Step, dip, and bell-shape traveling waves in a (2 + 1)-chemotaxis model with traction and long-range diffusion

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Objectives

- Provide the existence conditions of traveling waves in chemotaxis.
- Study the dynamical behaviors of 2D a chemotactic system.

Introduction

Chemotaxis is the ability of cells/bacteria to move towards (away) attractive (harmful or repulsive) chemicals/regions. This phenomenon has been studied in media with no dynamics of their own [1], though recent recent experimental evidences suggest otherwise [2]. Chemotaxis may be used to describe large scale behaviors, and to tackle fertility issues, perform organ repair, manufacture devices with the potential of performing specific tasks such as fat elimination, foraging just to name a few.

Though experimental evidences of traveling bands of matter in chemotactic systems has become a paradigm, theoretical description of these traveling structures akin to traveling waves is still under investigation. It is not exactly very clear how cells aggregate, form patterns and collectively move, but based on known biology, mathematical models describing their spatiotemporal evolutions can be assessed. Keller and Segel [3] were the first to derive a mathematical model for chemotaxis. The solutions of their model shows some drawbacks such as existence of analytical solutions only for singular chemotactic sensitivity, a very problematic assumption since it implies bacterial densities and velocities may grow to infinity. In such a case solutions are unphysical.

To circumvent these issues, a chemotaxis model accounting for traction forces, long-range diffusion, advection, bacterial proliferation and chemoattractant kinetics is considered. Analytical solutions are constructed, their existence conditions provided and their stability is numerically tested with success.

Hypotheses and model description

In our manufacturing, we are interested in describing the collective motion of bacteria placed in fluid presenting a uniform flow rate. The bulk velocity of the medium comprises a contribution emanating from cells while moving. Let denote by $n(\vec{r},t)$ the bacterial density and $c(\vec{r},t)$ the chemoattractant concentration. The conservation laws permit to write [1, 3, 4]

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot\left(n\vec{\delta}_0\right) = D_1\nabla^2 n - D_2\nabla^4 n - \tau_0\vec{\nabla}\cdot\left(n\vec{\nabla}n\right) - \chi_0\vec{\nabla}\cdot\left(n\vec{\nabla}c\right) + rn(\sigma - n), \quad (1a)$$

$$\frac{\partial c}{\partial t} + \vec{\nabla}\cdot\left(c\vec{\delta}_0\right) = D_3\nabla^2 c - \tau_0\vec{\nabla}\cdot\left(c\vec{\nabla}n\right) + \beta_1 n - \beta_3 n^3 - \beta nc. \quad (1b)$$

