

Look Elsewhere Effect

Look Elsewhere Effect



E.G., O. Vitells “Trial factors for the look elsewhere effect in high energy physics”,

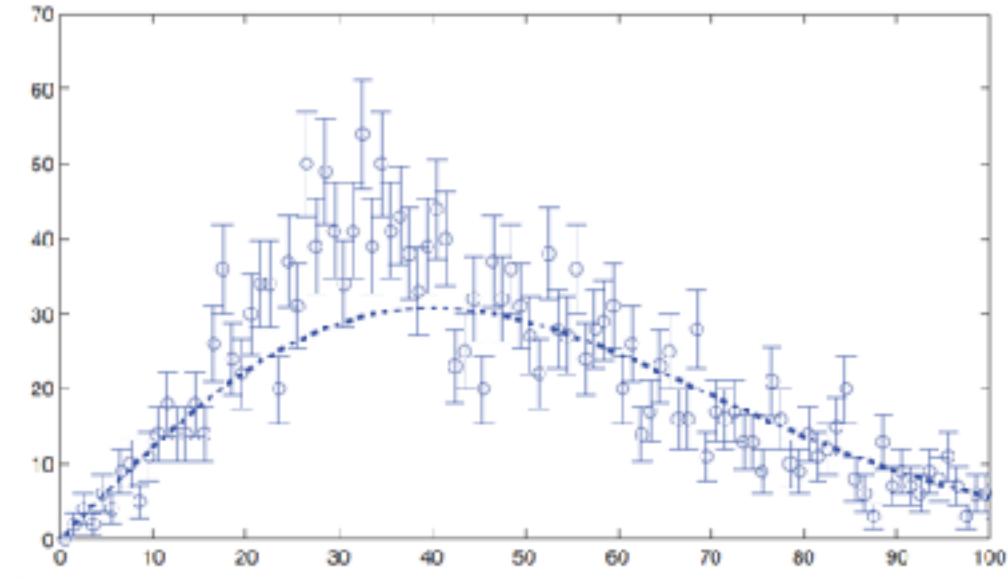
Eur. Phys. J. C 70 (2010) 525

O. Vitells and E. G., Estimating the significance of a signal in a multi-dimensional search,

1669 Astropart. Phys. 35 (2011) 230, arXiv:1105.4355

Look Elsewhere Effect

- Is there a signal here?

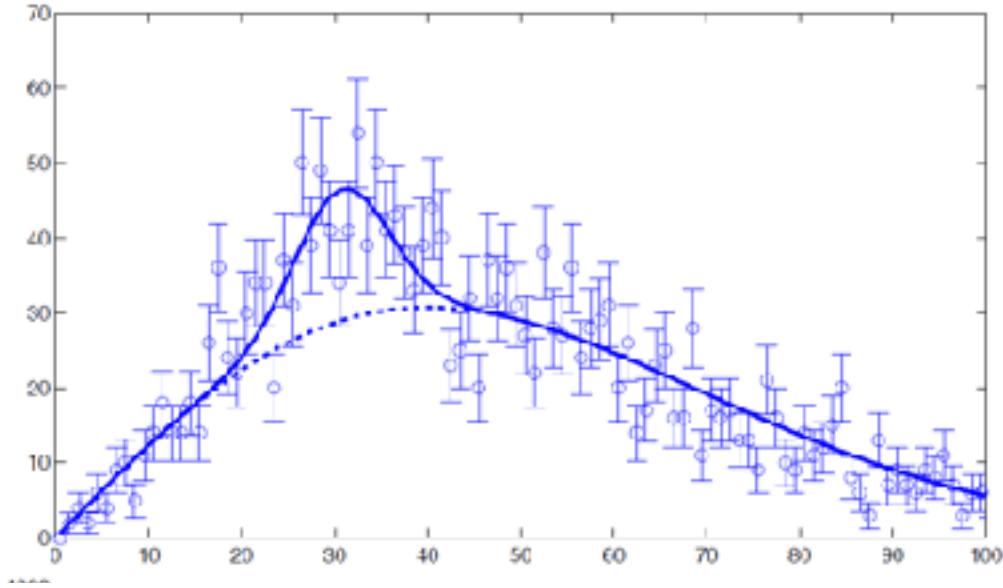


Look Elsewhere Effect

- Looks like a signal at $m=30$
- What is its significance?

Test the BG hypothesis
At $m=30$

$$q_0(\theta) = \begin{cases} -2 \log \frac{L(\mu = 0)}{L(\hat{\mu}, \theta)} \\ 0 \end{cases}$$



$$q_{fix,obs} = -2 \ln \frac{L(b)}{L(\hat{\mu}_{s(m=30)} + b)} \quad Z = \sqrt{q_{0,fix,obs}}$$

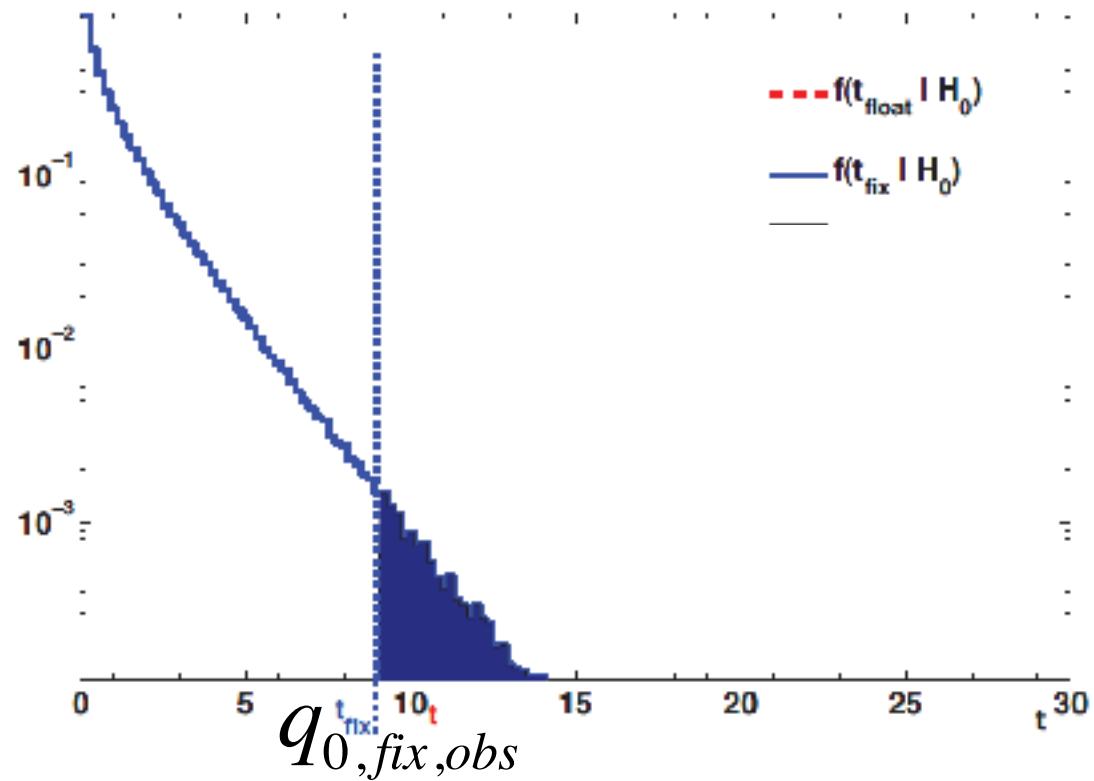


Look Elsewhere Effect

$$q_{0,fix} = -2 \ln \frac{L(\mu=0)}{L(\hat{\mu}s(30)+b)}$$

$$f(q_{0,fix} | H_0) \sim \chi^2$$

$$p_{fix} = \int_{q_{fix,obs}}^{\infty} f(q_0 | H_0) dq_0$$



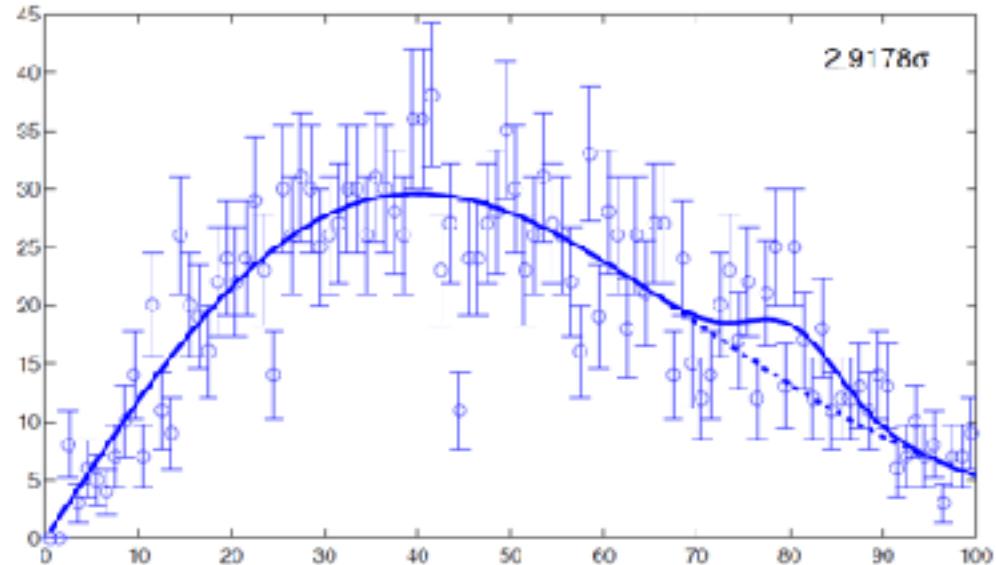
p_{fix} answers the question :

What is the probability to have a fluctuation as or bigger than the observed one?



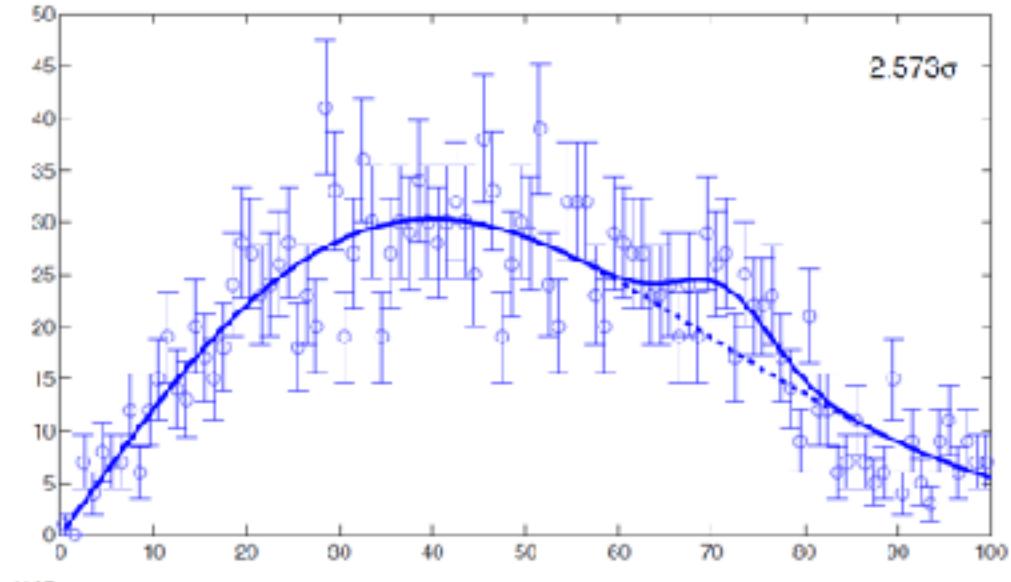
Look Elsewhere Effect

- Would you ignore this signal, had you seen it?



