African School of Fundamental Physics and Applications

Quantum Information

Lecture 3 : Quantum Cryptography and Quantum Computing

Mourad Telmini
mourad.telmini@fst.utm.tn

Department of Physics
Faculty of Science of Tunis
University of Tunis El Manar

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Outline

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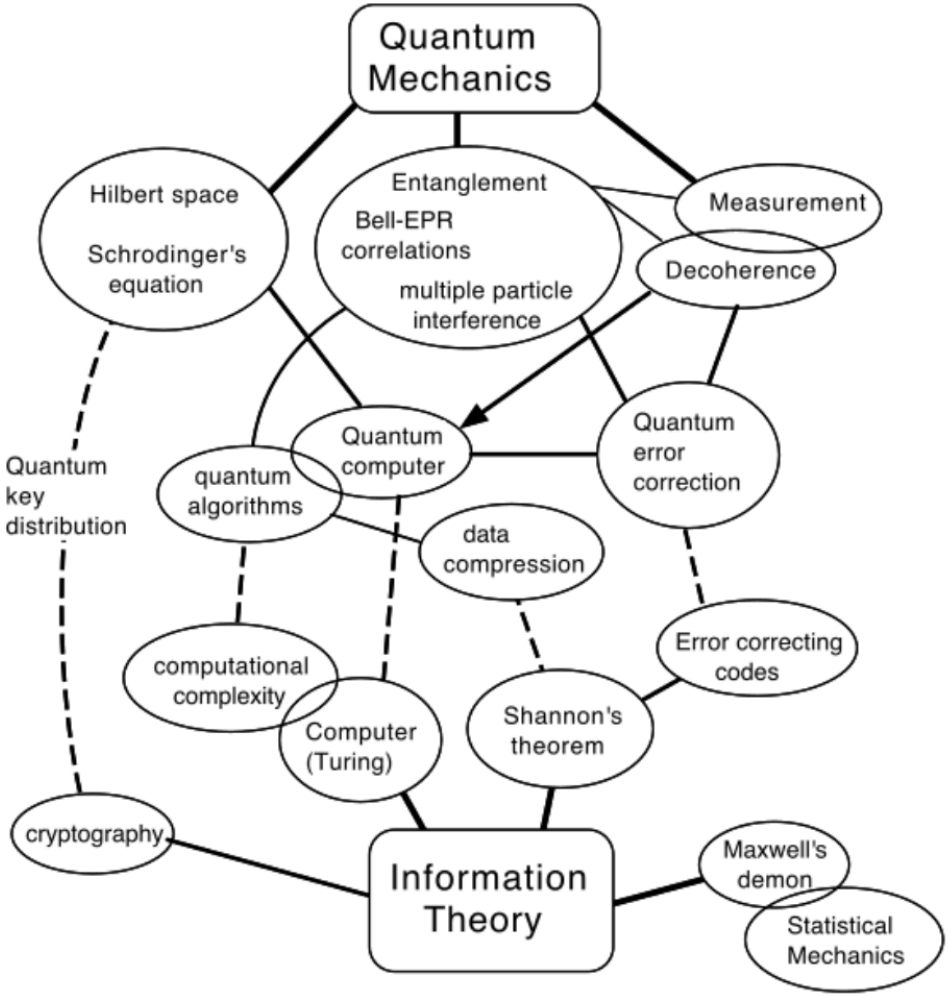
Quantum Mechanics and Information Theory



[1] Andrew Steane 1998 Rep. Prog. Phys. 61 117

Quantum Supremacy

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Article | Published: 23 October 2019

Quantum supremacy using a programmable superconducting processor

Frank Arute, Kunal Arya, [...] John M. Martinis ✉

Nature **574**, 505–510(2019) | Cite this article

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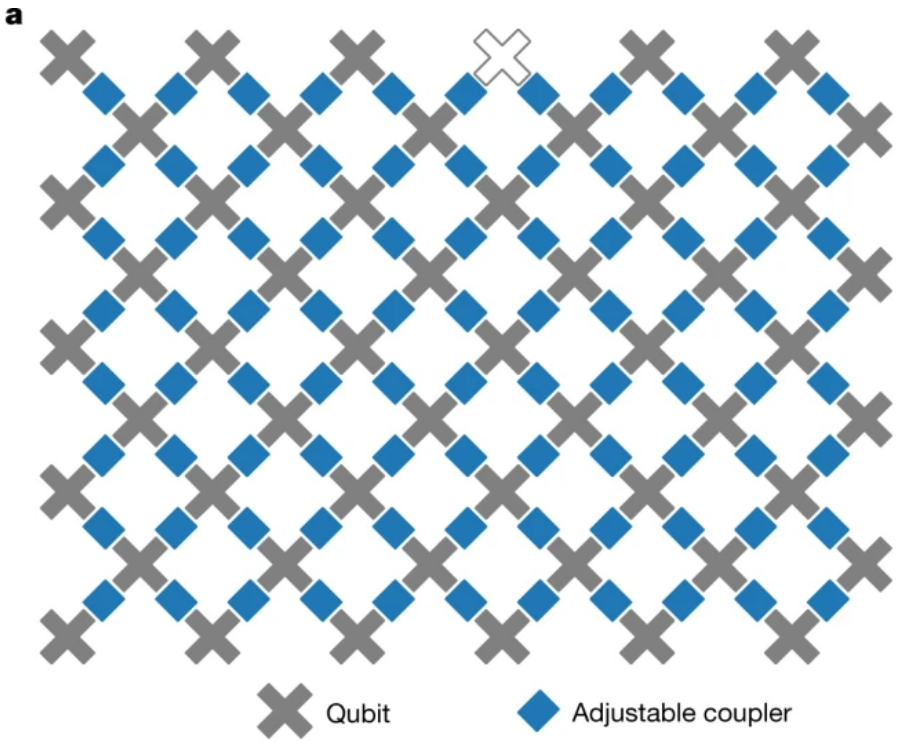
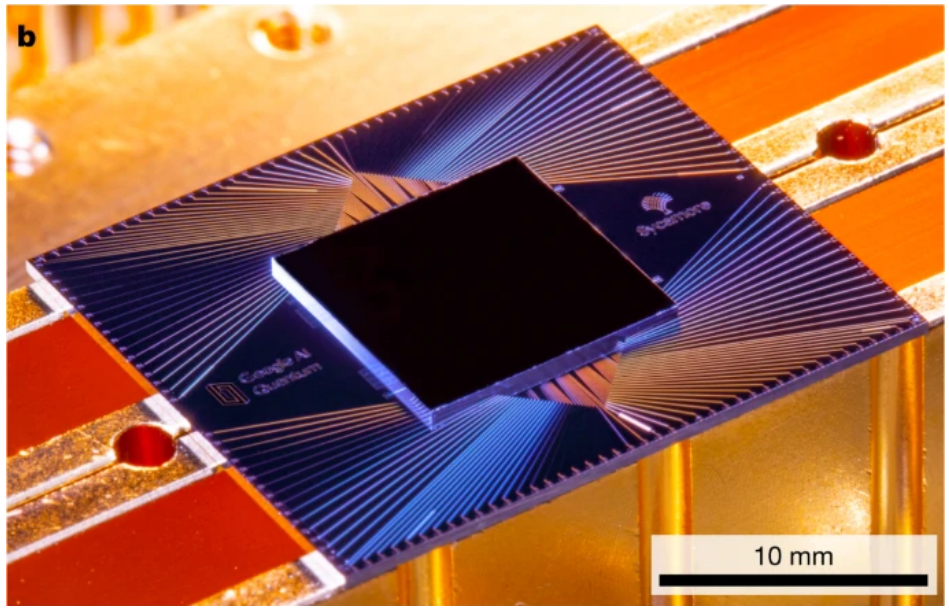
- Here we report the use of a processor with programmable superconducting qubits to create quantum states on 53 qubits, corresponding to a computational state-space of dimension 253 (about 1016).
- Our Sycamore processor takes about 200 seconds to sample one instance of a quantum circuit a million times—our benchmarks currently indicate that the equivalent task for a state-of-the-art classical supercomputer would take approximately 10,000 years.

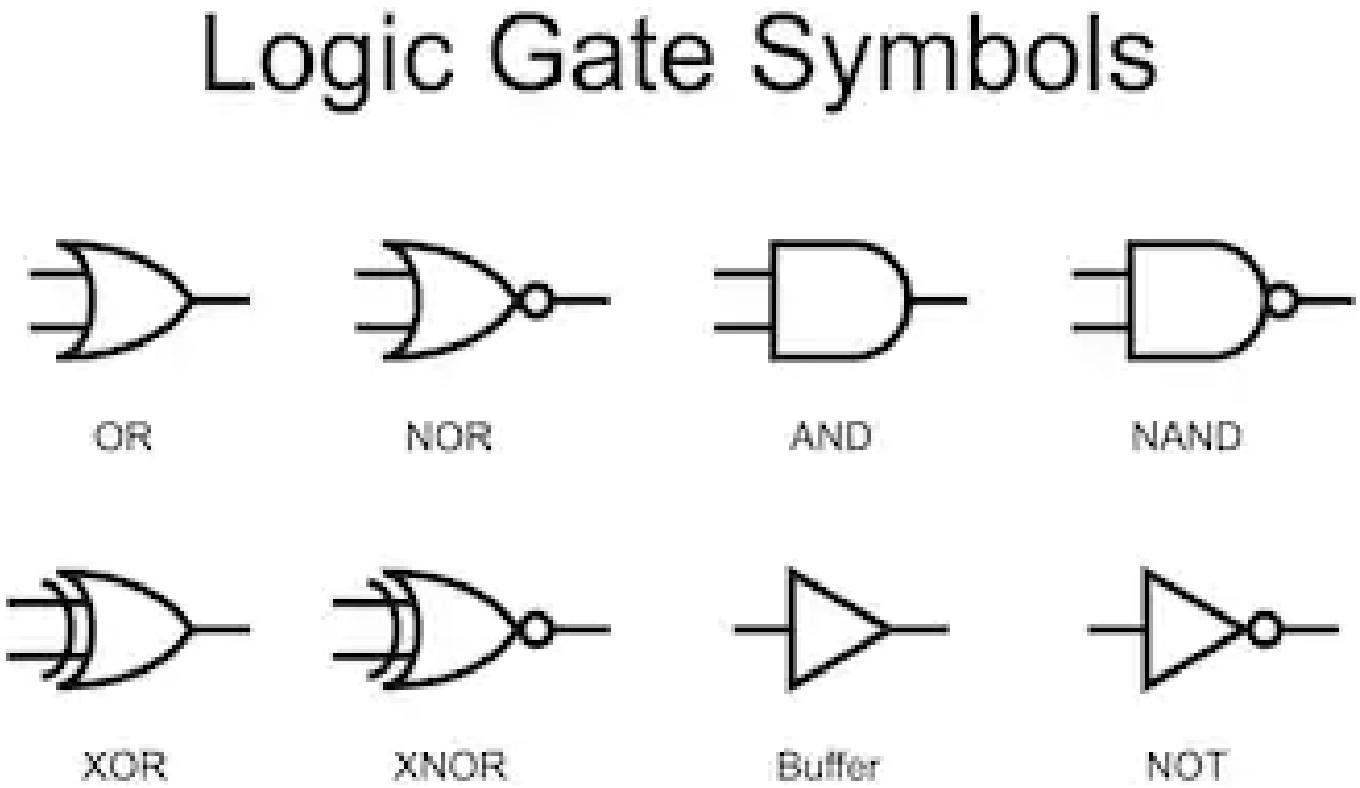
Sycamore processor





The Sycamore processor is made up of 53 qubits.

