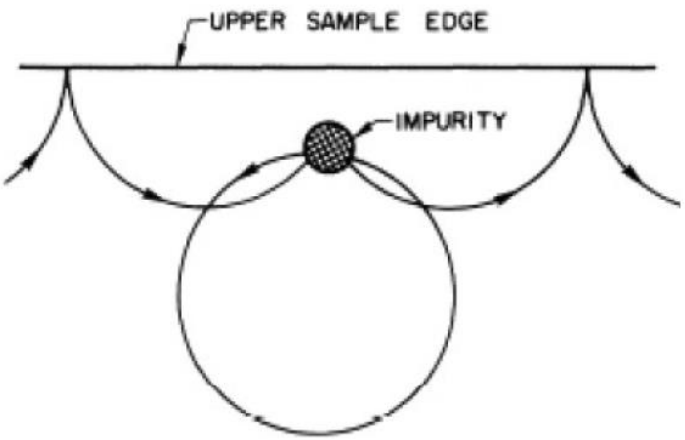


IQHE importance of disorder

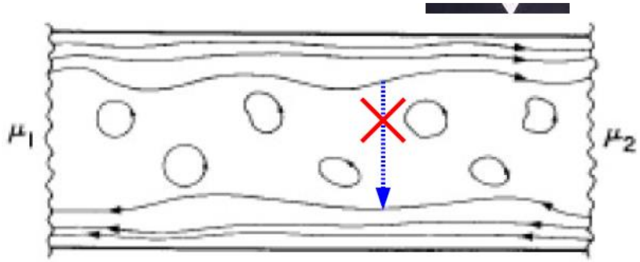
$$\sigma_{xy} = (e^2/h)\, n$$

σ_{xy} not sensitive to

- disorder
- to the sample geometry



M. Büttiker, Phys. Rev. B **38**, 9375 (1988)



Topological character of σ_{xy}

n: Chern number (topological invariant)

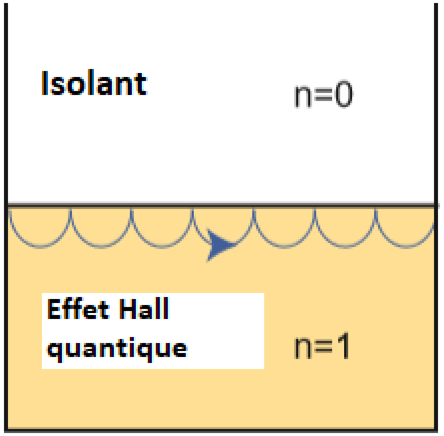
IQHE is topological

Definition 1: in an ordered topological phase, the response functions are expressed in terms of topological invariants

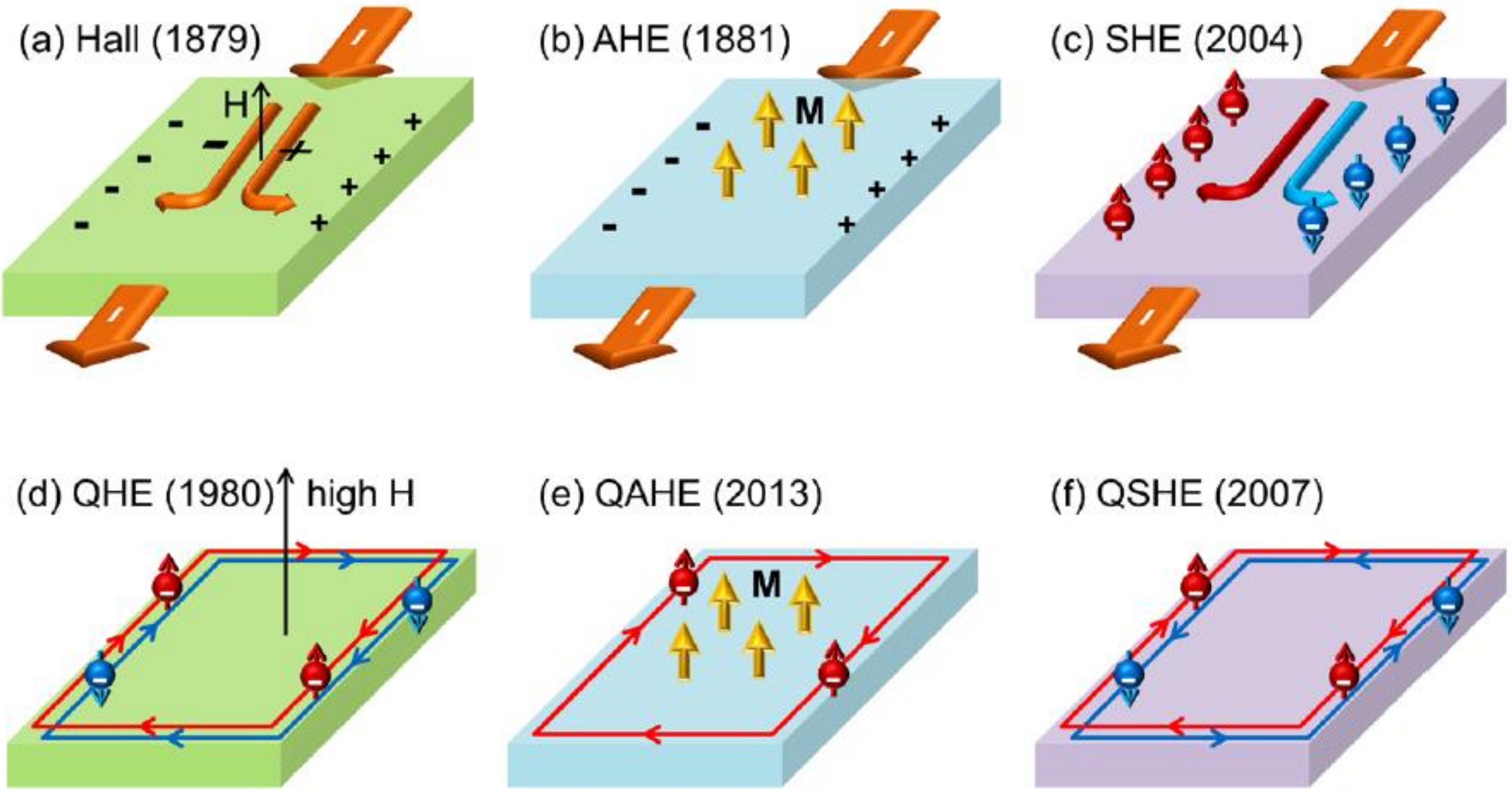
What is a topological invariant?

