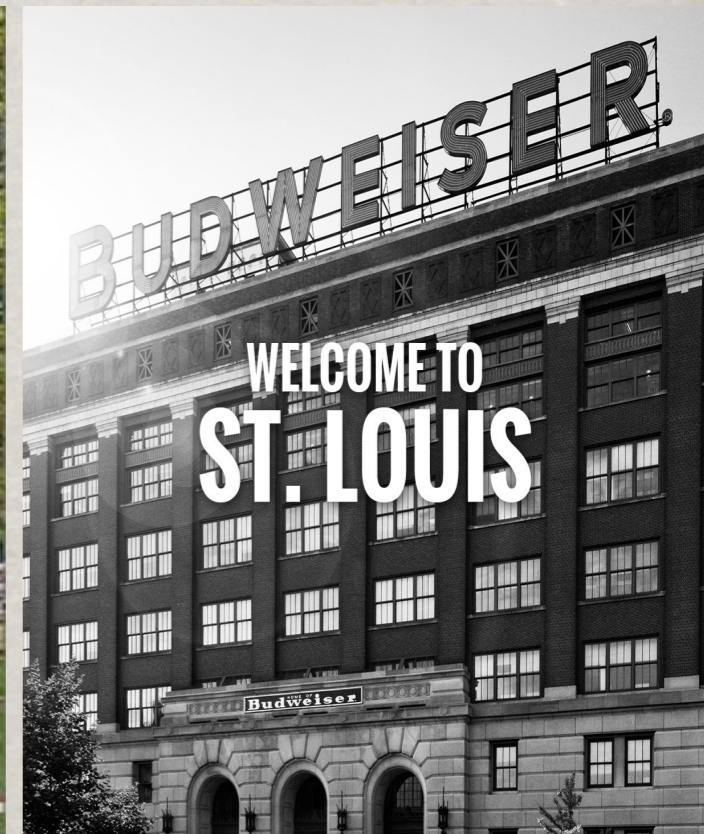


# OVERVIEW OF NEUTRINO PHYSICS AT COLLIDERS OR TESTING SEESAW

Tao Han, University of Pittsburgh

NTN Workshop on Neutrino NSI

Washington Univ., St. Louis, May 29, 2019





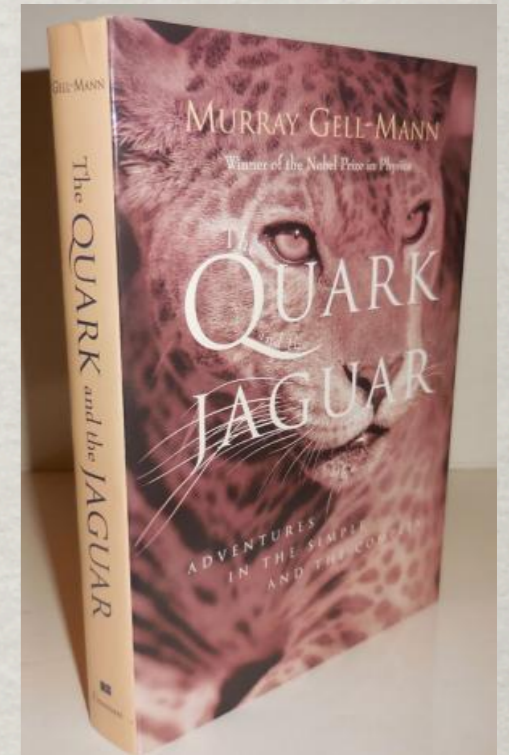
On May 24, 2019:

## Murray Gell-Mann, Nobel Prize-winning physicist who named quarks, dies at 89

- 1969 Nobel laureate helped discover subatomic particles
- Death confirmed by Santa Fe Institute he co-founded



▲ Murray Gell-Mann, seen Santa Fe Institute in 2003. Photograph: Jane Bernard/AP



2010 in Aspen

His “Totalitarian Principle” argument made a Majorana mass term “compulsory” (almost)



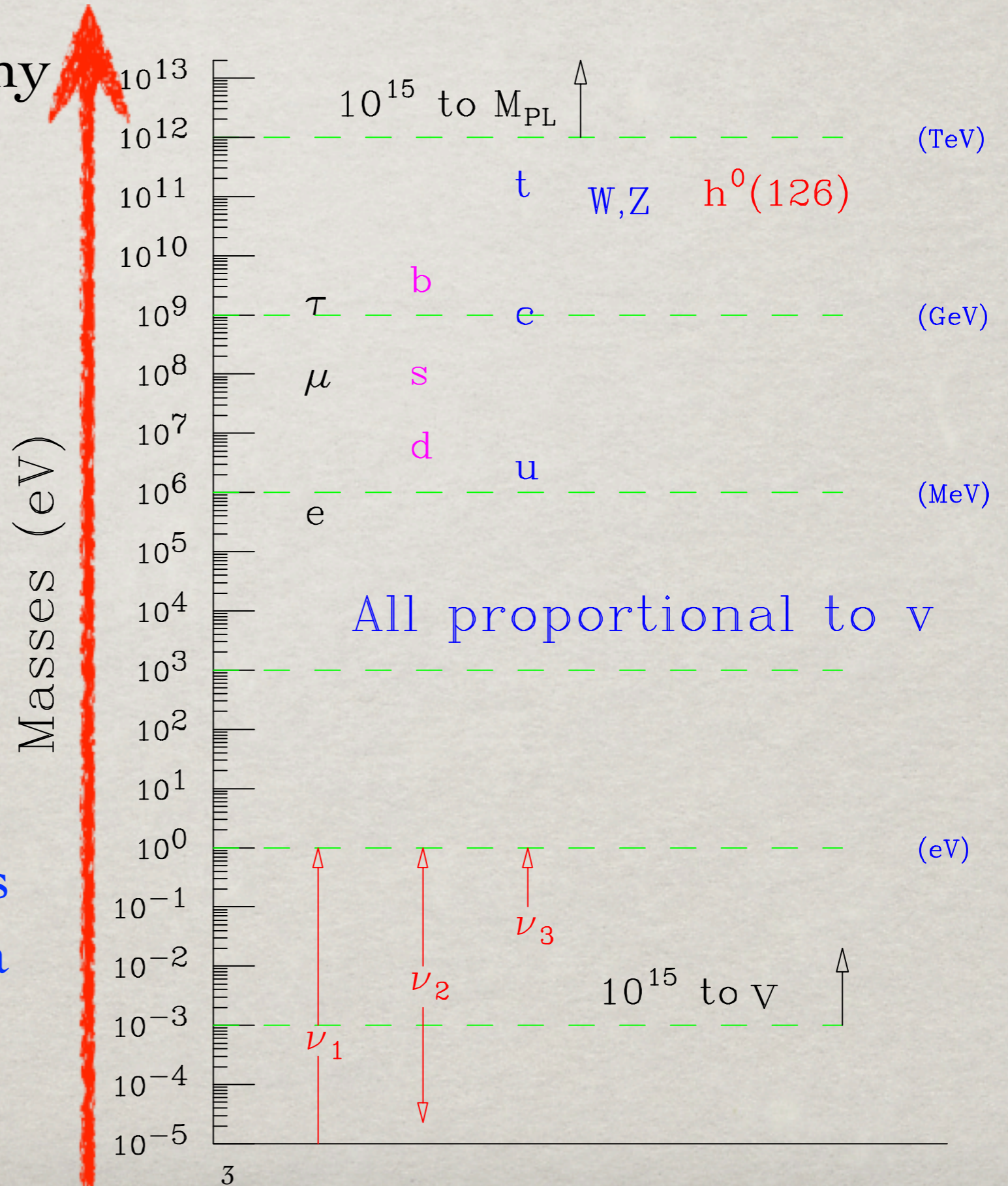


# 1. Flavor Puzzle is a much<sup>n</sup> harder problem

- Particle mass hierarchy
- Patterns of quark-neutrino-mixings
- Neutrino mass: Dirac (Higgs) vs. Majorana (seesaw)
- New CP-violation sources

## Nu-physics:

One of the best chances for Nature to teach us a lesson!





# On the theory side:

- “Technically natural” in t’Hooft sense:  
small values are protected by symmetry.  
At a “new physics” (cut-off) scale  $\Lambda$  :  
“natural”:  $\delta m_f \sim g^2/(16\pi^2) m_f \ln(\Lambda^2/m_f^2)$   
in contrast to “unnatural”:  $\delta m_H^2 \sim -y_t^2/(8\pi^2) \Lambda^2$
  - **Two ways to generate small values naturally:**
  - Suppression by integrating out heavy states:  
 $\sim g^2 E^2/M^2$   
the higher dimension  $1/\Lambda^n$ , the lower  $\Lambda$  can be.
  - Suppression by loop radiative generation:  
the higher loops  $1/(16\pi^2)^n$ , the lower  $m_\nu$  can be.
- **Scale and couplings wide open in theory space.**



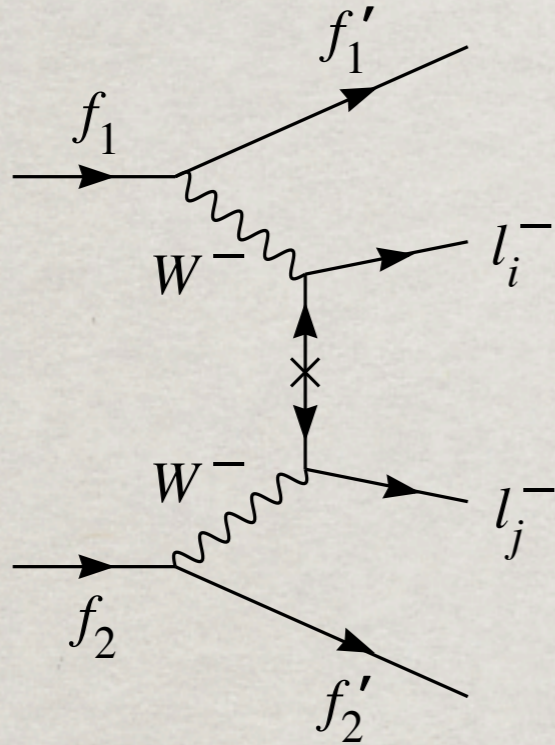
# On the phenomenology side:

- Will search for ANYTHING  
new states of mass  $M$ , new couplings/mixings  $k, V_{ij} \dots$
- Will search EVERY WHERE  
low-energy & high-energy regimes.
  - Today, primarily,  
target on Majorana nature:  $\Delta L=2$
  - Equally important,  
charged lepton flavor transition:  $\Delta L=0$
  - Observables  $\leftrightarrow$  Theory connections



## 2. The most-wanted process: $\Delta L=2$

The fundamental diagram:



$$U_{iN} \frac{\not{p} + m_N}{p^2 - m_N^2 + i\epsilon} U_{jN}$$

The crossing diagrams can probe different processes and new physics of  $N/T^0$ ,  $W^+_R$ ,  $H^{++}$

The transition rates are proportional to

$$|\mathcal{M}|^2 \propto \begin{cases} \langle m \rangle_{l_1 l_2}^2 = \left| \sum_{i=1}^3 U_{l_1 i} U_{l_2 i} m_i \right|^2 & \text{for light } \nu; \\ \frac{|\sum_i^n V_{l_1 i} V_{l_2 i}|^2}{m_N^2} & \text{for heavy } N; \\ \frac{\Gamma(N \rightarrow i) \Gamma(N \rightarrow f)}{m_N \Gamma_N} & \text{for resonant } N \text{ production.} \end{cases}$$

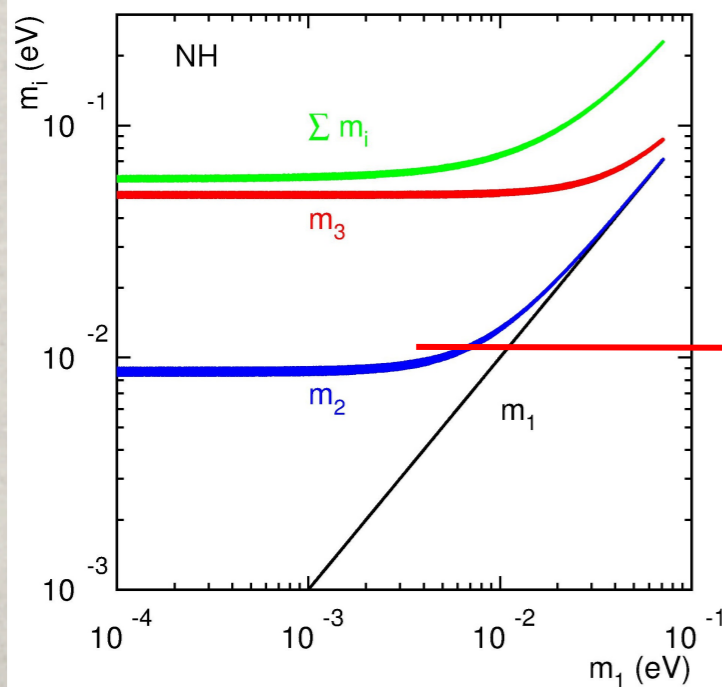


# 3. Neutrino-less double-beta decay

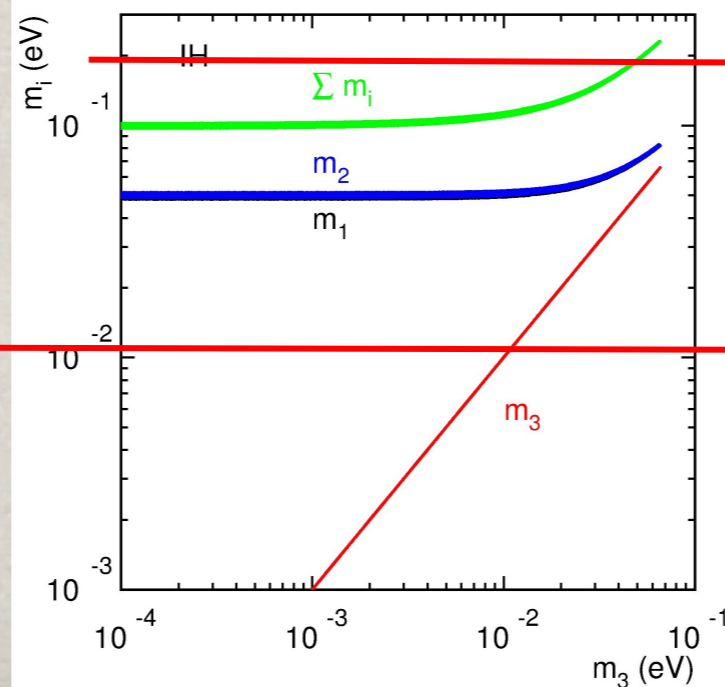
arXiv:1902.04097, M. Dolinski, A. Poon, W. Rodejohann

Isotope	$T_{1/2}^{0\nu}$ ( $\times 10^{25}$ y)	$\langle m_{\beta\beta} \rangle$ (eV)	Experiment	Reference
$^{48}\text{Ca}$	$> 5.8 \times 10^{-3}$	$< 3.5 - 22$	ELEGANT-IV	(157)
$^{76}\text{Ge}$	$> 8.0$	$< 0.12 - 0.26$	GERDA	(158)
	$> 1.9$	$< 0.24 - 0.52$	MAJORANA DEMONSTRATOR	(159)
$^{82}\text{Se}$	$> 3.6 \times 10^{-2}$	$< 0.89 - 2.43$	NEMO-3	(160)
$^{96}\text{Zr}$	$> 9.2 \times 10^{-4}$	$< 7.2 - 19.5$	NEMO-3	(161)
$^{100}\text{Mo}$	$> 1.1 \times 10^{-1}$	$< 0.33 - 0.62$	NEMO-3	(162)
$^{116}\text{Cd}$	$> 1.0 \times 10^{-2}$	$< 1.4 - 2.5$	NEMO-3	(163)
$^{128}\text{Te}$	$> 1.1 \times 10^{-2}$	—	—	(164)
$^{130}\text{Te}$	$> 1.5$	$< 0.11 - 0.52$	CUORE	(124)
$^{136}\text{Xe}$	$> 10.7$	$< 0.061 - 0.165$	KamLAND-Zen	(165)
	$> 1.8$	$< 0.15 - 0.40$	EXO-200	(166)
$^{150}\text{Nd}$	$> 2.0 \times 10^{-3}$	$< 1.6 - 5.3$	NEMO-3	(167)

Current bound:  
 $\langle m_{ee} \rangle \sim 0.2$  eV



(a)



(b)

Future expts:

- SNO+
  - SuperNEMO
  - nEXO
  - CUPID
  - LEGEND100
- Future:  
 $\langle m_{ee} \rangle \sim 0.01$  eV



## Already severe bounds:

$$|\mathcal{M}| \propto \left\{ \begin{array}{l} G_F^2 \frac{\langle m \rangle_{ee}}{q^2} \quad \text{for light } \nu \Rightarrow \langle m \rangle_{ee} \sim \mathcal{O}(0.2 \text{ eV}) \text{ (current)} \\ G_F^2 \frac{|\sum_i^n V_{ei} V_{ei}|}{m_N} \quad \text{for heavy } N \Rightarrow m_N > |V_{eN}/10^{-2}|^2 \text{ TeV} \\ G_F^2 \frac{M_W^4}{M_R^4} \frac{1}{m_N} \quad \text{for } W_R, M_R \sim m_N \Rightarrow M_R > 1 \text{ TeV} \\ G_F^2 \frac{v'}{M_{H^{++}}^2} \quad \text{for doubly charged Higgs} \Rightarrow M_{H^{++}} > \sqrt{v'/10 \text{ MeV}} \text{ TeV} \end{array} \right.$$

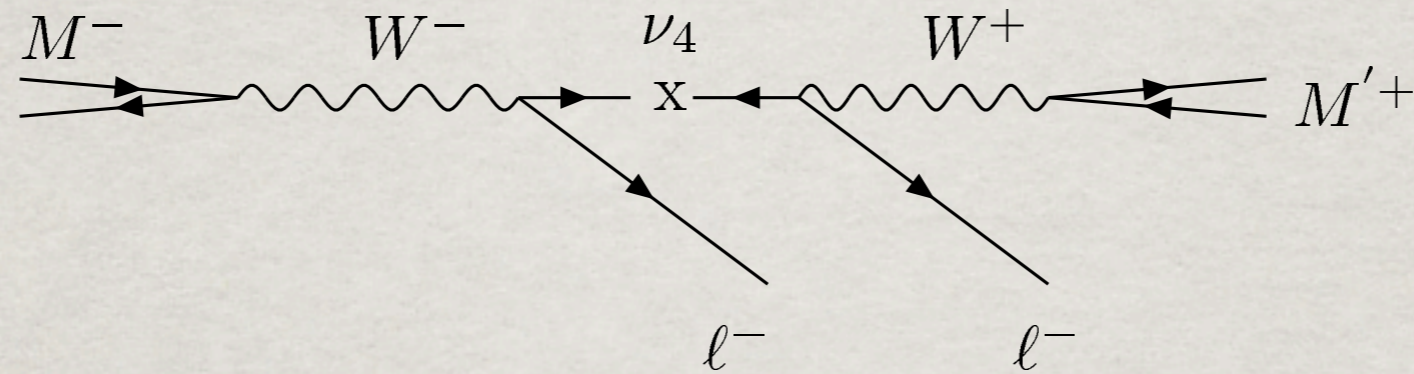
Remain to be most sensitive:

- but for ee final state only!
- What about other models?



# 4. Meson decays

## $N$ Resonance Production and Decay



$$|\mathcal{M}|^2 \propto \frac{\Gamma(N \rightarrow i) \Gamma(N \rightarrow f)}{m_N \Gamma_N}$$

On resonance at  $m_N$ , only  $V_{41}^2$  suppressed!

