

6th edition of the biennial African School of
Fundamental Physics and Applications.

Optics and Photonics

Dr. Rim Cherif
University of Carthage, Tunisia

July 19-30, 2021



What is the difference between Optics and Photonics?

- Optics is a general area of physics manipulating light through optical grade materials made from glass, plastic, or metallic/dielectric coatings.



Optical Lenses



Optical Mirrors



Windows and Diffusers

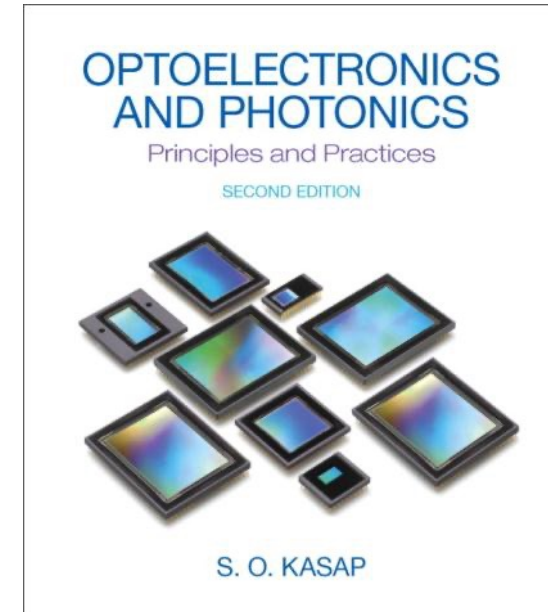
- Photonics is primarily concerned with the generation, "processing", and sensing of light.



Outline

Part 1. Wave nature of light

Part 2. Applications



ISBN-10: 0133081753
Second Edition Version 1.0337
[6 February 2015]



What is Light?

1. Ray

The basic element in geometrical **optics** is the light ray, a **hypothetical construct** that indicates the direction of the propagation of light at any point in space.

2. Wave

A wave is an undulation that propagates from one point to another, and as it travels it carries the energy of the electromagnetic spectrum.

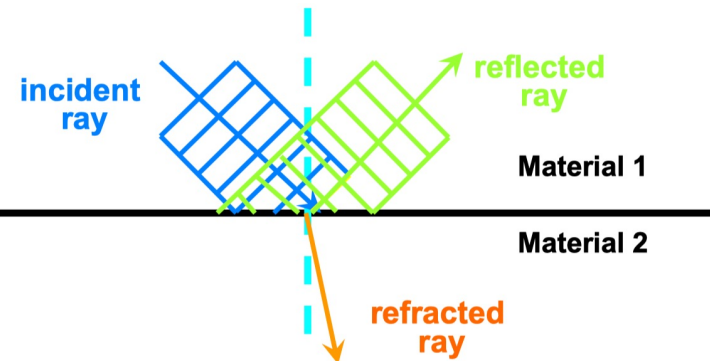
3. Particle

A *photon* is the smallest possible particle of electromagnetic energy.

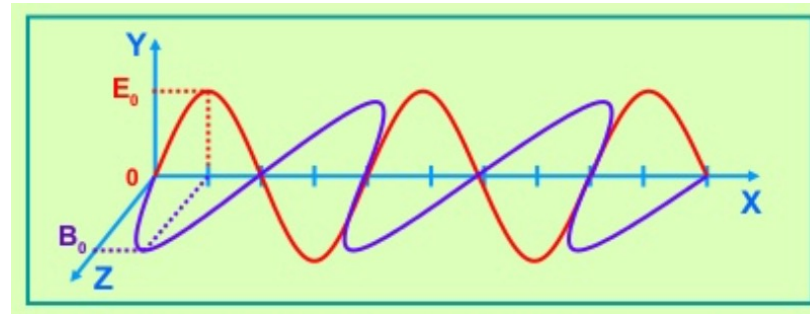


How to study light propagation?

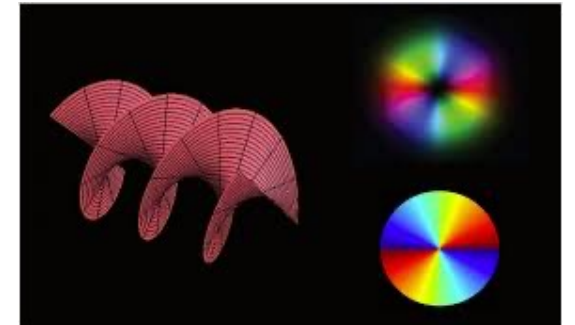
Geometric Optics



Wave Optics



Quantum Optics



Light is an electromagnetic wave

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

E_x = Electric field along x at position z at time t

k = **Propagation constant** = $2\pi/\lambda$

λ = Wavelength

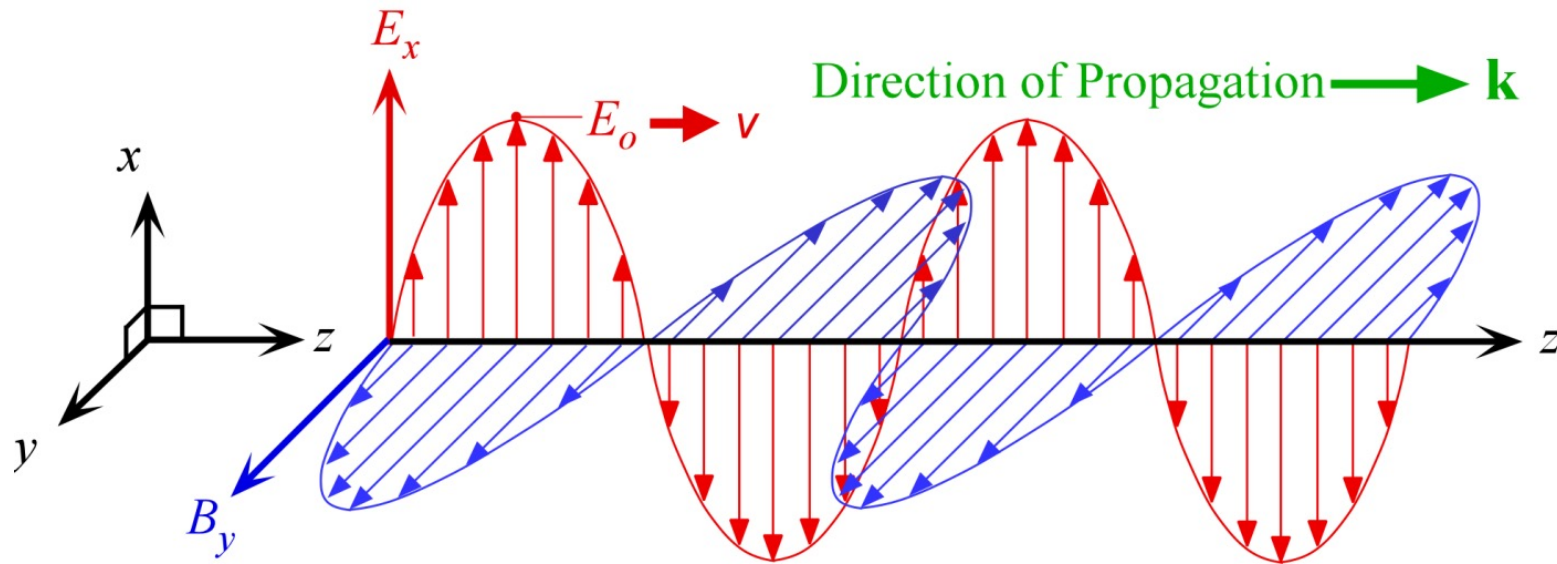
ω = Angular frequency = $2\pi\nu$ (ν = frequency)

E_o = Amplitude of the wave

ϕ_o = Phase constant; at $t = 0$ and $z = 0$, E_x may or may not necessarily be zero depending on the choice of origin.

$(\omega t - kz + \phi_o) = \phi$ = **Phase of the wave**

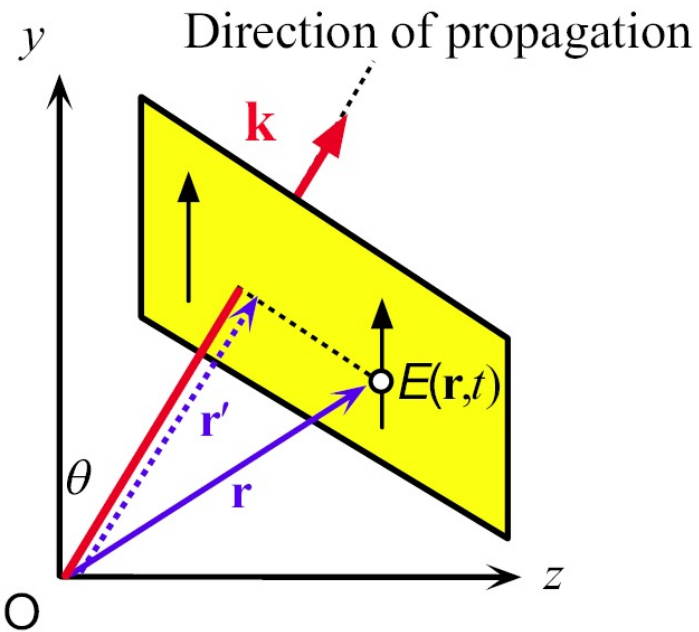
This is a **monochromatic plane wave** of *infinite extent* traveling in the positive z direction.