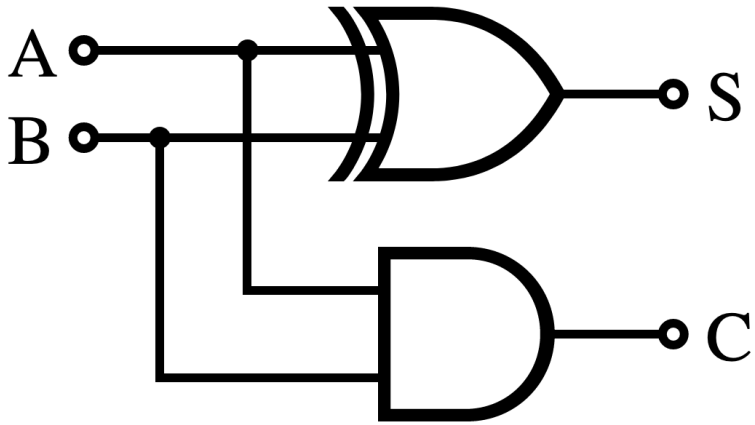
Summary of logic gates



See Appendix 1 for more details about classical logic gates

Half-adder circuit

- The half-adder takes two bits E_1 and E_2 as input and delivers 2 outputs, the sum S and the carry R, in accordance with the truth table presented above.



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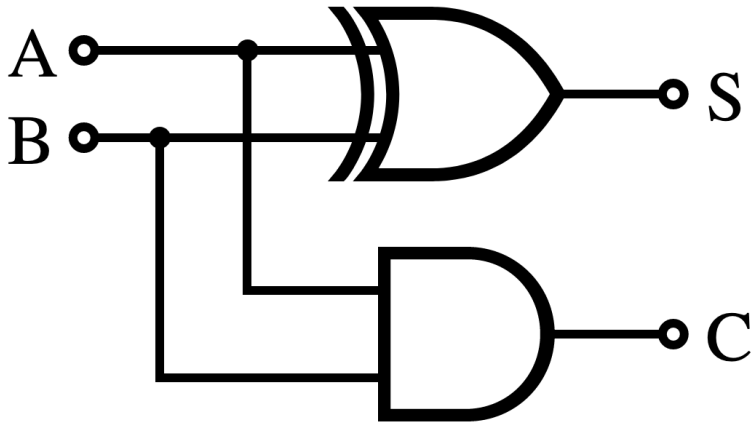
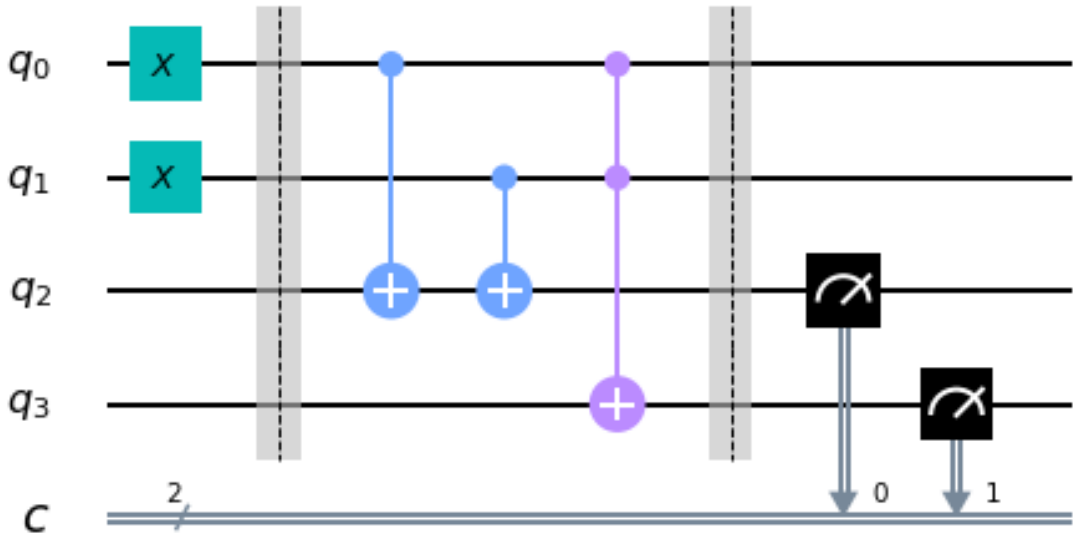


- See Appendix 1 full-adder circuit

Half-adder quantum circuit

- Here's what a half-adding quantum circuit looks like:



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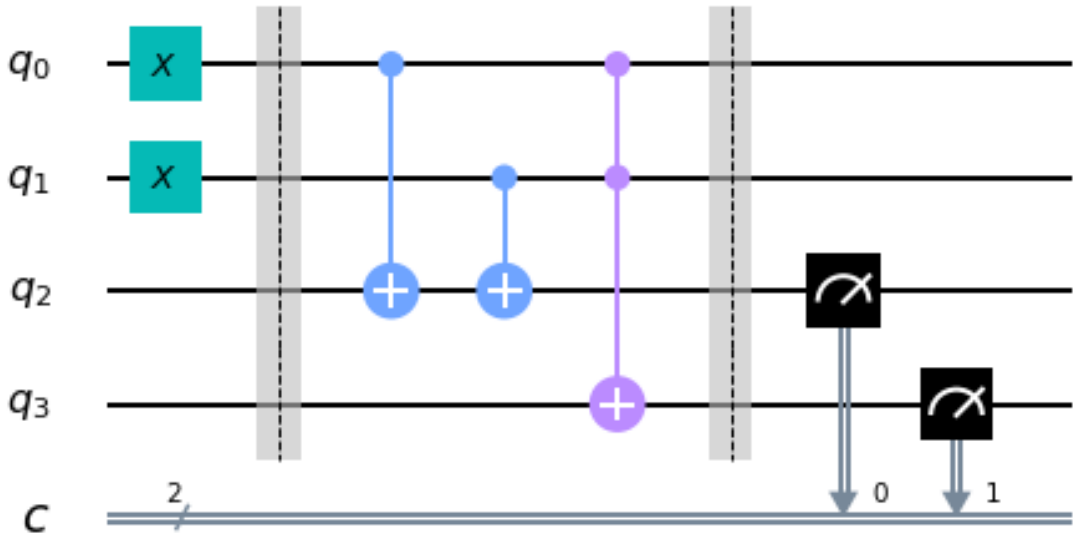


- How to build this Quantum circuit ?

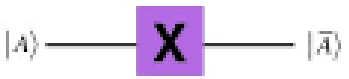
1-qubit quantum gates

- In quantum computation, a 1-qubit quantum gate is a unit transformation that transforms an the quantum state of input qubit into another quantum state or an output qubit.

- 1-qubit quantum gates are represented by unitary matrices 2×2.

A	$\bar{A}$
0	1
1	0

PAULI X GATE



$\|A\rangle$	$\|\bar{A}\rangle$
0	1
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Quantum Logic Gates

X (NOT)
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{array}$$

Y
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & 0 & -i \\ 1 & i & 0 \end{array}$$

Z
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \end{array}$$

S
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & i \end{array}$$

T
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & e^{i\pi/4} \end{array}$$

Hadamard (H)
$$\begin{array}{c|cc} & 0 & 1 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{array}$$

Sqrt NOT
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- We will use for the graphs, those of the IBM Qiskit environment (www.qiskit.org) where a 1 qubit gate (for example the X gate) is represented as follows:

Pauli Gates

- The Pauli gates X, Y and Z are represented by the 3 Pauli matrices σ_x, σ_y and σ_z.

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- here we adopted a specific notation with parentheses [ ] rather than the traditional parentheses ()

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$$H|0\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) = |+\rangle$$

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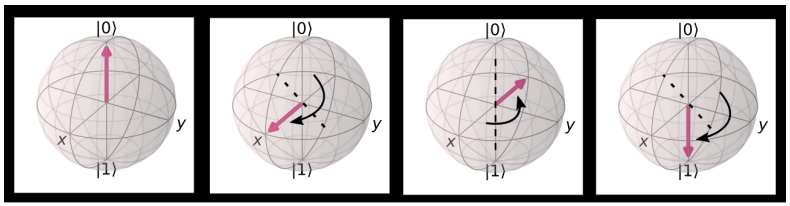
- The states $|+\rangle$ and $|-\rangle$ are particularly important in quantum cryptography.
- Exercice : verify the identity $X = HZH$



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- We will see below the main gates at n qubits, focusing our attention on the central cases $n = 2.3$, with in particular the introduction of the CNOT gate (2 qubits) and the Toffoli gate (3 qubits).

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- We will see below the main gates at n qubits, focusing our attention on the central cases $n = 2.3$, with in particular the introduction of the CNOT gate (2 qubits) and the Toffoli gate (3 qubits).
- Thus, with 1, 2 and 3 qubit gates, we will be able to build circuits capable of performing quantum calculations (Quantum Computing).