- Active searches:\*

$$\tau, K, D, B \text{ decays: } M^+ \rightarrow \ell_i^+ \ell_j^+ M^- \text{ via } N$$

- Other processes to look for:

$$D^+, B^+ \rightarrow \ell^+ \ell^+ K^*,$$

$$B^+ \rightarrow \tau^+ e^+ M^-, \tau^+ \mu^+ M^-, \tau^+ \tau^+ M^-.$$

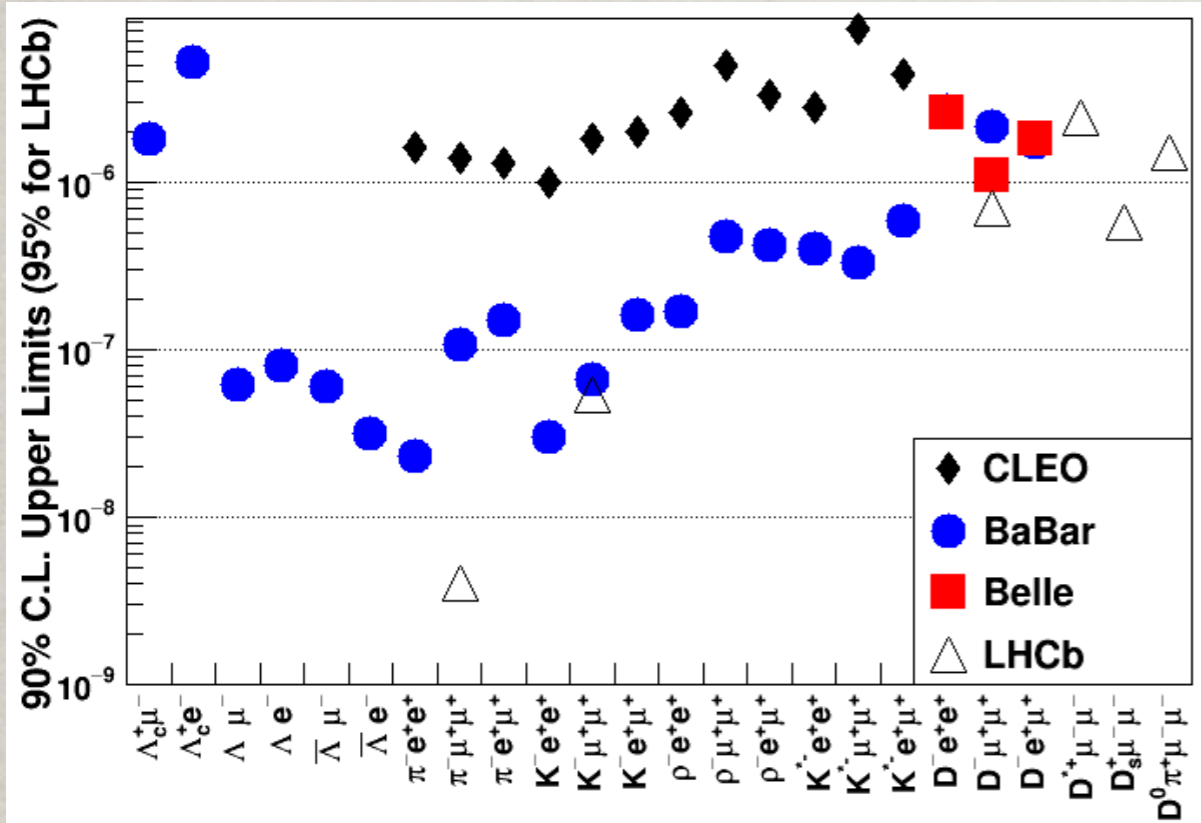
Atre, TH, Pascoli, Zhang, arXiv:0901.3589



# $\tau, K, D, B$ decays: $M^+ \rightarrow \ell_i^+ \ell_j^+ M^-$ via $N$

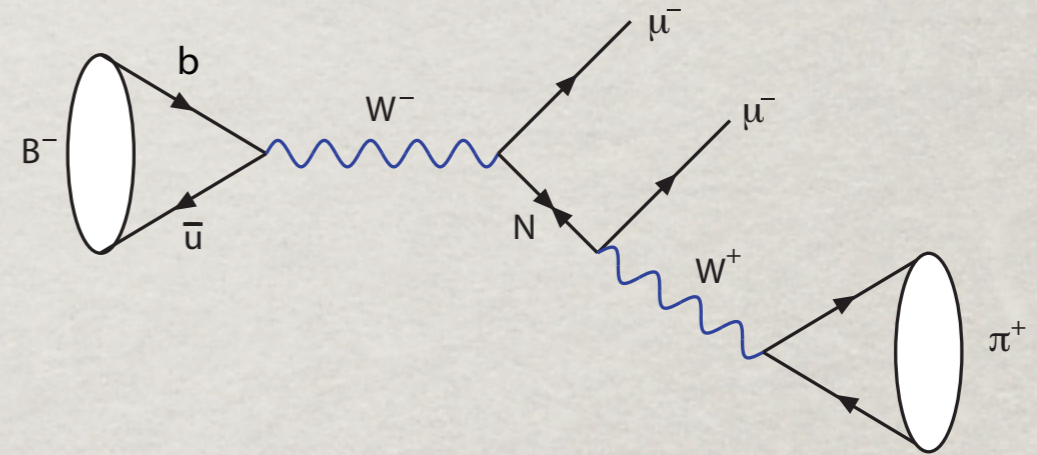
BaBar collaboration: arXiv:1503.08267v1.

CERN NA62, arXiv:1905.07770



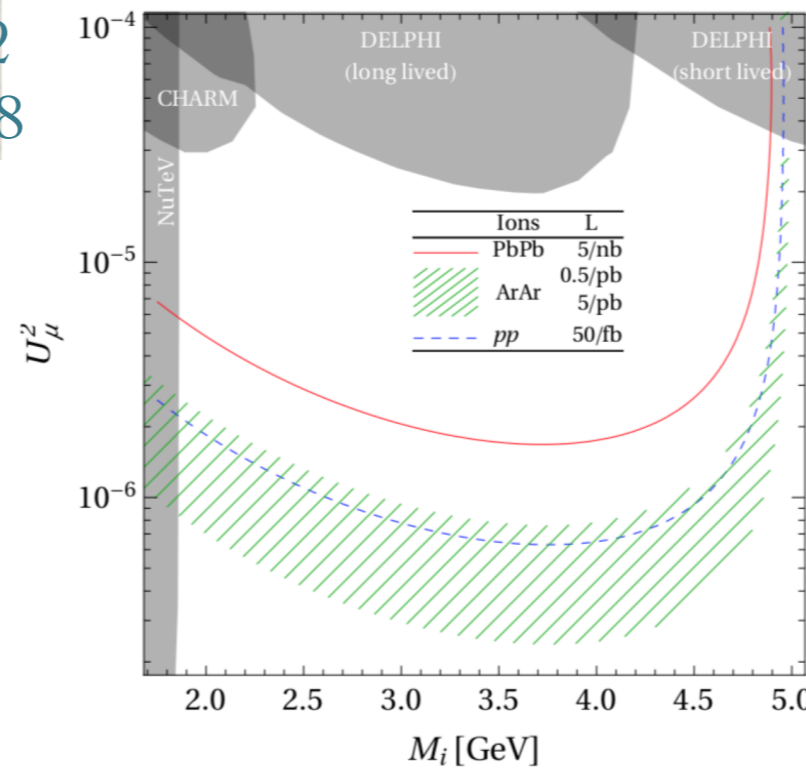
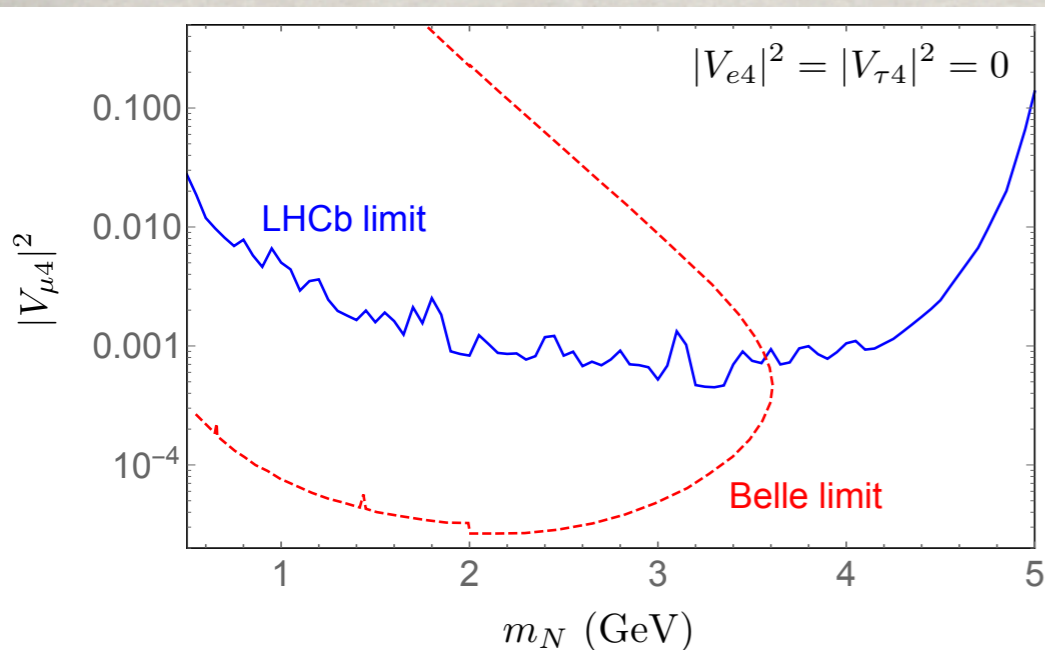
$$\mathcal{B}(K^+ \rightarrow \pi^- e^+ e^+) < 2.2 \times 10^{-10},$$

$$\mathcal{B}(K^+ \rightarrow \pi^- \mu^+ \mu^+) < 4.2 \times 10^{-11}.$$

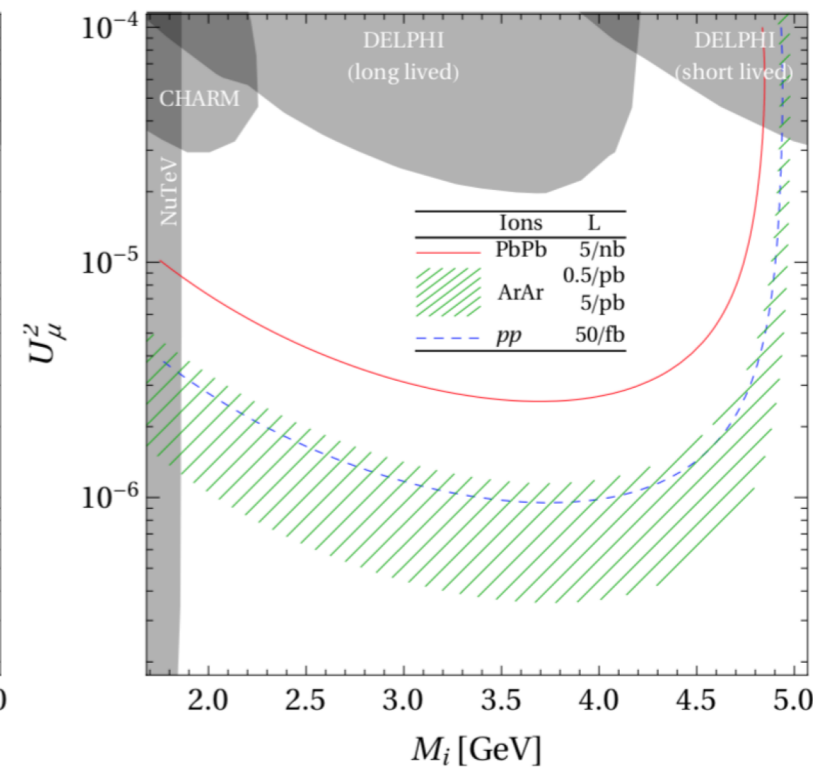


M. Drewes, J. Hajer et al., arXiv:1905.19828  
Heavy ion with low trigger threshold

LHCb collaboration: arXiv:1401.5361v2  
B. Shuve & M. Peskin, arXiv:1607.04258



(a) Exclusion with  $2\sigma$ .

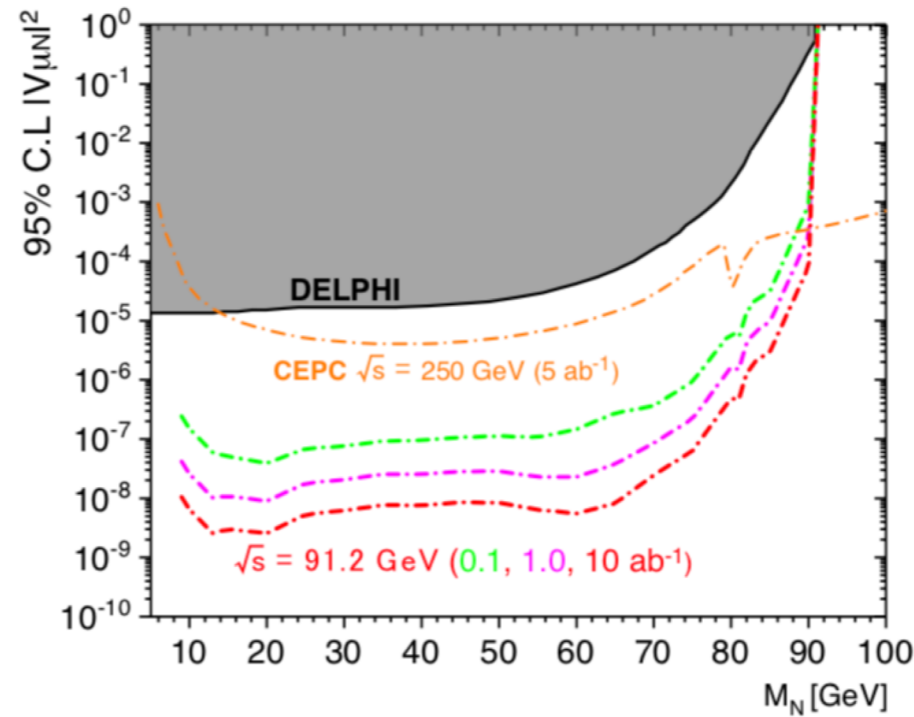
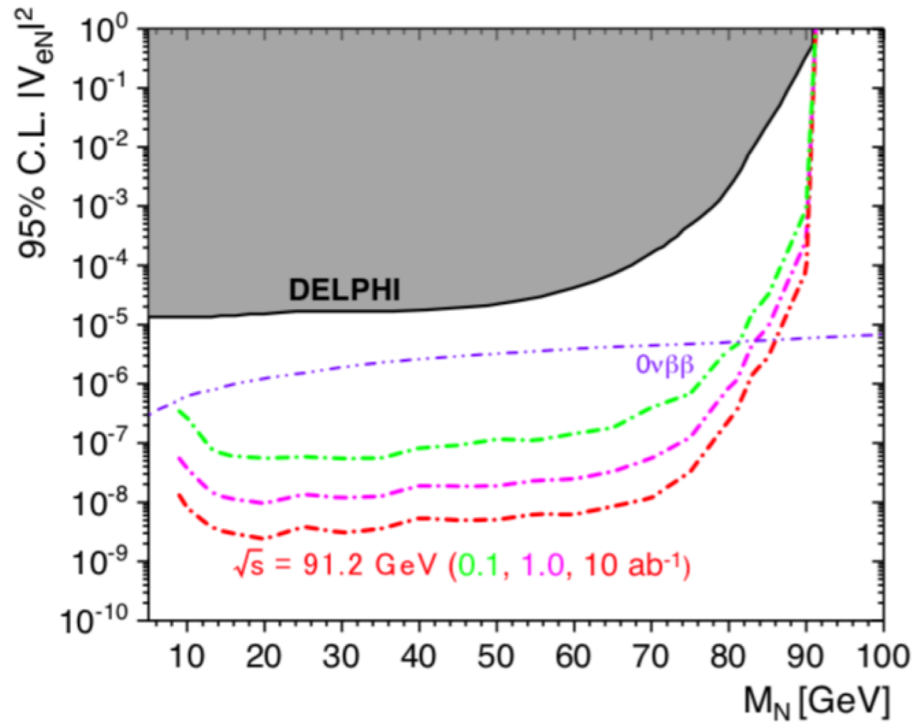
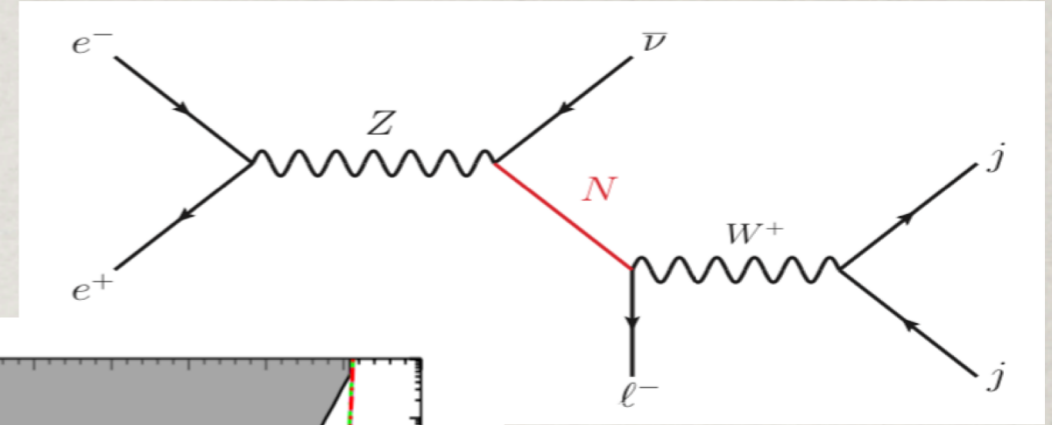


(b) Discovery with  $5\sigma$ .



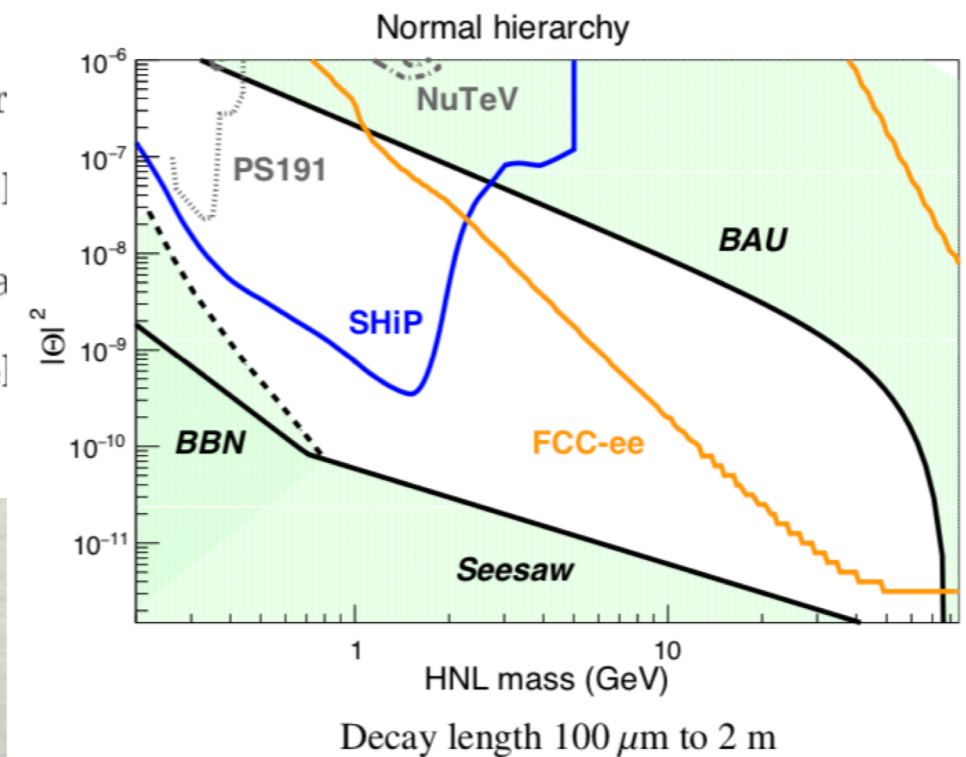
# 5. “Type I” at Lepton Colliders

Jian-Nan Ding, Qin Qin, Fu-Sheng Yu:  
arXiv:1903.02570



FCCee CDR Vol.2

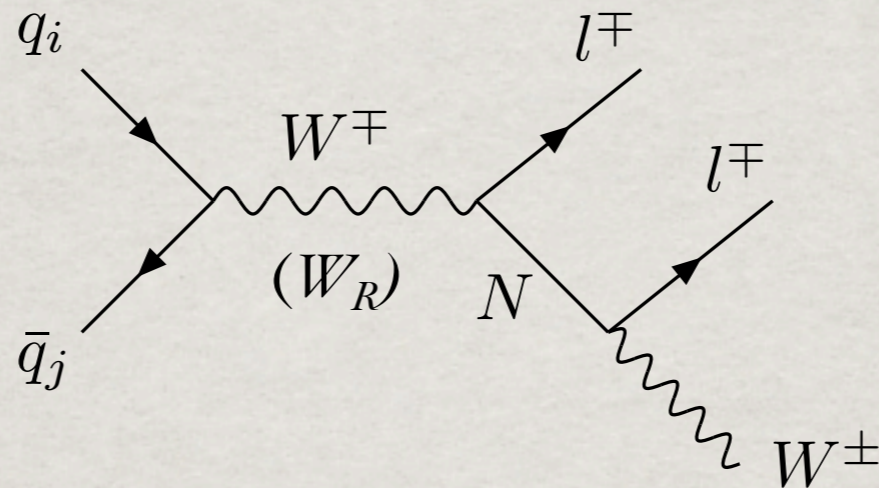
FIG. 6: The upper bounds on the mixing parameters  $|V_{eN}|^2$  (left) and  $|V_{\mu N}|^2$  (right) at 95% CL, compared to the upper bounds given by DELPHI [18], tritium experiments [19–21] and the CEPC as a Higgs factory [32]. The green, pink and red lines correspond to integrated luminosities of  $0.1 \text{ ab}^{-1}$ ,  $1.0 \text{ ab}^{-1}$  and  $10 \text{ ab}^{-1}$ , respectively. See [32] for details.





