Time dependent weighting fields for signals in silicon sensors

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W. Riegler, CERN
The current induced on a grounded electrode by a moving point charge \( q \) is given by

\[
I_n(t) = -\frac{q}{V_w} \vec{E}_n(\vec{x}(t)) \cdot \vec{v}(t)
\]

Where the weighting field \( E_n(x) \) is defined by removing the point charge, applying the potential \( V_w \) to the electrode in question and leaving the other electrodes grounded.

Removing the charge means that we just have to solve the Laplace equation and not the Poisson equation.
Ramo-Shockley Theorem

To find the voltages induced on electrodes that are not grounded we have to

- First calculate the currents induced on grounded electrodes.
- Then place these currents as ideal current sources on a circuit containing the discrete components and the mutual electrode capacitances.

The second step is typically performed by using an analog circuit simulation program.
Extensions of the theorem for conductive media

Resistive Plate Chambers

- 2mm Bakelite, layer $\rho \sim 10^{10} \Omega\text{cm}$
- 3mm glass, layer $\rho \sim 2 \times 10^{12} \Omega\text{cm}$
- 0.4mm glass, layer $\rho \sim 10^{13} \Omega\text{cm}$

P. Fonte, V. Peskov et al.

M.C.S. Williams et al.

R. Santonico, R. Cardarelli

Silicon Detectors

- Undepleted layer $\rho \sim 5 \times 10^3 \Omega\text{cm}$
- Depletion layer

E.g.

Irradiated silicon sensors (typically has larger volume resistance).

CMOS sensors with un-depleted regions

AC-LGADs

VCI2004

Werner Riegler, CERN
Extensions of the theorem for conductive media

Induced signals in resistive plate chambers

Werner Riegler

EP Division, CERN, CH-1211 Geneva 23, Switzerland

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Abstract

We derive theorems for induced signals on electrodes embedded in a medium with a position and frequency-dependent permittivity $\varepsilon(x, s)$ and conductivity $\sigma(x, s)$ that are connected with arbitrary discrete elements. The problem is treated using the quasi-static approximation of Maxwell's equations for weakly conducting media. The induced signals can be derived by time-dependent weighting fields and potentials and the result is the same as the one given in Gatti et al. (Nucl. Instr. and Meth. 193 (1982) 651). We also show how these time-dependent weighting fields can be derived from electrostatic solutions. Finally, we will apply the results to Resistive Plate Chambers where we discuss the effects of the resistive plates and thin resistive layers on the signals induced on plane electrodes and strip electrodes.

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Extended theorems for signal induction in particle detectors VCI 2004

W. Riegler*

CERN, PH Division, Rt. De Meyrin, Geneva 23CH-1211, Switzerland

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Abstract

Most particle detectors are based on the principle that charged particles leave a trail of ionization in the detector and that the movement of these charges in an electric field induces signals on the detector electrodes. Assuming detector elements that are insulating and electrodes with infinite conductivity one can calculate the signals with an electrostatic approximation using the so-called ‘Ramo theorem’. This is the standard way for the calculation of signals e.g. in wire chambers and silicon detectors. In case the detectors contain resistive elements, which is, e.g. the case in resistive plate chambers or underdepleted silicon detectors, the time dependence of the signals is not only given by the movement of the charges but also by the time-dependent reaction of the detector materials. Using the quasistatic approximation of Maxwell’s equations we present an extended formalism that allows the calculation of induced signals for detectors with general materials by time dependent weighting fields. As examples, we will discuss the signals in resistive plate chambers and underdepleted silicon detectors.

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PACS: Signal theory; Particle detectors; RPCs; Silicon detectors

Keywords: 29.40

Time dependent weighting fields, induced currents, voltages, admittance and impedance matrix, examples...

Fig. 6. A geometry representing a resistive plate chamber or underdepleted silicon detector.

electrode 1 we find (in the Laplace domain):

\[
E_{2}(s) = \frac{-\varepsilon_{o}V_{0}}{\varepsilon_{o}d_{2} + \varepsilon_{o}d_{1}} = \frac{-V_{0}d_{2}}{(d_{1} + d_{2}\varepsilon_{o})} \times \frac{s + 1/\tau_{1}}{s + 1/\tau_{2}}, \quad z > 0
\]

\[
e_{1}V_{0} = \frac{-\varepsilon_{o}V_{0}}{\varepsilon_{o}d_{2} + \varepsilon_{o}d_{1}} = \frac{-V_{0}}{(d_{1} + d_{2}\varepsilon_{o})} \times \frac{s}{s + 1/\tau_{2}}, \quad z < 0
\]

\[
\tau_{1} = \frac{\varepsilon_{o}d_{0}}{\sigma}, \quad \tau_{2} = \frac{\varepsilon_{o}d_{0}}{\sigma} \left( \frac{d_{1} + d_{2}\varepsilon_{o}}{d_{2}} \right)
\]