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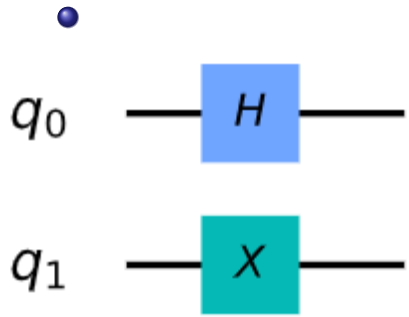
- We can construct operations which act on the tensor product, with the tensor products of 1-qubit gates.
- For example, with a gate H acting on $|q_0\rangle$ and a gate X acting on $|q_1\rangle$, we construct:

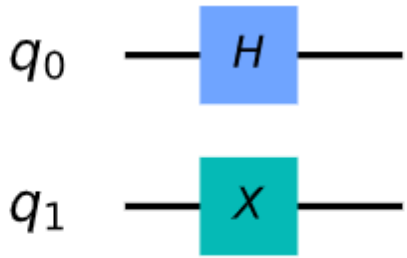
$$(X \otimes H)|q_1 q_0\rangle = X|q_1\rangle \otimes H|q_0\rangle$$



Action of 1-qubit gates on n-qubits register

- In accordance with the rules of the tensor product of 2 matrices, we have:

$$(X \otimes H) = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$



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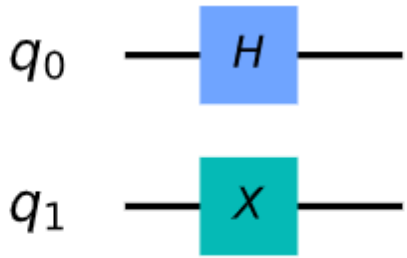


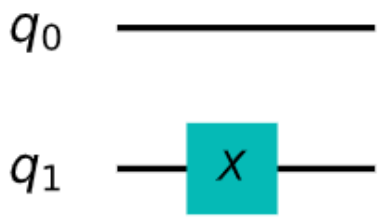
- By noting that the blocks 2×2 are none other than the gate H (by including the factor $(1/\sqrt{2})$, we can use the more compact notation for the same 2 qubit gate :

$$(X \otimes H) = \begin{bmatrix} 0 & H \\ H & 0 \end{bmatrix}$$

Action of 1-qubit gates on n-qubits register

- If we want to apply a single gate to 1 qubit on one of the 2 qubits, we use the tensor product with the identity operator:

$$(X \otimes I) = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$



CNOT gate

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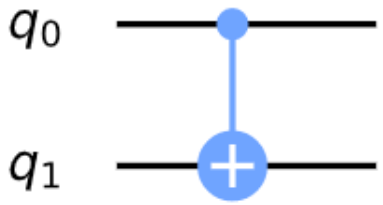

CNOT gate

- The CNOT gate is a 2-qubits gate
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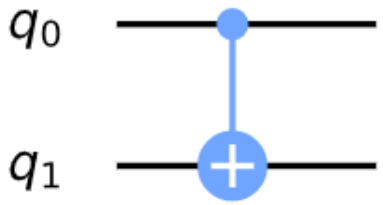
$$(CNOT) = \begin{bmatrix} I & 0 \\ 0 & X \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



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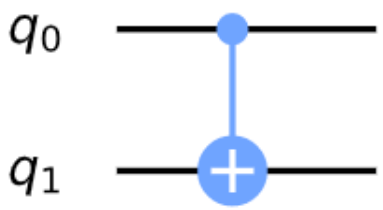

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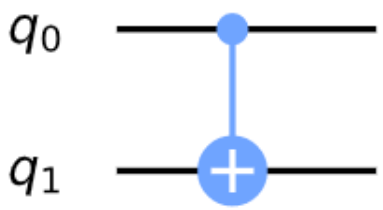

- It is a conditional gate which causes a gate X to act on the second qubit $|q_1\rangle$ (target), if the first qubit $|q_0\rangle$ (control) is in state $|1\rangle$.
- In this case, the CNOT gate reverses the amplitudes of the target qubit. Otherwise, she does not change her condition.

First Quantum Circuit

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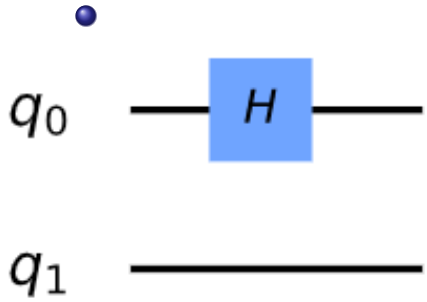

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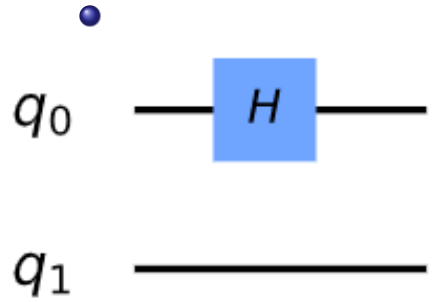

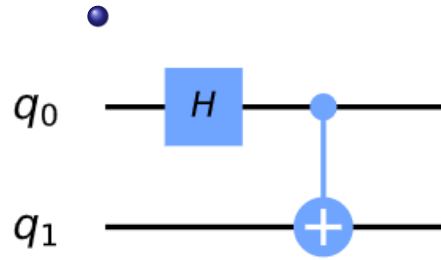

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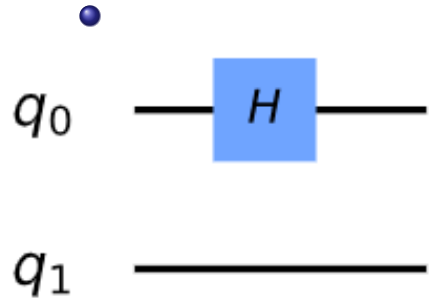
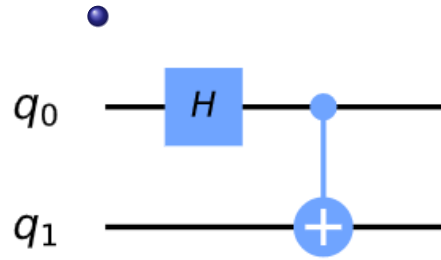
- Then the CNOT gate is applied, which amounts to creating a first quantum circuit.
- The action of this circuit on the 2 qubit system initially in the state $|00\rangle$ is:

$$(CNOT.(H \otimes I))|00\rangle = CNOT|0+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

which is a Bell state.





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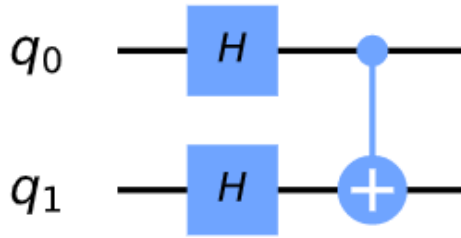

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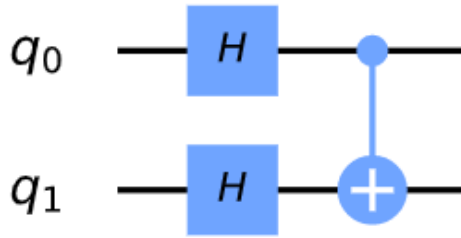

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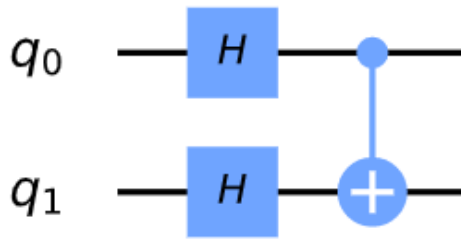
- If we put the target in the state $|-\rangle$, by applying a gate X after the gate H, and we add a gate CNOT, the circuit produces the state

$$|--\rangle = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$$



Other quantum circuits

- We are going to complicate the previous circuit a bit, by adding a H gate on the second qubit.
- This circuit produces at output the state:

$$|++\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

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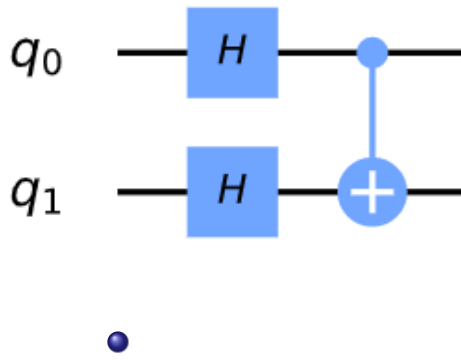
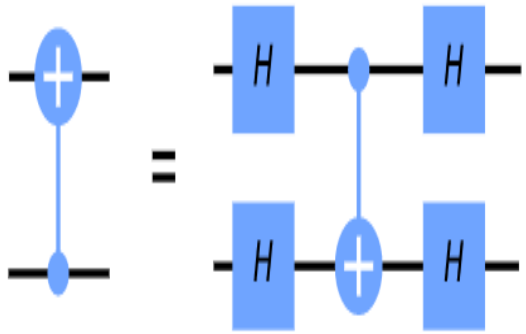
- Note that this circuit changes the state of the control qubit, without modifying that of the target qubit.



Phase kickback

- This circuit makes it possible to create a kind of CNOT gate upside down.



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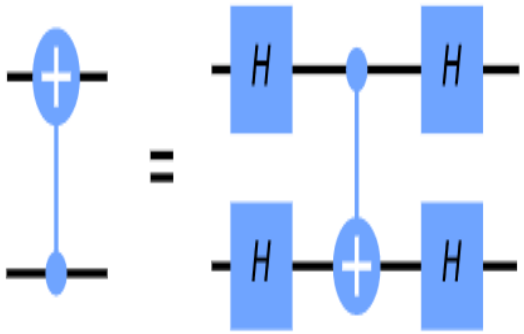

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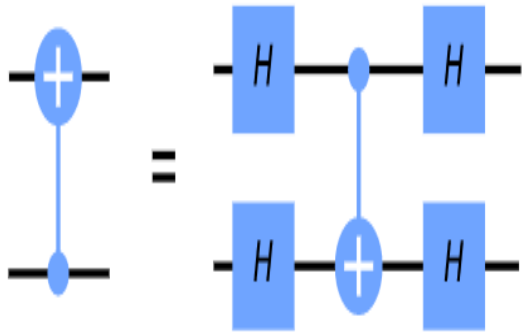
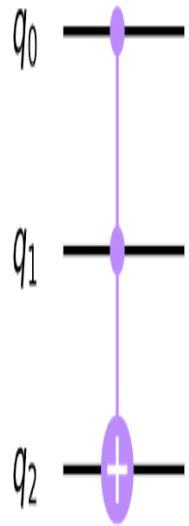
- Phase reversal is useful in some quantum circuits, where one needs to change the control qubit of the CNOT gate



Toffoli gate

- The Toffoli gate is a 3 qubit gate, of which 2 are control qubits and 1 target qubit.