# 6. "Type I" at Hadron Colliders

At hadron colliders:  $\S$   $pp(\bar{p}) \rightarrow \ell^\pm \ell^\pm jj X$



$$\sigma(pp \rightarrow \mu^\pm \mu^\pm W^\mp) \approx \sigma(pp \rightarrow \mu^\pm N) Br(N \rightarrow \mu^\pm W^\mp) \equiv \frac{V_{\mu N}^2}{\sum_l |V_{\ell N}|^2} V_{\mu N}^2 \sigma_0.$$

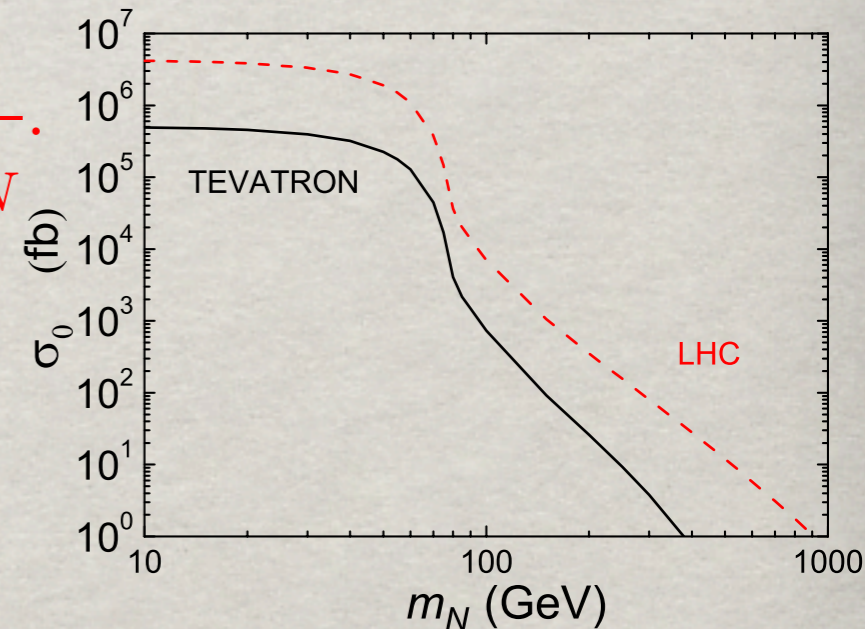
Factorize out the mixing couplings:  $\dagger$

$$\sigma(pp \rightarrow \mu^\pm \mu^\pm W^\mp) \equiv S_{\mu\mu} \sigma_0,$$

$$S_{\mu\mu} = \frac{V_{\mu N}^4}{\sum_l |V_{\ell N}|^2} \approx \frac{V_{\mu N}^2}{1 + V_{\tau N}^2/V_{\mu N}^2}.$$

A very clean channel:

- like-sign di-muons plus two jets;
- no missing energies;
- $m(jj) = M_W$ ,  $m(jj\mu) = m_N$ .



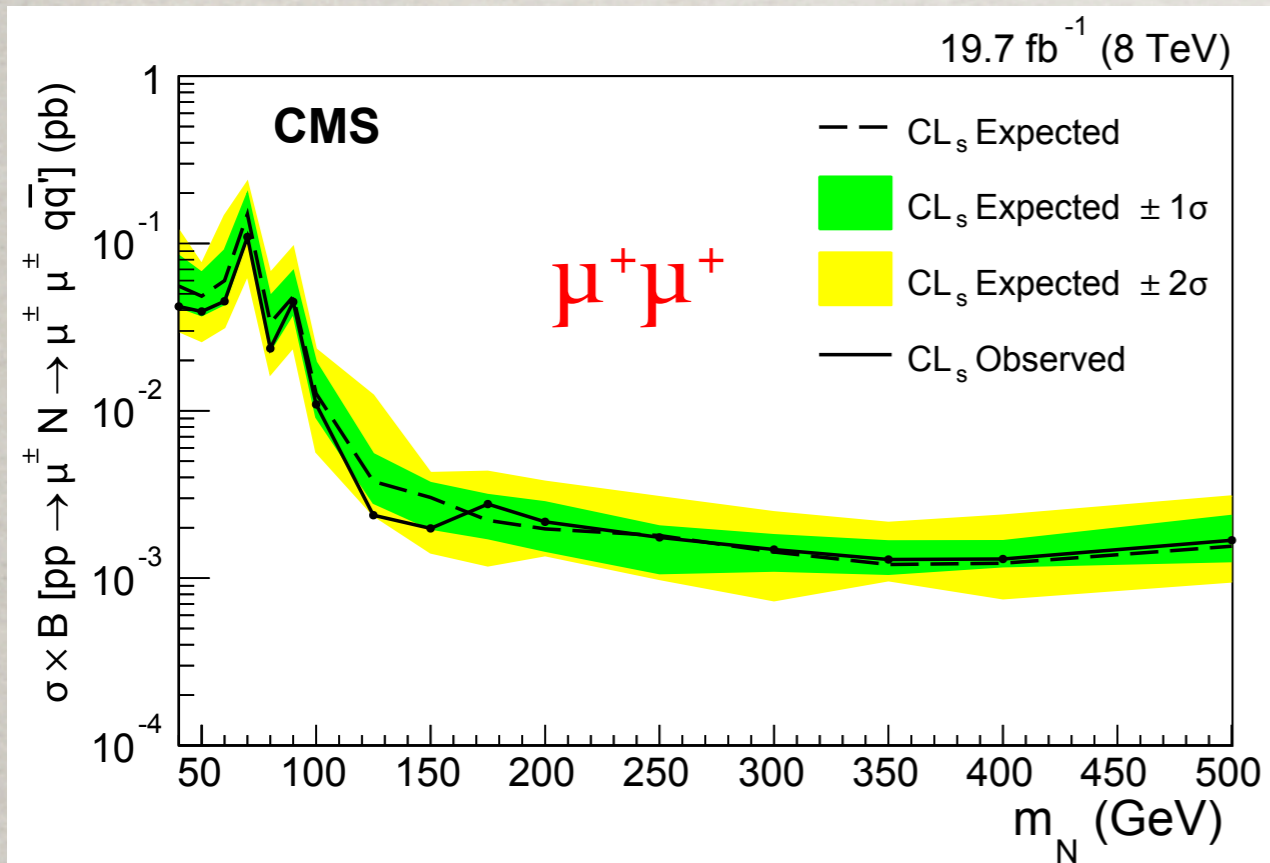
$\S$ Keung, Senjanovic (1983); Dicus et al. (1991); A. Datta, M. Guchait, A. Pilaftsis (1993); ATLAS TDR (1999); F. Almeida et al. (2000); F. del Aguila et al. (2007).

$\dagger$ T. Han and B. Zhang, hep-ph/0604064, PRL (2006).



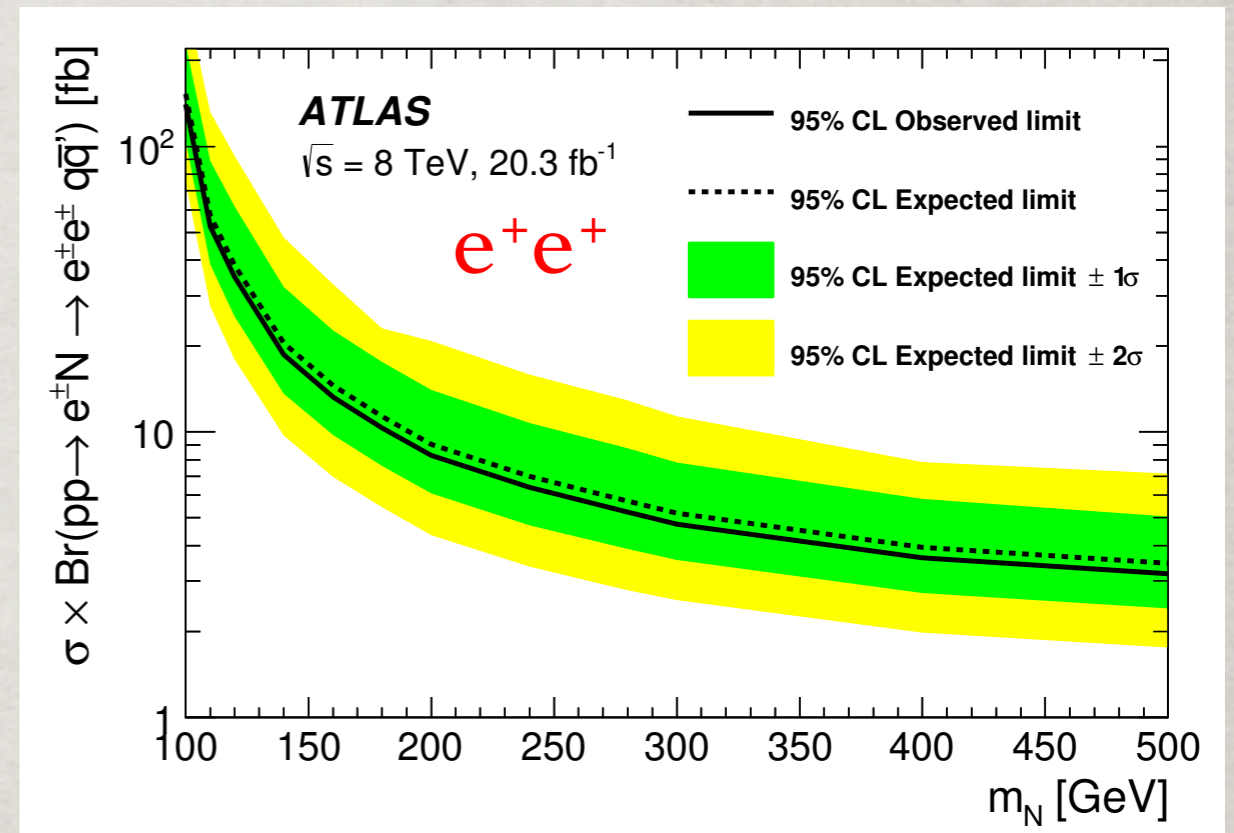
# CMS:

CMS collaboration: arXiv:1501.05566v1.



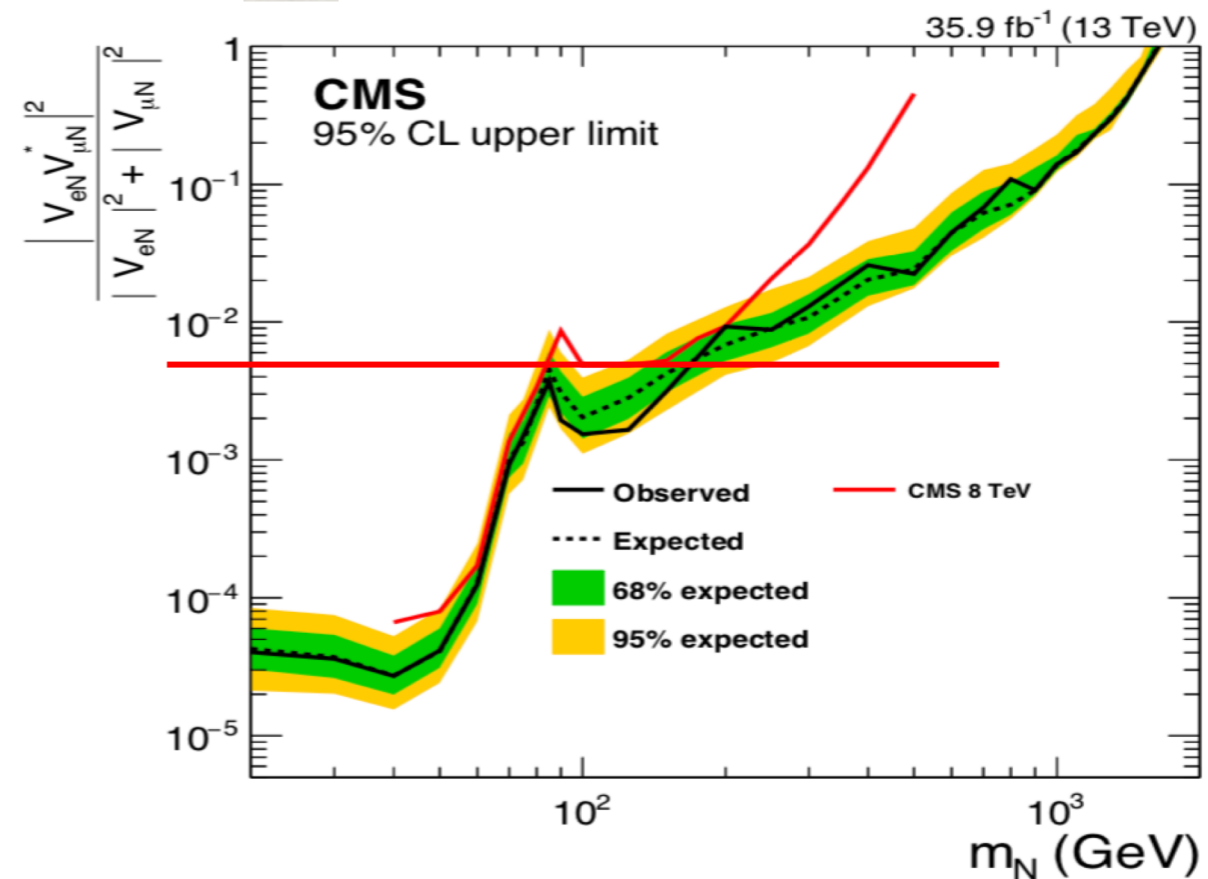
# ATLAS:

ATLAS collaboration: arXiv:1506.06020v2.



CMS collaboration update:  
arXiv:1806.10905.

Insensitive to low mass:  
There is a trigger threshold  
~ 20 GeV!

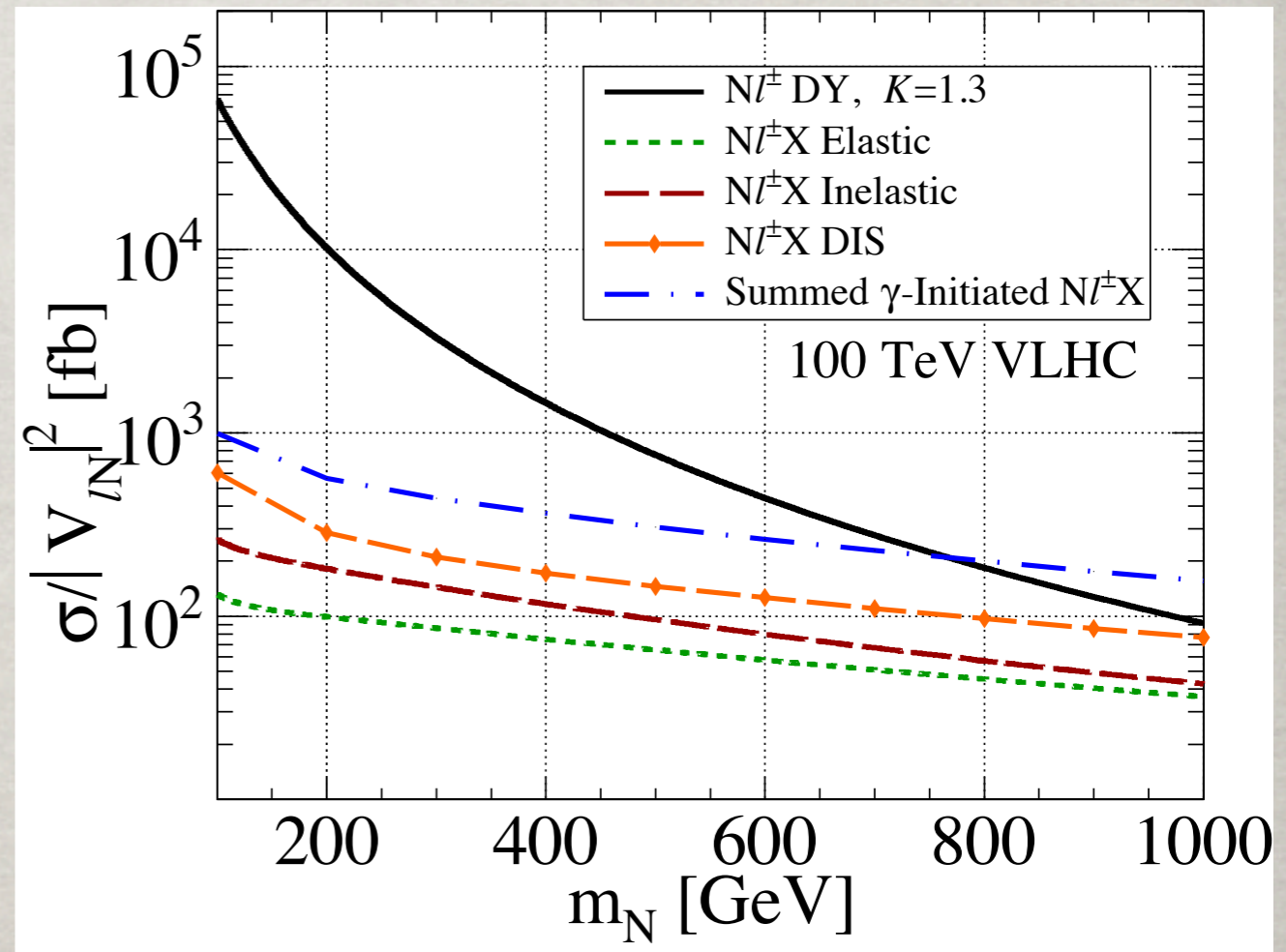
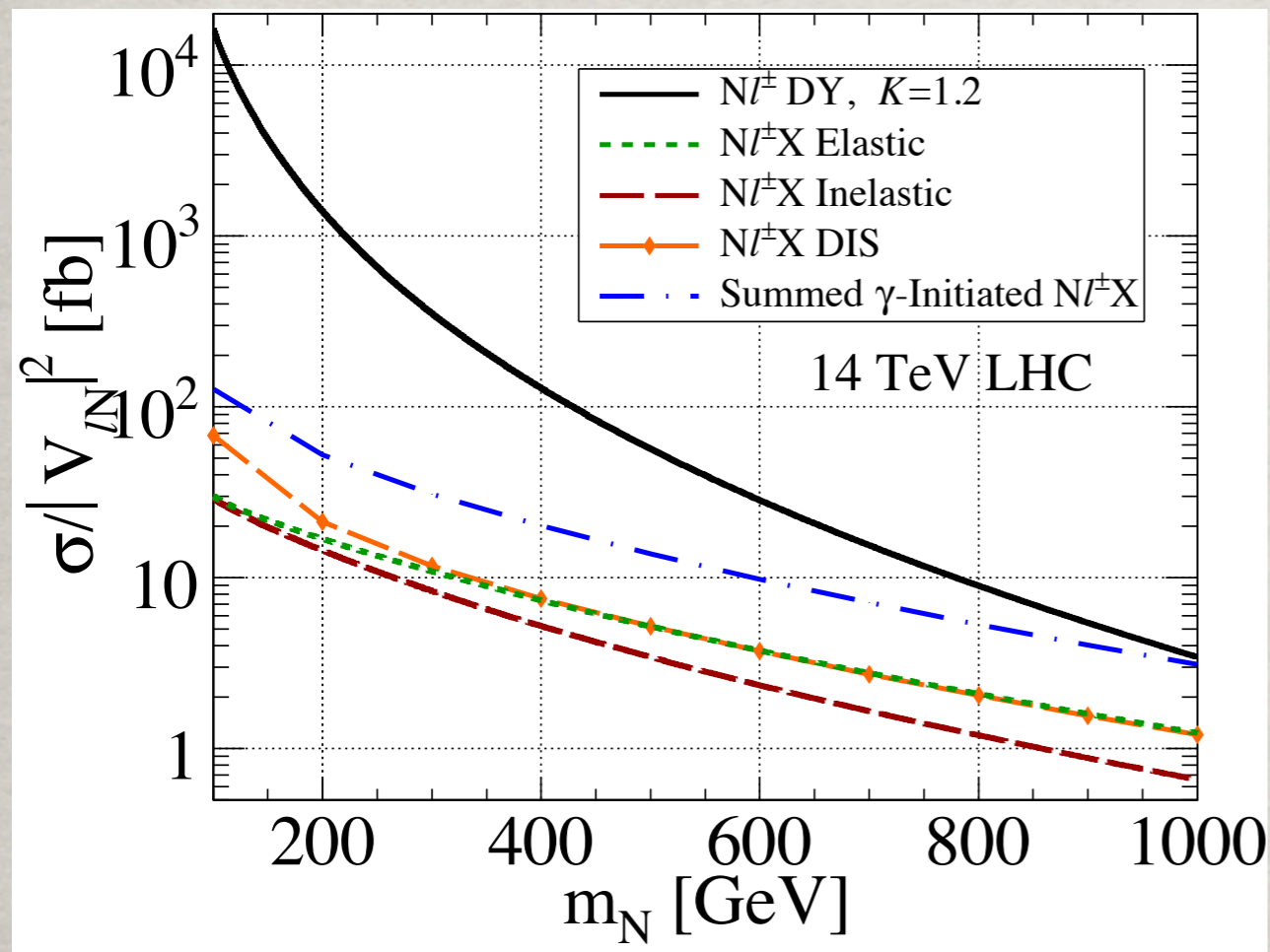
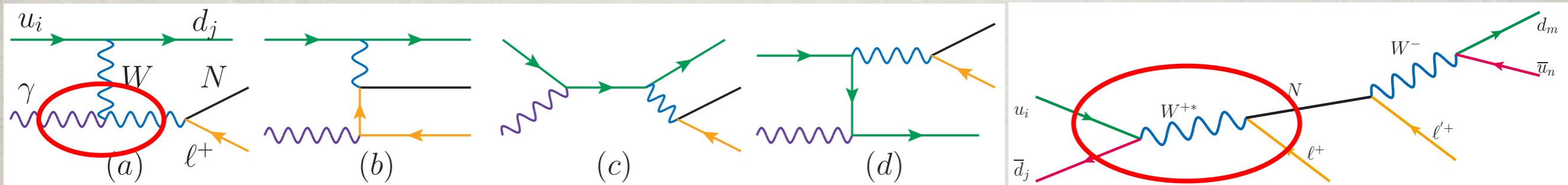