The weighting field of the other electrode is given by \( E_{1}(s) = -E_{2}(s) \), so we have \( I_{1}(t) = -I_{2}(t) \). Using the particle trajectory \( z(t) = d_{2} - vt, t < T = d_{2}/v \) and evaluating Eq. (18) we find the induced current of

\[
I_{2}(t) = -q\varepsilon_{0} \frac{d_{1}}{d_{1} + \varepsilon_{o}d_{2}} \left[ \frac{d_{1}}{d_{2}}e^{-t/\tau_{2}} - \frac{1}{\varepsilon_{o}d_{2}}(1 - e^{-t/\tau_{2}}) \right], \quad t < T
\]

\[
= -q\varepsilon_{0} \frac{1}{d_{1} + \varepsilon_{o}d_{2}} \frac{d_{1}}{d_{2}} \left( e^{t/\tau_{2}} - 1 \right)e^{-t/\tau_{2}}, \quad t > T.
\]
Formulation of the problem

At t=0, a pair of charges +q, -q is produced at some position in between the electrodes. From there they move along trajectories \(x_0(t)\) and \(x_1(t)\).

What are the voltages induced on electrodes that are embedded in a medium with position and frequency dependent permittivity and conductivity, and that are connected with arbitrary discrete elements?

Quasistatic approximation of Maxwell’s equations

\[
\nabla \left[ \varepsilon_{\text{eff}}(\vec{x}, s) \nabla \right] \phi(\vec{x}, s) = -\rho_{\text{ext}}(\vec{x}, s)
\]

\[
\varepsilon_{\text{eff}}(\vec{x}, s) = \varepsilon(\vec{x}, s) + \frac{1}{\sigma}(\vec{x}, s)
\]

Extended version of Green’s 2nd theorem

\[
\int_{A} \left[ \psi(\vec{x}) f(\vec{x}) \nabla \phi(\vec{x}) - \phi(\vec{x}) f(\vec{x}) \nabla \psi(\vec{x}) \right] d\vec{A}
\]

\[
= \int_{V} \left[ \psi(\vec{x}) \nabla f(\vec{x}) \nabla \phi(\vec{x}) - \phi(\vec{x}) \nabla f(\vec{x}) \nabla \psi(\vec{x}) \right] d^3 x
\]
Remove the charges and the discrete elements and calculate the weighting fields of all electrodes by putting a voltage $V_0 \delta(t)$ on the electrode in question and grounding all others.

In the Laplace domain this corresponds to a constant voltage $V_0$ on the electrode.

Calculate the (time dependent) weighting fields of all electrodes

$$\nabla \left[ \varepsilon_{eff}(\bar{x}, s) \nabla \phi(\bar{x}, s) \right] = 0 \quad \phi_n(\bar{x}, s)|_{\bar{x}=\bar{r}_m} = V_0 \delta_{nm}$$

$$\vec{E}_n(\bar{x}, s) = -\nabla \phi_n(\bar{x}, s) \quad \vec{E}_n(\bar{x}, t) = \mathcal{L}^{-1} \left[ \vec{E}_n(\bar{x}, s) \right]$$

Calculate induced currents in case the electrodes are grounded

$$I_n(t) = -\frac{q}{V_0} \int_0^t \vec{E}_n \left[ \vec{x}_0(t'), t - t' \right] \vec{x}_0(t') dt'$$

$$+ \frac{q}{V_0} \int_0^t \vec{E}_n \left[ \vec{x}_1(t'), t - t' \right] \vec{x}_1(t') dt'$$
Extended theorems

Add the impedance elements to the original circuit and the impedance circuit representing the medium and place the calculated currents on the nodes to find the induced voltages.
Sensors using non-linear materials of finite conductivity

For linear media, i.e. for media where the material properties i.e. conductivity and permittivity, do not depend on the applied voltages, the weighting fields are calculated by applying delta voltages or delta currents to the electrodes when they are uncharged or at zero potential.