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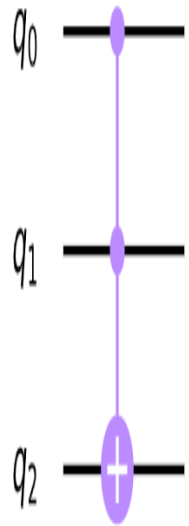

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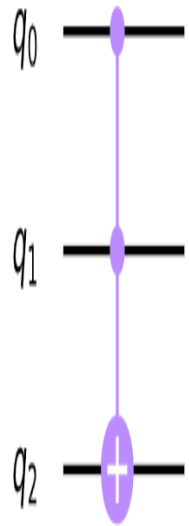

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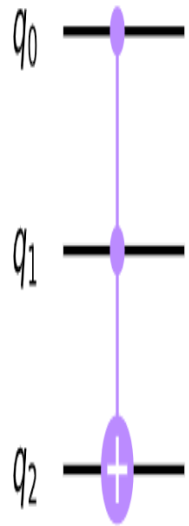

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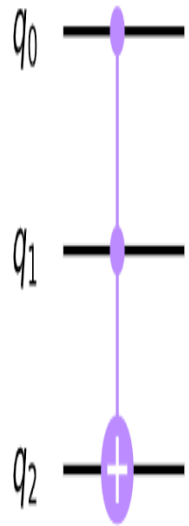

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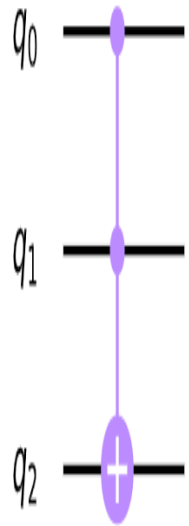
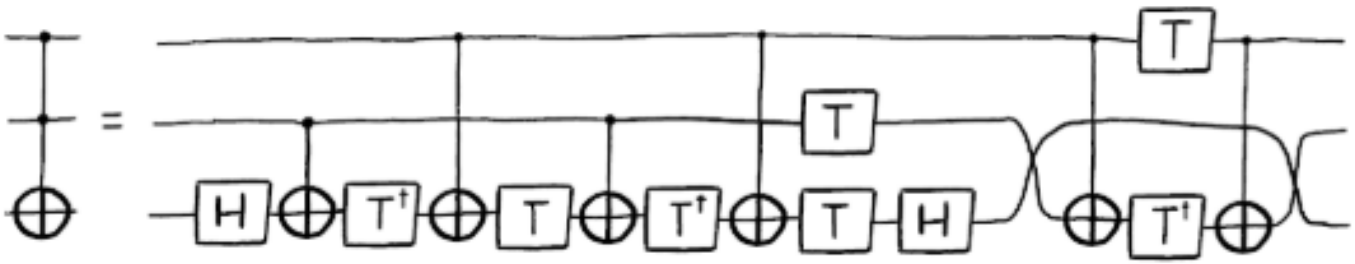
- Exercise ☺: Check the equivalence scheme below





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  - writing of the measurement results in conventional classical registers, either to read them directly as results of quantum computation, or to reuse them in a new iterative computation cycle.

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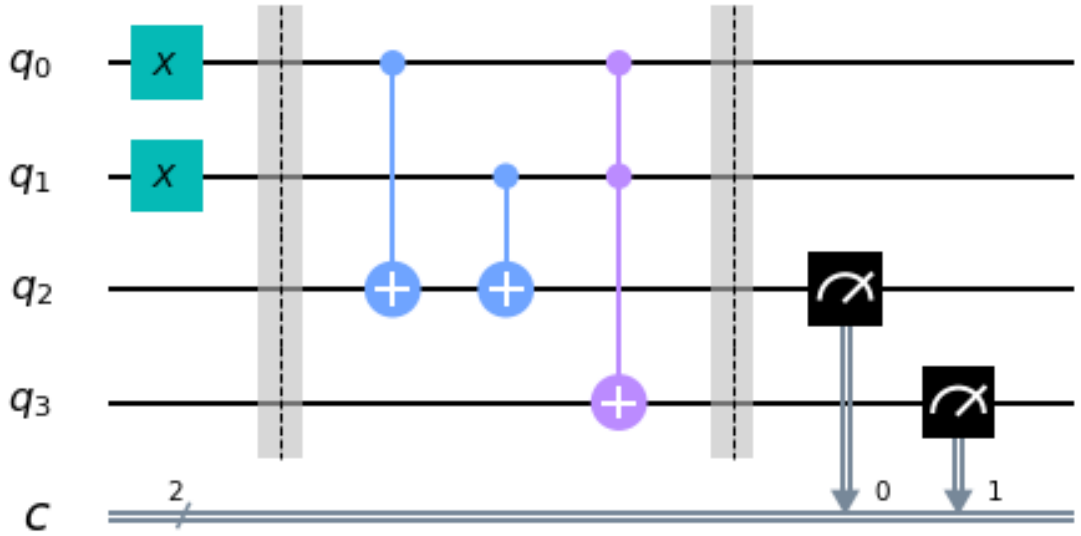
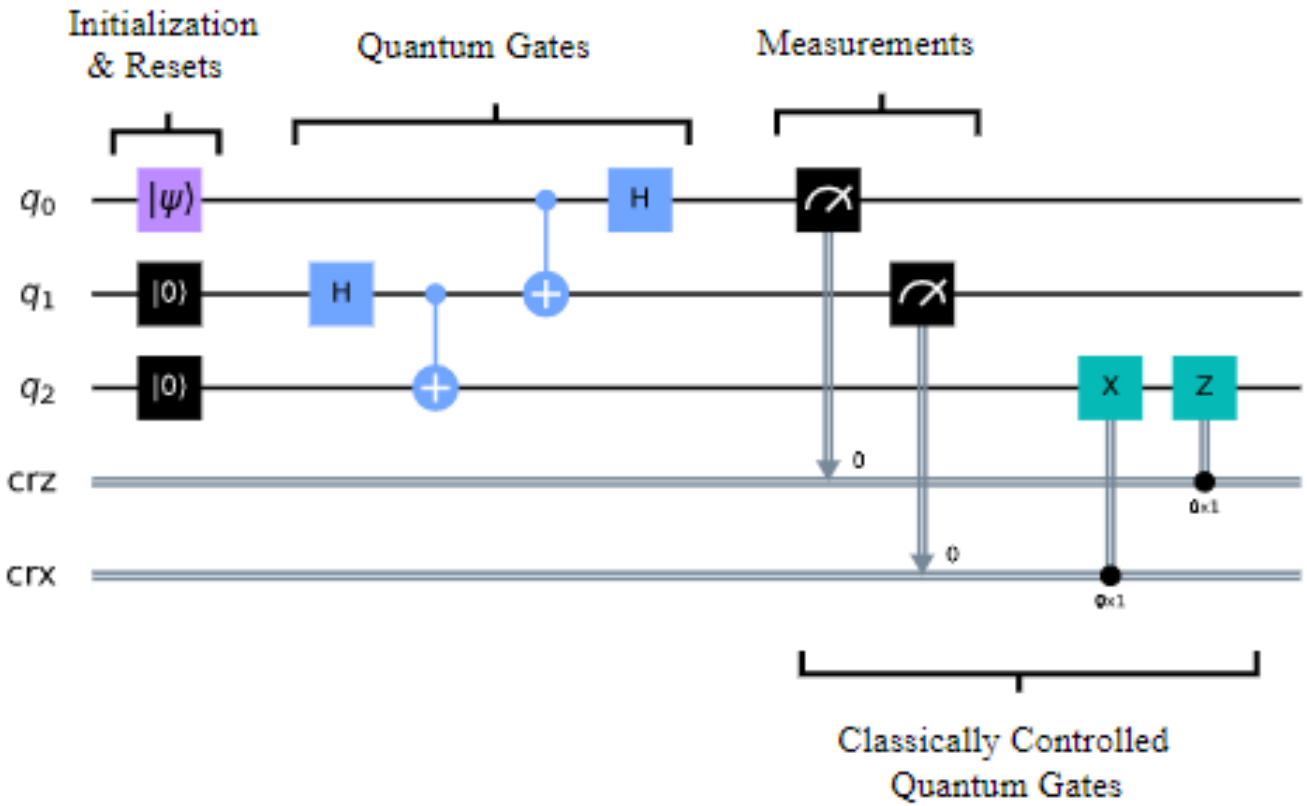
Example 1: Half-adder circuit

- Here again is a half-adder quantum circuit:



Example 2: Teleportation circuit



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- In most algorithms, we measure the qubits so that we can send the results through stable classical channels instead of noisy and fragile quantum channels.
- Finally, we need to use the results of quantum computation in a classical form that we can use in practice.

Quantum algorithms

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  - Deutsch's and Deutsch-Jozsa Algorithms
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  - Simon's Algorithm
  - Quantum counting
  - Quantum Teleportation
  - Shor's Algorithm
  - Grover's Algorithm
  - Quantum Key Distribution (Quantum Cryptography)
  - Quantum Fourier Transform (QFT)
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- In this lecture, we will have time to see some of these algorithms: Quantum Teleportation and Quantum Cryptography

Quantum Teleportation & non-cloning theorem

- Alice wants to send a qubit q_0 to Bob (more precisely the quantum state of the qubit q_0). However, the quantum non-cloning theorem prohibits this operation if the only parties involved are the sender (Alice) and the receiver (Bob).
- If $|\chi\rangle$ is any state of system A (for example a qubit), it is not possible to clone it, *i.e.* to copy it to a system B (for example another qubit).

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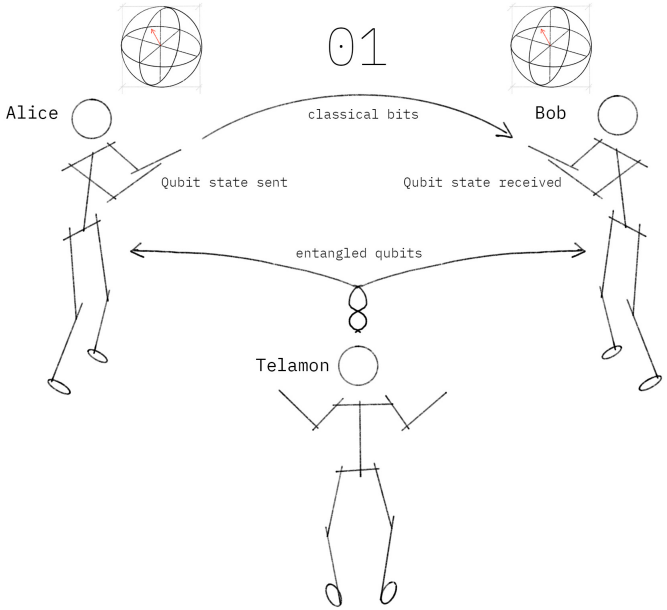
- See Appendix 4 for proof.