topological invariant = quantity which does not change under continuous deformations (genus in Math)

Definition 2: a topological phase is insulating but has surface/edge metallic states if it is close to a trivial (ordinary) phase

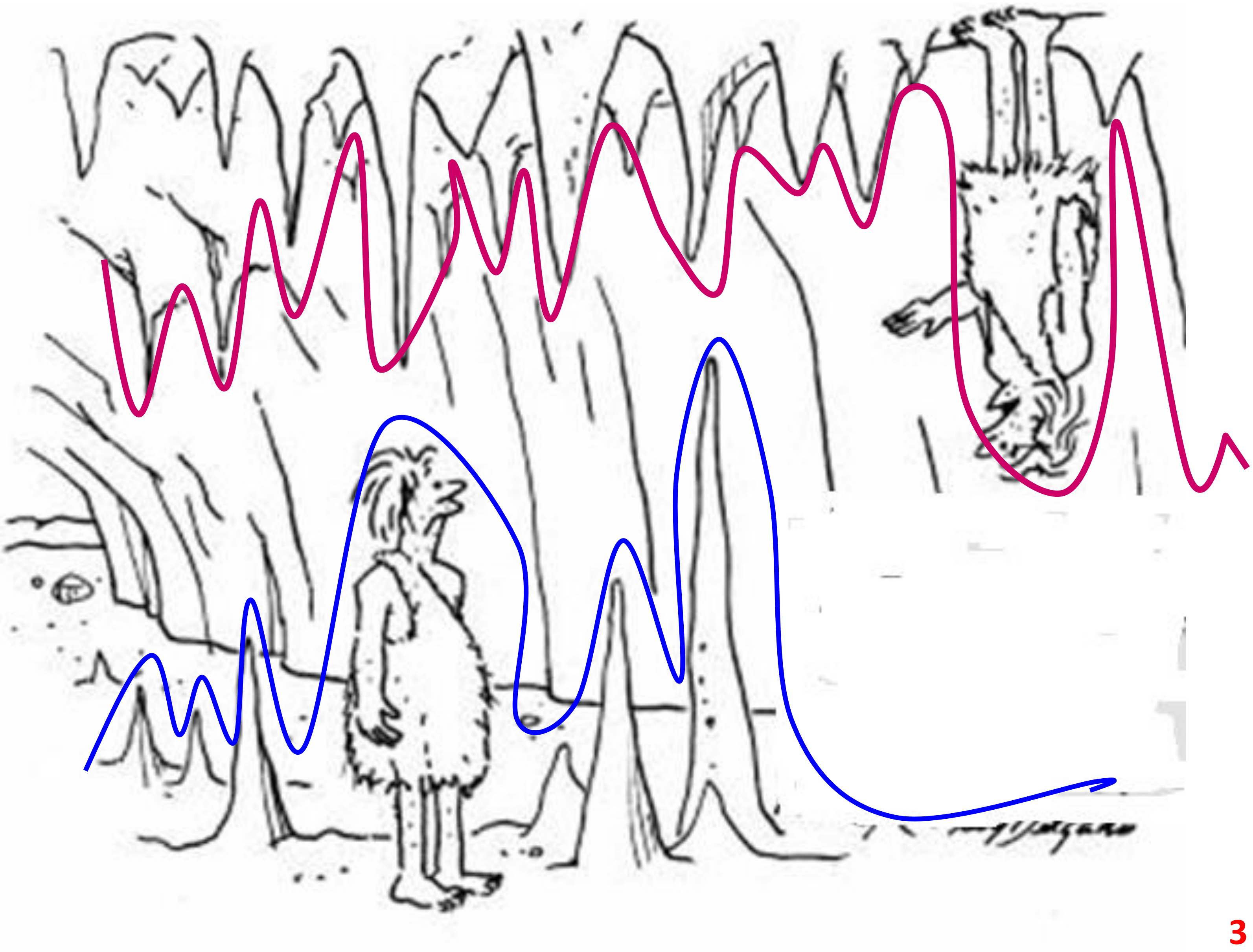


Review



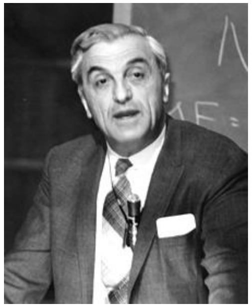
Chang and Li, J. Phys.: Condens. Matter **28** (2016) 123002