An electromagnetic wave is a traveling wave that has time-varying electric and magnetic fields that are perpendicular to each other and the direction of propagation z .



Wave Vector or Propagation Vector



Direction of propagation is indicated with a vector \mathbf{k} , called the **wave vector**, whose magnitude is the *propagation constant*, $k = 2\pi/\lambda$. \mathbf{k} is *perpendicular* to constant phase planes.

When the electromagnetic (EM) wave is propagating along some arbitrary direction \mathbf{k} , then the electric field $E(\mathbf{r}, t)$ at a point \mathbf{r} on a plane perpendicular to \mathbf{k} is

$$E(\mathbf{r}, t) = E_o \cos(\omega t - \mathbf{k} \cdot \mathbf{r} + \phi_o)$$

The equations of Optics are Maxwell's Equations

(first written down in 1864)

$$\begin{aligned}\vec{\nabla} \cdot \vec{E} &= \rho / \epsilon & \vec{\nabla} \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \vec{\nabla} \cdot \vec{B} &= 0 & \vec{\nabla} \times \vec{B} &= \mu \vec{J} + \mu \epsilon \frac{\partial \vec{E}}{\partial t}\end{aligned}$$



James Clerk Maxwell
(1831-1879)

where E is the electric field, B is the magnetic field, ρ is the charge density, J is the current density, ϵ is the permittivity, and μ is the permeability of the medium.

Often, in optics, there are no free charges or currents, so mostly we can assume that $\rho = 0$ and $J = 0$.



A vector wave equation for the electric field

A light wave can propagate in any direction in space. So we must allow the space derivative to be 3D:

in one dimension:

$$\frac{\partial^2 E}{\partial x^2} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0$$

in three dimensions:

$$\vec{\nabla}^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

Note the vector symbol over the E .

Questionnaire:

1. Prove this equation ?? → Need to know Div, grad, curl and all that
2. If E and B satisfy the same equation; does that mean they're equal?
3. How E and B fields are aligned always?



Derivation of the 3D wave equation

Steps involved:

- 1) Take cross product of “Del” with “Curl of E”.
- 2) Change the differentiation on the RHS and substitute the value of “Curl of B” on RHS.
- 3) Substitute LHS to be:

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$$

For any function at all,

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F} = \vec{\nabla} \times [\vec{\nabla} \times \vec{F}]$$

See: <https://www.youtube.com/watch?v=P4edqL1r4DQ>

Here assuming zero charge density,

Permeability and permittivity to be independent of time

We're left with the WAVE EQUATION !!



1. Expanding the Laplacian, we find:

$$\frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

2.

First: take $\vec{\nabla} \times$ of this one: $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = \vec{\nabla} \times \left[-\frac{\partial \vec{B}}{\partial t}\right]$$

Next: change the order of differentiation on the right-hand side:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} [\vec{\nabla} \times \vec{B}]$$

3.

But: $\vec{\nabla} \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t}$

Next: substituting for $\vec{\nabla} \times \vec{B}$, we have:

$$\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} [\vec{\nabla} \times \vec{B}] \Rightarrow \vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\frac{\partial}{\partial t} \left[\mu\epsilon \frac{\partial \vec{E}}{\partial t}\right]$$

Or: $\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

assuming that μ and ϵ are both independent of time.

4.

We are up to here: $\vec{\nabla} \times [\vec{\nabla} \times \vec{E}] = -\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$

Now, it can be shown that this: $\vec{\nabla} \times [\vec{\nabla} \times \vec{E}]$

is the same as this: $\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E}$ For any function at all,
 $\vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F} = \vec{\nabla} \times [\vec{\nabla} \times \vec{F}]$

See: <https://www.youtube.com/watch?v=P4edqL1r4DQ>

If we now assume zero charge density: $\rho = 0$, then

$$\vec{\nabla} \cdot \vec{E} = 0$$

and we're left with the Wave Equation! $\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$



EM propagation in homogeneous materials

- The speed of an EM wave in free space is given by: $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

ϵ_0 = permittivity of free space, μ_0 = magnetic permeability of free space

- How are Maxwell's equations in matter different?**
 - $\epsilon_0 \rightarrow \epsilon$, $\mu_0 \rightarrow \mu$
 - Generally increased by the presence of matter – especially ϵ*
 - => Speed of light in matter v related to the speed of light in vacuum c by:

$$v_{\text{medium}} = \frac{c}{n_{\text{medium}}}$$

What is n ?



Refractive Index

TABLE 1.1 Low-frequency (LF) relative permittivity $\epsilon_r(\text{LF})$ and refractive index n

Material	$\epsilon_r(\text{LF})$	$[\epsilon_r(\text{LF})]^{1/2}$	n (at λ)	Comment
Si	11.9	3.44	3.45 (at 2.15 μm)	Electronic bond polarization up to optical frequencies
Diamond	5.7	2.39	2.41 (at 590 nm)	Electronic bond polarization up to UV light
GaAs	13.1	3.62	3.30 (at 5 μm)	Ionic polarization contributes to $\epsilon_r(\text{LF})$
SiO ₂	3.84	2.00	1.46 (at 600 nm)	Ionic polarization contributes to $\epsilon_r(\text{LF})$
Water	80	8.9	1.33 (at 600 nm)	Dipolar polarization contributes to $\epsilon_r(\text{LF})$, which is large





n depends on the wavelength λ

Dispersion relation: $n = n(\lambda)$

The simplest electronic polarization gives

$$n^2 = 1 + \left(\frac{N_{\text{at}} Z e^2}{\epsilon_o m_e} \right) \left(\frac{\lambda_o}{2\pi c} \right)^2 \frac{\lambda^2}{\lambda^2 - \lambda_o^2}$$

N_{at} = Number of atoms per unit volume
 Z = Number of electrons in the atom (atomic number)

λ_o = A “resonant frequency”

Sellmeier Equation

$$n^2 = 1 + \frac{A_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3 \lambda^2}{\lambda^2 - \lambda_3^2}$$



n depends on the wavelength λ

TABLE 1.2 Sellmeier and Cauchy coefficients

Sellmeier	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO ₂ (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO ₂ -13.5%GeO ₂	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO ₂	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	–	0.1750	0.1060	–

Cauchy	Range of $h\nu$ (eV)	n_{-2} (eV ²)	n_0	n_2 (eV ⁻²)	n_4 (eV ⁻⁴)
Diamond	0.05–5.47	-1.07×10^{-5}	2.378	8.01×10^{-3}	1.04×10^{-4}
Silicon	0.002–1.08	-2.04×10^{-8}	3.4189	8.15×10^{-2}	1.25×10^{-2}
Germanium	0.002–0.75	-1.0×10^{-8}	4.003	2.2×10^{-1}	1.4×10^{-1}

Source: Sellmeier coefficients combined from various sources. Cauchy coefficients from D. Y. Smith *et al.*, *J. Phys. CM*, 13, 3883, 2001.





EXAMPLE 1.2.1 Sellmeier equation and diamond

Using the Sellmeier coefficients for diamond in Table 1.2, calculate its refractive index at 610 nm (red light) and compare with the experimental quoted value of 2.415 to three decimal places.

Solution

The Sellmeier dispersion relation for diamond is

$$n^2 = 1 + \frac{0.3306\lambda^2}{\lambda^2 - 175 \text{ nm}^2} + \frac{4.3356\lambda^2}{\lambda^2 - 106 \text{ nm}^2}$$
$$n^2 = 1 + \frac{0.3306(610 \text{ nm})^2}{(610 \text{ nm})^2 - (175 \text{ nm})^2} + \frac{4.3356(610 \text{ nm})^2}{(610 \text{ nm})^2 - (106 \text{ nm})^2} = 5.8308$$

So that

$$n = 2.4147$$

which is 2.415 to three decimal places and matches the experimental value.

Diffraction

Interference

Reflection

Refraction

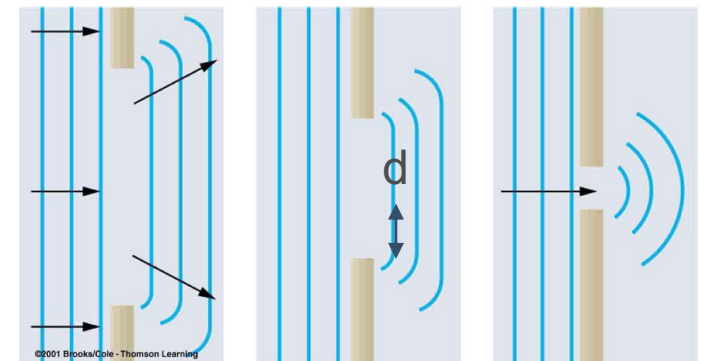
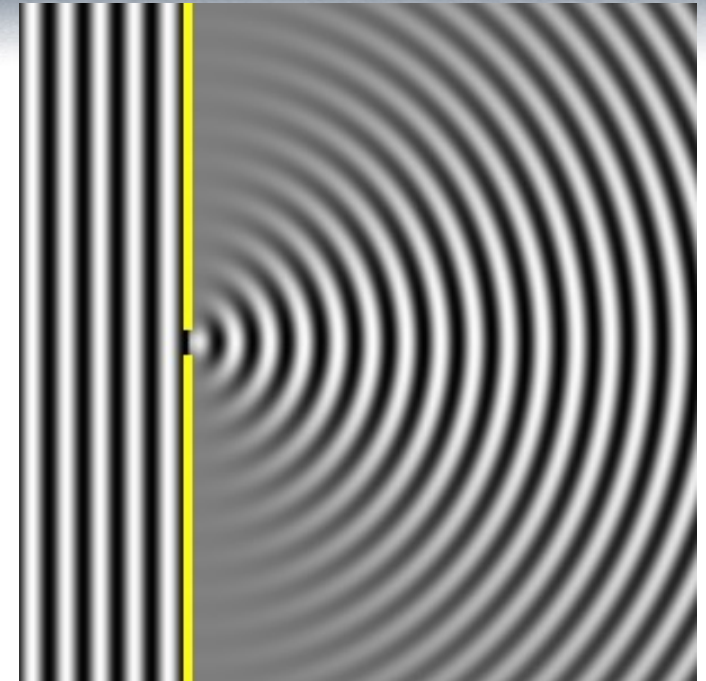
Dispersion

Polarization



Characteristic of waves: Diffraction

- **Diffraction** refers to various phenomena that occur when a wave encounters an obstacle or opening (slit).
- It is defined as the **bending of waves** around the corners of an obstacle or through an aperture into the region of geometrical shadow of the obstacle/aperture.
- Diffraction depends on **slit width**: the smaller the width, relative to wavelength, the more bending and diffraction.



