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Gauge field theory is the foundation of modern physics, including general relativity and the standard model of physics. It describes how a theory of physics should transform under symmetry transformations. For instance, in electrodynamics, electric forces may transform into magnetic forces if we transform a static observer to one that moves at constant speed. Similarly, in general relativity acceleration and gravity are equated to each other under symmetry transformations. Gauge fields also play a crucial role in modern quantum field theory and the standard model of physics, where they describe the forces between particles that transform into each other under (abstract) symmetry transformations. In this work we describe how the mathematics of gauge groups becomes inevitable when you are interested in deep learning on manifolds. Defining a convolution on a manifold involves transporting geometric objects such as feature vectors and kernels across the manifold, which due to curvature become path dependent. As such it becomes impossible to represent these objects in a global reference frame and one is forced to consider local frames. These reference frames are arbitrary and changing between them is called a (local) gauge transformation. Since we do not want our computations to depend on the specific choice of frames we are forced to consider equivariance of our convolutions under gauge transformations. These considerations result in the first fully general theory of deep learning on manifolds, with gauge equivariant convolutions as the necessary key ingredient.