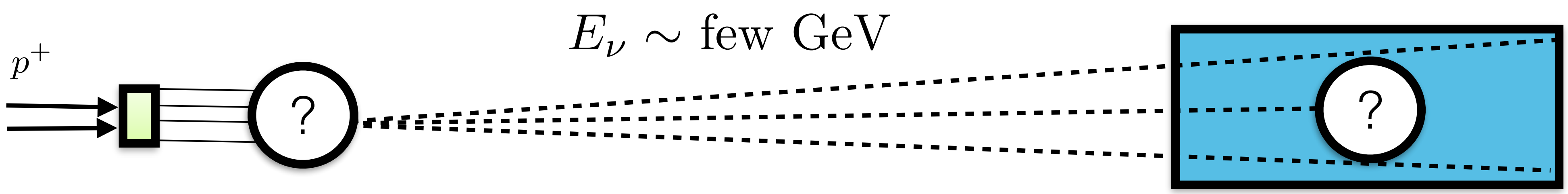
NEW PHYSICS IN RARE NEUTRINO SCATTERING

Matheus Hostert
IPPP, Durham University, UK

P. Ballett, M.H., S. Pascoli, Y. F. Perez-Gonzalez, Z. Tabrizi and R. Z. Funchal
arXiv:1807.10973 and arXiv:1902.08579

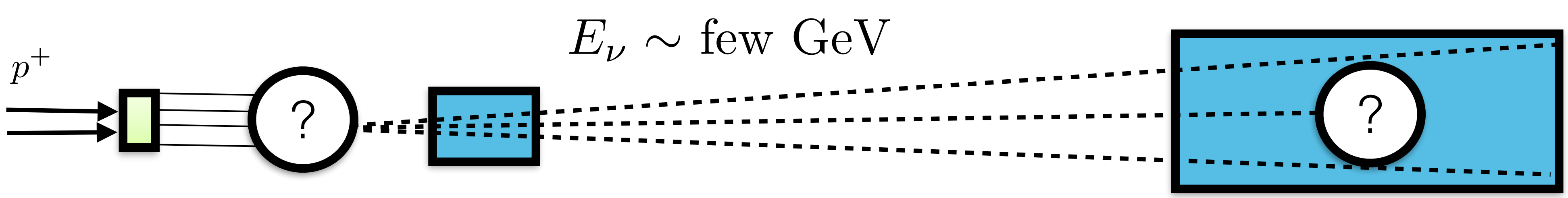
Oscillation experiments — precision era



Flux uncertainties are large
(hadro-production and focusing)
~9% at the NuMI beam

Neutrino-nucleus cross sections:
nuclear physics + lack of data

Oscillation experiments — precision era



Flux uncertainties are large
(hadro-production and focusing)
~9% at the NuMI beam

Neutrino-nucleus cross sections:
nuclear physics + lack of data

Beating systematics for
precision on oscillation
→
Near detectors
Typically $> 10^5$ interactions

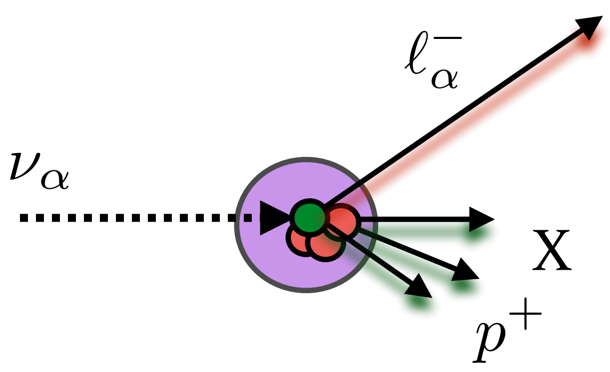
DUNE: expect $> 10^8$ total neutrino interactions in a 75t LAr near detector

Neutrino scattering — weak force in isolation!

Neutrino-hadron interactions

Neutrino-nucleus

- Large statistics (# of targets)
- CC and NC on event-by-event
- Nuclear effects

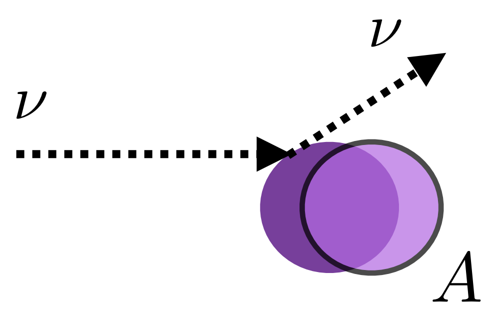


$$\sigma \sim 10^{-38}\ \mathrm{cm}^2$$

Hard to model, but suitable for oscillation physics

CEvNS

- Large cross section
- NC only
- Low energy recoil



$$\sigma \sim 10^{-38}\ \mathrm{cm}^2$$

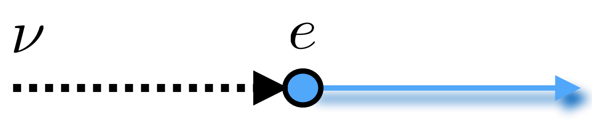
Great for light new physics or NSIs

See talks by X. Xu and B. Dutta.

(Semi-)Leptonic neutrino interactions

Neutrino-electron

- Low statistics
- NC or NC+CC
- Fundamental couplings only

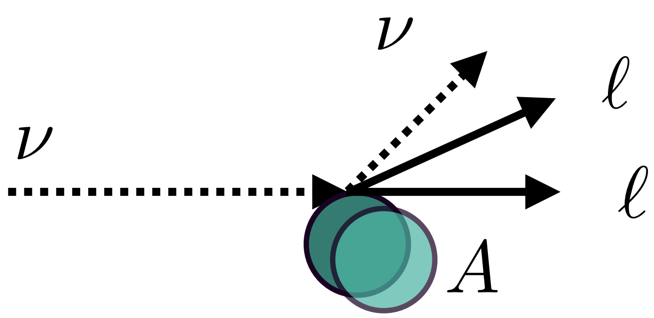


$$\sigma \sim 10^{-41} \ \mathrm{cm}^2$$

Clean and rare process
(couplings to e-)

Neutrino trident production

- Low statistics
- NC or NC+CC
- Well understood



$$\sigma \sim 10^{-44} \ \mathrm{cm}^2$$

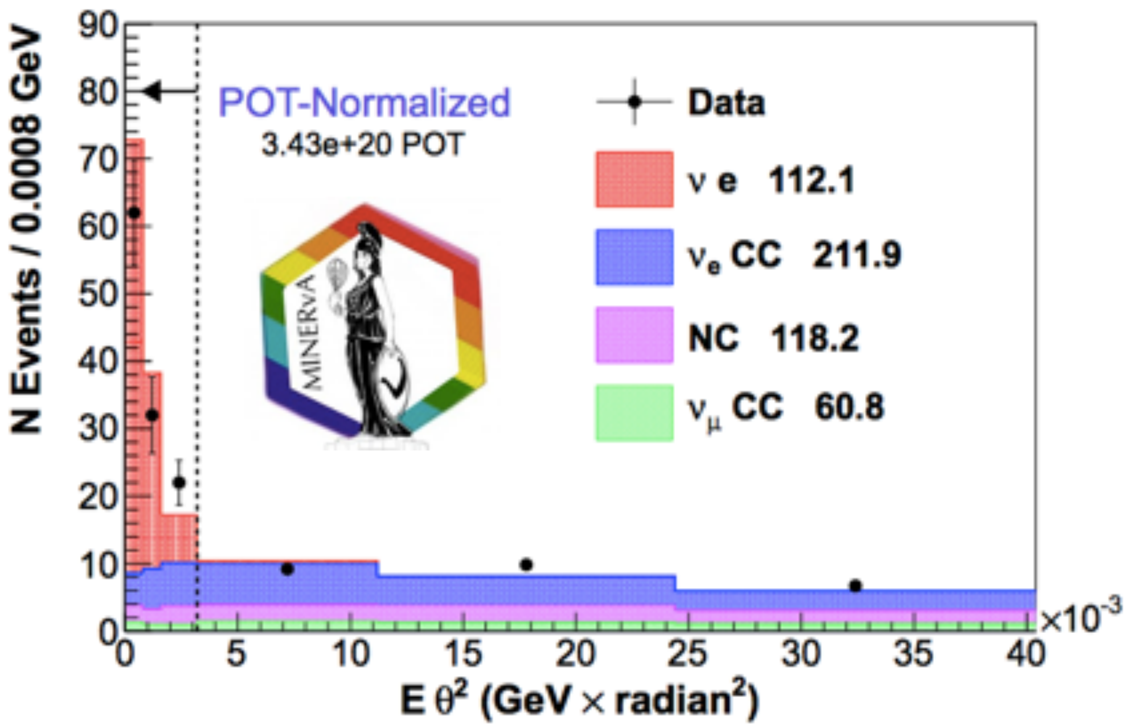
Clean, **very rare** process with
a rich **flavour structure**.

Neutrino-electron scattering

Very simple cross section

$$\frac{d\sigma_{\nu_\alpha - e}}{dT_e} = \frac{2 m_e G_F^2}{\pi}\left[(C_\alpha^{\rm L})^2 + (C_\alpha^{\rm R})^2\left(1 - \frac{T_e}{E_\nu}\right)^2 - C_\alpha^{\rm L} C_\alpha^{\rm R}\, m_e \frac{T_e}{E_\nu^2}\right]$$

Minerva LE measurement (see C. Argüelles talk)



$$E\theta^2 < 2 m_e$$

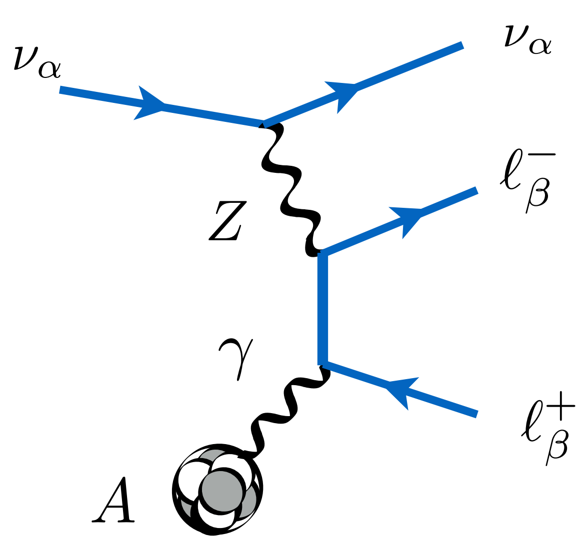
$$T_{\rm th} \leq T_e \leq \frac{2E_\nu^2}{m_e + 2E_\nu}$$

Used to normalise the neutrino flux

Reduces flux uncertainty from 9% to 6% at LE NuMI

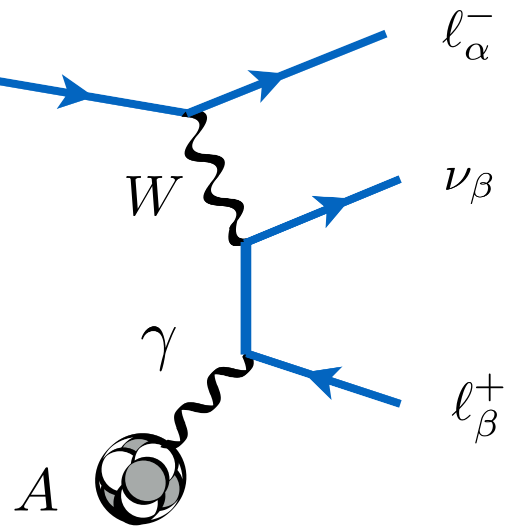
$\frac{S}{B} \sim \frac{127}{30}$ — expect DUNE ND to do better.

Neutrino trident production



Neutrino trident production/scattering:

Neutrino charged lepton production in the Coulomb field of a nucleus.



Very crude approximation for the total cross section:

$$\sigma_\Psi \approx (g_V^2 + g_A^2) \frac{Z^2 \alpha^2 G_F^2}{9\pi^3} \log\left(\frac{2E_\nu \Lambda_{\rm QCD}}{4 m_\ell^2 A^{1/3}}\right) \frac{2E_\nu \Lambda_{\rm QCD}}{A^{1/3}}$$

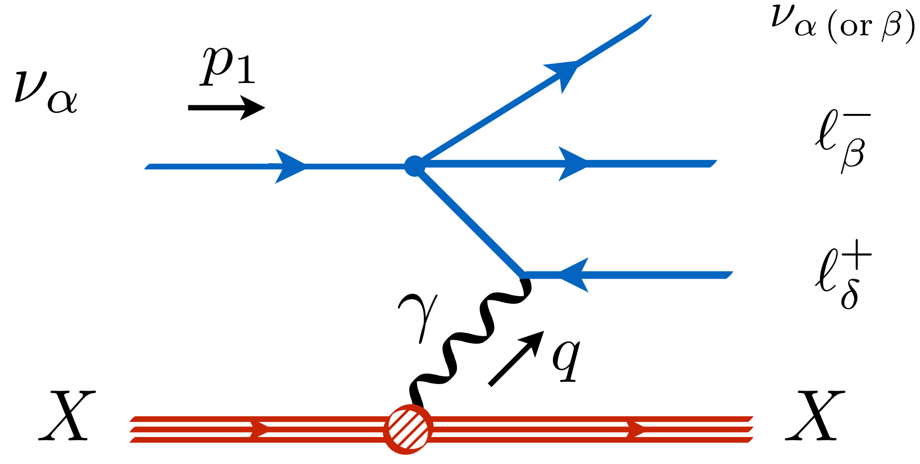
Channel	SM Contributions	g_V	g_A
$\nu_\mu \to \nu_\mu \mu^+ \mu^-$	CC, NC	$1/2 + 2s_{\rm w}^2$	$1/2$
$\nu_\mu \to \nu_\mu e^+ e^-$	NC	$-1/2 + 2s_{\rm w}^2$	$-1/2$
$\nu_\mu \to \nu_e e^+ \mu^-$	CC	1	1

$$\frac{(g_V^2 + g_A^2)_{\rm CC+NC}}{(g_V^2 + g_A^2)_{\rm CC}} \approx 0.60 \longrightarrow$$ NC+CC destructive interference.

Process in coherent, incoherent or DIS limit:

$$\frac{d^2\sigma_{\nu X}}{dQ^2 d\hat{s}} = \frac{1}{32\pi^2 (s - M_X^2)^2} \frac{H_X^{\mu\nu} L_{\mu\nu}}{Q^4}$$

where $\hat{s} = 2\,(p_1 \cdot q)$ and $Q^2 = \sqrt{-q^2}\,.$



Universal leptonic T and L cross sections.

$$\frac{d^2\sigma_{\nu X}}{dQ^2 d\hat{s}} = \frac{1}{32\pi^2} \frac{1}{\hat{s}\,Q^2} \left[h_X^T(Q^2, \hat{s})\, \sigma_{\nu\gamma}^T(Q^2, \hat{s}) + h_X^L(Q^2, \hat{s})\, \sigma_{\nu\gamma}^L(Q^2, \hat{s}) \right]$$

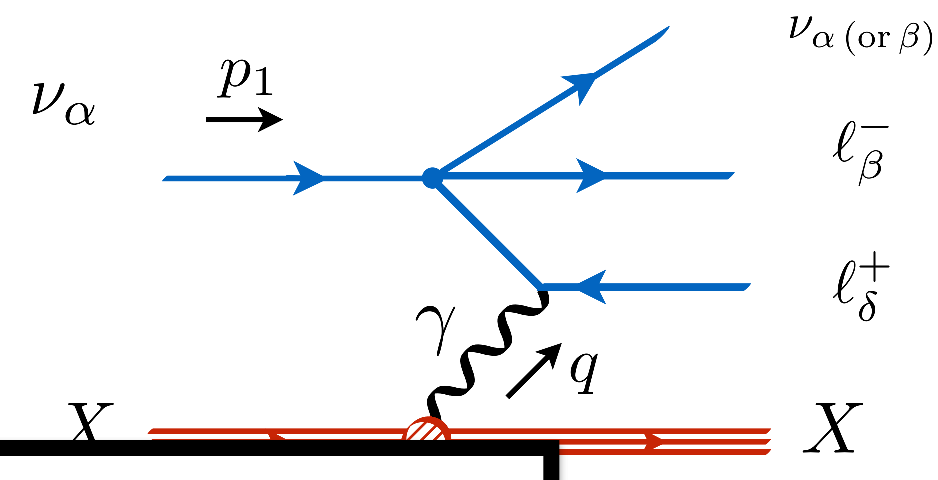
T and L photon flux function (**target** and **regime** dependent)

P. Ballett et al, JHEP 1901 (2019) 119.

*On-shell photon (EPA or Weiszacker-Williams) is a bad approximation at LE
(factors of 2 or 3 difference — unsuitable for incoherent scattering)

Low energy cross sections

Process in coherent, incoherent or DIS limit:



$$\frac{\mathrm{d}^2\sigma_{\nu X}}{\mathrm{d}Q^2\mathrm{d}\hat{s}} = \frac{1}{32\pi^2(s-M_X^2)^2}\frac{H_X^{\mu\nu}L_{\mu\nu}}{Q^4}$$

where $\hat{s} = 2\,(p_1 \cdot q)$ ar

$$\frac{\mathrm{d}^2\sigma_{\nu X}}{\mathrm{d}Q^2\mathrm{d}\hat{s}} =$$

Well understood process.

Electromagnetic nuclear form factors

+

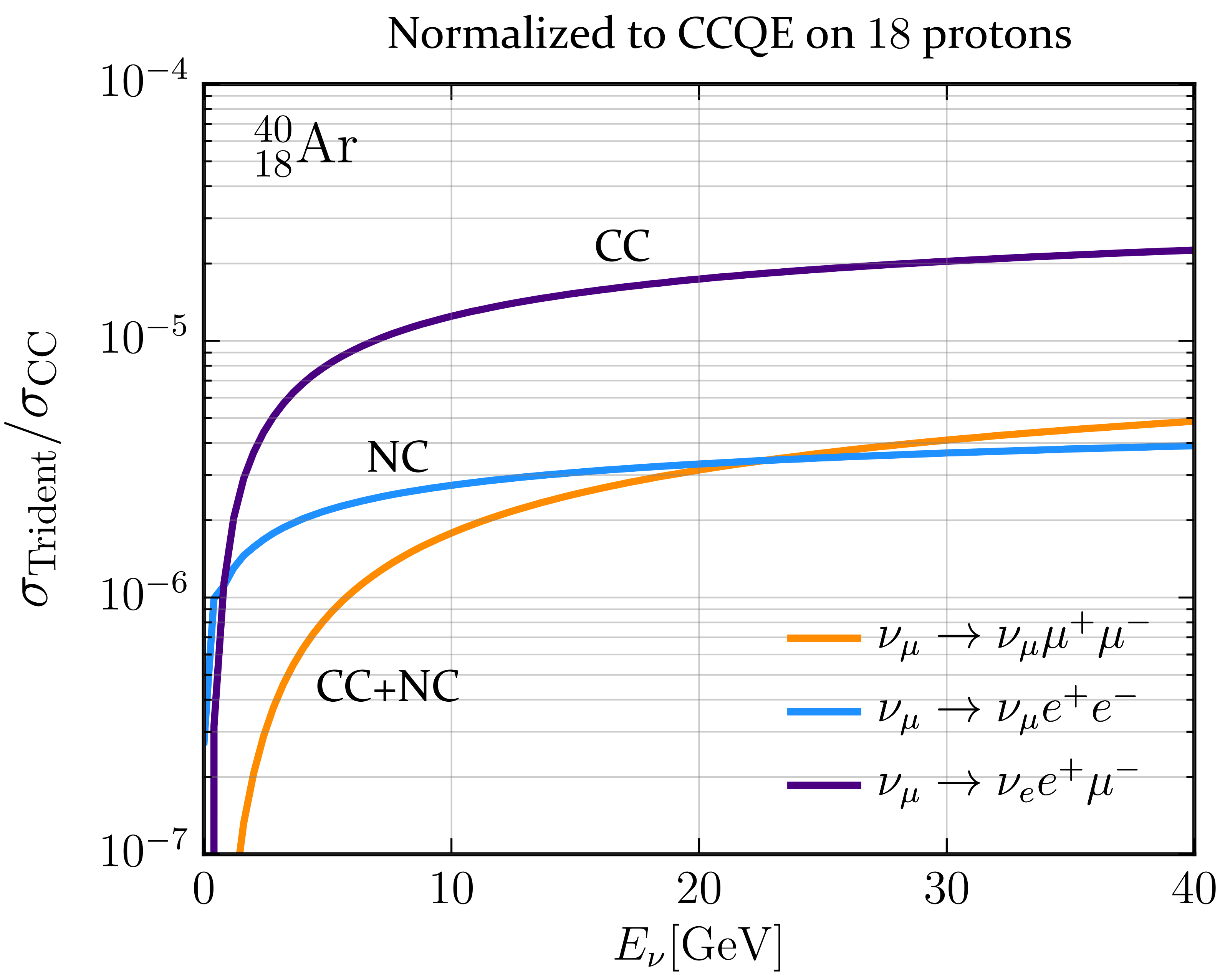
fundamental neutrino-lepton scattering

T and L photon flux function (**target** and **regime** dependent)

P. Ballett et al, JHEP 1901 (2019) 119.

*On-shell photon (EPA or Weiszacker-Williams) is a bad approximation at LE (factors of 2 or 3 difference — unsuitable for incoherent scattering)

Cross sections



Need > 10^6 CCQE events to join the game at LE.