# A recent update:

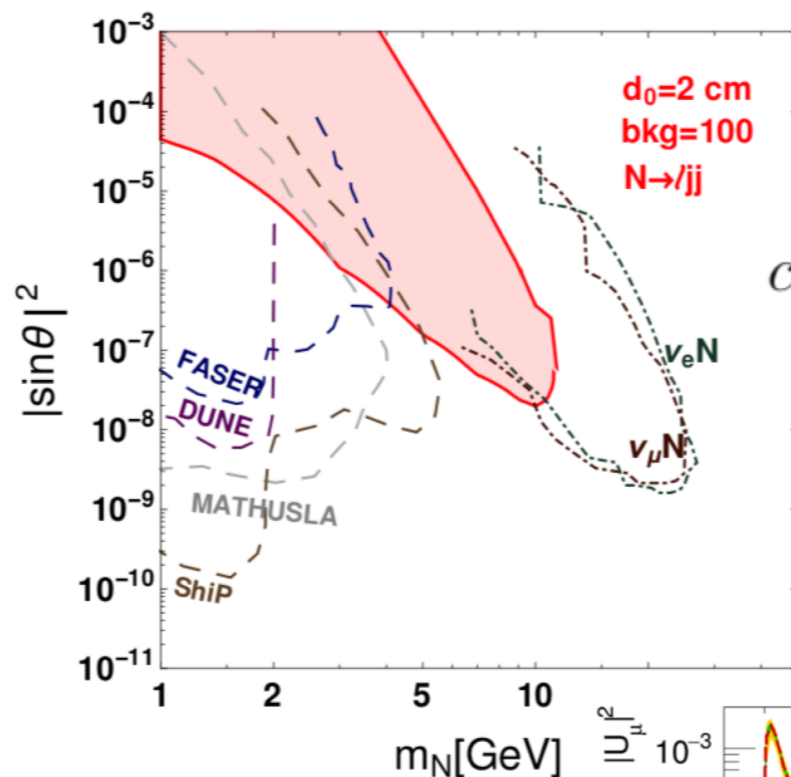
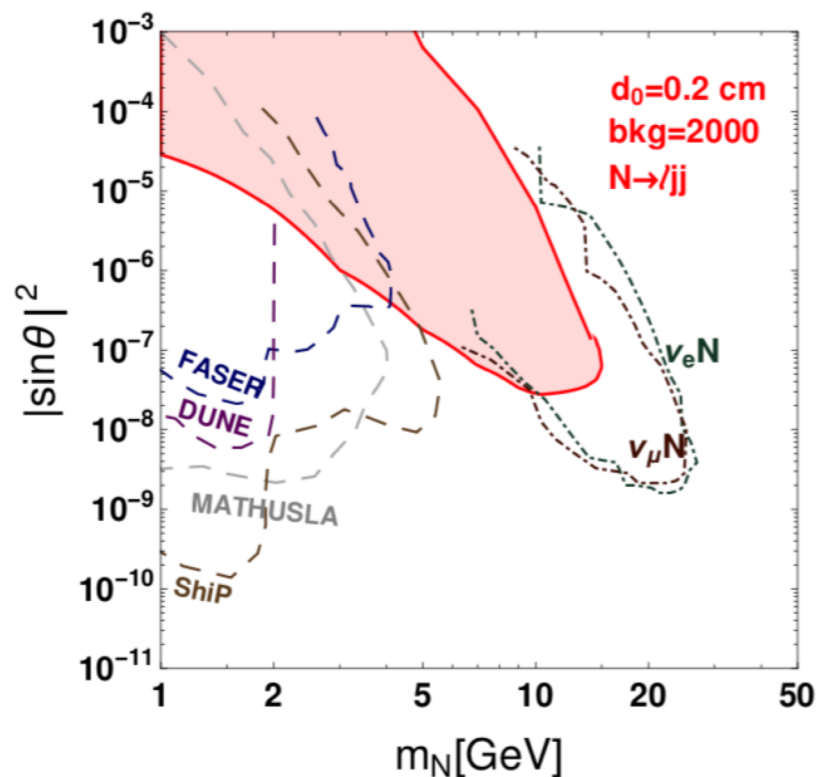
## VBF ( $W\gamma$ ) contributions and NNLO QCD effects

P. S. B. Dev, A. Pilaftsis and U.-k. Yang,  
*Phys. Rev. Lett.* 112 (2014) 081801;  
 Alva, TH, Ruiz: arXiv:1411.7305



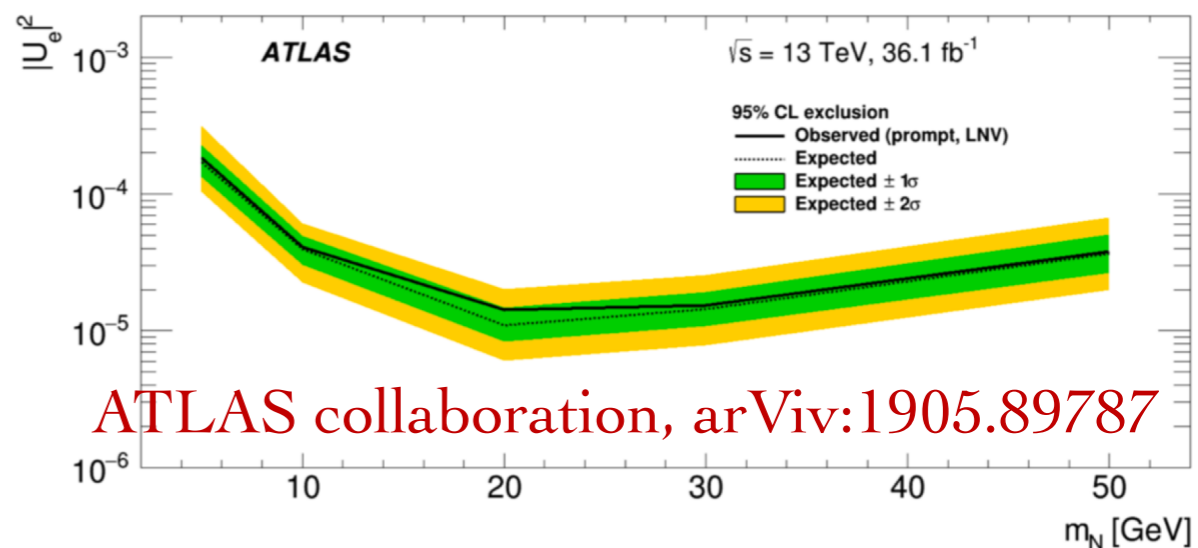
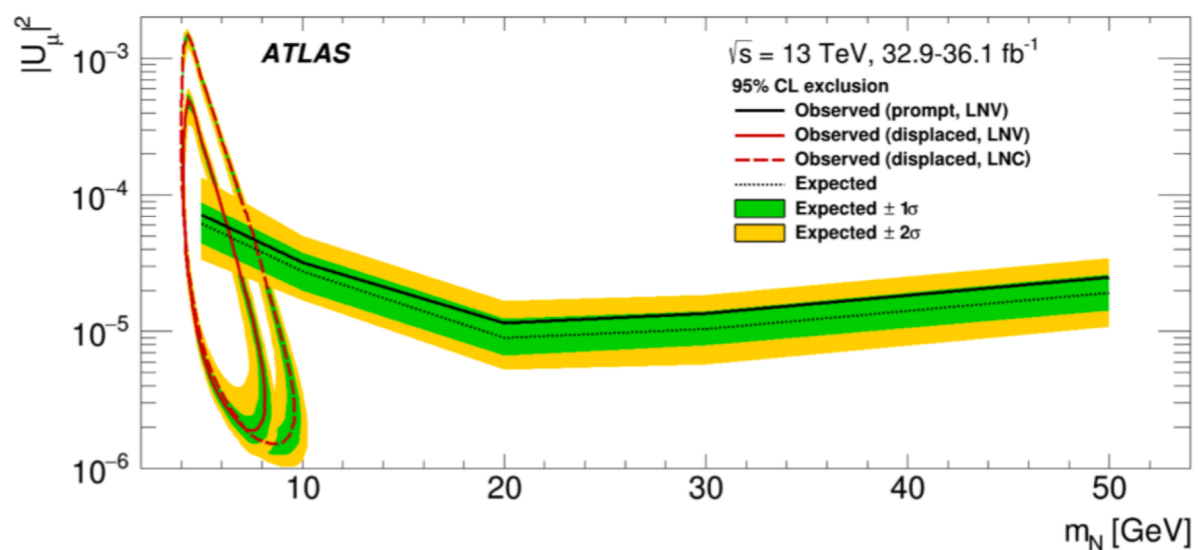


# New Strategy: Long Lived Particles @ Low mass



$$c\tau \simeq 12 \text{ km} \times \left( \frac{10^{-12}}{\sin^2 \theta} \right) \left( \frac{10 \text{ GeV}}{m_N} \right)^5$$

FIG. 6. The 95% C.L. reach for sterile neutrino from W gauge boson decay, plotted in the  $|\sin\theta|^2$  vs  $m_N$  plane. The sensitivities for “ $\nu_e N$ ” and “ $\nu_\mu N$ ” come from prompt lepton triggered displaced vertex projection at HL-LHC [70] assuming zero background. The projected reach for MATHUSLA [45], FASER [44], DUNE [29] and SHiP [30, 71] are also shown.



M. Drewes and J. Hajer, arXiv:1903.06100;  
 J. Liu, Z. Liu, L.-T. Wang, X. Wang,  
 arXiv:1904.01020.

ATLAS collaboration, arXiv:1905.89787



Type I, in general, too many theory parameters,

as the Casas-Ibarra parametrization

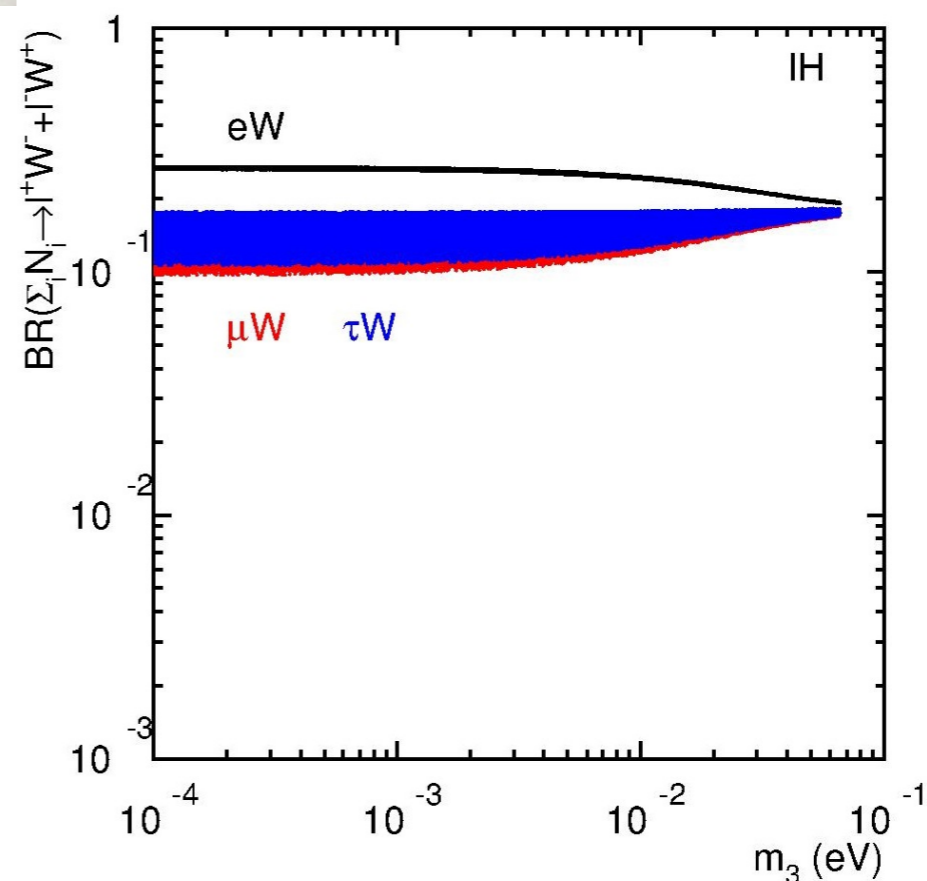
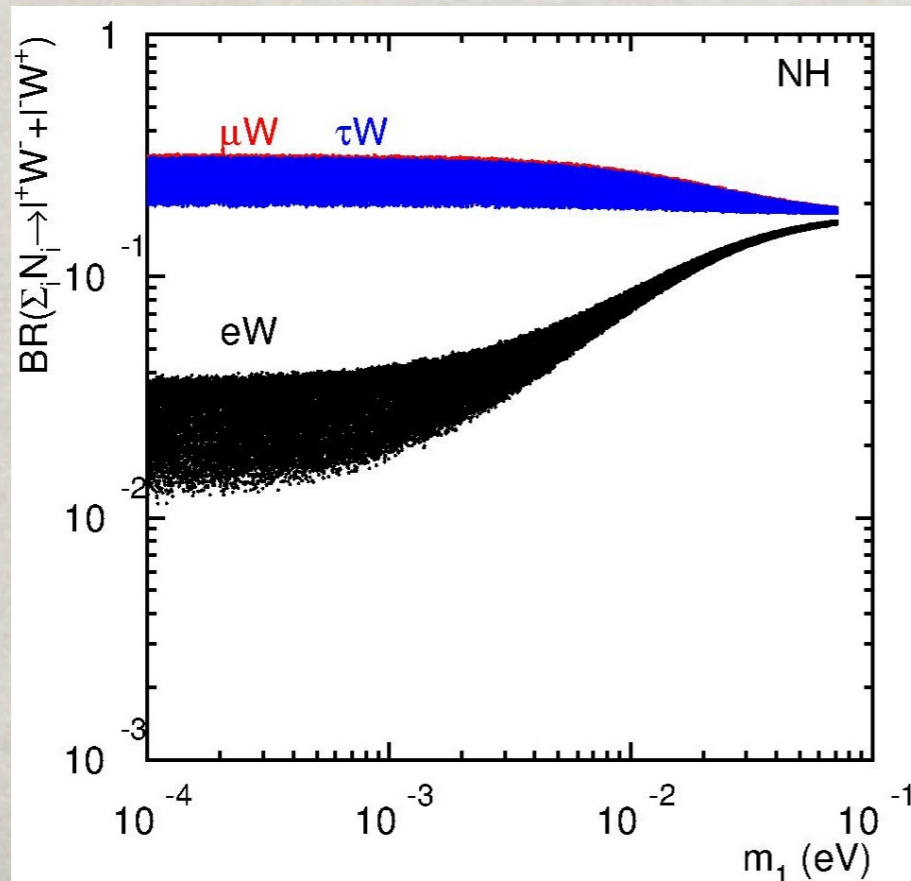
$$V_{\ell N} = U_{PMNS} m_\nu^{1/2} \Omega M_N^{-1/2},$$

If assuming degenerate  $N_i$

$$M_N \sum_N (V_{\ell N}^*)^2 = (U_{PMNS}^* m_\nu U_{PMNS}^\dagger)_{\ell\ell}.$$

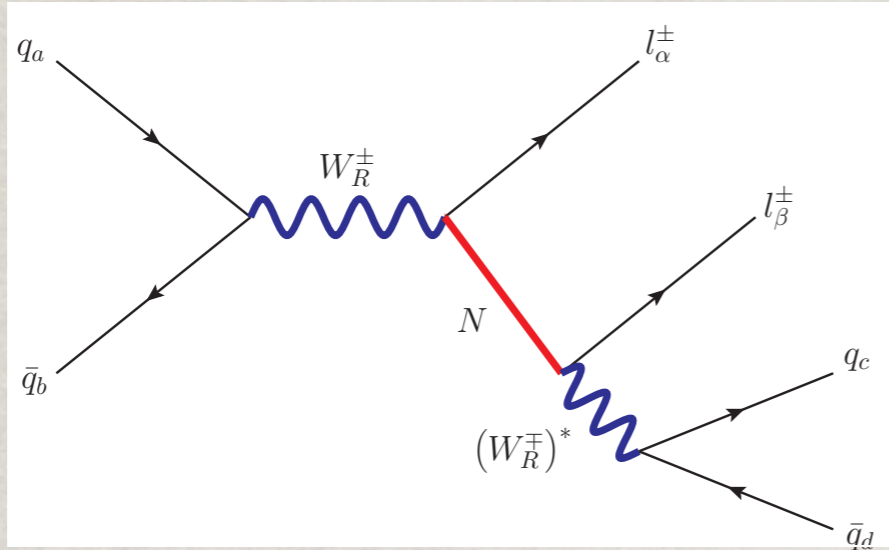
$$\sum_N |V_{eN}|^2 \ll \sum_N |V_{\mu N}|^2, \sum_N |V_{\tau N}|^2 \quad \text{for NH,}$$

$$\sum_N |V_{eN}|^2 > \sum_N |V_{\mu N}|^2, \sum_N |V_{\tau N}|^2 \quad \text{for IH.}$$