Silicon sensors do however not represent linear media. The material properties i.e. conductivity and permittivity do depend on the voltages applied to the electrodes. For this case one can define the weighting fields by adding infinitesimal voltage or current pulses to the electrodes in question and subtracting the static field from the time dependent weighting field.

This has a very practical application when using TCAD simulations.

The weighting field formalism applies as long as the electric fields caused by the moving charges do not alter the distribution of conductivity in the sensor.

In case the electric field from the charges has an influence on the movement of the charges, the theorems still hold. The calculation of the movement does of course get more involved ...

\[ W_n(\vec{x}, t) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} [\vec{E}_{Dn}(\vec{x}, t, \Delta t) - \vec{E}_D(\vec{x})] \]
An application of extensions of the Ramo-Shockley theorem to signals in silicon sensors

W. Riegler\textsuperscript{a}

\textsuperscript{a}CERN, Geneva, Switzerland

Abstract

We discuss an extension of the Ramo-Shockley theorem that allows the calculation of signals in detectors that contain non-linear materials of arbitrary permittivity and finite conductivity (volume resistivity) as well as a static space-charge. The readout-electrodes can be connected by an arbitrary impedance network. This formulation is useful for the treatment of semiconductor sensors where the finite volume resistivity in the sensitive detector volume cannot be neglected. The signals are calculated by means of time dependent weighting fields and weighting vectors. These are calculated by adding voltage or current signals to the electrodes in question, which has a very practical application when using semiconductor device simulation programs. An analytic example for an un-depleted silicon sensor is given.
Static electric field in a biased silicon sensor

Metal electrodes embedded in a medium with

- Static space-charge $\rho_0(x)$
- Position and frequency dependent permittivity $\varepsilon(x, s)$
- Position and frequency dependent conductivity $\sigma(x, s) = 1/\rho(x, s)$ ($\rho$ ... volume resistivity)
- Connection of the electrodes with discrete impedance elements $z_{mn}(s)$
- Nonlinear material i.e. $\rho_0(x), \varepsilon(x, s), \sigma(x, s)$ can depend on the voltages $V_n$ applied to the electrodes.

$\Rightarrow$ Silicon sensor!
2.1. Induced voltage

This weighting field is defined by placing a ‘step charge’ or ‘delta current’ on the electrode in question and calculating the resulting electric field.

This theorem is very well suited for calculation of signals with TCAD simulation programs.

One can add the entire discrete circuitry like biasing network, amplifier etc. to the TCAD model and directly find the voltages induced on the nodes.

In TCAD one can e.g. use a triangular current pulse with duration $T$ and peak value $I_p$ and then use $Q_0 = I_p \times T/2$, where $T$ must be chosen much smaller than the reaction time of the medium.

In general any current $I(t)$ with $Q_0 = \int I(t) \, dt$ can be used, as long as the duration is much smaller than the reaction time of the medium.

\begin{equation}
I(t) \quad I_p \quad t
\end{equation}
2.2. Induced charge on grounded electrode

This weighting field is defined by placing a step voltage on the electrode in question and calculating the resulting electric field.

This theorem is well suited for calculation of signals with TCAD simulation programs when the input impedance of the amplifier that connects to the electrode is negligible with respect to the other impedances in the circuit.

In that case \( I_n(t) = -\frac{dQ_n(t)}{dt} \) directly gives the input current to the amplifier.
2.3. Induced current on grounded electrode

The induced current can also be calculated by a ‘weighting field’ or ‘weighting vector’ \( W_n \) that is caused by a small voltage pulse on the electrode in question.

\( W(x, t) \) has units of V/cm\(^2\)s and therefore does not represent an electric field.

Using this weighting vector the induced current can be calculated directly.

This weighting vector will always have a ‘prompt’ component that follows the short pulse and a ‘delayed’ component that includes the reaction of the medium.

When using this weighting field for numerical simulations it is useful to use these two components separately to avoid numerical issues.
Induced Currents and Voltages

The charges $Q^\text{ind}_n(t)$ and current $I^\text{ind}_n(t) = -\frac{dQ^\text{ind}_n(t)}{dt}$ are related to the voltages $V^\text{ind}_n(t)$ induced on the electrodes that are connected by the discrete impedance elements $z_{mn}(s)$ through the admittance matrix $y_{mn}(s)$.

Let us assume we have calculated the weighting field $H(x, t)$ or the weighting vector $W(x, t)$ for the induced charge or the induced current, for the case where the other electrodes are held at fixed potentials i.e. the interconnecting impedance elements $z_{mn}(s)$ do not play a role.

