Precision electroweak calculations in the Standard Model

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1. Canonical electroweak precision observables
2. Observables and pseudo-observables
3. Multi-loop calculations
4. Opportunities and challenges for future colliders

Canonical electroweak precision observables

Structure of SU(2)$\times$U(1) interactions well understood

Open questions of the Standard Model:

- Is the Higgs boson part of a more complex sector?
- Is there an extended/unified symmetry group?
- What is dark matter?
- Why is there more matter than anti-matter in the universe?
- ...

$\longrightarrow$ **Phyiscs beyond the Standard Model**

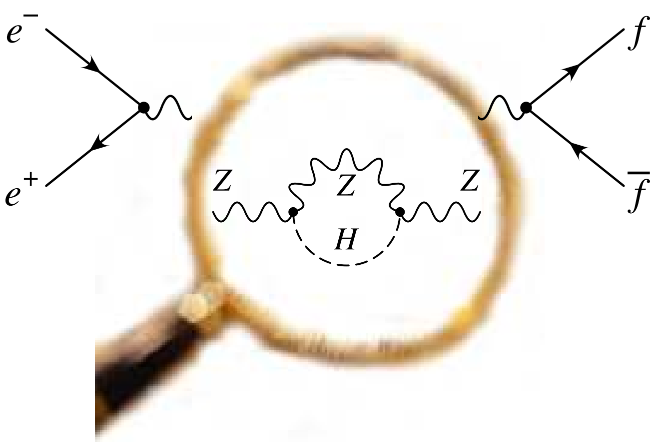
$\longrightarrow$ Direct searches at high-energy colliders (LHC)

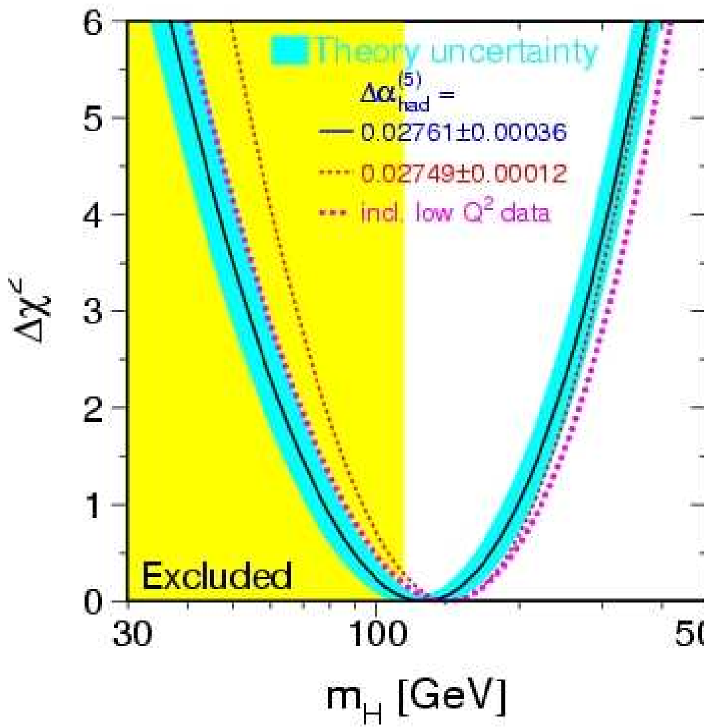
$\longrightarrow$ Astro-physics searches (e.g. DM direct / indirect detection)

$\longrightarrow$ Indirect evidence from precision measurements

Quantum effects

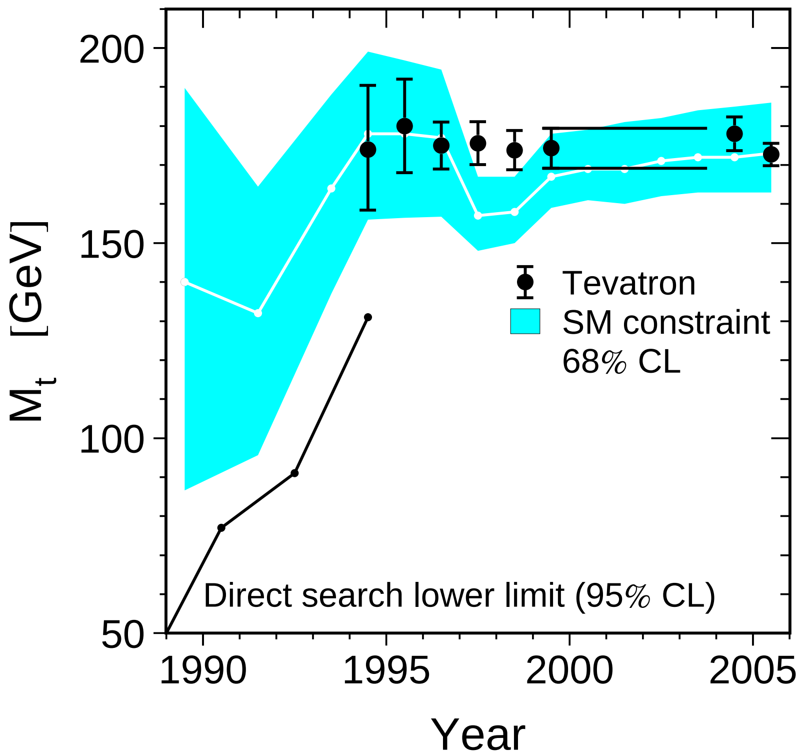
Quantum fluctuations are sensitive to **heavy SM particles** and **new physics**