Production of taus is very small… not visible in the plot. Large Q + three propagator suppression.

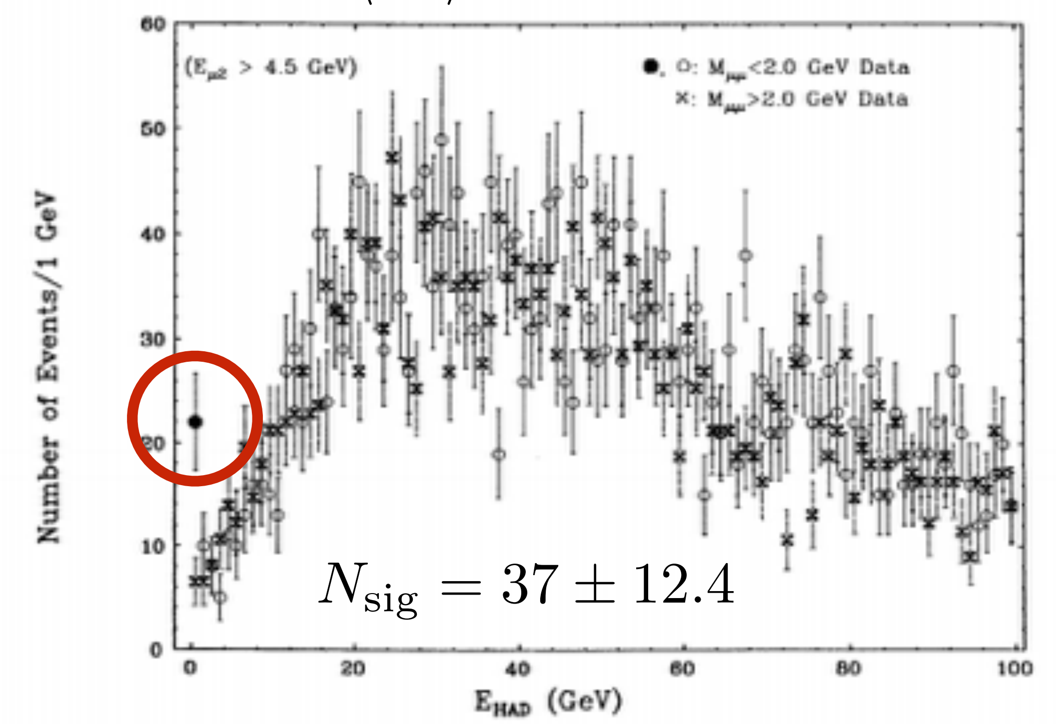
Measurements

Dimuon tridents in muon neutrino beam

CCFR

Lab E detector at FNAL

$\langle E_\nu \rangle = 160$ GeV



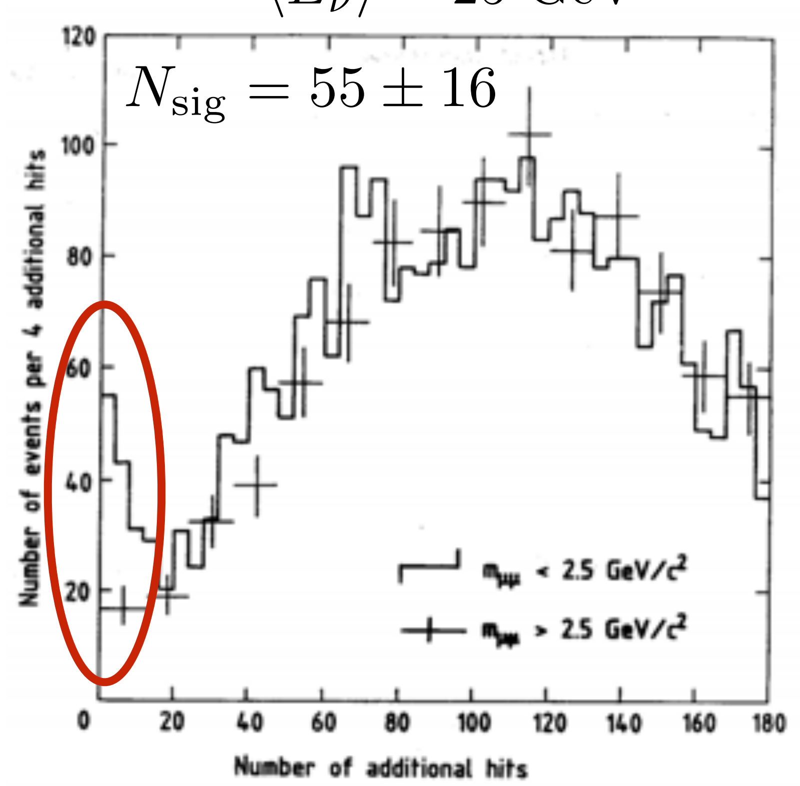
$$\frac{\sigma_{\rm CCFR}}{\sigma_{\rm SM}} = 0.82 \pm 0.28$$

Phys. Rev. Lett. 66, 3117 (1991).

CHARM-II

WANF beam (CERN)

$\langle E_\nu \rangle = 25$ GeV



$$\frac{\sigma_{\rm CHARM-II}}{\sigma_{\rm SM}} = 1.58 \pm 0.57$$

Phys. Lett. B 245, 271 (1990)

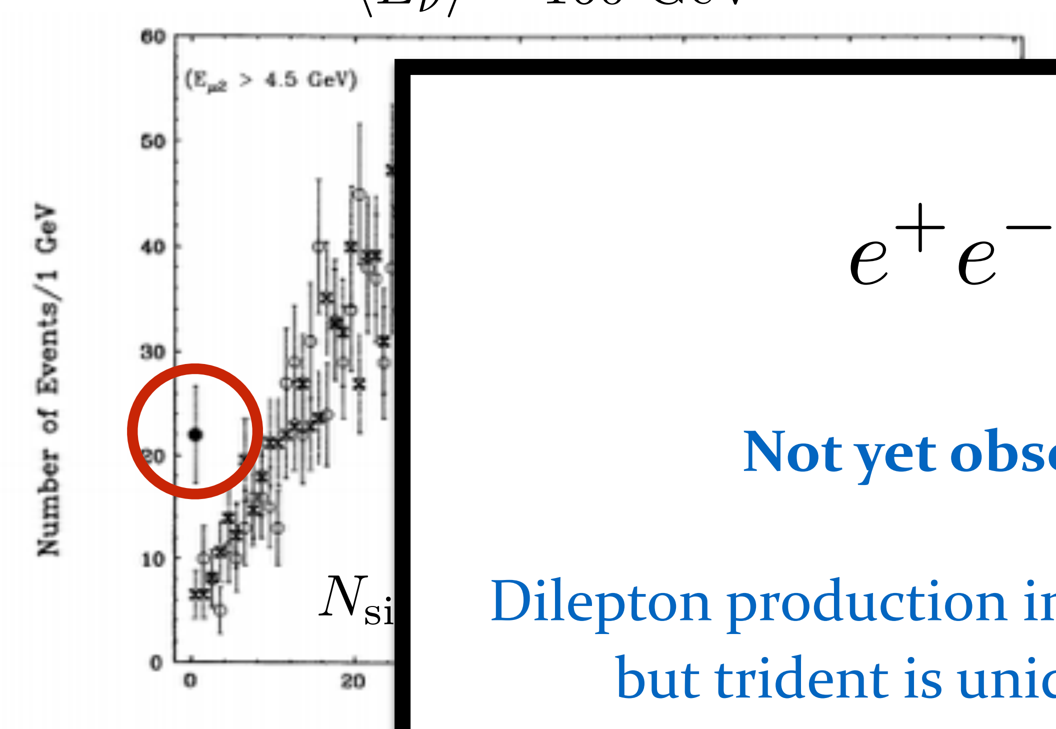
Measurements

Dimuon tridents in muon neutrino beam

CCFR

Lab E detector at FNAL

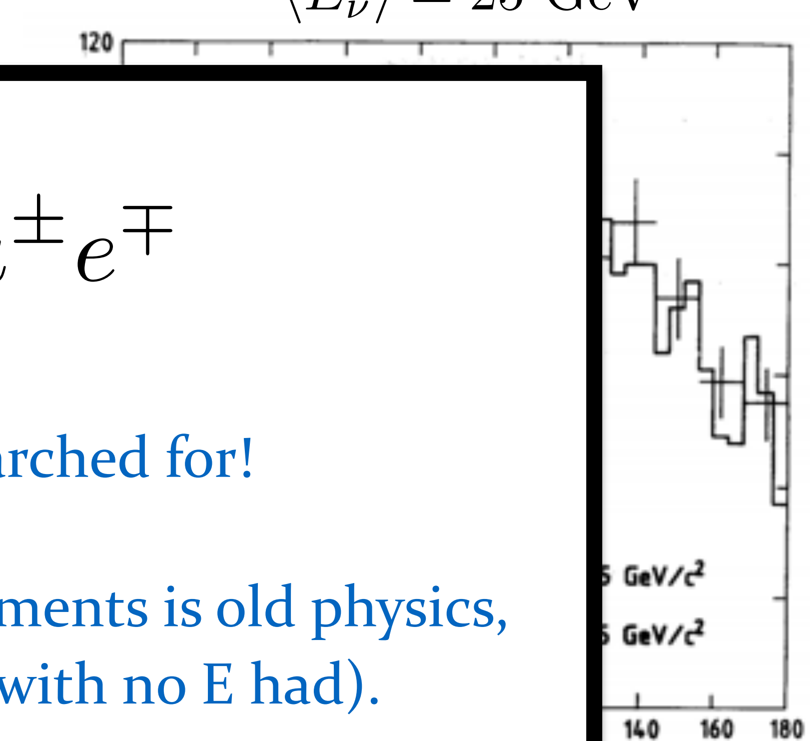
$\langle E_\nu \rangle = 160$ GeV



CHARM-II

WANF beam (CERN)

$\langle E_\nu \rangle = 25$ GeV



e^+e^- $\mu^\pm e^\mp$

Not yet observed or even searched for!

Dilepton production in neutrino experiments is old physics, but trident is unique (clean signal with no E had).

See G. Magill and R. Plestid, arXiv:1612.05642

$\frac{\sigma_{\rm CCF}}{\sigma_{\rm SM}}$ … 8±0.57

Phys. Rev. Lett. 66, 3117 (1991).

Phys. Lett. B 245, 271 (1990)

Why is it hard?

Dilepton production is rare, usually alongside heavy resonances and E had, but…

PID $\begin{cases} \mu^\pm/\pi^\pm \\ e^\pm/\gamma \end{cases}$

$\underline{\mu^+\mu^-}$

$\mathrm{CC}1\pi^\pm$ misID $\pi^\pm$.

$\underline{e^+e^-}$

$\mathrm{NC}1\pi^0$ and $\nu_e\mathrm{CC}\pi^0$.

Even if misID is few %, there are just too many, we have to rely on kinematics.

$\underline{e^+\mu^-}$

$\mathrm{CC}1\pi^0$ with misID γ .

See P. Ballett et al, JHEP1901(2019)119 for preliminary analysis in LAr for all channels
W. Altmannshoffer et al, arXiv:1902.06765 for more sophisticated dimuon projection in DUNE.

DUNE ND strategy

A. Bross
12/3/2018
PONDD

Multi-pronged approach

- Prong I: State-of-the-art Ar detectors:
  - LAr (~75t fiducial target mass), non-magnetized
    - Pixelated (raw 3D data)
    - Optically segmented
      - Neutron tagging
  - Multi-purpose Detector (MPD)
    - High-Pressure (10ATM) gas TPC (HPgTPC) (1t fiducial target mass)
    - In ~0.5T field (magnetic spectrometer)
    - Surrounded by high-performance ECAL and muon tagger
- Prong II: DUNE-PRISM
  - Move LAr and possible MPD off axis
- Prong III: 3 dimensional scintillator (CH) tracker (3DST) (4t)
  - Interactions on protons and carbon
  - Magnetized
  - With external tracking and ECAL

17 12/3/2018 A. Bross I The DUNE Near Detector Complex

Event rates at the DUNE ND

A. Bross
12/3/2018
PONDD

- Prong I: State-of-the-art Ar detectors:
  - LAr (~75t fiducial target mass), non-magnetized
    - Pixelated (raw 3D data)
    - Optically segmented
      - Neutron tagging
  - Multi-purpose Detector (MPD)
    - High-Pressure (10ATM) gas TPC (HPgTPC) (1t fiducial target mass)
    - In ~0.5T field (magnetic spectrometer)
    - Surrounded by high-performance ECAL and muon tagger

Assume Prong I

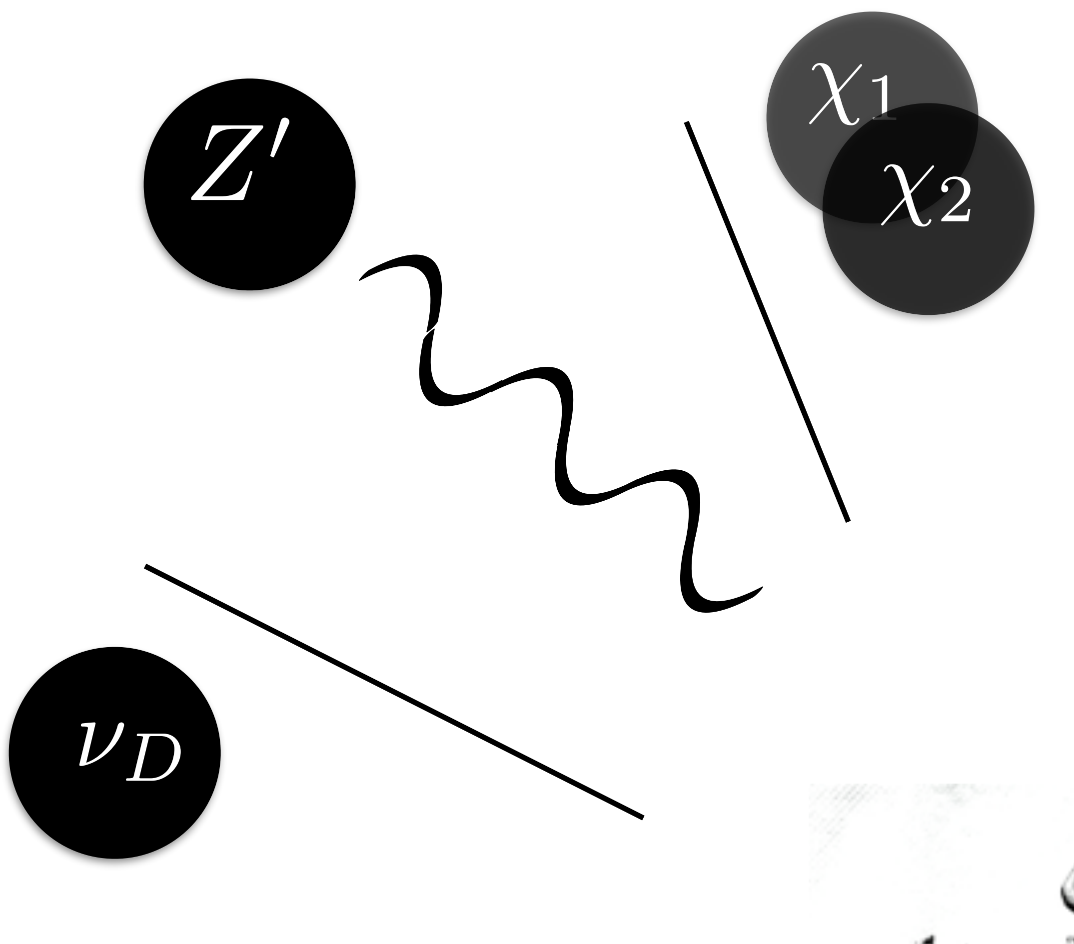
75t LAr
+
Magnetized HPgTPC

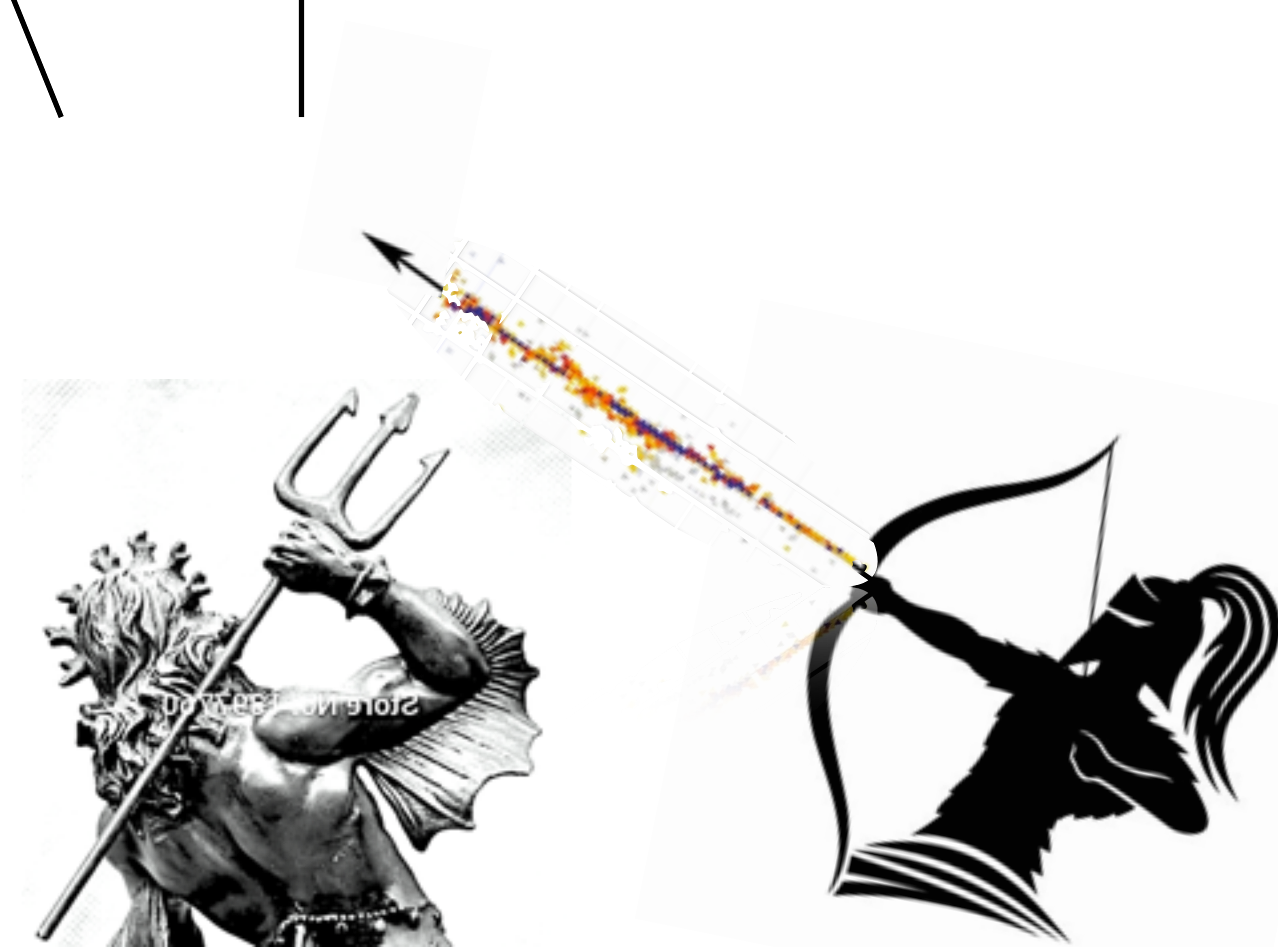
Design	Mode	$\mu^+\mu^-$ trident	e^+e^- trident	$\nu - e$ scattering	POTs/year
120 GeV p^+	ν	47.6	110	8930	1.1×10^{21}
	$\overline{\nu}$	40.7	97.6	6450	1.1×10^{21}
ν_τ app optm	ν	210	321	24900	1.1×10^{21}
	$\overline{\nu}$	156	243	14700	1.1×10^{21}

Higher energy neutrinos!