Young's double slit interference



How the two waves will overlap?

1. Locate the bright spots
2. Locate the dark spots
3. Trace the path length difference:

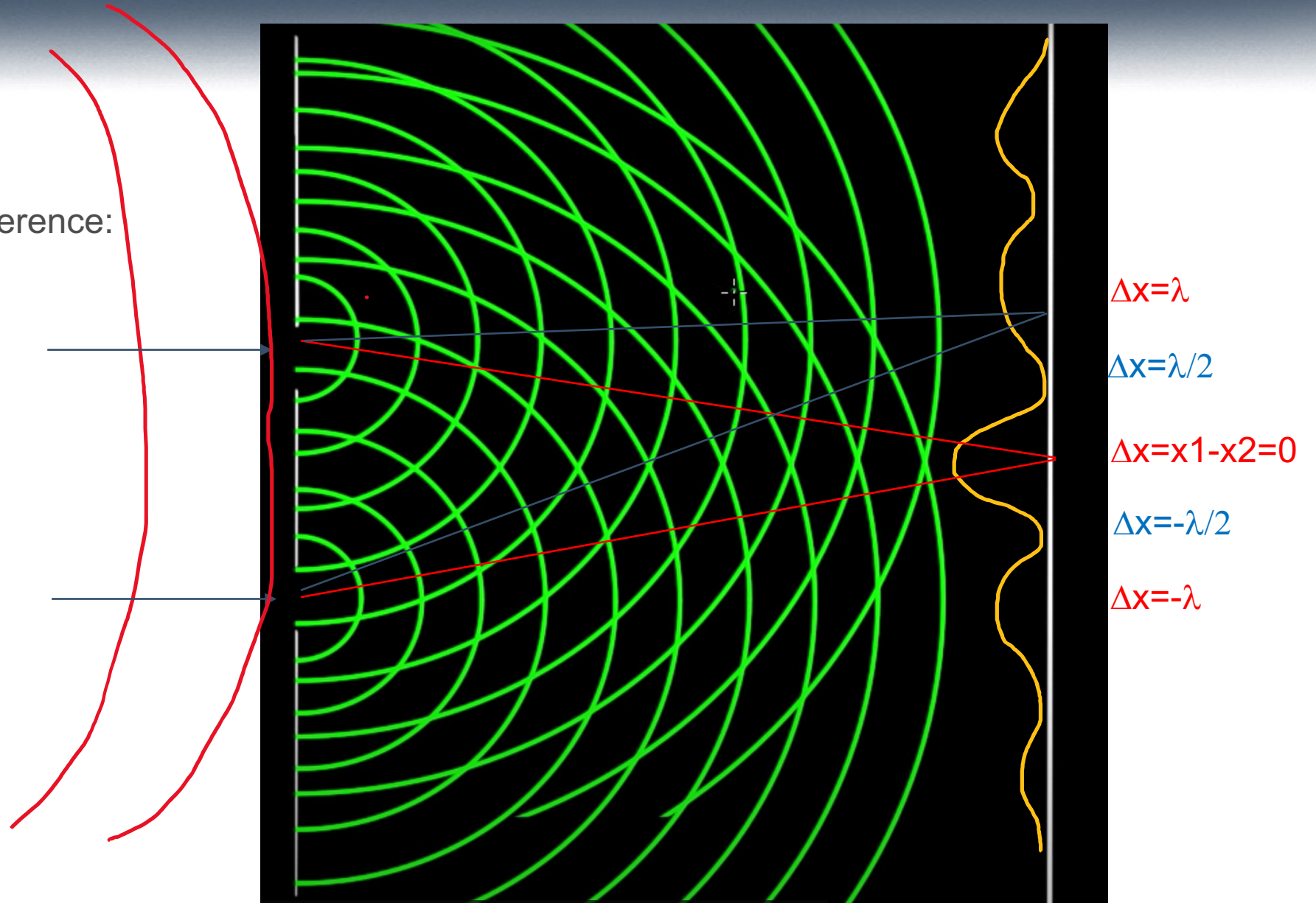
$$\Delta x = X_1 - X_2$$

- Constructive interference (bright fringes) at:

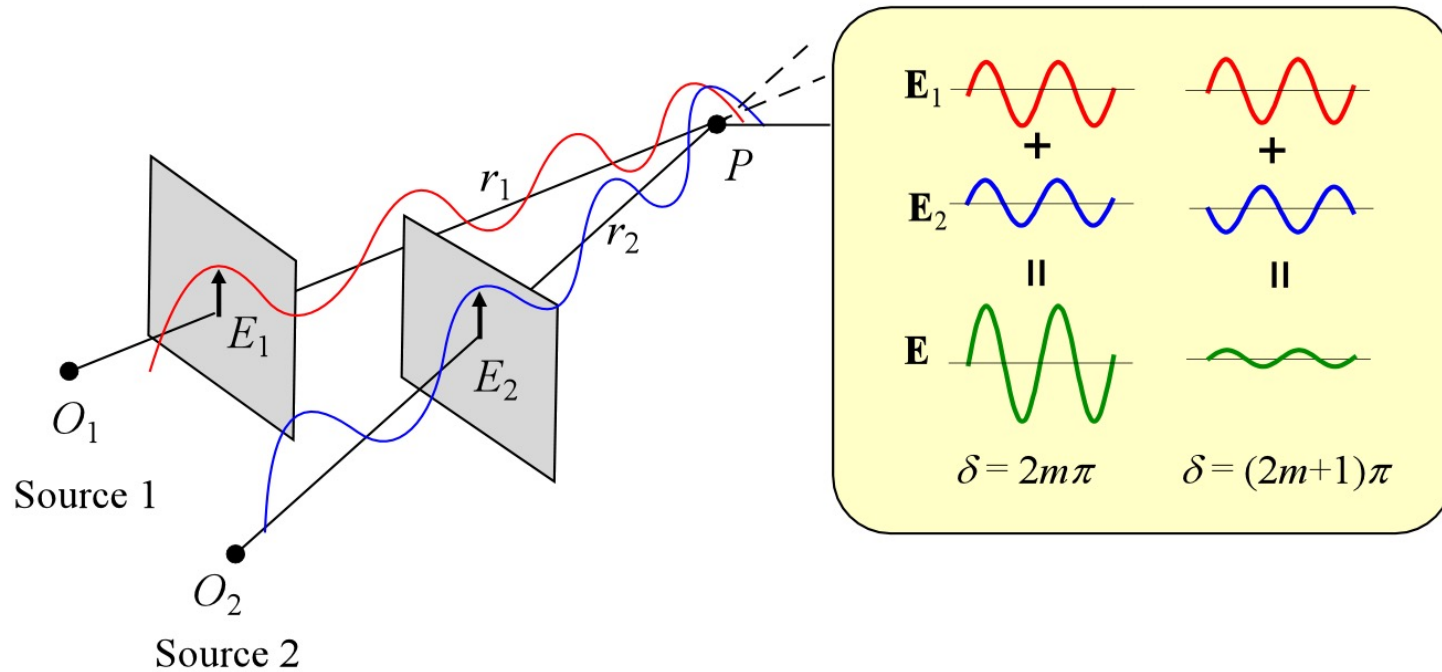
$$\Delta x = 0, \lambda, 2\lambda \dots$$