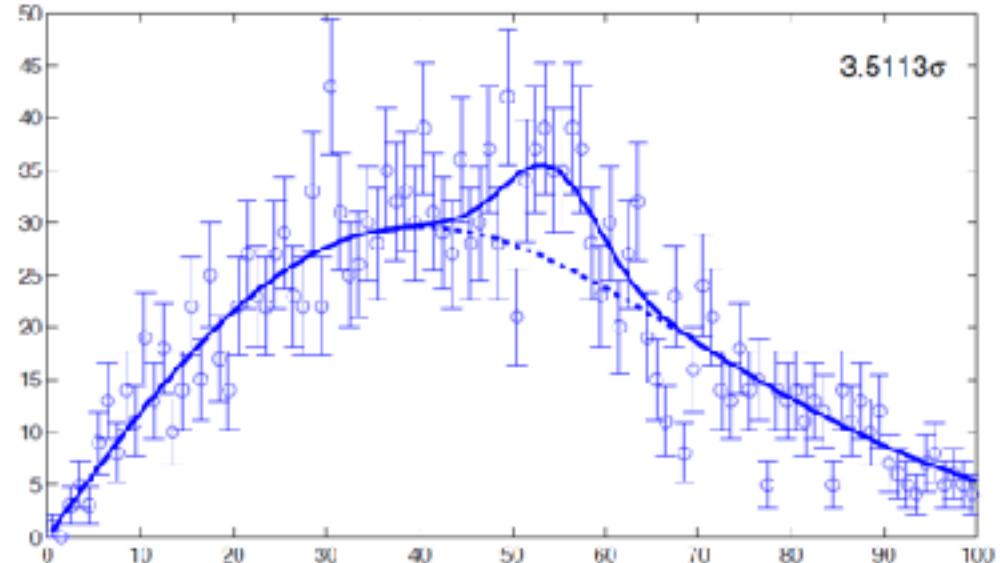
Look Elsewhere Effect

- Or this?



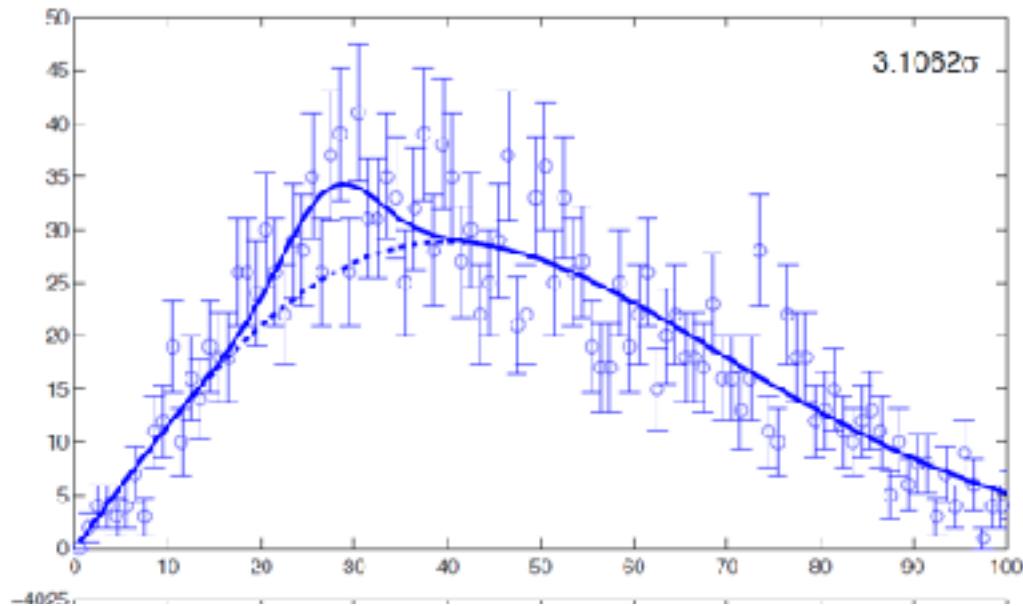
Look Elsewhere Effect

- Or this?



Look Elsewhere Effect

- Or this?
- Obviously NOT!
- ALL THESE "SIGNALS" ARE BG FLUCTUATIONS



The right question :

What is the probability to have a fluctuation as or bigger than the observed one

ANYWHERE in the mass search range ?



Look Elsewhere Effect

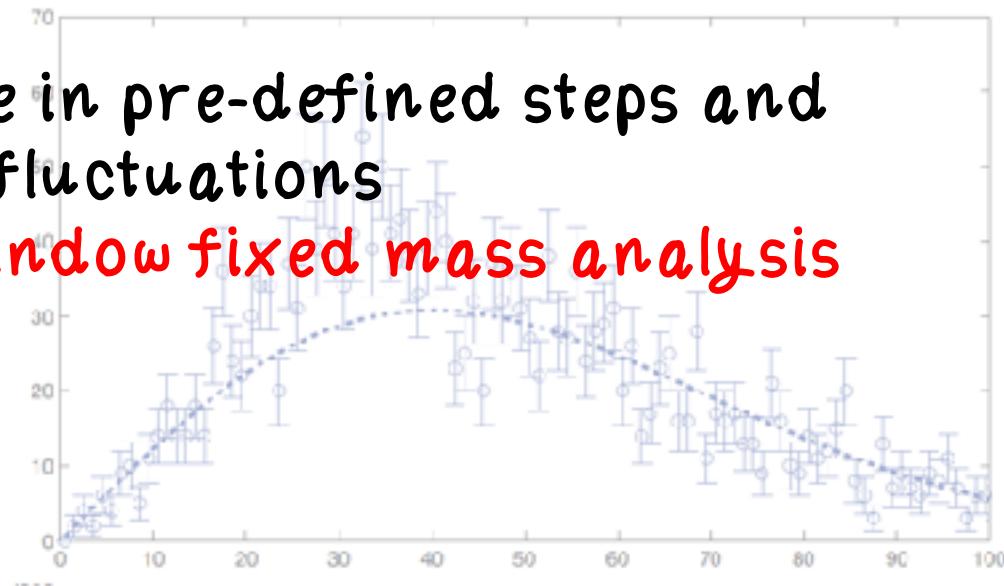
- Having no idea where the signal might be there are two equivalent options

- OPTION I:**

scan the mass range in pre-defined steps and test any disturbing fluctuations

Perform a sliding window fixed mass analysis

$$q_{0, \text{float}} = \max_m (q_0(m))$$



- OPTION II:**

Perform a floating mass analysis

$$q_{0, \text{float}} = q_0(\hat{m}) = -2 \ln \frac{L(b)}{L(\hat{\mu}s(\hat{m}) + b)}$$

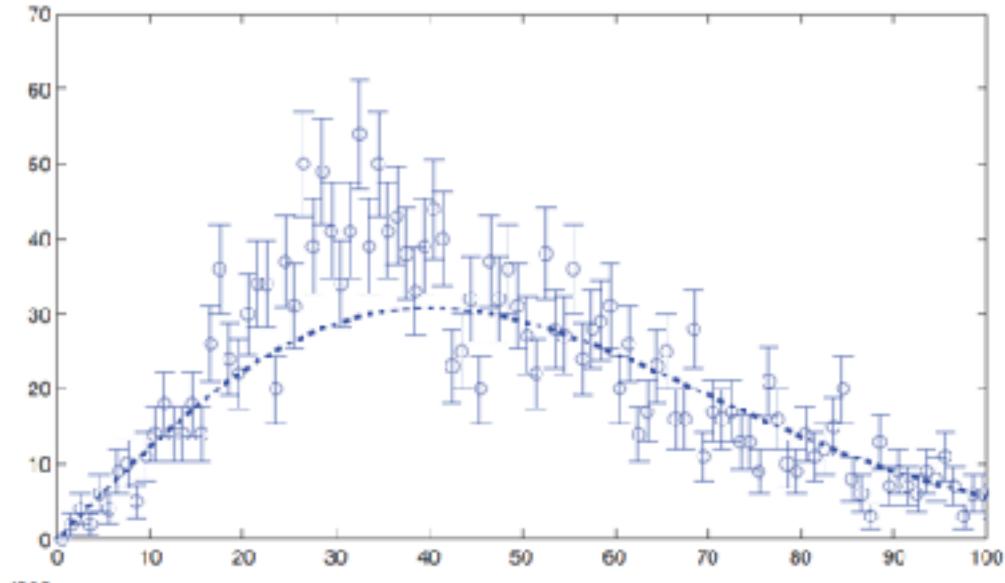
$$p_{\text{float}} = \int_{q_{\text{float}, \text{obs}}}^{\infty} f(q_{0, \text{float}} | H_0) dq_{0, \text{float}}$$



Sliding Window

- Scan and perform a fixed mass analysis at each point

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



- The scan resolution must be less than the signal mass resolution



Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$

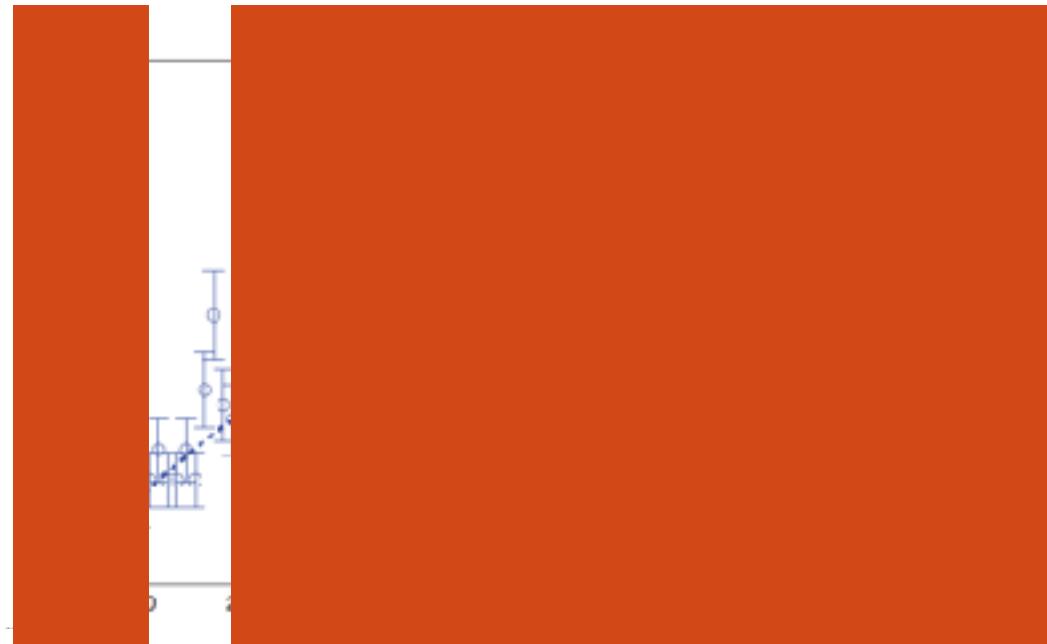


1/30/18



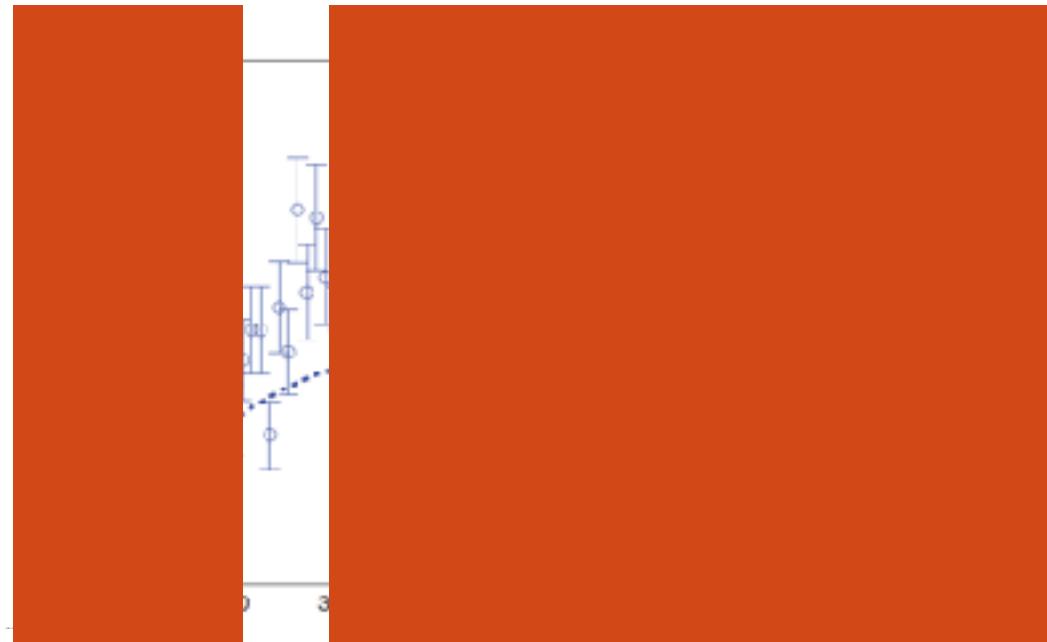
Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$