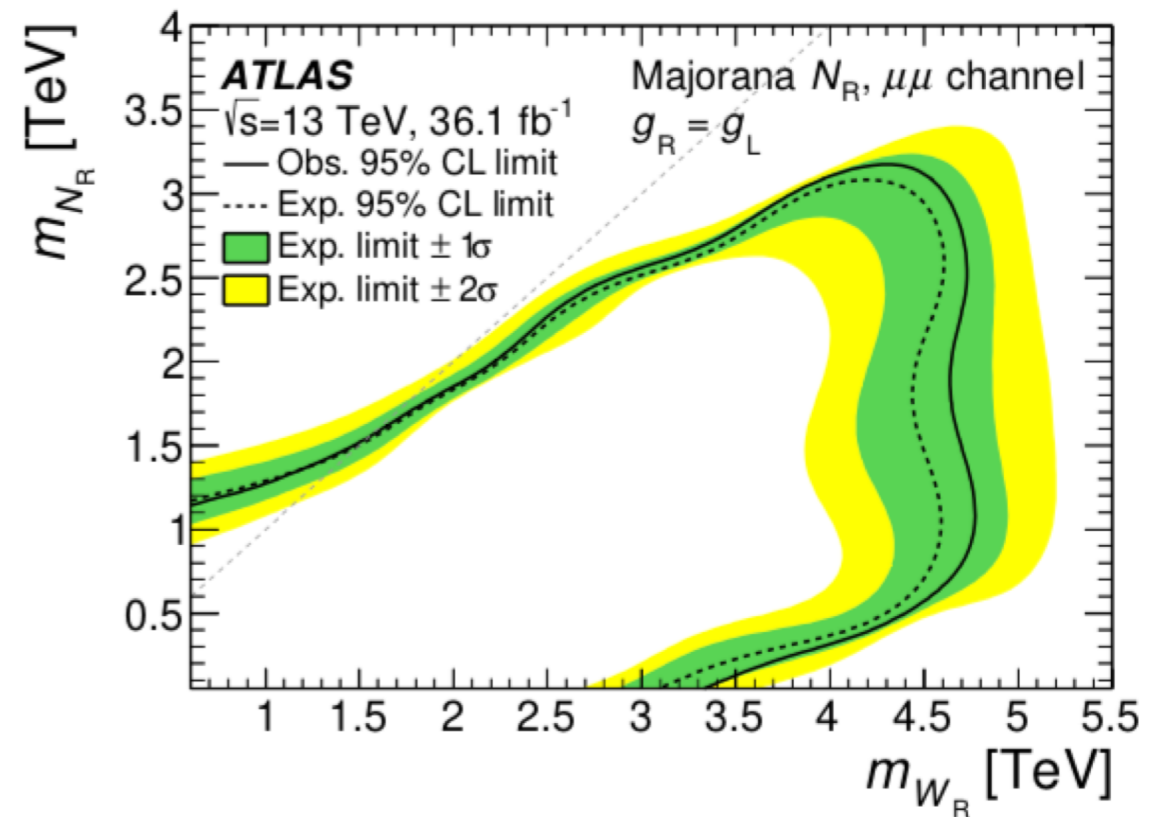
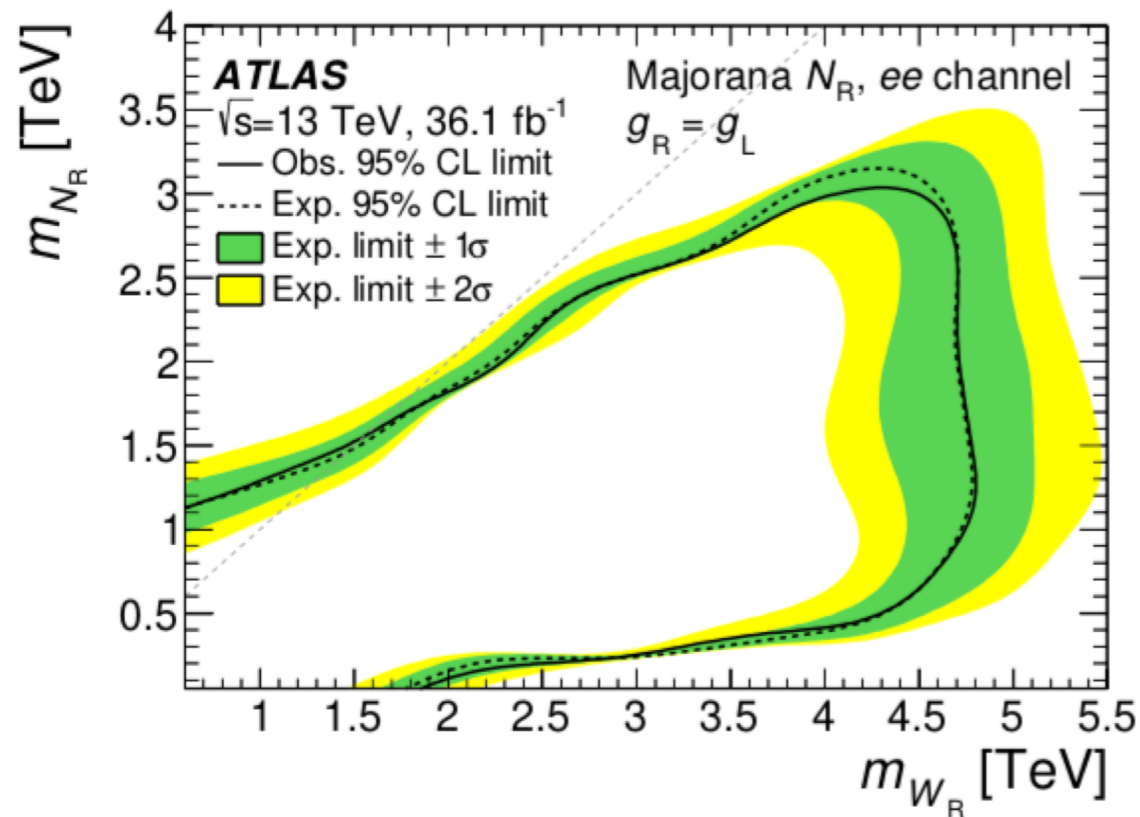


# 7. $W_R$ & $N$ @ Hadron Colliders



G. Senjanovic & W. Keung,  
PRL 50 (1983) 1427

- No mixing suppression
- New unknown mass scale  $M_R$

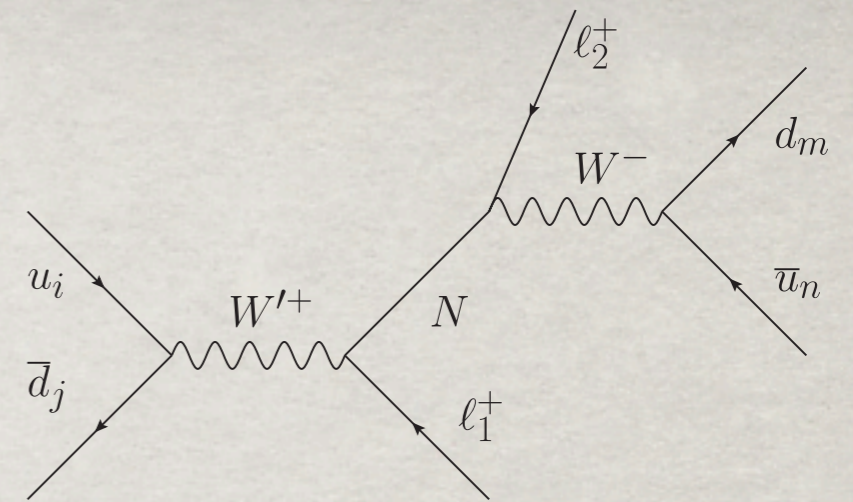


ATLAS collaboration: [arXiv:1809.11105](https://arxiv.org/abs/1809.11105).



# $W_R$ & $N_R$ Properties:

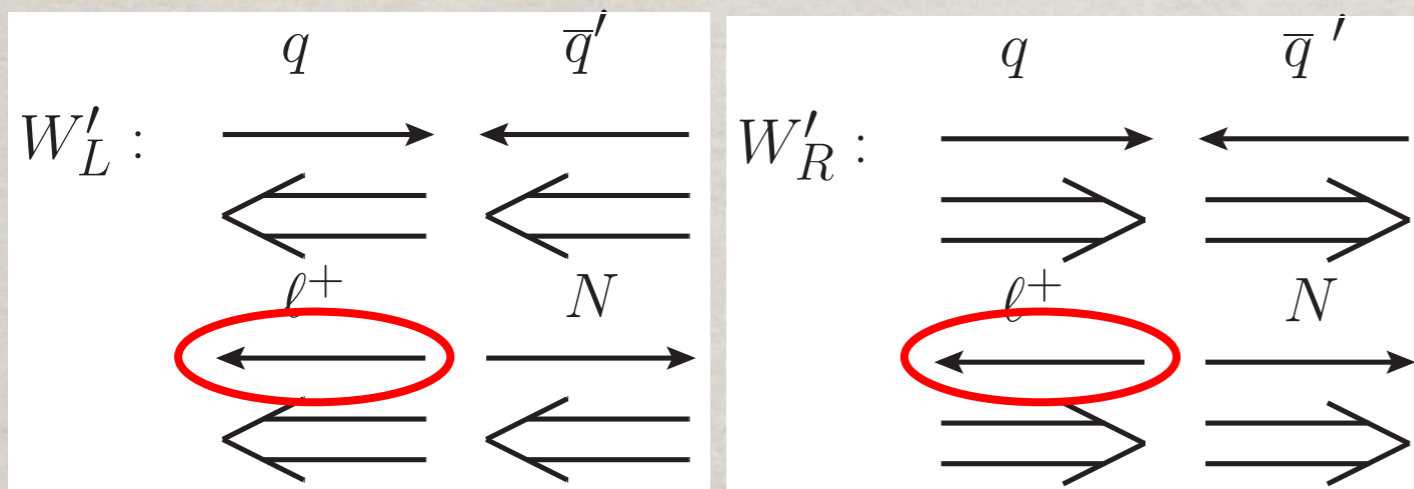
$SU(2)_L \otimes SU(2)_R$  symmetric model:



A clean channel with rich physics:<sup>†</sup>

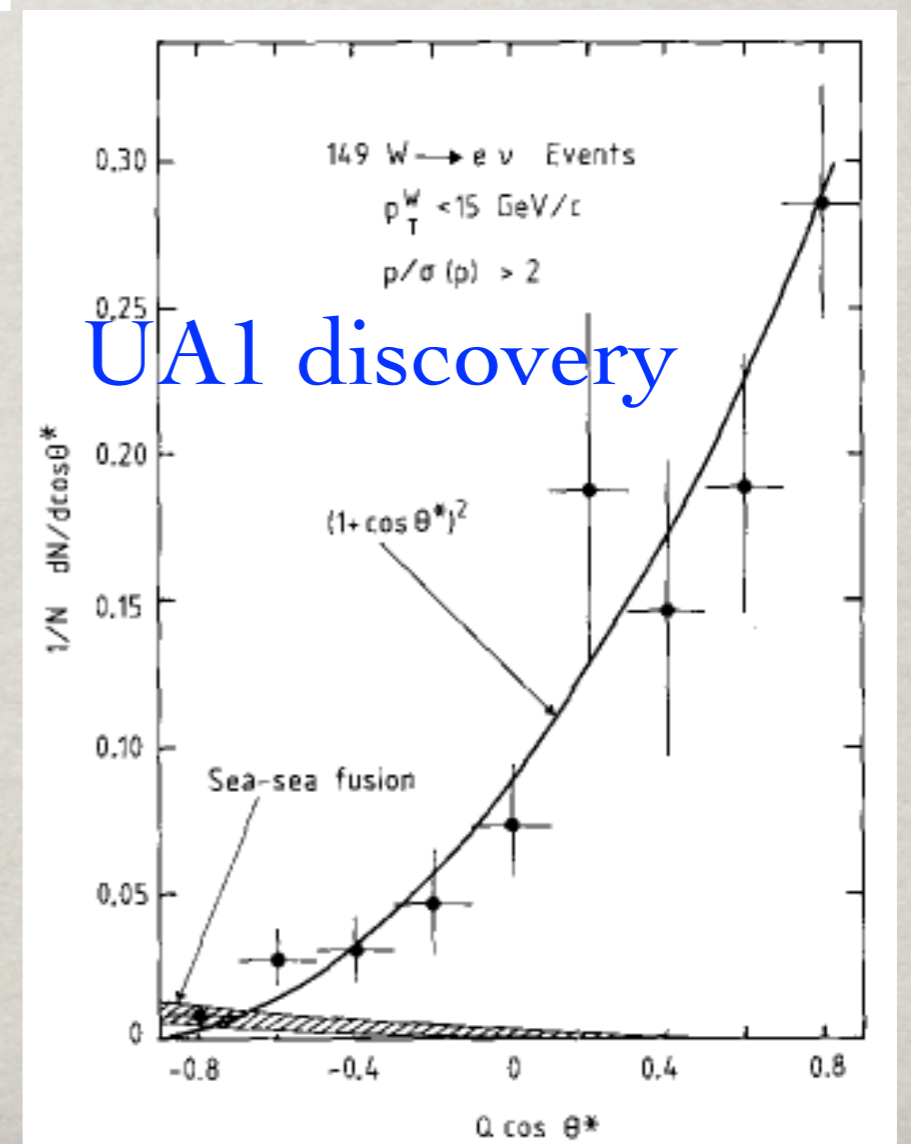
- Significantly enhanced rate at  $W_R$  resonance; ¶
- If observed, determine  $N$ 's nature:  $\Delta L = 2$ , azimuthal angle ...
- and determine  $W'$  chiral coupling to  $\ell - N_{R,L}$  and  $q - \bar{q}$ .

The primary lepton does not provide L-R discrimination:



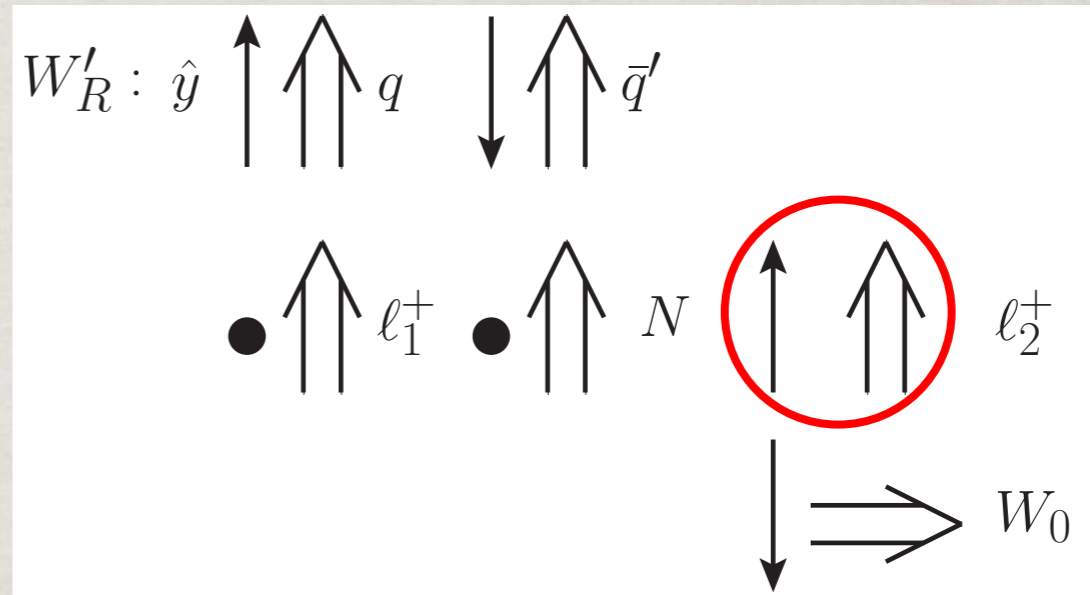
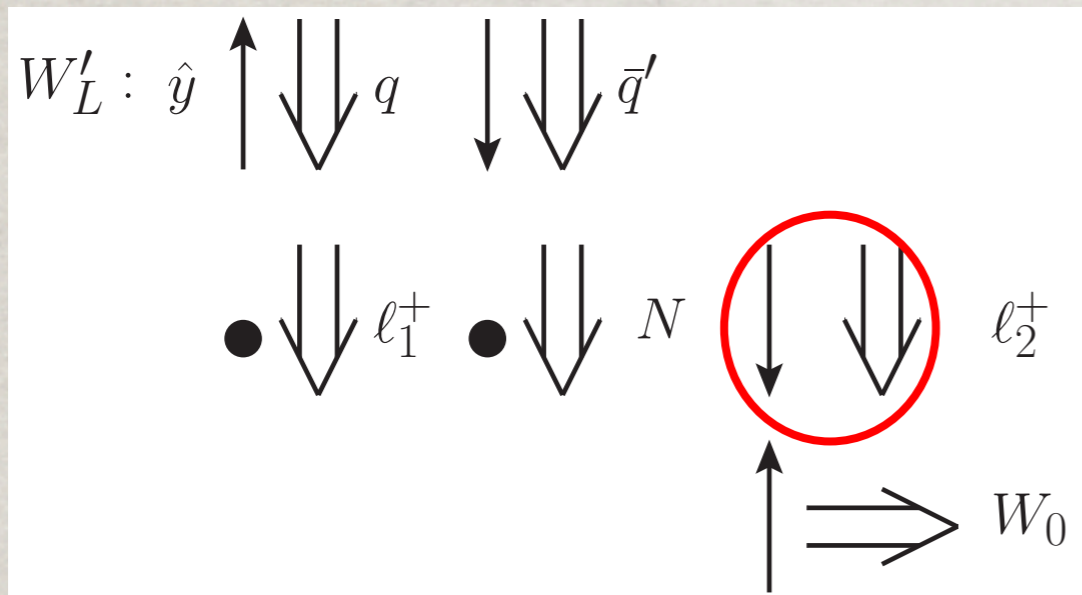
Keung & Senjanovic, PRL (1983).