Teleportation protocol

- Alice and Bob call on a third partner (Telamon) who sends each a qubit that is part of a pair of entangled qubits (q_1 for Alice and q_2 for Bob).



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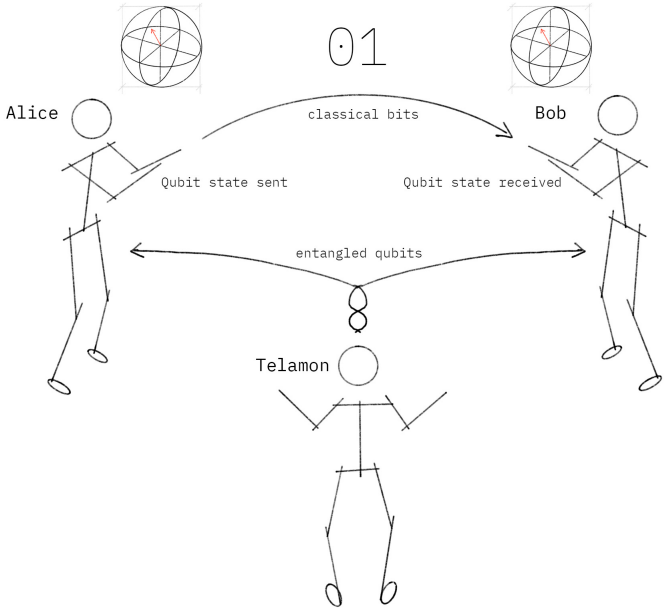
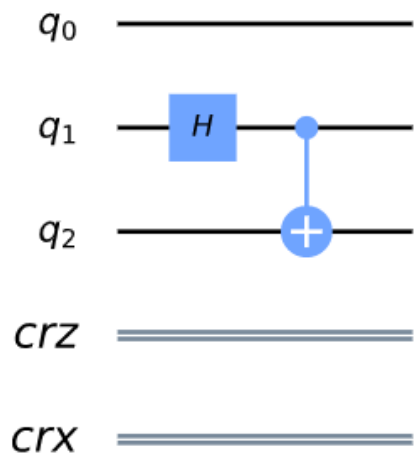
- Telamon uses a special pair which is a Bell pair, in which both qubits are in a Bell entangled state.

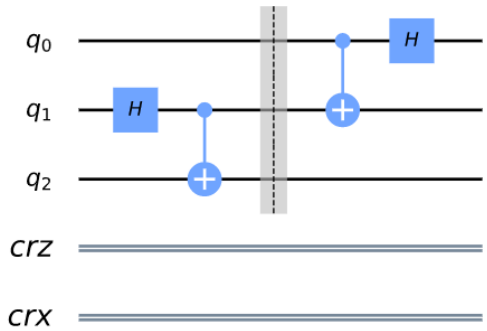


Teleportation protocol

- The translation in terms of quantum circuit of this preparation step is the first piece :
- The qubit q_0 passes a Hadamard gate which creates a $|+\rangle$ state. Then apply a $CNOT$ gate on the other qubit q_1 controlled by q_0.

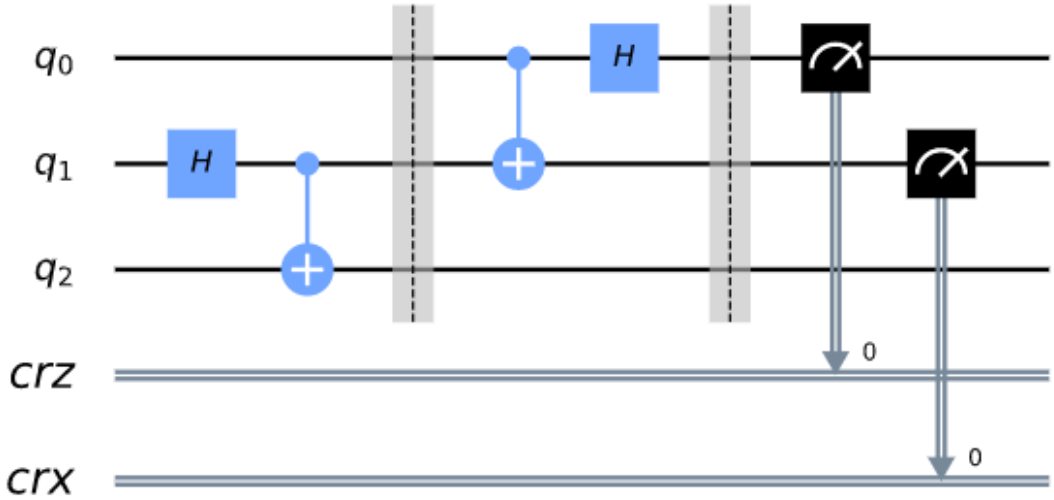


- Alice applies a $CNOT$ gate to q_1 controlled by q_0. Next, a Hadamard gate on q_0 that she wants to send to Bob.



Teleportation protocol

Then, Alice measures the 2 qubits q_1 and q_0 and records the results in two standard bits. Then she sends these 2 classic bits to Bob through a classic channel.



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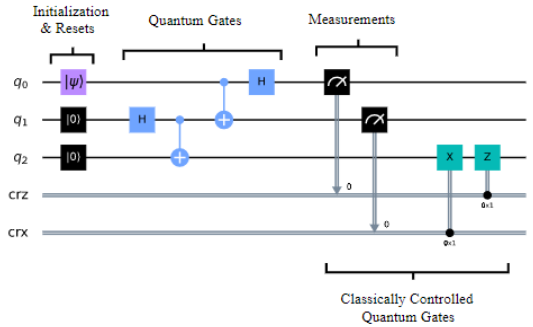
- Bob, who had already received the qubit q_2 from Telamon, applies one of the following gates to it depending on the state of the classic bit sent by Alice:



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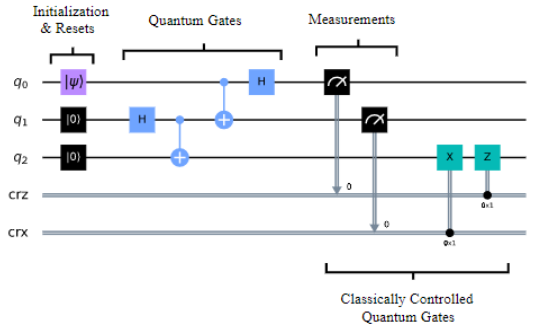

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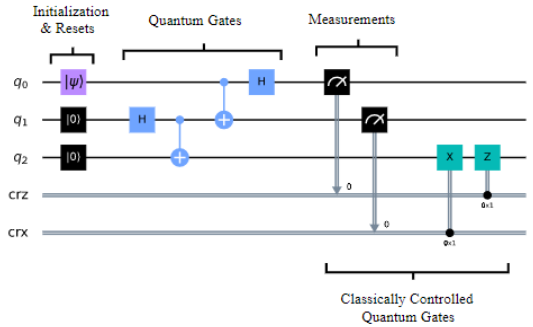

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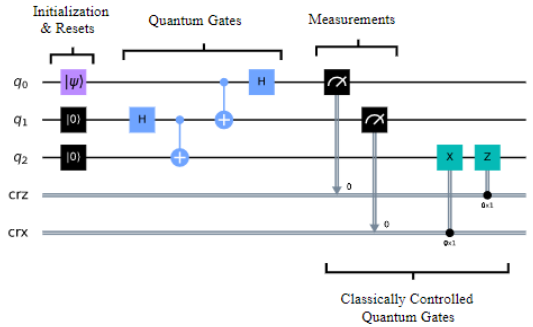

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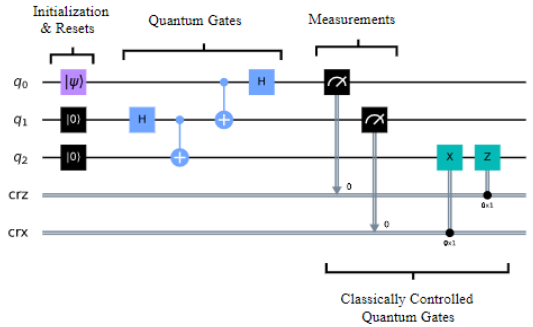

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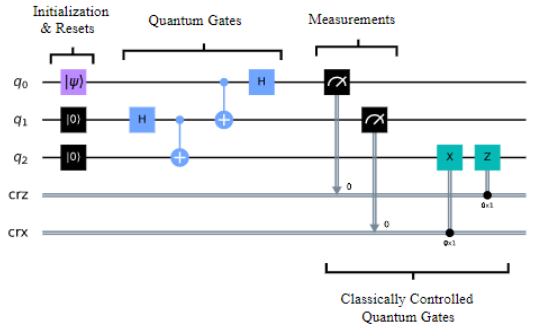

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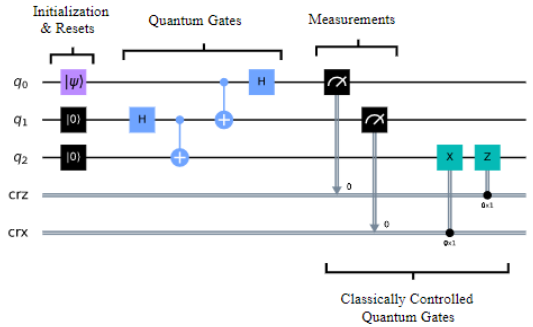

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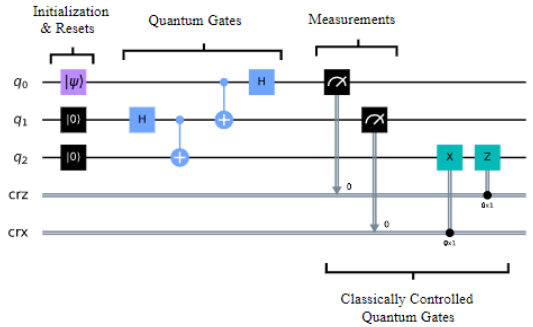

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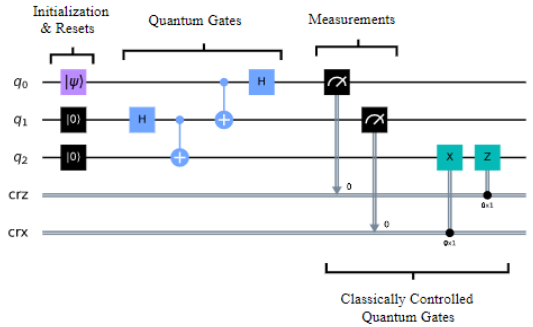

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- Specifically, Bob reconstructed the quantum state that Alice sent him, thanks to the invaluable help of Telamon and his entangled qubits.