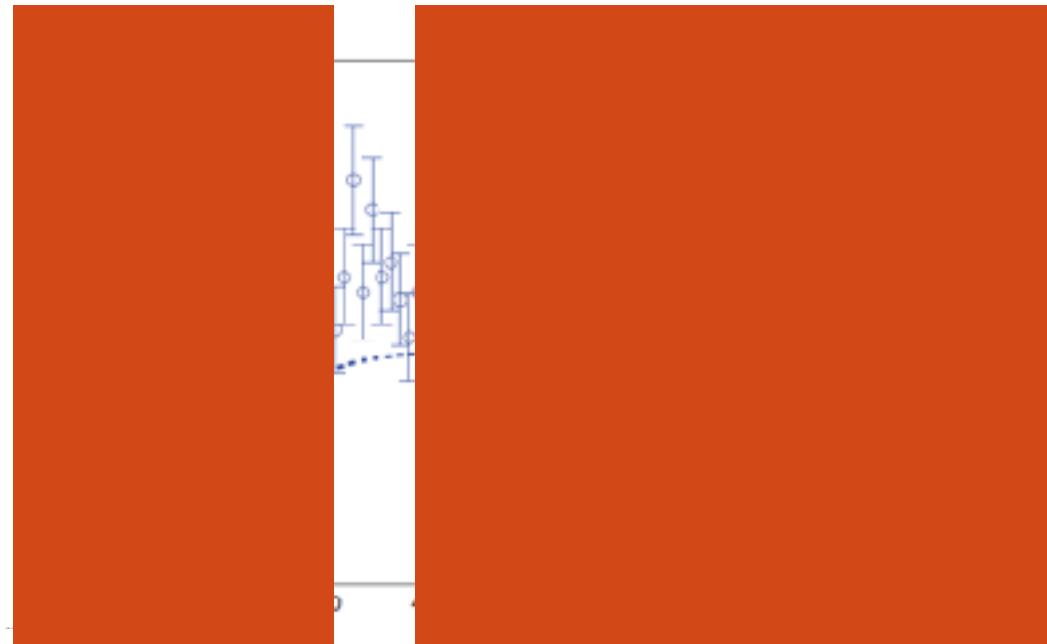


1/30/18



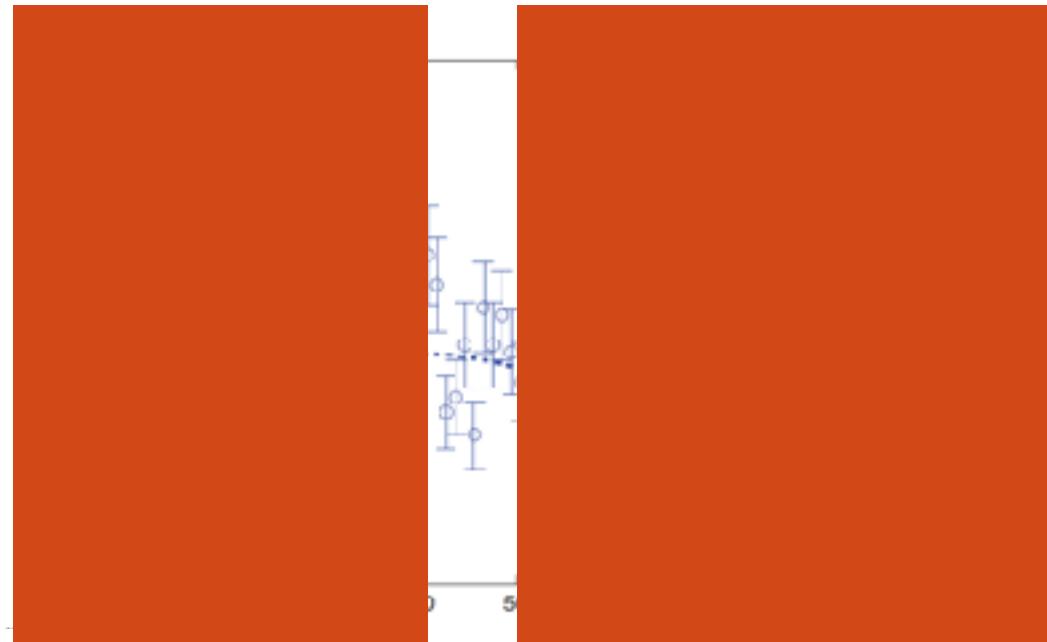
Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



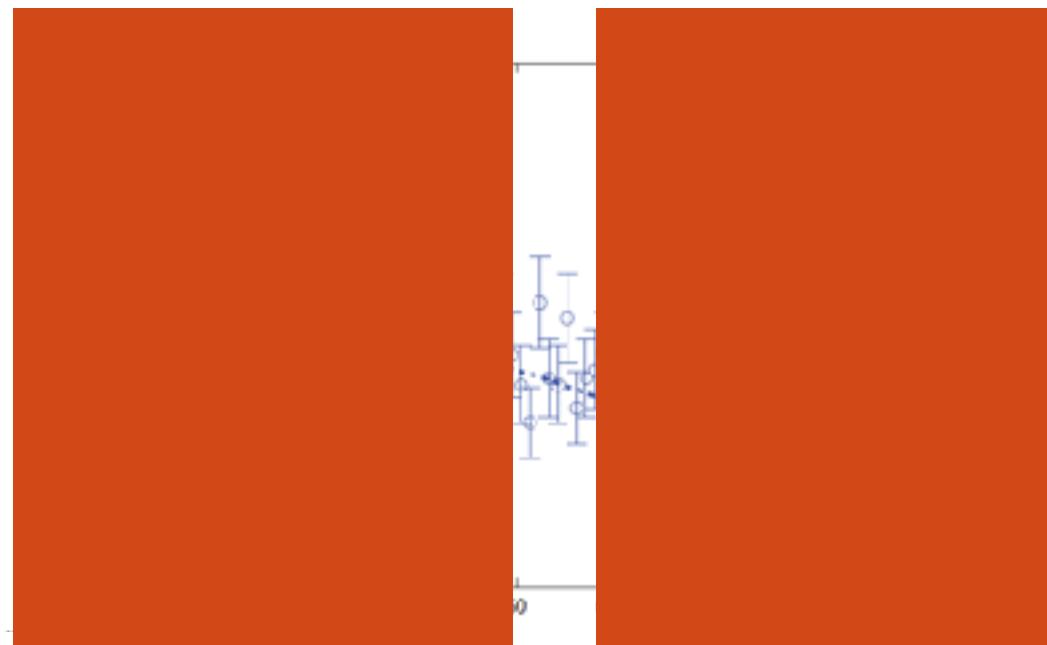
Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



Sliding Window

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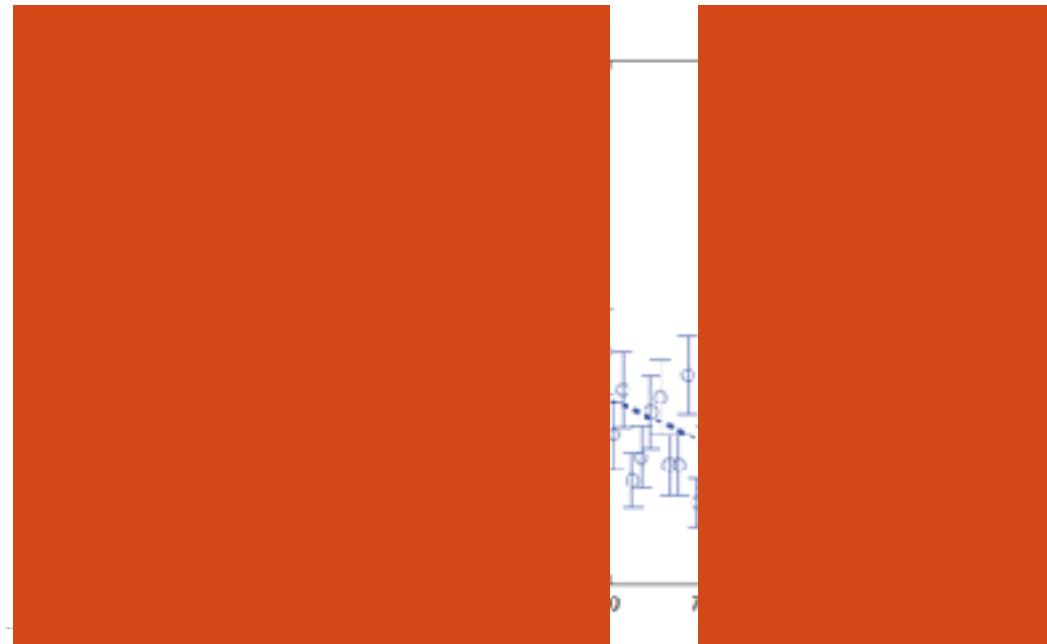


1/30/18



Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$

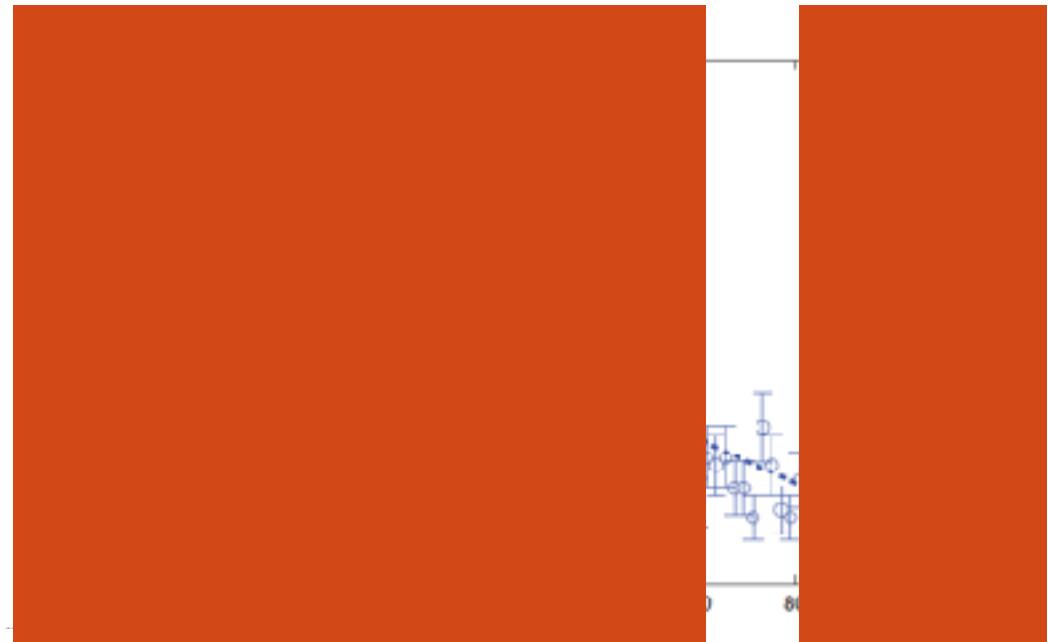


1/30/18



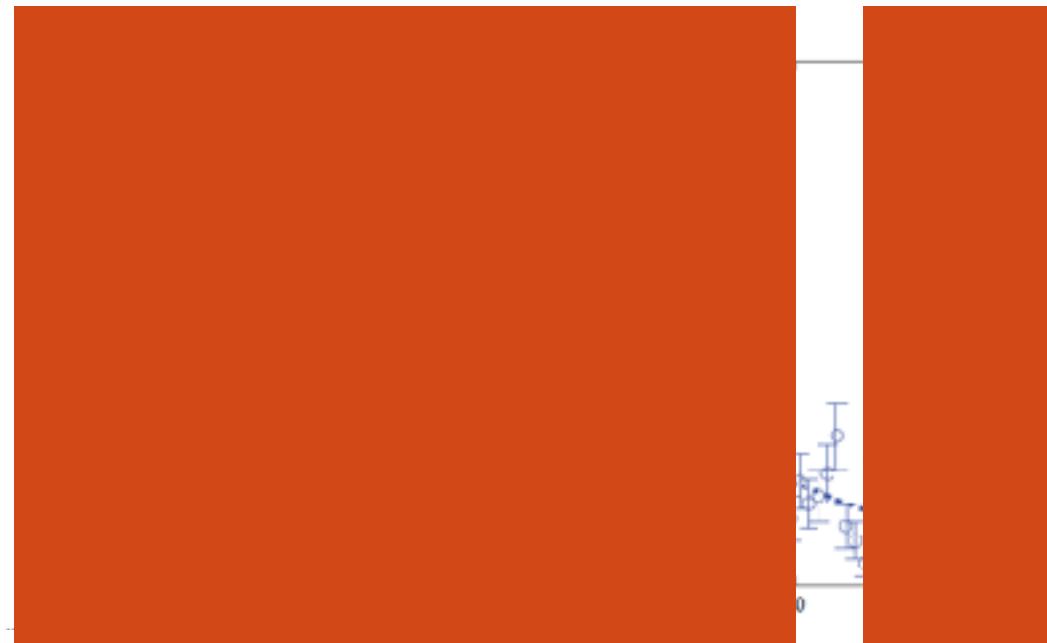
Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