- Destructive interference (dark fringes) at:

$$\Delta x = \lambda/2, 3\lambda/2 \dots$$



Interference

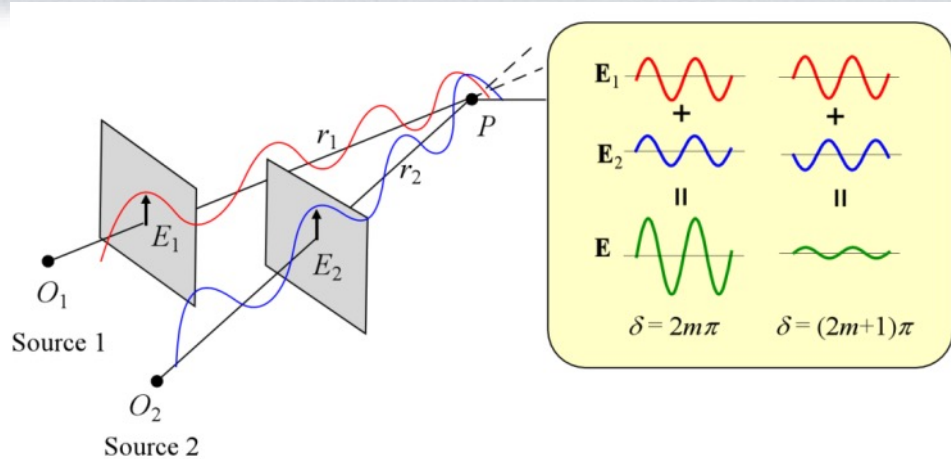


$$\mathbf{E}_1 = \mathbf{E}_{o1} \sin(\omega t - kr_1 - \phi_1) \quad \text{and} \quad \mathbf{E}_2 = \mathbf{E}_{o2} \sin(\omega t - kr_2 - \phi_2)$$

Interference results in $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

$$\overline{\mathbf{E} \cdot \mathbf{E}} = \overline{(\mathbf{E}_1 + \mathbf{E}_2) \cdot (\mathbf{E}_1 + \mathbf{E}_2)} = \overline{\mathbf{E}_1^2} + \overline{\mathbf{E}_2^2} + 2\overline{\mathbf{E}_1 \mathbf{E}_2}$$

Interference



Resultant intensity I is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\phi_2 - \phi_1)$$

Phase difference due to optical path difference

Constructive interference

$$I_{\max} = I_1 + I_2 + 2(I_1 I_2)^{1/2} \quad \text{and}$$

Destructive interference

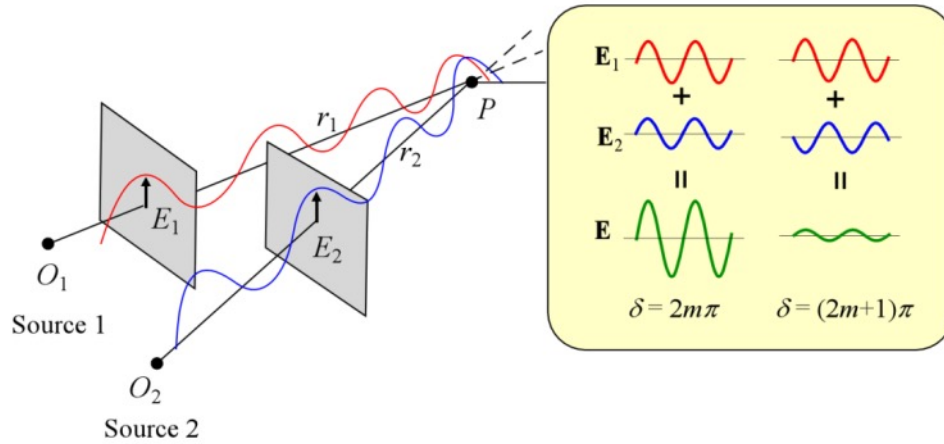
$$I_{\min} = I_1 + I_2 - 2(I_1 I_2)^{1/2}$$

If the interfering beams have equal irradiances, then

$$I_{\max} = 4I_1$$

$$I_{\min} = 0$$

Interference between coherent waves



Resultant intensity I is

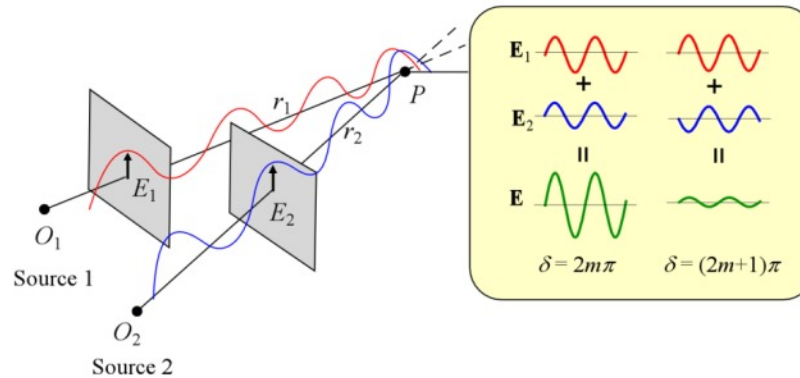
$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\phi_2 - \phi_1)$$

Interference between **incoherent** waves

$$I = I_1 + I_2$$

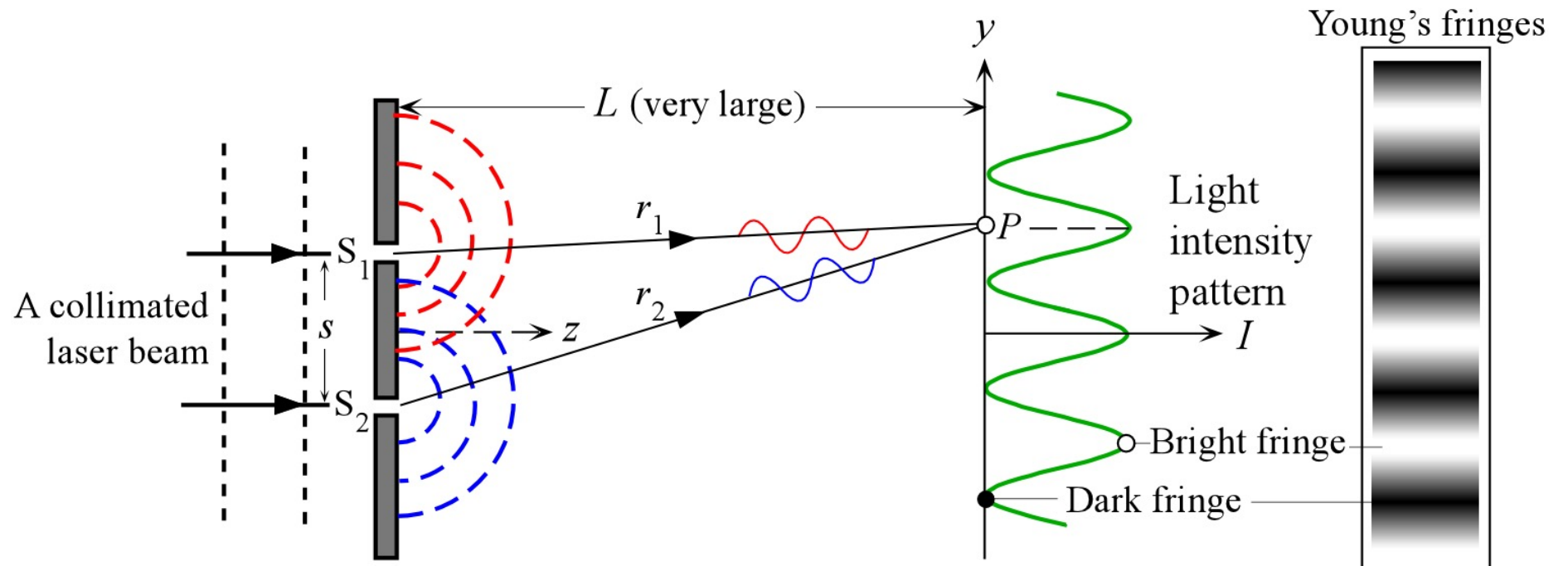
Interference between coherent waves



Resultant intensity I is

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \delta$$

$$\delta = k(r_2 - r_1) + (\phi_2 - \phi_1)$$



Snell's Law or Descartes's Law?



Willebrord Snellius (Willebrord Snel van Royen, 1580–1626) was a Dutch astronomer and a mathematician, who was a professor at the University of Leiden. He discovered his law of refraction in 1621 which was published by René Descartes in France 1637; it is not known whether Descartes knew of Snell's law or formulated it independently. (*Courtesy of AIP Emilio Segre Visual Archives, Brittle Books Collection.*)



René Descartes (1596–1650) was a French philosopher who was also involved with mathematics and sciences. He has been called the “Father of Modern Philosophy.” Descartes was responsible for the development of Cartesian coordinates and analytical geometry. He also made significant contributions to optics, including reflection and refraction. (*Courtesy of Georgios Kollidas/Shutterstock.com.*)



Fermat's principle of least time

Fermat's principle of least time in simple terms states that:

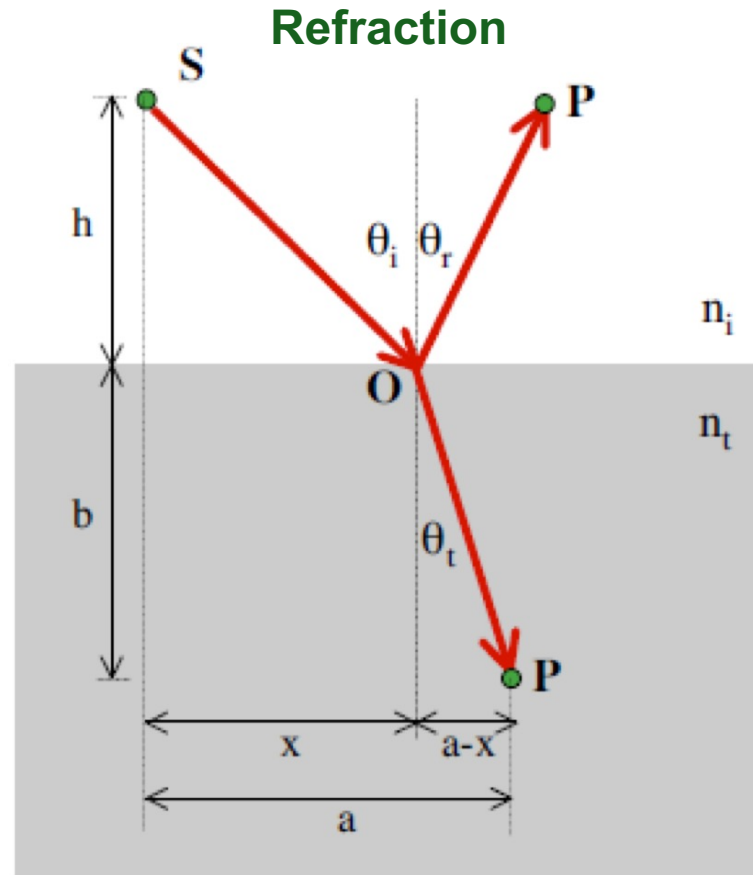
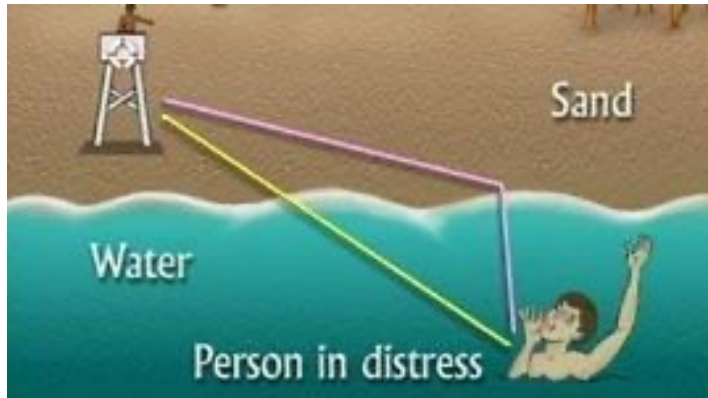
When light travels from one point to another it takes a path that has the shortest time.



Pierre de Fermat (1601–1665) was a French mathematician who made many significant contributions to modern calculus, number theory, analytical geometry, and probability. *(Courtesy of Mary Evans Picture Library/Alamy.)*

Fermat's Principle and the laws of Reflection and Refraction

The path actually taken by light in going from point S to a point P is the shortest optical path length (OPL)



$$OPL = n_i \cdot \overline{SO} + n_t \cdot \overline{OP}$$

$$= n_i \cdot \sqrt{h^2 + x^2} + n_t \cdot \sqrt{b^2 + (a-x)^2}$$

$$\frac{dOPL}{dx} = 0 \text{ to minimize } OPL$$

$$n_i \cdot \frac{x}{\sqrt{h^2 + x^2}} + n_t \cdot \frac{-(a-x)}{\sqrt{b^2 + (a-x)^2}} = 0$$



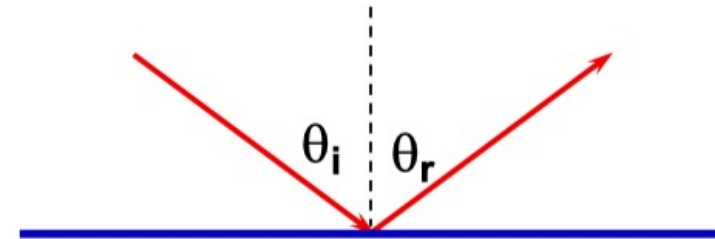
$$\frac{n_i}{n_t} = \frac{\sin(\theta_t)}{\sin(\theta_i)}$$

Snell's law

Reflection

- Angle of incidence = angle of reflection
both angles are measured from the normal

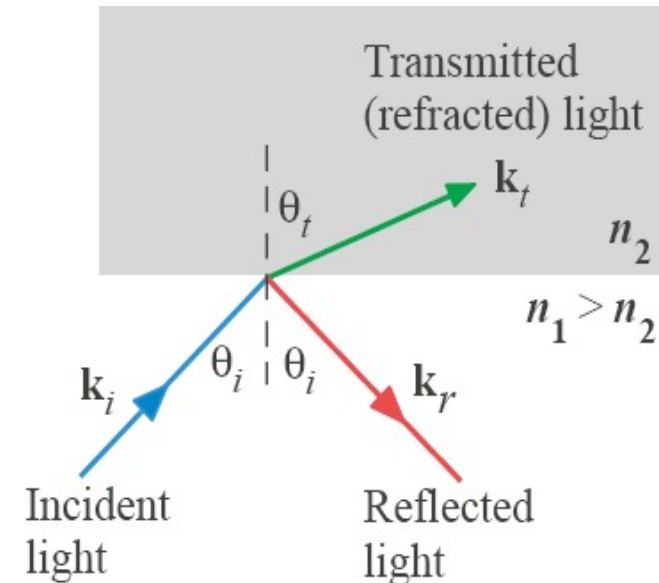
$$\theta_i = \theta_r$$



Refraction

- How is the angle of refraction related to the angle of incidence?

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$



Thin films interference

- A wave traveling from a medium of index of refraction n_1 toward a medium of index of refraction n_2 undergoes a 180° phase change upon reflection when $n_2 > n_1$ and undergoes no phase change if $n_2 < n_1$.

- The wavelength of light λ_n in a medium whose index of refraction is n is:

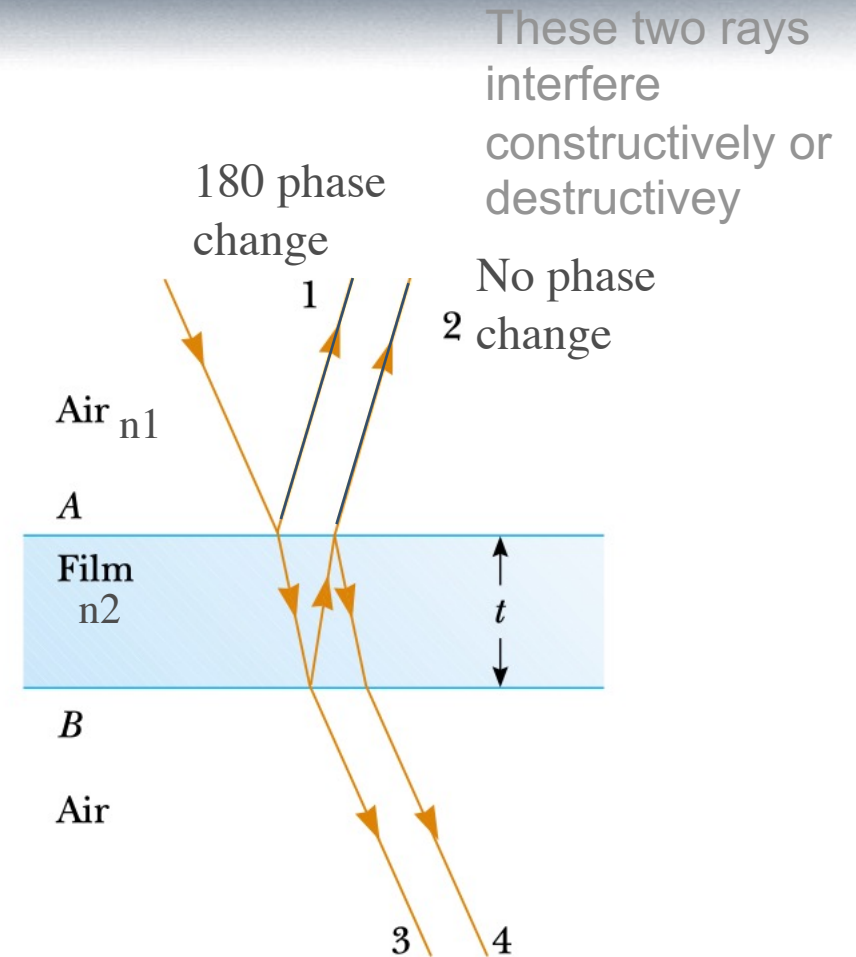
$$\lambda_n = \frac{\lambda}{n}$$

$$\Delta x = L_2 - (L_1 + \lambda/2)$$

- For constructive interference: $\Delta x = m\lambda$

$$2t = (m + \frac{1}{2})\lambda_n \quad (m = 0, 1, 2, \dots)$$

Approximation: we are considering light rays that are close to normal to the surface



Problem

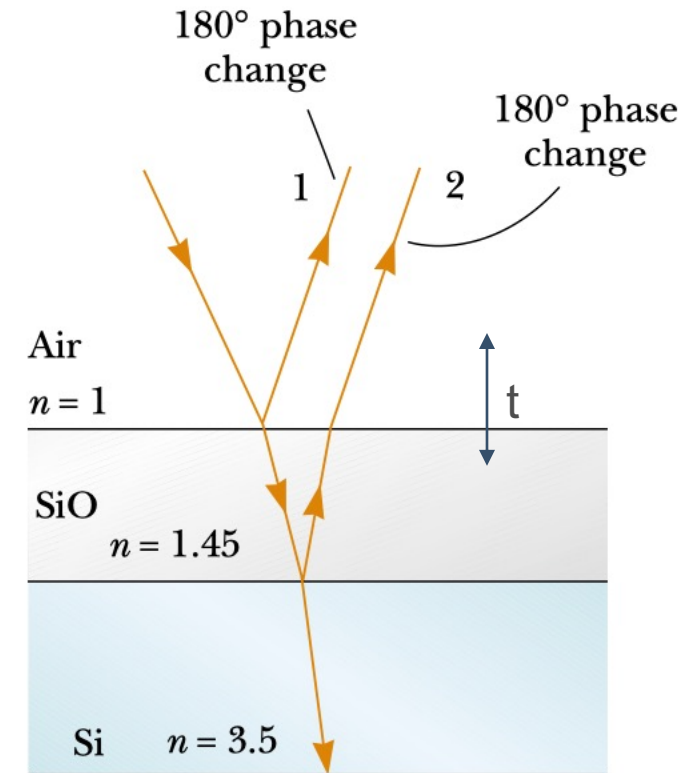
We consider a solar cell which generates electricity when exposed to sunlight. They are coated with a transparent thin film of SiO to minimize reflective losses from the surface. Suppose that a silicon solar cell ($n=3.5$) is coated with a thin film of SiO for this purpose, determine the minimum film thickness that produces the least reflection at a wavelength of 550 nm, near the center of the visible spectrum.