Topological band structure

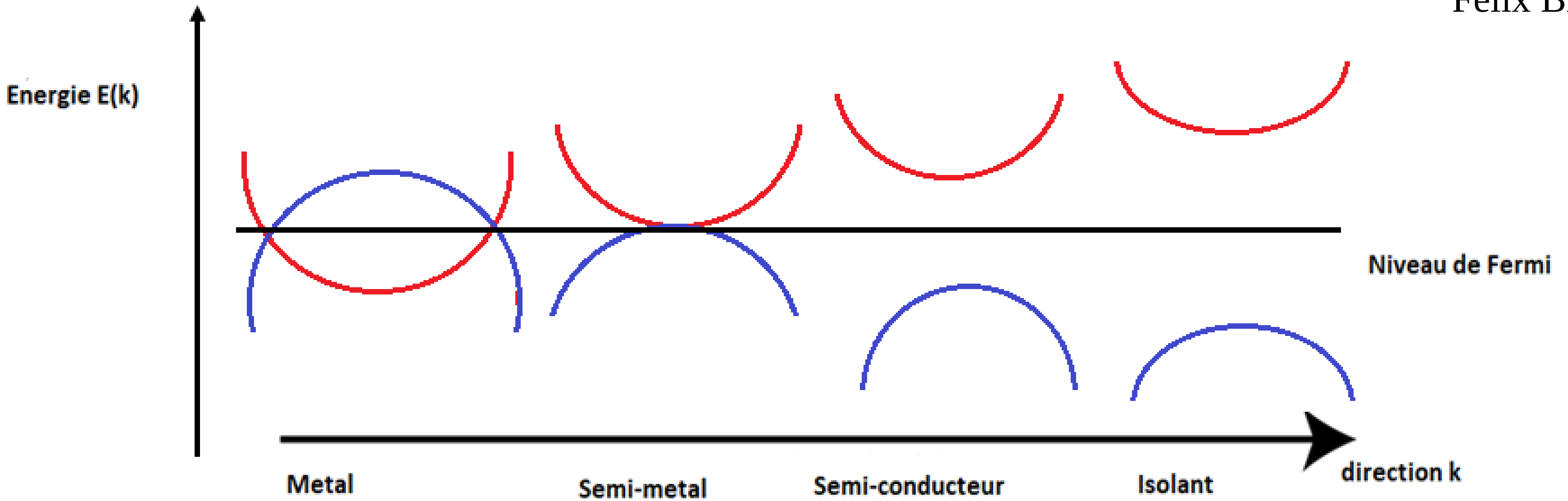


" I can never remember which is a stalagmite and which is a stalactite. "

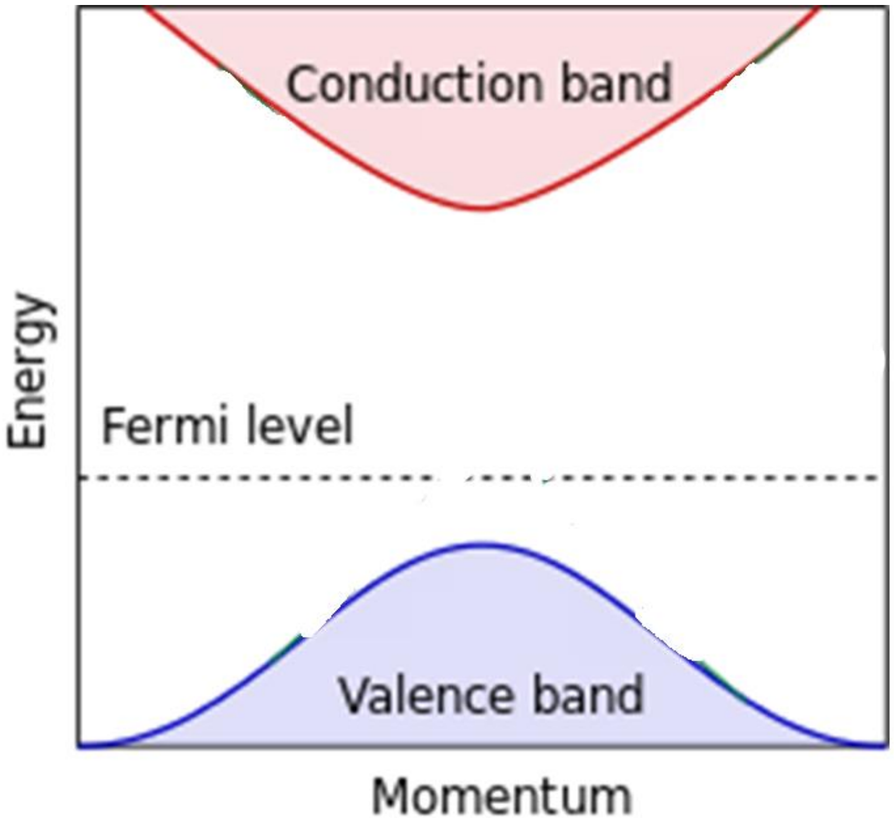
Bloch theorem (1950) → **Electronic band structure**