We perform the Laplace transform and have $H_n(x, s)$ and $W_n(x, s)$.

We define the admittance matrix $y_{mn}(s)$ by integrating these weighting fields over the electrode surfaces.

$$i^\text{ind}_n(t) = -\frac{q}{V_0} \int_0^t \tilde{W}_n([\tilde{x}_1(t'), t - t'] \tilde{x}_1(t') dt'$$

$$+ \frac{q}{V_0} \int_0^t \tilde{W}_n([\tilde{x}_2(t'), t - t'] \tilde{x}_2(t') dt'$$

$$y_{mn}(s) = \frac{s}{V_0} \int_{A_m} [\varepsilon(\vec{x}, s) + \sigma(\vec{x}, s)/s] \tilde{W}_n(\vec{x}, s)dA$$

$$Z_{nm}(s) = -\frac{1}{y_{nm}(s)} \quad m \neq n$$

$$Z_{nn}(s) = \frac{1}{\sum_{m=1}^{N} y_{mn}(s)}$$
The electrodes and the medium can be represented by nodes that are connected by impedance elements.

The induced voltage signals for the case where the electrodes are connected by arbitrary impedance elements can then be calculated by the induced currents on grounded electrodes together with the equivalent circuit diagram.

In case the medium has no conductivity, i.e. $\sigma=0$, these impedance elements are $Z_{nm}=1/sC_{nm}$ with $C_{nm}$ being the mutual electrode capacitances.

This second method for calculating the induced voltages has the advantage that one calculates the currents $I_{\text{ind}}^{\text{m}}(t)$ and the impedance elements $Z_{\text{mn}}(s)$ once and can then perform all further calculations for different readout and biasing circuits in a separate SPICE simulation.
The admittance matrix can be measured by adding a small sine wave voltage to an electrode and measuring the currents on all the other electrodes (I-V curves).
Example un-depleted silicon sensor

Example 5.3 page 103 for a bias voltage smaller than the depletion voltage.
Example un-depleted silicon sensor

\[ V_{dep} = \frac{qN_D d^2}{2\varepsilon_1} \quad d_0 = d \sqrt{\frac{V}{V_{dep}}} \quad \text{for} \quad 0 < V < V_{dep} \]  

(20)

where \( q \) is the elementary charge and \( \varepsilon_1 = \varepsilon_r \varepsilon_0 \) is the dielectric permittivity of silicon. The static space charge density \( \rho_0 \) of the depleted layer and the conductivity \( \sigma \) (the inverse of the volume resistivity) of the un-depleted bulk layer are given by

\[ \rho_0 = qN_D = \frac{2V_{dep}\varepsilon_1}{d^2} \quad \sigma = q\mu_e N_D \]  

(21)

where \( \mu_e \) is the electron mobility.

\[ E_D(z) = -\frac{2V}{d_0} \left( 1 - \frac{z}{d_0} \right) \quad 0 < z < d_0 \]  

(22)

\[ \frac{dz_e(t)}{dt} = -\mu_e E_D(z_e(t)) \quad \frac{dz_h(t)}{dt} = \mu_h E_D(z_h(t)) \quad z_e(0) = z_h(0) = z_0 \]  

(23)

with the solution

\[ z_e(t) = d_0 - (d_0 - z_0)e^{-t/\tau_e} \quad \tau_e = \frac{d^2}{2\mu_e V_{dep}} \quad 0 < t < \infty \]  

(24)

\[ z_h(t) = d_0 - (d_0 - z_0)e^{t/\tau_h} \quad \tau_h = \frac{d^2}{2\mu_h V_{dep}} \quad 0 < t < t_h \]  

(25)

The holes take the time \( t_h(z_0) = -\tau_h \ln \left( 1 - \frac{z_0}{d_0} \right) \) to arrive at \( z = 0 \), while the electrons take an infinite amount of time to arrive at \( z = d_0 \) since the electric field is zero at this position. The related velocities are:

\[ v_e(t) = \frac{dz_e(t)}{dt} = \frac{d_0 - z_0}{\tau_e} e^{-t/\tau_e} \]  

(26)

\[ v_h(t) = \frac{dz_h(t)}{dt} = -\frac{d_0 - z_0}{\tau_h} e^{t/\tau_h} \Theta(t_h - t) \]  