Sliding Window

$$q_0 = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$



1/30/18



Sliding Window

- Assuming the signal can be only at one place
- pick the one with the MAXIMUM SIGNIFICANCE



$$q_{0, \text{float}} = \max_m (q_0(m))$$

1/30/18

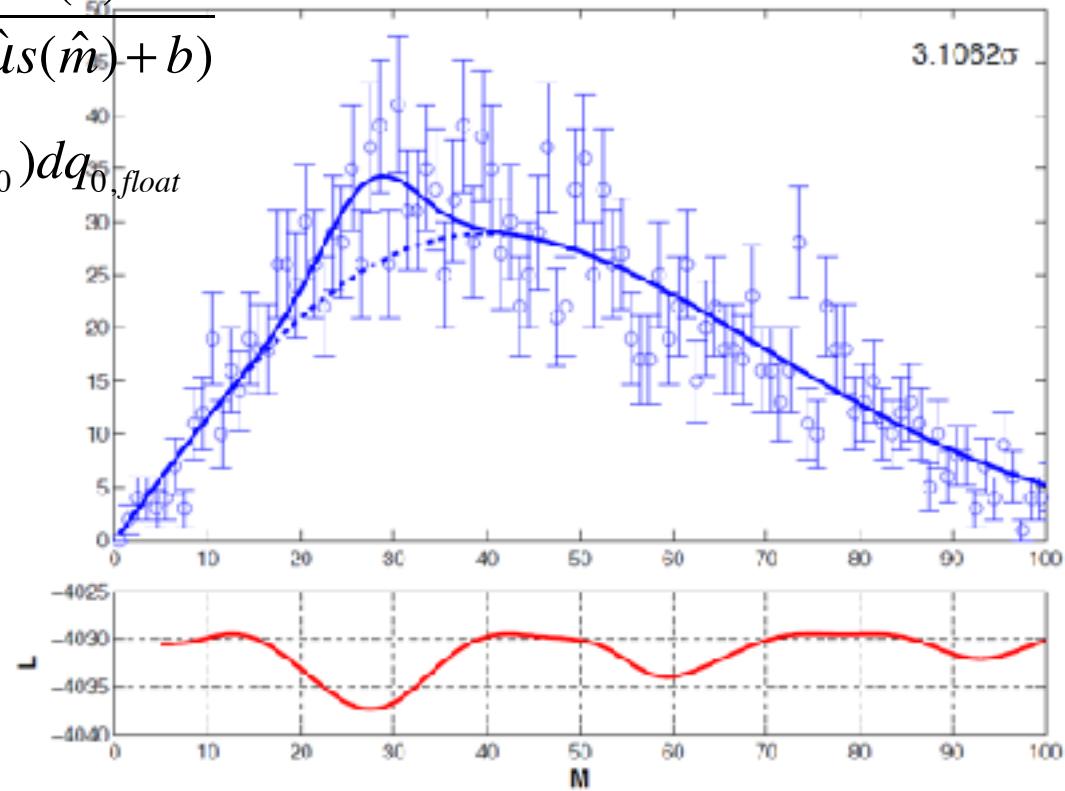


Look Elsewhere Effect: Floating Mass

OPTION II

$$q_{0, \text{float}} = q_0(\hat{m}) = -2 \ln \frac{L(b)}{L(\hat{\mu}_s(\hat{m}) + b)}$$

$$p_{\text{float}} = \int_{q_{\text{float,obs}}}^{\infty} f(q_{0, \text{float}} \mid H_0) dq_{0, \text{float}}$$

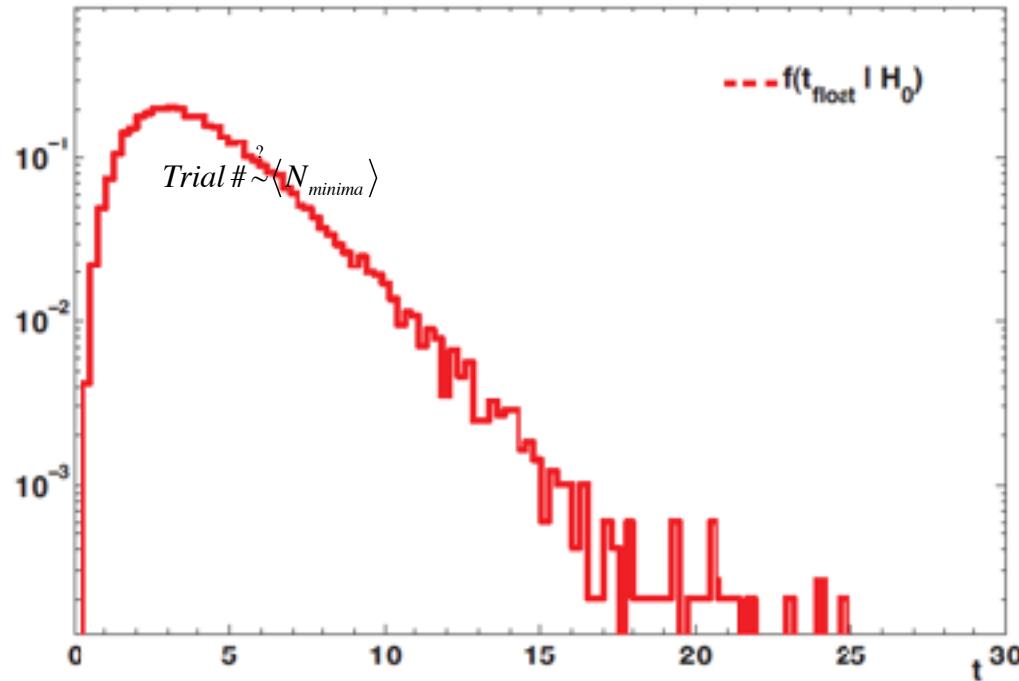


Look Elsewhere Effect

- The distribution $f(q_{\text{float}} | H_0)$ does not follow a chi-squared with 2dof because the mass parameter is not defined under the null hypothesis

for any m_{fix} $q_0(\hat{m}) \geq q_0(m_{\text{fix}})$

The χ^2 distribution is pushed to the right



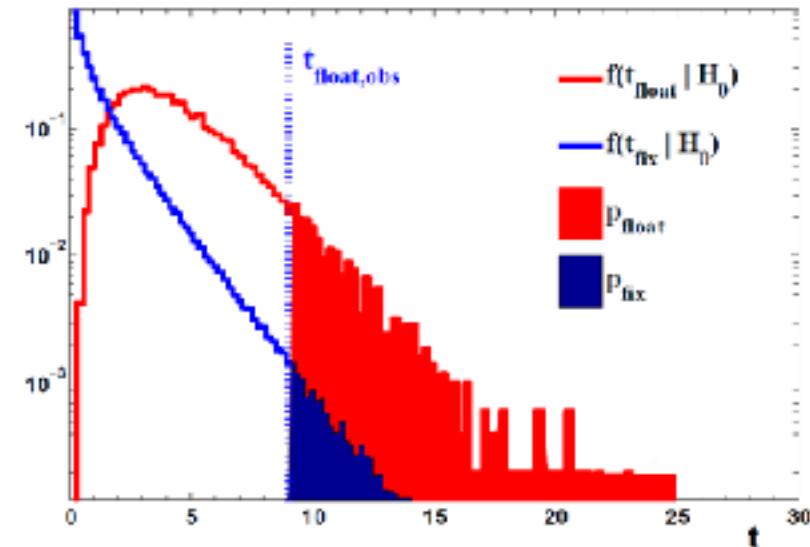
trial#

- Assume a maximal local fluctuation at mass $\hat{m} = 30$
- The observed q_0 is given by

$$q_{0,obs} = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}s(m) + b)}$$

$$p_{fix} = \int_{q_{0,obs}}^{\infty} f(q_{0,fix} | H_0) dq_{0,fix}$$

$$p_{float} = \int_{q_{0,obs}}^{\infty} f(q_{0,float} | H_0) dq_{0,float}$$



$$\text{trial } \# = \frac{p_{float}}{p_{fix}}$$

Can we calculate analytically the floating mass p-value



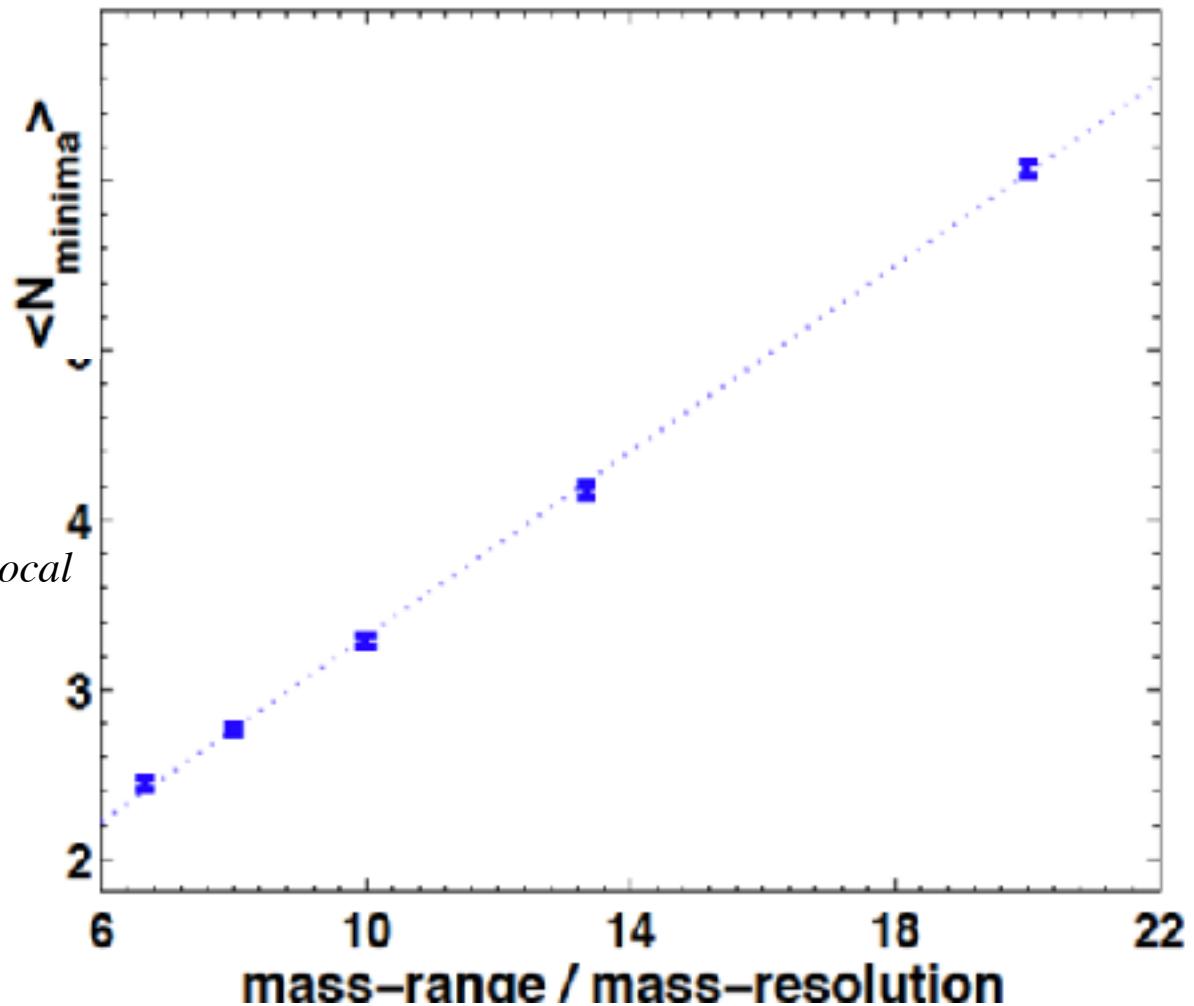
(Wrong) Thumb Rule

$$\langle N_{minima} \rangle \sim \frac{\text{Mass Range}}{\text{Mass Resolution}}$$

Trial # $\sim \langle N_{minima} \rangle$

Trial # $= \langle N_{minima} \rangle p_{local}$?