SM event rate **/year/75t**

Smiting New Physics





Leptophilic Z'

$$B, \quad L_e, \quad L_\mu, \quad L_\tau$$

Accident tree-level symmetries of the SM

(L_α, approx symmetry with massive neutrinos)

Without extra particle content,

$$L_\alpha - L_\beta, \; (\alpha \neq \beta)$$

is anomaly-free in the SM.

Promote it to an abelian gauge symmetry

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{L_\alpha - L_\beta}$$

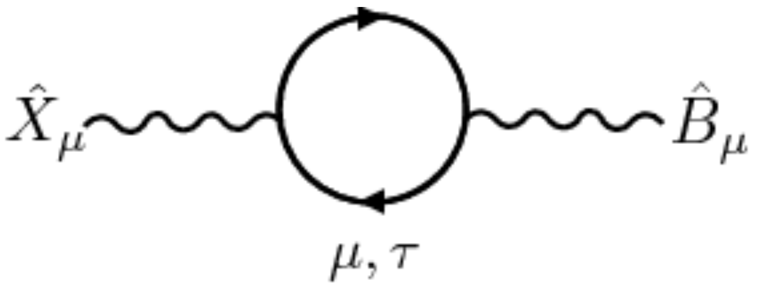
For updated bounds see M. Bauer et al, arXiv:1803.05466

Kinetic mixing

Kinetic mixing term allowed by the symmetries

$$\mathcal{L}_{\rm mix} = -\frac{\varepsilon}{2} F_{\kappa\rho} F'^{\,\kappa\rho}$$

We assume this term is **not present at tree level**, but…always generate it, even if at 1-loop:



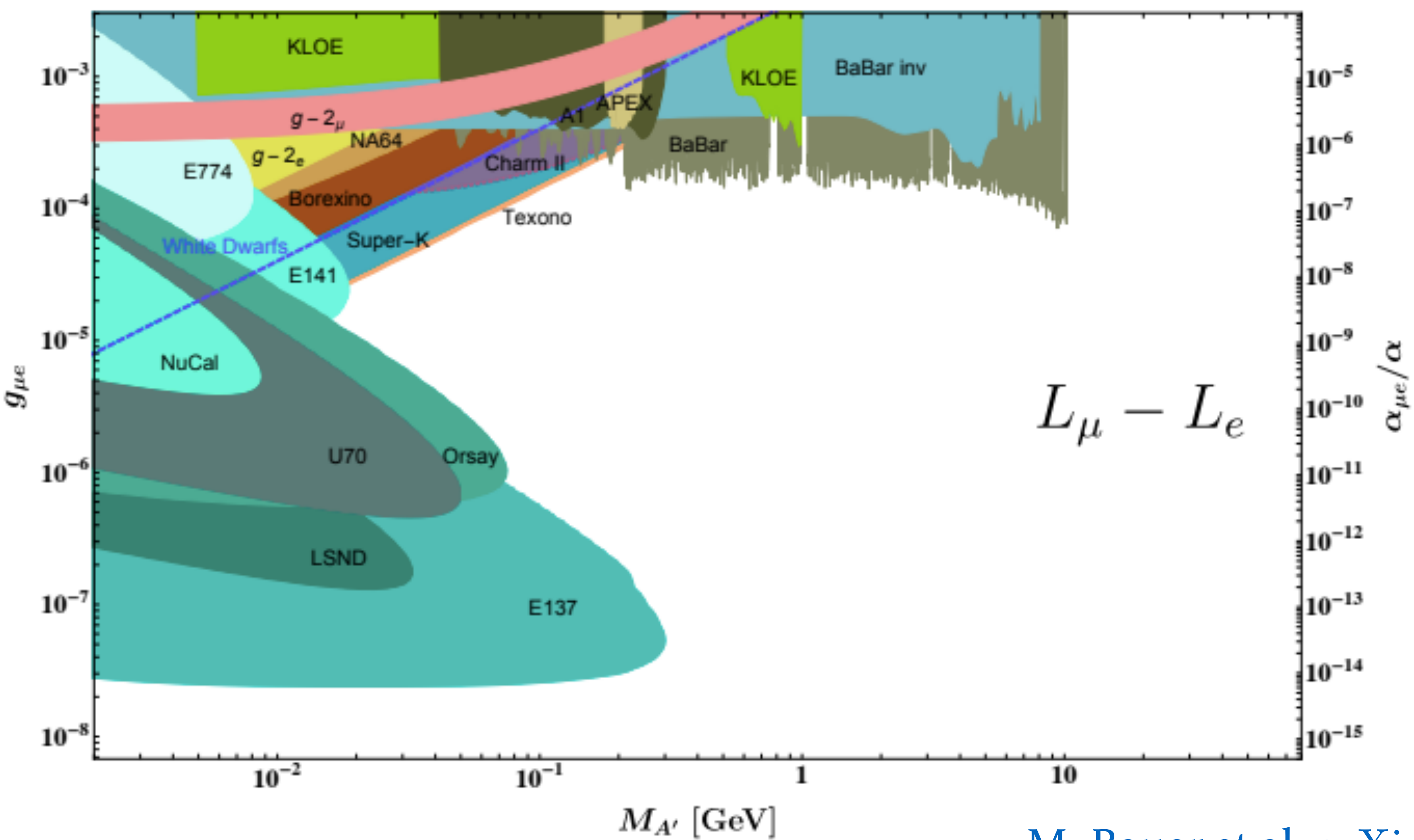
$$\varepsilon(q^2) = -\frac{3eg'}{4\pi^2}\int_0^1 {\rm d}x\, x(1-x)\ln\frac{m_\tau^2 - x(1-x)q^2}{m_\mu^2 - x(1-x)q^2}$$

"Pessimistic" scenario to probe these models — coupling to hypercharged particles at 1-loop.

$L_e - L_\mu$ model

$$\text{SM} \times U(1)_{L_e - L_\mu}$$

Admittedly not so motivated, but good benchmark model for NSI on electrons
(see also long-range matter potentials for neutrinos)



M. Bauer et al, arXiv:1803.05466

$L_\mu - L_\tau$ model

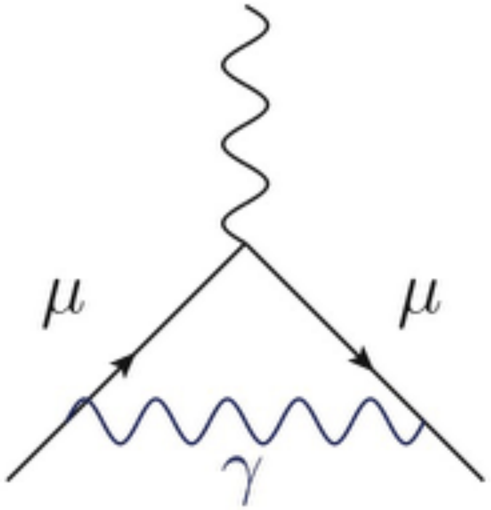
$$\mathrm{SM} \times U(1)_{L_\mu - L_\tau}$$

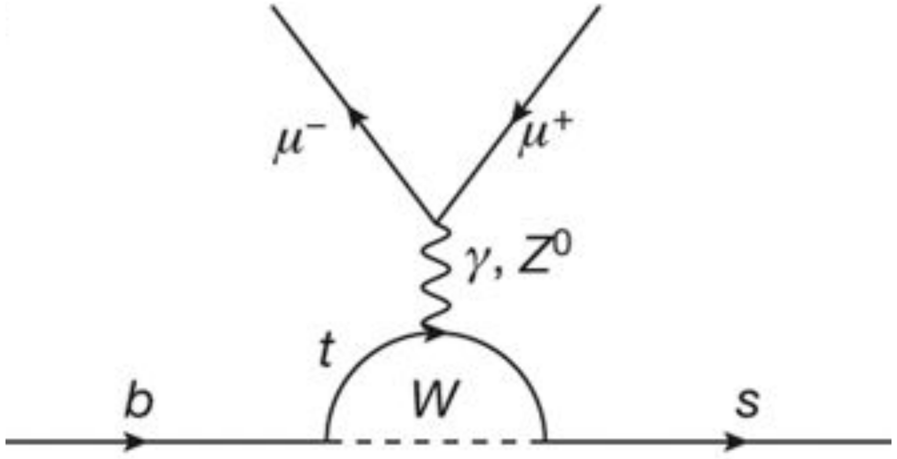
The model that revived the interest in dimuon tridents

W. Altmannshoffer et al, PRL113(2014)091801

$$\mathscr{L}_{\mathrm{int}} \supset g' Z'_\alpha \left(\overline{L}_\mu \gamma^\alpha L_\mu - \overline{L}_\tau \gamma^\alpha L_\tau + \overline{\mu}_R \gamma^\alpha \mu_R - \overline{\tau}_R \gamma^\alpha \tau_R\right)$$

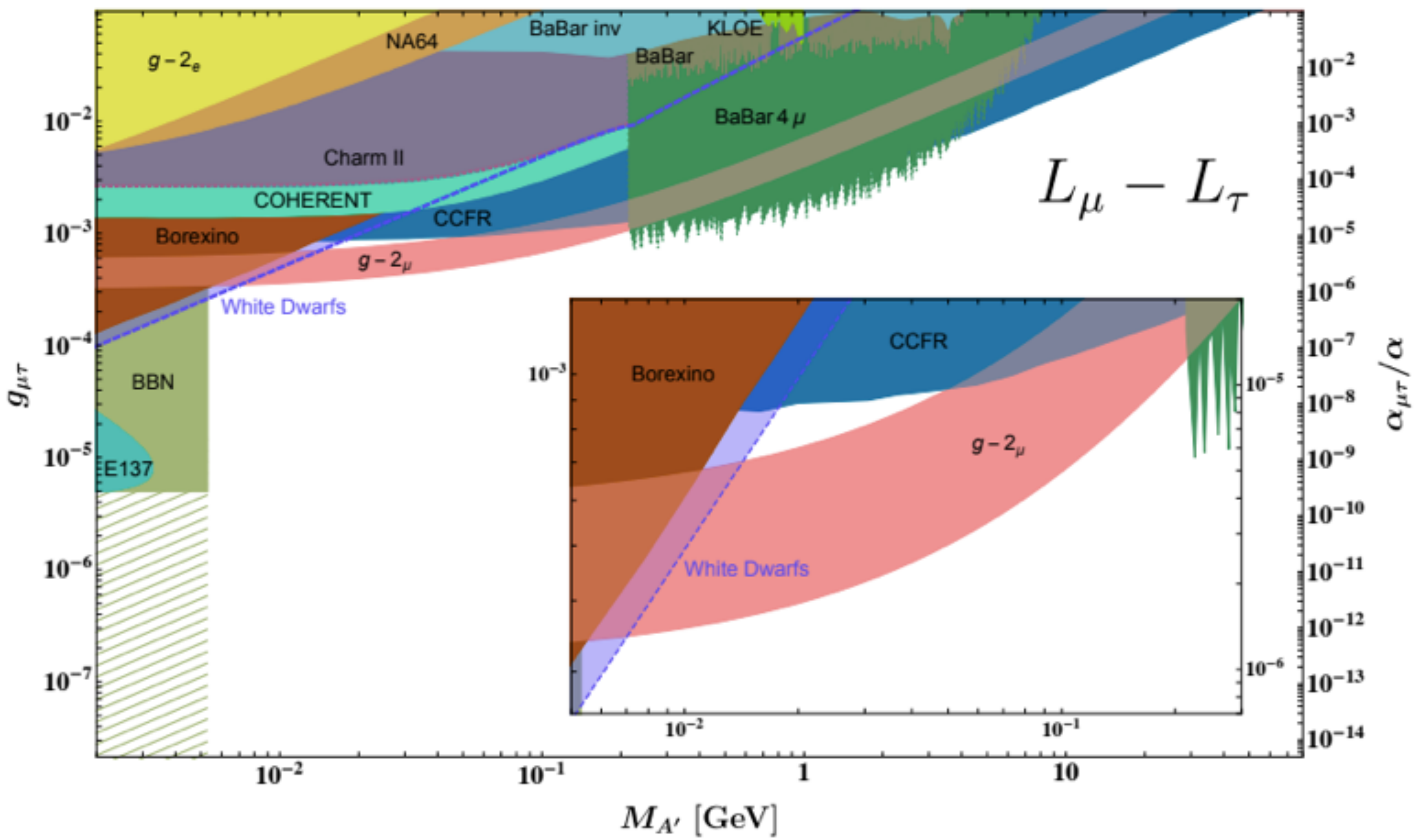
Minimal **anomaly-free** extension that:
— can explain the muon (g-2) discrepancy.
— contains lepton non-universality (can be related to **b** anomalies)
— compatible with neutrino oscillation data!





$L_\mu - L_\tau$ model

$$\text{SM} \times U(1)_{L_\mu - L_\tau}$$

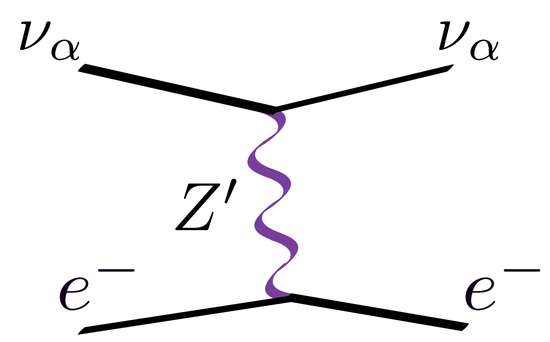


M. Bauer et al, arXiv:1803.05466

BSM neutrino-electron scattering

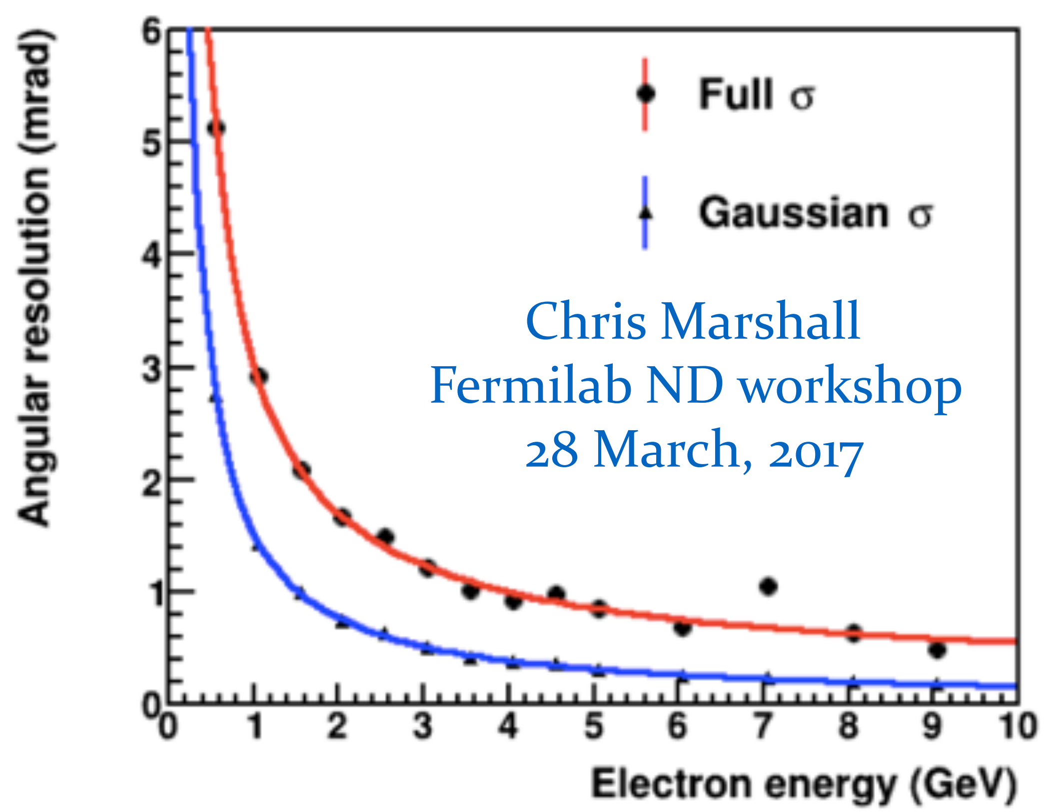
Interference term is always positive (constructive) for ν_μ, and negative (destructive) for $\overline{\nu}_\mu$.

$$\frac{d\sigma_{\nu_\alpha - e}}{dT_e} = \frac{2m_e G_F^2}{\pi}\left[(C_\alpha^{\mathrm{L}})^2 + (C_\alpha^{\mathrm{R}})^2\left(1-\frac{T_e}{E_\nu}\right)^2 - C_\alpha^{\mathrm{L}} C_\alpha^{\mathrm{R}}\, m_e \frac{T_e}{E_\nu^2}\right]$$