Results II: The wave profiles

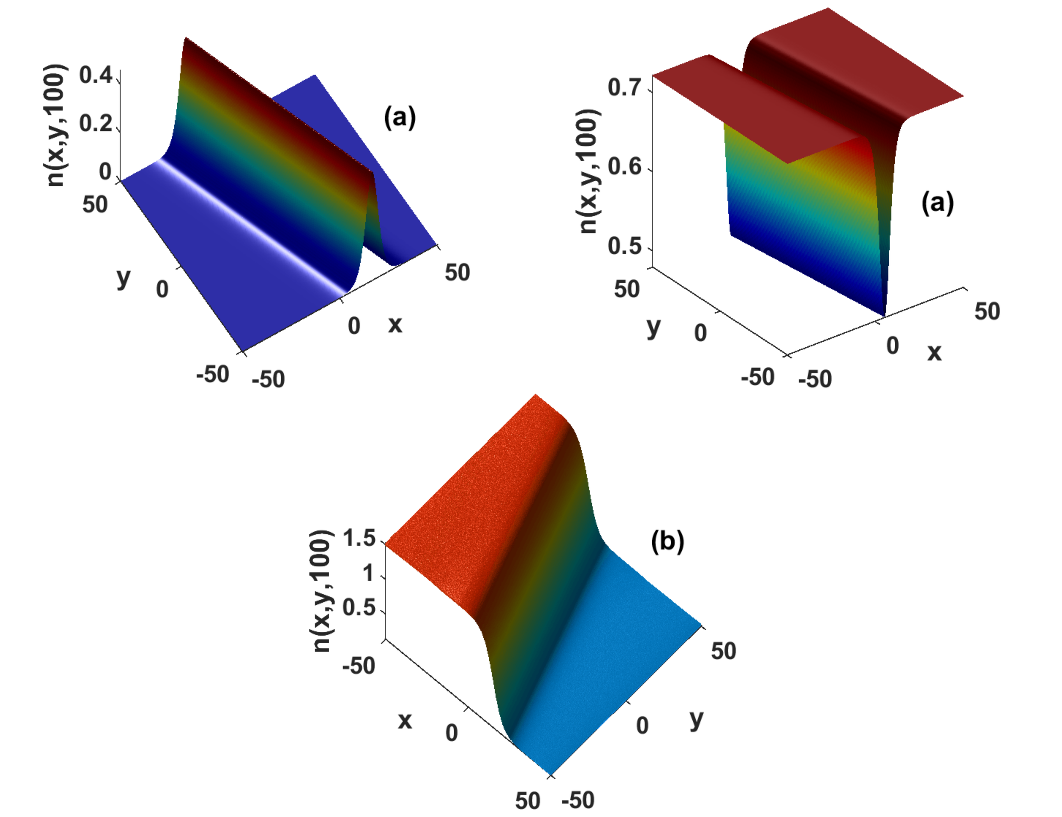

Figure 1: bacterial wave constructed are bell-shape, dip and step traveling waves profiles

Important Results

- In addition to the fact that traction completely modifies the system, traction, long-range diffusion and chemotaxis present competing effects.
- Traction and long-range diffusion slow down the waves and entail the transport of a small number of particles. Long-range diffusion increases the thickness of the wave but does not alter the magnitude.
- We prove that while dip waves travel faster (better candidates to explaining fast bacterial coordination), step waves have the potential of carrying a higher number of cells, hence may be considered as robust structures to perform transport in a highly dense medium.