T. Han, I. Lewis, R. Ruiz, Z. Si, arXiv:1211.6447v2

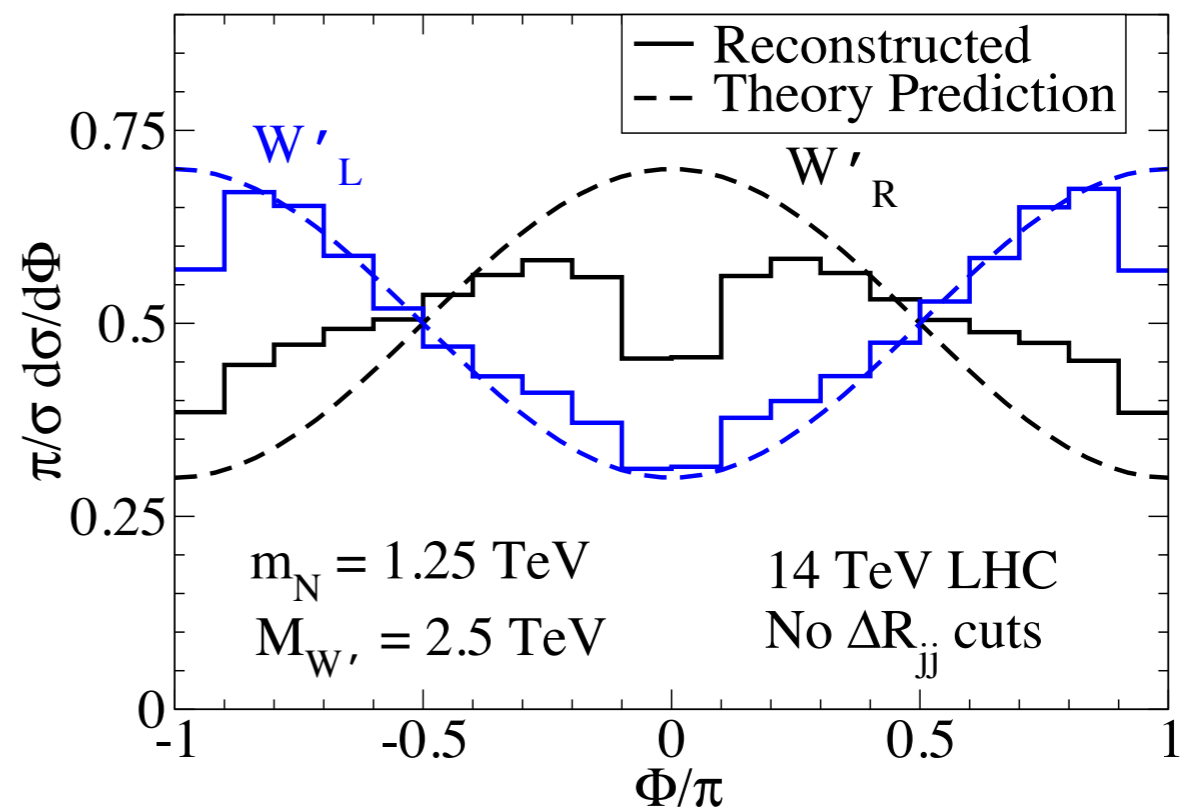
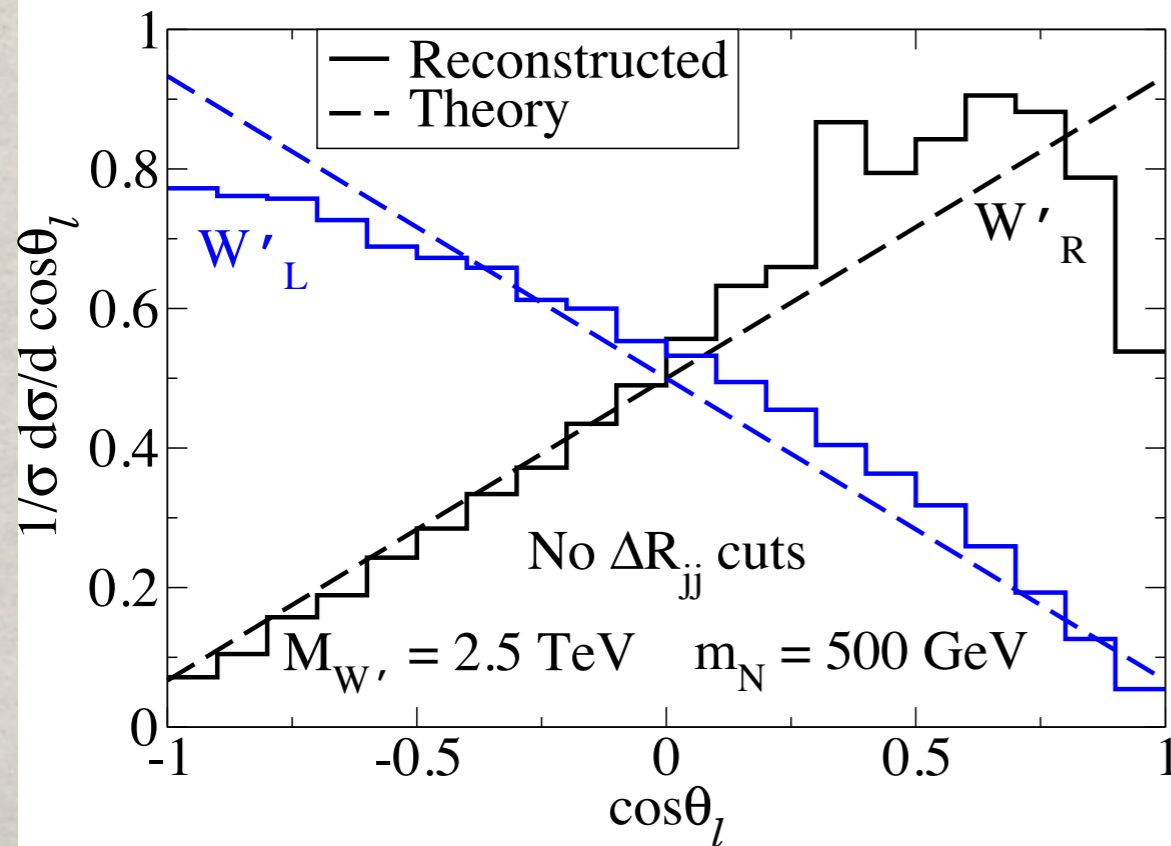




# $W_{L,R}$ Discrimination via $N_{L,R}$ Decay:



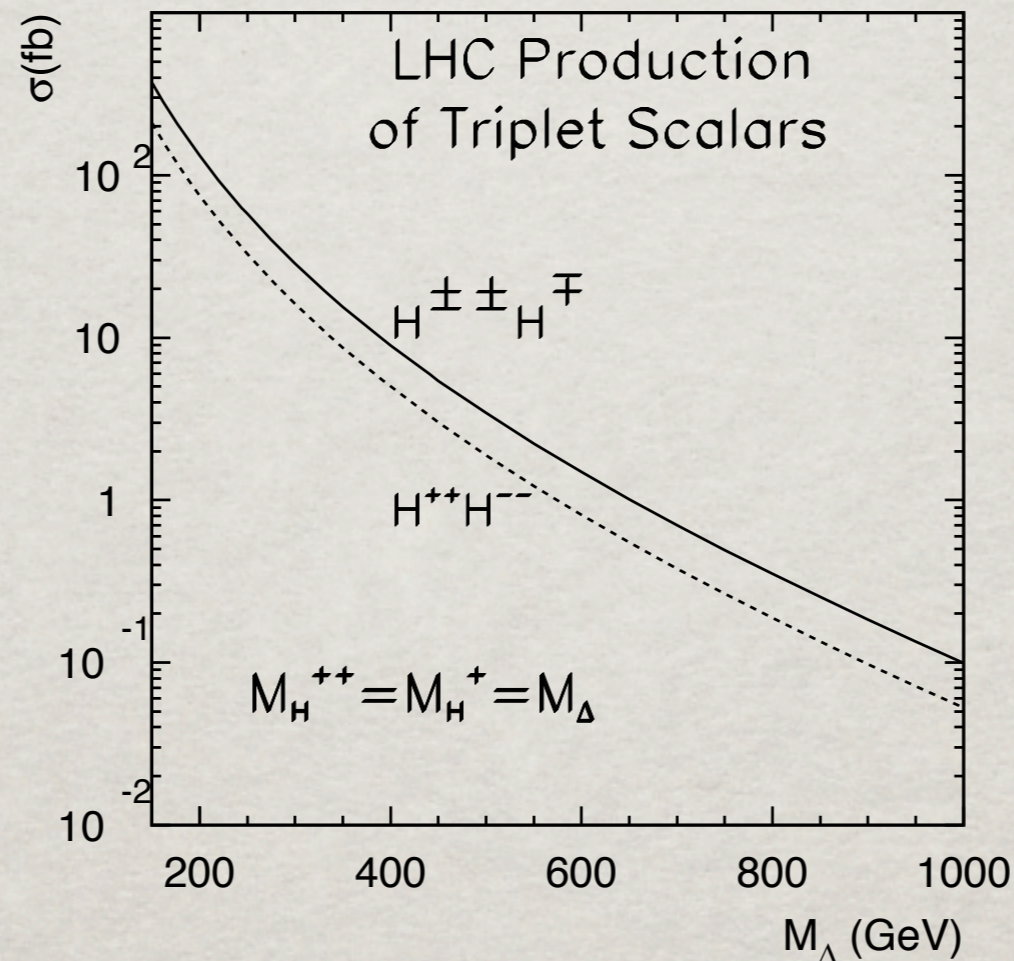
$$N_{L,R} \rightarrow \ell^+ W^- \rightarrow \ell^+ q \bar{q}'$$





## 8. Type II Seesaw: $H^{\pm\pm}$ & $H^\pm$

$H^{++}H^{--}$  production at hadron colliders: †  
Pure electroweak gauge interactions



Akeroyd, Aoki, Sugiyama, 2005, 2007.

$\gamma\gamma \rightarrow H^{++}H^{--}$  10% of the DY.

†Revisit, T.Han, B.Mukhopadhyaya, Z.Si, K.Wang, arXiv:0706.0441.

Recently, a new model: J. Gehrlein, D. Goncalves, P. Machado,  
Y. Perez-Gonzalez: arXiv:1804.09184.

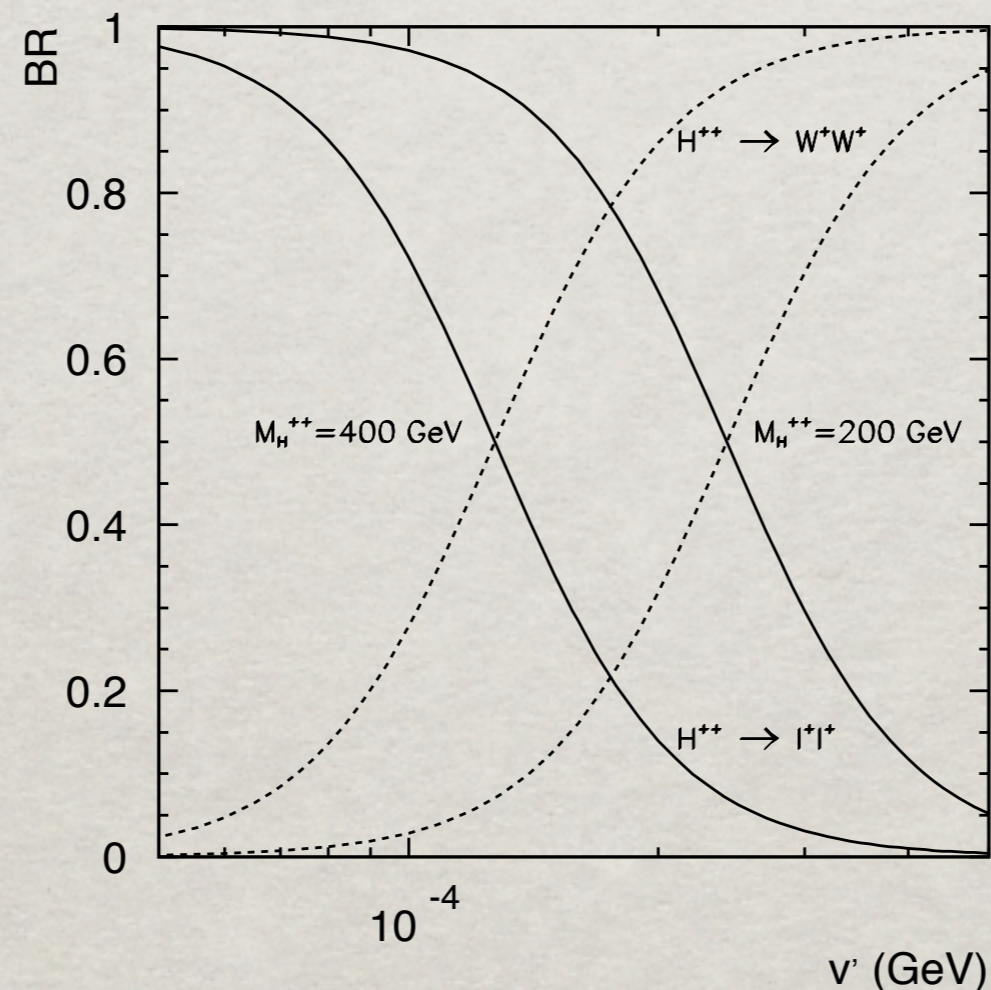


# Type II Seesaw: Complimentary Decays

$$\Gamma(\phi^{++} \rightarrow \ell^+ \ell^+) \propto Y_{ij}^2 M_\phi$$

$$\Gamma(\phi^{++} \rightarrow W^+ W^+) \propto \frac{v'^2 M_\phi^3}{v^4},$$

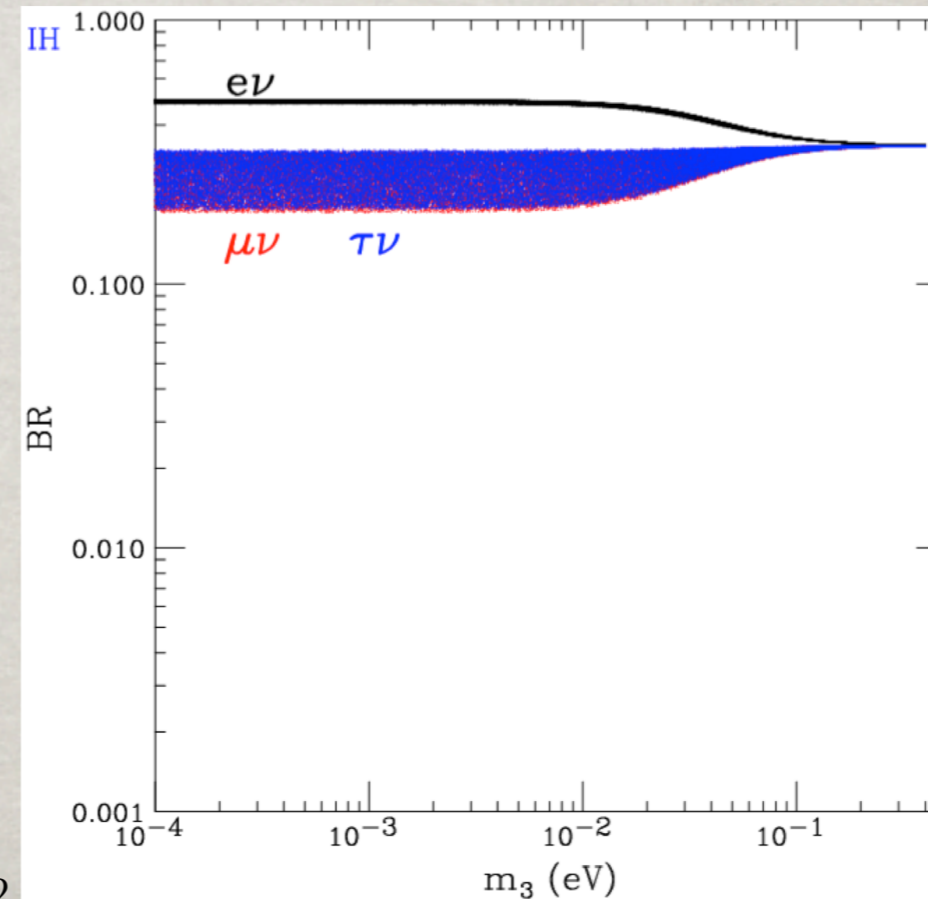
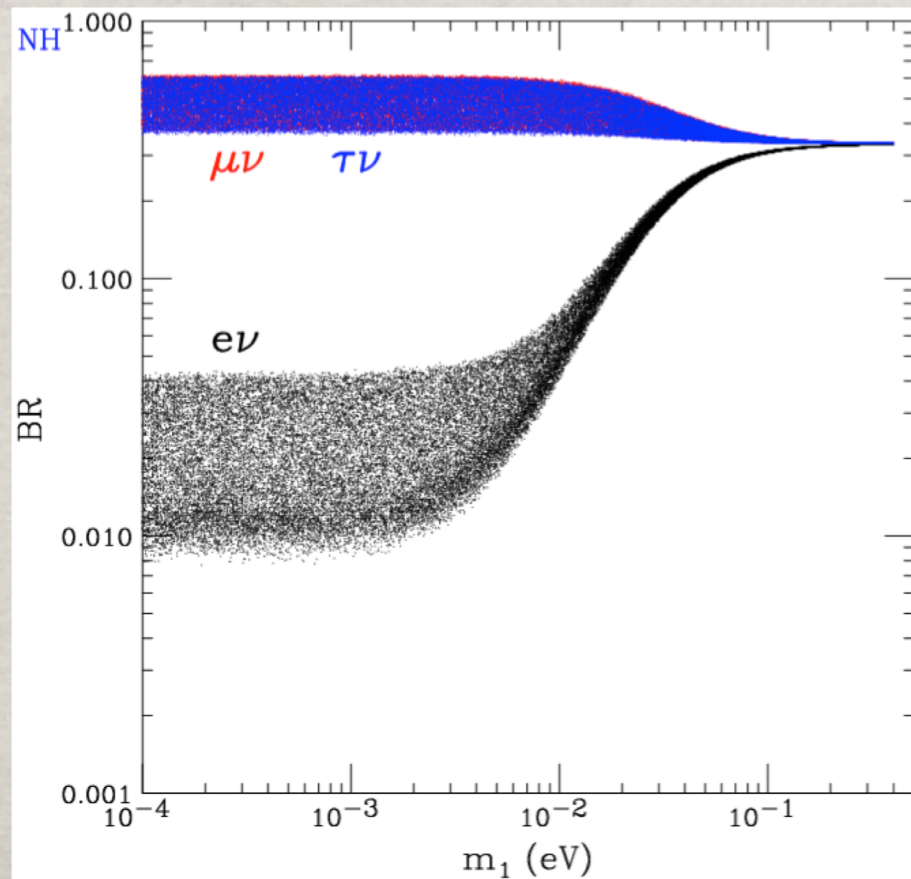
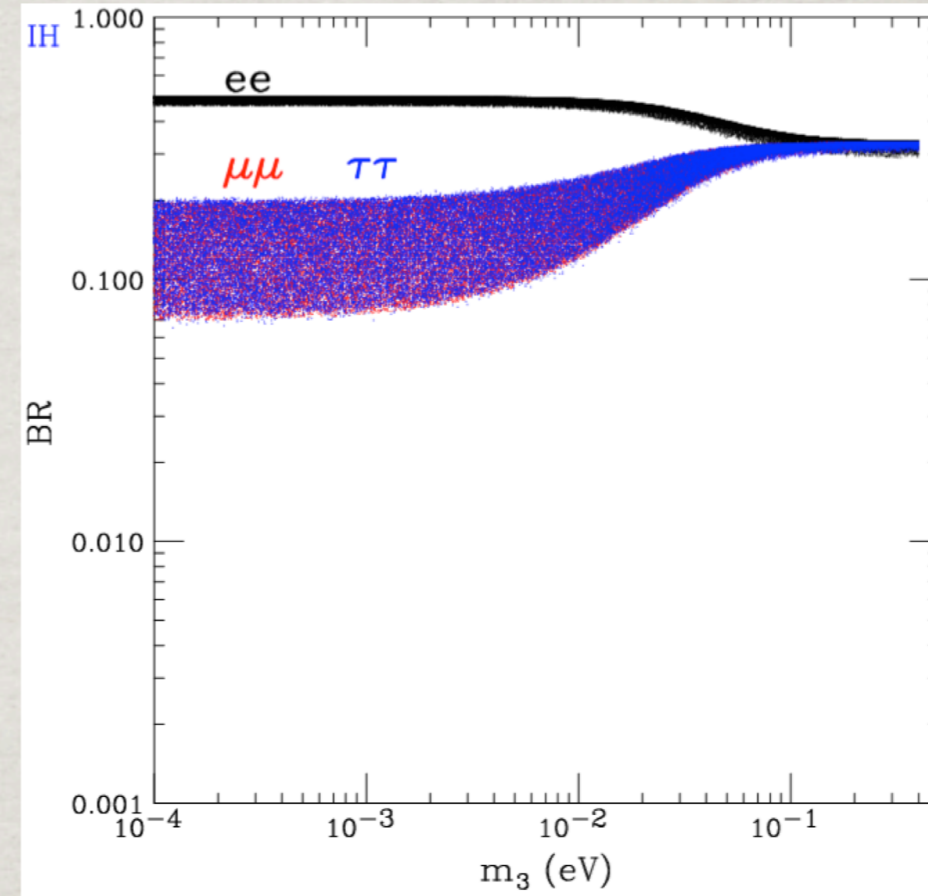
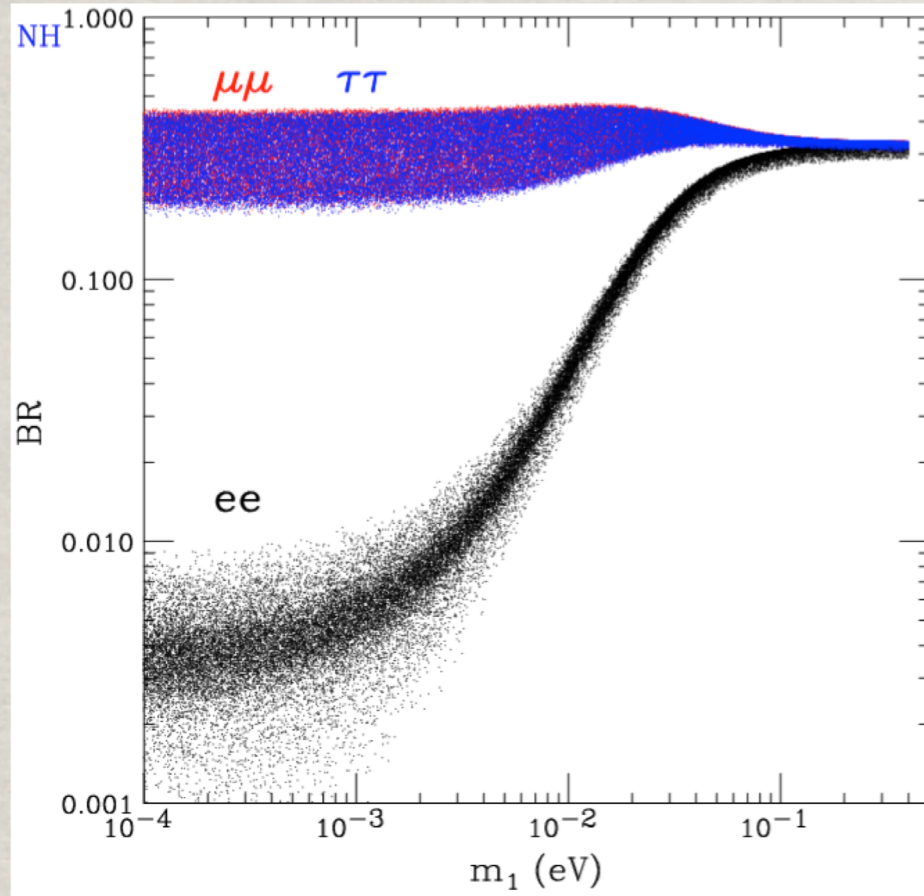
with  $Y_{ll} v' \approx m_\nu$  (eV)  $\Rightarrow v' \approx 2 \times 10^{-4}$  GeV the division.