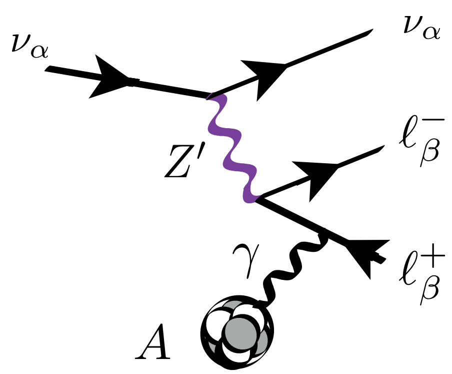
$$C_\alpha^{\mathrm{V}} = C_{\mathrm{SM}}^{\mathrm{V}} + \frac{Q_e^{\mathrm{V}} Q_\alpha^{\mathrm{L}}}{2\sqrt{2}G_F}\frac{(g')^2}{M_{Z'}^2 + 2m_e T_e}$$

Realistic thresholds of 600 MeV
in order to reject backgrounds with
E theta^2 cut



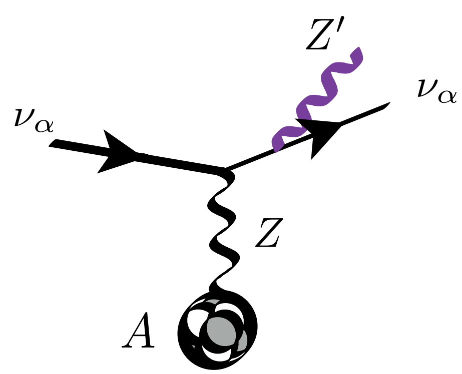
BSM tridents

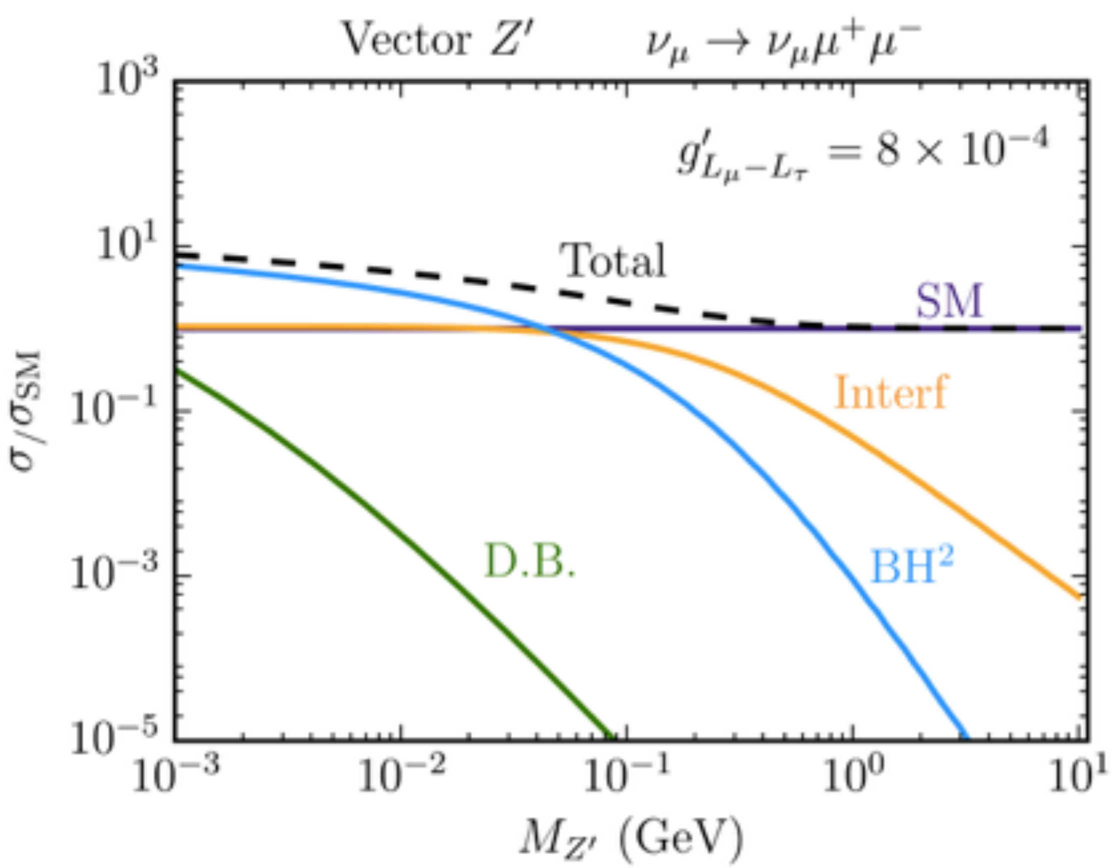
Bethe-Heitler:



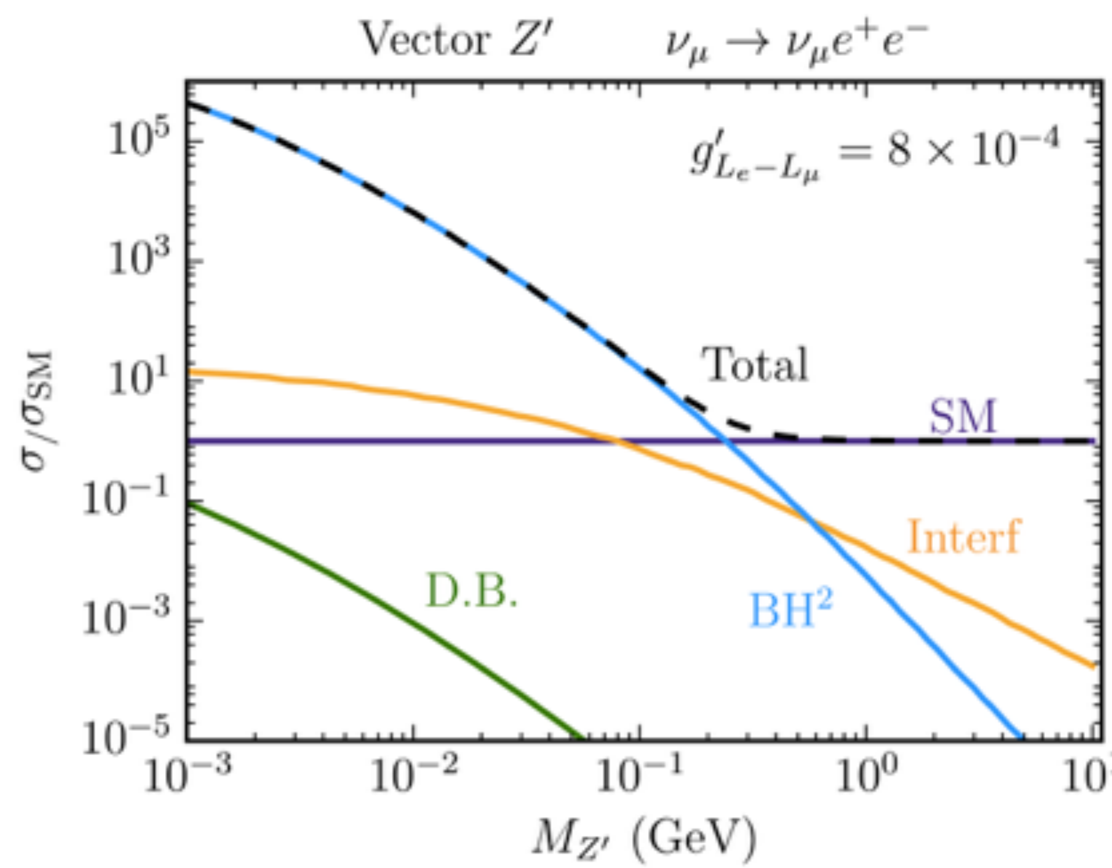
Dark-Brems.:

(negligible)





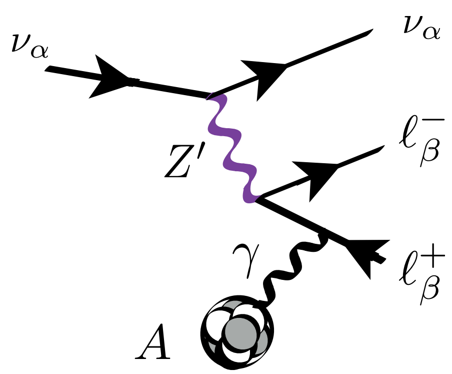
$L_\mu - L_\tau$



$L_e - L_\mu$

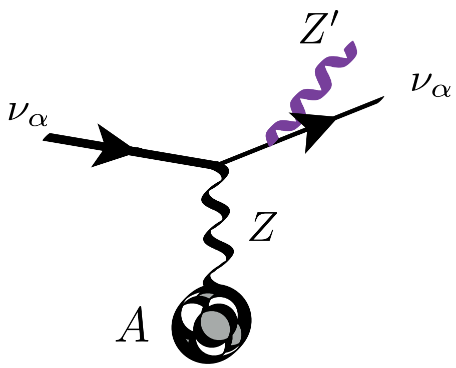
BSM tridents

Bethe-Heitler:

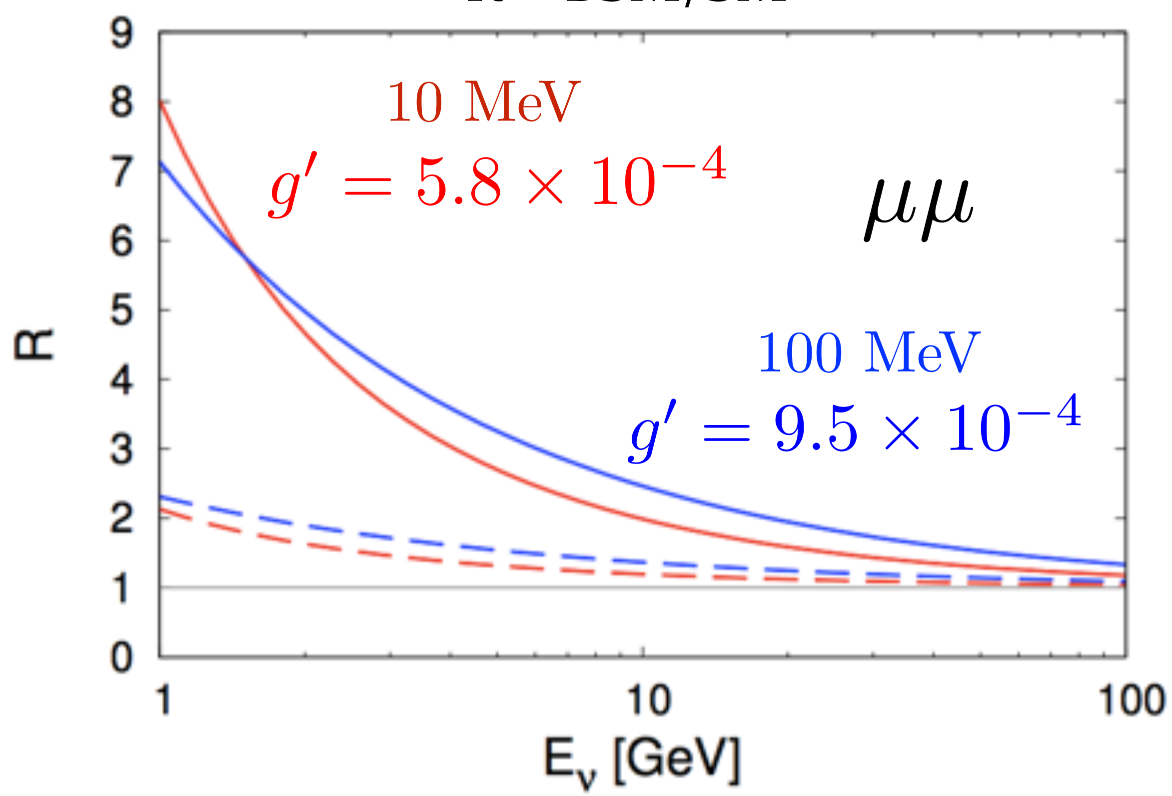


Dark-Brems.:

(negligible)



R = BSM/SM

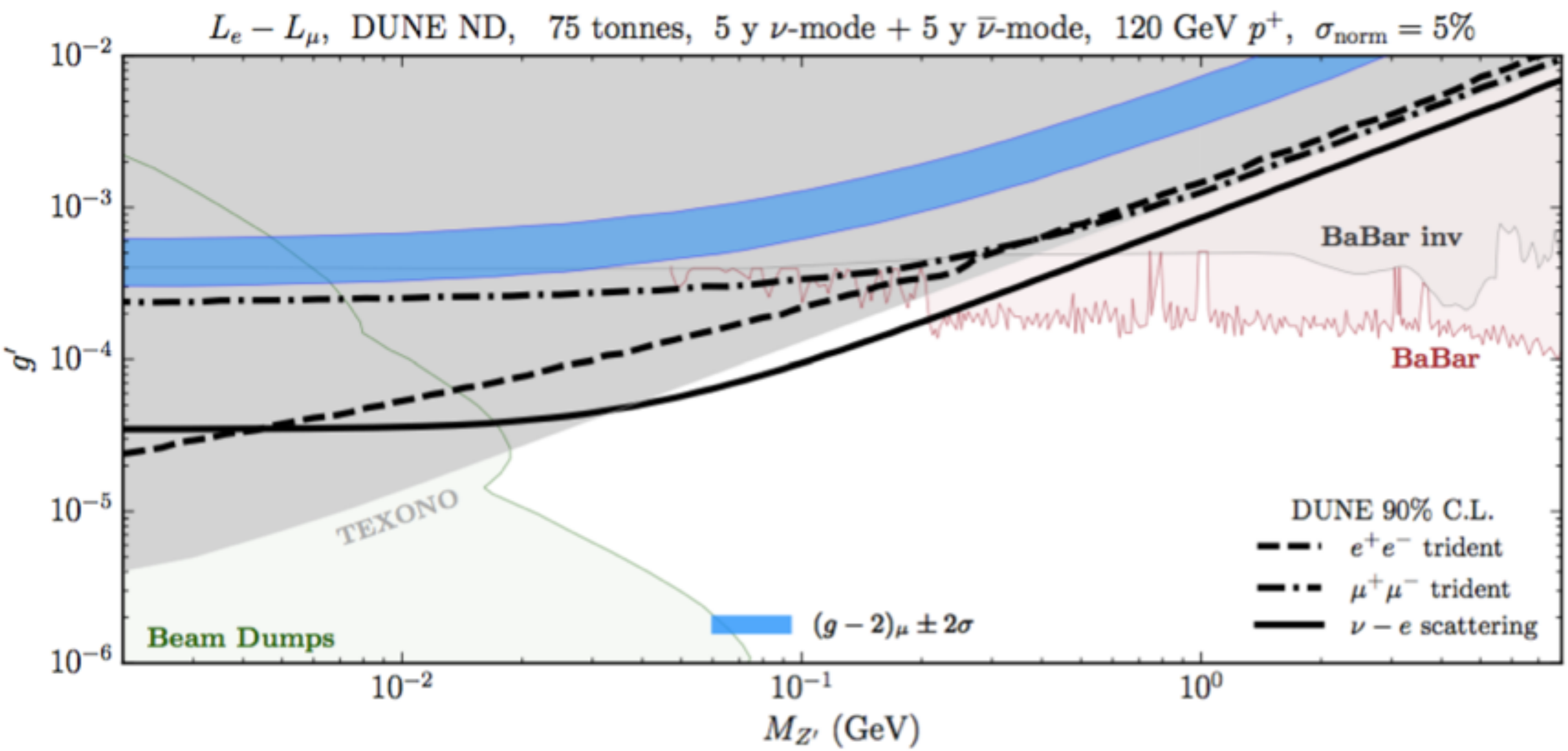


$L_\mu - L_\tau$

Measurement at low energies more sensitive

Y. Kaneta et al, PTEP 2017 (2017) no.5, 053B04

$L_e - L_\mu$ at DUNE

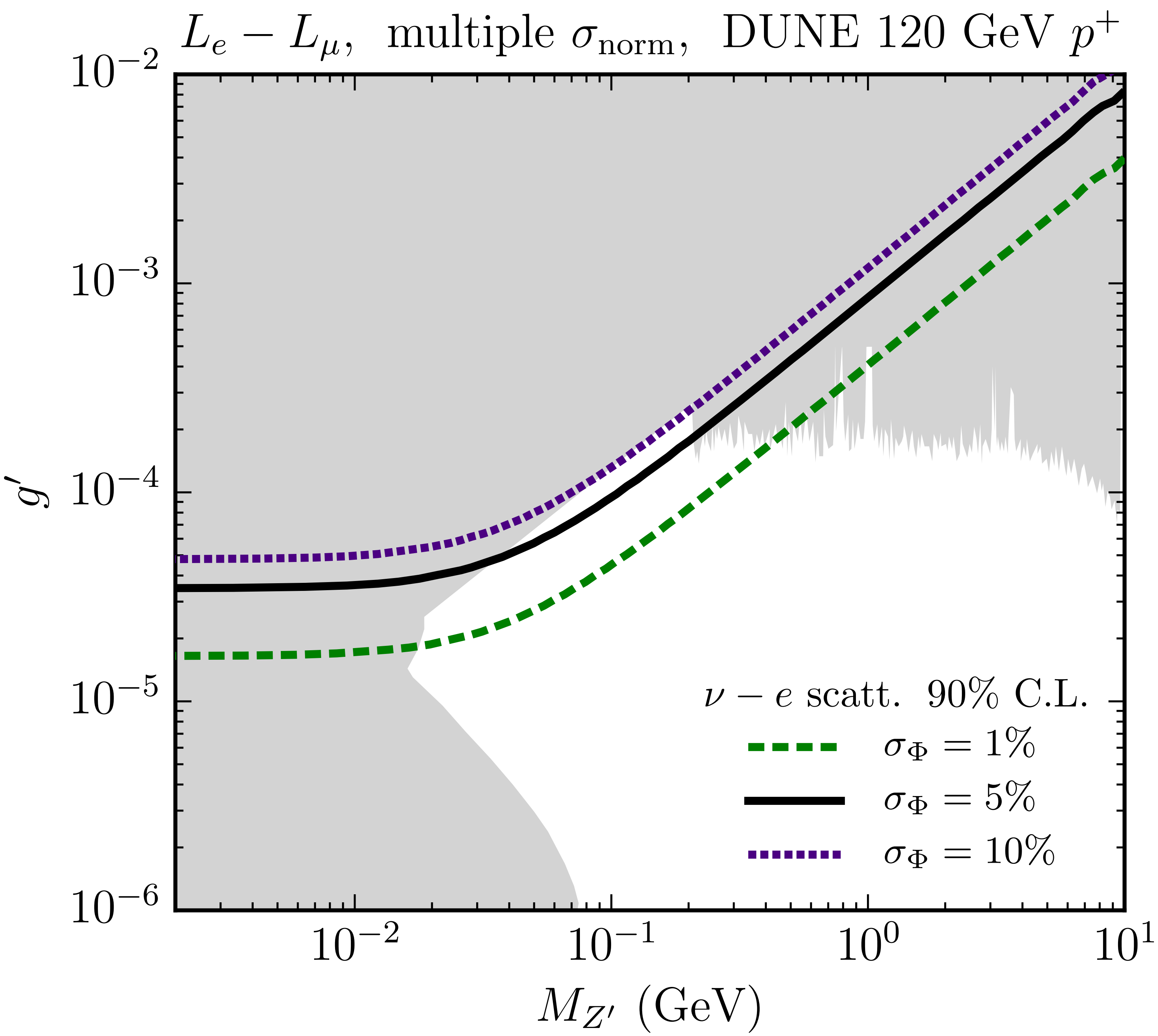


No backgrounds, but kin. cuts on the trident signal.