The answer is NO



The right question :

*What is the probability to have a fluctuation
as or bigger than the observed one
ANYWHERE in the mass search range?*

*Let θ be a nuisance parameter
undefined under the null hypothesis.*

Define $q(\hat{\theta}) = \max_{\theta}(q(\theta))$

Davies (1987) finds, for $c \gg 1$

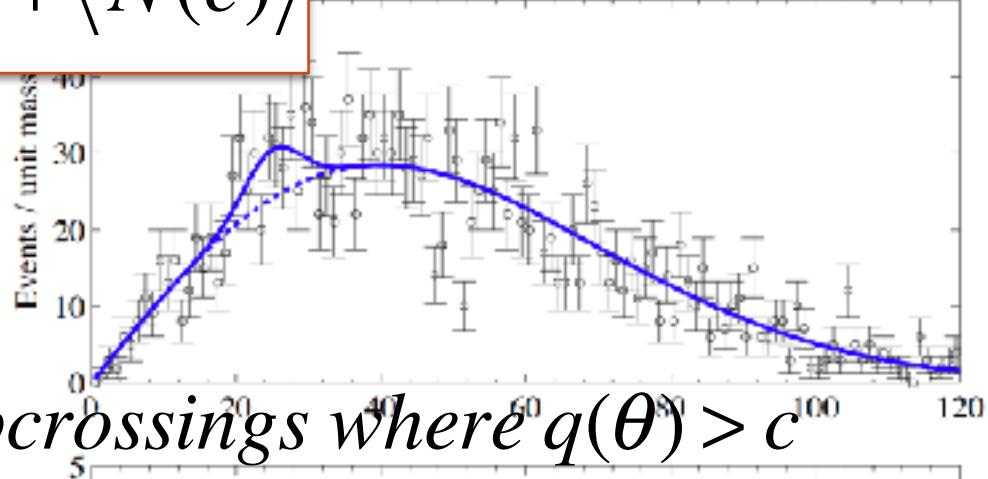
$$P(q(\hat{\theta}) > c) \sim P(\chi^2_1 > c) + \langle N(c) \rangle$$

$\langle N(c) \rangle = \text{Number of upcrossings } q(\theta) > c$



Davies Formula

$$P\left(q(\hat{\theta}) > c\right) \sim P(\chi^2_1 > c) + \langle N(c) \rangle$$



$\langle N(c) \rangle = \text{Number of upcrossings where } q(\theta) > c$

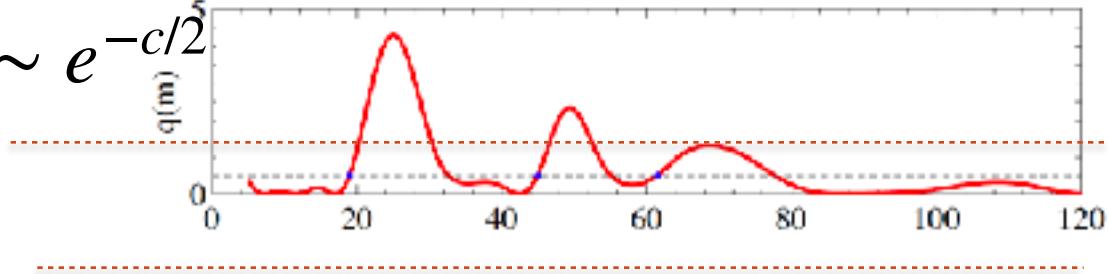
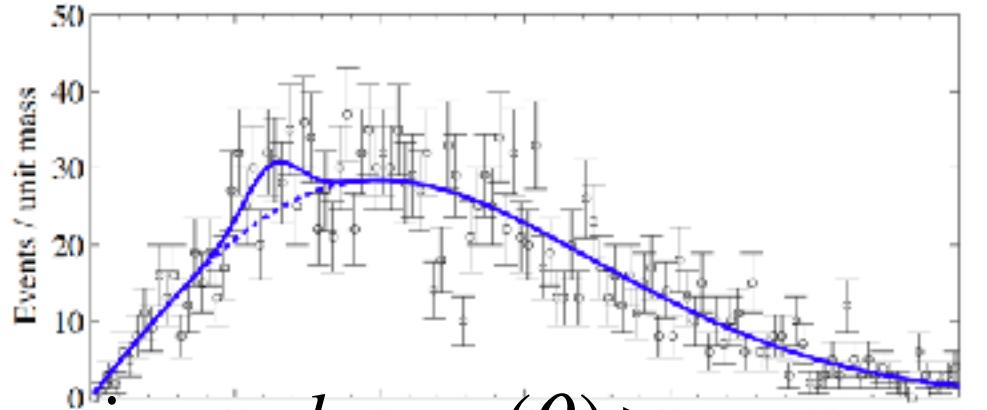
for $c \gg 1 \rightarrow \langle N(c) \rangle \ll 1$



Davies Formula

$\langle N(c) \rangle = \text{Number of upcrossings where } q(\theta) > c$

$$\langle N(c) \rangle \sim P(\chi^2_2 > c) \sim e^{-c/2}$$



$$P(q(\hat{\theta}) > c) \sim P(\chi^2_1 > c) + \langle N(c) \rangle$$

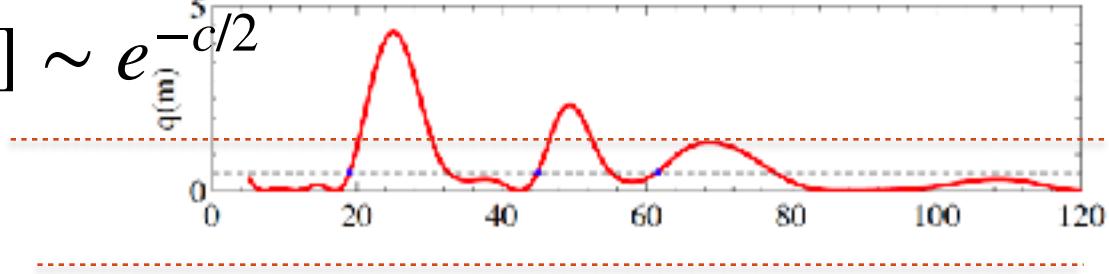
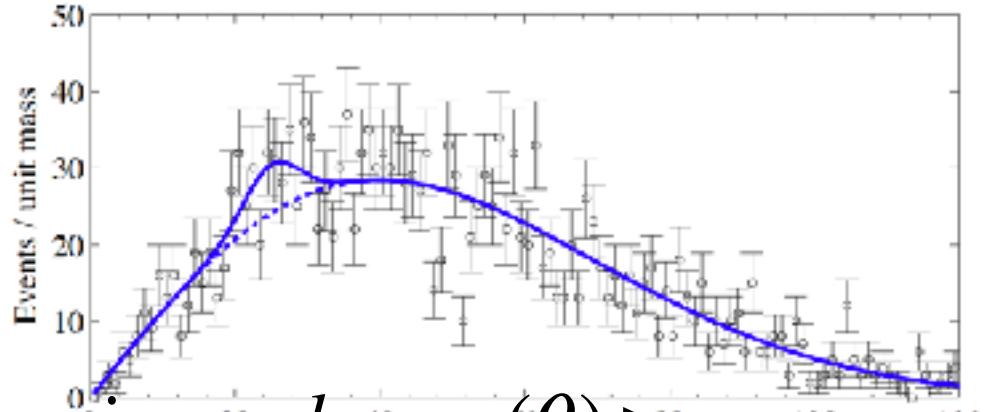
$$P(q(\hat{\theta}) > c) \sim P(\chi^2_1 > c) + \mathcal{N}P(\chi^2_2 > c)$$



Davies Formula

$\langle N(c) \rangle = \text{Number of upcrossings where } q(\theta) > c$

$$\langle N(c) \rangle \sim [P(\chi^2_2) > c] \sim e^{-c/2}$$

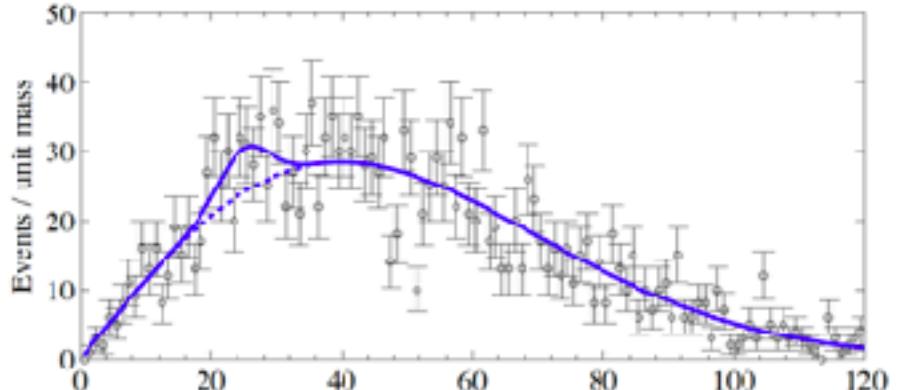


$$\langle N(c) \rangle = \frac{\langle N(c) \rangle}{\langle N(c_0) \rangle} \langle N(c_0) \rangle = e^{-(c-c_0)/2} \langle N(c_0) \rangle$$



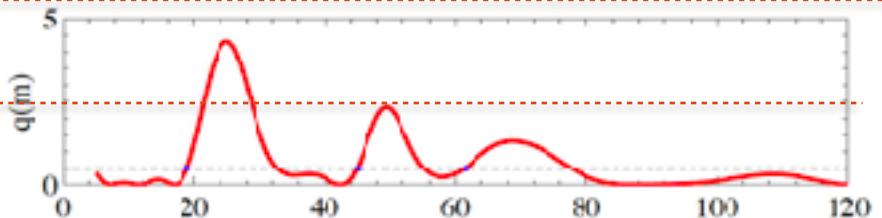
Making Davies Formula Accessible

$$\langle N(c) \rangle = \frac{\langle N(c) \rangle}{\langle N(c_0) \rangle} \langle N(c_0) \rangle = e^{-(c-c_0)/2} \langle N(c_0) \rangle$$



$$\langle N(c) \rangle \ll 1$$

$$\langle N(c) \rangle \sim e^{-c/2}$$



$$P(\hat{\theta} > c) \sim P(\chi^2_1 > c) + \langle N(c_0) \rangle \frac{\langle N(c) \rangle}{\langle N(c_0) \rangle}$$

$$P(\hat{\theta} > c) \sim P(\chi^2_1 > c) + \langle N(c_0) \rangle e^{-(c-c_0)/2}$$

Gross Vitells
Formula



Trial

$$P(\chi^2_1 > c) \xrightarrow{c \gg 1} \sqrt{\frac{2}{c}} \frac{e^{-c/2}}{\Gamma\left(\frac{1}{2}\right)}$$
$$P(\chi^2_2 > c) \xrightarrow{c \gg 1} e^{-c/2}$$

$$\text{trial } \# = \frac{P(q(\hat{\theta}) > c)}{P(q(\theta) > c)} \approx$$
$$\approx 1 + \mathcal{N} \frac{P(\chi^2_2 > c)}{P(\chi^2_1 > c)} \Rightarrow$$

$$\text{trial } \# \approx 1 + \mathcal{N} \sqrt{\frac{c}{2}} \Gamma(1/2) \Rightarrow$$

$$\text{trial } \# \approx 1 + \sqrt{\frac{\pi}{2}} \mathcal{N} Z_{fix}$$



Example: The 750 GeV Resonance

Spin 0 2015

Largest significance

$m_x \sim 750 \text{ GeV}$, $\Gamma_x \sim 45 \text{ GeV}$ (6)