(27)
Weighting field and current induced by a single e-h pair

\[ W_b(t) = \frac{V_0}{d} \left( \delta(t) - \frac{1}{\tau} e^{-t/\tau} \right) \quad \tau = \frac{\varepsilon_1 d}{d_0 \sigma} \quad (29) \]

\[ W_a(t) = \frac{V_0}{d} \left( \delta(t) + \frac{d-d_0}{d_0} \frac{1}{\tau} e^{-t/\tau} \right) \]

\[ i_e(t) = \frac{q}{V_0} \int_0^t W_a(t-t')v_e(t')dt' \]

\[ i_h(t) = -\frac{q}{V_0} \int_0^t W_a(t-t')v_h(t')dt' \quad 0 < t < t_h \]

\[ = -\frac{q}{V_0} \int_0^{t_h} W_a(t-t')v_h(t')dt' \quad t > t_h \]

\[ i_e(t, z_0) = \frac{q d_0 - z_0}{d} \left[ \frac{d-d_0}{d_0(\tau + \tau_e)} \left( e^{-t/\tau} - e^{-t/\tau_e} \right) + \frac{1}{\tau_e} e^{-t/\tau_e} \right] \]

\[ i_h(t, z_0) = \frac{q d_0 - z_0}{d} \left[ \frac{d-d_0}{d_0(\tau + \tau_h)} \left( e^{t/\tau_h} - e^{-t/\tau} \right) + \frac{1}{\tau_h} e^{t/\tau_h} \right] \quad t < t_h \]

\[ = \frac{(d-d_0)(d_0 - z_0)}{d_0d(\tau + \tau_h)} \left( e^{t_h(z_0)/\tau + t_h(z_0)/\tau_h} - 1 \right) e^{-t/\tau} \quad t > t_h \]

The charge induced by the electrons and the holes is given by

\[ Q_e = \int_0^\infty i_e(t)dt = q \left( 1 - \frac{z_0}{d_0} \right) \quad Q_h = \int_0^\infty i_h(t)dt = q \frac{z_0}{d_0} \]
Current induced by a single e-h pair

Figure 8: Induced currents from a) a single electron b) a single hole c) an single electron+hole pair, for a sensor of 300\,\mu m thickness with a depletion voltage of 56.8\,V. The applied voltage is V=-25.2\,V resulting in a depleted region of d_0 = 200\,\mu m thickness. The e-h pair is deposited at z_0 = 150\,\mu m at t = 0. The dotted line assumes zero volume resistivity of the un-depleted layer and the dashed line assumes infinite volume resistivity.
Electrode Impedance

The impedance of the silicon sensor is equal to a capacitance $C_1$ in series with a capacitance $C_2/\sigma$.

$$1/z_{11}(s) = \frac{s}{V_0} \varepsilon_1 W_a(s) A = \frac{\varepsilon_1 s (1 + \varepsilon_1 s/\sigma)}{d_0 + d\varepsilon_1 s/\sigma} A$$

The weighting field $K$ for calculation of the induced voltage is then

$$K_1(s) = \frac{Q_0}{d_0} \frac{1}{sC_1} \frac{Z_2(s)}{z_{11}(s) + Z_2(s)}$$
Weighting field of a ‘double junction’ sensor

\[ E_1 = E_3 = \frac{V_0}{d} \left( \delta(t) + \frac{d_2}{d_1 + d_3} \frac{1}{\tau} e^{-t/\tau} \right) \]

\[ E_2 = \frac{V_0}{d} \left( \delta(t) - \frac{1}{\tau} e^{-t/\tau} \right) \]

\[ \tau = \frac{d}{d_1 + d_3} \frac{\varepsilon}{\sigma} \]
Strip Example

\[ T << \tau \]
\[ T = \tau \]
\[ T = 10\tau \]
\[ T = 50\tau \]
\[ T = 500\tau \]

The conductive layer ‘spreads’ the signals across the strips.

\[ \tau = \frac{\varepsilon_0}{\sigma} \]
Weighting field and impedance for a general ‘one-dimensional’ silicon sensor

A silicon pad with a size significantly larger than the thickness can be modelled as a one-dimensional sensor.

At full depletion the weighting field is just that of a parallel plate chamber, but after irradiation there can be zones of finite conductivity.

Knowing the weighting field and the impedance for a sensor of arbitrary conductivity $\sigma(z)$ we can try to make contact with I-V curve and TCT measurements and extract parameters of the sensor.