5% normalization sys is optimistic — can rely on low-nu technique.

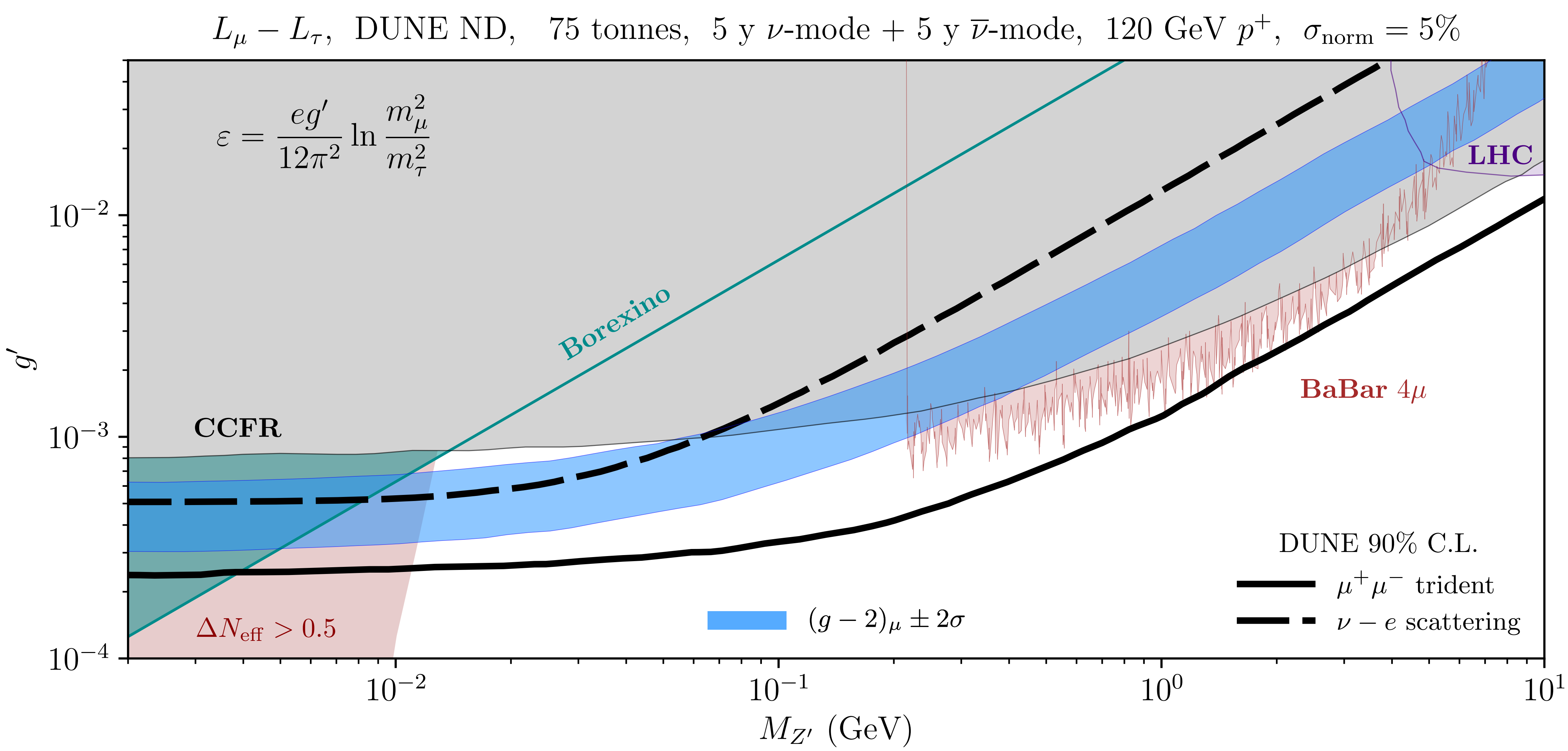
P. Ballett et al, arXiv:1902.08579

$L_e - L_\mu$ at DUNE



P. Ballett et al, arXiv:1902.08579

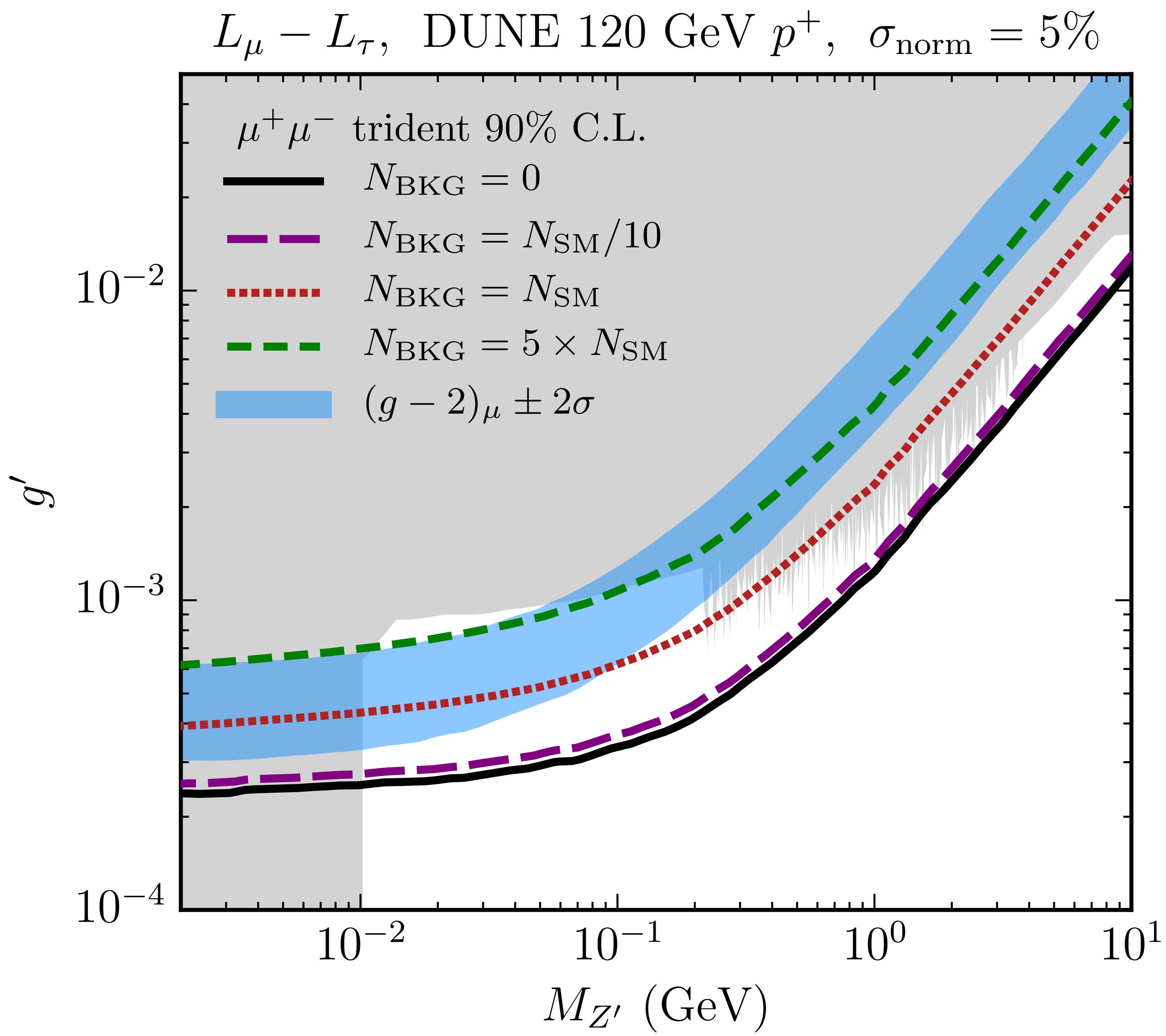
$L_\mu - L_\tau$ at DUNE



No backgrounds, but kin. cuts on signal, eg. $\theta_{\mu\mu} < 20^\circ$

P. Ballett et al, arXiv:1902.08579

$L_\mu - L_\tau$ at DUNE

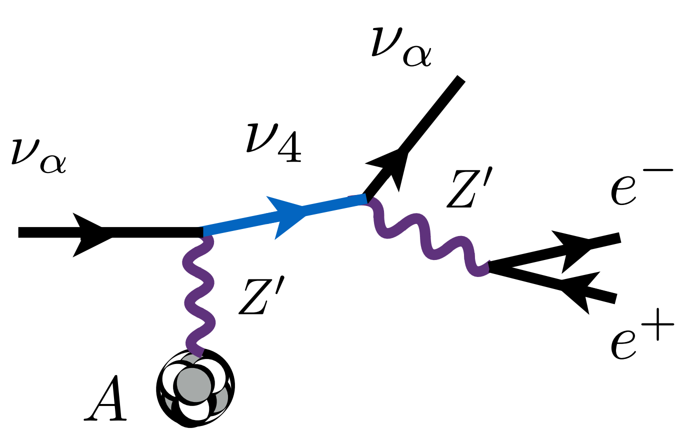


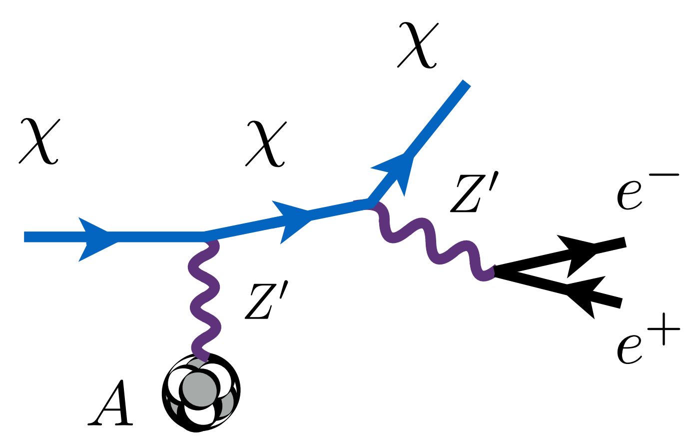
P. Ballett et al, arXiv:1902.08579

Dielectron signatures

New forces that talk directly to electrons more directly probed by neutrino-electron scattering data…

… but BSM e^+e^- signatures still predicted by many models.





Dark neutrinos (U’(1) charged heavy neutrinos)
— MiniBooNE explanations
— Neutrino mass generation at low scales

Talks by S. Jana and C. Argüelles

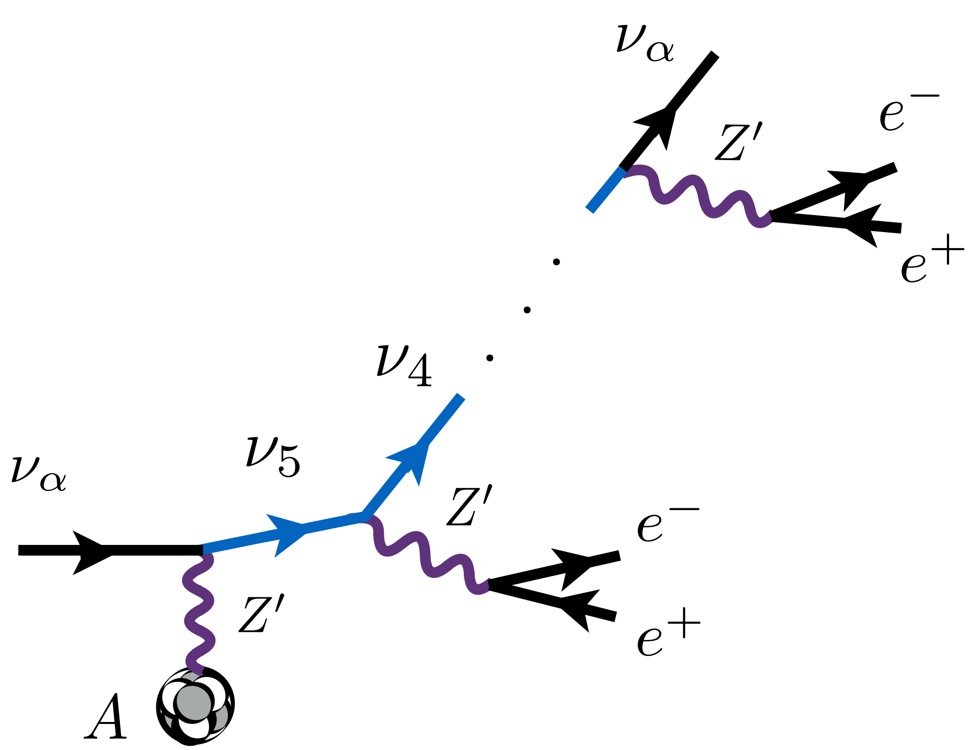
E. Bertuzzo et al, PRL121.241801
P. Ballett et al, PRD 99,071701(R)

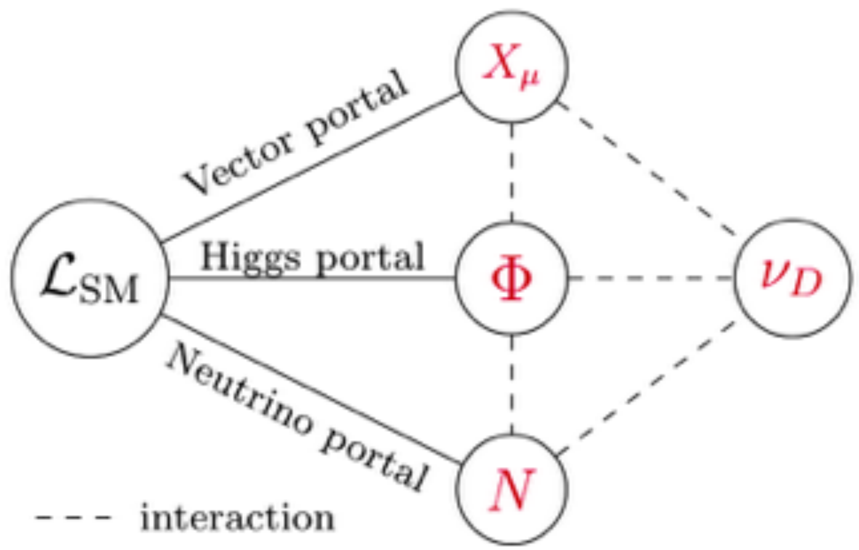
Light DM (and Inelastic DM)
— produced in the beam
— scattering or decay signature

Don’t see A. de Gouvea's talk

E. Izaguirre, PRD96(2017)no.5, 055007
A. de Gouvea et al, JHEP1901(2019)001

A light dark neutrino sector





Explains "Minimal" radiative ISS texture

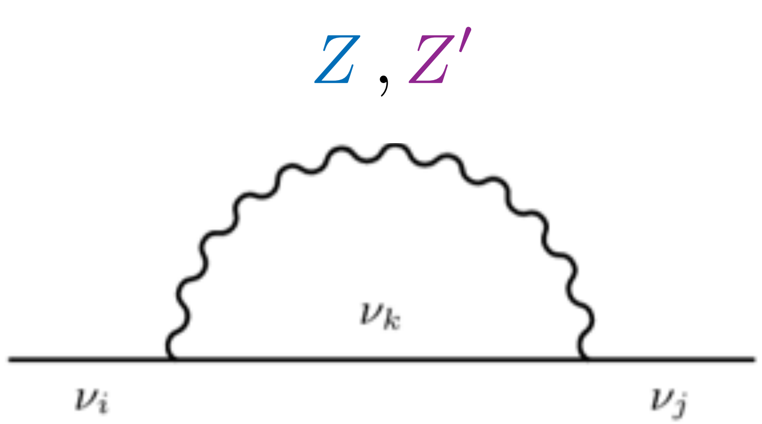
P.S. Bhupal Dev et al, arXiv:1209.4051

Dark neutrinos (U'(1) charged heavy neutrinos)

— MiniBooNE explanations

— **Neutrino mass generation at low scales**

$$\frac{1}{2}\begin{pmatrix}\overline{\nu_\alpha} & \overline{N} & \overline{\nu_D}\end{pmatrix}\begin{pmatrix}0 & m_D & 0\\ m_D & \mu' & \Lambda\\ 0 & \Lambda & 0\end{pmatrix}\begin{pmatrix}\nu_\alpha^c\\ N^c\\ \nu_D^c\end{pmatrix}$$



P. Ballett, MH, S. Pascoli, 1903.07589

P. Ballett, MH, S. Pascoli, PRD99,091701(R)

Conclusions

We have a chance to return to **rare neutrino scattering** physics with DUNE

Conclusions

We have a chance to return to **rare neutrino scattering** physics with DUNE

Leptophilic Z's a clear target — study clean leptonic scattering cross sections
(nu-e and trident are **not statistically limited!**)

Conclusions

We have a chance to return to **rare neutrino scattering** physics with DUNE

Leptophilic Z's a clear target — study clean leptonic scattering cross sections
(nu-e and trident are **not statistically limited!**)

Improvement on Le-Lmu requires to do better at flux systematics.
Lmu-Ltau is promising. but relies on bkg rejection.

Conclusions

We have a chance to return to **rare neutrino scattering** physics with DUNE

Leptophilic Z's a clear target — study clean leptonic scattering cross sections
(nu-e and trident are **not statistically limited!**)

Improvement on Le-Lmu requires to do better at flux systematics.
Lmu-Ltau is promising. but relies on bkg rejection.

Beyond tridents, what else are neutrinos doing at the
per mille of a per mille level of CCQE?

Questions

Neutrino-electron scattering

- how large are backgrounds at low recoil energies E < 600 MeV?
- Flux uncertainties: can we expect improvements at accelerator experiments?
- If B-L, no low-nu technique to rely on.

Dimuon tridents

- Limited by backgrounds — alternative: use **HPgTPC** as it provides better PID?
- Do we need to wait for DUNE? — Searches at MINERvA, MINOS or SBND?