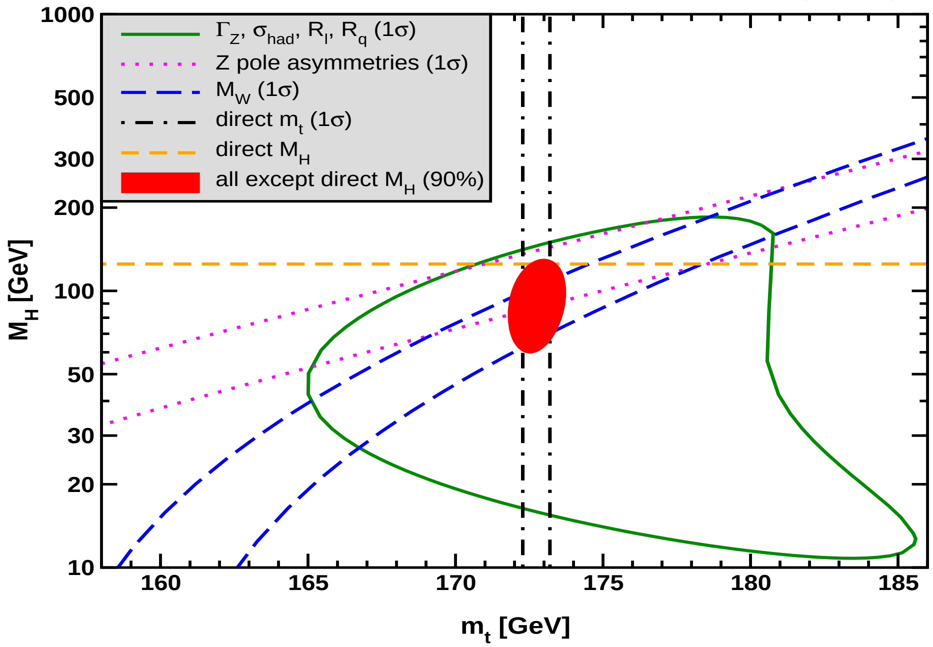


Constraints from fit of SM to *all* electroweak precision observables:

LEP EWWG '05

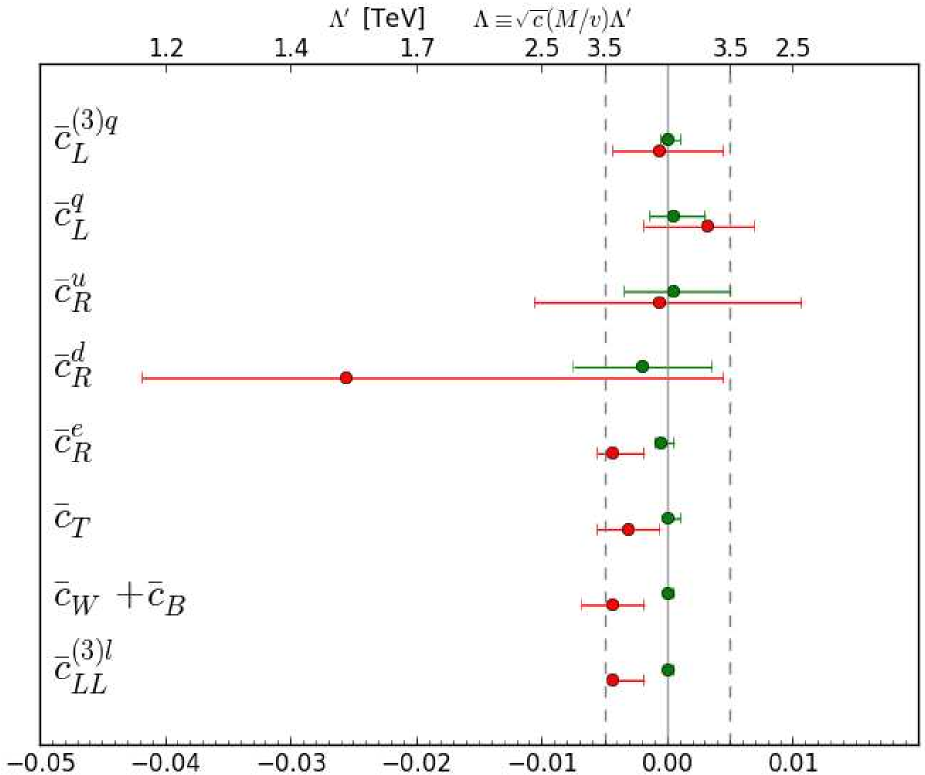


Erler '18



Sensitivity to new physics

Effective operator approach: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}_i + \mathcal{O}(\Lambda^{-3}) \qquad (\Lambda \gg M_Z)$