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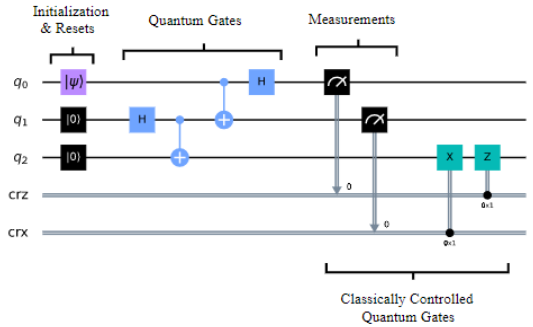

- Note here that this information transfer is purely classical.
- That's it ! At the end of the protocol, Alice's qubit q_0 was teleported to Bob. Let us insist on the fact that it is not the qubit itself which has been teleported, but a state of the qubit.
- Specifically, Bob reconstructed the quantum state that Alice sent him, thanks to the invaluable help of Telamon and his entangled qubits.
- So: No entanglement, no Quantum Teleportation!

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- At the same time, the idea germinated to use the quantum properties themselves (superposition / entanglement) to design quantum cryptography algorithms which would thus be almost inviolable.

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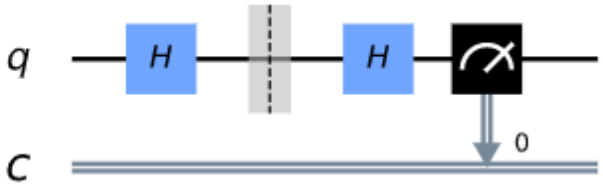
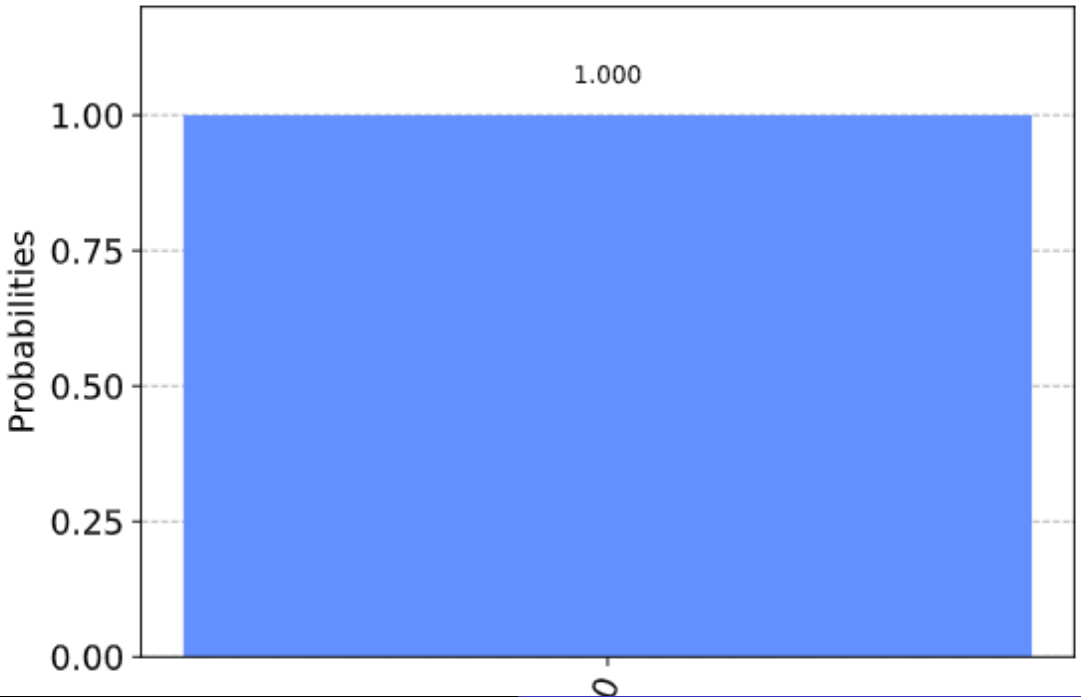
- Schematically, this is based on the fact that on a quantum system, spying, which results in a measurement, alters the quantum state of the qubit, unlike a conventional channel and this alteration can be detected by Bob.

How does it work ?

- If Alice prepares the qubit in the state $|+\rangle$, which is the eigenstate $|0\rangle$ of the gate X, and if Bob measures the state in the same base, he is certain always get the result $|0\rangle$ with a probability of 100 %.





How it works ?

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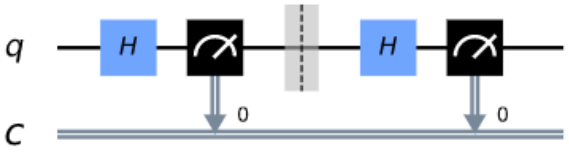

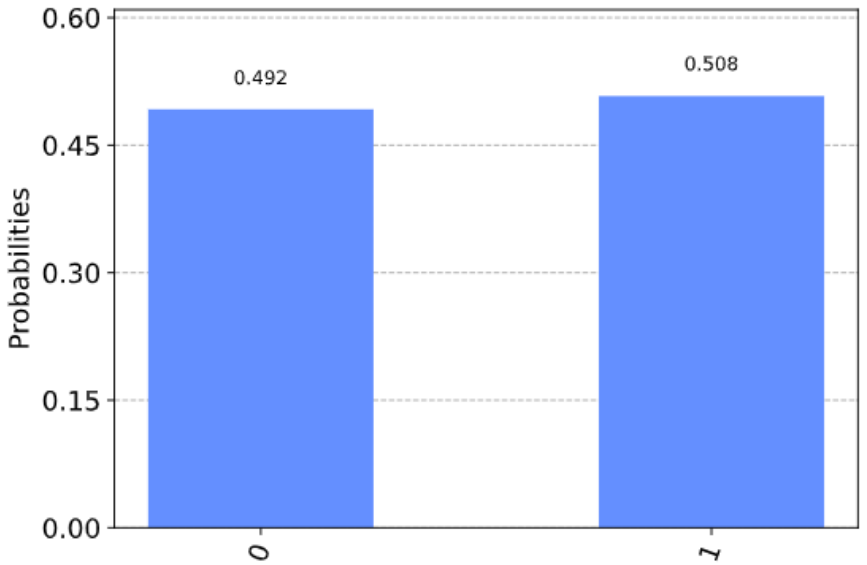

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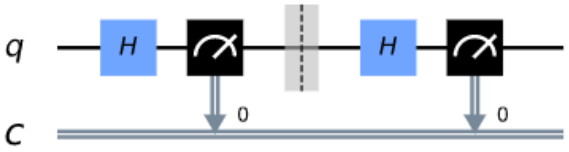

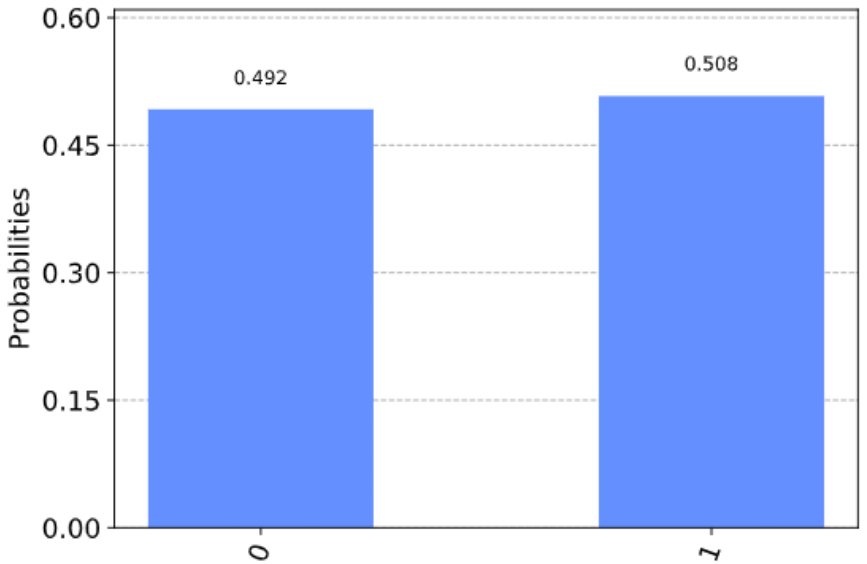

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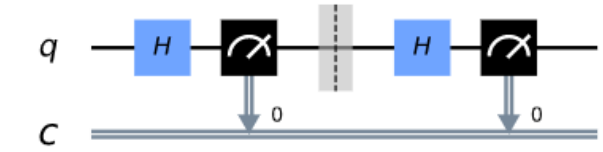

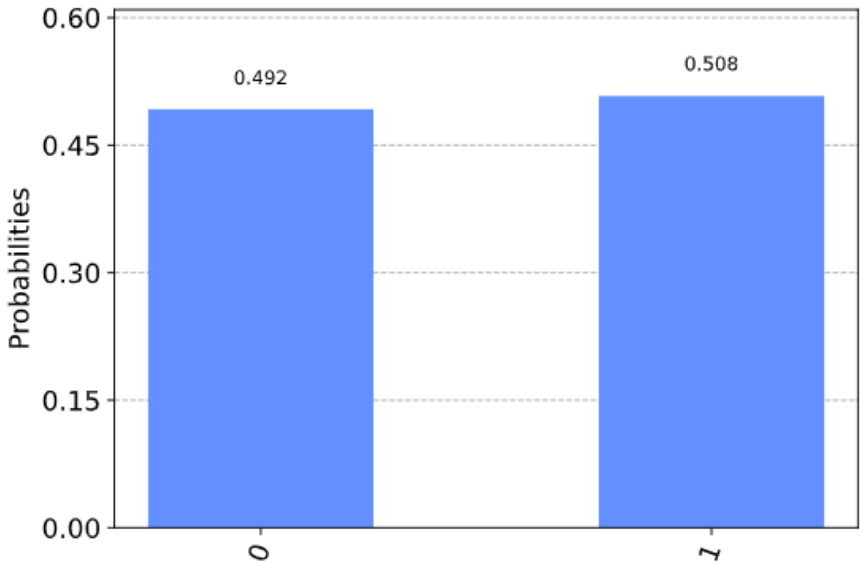

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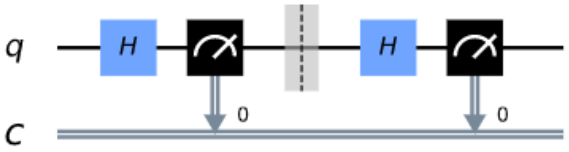