Local $Z = 3.9\sigma$

Any peak with $Z > 3.8\sigma$
with $m=500-2000$ will draw our attention

$$P_{global}(u) \approx p_{local}(u) + E(n_{u_0})e^{\frac{u_0-u}{2}}$$

$$p_{local} = 5 \cdot 10^{-5}$$

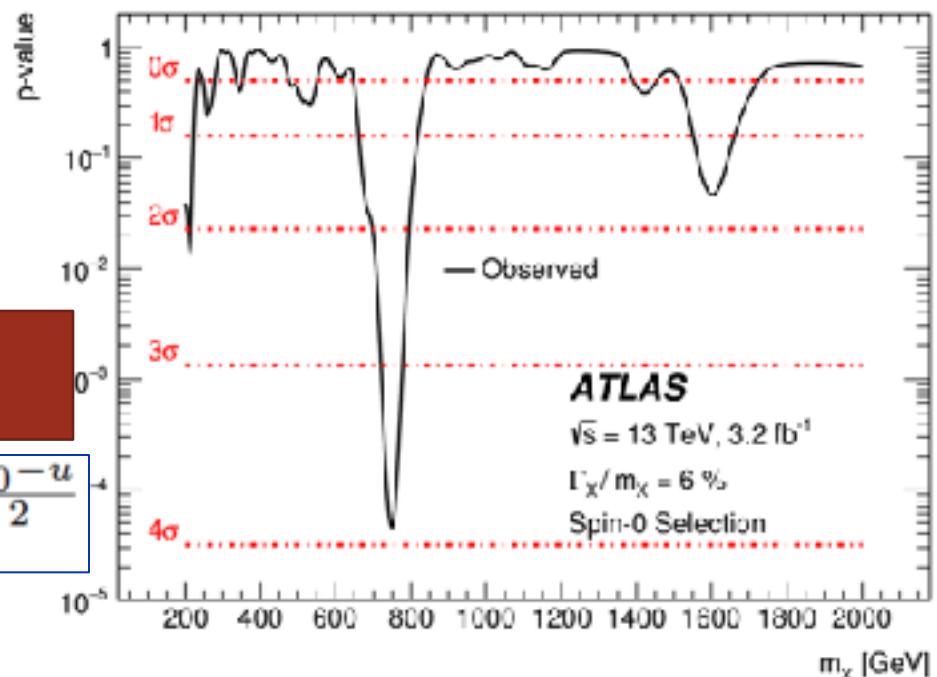
$$u_0 = 0$$

$$n_{u_0} = 7 \pm 2.6$$

$$u = Z^2 = 3.9^2 = 15.2$$

$$p_{global} = 5 \cdot 10^{-5} + (7 \pm 2.6)e^{-15.2/2} = (2.2 - 4.8)10^{-3}$$

$$Z_{global} \sim 2.7 \pm 0.1\sigma$$



The LEE is even stronger when you consider another dimension
(the width range (0-10% of m) should also be taken into account)

A real life example

$$P(q_0 > u) \leq E[N_u] + P(q_0(0) > u)$$

$$E[N_u] = N_1 e^{-u/2}$$

$$N_1 \cong \langle N_{u_0} \rangle e^{u_0/2}$$

$$P(q_0 > u) = N_1 e^{-u/2} + \frac{1}{2} P(\chi^2_1 > u)$$

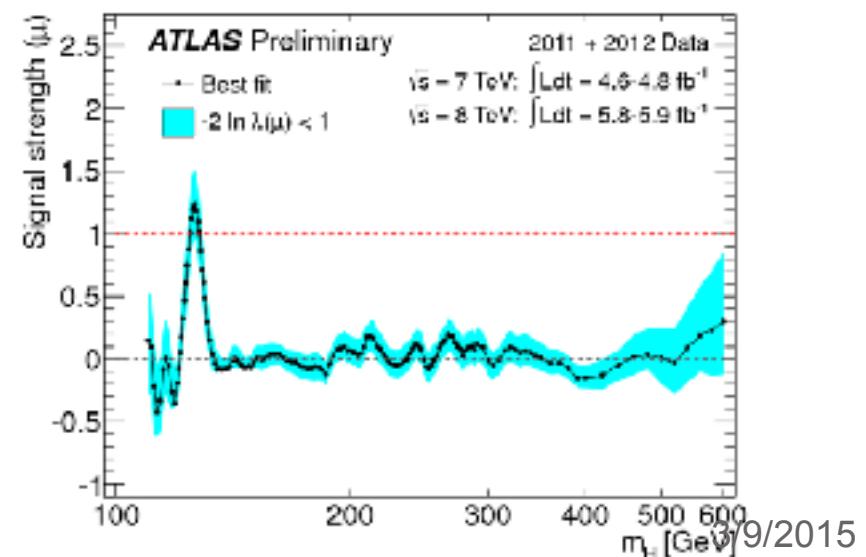
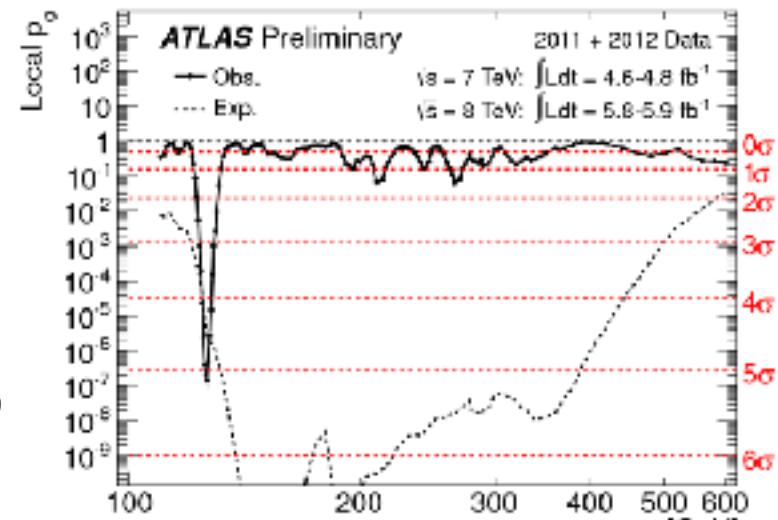
$$p_{global} = N_1 e^{-u/2} + p_{local}$$

$$p_{global} = \langle N_{u_0} \rangle e^{\frac{u_0 - u}{2}} + p_{local}$$

$$N_{u_0=0} = 9 \pm 3$$

$$p_{global} = 9 \cdot e^{-25/2} + O(10^{-7}) = 3.3 \cdot 10^{-5}$$

$5\sigma \rightarrow 4\sigma$ trial#~100



The 2D LEE



Define the Problem

- Let $n = \mu s(m, \Gamma) + b$
- m, Γ are nuisance parameters undefined under the null hypothesis $\mu = 0$
- What is the pdf of

$$\hat{q}_0 \equiv q_0(\hat{m}, \hat{\Gamma}) = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}, \hat{m}, \hat{\Gamma})} = \max_{m, \Gamma} q_0(m, \Gamma)$$

under the null hypothesis



Define the Problem

- To generalize the problem , let Θ be the nuisance parameter, undefined under the null hypothesis, and let us try to find out the pdf of

$$\hat{q}_0 \equiv q_0(\hat{\theta}) = -2 \ln \frac{L(\mu = 0)}{L(\hat{\mu}, \hat{\theta})} = \max_{\theta} q_0(\theta)$$

for which we want to calculate

$$p-value = P\left(\max_{\theta} [q_0(\theta)] \geq u\right), \quad u = Z^2$$



Chi Squared Random Field

- For fixed θ $q_0(\theta) = -2 \ln \frac{L(\mu=0)}{L(\hat{\mu},\theta)} \sim \chi^2_1$
- $q_0(\theta)$ is a chi squared random field over the space of θ
(a random variable indexed by a continuous parameter(s))
- We are interested in

$$\hat{q}_0 \equiv q_0(\hat{\theta}) = -2 \ln \frac{L(\mu=0)}{L(\hat{\mu},\hat{\theta})} = \max_{\theta} q_0(\theta)$$

for which we want to calculate

$$p-value = P\left(\max_{\theta} [q_0(\theta)] \geq u \right), \quad u = Z^2$$



Chi Squared Random Field

- We are only interested in positive signals
(downward fluctuations of the background are not considered as an evidence against the background)

$$q_0(\theta) = \begin{cases} -2\log \frac{\mathcal{L}(\mu = 0)}{\mathcal{L}(\hat{\mu}, \theta)} & q_0(\theta) \sim \frac{1}{2} \chi^2_1 \\ 0 & \end{cases}$$

[H. Chernoff, Ann. Math. Stat. 25, 573578 (1954)]



Chi Squared Random Field

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$$q_0(\theta) = \begin{cases} -2 \log \frac{\mathcal{L}(\mu = 0)}{\mathcal{L}(\hat{\mu}, \theta)} & q_0(\theta) \sim \frac{1}{2} \chi^2_1 \\ 0 & \end{cases}$$

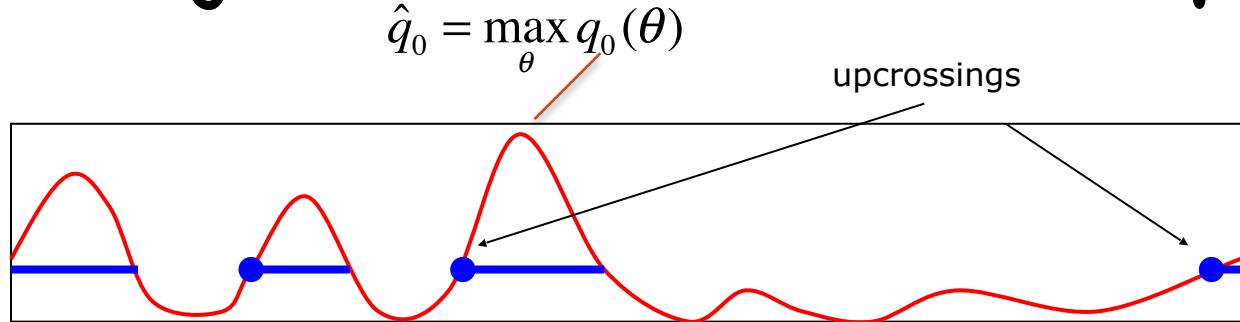
[H. Chernoff, Ann. Math. Stat. 25, 573578 (1954)]

- $q_0(\theta) = \left(\frac{\hat{\mu}(\theta)}{\sigma} \right)^2$ $\hat{\mu}(\theta)$ is a Gaussian Random Field over θ



1-D Random Fields

- In 1-D points where the field becomes larger than u are called upcrossings.



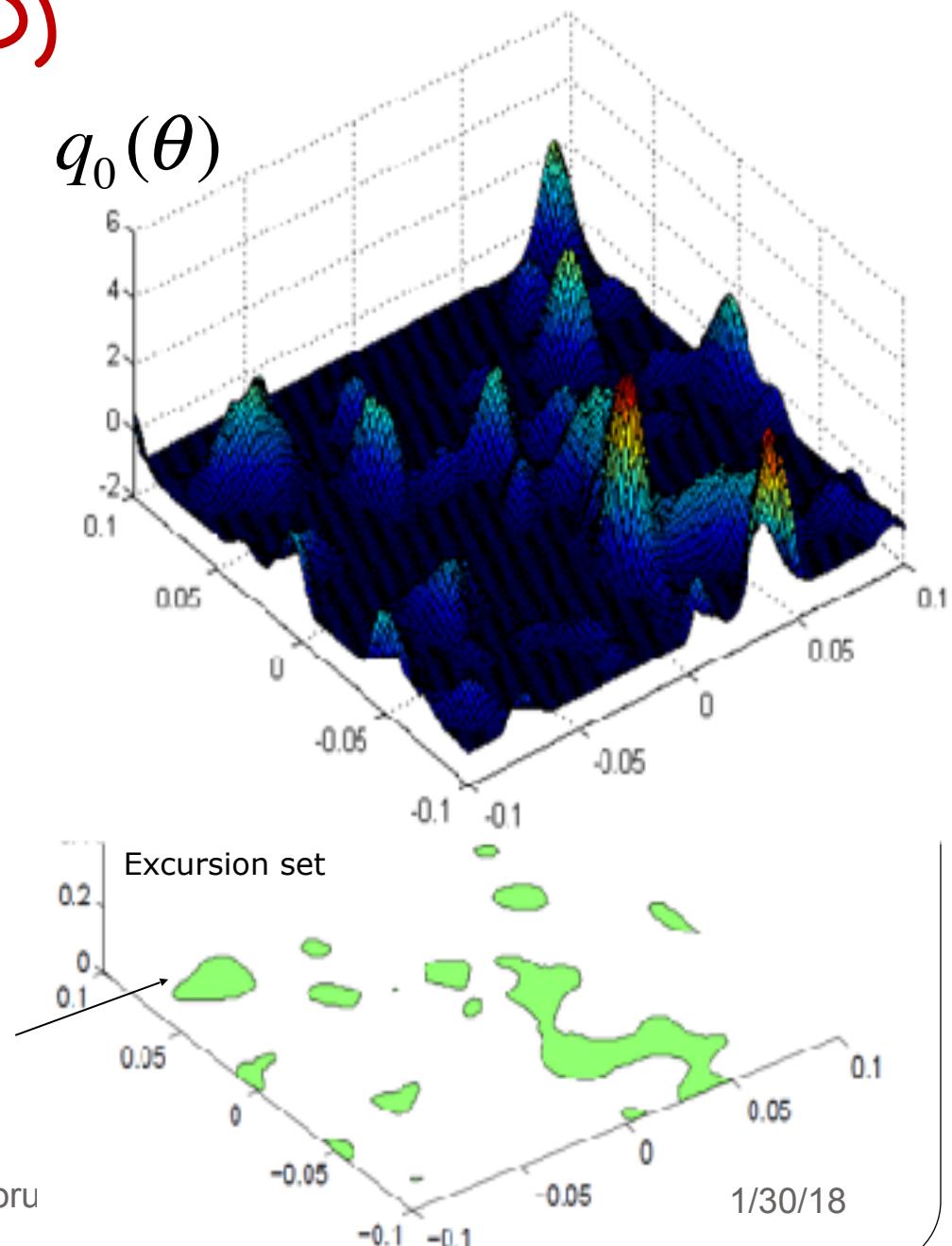
- The probability that the global maximum is above the level u is called **exceedance probability**.