Solution

The reflected light is a minimum when rays 1 and 2 meet the condition of destructive interference.

In this situation, both rays undergo a 180° phase change upon reflection—ray 1 from the upper SiO surface and ray 2 from the lower SiO surface. The net change in phase due to reflection is therefore zero, and the condition for a reflection minimum requires a path difference of $\lambda_n/2$, where λ_n is the wavelength of the light in SiO.

Hence $2t = \lambda/2n$, where λ is the wavelength in air and n is the index of refraction of SiO.

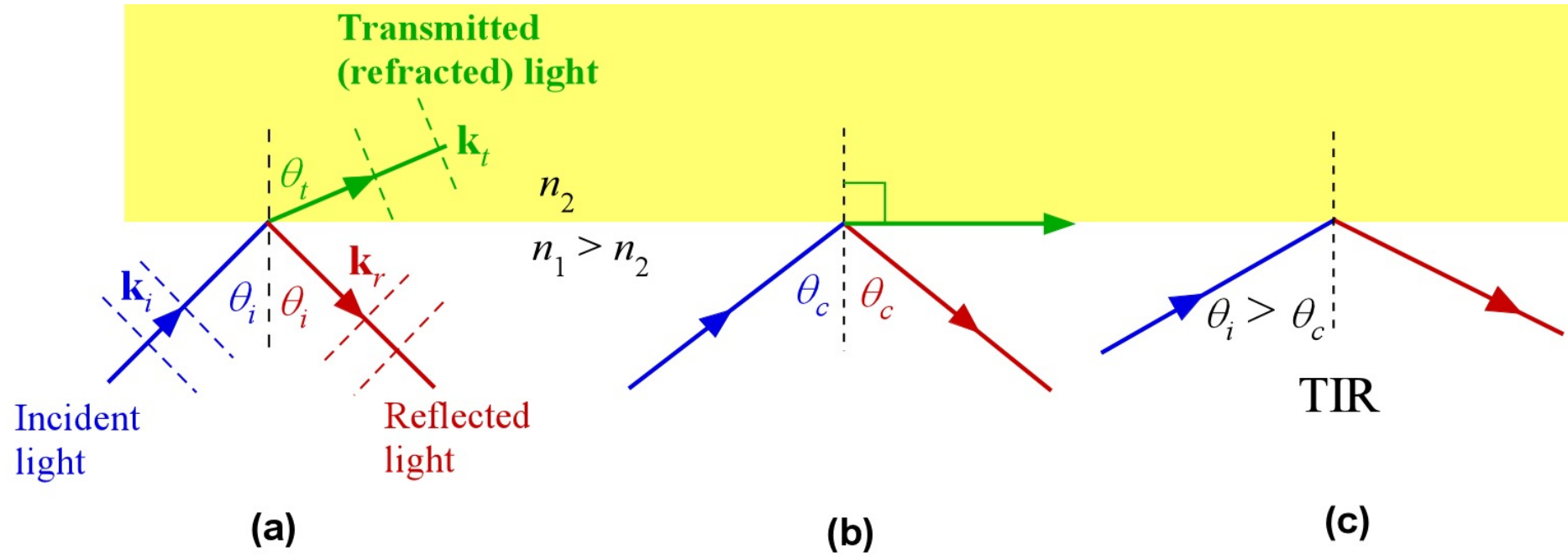


The required thickness is:

$$t = \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4(1.45)} = 94.8 \text{ nm}$$



Total Internal Reflection

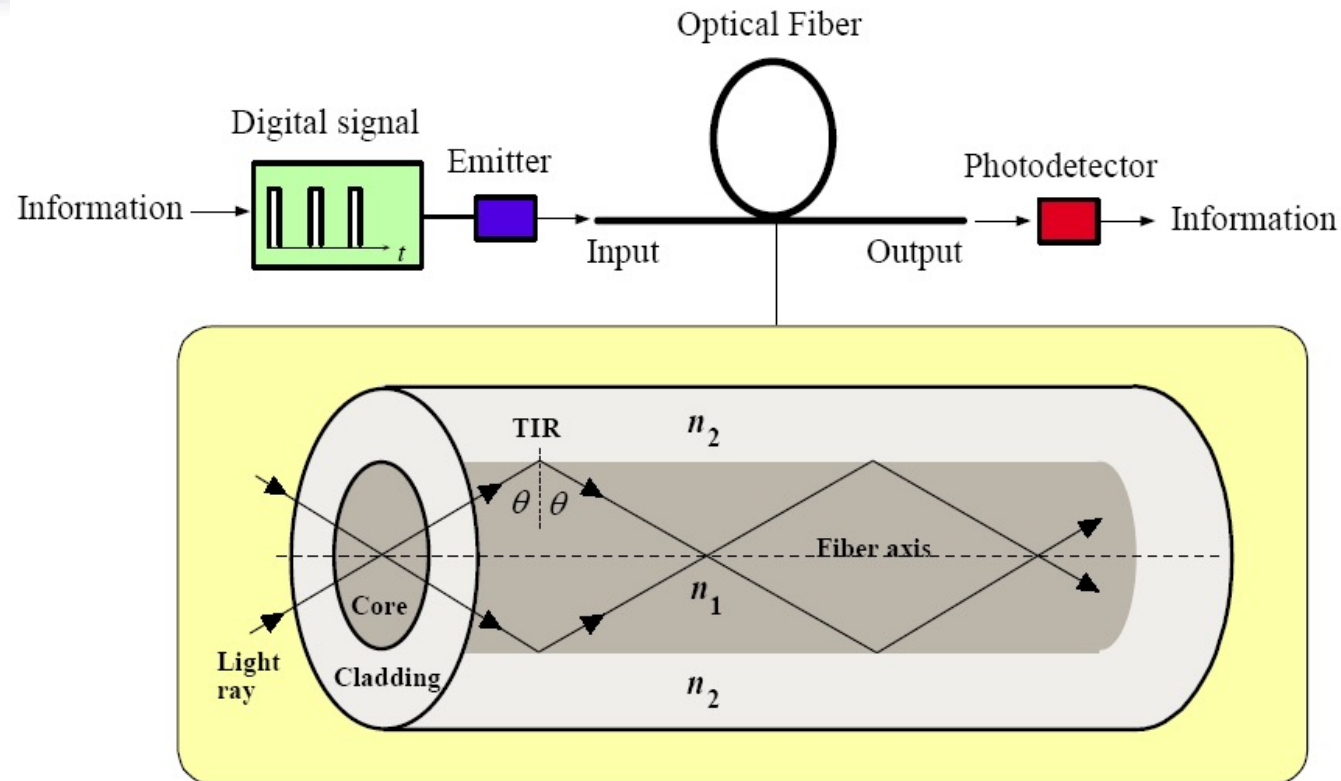


Light wave traveling in a more dense medium strikes a less dense medium. Depending on the incidence angle with respect to θ_c , which is determined by the ratio of the refractive indices, the wave may be transmitted (refracted) or reflected.

(a) $\theta_i < \theta_c$ (b) $\theta_i = \theta_c$ (c) $\theta_i > \theta_c$ and total internal reflection (TIR).