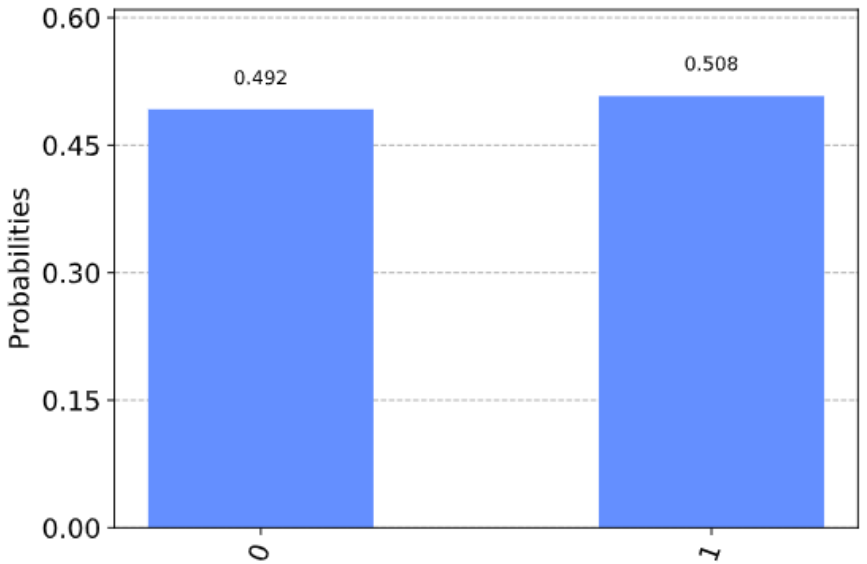

  - We see that Bob in this case has a 50% chance of measuring 0 (or 1), so Alice and Bob realize the interception of the message.
  - The quantum key distribution protocol relies on repeating this process enough times that Eve has virtually no chance of making her interception undetectable.

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  - If the keys match, they can be sure (with a small margin of error though) that the transmission is successful and secure.

Post-Quantum era

THE NATIONAL LAW REVIEW

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Volume XI, Number 211

65
NEW ARTICLES

Preparing for the Post-Quantum Migration: A Race to Save the Internet

Thursday, July 29, 2021

Most people don't know or care to know, about cryptography. Without cryptography, the internet privacy that we all rely on for transmitting virtually all forms of digital communication would be insecure from attackers. Our current encryption methods are threatened by the breakthrough in quantum computing. Unless proactive steps are taken to mitigate this threat, large-scale quantum computers will tear down the backbone of the internet, secure communications.

Article By

Robert M. Botkin

Womble Bond Dickinson (US) LLP
HeyDataData

WOMBLE BOND DICKINSON

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- You are most welcome to join the African Quantum community through various initiatives : OneQuantum Africa, ASFAP, QWorld branches, ...

Quantum Africa

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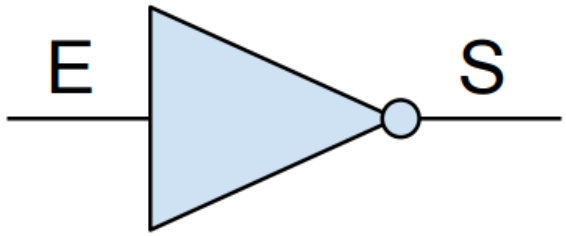
Appendix 1 : classical logic gates, NOT gate

- The NOT gate takes an input E and gives an output S. Its symbol is:



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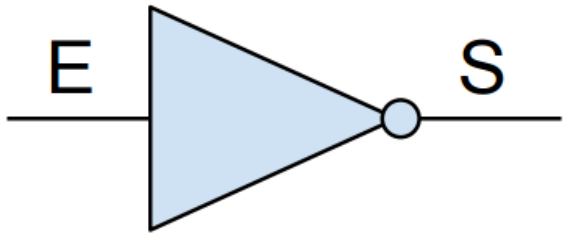

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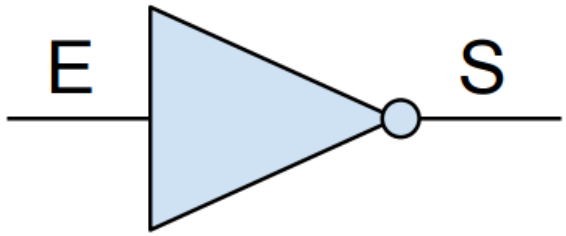

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- This gate is also called an inverter because it inverts the values of bits 0 and 1.

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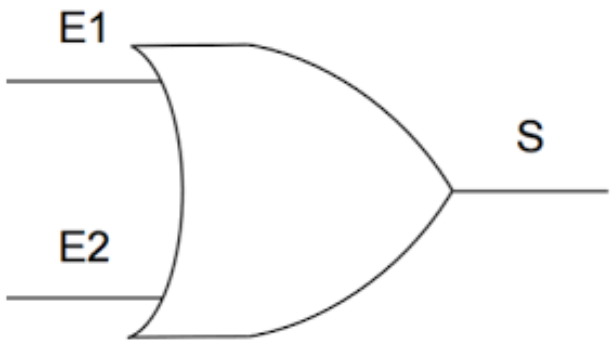

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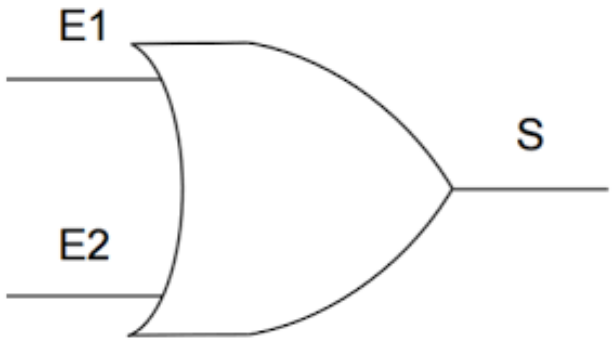

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E_1	E_2	S
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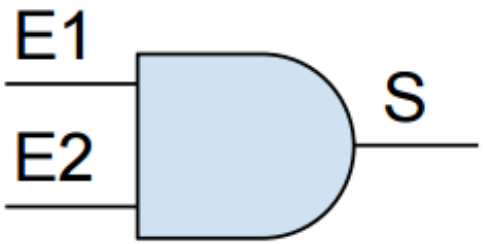

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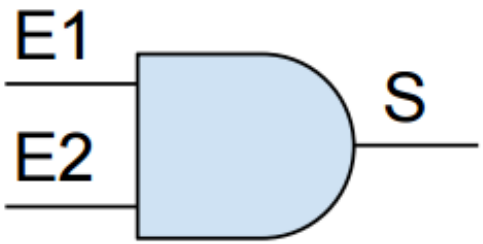

- The truth table is written :

E_1	E_2	S
0	0	0
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Appendix 1 : Composite logic gates

- By combining the 3 simple gates NOT, OR and AND, we define the composite gates NAND, NOR, XOR and XNOR

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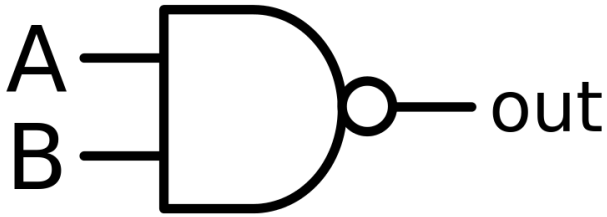

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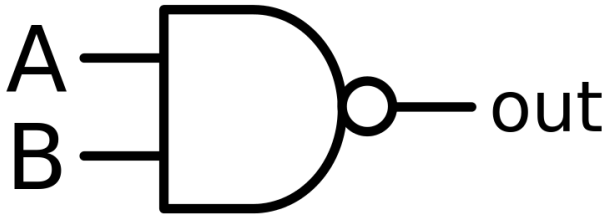

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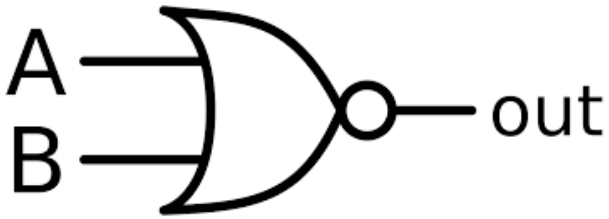

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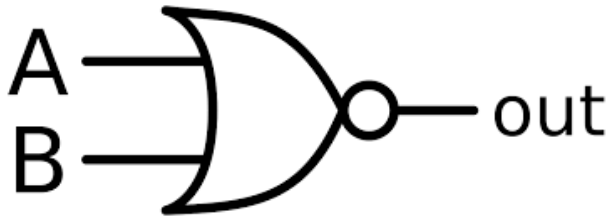

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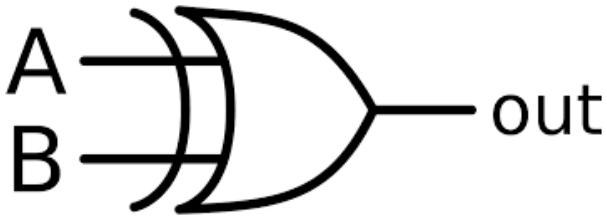

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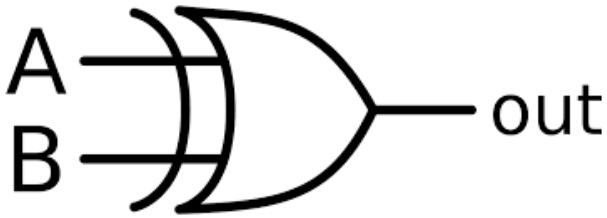

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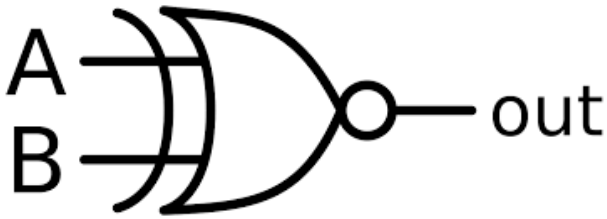