Dielectron tridents

- Also use HPgTPC? — pi0s are not a problem there! Magnetized!
- Clear SM search/measurement provides bounds for a plethora of dark models
- eg. dark ………… (photon, neutrinos, scalars, matter)

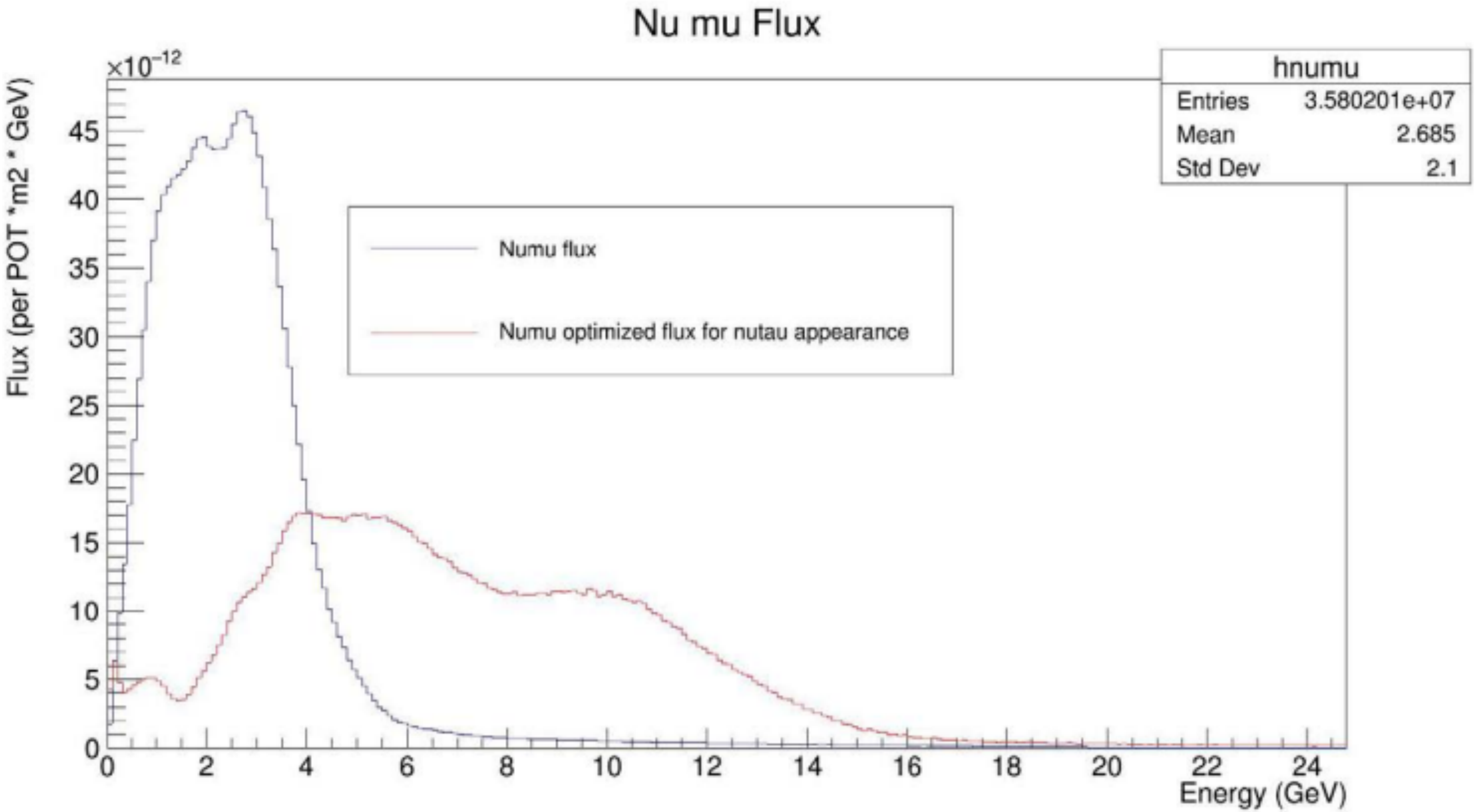
Le-Lmu

- Matter effects vs scattering.
- Improvements from reactor nu-e measurements

THANK YOU

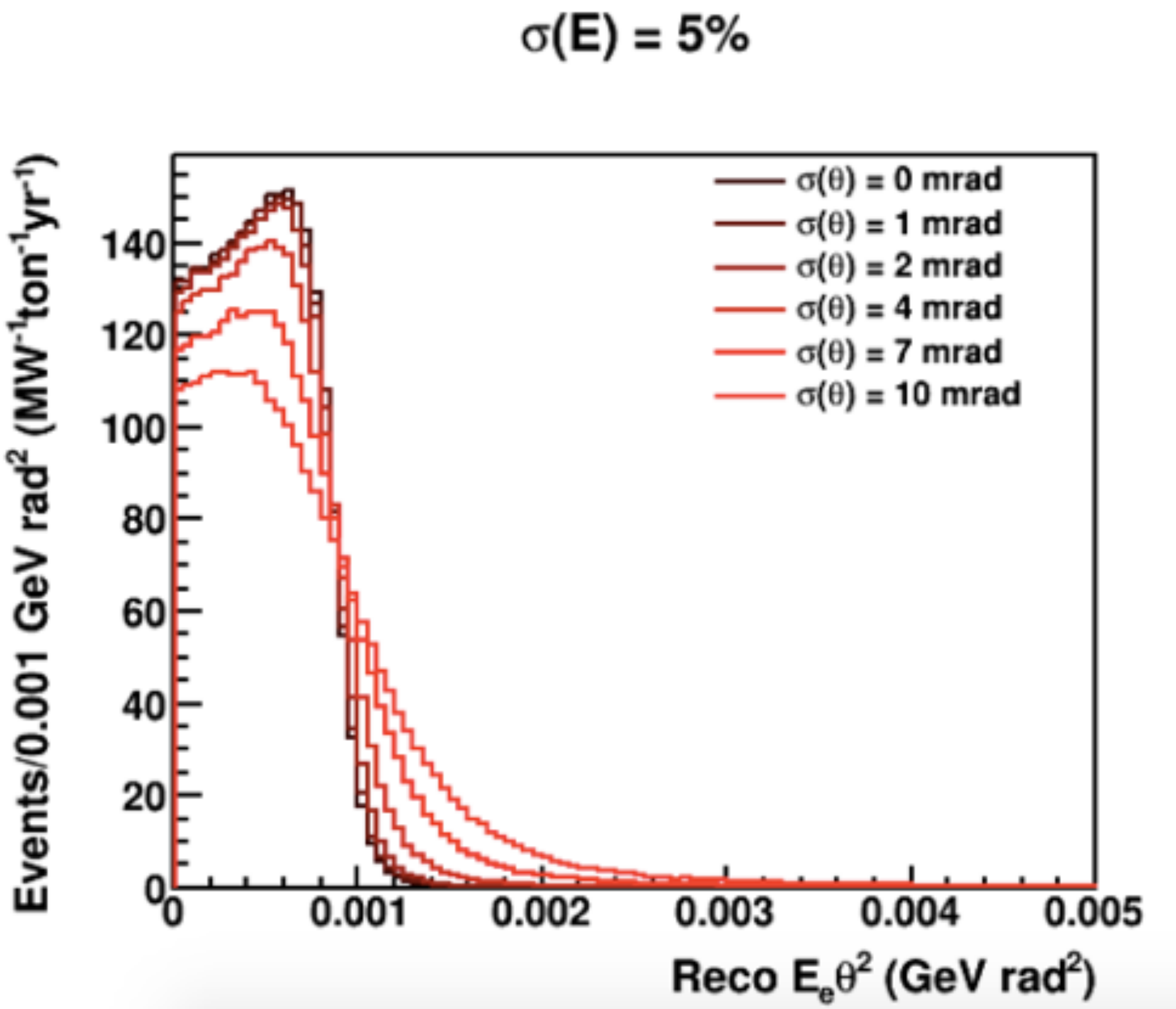
APPENDIX

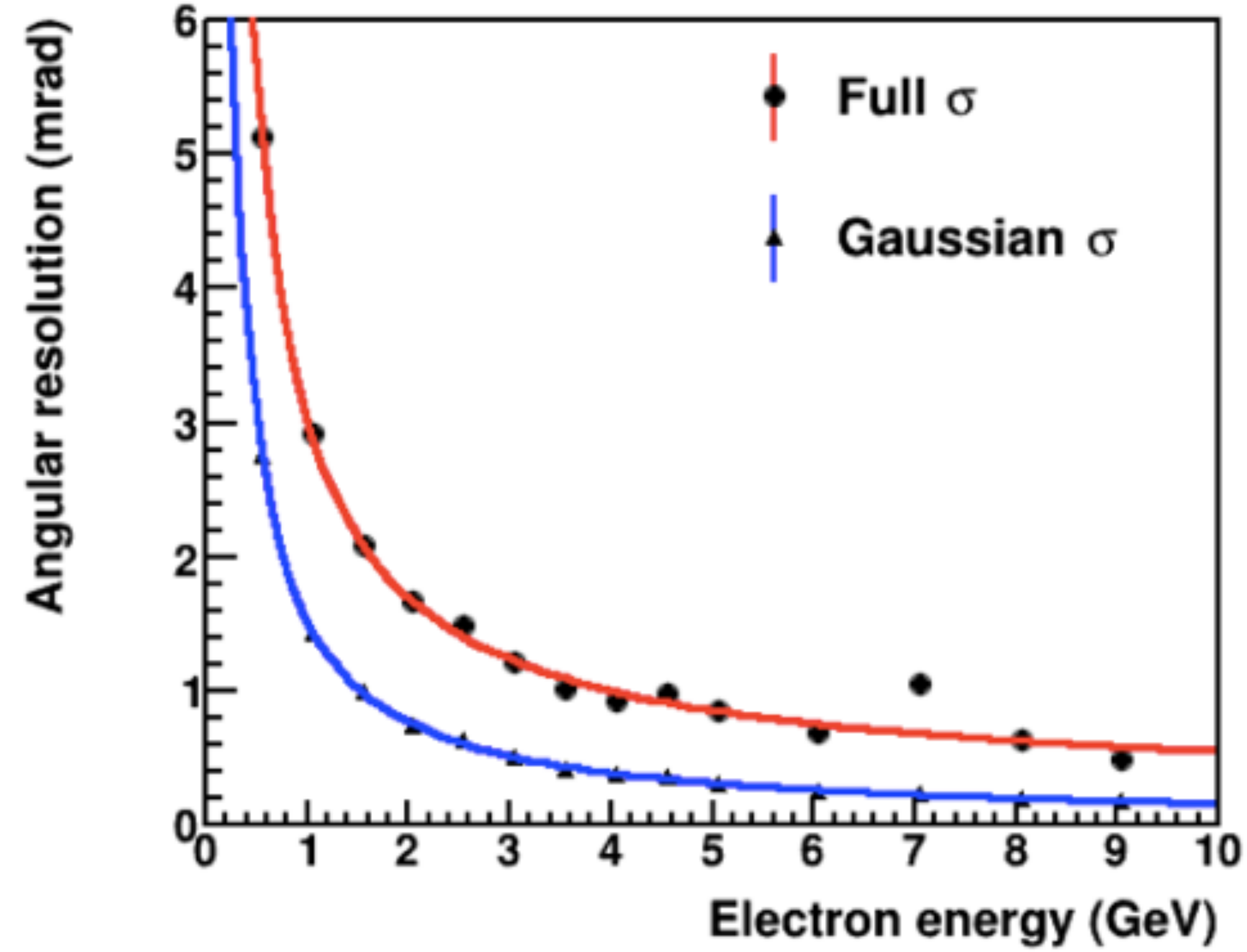
DUNE fluxes



A. Bross — 12/3/2018 — PONDD

Electron angular resolution

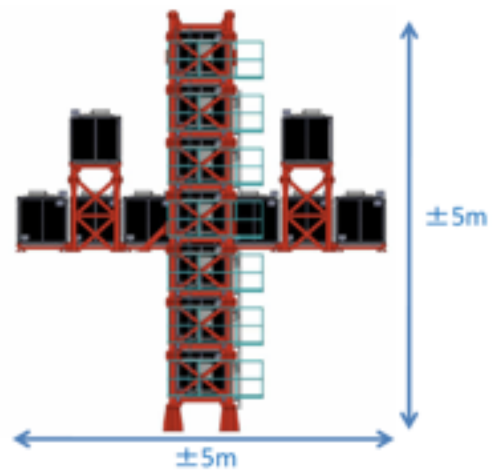


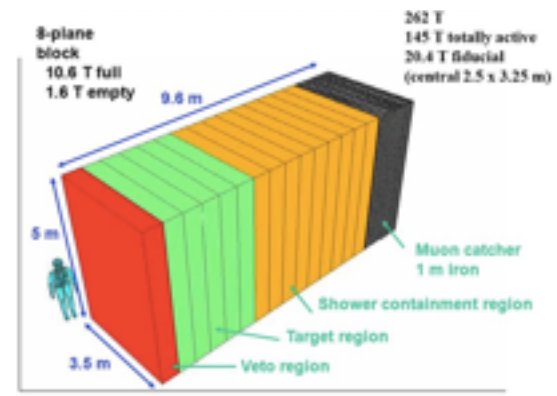


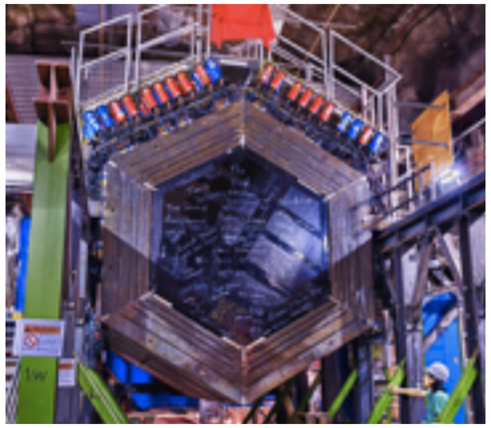
Flux measurements with DUNE ND: ν+e elastic & low-ν
Chris Marshall
Fermilab ND workshop
28 March, 2017

Present

INGRID	MINOS/+	NOvA	MINERvA



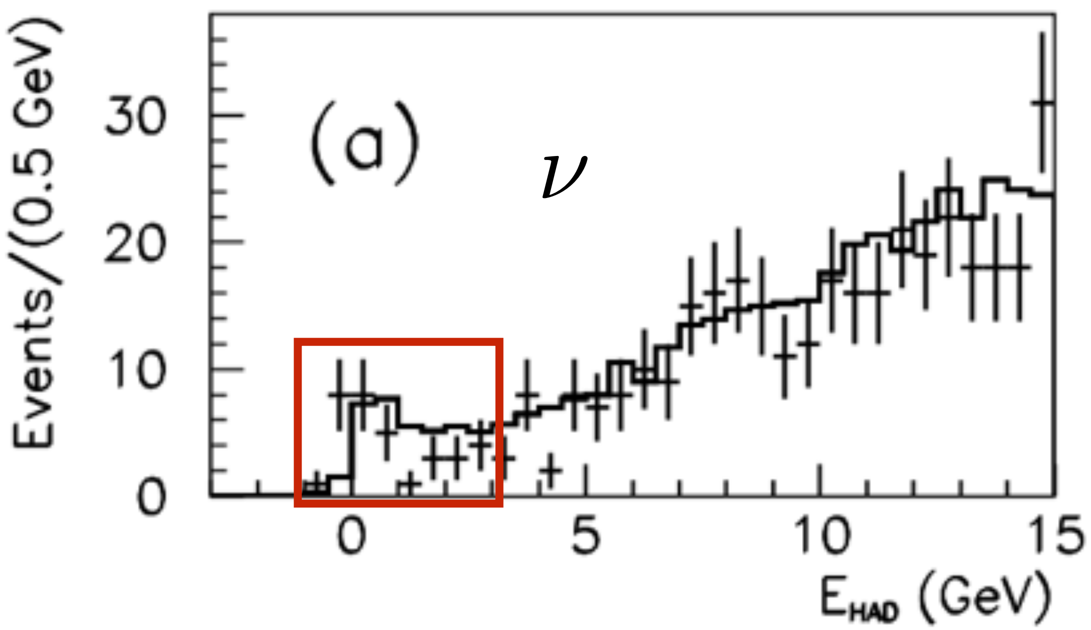




Channel	**T2K-I**	**T2K-II**	**MINOS**	**MINOS+**	**NOνA-I**	**NOνA-II**	**MINERνA**
Total $e^{\pm}\mu^{\mp}$	563	1444	222 (56)	730	83 (72)	340 (374)	149 (102)
	96	246	46 (11)	151	25 (22)	102 (114)	56 (39)
Total e^+e^-	277	711	61 (15)	62	29 (22)	119 (114)	39 (27)
	24	62	9 (2)	8	4 (4)	16 (21)	10 (7)
Total $\mu^+\mu^-$	30	76	26 (6)	86	9 (9)	37 (47)	18 (13)
	21	54	15 (3)	49	8 (8)	34 (36)	18 (13)

100% efficiencies. Detector capability is the name of the game.

NuTeV
Lab E detector at FNAL

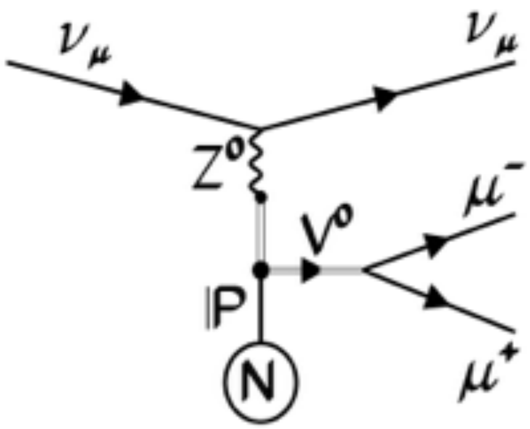


$$\langle E_\nu \rangle = 160 \text{ GeV}$$

$$\frac{\sigma_{\text{NuTeV}}}{\sigma_{\text{SM}}} = 0.72^{+1.73}_{-0.72}$$

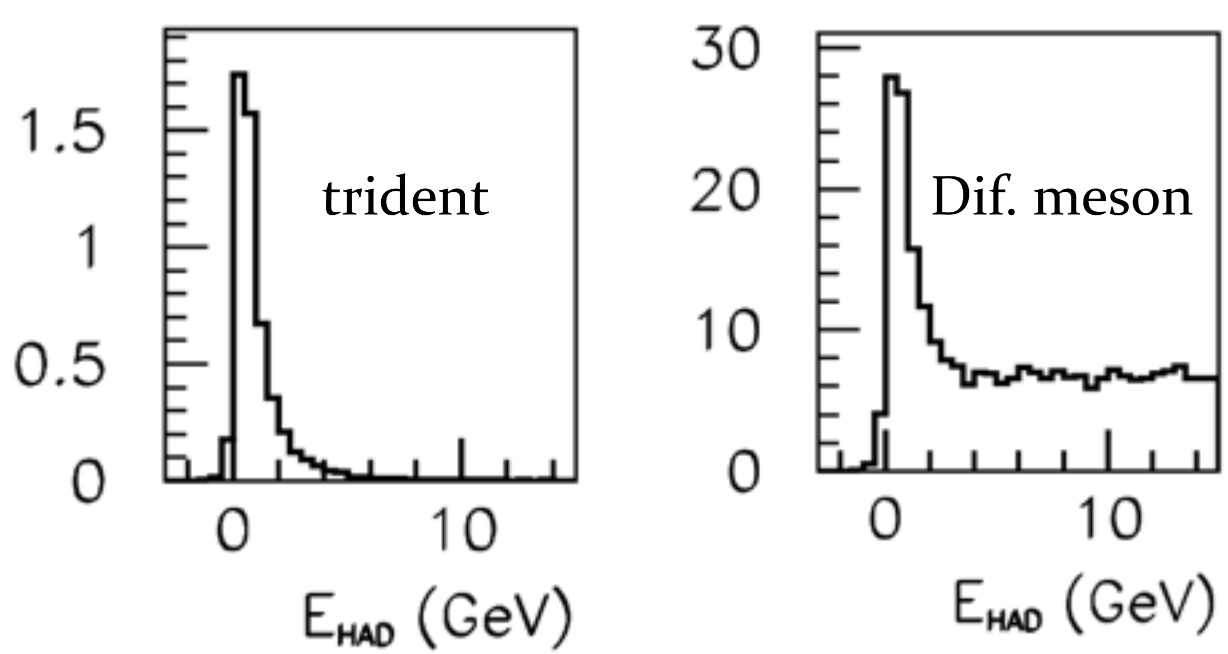
Inconclusive

Phys. Rev. D 61, 092001 (2000)