$\mathcal{O}_{\mathrm{T}} = (D_\mu\Phi)^\dagger\Phi\,\Phi^\dagger(D^\mu\Phi)$

$\mathcal{O}_{\mathrm{BW}} = \Phi^\dagger B_{\mu\nu}W^{\mu\nu}\Phi$

$O_{\mathrm{LL}}^{(3)e} = (\bar{L}_{\mathrm{L}}^e\sigma^a\gamma_\mu L_{\mathrm{L}}^e)(\bar{L}_{\mathrm{L}}^e\sigma^a\gamma^\mu L_{\mathrm{L}}^e)$

$O_{\mathrm{R}}^f = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{f}_{\mathrm{R}}\gamma^\mu f_{\mathrm{R}})$

$O_{\mathrm{L}}^F = i(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi)(\bar{F}_{\mathrm{L}}\gamma^\mu F_{\mathrm{L}})$

$O_{\mathrm{L}}^{(3)F} = i(\Phi^\dagger \overleftrightarrow{D}_\mu^a \Phi)(\bar{F}_{\mathrm{L}}\sigma_a\gamma^\mu F_{\mathrm{L}})$

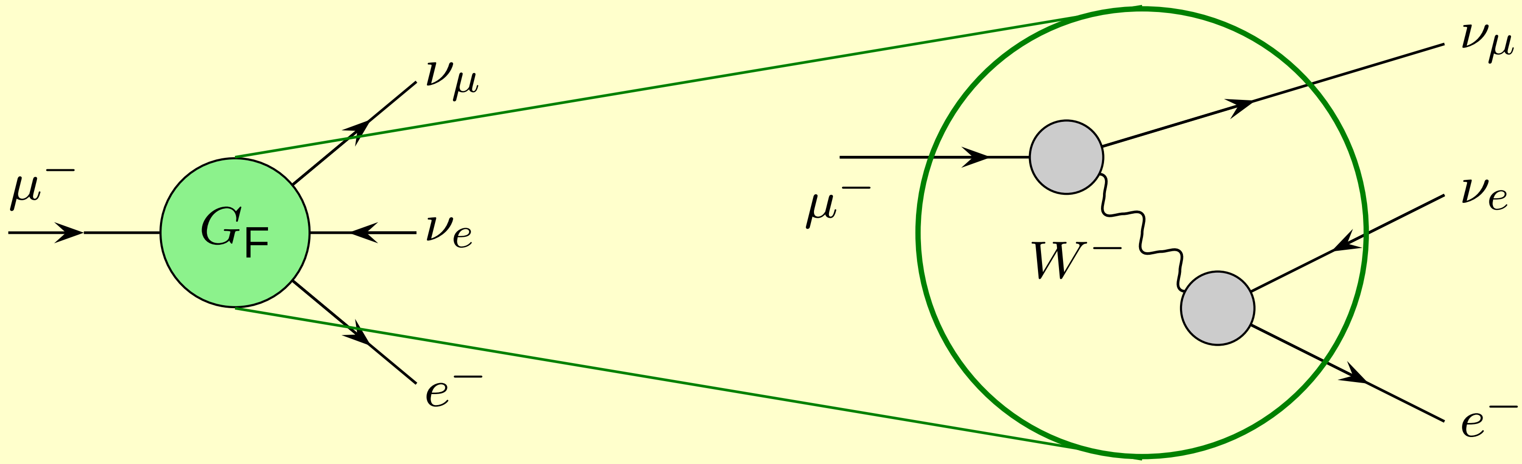
Pomarol, Riva ’13
Ellis, Sanz, You ’14

Suppression Q^2/M^2 requires **high precision** in experiment and theory

few %	1-loop electroweak
few $\times 10^{-3}$	2-loop
few $\times 10^{-4}$	3-loop

W mass prediction

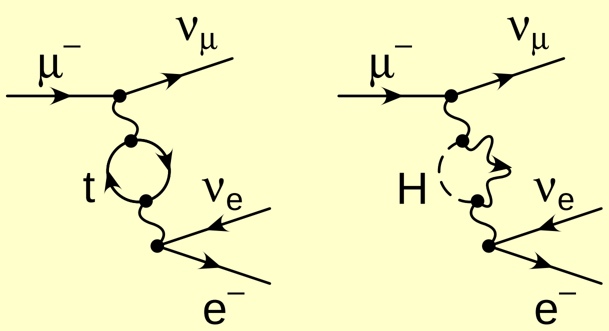
W-boson mass can be calculated from muon decay rate:



μ decay in Standard Model

$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

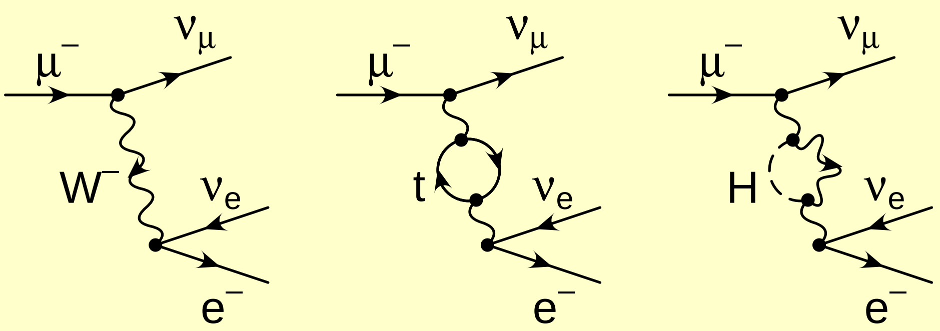
electroweak corrections



- Very small exp. uncertainty

W mass

μ decay in Standard Model:



$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}\Big(1 + \Delta r(M_Z, M_H, m_t, \ldots)\Big)$$

electroweak corrections (few %)

Can solve for

$$M_W = M_W(G_F, M_Z, M_H, m_t, \ldots)$$

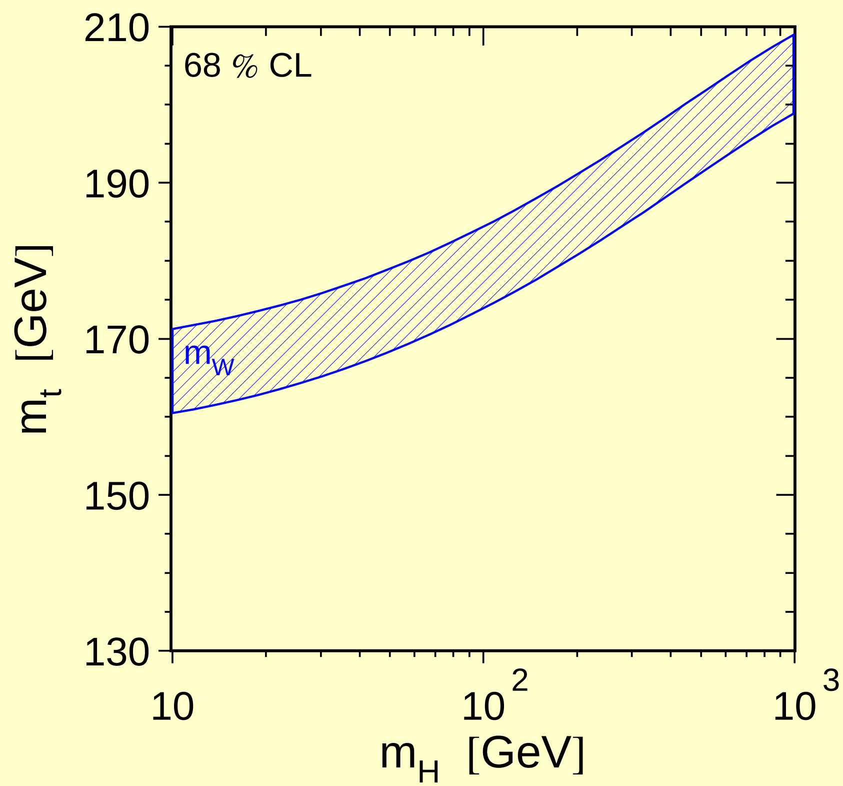
Experiment: Particle Data Group ’18

$G_F = 1.1663787 \times 10^{-5}\ \text{GeV}^{-2}$

$M_W = 80.376(33)$ GeV (LEP)

$80.387(16)$ GeV (TEV)

$80.370(19)$ GeV (LHC)



Z cross section and branching fractions

$e^+e^- \to f\bar{f}$ for $E_{\text{CM}} \sim M_{\text{Z}}$:

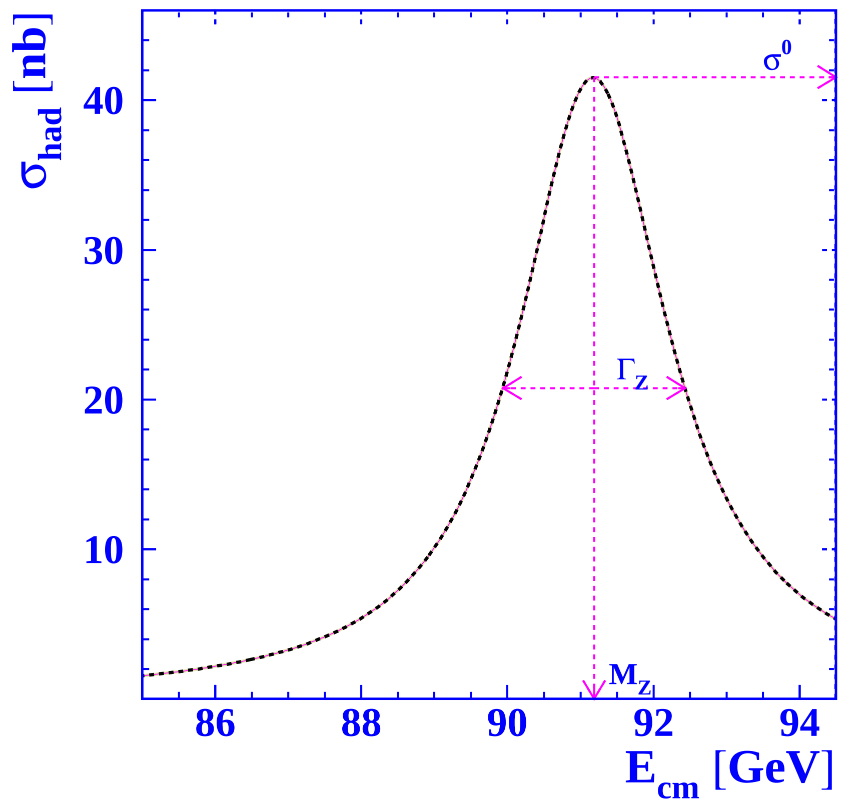
- Mass M_{Z}
- Width $\Gamma_{\text{Z}} = \sum_f \Gamma_{ff}$
- Braching ratio $R_f = \Gamma_{ff}/\Gamma_{\text{Z}}$
- $\sigma^0 \approx \frac{12\pi\, \Gamma_{ee}\Gamma_{ff}}{(s-M_{\text{Z}}^2)^2+M_{\text{Z}}^2\Gamma_{\text{Z}}^2} = \frac{12\pi}{M_{\text{Z}}^2} R_e R_f$

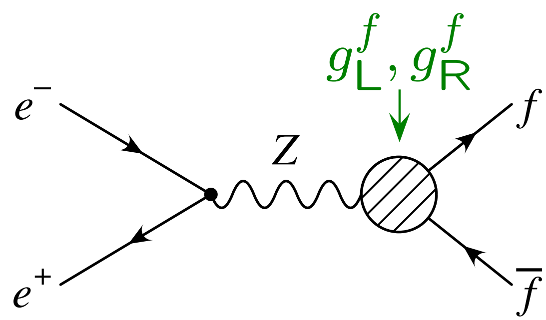
$$\Gamma_{ff} = C\left[(g_{\text{L}}^f)^2 + (g_{\text{R}}^f)^2\right]$$

$\longrightarrow$ Dependence on α_{s}, m_{t}, M_{Z}, ...

Main exp. systematic uncertainties:

- Beam energy calibration
- Particle ID and acceptance





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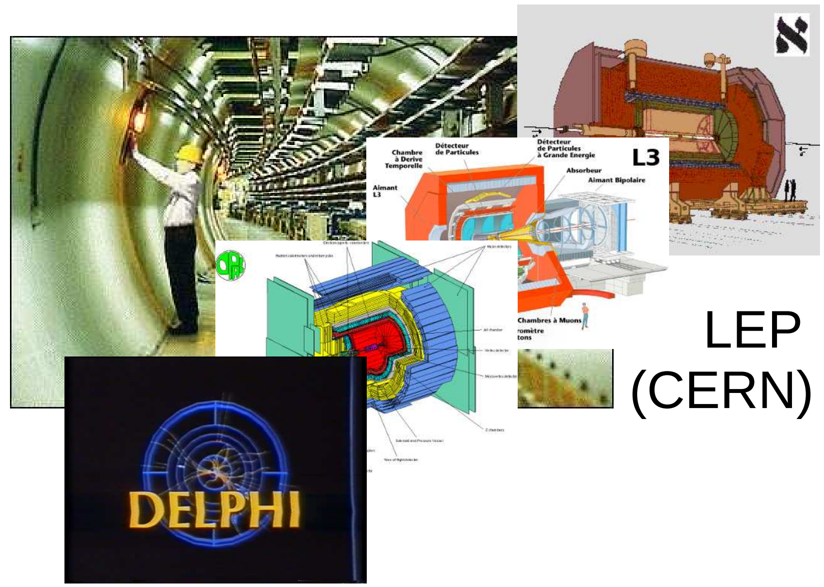
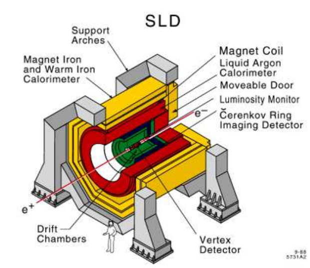


LEP (CERN)



SLC (SLAC)

Z cross section and branching fractions

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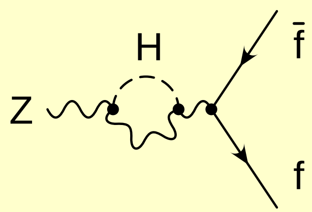
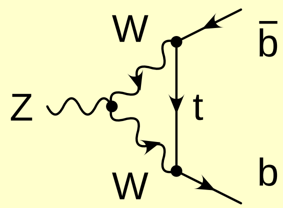