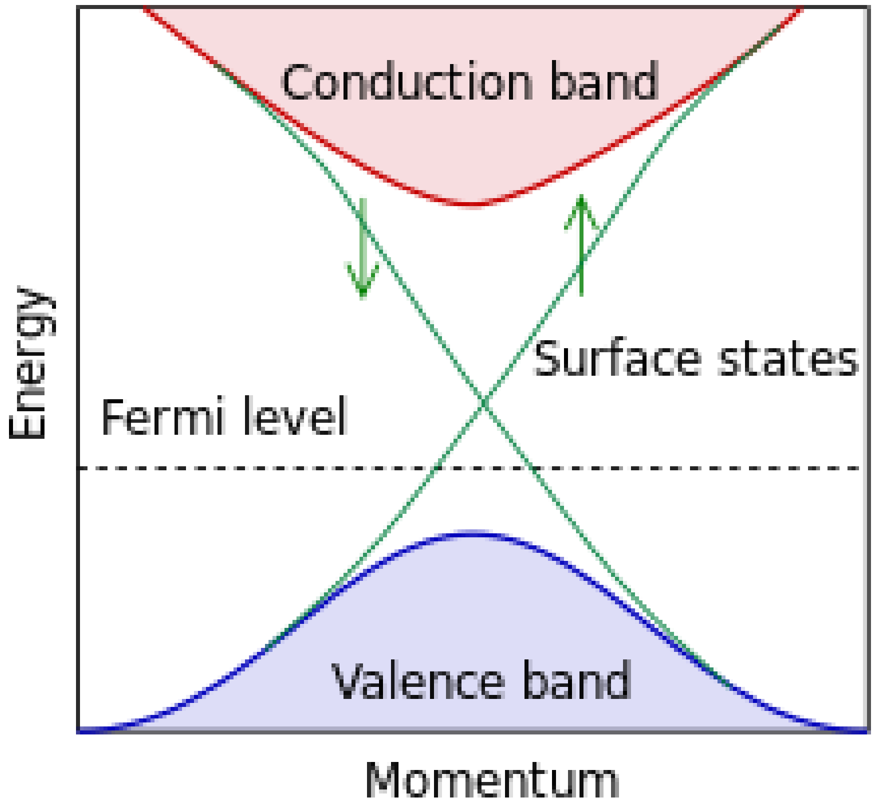
Felix Bloch



There are not only metal, semi-conductors and insulators in life…



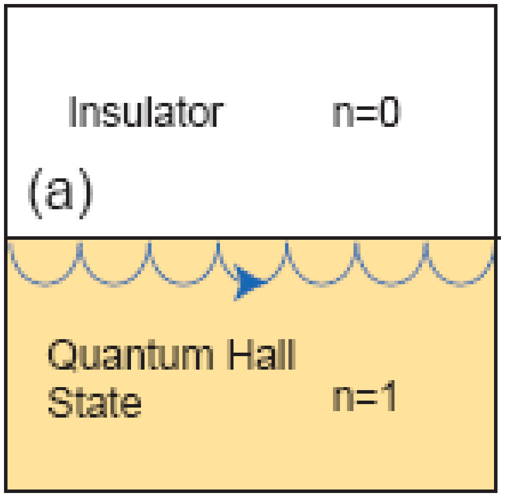
***Trivia*l band insulator**



Topological insulator (2005)

5

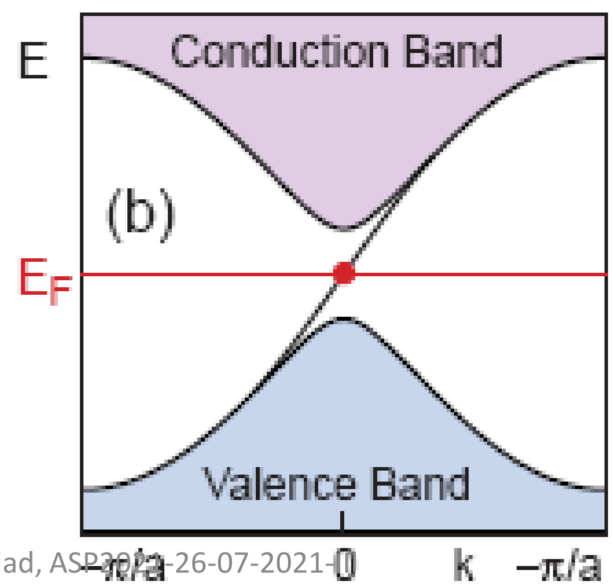
Origin of edge states



At the interface
The topological invariant n passe changes from 0 à 1

Gap should close at some points

Appearance of edge states inside the gap



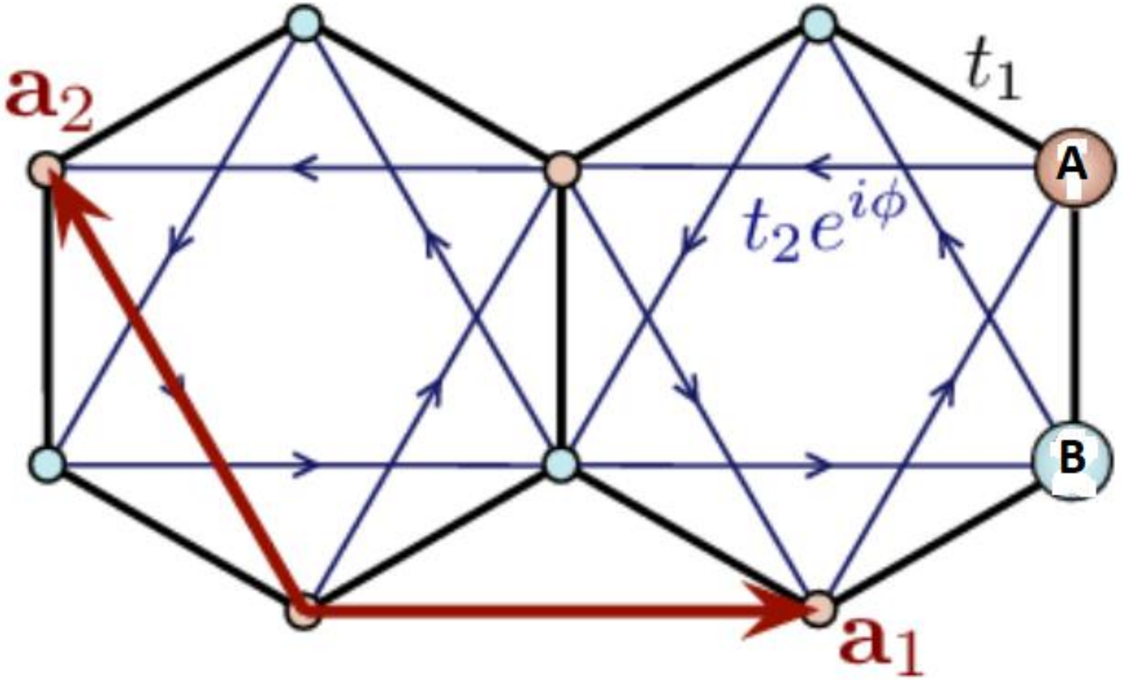
Topological insulator

Key ingredients:

F. D. M. Haldane

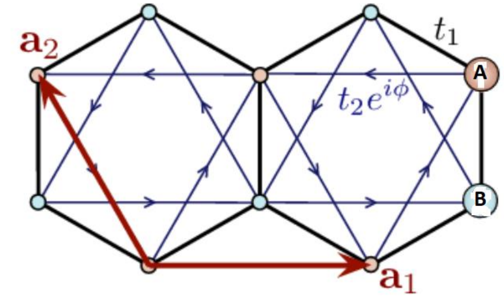
Haldane, PRL 61,2015 (1988)

- Honeycomb lattice at zero magnetic filed with…
- locally broken Time Reversal Symmetry (TRS) by local magnetic fluxes
- Complex second neighbor hopping parameters



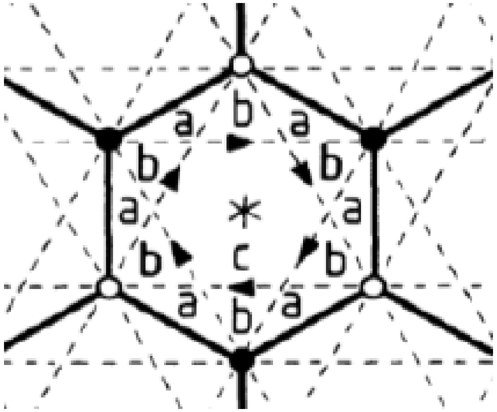
Haldane model

Haldane hamiltonian



$$H_{Haldane}(\vec{k}) = H_{grahene}(\vec{k}) + \left[M - 2t_2 \sin\phi \sum_{i=1}^{3} (\sin \vec{k}.\vec{b}_i)\right]\sigma_z$$

M: mass term breaks inversion symmetry



Φ: breaks Time Reversal Symmetry (TRS): $\phi \rightarrow \vec{A} \rightarrow \vec{B}_{local}$

Total flux: $\oiint \vec{B}.\,d\vec{s} = 0$

➡ $\sigma_{xy} = (e^2/h)\, C$, where C Chern number

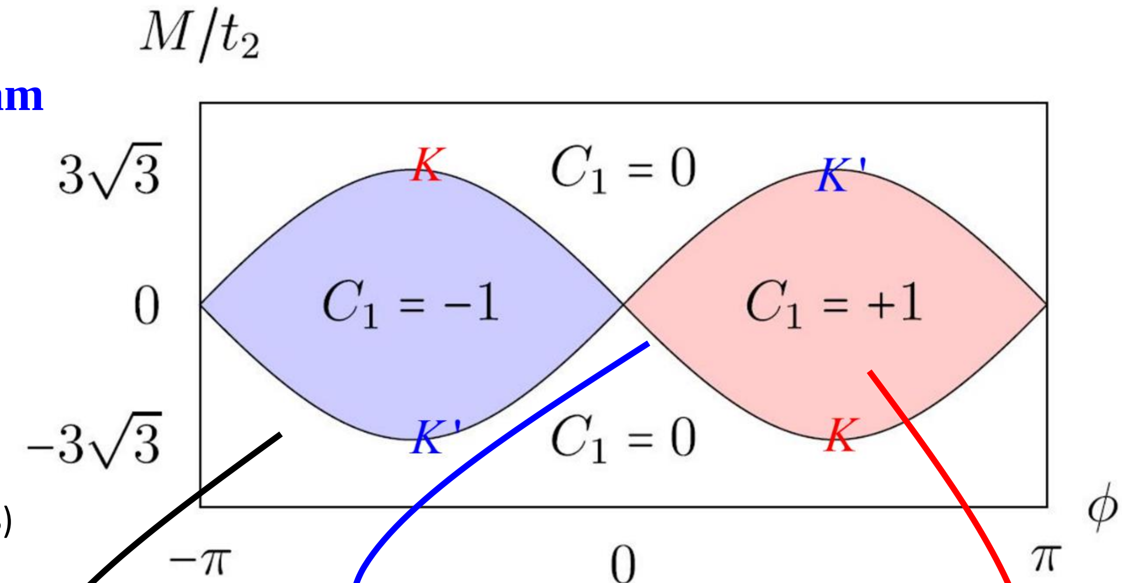
$$C = \frac{1}{2}[sign(m_+) - sign(m_-)]$$

Quantum Anomalous Hall effect (QAH)