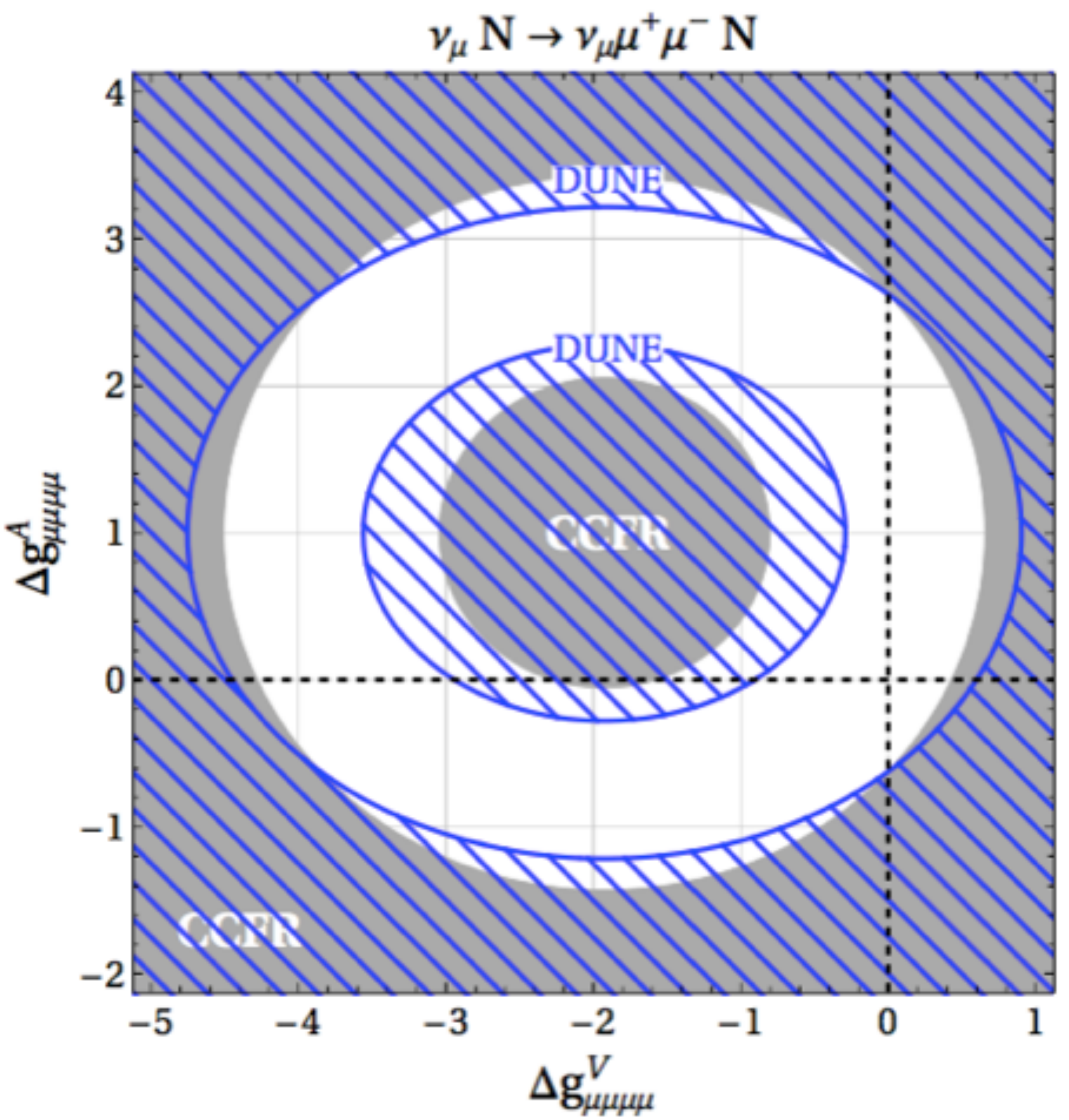
Diffractive vector meson contribution.

NuTeV MC



See discussion in
G. Krnjaic, arXiv:1902.07715

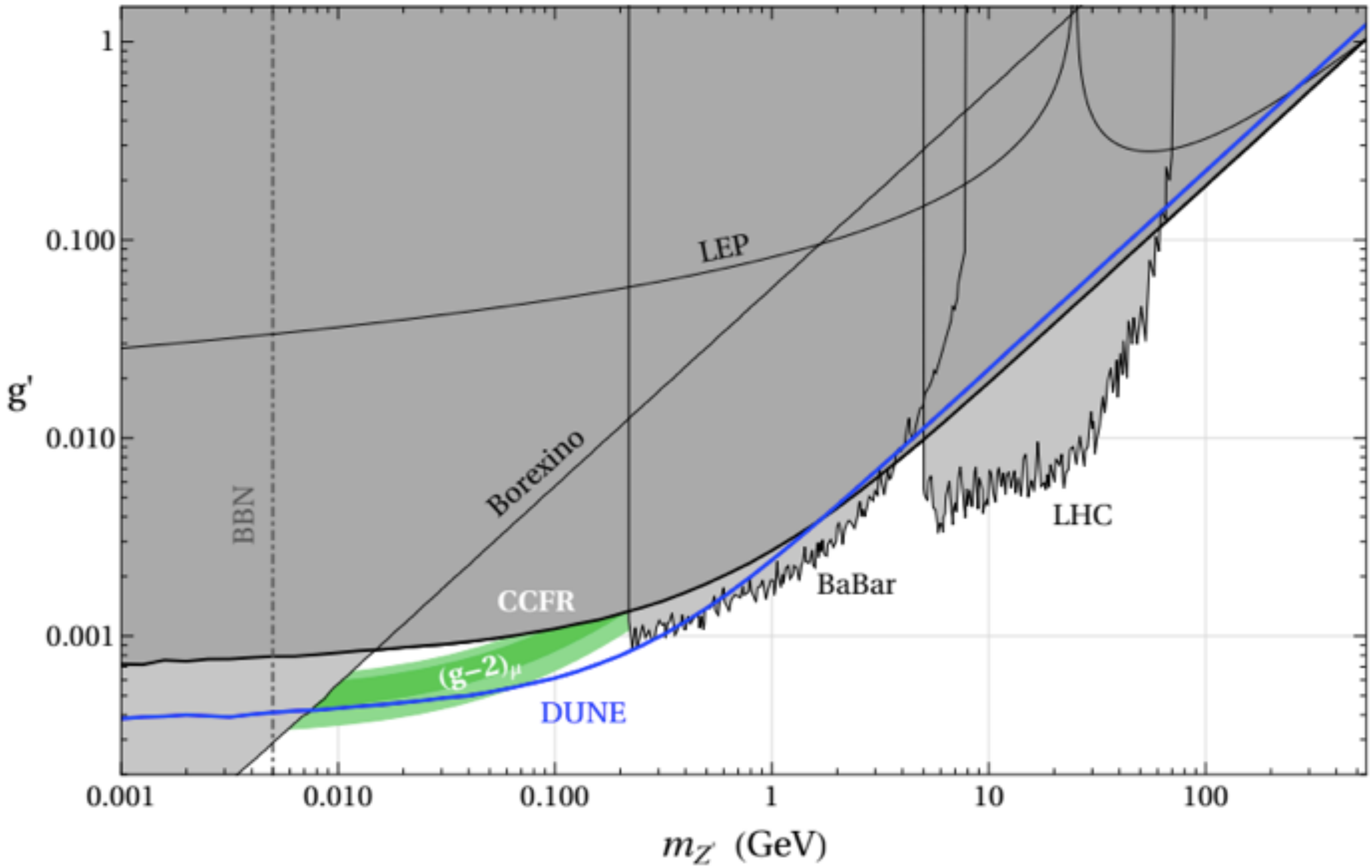
Precision physics?



$$2g_V = 1 + 4\sin^2\theta_W + \Delta g^V_{\mu\mu\mu\mu} \qquad -2g_A = -1 + \Delta g^A_{\mu\mu\mu\mu}$$

W. Altmannshoffer et al, arXiv:1902.06765

DUNE



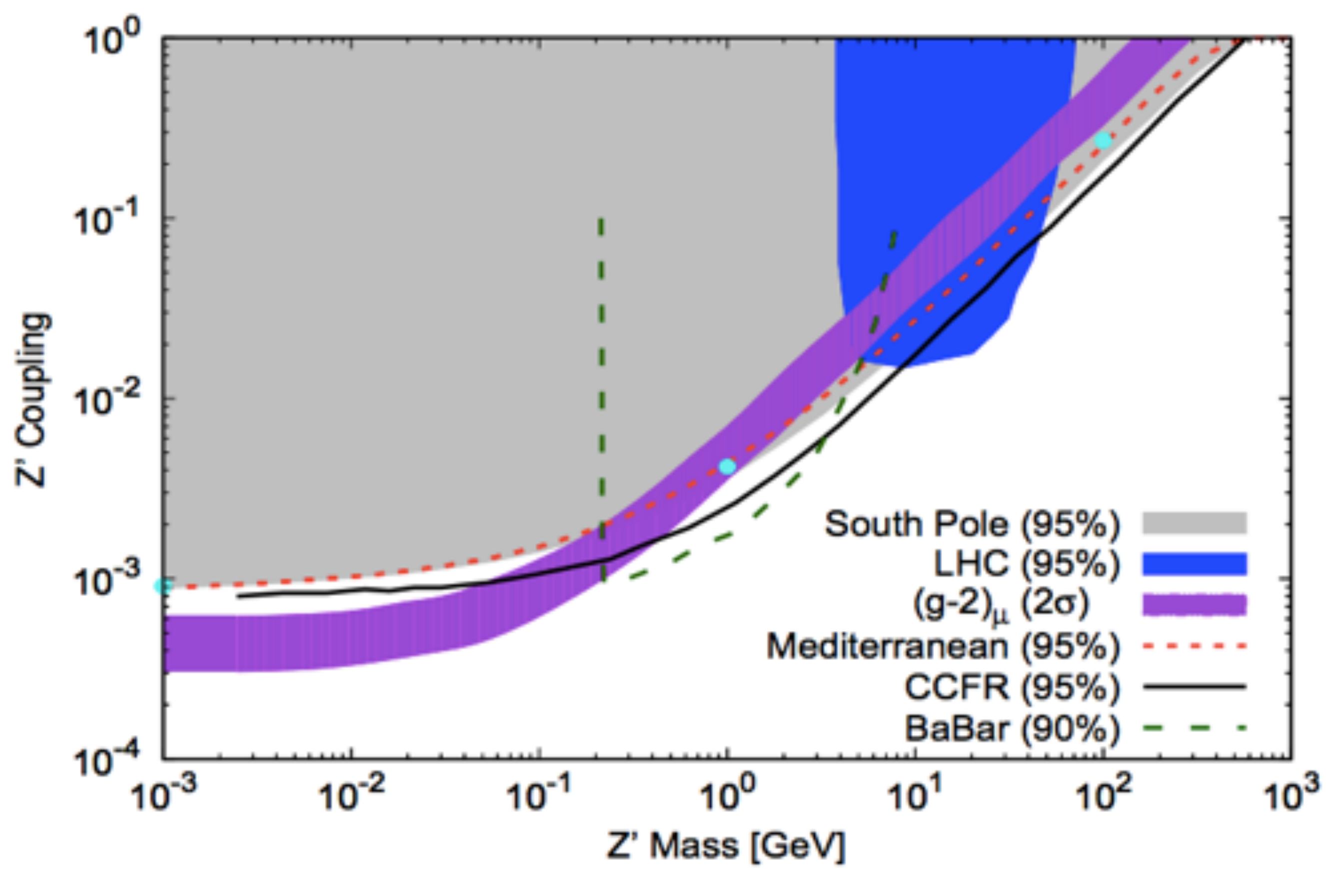
Backgrounds: $S/\sqrt{B} \sim 1.6$ per year, 25% measurement in ~ 6.5 years.

Tight kin. cuts (eg. $\theta_{\mu\mu} < 5^\circ$). Reducing pion misID can loosen it

W. Altmannshoffer et al, arXiv:1902.06765

Atmospheric neutrinos in the game

Trade off between # of events and BSM enhancement.



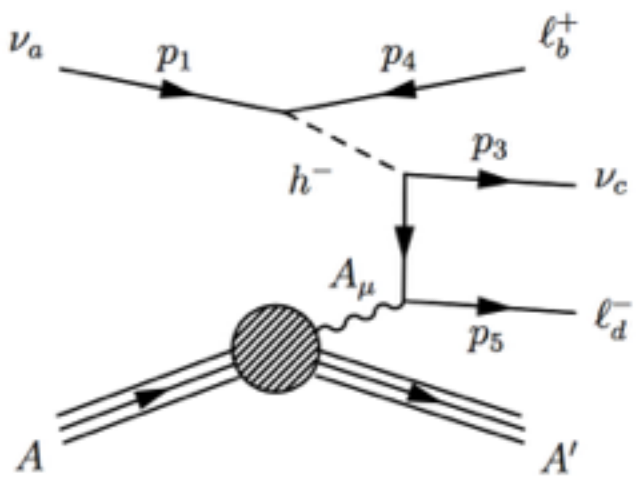
Total SM rate expected in 10 years:

7.4 (17, 63) at PINGU (DeepCore, IceCube)
16 (23) at ORCA (ARCA)

SF. Ge et al, PLB772(2017)164-168

Code available at:
https://gitlab.com/gesf/NuTrident_CompHEP

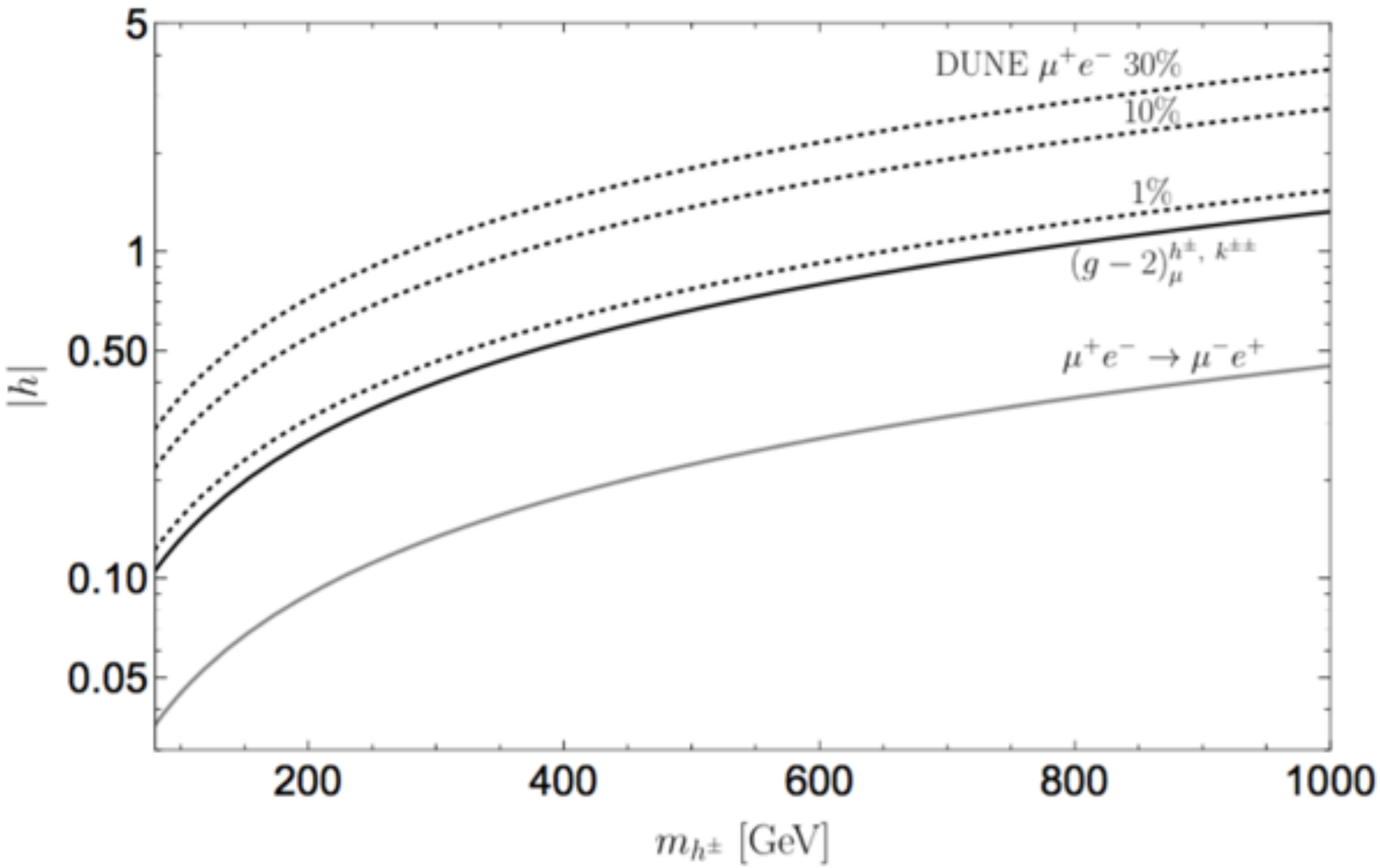
New physics in charge-current channels



G. Magill et al, PRD97 (2018)no.5,055003

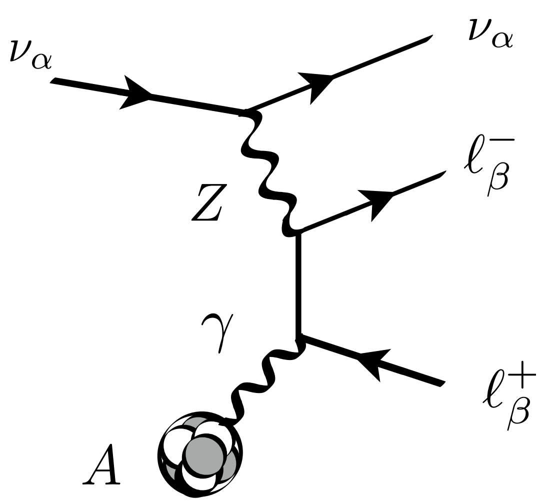
Much harder — new physics more easily constrained otherwise.

$$\mathcal{L} \supset |\partial_\mu h|^2 - m_h^2|h|^2 + \sqrt{2}h_{ab}\nu^a\ell^b h + k_{ab}\ell^a\ell^b k + c.c.$$

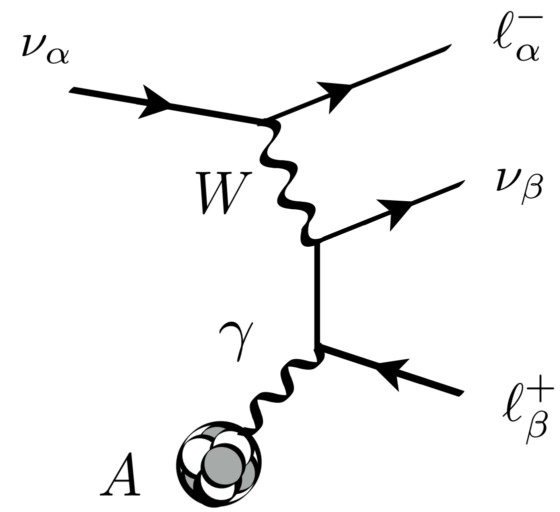


...but more data cannot harm!