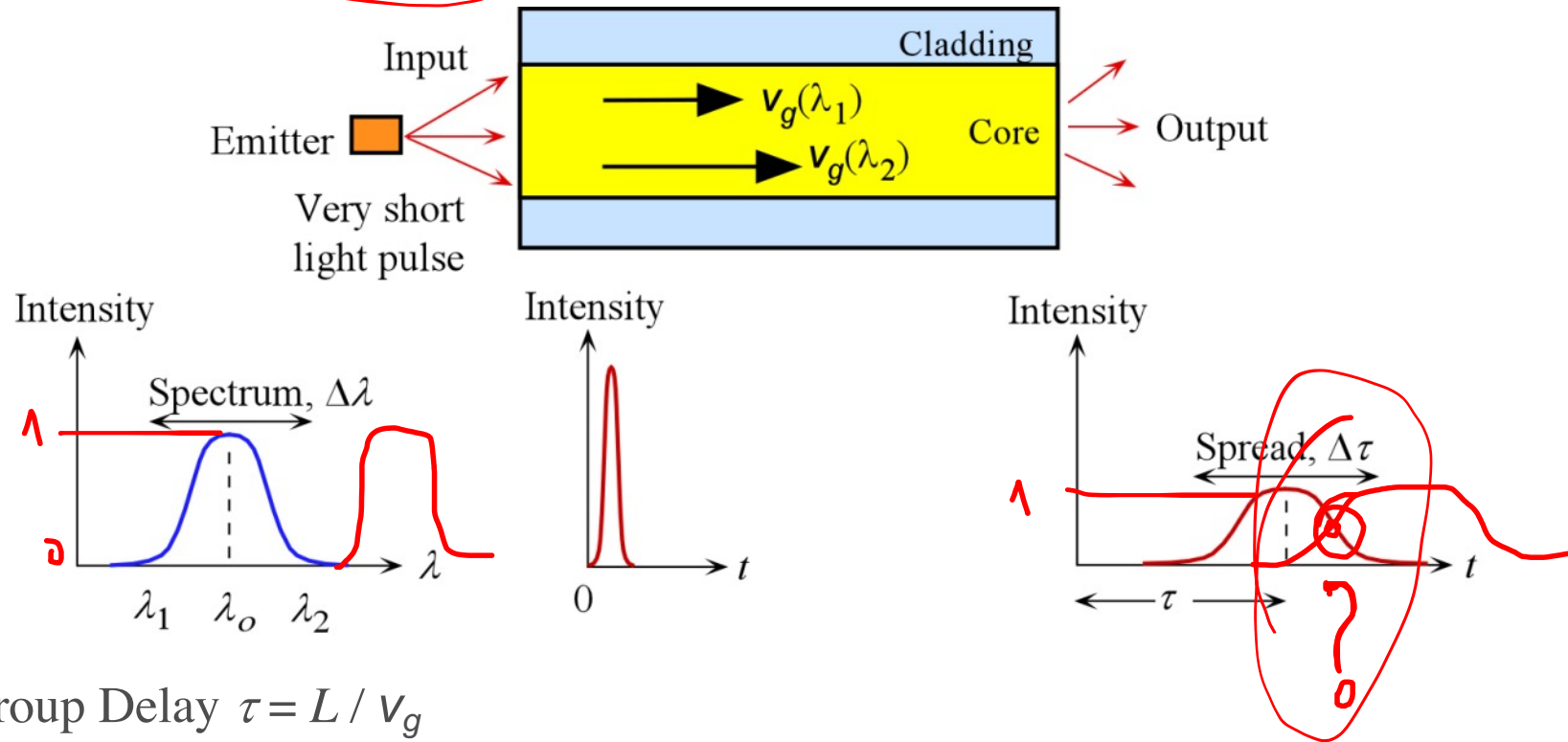
Light travels by TIR in optical fibers



An optical fiber link for transmitting digital information in communications. The fiber core has a higher refractive index so that the light travels along the fiber inside the fiber core by total internal reflection at the core-cladding interface.

Intramode Dispersion (SMF)

Dispersion in the fundamental mode



Group Delay $\tau = L / v_g$

Group velocity v_g depends on

Refractive index $= n(\lambda)$

V-number $= V(\lambda)$

Δ $= (n_1 - n_2)/n_1 = \Delta(\lambda)$

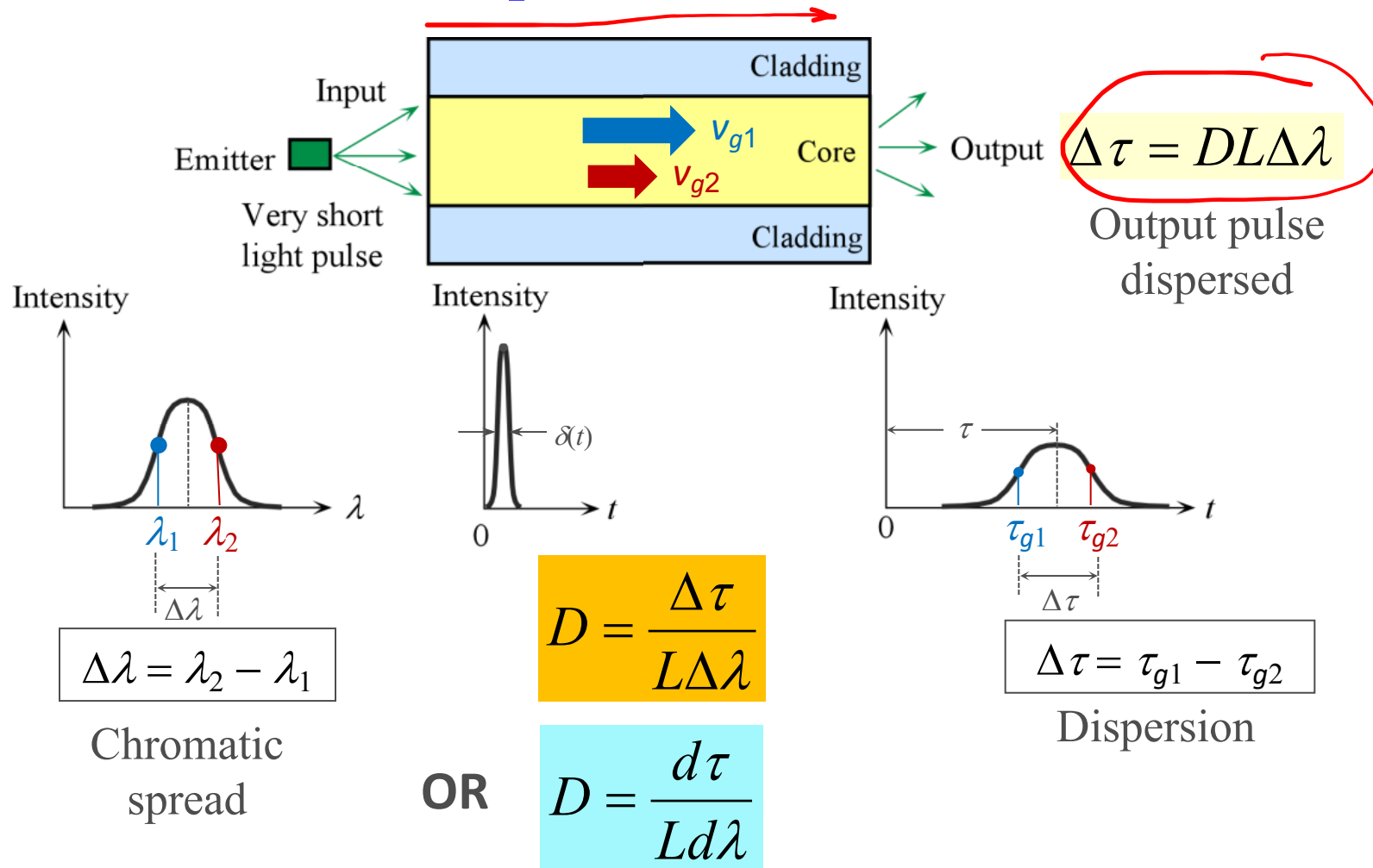
Material Dispersion

Waveguide Dispersion

Profile Dispersion

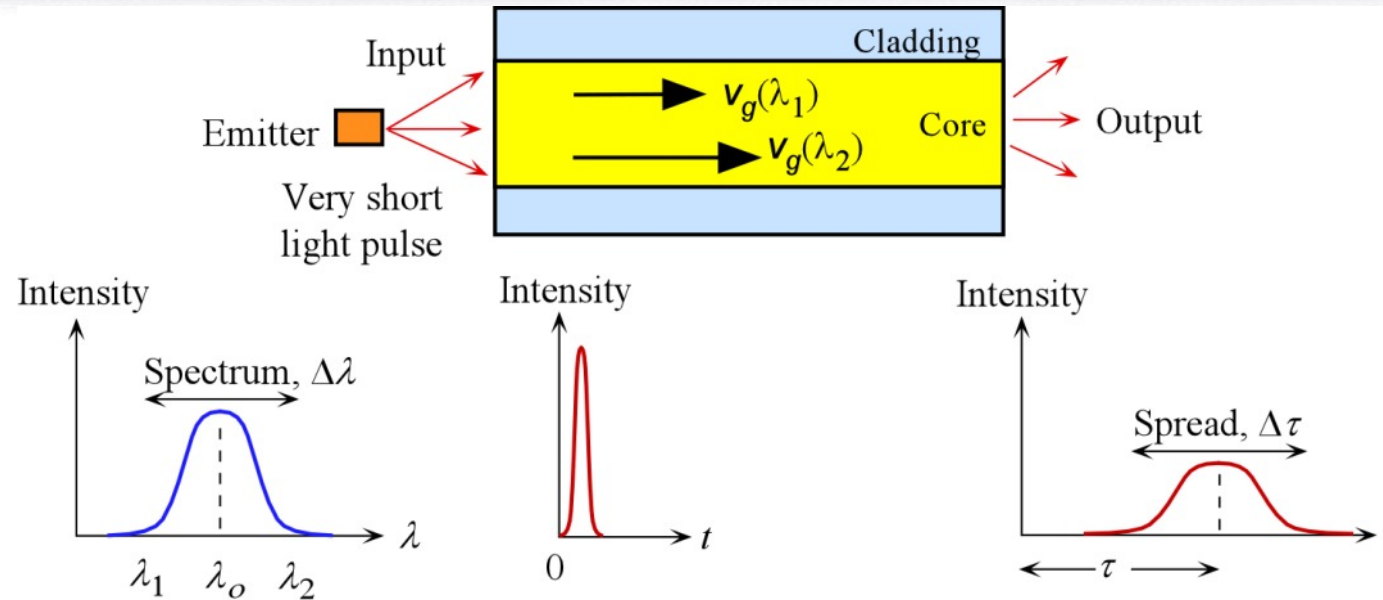
Intramode Dispersion (SMF)