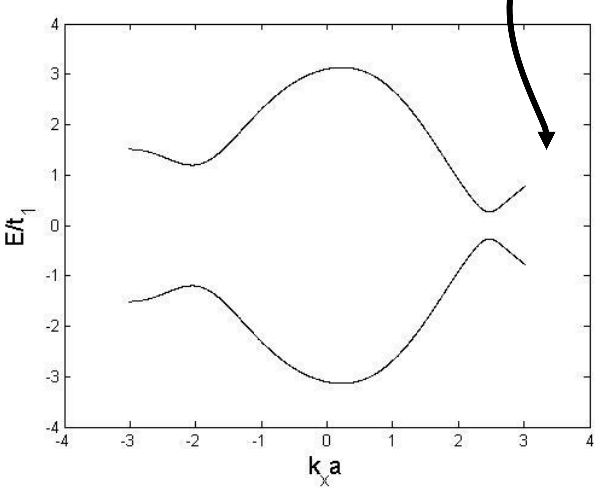
$$m_\pm = M \mp 3t_2\sqrt{3}\sin\phi$$

Haldane model

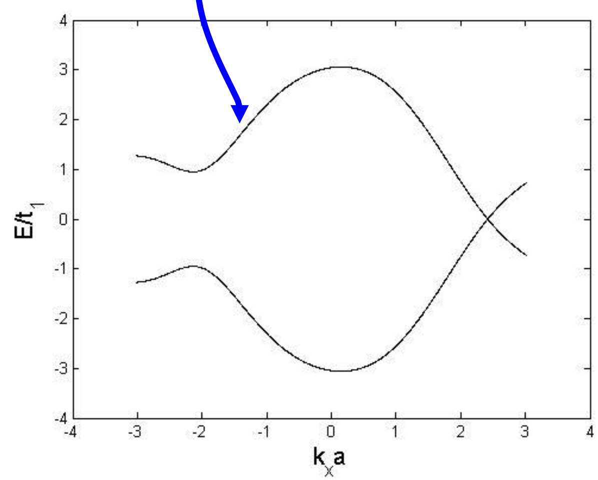
Haldane phase diagram



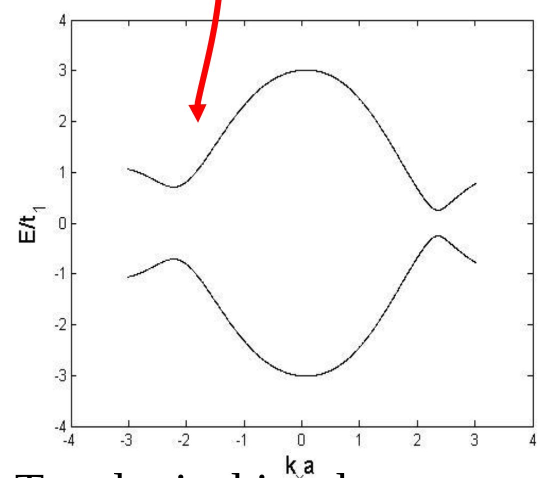
Haldane, PRL 61,2015 (1988)



Band insulator(trivial)