Standard Model contributions

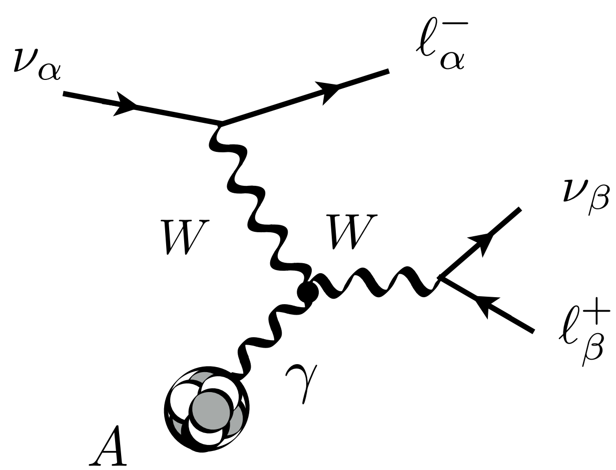


+ crossed boson lines

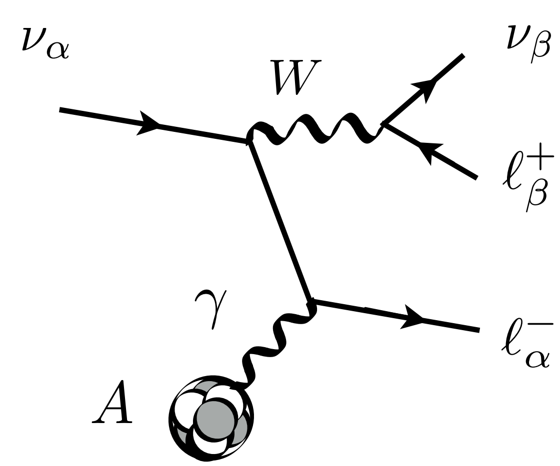


Z exchange boson lines
—Bethe-Heitler resembling photon pair production—

W exchange



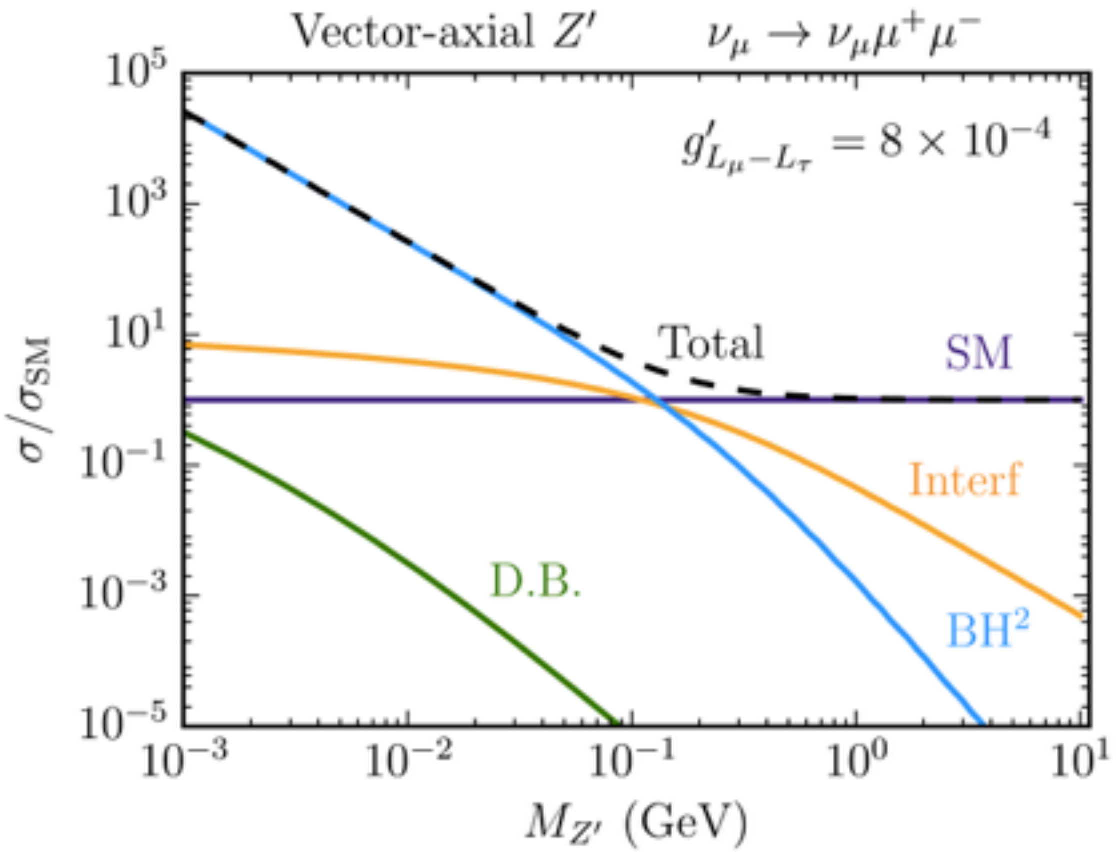
Resonant at high energies

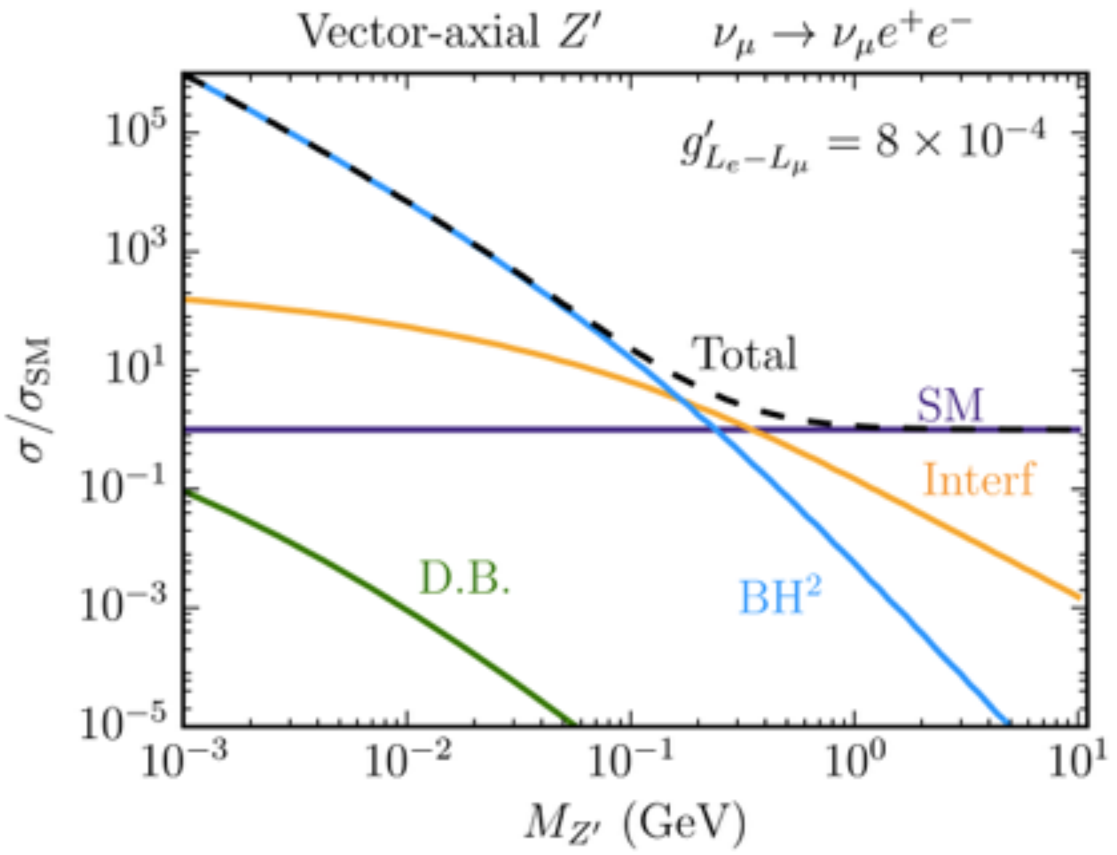


Triple gauge boson coupling
(order M_W^4)

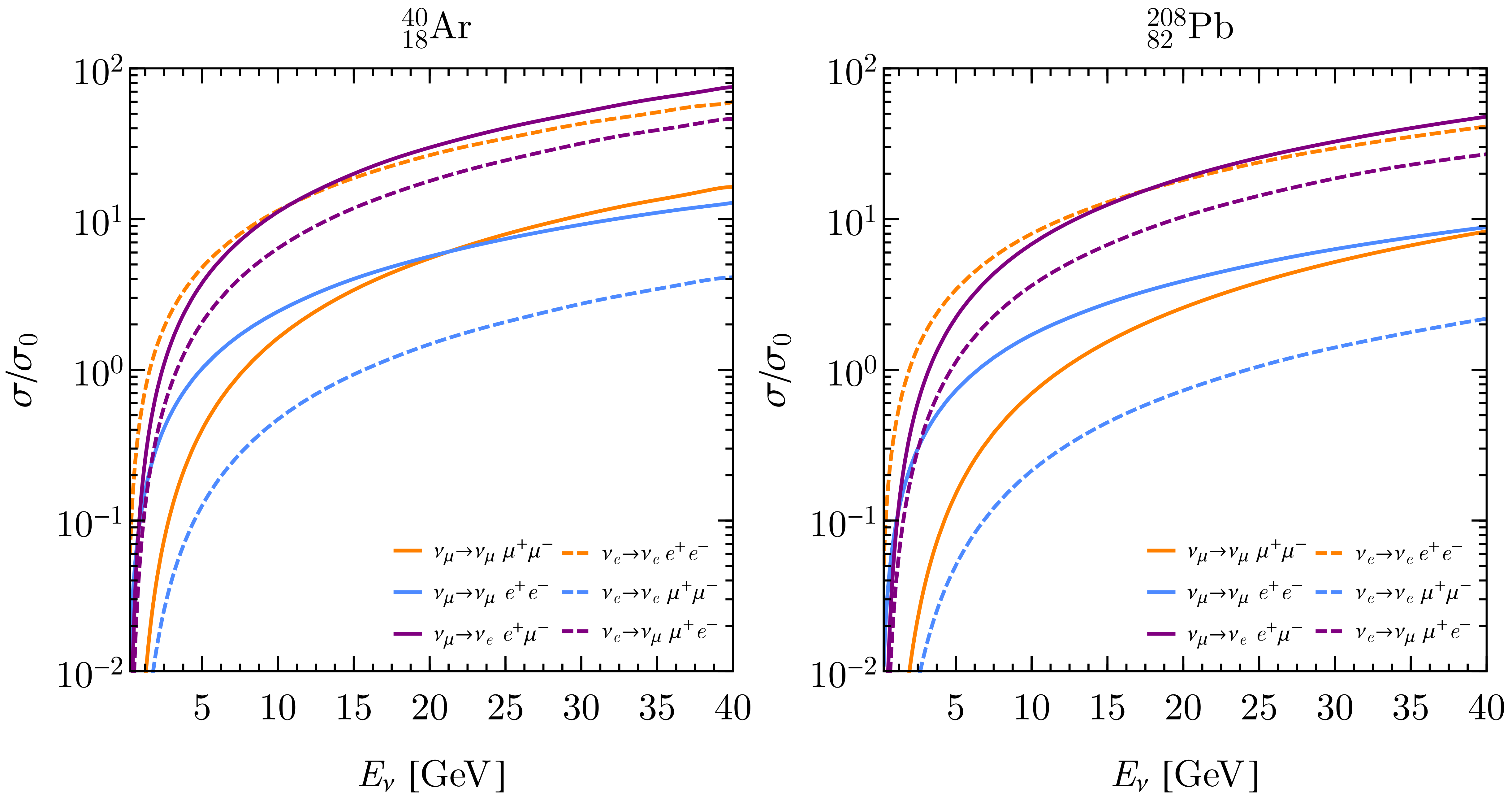
W radiation

Axial-vector mediators



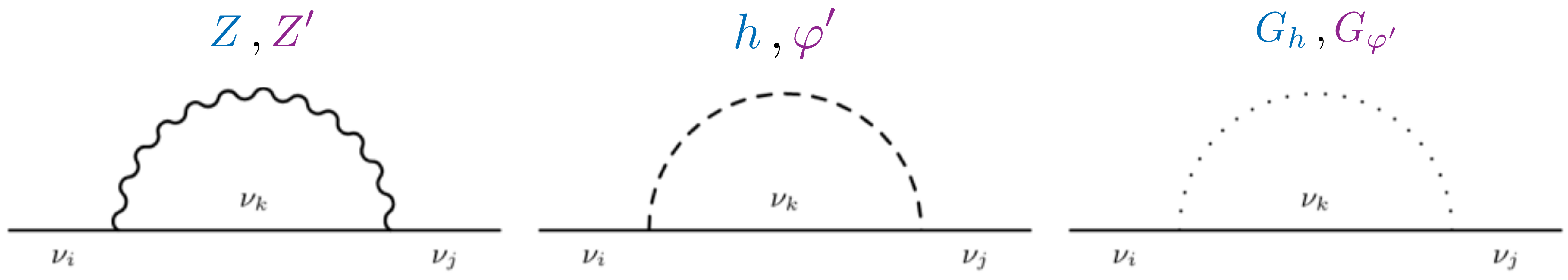


Coherent cross sections



$$\sigma_0 = Z^2 \, 10^{-44} \ \text{cm}^2$$

Neutrino masses at one-loop level



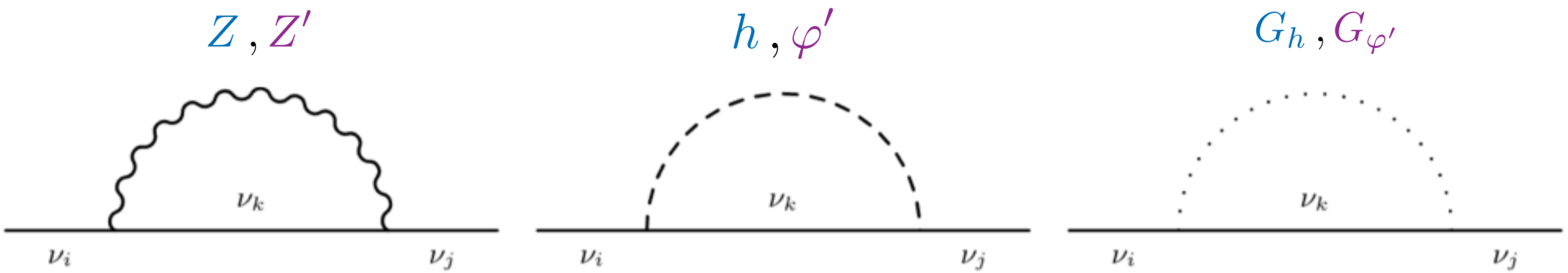
$$m_{ij} = \frac{1}{4\pi^2}\sum_{k=4}^{5}\left[C_{ik}C_{jk}\frac{m_k^3}{m_Z^2}F(m_k^2, m_Z^2, m_h^2) + D_{ik}D_{jk}\frac{m_k^3}{m_{Z'}^2}F(m_k^2, m_{Z'}^2, m_{\varphi'}^2)\right],$$

SM BSM

$$C_{ik} \equiv \frac{g}{4c_W}\sum_{\alpha=e}^{\tau} U_{\alpha i}^* U_{\alpha k} \qquad D_{ik} \equiv \frac{g'}{2} U_{Di}^* U_{Dk}.$$

In general, both contributions are important, but if NP is light, it dominates.

Neutrino masses at one-loop level



With a single pair of heavy states, at least one light neutrino remains massless.

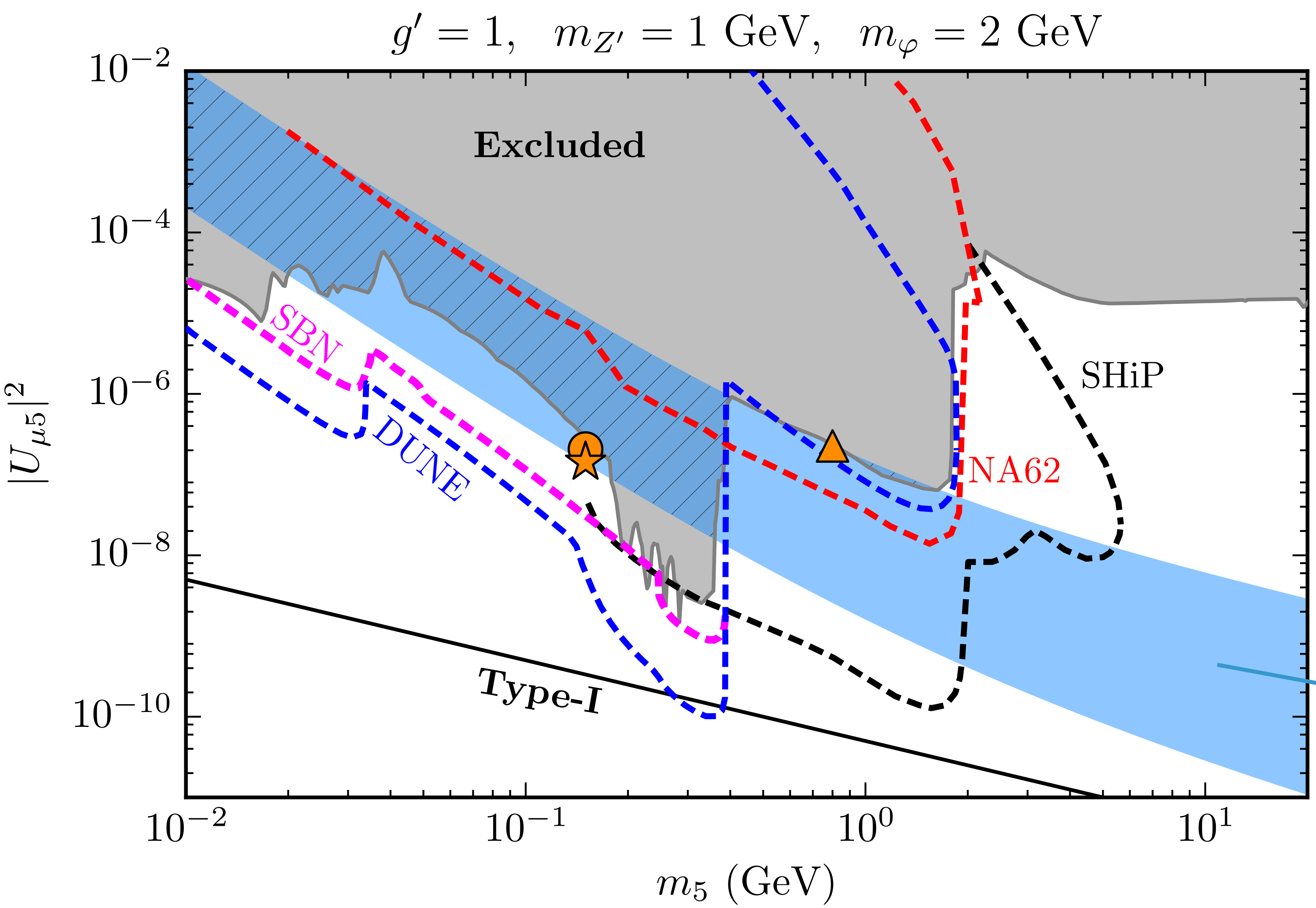
$$m_{ij} = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix} + \begin{pmatrix} a'^{\,2} & a'b' & a'c' \\ a'b' & b'^{\,2} & b'c' \\ a'c' & b'c' & c'^{\,2} \end{pmatrix}$$

If only SM/BSM contribution => only one massive neutrino

If both => two massive neutrinos (**highly predictive**, but further study needed.)

Testing the mechanism

Reproducing the right scale for neutrino masses



Prediction of the model:

$$R = \frac{m_4}{m_5} = -\frac{U_{\alpha 5}^2}{U_{\alpha 4}^2}$$

Blue band:

$$m_3 = \sqrt{\Delta m^2_{\rm atm}}$$

$$1\% < R < 99\%$$