Results I: The wave velocity

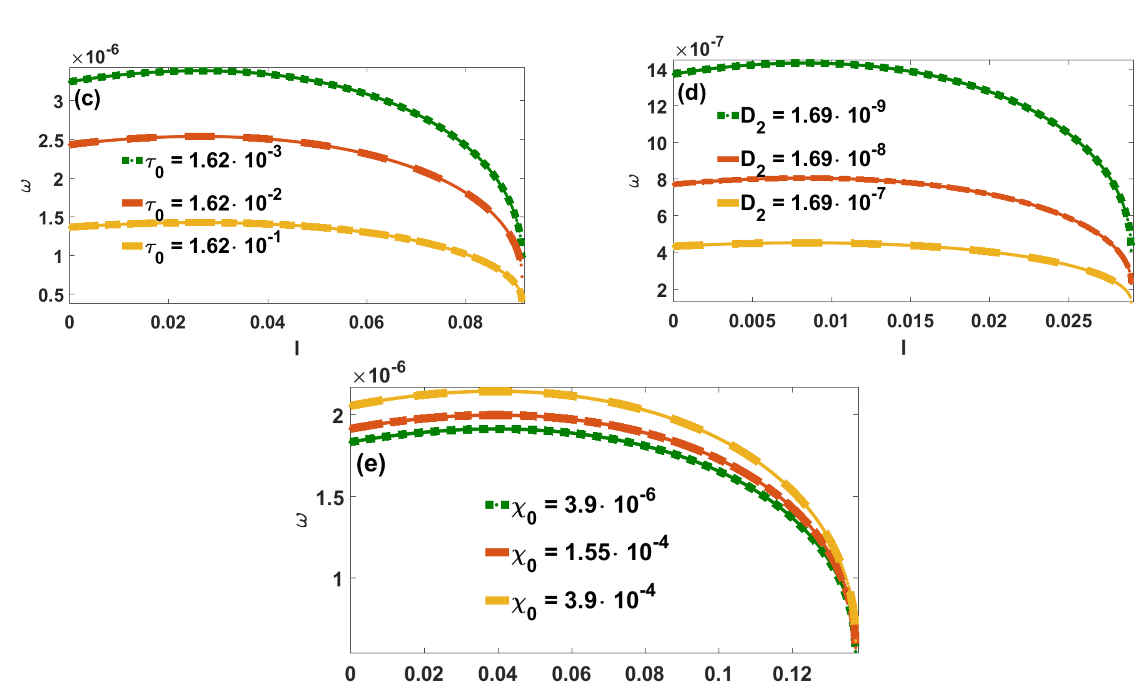

Figure 2: Influence of traction (c), long-range diffusion (d) and chemotaxis (e) on the wave velocity

Results III: The wave thickness, amplitude

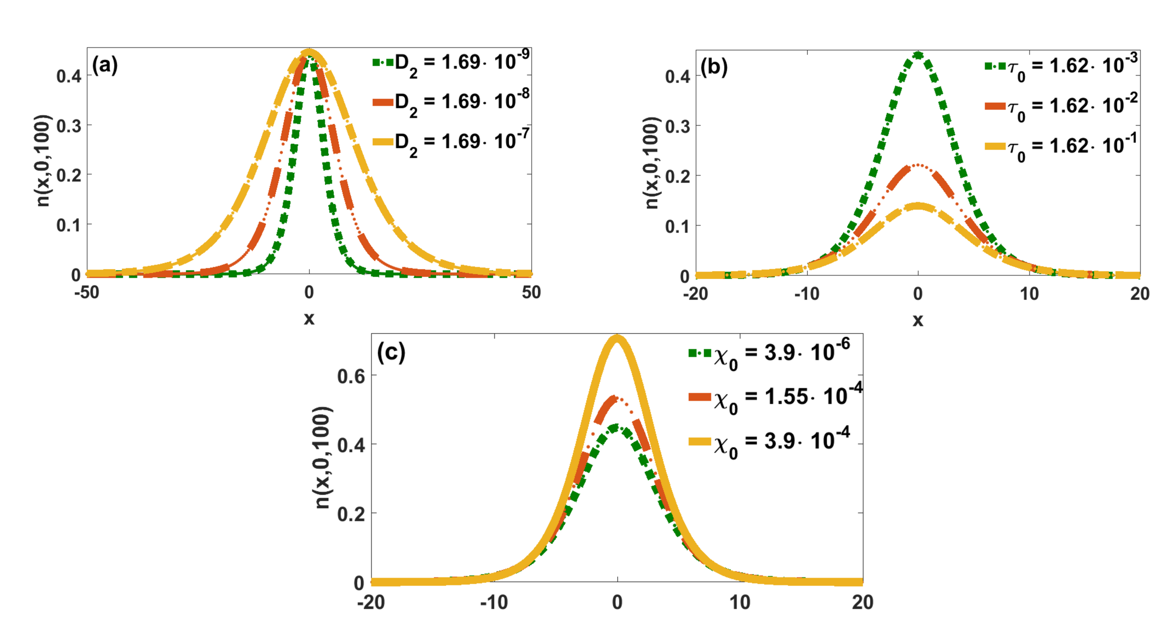

Figure 3: Influence of long-range diffusion (a), traction (b), and chemotaxis (c) on the wave characteristics

Conclusion

In summary, we have introduced a new mathematical model for chemotaxis. Applying the F-expansion method, we construct solutions of our model and use them to study the dynamical behaviors of the bulk system. We provided the conditions within which solutions constructed exist and hence are physical objects that may be expected in real experiments. The stability of solutions constructed is analyzed through direct numerical integration. Both numerical and analytical solutions remain very close hence ascertaining our theoretical predictions.

References

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