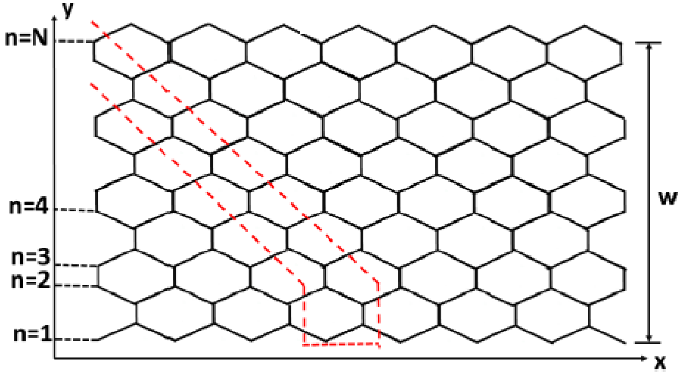
Semi-metal

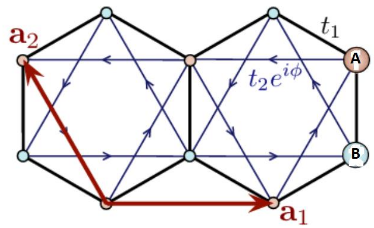


Topological insulator

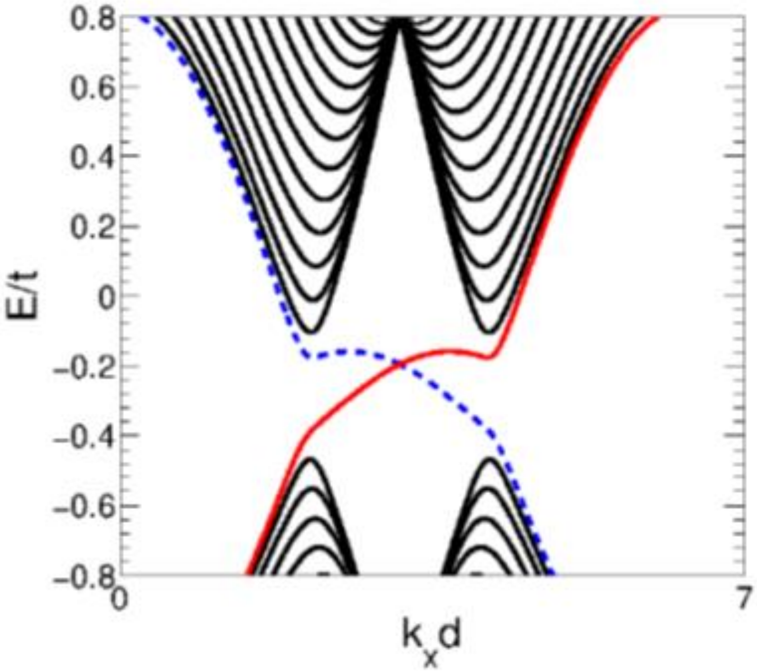
Haldane model

Zigzag graphene nanoribbon

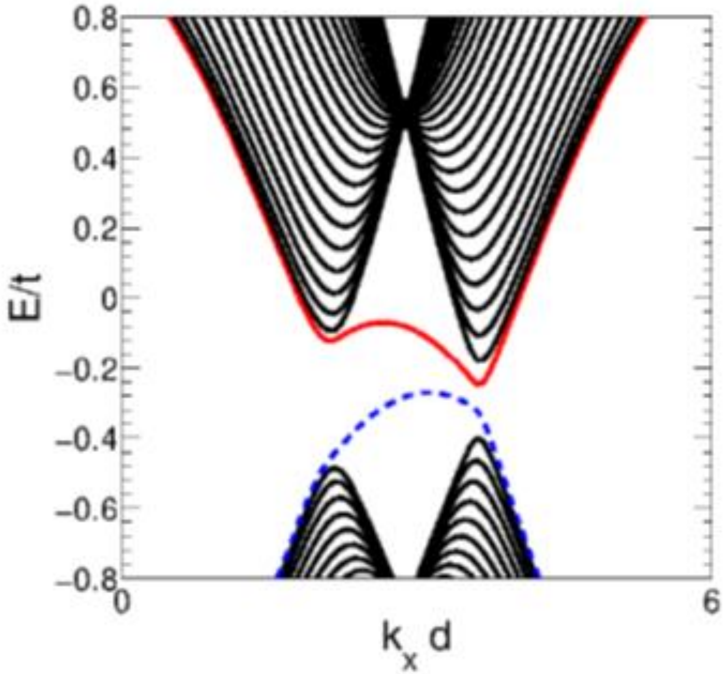




Topological phases **Edge states**



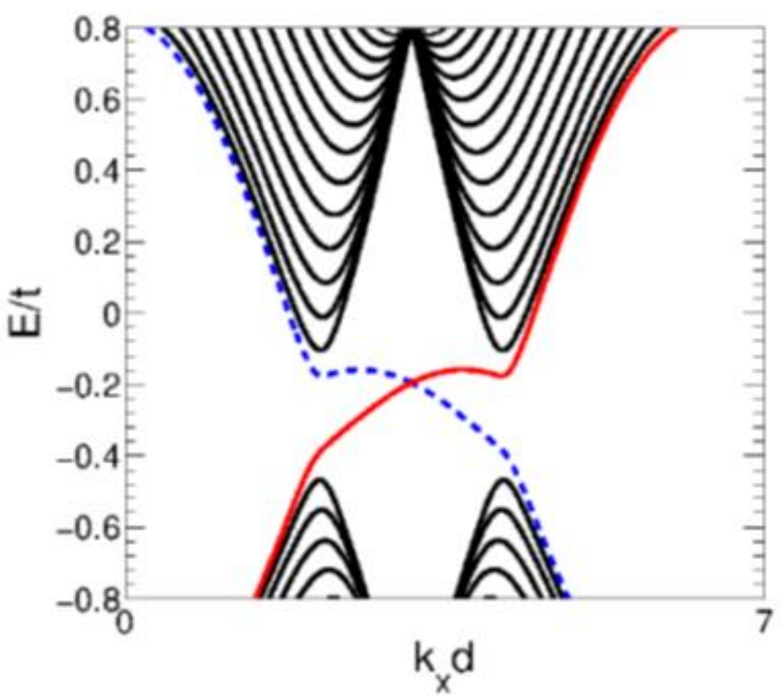
Trivial phases no **edge states**



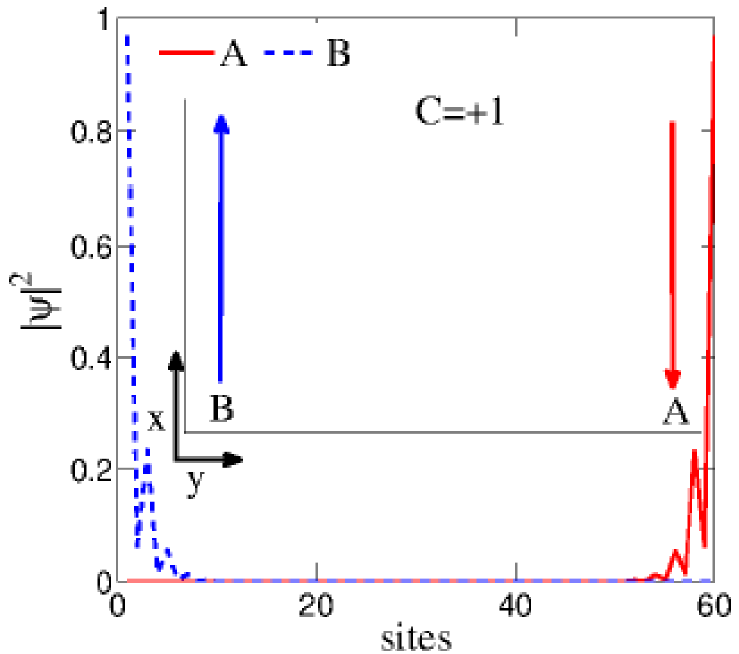
Haldane model

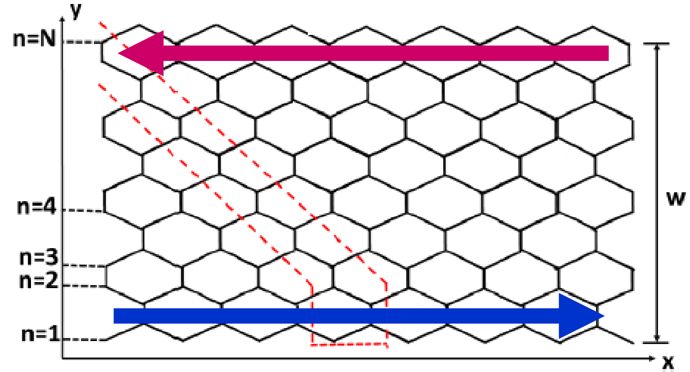