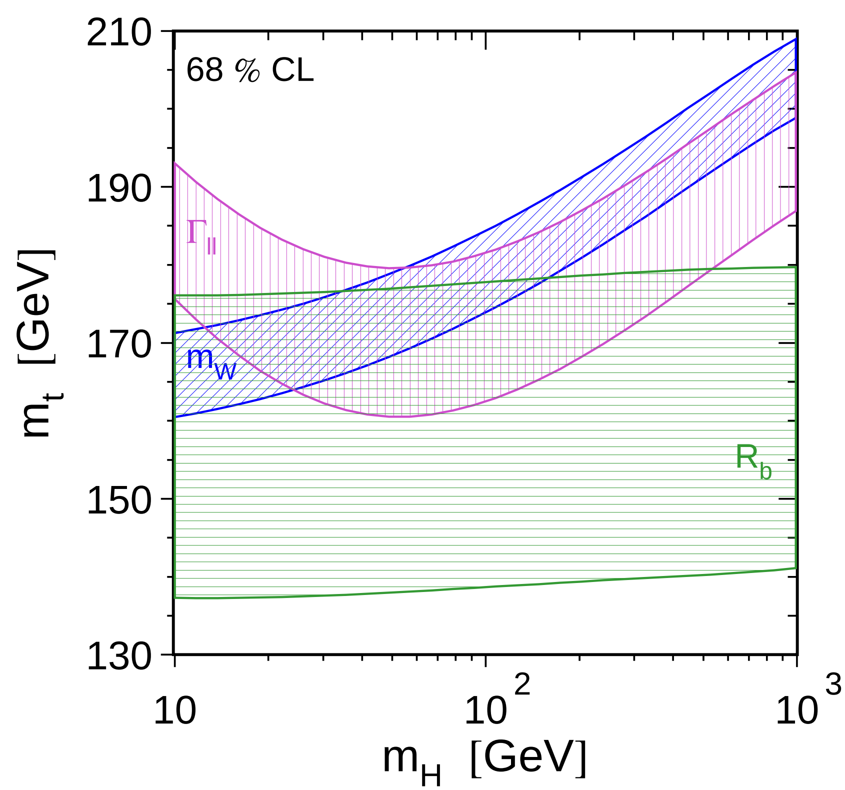


Comparison with experiment:

$\Gamma_{ll} = 83.984(86)$ MeV

$R_b = 0.2163(7)$

Particle Data Group '18



Z-pole asymmetries

Parity violation in $Zf\bar{f}$ couplings:

$g_{\mathrm{L}}^f \neq g_{\mathrm{R}}^f$

Left-right asymmetry:

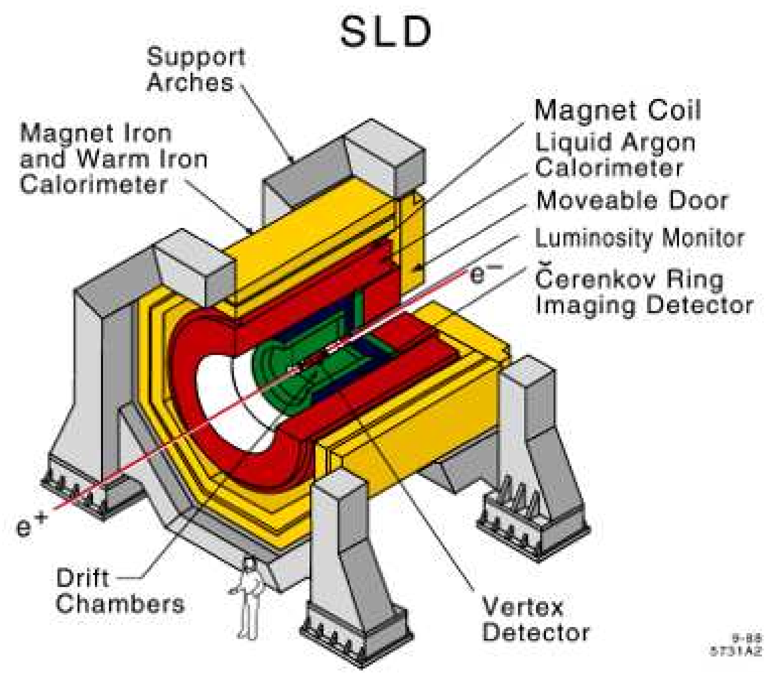
$$A_{\mathrm{LR}} \equiv \frac{1}{P_{e^-}} \frac{\sigma_{\mathrm{L}} - \sigma_{\mathrm{R}}}{\sigma_{\mathrm{L}} + \sigma_{\mathrm{R}}} = \mathcal{A}_e$$

$$\mathcal{A}_f = \frac{2(1-4\sin^2\theta_{\mathrm{eff}}^f)}{1+(1-4\sin^2\theta_{\mathrm{eff}}^f)^2}$$

$$\sin^2\theta_{\mathrm{eff}}^f = \frac{g_R^f}{2|Q_f|(g_R^f - g_L^f)}$$

- Limited by systematic error of P_{e^-}
  0.5% at SLD, 0.1% possible in future

Karl, List '17



Z-pole asymmetries

Blondel scheme: (if e^- and e^+ polarization available) Blondel '88

Four independent measurements for $P_{e^+}/P_{e^-} = ++, +-, -+, --$

$$A_{\mathrm{LR}} = \sqrt{\frac{(\sigma_{++}+\sigma_{-+}-\sigma_{+-}-\sigma_{--})(-\sigma_{++}+\sigma_{-+}-\sigma_{+-}+\sigma_{--})}{(\sigma_{++}+\sigma_{-+}+\sigma_{+-}+\sigma_{--})(-\sigma_{++}+\sigma_{-+}+\sigma_{+-}-\sigma_{--})}}$$

Note: No need to know $|P_{e^\pm}|$!