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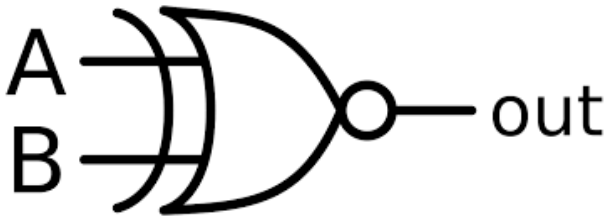

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Appendix 2 : More 1-qubit Quantum gates, Gate R_ϕ

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$$R_\Phi = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix}$$

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- We will see later that the gates S and T are also special cases of the gate R_ϕ

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- We can also notice that the gate U_1 is nothing more than a gate R_ϕ $(\phi = \lambda)$. Indeed :

$$U_1 = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{bmatrix}$$

Appendix 3 : Comparator circuit

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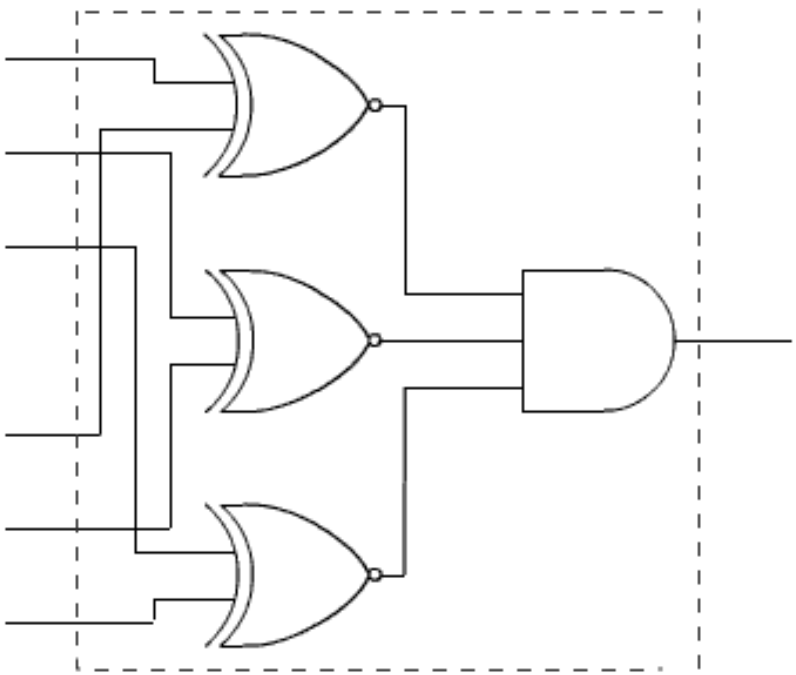
- It is formed by n XNOR gates connected in parallel and an AND gate with n inputs (below is the circuit diagram for $n = 3$).



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- The sum of the next two bits requires the carryover to be taken into account (indicated at the top in red), which amounts to performing 2 additions instead of just one for the first pair of bits ($1 + 0 + 1 = 0$) with a carryover of 1 for the next column.

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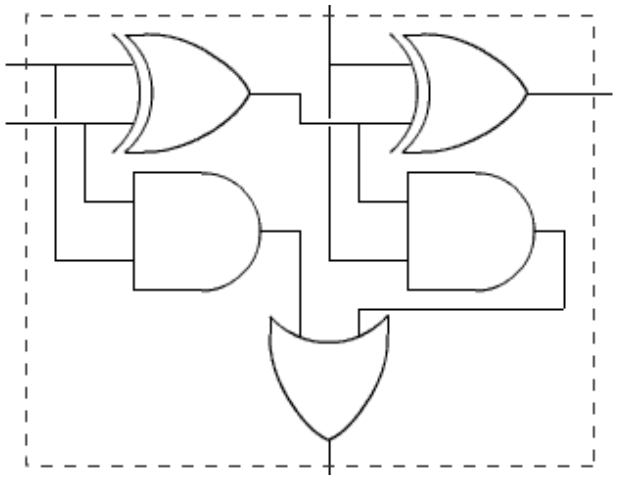

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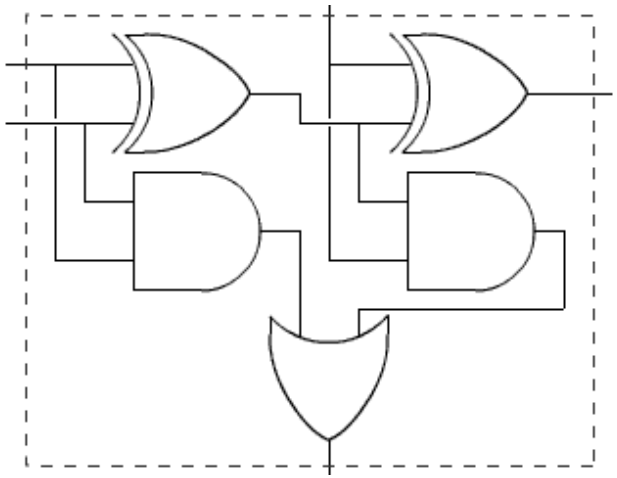

- The circuit which will realize the total addition will be a cascade of full-adder circuits.

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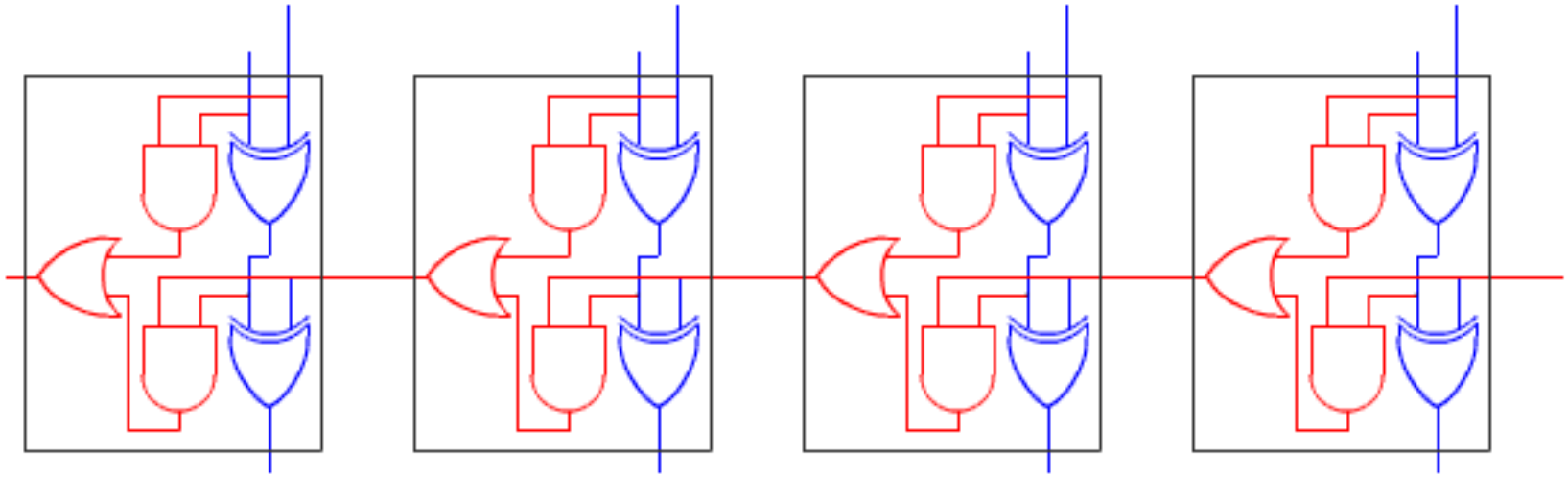
- Cascading of 4 adders for the addition of two 4-bit numbers.



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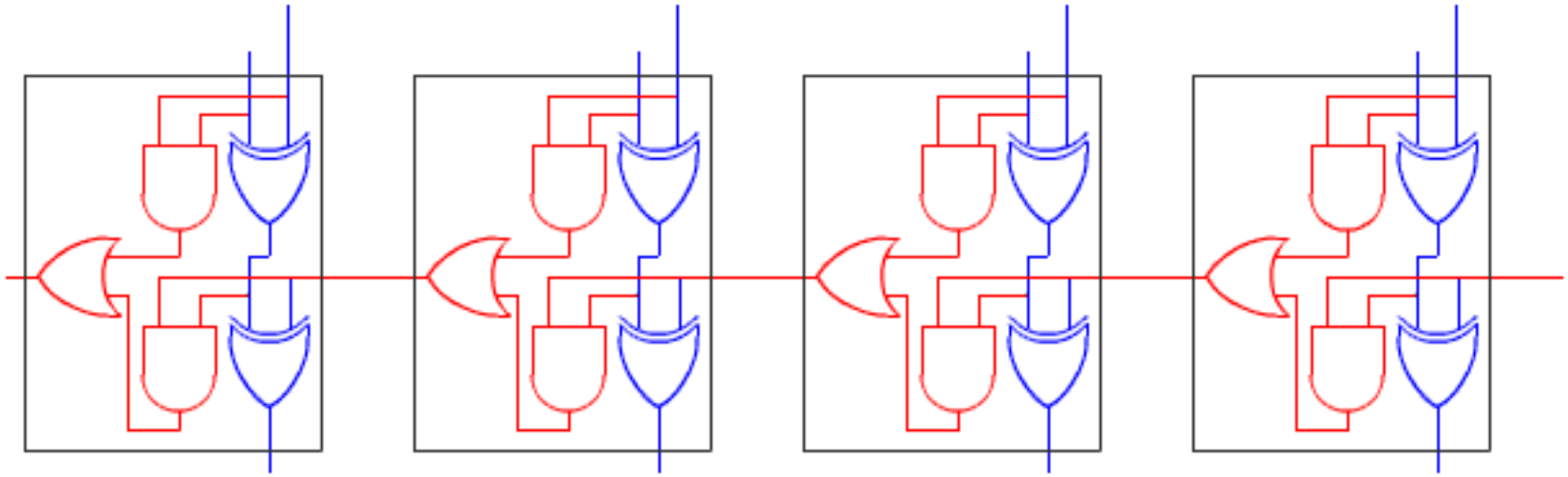

- Exercise: Check the correct implementation of the 4-bit addition given as an example above.

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- If such a gate exists, then it must be able to copy any state from A to B, *i.e.* for 2 states χ_{1A} and χ_{2A} :

$$U|\chi_{1A} \otimes \phi_B\rangle = |\chi_{1A} \otimes \chi_{1B}\rangle$$
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  - $X = 1 \Rightarrow |\chi_1\rangle = |\chi_2\rangle$.
  - $X = 0 \Rightarrow |\chi_1\rangle \perp |\chi_2\rangle$.