Zigzag graphene nanoribbon

Edge states



Edge currents

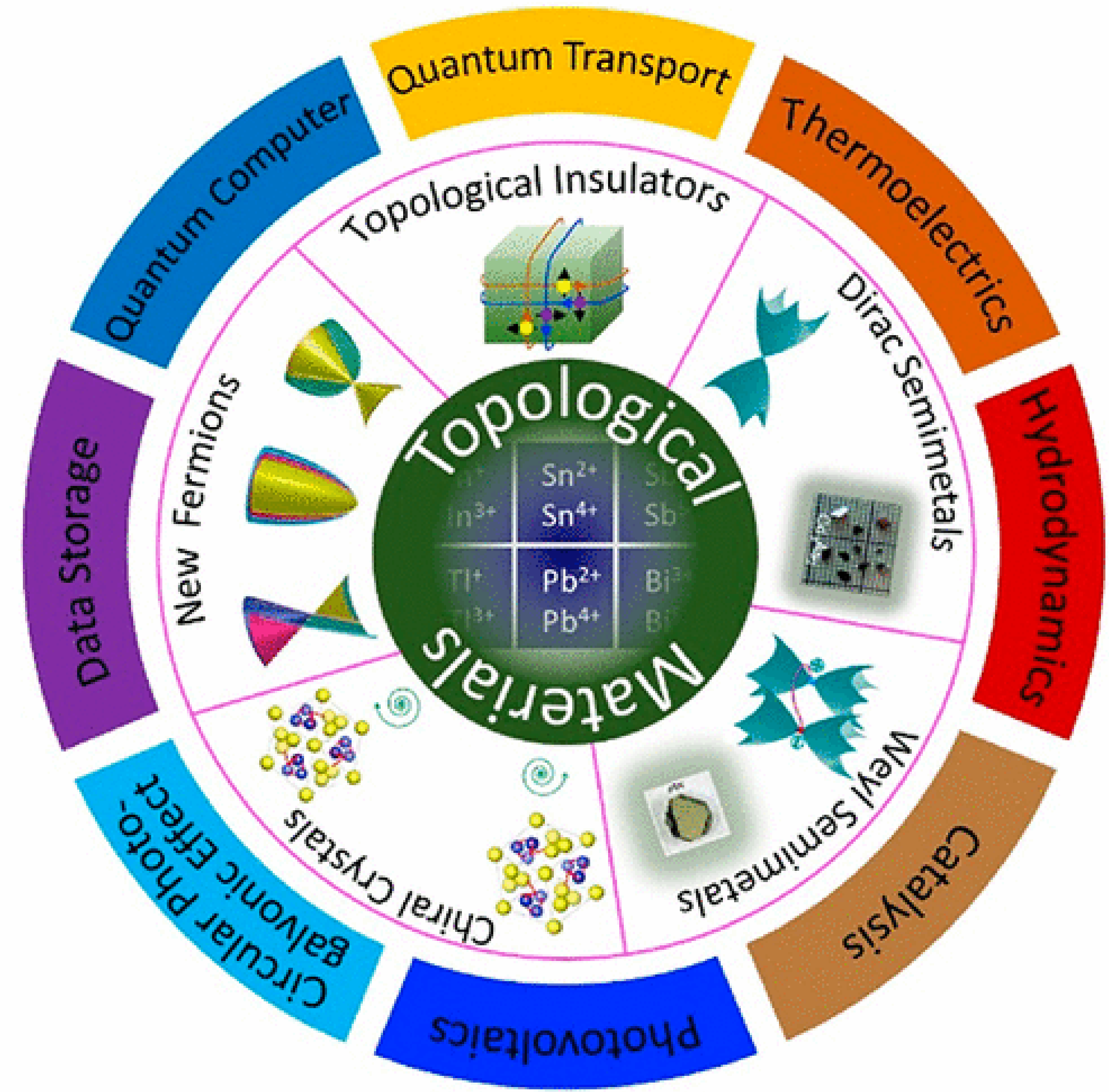




Lecture III: Introduction to topological materials: Summary

TED Ed

LESSONS WORTH SHARING



Topological Quantum Materials from the Viewpoint of Chemistry

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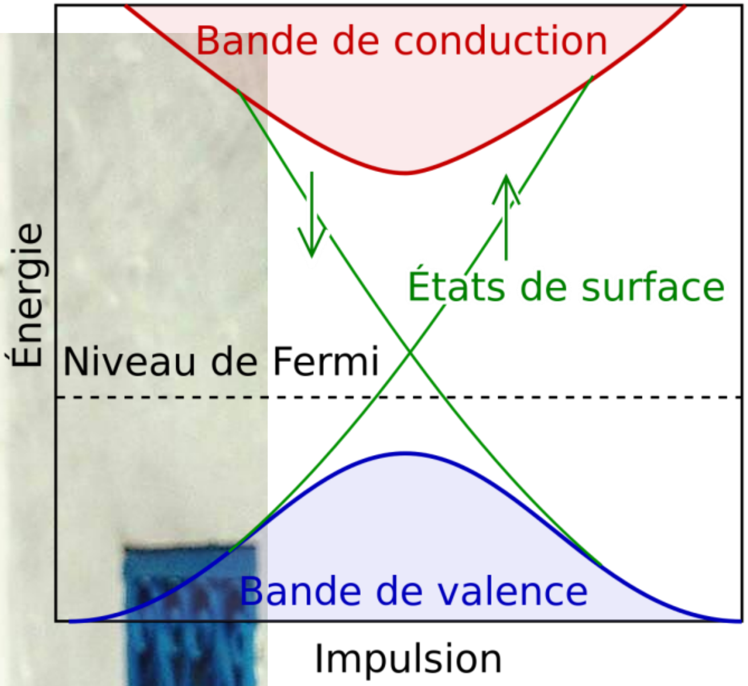
Topological band structure



Thank you for your attention

Any questions?