We start with a sensor that has $N$ discrete layers of permittivity $\varepsilon_n$, conductivity $\sigma_n$ and thickness $d_n$. 
Weighting field for a general ‘one-dimensional’ silicon sensor

In case the conductivities of the layers are zero we have

$$\sum_{n=1}^{N} d_n E_n = V \quad \varepsilon_n E_n = \varepsilon_{n+1} E_{n+1} \quad \rightarrow \quad E_n = \frac{V}{\varepsilon_n} \left( \sum_{m=1}^{N} \frac{d_m}{\varepsilon_m} \right)^{-1}$$

In case the conductivities of the layers $\sigma_n$ are different from zero and a step voltage $V/s$ is placed on the electrode, we replace $\varepsilon_n$ by $\varepsilon_n + \frac{\sigma_n}{s}$ and and defining $\tau_n = \varepsilon_n/\sigma_n$ we have

$$E_n(s) = \frac{V}{\varepsilon_n s (s + 1/\tau_n)} D(s) \quad D(s) = \sum_{m=1}^{N} \frac{d_m}{\varepsilon_m} \frac{1}{s (s + 1/\tau_m)}$$

(9)

The weighting field of layer $n$ is then given by $W_n(s) = s E_n(s)$

$$W_n(s) = \frac{V}{\varepsilon_n s (s + 1/\tau_n)} D(s)$$

The admittance matrix of the electrode is

$$y_{11}(s) = \frac{s}{V} \int (\varepsilon_1 + \sigma_1/s) W_1(s) dA = \frac{A}{D(s)} \quad \rightarrow \quad Z_{11}(s) = \frac{1}{A} D(s)$$

$$Z(i\omega) = \frac{1}{A} \sum_{m=1}^{N} \frac{d_m}{\varepsilon_m} \frac{1}{i\omega + 1/\tau_m}$$
Weighting field for a general ‘one-dimensional’ silicon sensor

\[
\frac{\partial}{\partial z} \varepsilon(z) \frac{\partial \varphi(z)}{\partial z} = 0 \quad \frac{\partial}{\partial z} \varepsilon(z) E_z(z) = 0 \quad \Rightarrow \quad E_z(z) = \frac{c_1}{\varepsilon(z)}
\]

\[
\int_0^d E(z) \, dz = V \quad \Rightarrow \quad c_1 = \frac{V}{\int_0^d \frac{dz}{\varepsilon(z)}} \quad \Rightarrow \quad E(z) = \frac{V}{\varepsilon(z) \int_0^d \frac{dz}{\varepsilon(z)}}
\]

\[
W_z(z, s) = \frac{V}{[\varepsilon(z, s) + \sigma(z, s)/s] \int_0^d \frac{dz}{\varepsilon(z, s) + \sigma(z, s)/s}}
\]

\[
y_{11}(s) = \frac{s}{V} \int (\varepsilon(0, s) + \sigma(0, s)/s) W_z(s, 0) \, dA = \frac{s A}{\int_0^d \frac{dz}{\varepsilon(z, s) + \sigma(z, s)/s}}
\]

\[
Z_{11}(s) = \frac{1}{A} \int_0^d \frac{dz}{s \varepsilon(z, s) + \sigma(z, s)}
\]
Time resolution of silicon pixel sensors

W. Rieglerr and G. Aglieri Rinella
CERN EP,
CH-1211 Geneve 23, Switzerland
E-mail: werner.riegler@cern.ch

Abstract: We derive expressions for the time resolution of silicon detectors, using the Landau theory and a PAI model for describing the charge deposit of high energy particles. First we use the centroid time of the induced signal and derive analytic expressions for the three components contributing to the time resolution, namely charge deposit fluctuations, noise and fluctuations of the signal shape due to weighting field variations. Then we derive expressions for the time resolution using leading edge discrimination of the signal for various electronics shaping times. Time resolution of silicon detectors with internal gain is discussed as well.

Analytic expressions for:

Time resolution of LGADs

Effects of weighting fields

Effects of reading out on different sides (iLGad)

Etc.
Summary

Time dependent weighting field theorems that allow the calculation of induced currents, induced voltages and electrode impedances in silicon sensors were presented. They are well suited for simulation of signals using TCAD device simulation programs.

The weighting fields and signals in an un-depleted silicon pad sensor were presented. These results can be used to benchmark TCAD + e.g. Garfield++ signals simulations.

The weighting field and impedance for a general one dimensional silicon sensor was presented, that might allow to relate I-V curve and edge TCT measurements to silicon sensor parameters after irradiation.