We will focus on the leptonic decays, with a small  $v'$ .



# $H^{++}, --, H^+, -$ Decays: Revealing the flavor pattern





# Neutrino – charged lepton correlations

Summarize the discovery modes:

Spectrum

Relations

Normal Hierarchy  
( $\Delta m_{31}^2 > 0$ )

$$\begin{aligned} \text{BR}(H^{++} \rightarrow \tau^+ \tau^+), \text{BR}(H^{++} \rightarrow \mu^+ \mu^+) &\gg \text{BR}(H^{++} \rightarrow e^+ e^+) \\ \text{BR}(H^{++} \rightarrow \mu^+ \tau^+) &\gg \text{BR}(H^{++} \rightarrow e^+ \mu^+), \text{BR}(H^{++} \rightarrow e^+ \tau^+) \\ \text{BR}(H^+ \rightarrow \tau^+ \bar{\nu}), \text{BR}(H^+ \rightarrow \mu^+ \bar{\nu}) &\gg \text{BR}(H^+ \rightarrow e^+ \bar{\nu}) \end{aligned}$$

Inverted Hierarchy  
( $\Delta m_{31}^2 < 0$ )

$$\begin{aligned} \text{BR}(H^{++} \rightarrow e^+ e^+) &> \text{BR}(H^{++} \rightarrow \mu^+ \mu^+), \text{BR}(H^{++} \rightarrow \tau^+ \tau^+) \\ \text{BR}(H^{++} \rightarrow \mu^+ \tau^+) &\gg \text{BR}(H^{++} \rightarrow e^+ \tau^+), \text{BR}(H^{++} \rightarrow e^+ \mu^+) \\ \text{BR}(H^+ \rightarrow e^+ \bar{\nu}) &> \text{BR}(H^+ \rightarrow \mu^+ \bar{\nu}), \text{BR}(H^+ \rightarrow \tau^+ \bar{\nu}) \end{aligned}$$

Quasi-Degenerate  
( $m_1, m_2, m_3 > |\Delta m_{31}|$ )

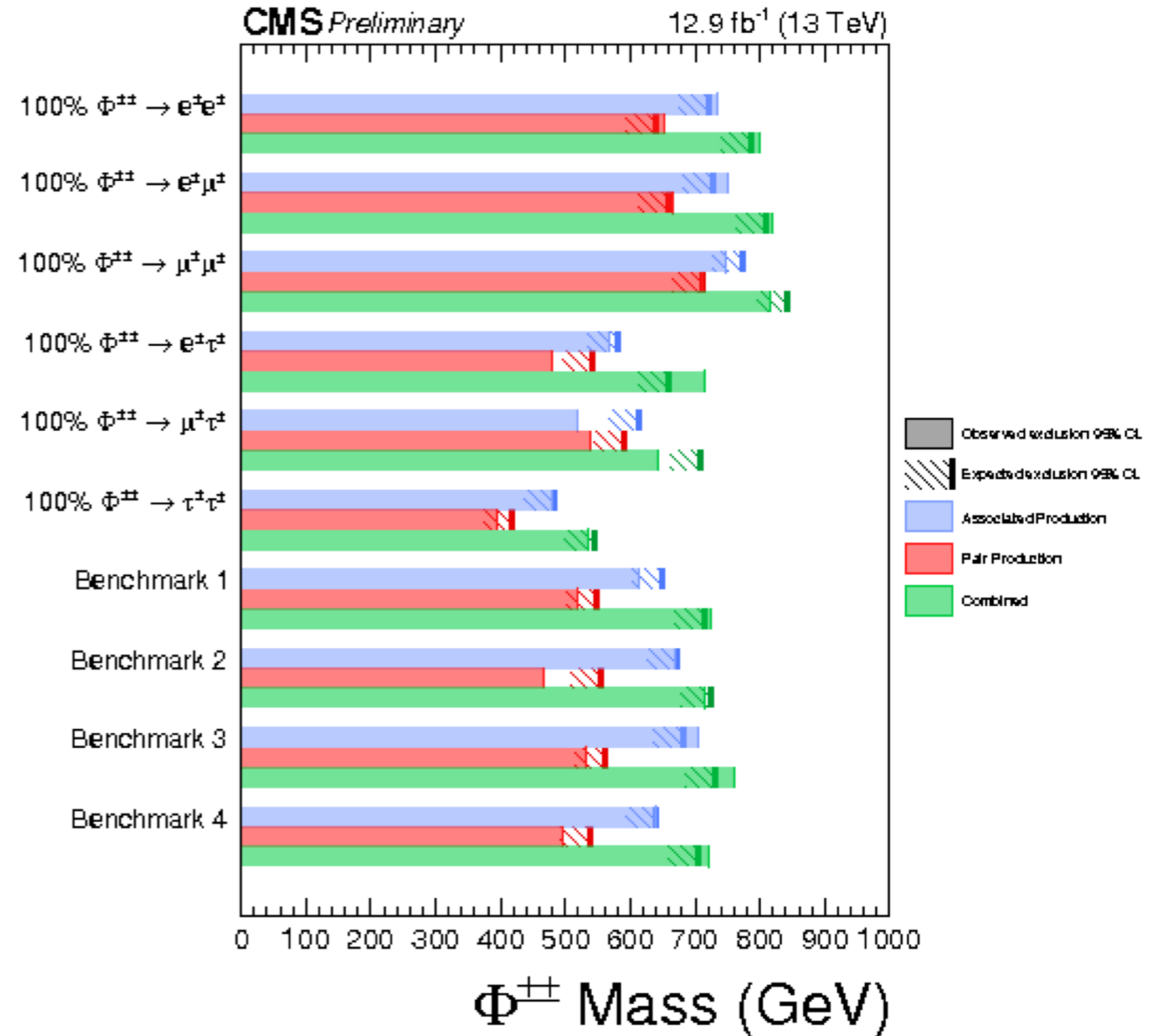
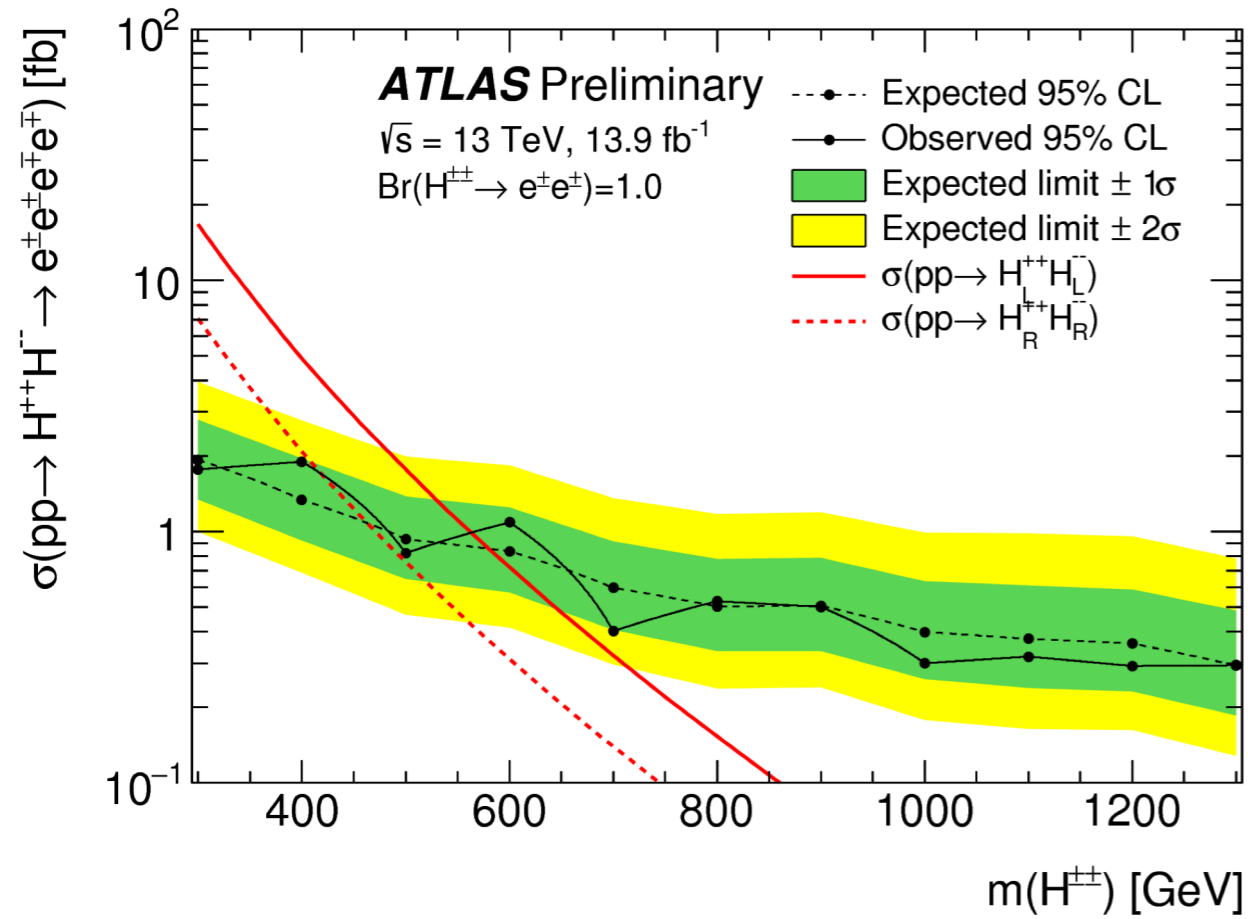
$$\begin{aligned} \text{BR}(H^{++} \rightarrow e^+ e^+) &\sim \text{BR}(H^{++} \rightarrow \mu^+ \mu^+) \sim \text{BR}(H^{++} \rightarrow \tau^+ \tau^+) \approx 1/3 \\ \text{BR}(H^+ \rightarrow e^+ \bar{\nu}) &\sim \text{BR}(H^+ \rightarrow \mu^+ \bar{\nu}) \sim \text{BR}(H^+ \rightarrow \tau^+ \bar{\nu}) \approx 1/3 \end{aligned}$$

† Pavel Fileviez Perez, Tao Han, Gui-Yu Huang, Tong Li, Kai Wang,  
arXiv:0803.3450 [hep-ph]



# Sensitivity to $H^{++}H^{--} \rightarrow l^+l^+, l^-l^-$ Mode:

## ATLAS Bounds: CMS-PAS-HIG-16-036



With  $300 \text{ fb}^{-1}$  integrated luminosity,  
 a coverage upto  $M_{H^{++}} \sim 1 \text{ TeV}$  even with  $BR \sim 40 - 50\%$ .

Possible measurements on  $BR$ 's.

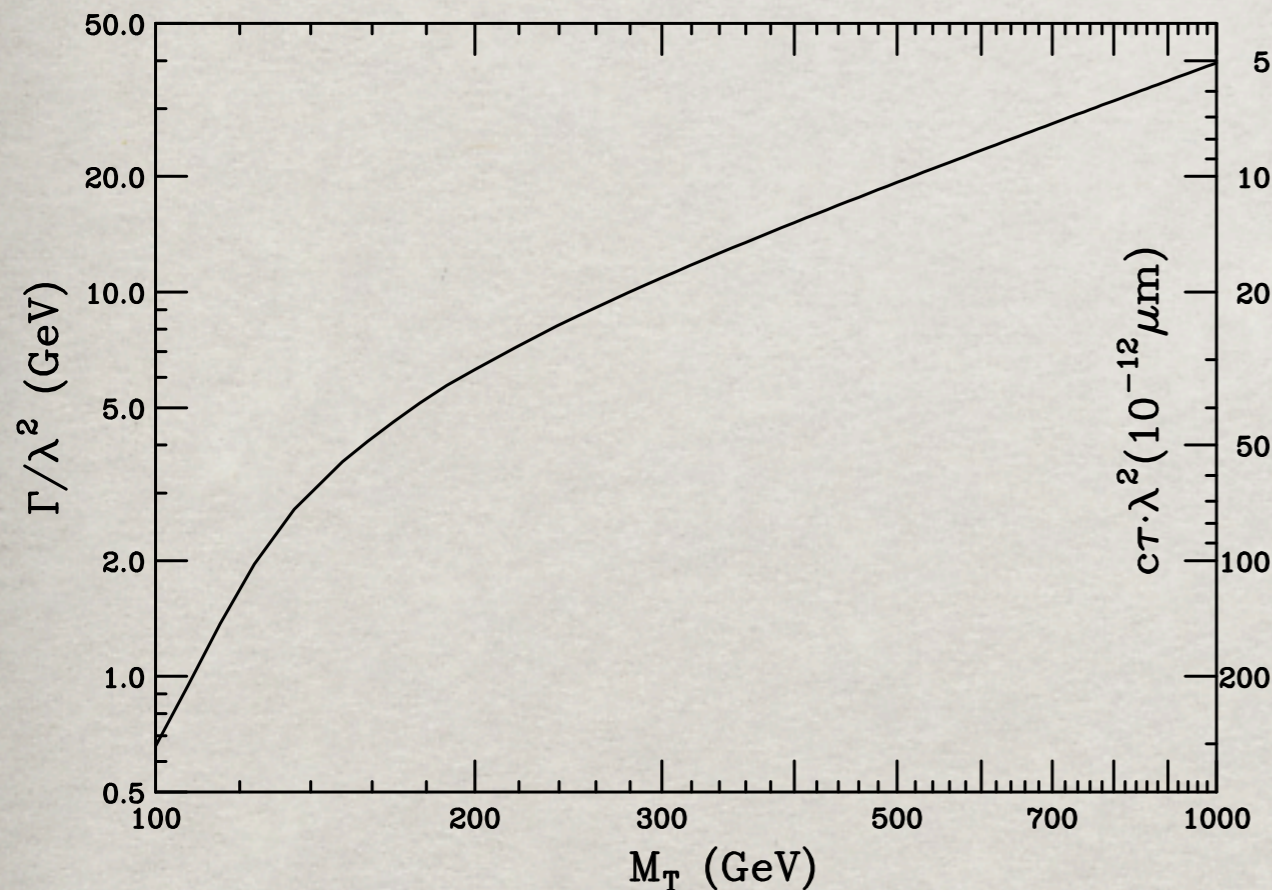


# 9. Type III Seesaw: $T^\pm$ & $T^0$

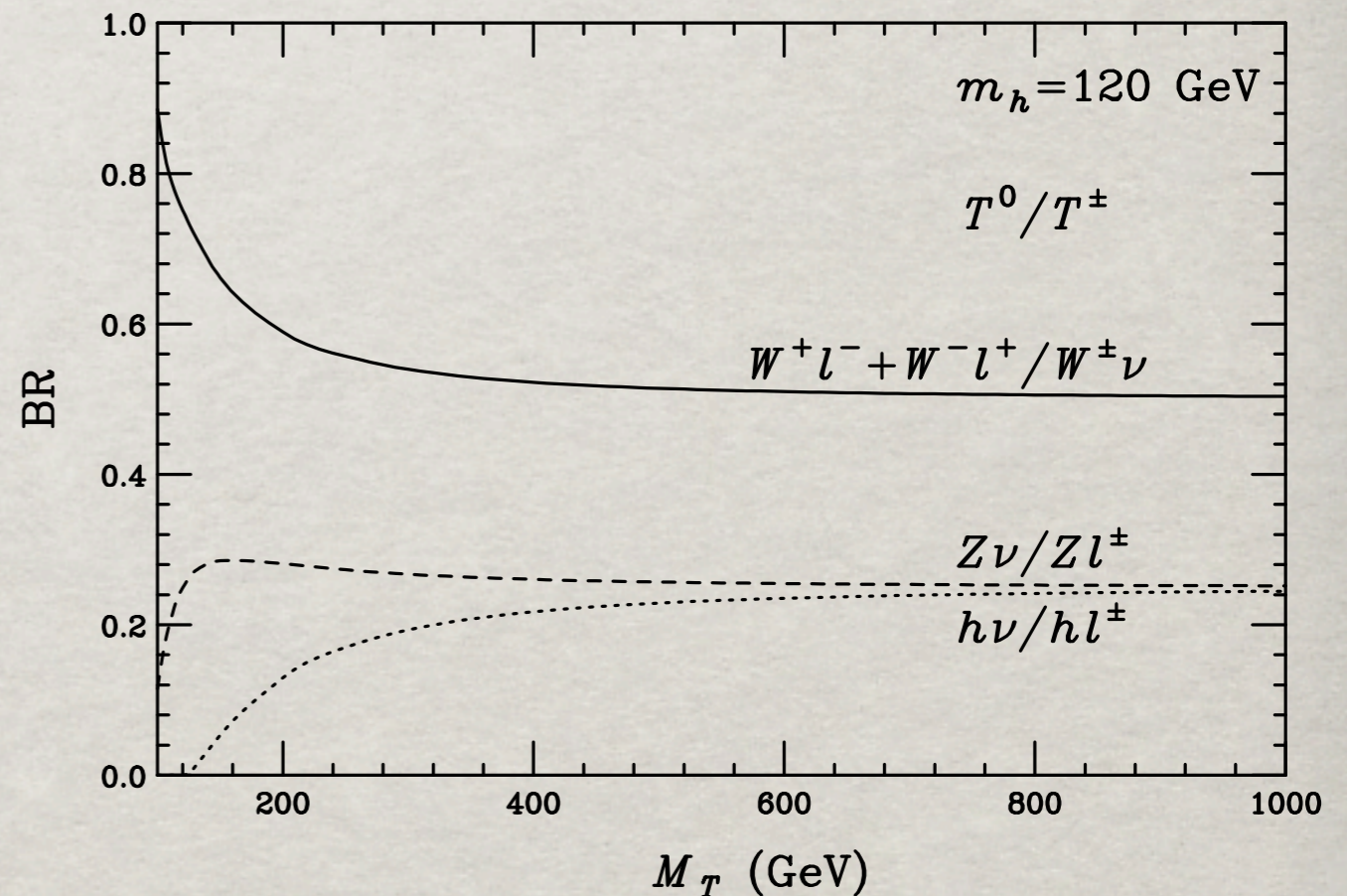
Consider their decay length:

$$\begin{aligned} \Gamma(T^+ \rightarrow W^+ \nu) &\approx 2\Gamma(T^+ \rightarrow Z\ell^+) \approx 2\Gamma(T^+ \rightarrow h\ell^+) \\ &\approx \Gamma(T^0 \rightarrow W^+ \ell^- + W^- \ell^+) \approx \frac{M_T}{16\pi} \sum_i |y_i|^2. \end{aligned}$$