Chromatic dispersion in the fundamental mode



Definition of Dispersion Coefficient

Material Dispersion



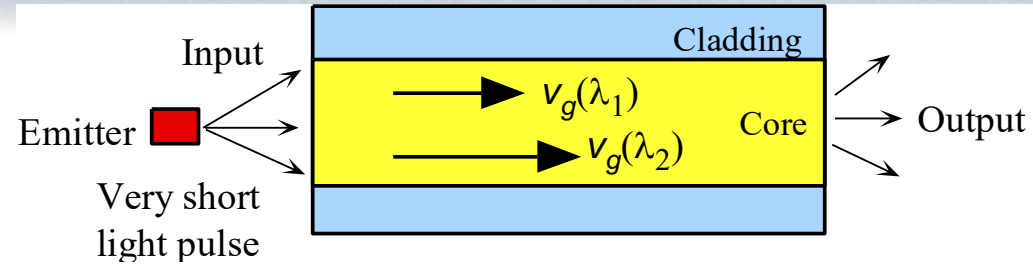
Emitter emits a spectrum $\Delta\lambda$ of wavelengths.

Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1 . The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

$$\frac{\Delta\tau}{L} = D_m \Delta\lambda$$

D_m = Material dispersion coefficient, $\text{ps nm}^{-1} \text{ km}^{-1}$

Material Dispersion



$$v_g = c / N_g$$

Group velocity

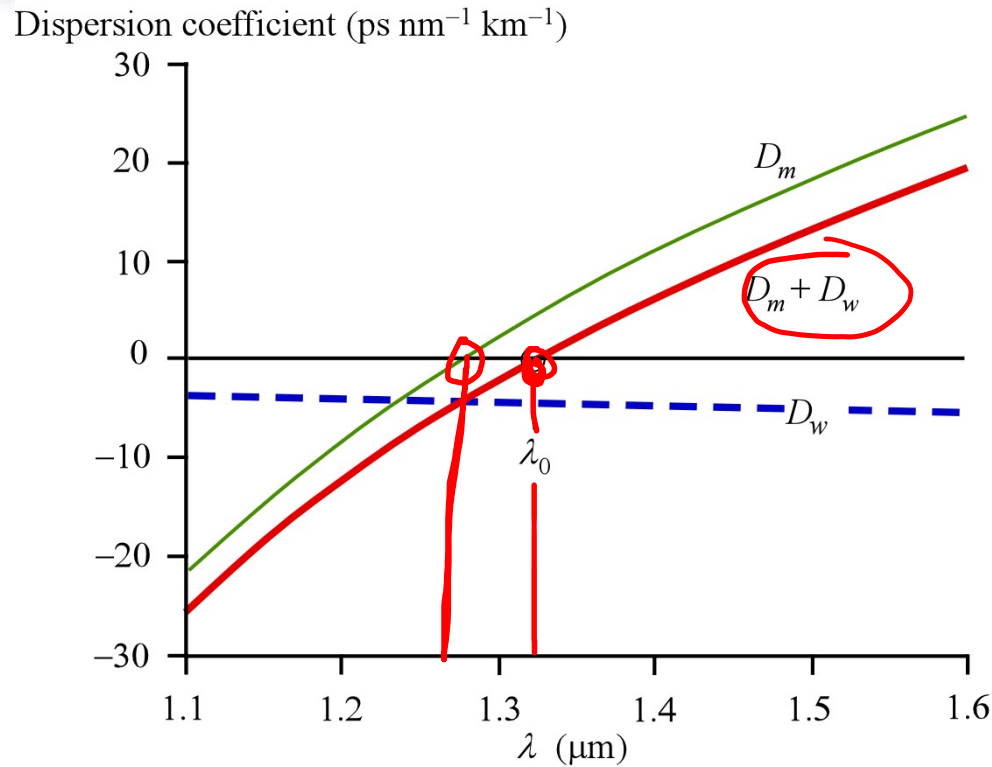
Depends on the wavelength

$$\frac{\Delta \tau}{L} = D_m \Delta \lambda$$

D_m = Material dispersion coefficient, ps nm⁻¹ km⁻¹

$$D_m \approx -\frac{\lambda}{c} \left(\frac{d^2 n}{d\lambda^2} \right)$$

Chromatic Dispersion



Material dispersion coefficient (D_m) for the core material (taken as SiO_2), waveguide dispersion coefficient (D_w) ($a = 4.2 \mu\text{m}$) and the total or chromatic dispersion coefficient $D_{ch} (= D_m + D_w)$ as a function of free space wavelength, λ

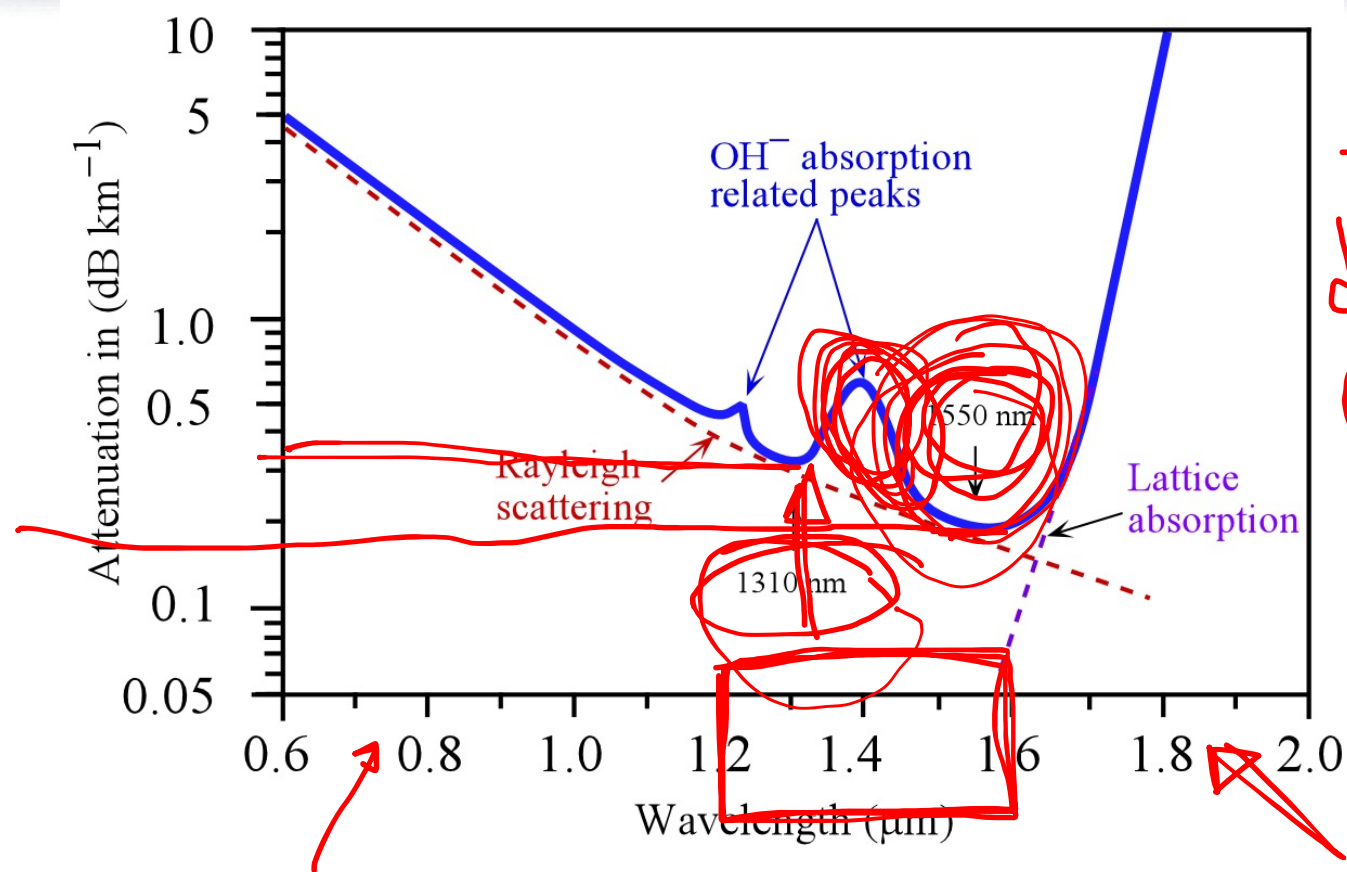
$$\lambda = 1.32 \mu\text{m} \quad ?$$

Chromatic = Material + Waveguide

$$\frac{\Delta \tau}{L} = (D_m + D_w) \Delta \lambda$$



Attenuation in Optical Fibers



β α

dist[↓] amplif

comp fiber

Cost?

Attenuation vs. wavelength for a standard silica based fiber.



Summary

1. What is light?
2. How can we study light propagation in a medium?
3. Optical fibers