$$(p\text{-value of } q_0(\hat{\theta})) \quad p = P\left(\max_{\theta} [q_0(\theta)] \geq u\right), \quad u = Z^2$$



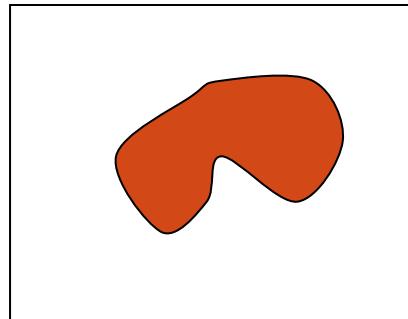
Random fields (>1 D)

- The set of points where the field has values larger than some number u is called the **excursion set** A_u above the level u .

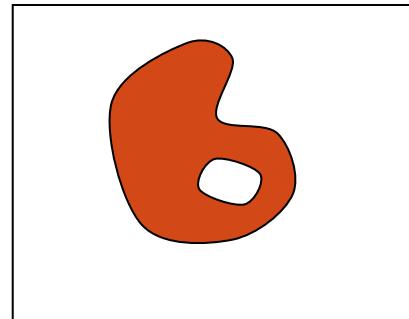


Euler characteristic

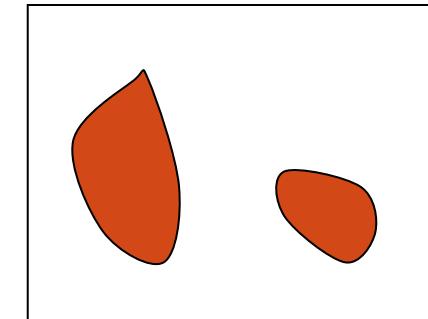
- Number of disconnected components minus number of 'holes'



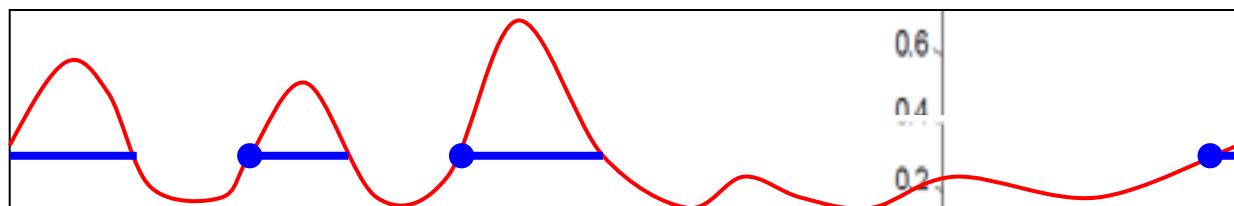
$\varphi=1$



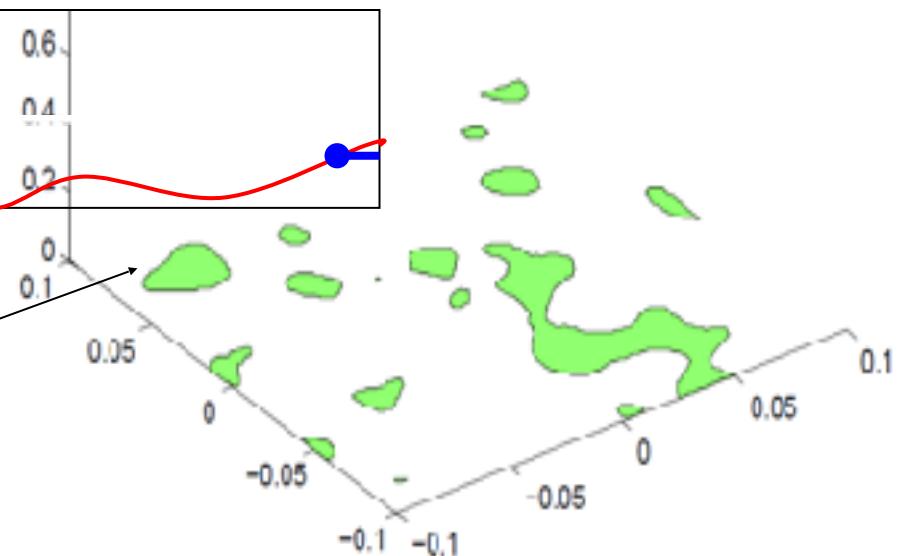
$\varphi=0$



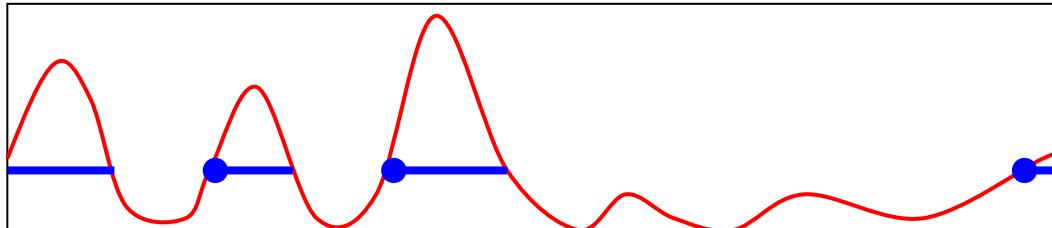
$\varphi=2$



Excursion set



1D Euler characteristic



In 1 dimension:

$$\varphi(A_u) = N_u + \mathbf{1}_{[q_0(0) > u]}$$

$$\begin{aligned} E[\varphi(A_u)] &= E[N_u] + P(q_0(0) > u) \\ &= N_0 P(\chi^2_1 > u) + N_1 e^{-u/2} \end{aligned}$$

$$N_0 = \varphi(\text{manifold}) = 1$$

$$E[\varphi(A_u)] = P(\chi^2_1 > u) + N_1 e^{-u/2}$$

This is Davies Formula

In general for high-level excursions

$$E[\varphi(A_u)] \xrightarrow{u \gg 1} P\left(\max_{\theta} [q_0(\theta)] \geq u\right)$$

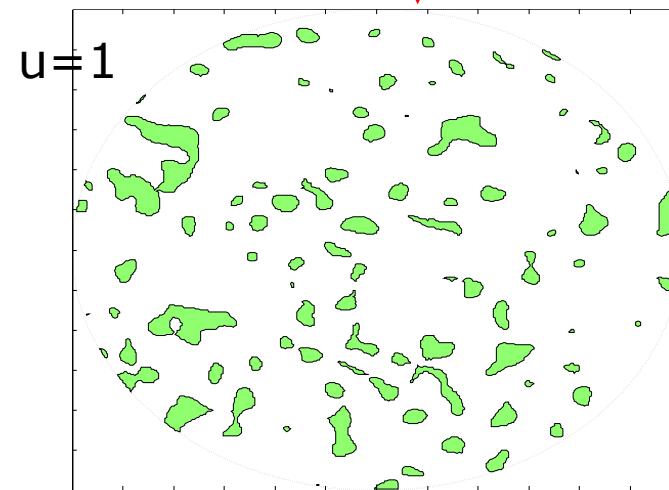
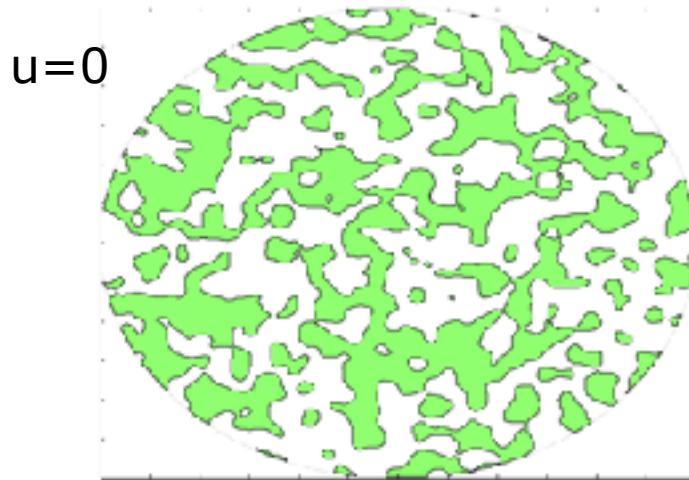


2-d example: search for neutrino sources (IceCube)

For a χ^2 field in 2 dimensions:

$$E[\vartheta(A_u)] = \frac{1}{2} P(\chi^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2 \sqrt{u}) e^{-u/2}$$

Estimate $E[\phi]$ at two levels, e.g. 0 and 1, and solve for N_1 and N_2



From 20 bkg. Simulations:

$$\langle \varphi_0 \rangle = 33.5 \pm 2$$

$$\langle \varphi_1 \rangle = 94.6 \pm 1.3$$

↓

$$N_1 = 33 \pm 2$$

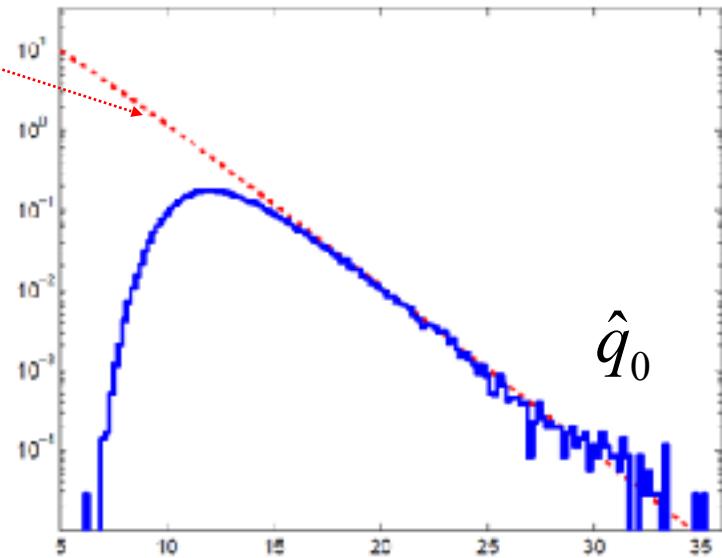
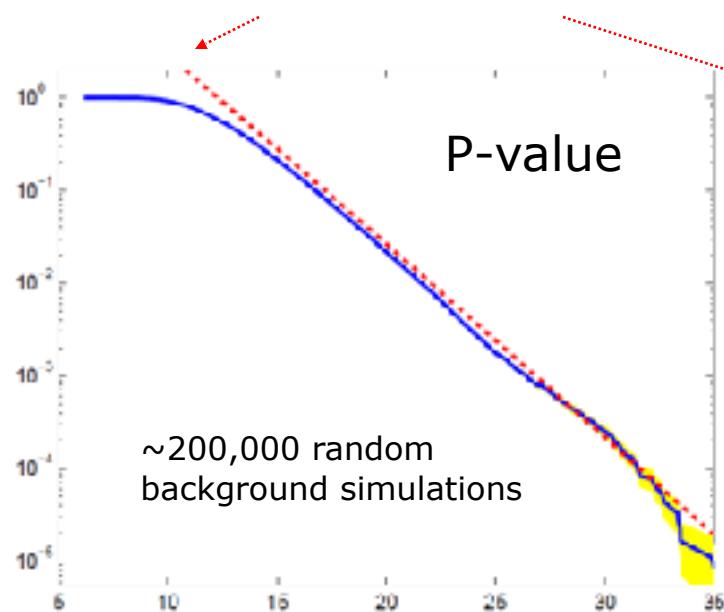
$$N_2 = 123 \pm 3$$