Width and Decay Length



Lepton Triplet Branching Fraction



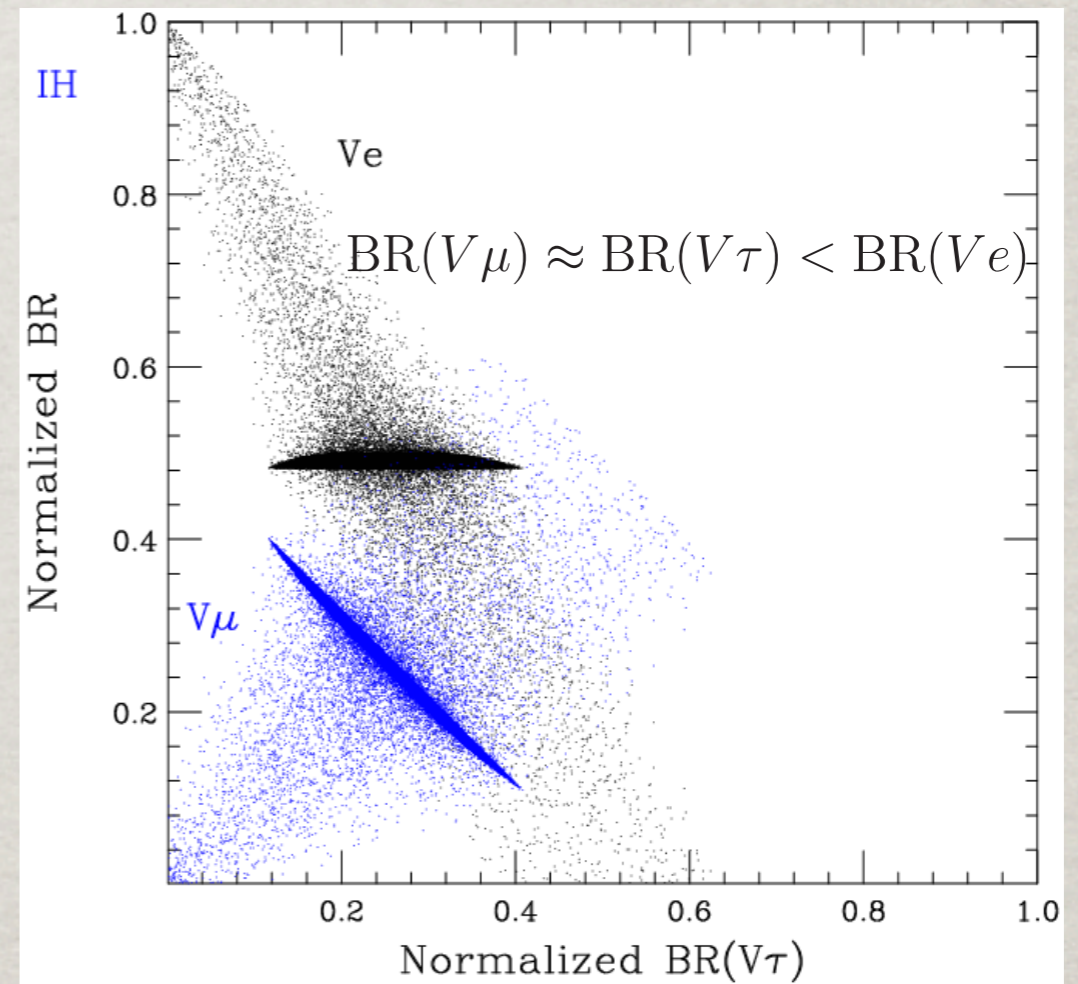
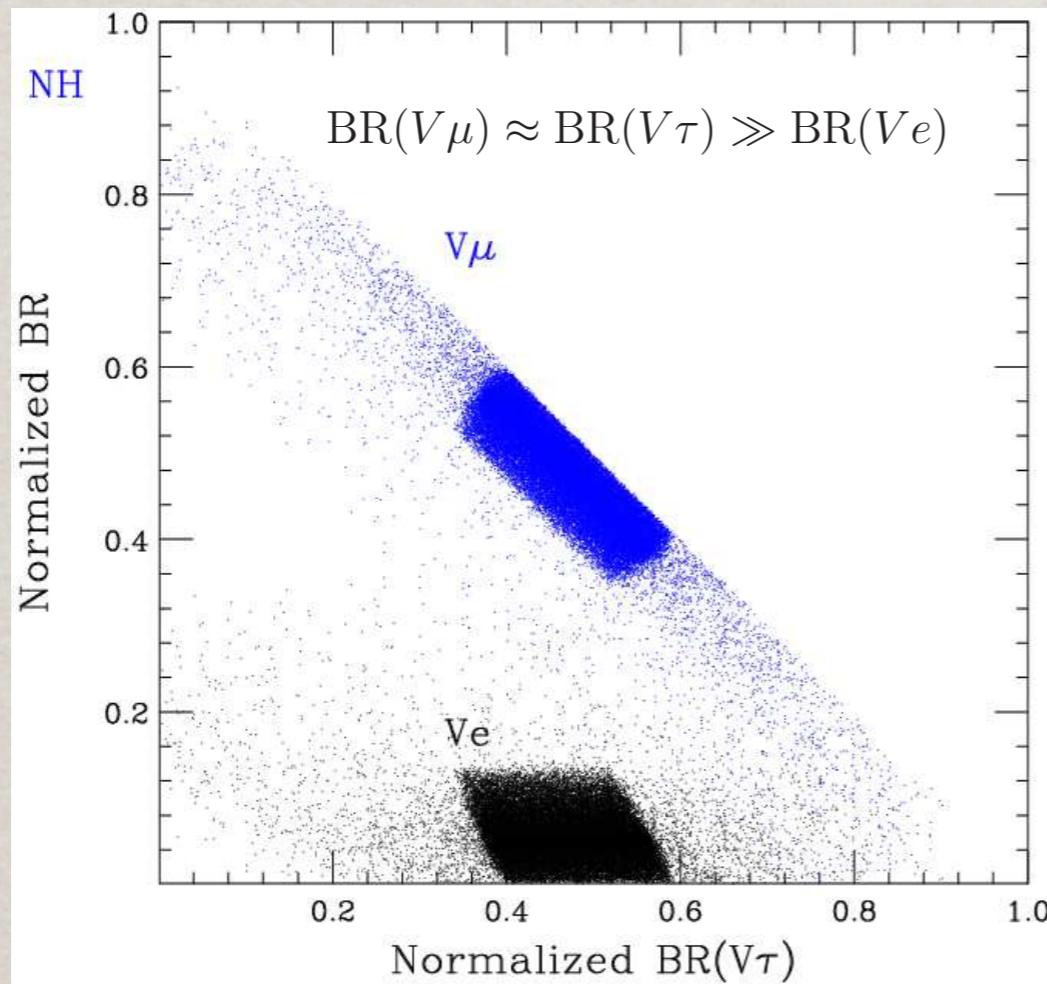
With  $\lambda^2 = y_j^2 \sim 10^{-16} - 10^{-12}$ , then  $c\tau \sim 10^{-2} - 10^{-4}$  m  
 Still not too long-lived, but possibly large displaced vertices.



# Type III Seesaw: $T^\pm$ & $T^0$

Lepton flavor combination determines the  $\nu$  mass pattern: †

$$m_\nu^{ij} \sim -v^2 \frac{y_T^i y_T^j}{M_T}, \quad BR(T^{\pm,0} \rightarrow W^\pm \ell, Z\ell) \sim y_T^2 \sim V_{PMNS}^2 \frac{M_T m_\nu}{v^2}.$$

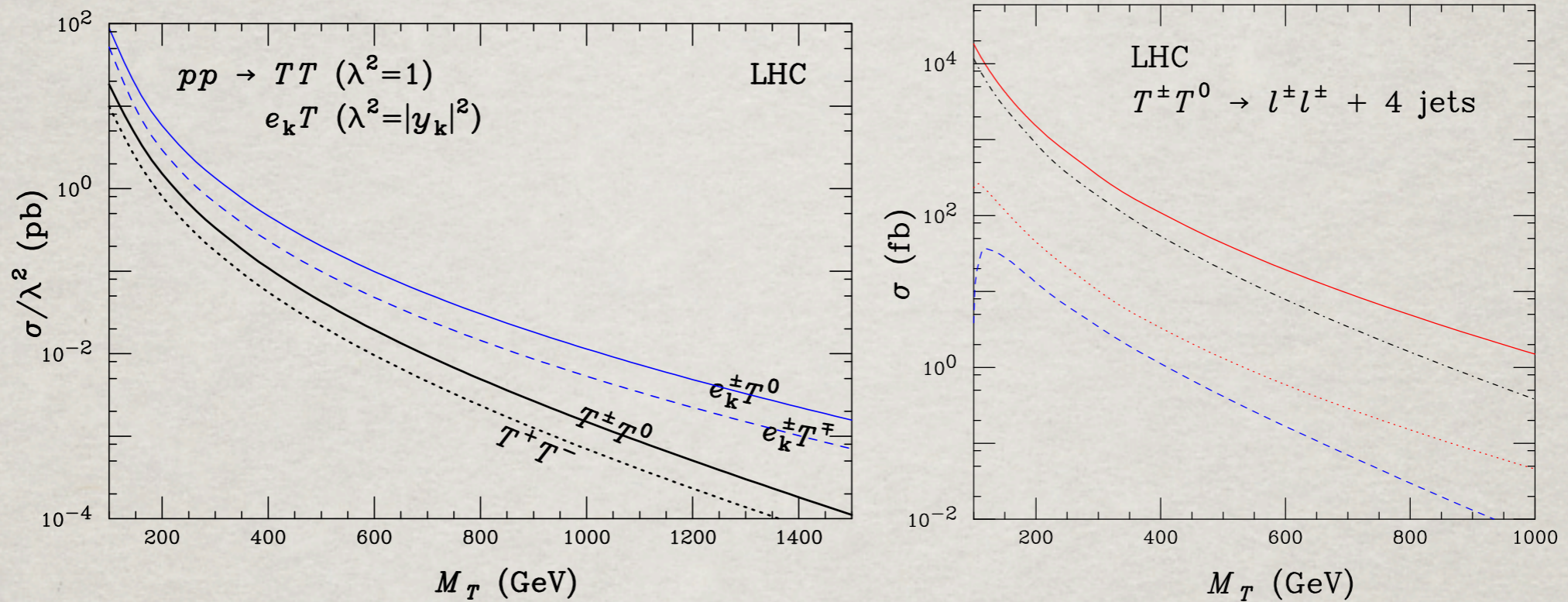


Lepton flavors correlate with the  $\nu$  mass pattern.

† Abdesslam Arhrib, Borut Bajc, Dilip Kumar Ghosh, Tao Han, Gui-Yu Huang, Ivica Puljak, Goran Sejanovic, arXiv:0904.2390.



# Type III Seesaw: $T^\pm$ & $T^0$



- Single production  $T^\pm l^\mp$ ,  $T^0 l^\pm$  :

Kinematically favored, but highly suppressed by mixing.

- Pair production with gauge couplings.

Example:  $T^\pm + T^0 \rightarrow l^\pm Z(h) + l^\pm W^- \rightarrow l^\pm jj(b\bar{b}) + l^\pm jj$ .

Low backgrounds.

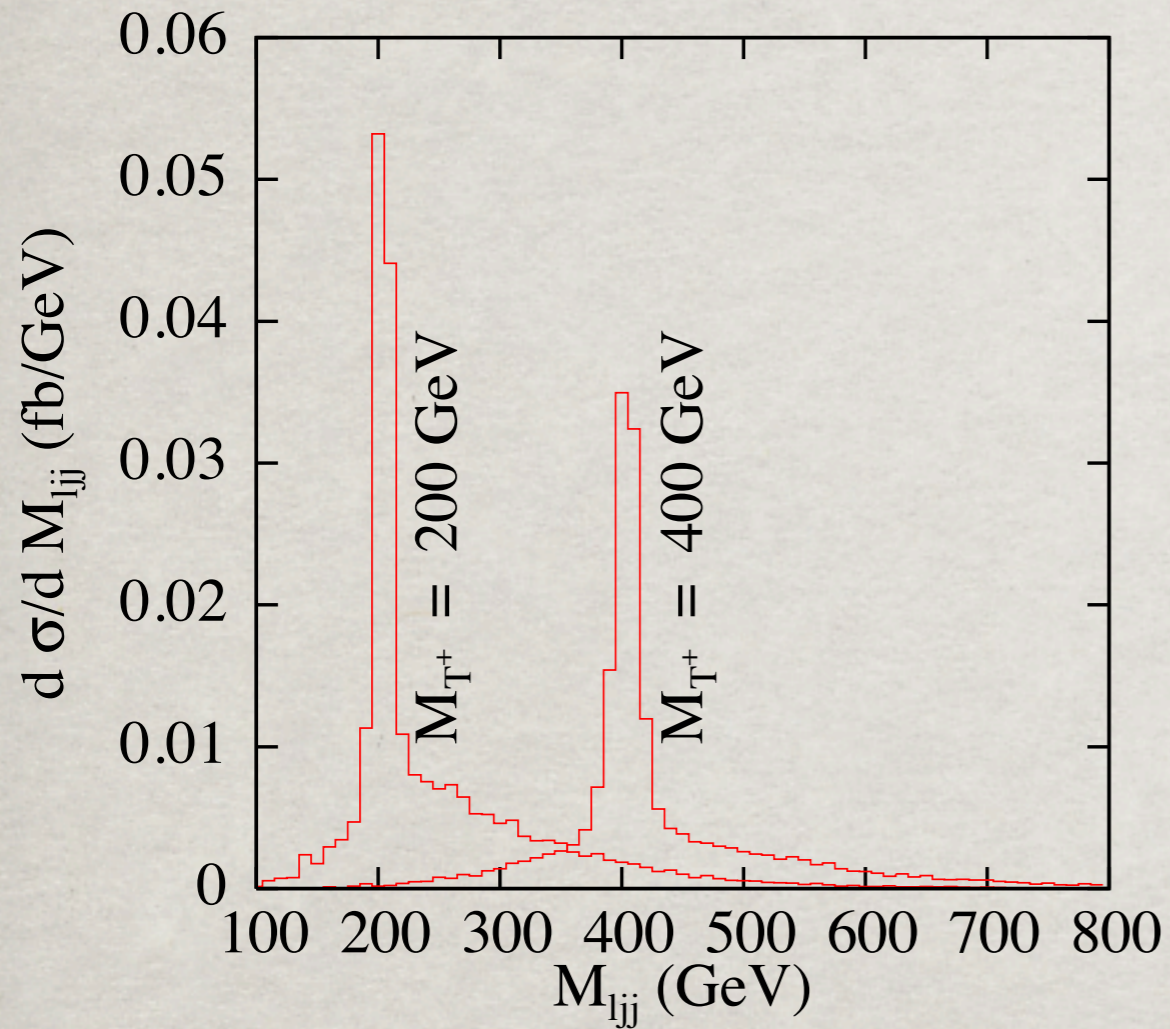
- LHC studies with Minimal Flavor Violation implemented. †

† Similar earlier work: Franceschini, Hambye, Strumia, arXiv:0805.1613.

‡ O. Eboli, J. Gonzalez-Fraile, M.C. Gonzalez-Garcia, arXiv:1108.0661 [hep-ph].



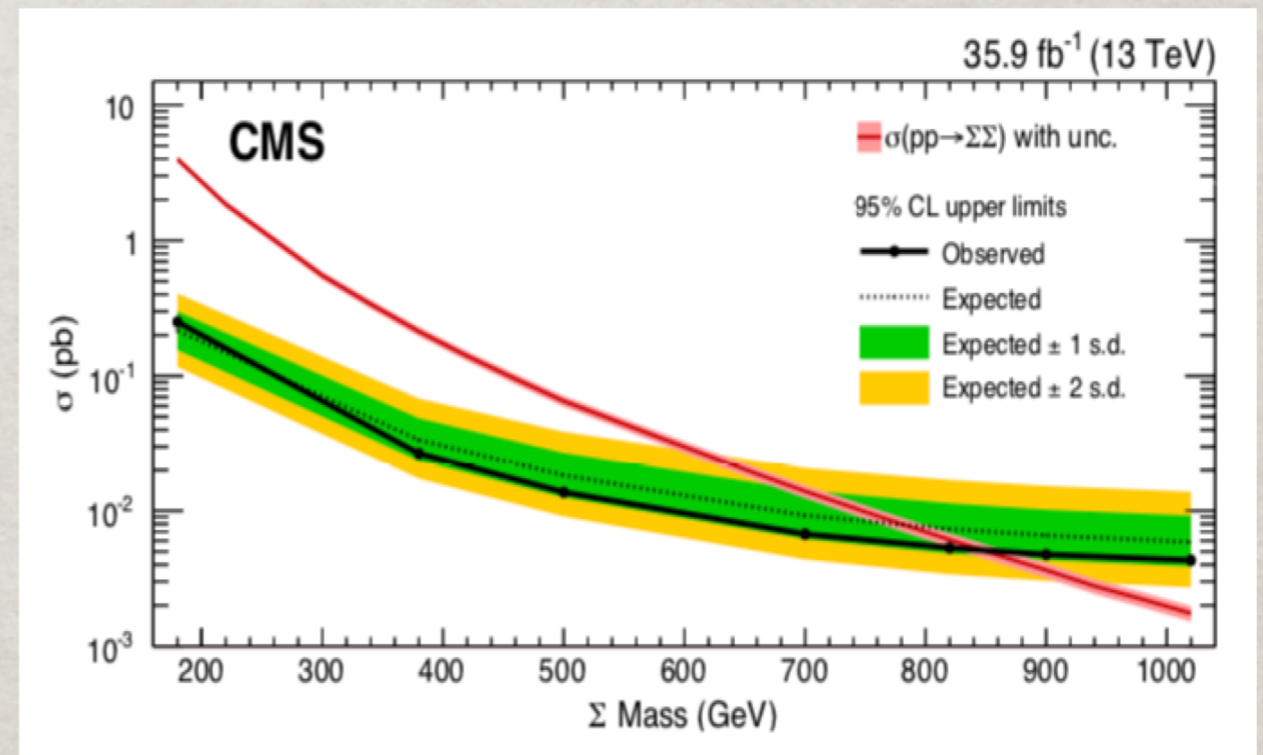
# $\Delta L=2$ & mass reconstruction for $T^\pm$ & $T^0$



Current LHC bounds:

$M_{T^{+-}} > 840$  GeV @ 95% CL

CMS: arXiv:1708.07962





# Summary

- It is of fundamental importance to test the Majorana nature of  $\nu$ 's.
- Type I See-saw:
  - $\tau, K, D, B$  rare decays sensitive to  
 $140 \text{ MeV} < m_4 < 5 \text{ GeV}, 10^{-9} < |V_{\ell 4}|^2 < 10^{-2};$
  - LHC sensitive:  $10 \text{ GeV} < m_4 < 400 \text{ GeV}, 10^{-6} < |V_{\mu 4}|^2 < 10^{-2}.$
  - Difficulty! May be helped with the “inverse seesaw” mechanism.
- Type II See-saw: for a scalar triplet  $\Phi^{\pm\pm}$ 
  - LHC sensitive:  $M_\phi \sim 600 - 1000 \text{ GeV}$  ( $\ell^\pm \ell^\pm$  or  $W^\pm W^\pm$ ).
  - Distinguish Normal/Inverted Hierarchy; Probe Majorana phases.
  - With  $W'^{\pm} \rightarrow N \ell^\pm$ , reach  $M_N < M_{W'} \sim 4 - 5 \text{ TeV}.$
- Type III See-saw: for a lepton triplet  $T^\pm, T^0$ 
  - LHC sensitive:  $M_T \sim 800 \text{ GeV}.$
  - Also distinguish Normal/Inverted Hierarchy.

Radiative seesaw  $\rightarrow$  rich physics in extended Higgs sector.

IF lucky, hadron colliders may serve as the discovery machine for Majorana nature of  $\nu$ 's.



# OTHER MODELS & PHENOMENOLOGY

Thus far, we only considered **Type I, II, III seesaw models**

Many models to account for the neutrino mass.\*

Another class of well-motivated models:

Radiative generation of neutrino masses.

- Zee (1986)-Babu (1988) Model:  
add singlet scalar fields  $m_\nu$  generate at 2-loop  
→ change Higgs physics
- Ma Models (2006):  
add singlet scalars +  $Z_2$  symmetry  
→ Dark matter
- ... ..

Typically, they introduce additional Higgs states and thus new (model-dependent) collider signatures.

\* For a review, see, M.C. Chen & J.R. Huang, arXiv:1105.3188v2.



# Summary: Fill up a Matrix:

	$0\nu 2\beta$	$\mu$ - $e$ conversion $\mu \rightarrow e\gamma$ etc.	rare decays $\tau, K, D, B$	colliders $e^+e^-, pp$	features
Type-I	✓	✓	✓	✓	$N$
Type-II	✓	✓	✓	✓	$H^{\pm\pm}, W_R^\pm$
Type-III	✓	✓	?	✓	$T^\pm$
Zee-Babu	?	✓	?	✓	$k^{\pm\pm}, h^\pm$
Ma models	?	✓	?	✓	scalars, DM
RPV/leptoquarks	?	✓	✓	✓	$\ell_q$
extra-dim	?	?	✓	✓	KK states
Inverse/linear	?	?	✓	✓	
Pseudo Dirac	?	?	✓	✓	
NSI	?	?	?	✓	mediators
... ..					

Please help to

- Fill the entries
- Expand on both sides

**More work to do !**