2-d example: search for neutrino sources (IceCube)

$$E[\vartheta(A_u)] = \frac{1}{2}P(\chi^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2 \sqrt{u}) e^{-u/2}$$

$$\begin{aligned}\mathcal{N}_1 &= 33 \pm 2 \\ \mathcal{N}_2 &= 123 \pm 3\end{aligned}$$



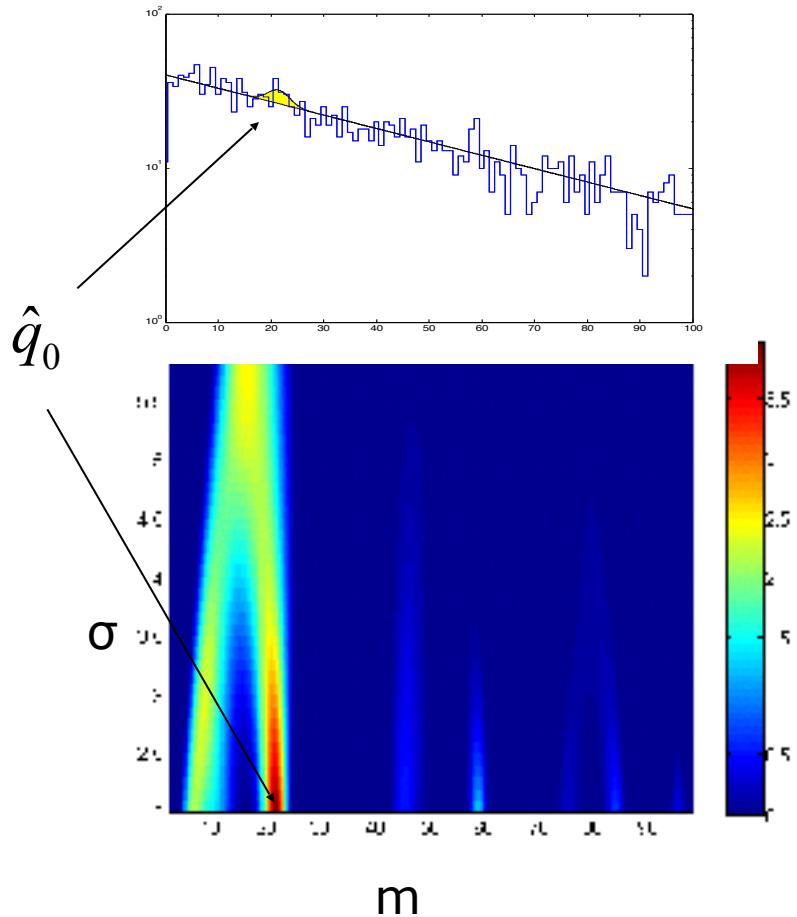
e.g.: $P(\max q_0 > 30) = (2.5 \pm 0.4) \times 10^{-4}$ (estimated)

E.C. Formula : $(2.28 \pm 0.06) \times 10^{-4}$



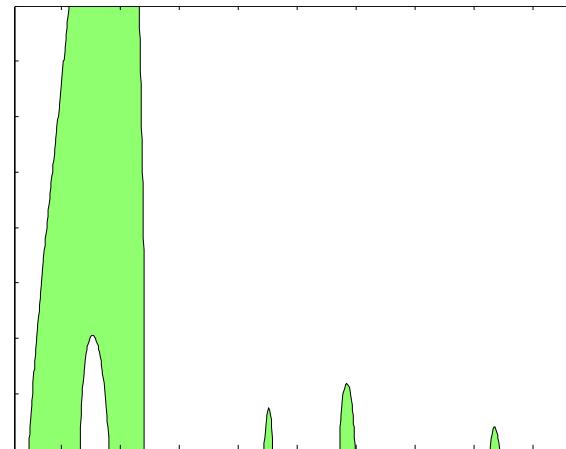
2-D example #2: resonance search with unknown width

- Gaussian signal on exponential background
- Toy model : $0 < m < 100$, $2 < \sigma < 6$
- Unbinned likelihood:

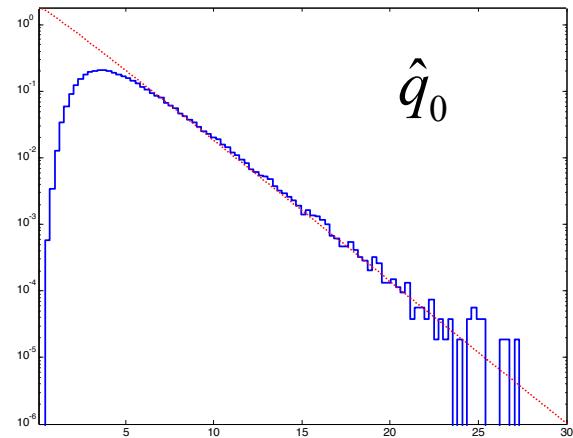


$$\mathcal{L} = \prod_i \frac{N_s f_s(x_i) + N_b f_b(x_i)}{N_s + N_b} \times \text{Poiss}(N | N_s + N_b)$$

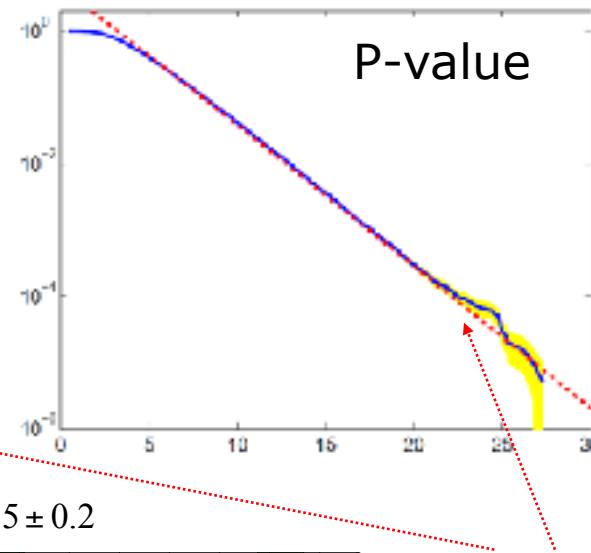
$$f_s(x; m, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-m)^2}{2\sigma^2}} \quad f_b(x) = ce^{-cx}$$



2-D example #2: resonance search with unknown width



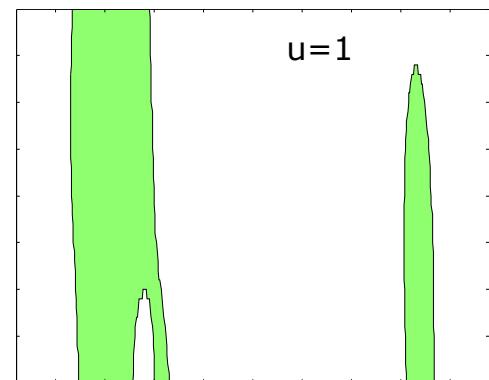
\hat{q}_0



P-value

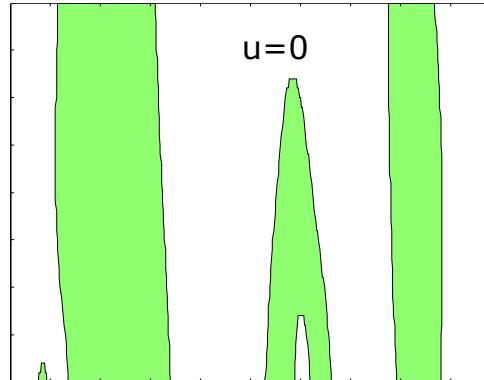
Excellent approximation above the $\sim 2\sigma$ level

$$\langle \varphi_1 \rangle = 3 \pm 0.16$$



$u=1$

$$\langle \varphi_0 \rangle = 4.5 \pm 0.2$$



$u=0$

$$E[\vartheta(A_u)] = \frac{1}{2} P(\chi^2_2 > u) + (\mathcal{N}_1 + \mathcal{N}_2 \sqrt{u}) e^{-u/2}$$

$$\mathcal{N}_1 = 4 \pm 0.2$$

$$\mathcal{N}_2 = 0.7 \pm 0.3$$



2015

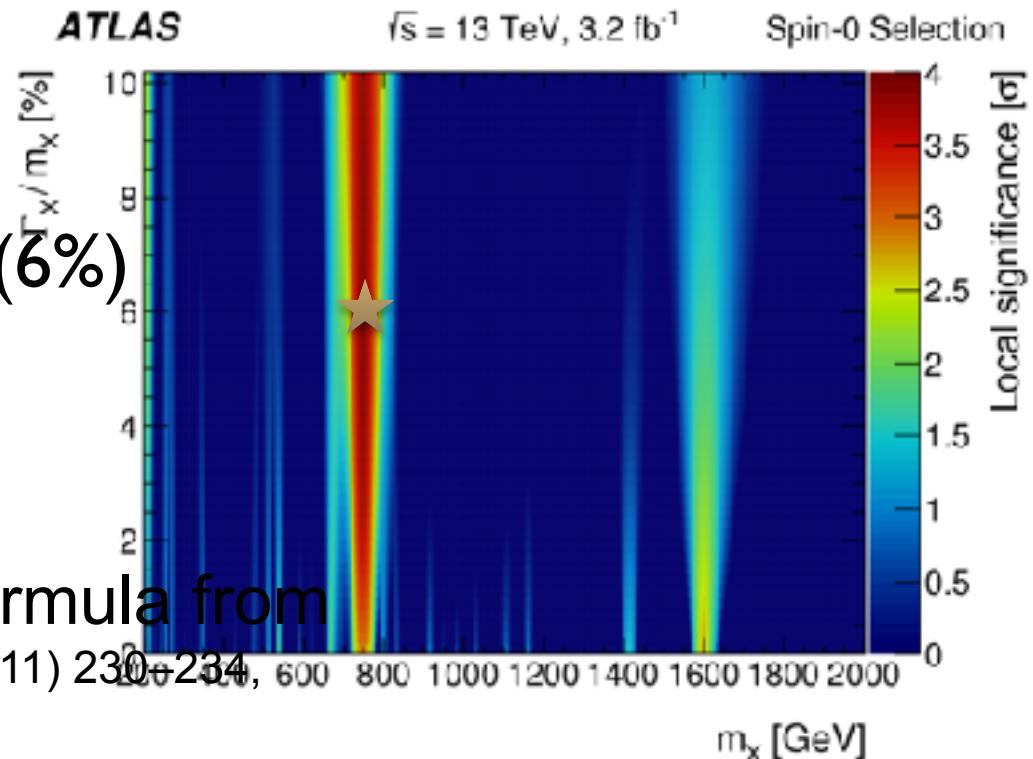
2D Scan

Largest significance

 $m_x \sim 750 \text{ GeV}, \Gamma_x \sim 45 \text{ GeV} (6\%)$ $\text{Local } Z = 3.9\sigma$ $m=200-2000 \text{ GeV}$ $\Gamma_x/m_x=0-10\%$

Use toys or asymptotic formula from

O. Vitells et. al. Astropart. Phys. 35 (2011) 230–234, arXiv:1105.4355



$$Z_{local} = 3.9\sigma$$

$$Z_{global} = 2.1\sigma \quad 2.1\sigma \text{ is not something to write home about}$$



Summary

$$p_{global}(s=1, D=1) \approx E[\vartheta(A_u)] = \frac{1}{2} P(\chi_1^2 > u) + \mathcal{N}_1 e^{-u/2}$$

$$p_{global}(s=1, D=2) \approx E[\vartheta(A_u)] = \frac{1}{2} P(\chi_2^2 > u) + (\mathcal{N}_1 + \mathcal{N}_2 \sqrt{u}) e^{-u/2}$$

- The procedure for estimating the p-value is simple and reliable.
- The Euler characteristic formula provides a practical way of estimating the look-elsewhere effect.
- It is easily expandable to s p.o.i and D NPs (undefined under the null hypothesis)



Thank You

Eilam Gross