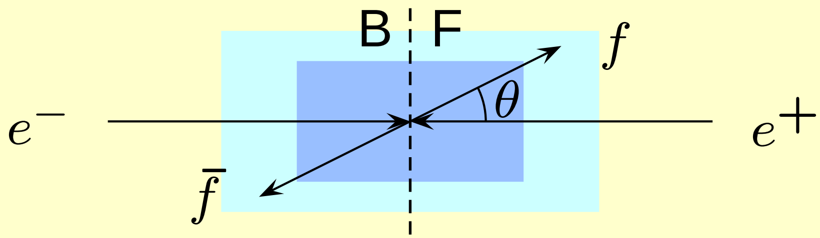
Main systematic uncertainties:

- Difference of $|P|$ for $P > 0$ and $P < 0$
- Difference of $\mathcal{L}$ for $P > 0$ and $P < 0$

$\delta A_{\mathrm{LR}} \approx 10^{-4} \quad \Rightarrow \quad \delta \sin^2\theta^\ell_{\mathrm{eff}} \approx 1.3 \times 10^{-5}$ Mönig, Hawkings '99

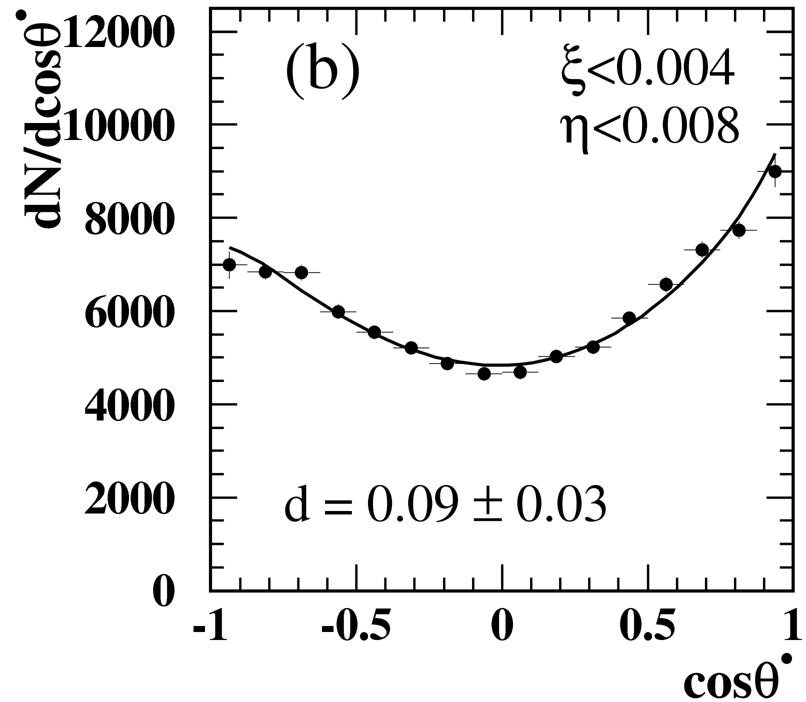
Forward-backward asymmetry:

$$A_{\mathrm{FB}} \equiv \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$$



Main systematic uncertainties:

- $f = b$: charge tag, jet clustering
- $f = \mu$: measurement of $\sqrt{s}$, $\mathcal{L}$ stability



OPAL collaboration ’01

Polarization asymmetry:

Average τ pol. in $e^+e^- \to \tau^+\tau^-$, $\langle \mathcal{P}_\tau \rangle = -\mathcal{A}_\tau$

Z-pole asymmetries

Forward-backward asymmetry:

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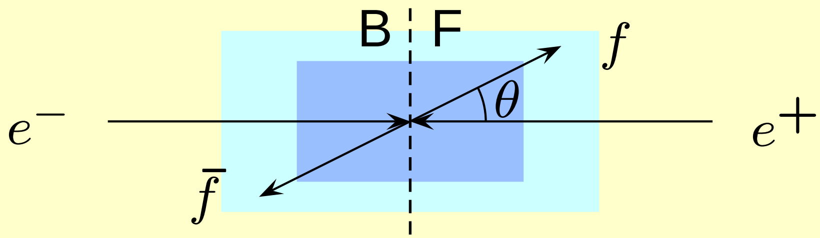

Main systematic uncertainties:

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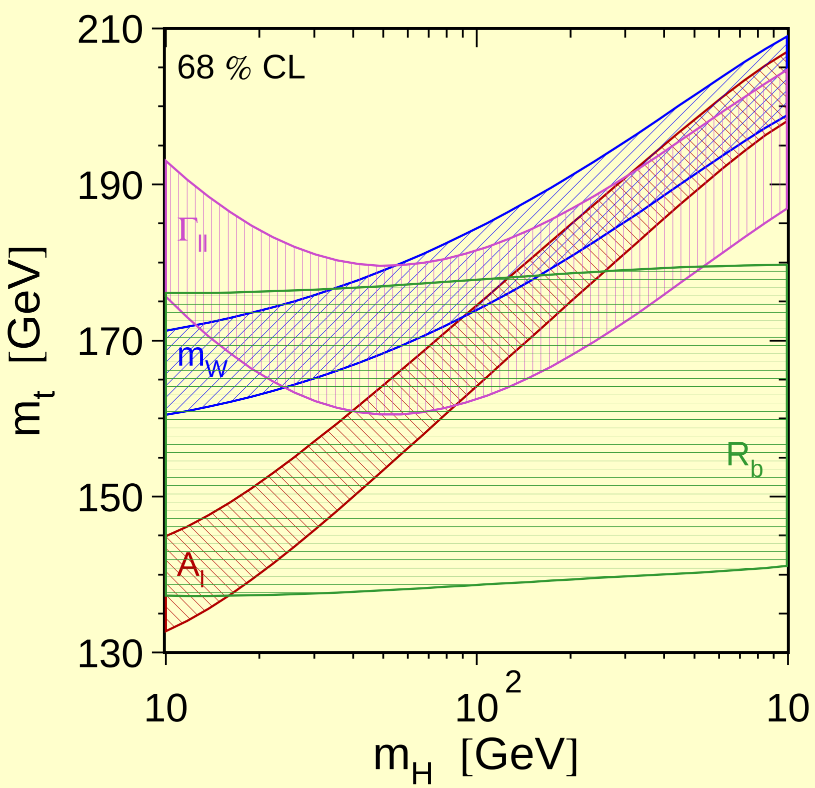
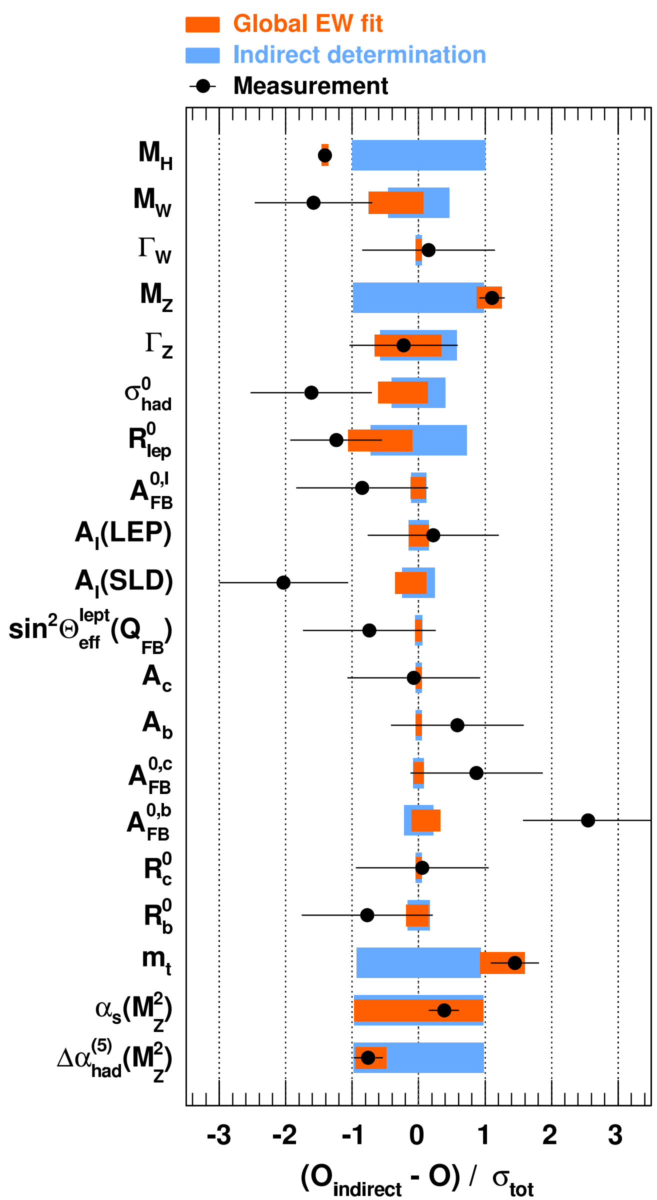
Comparison with experiment:

$\mathcal{A}_l = 0.1475(10)$

Particle Data Group '12



Global electroweak fit



Surprisingly good agreement:

$\chi^2/\text{d.o.f.} = 47.0/41$ ($p = 24\%$)

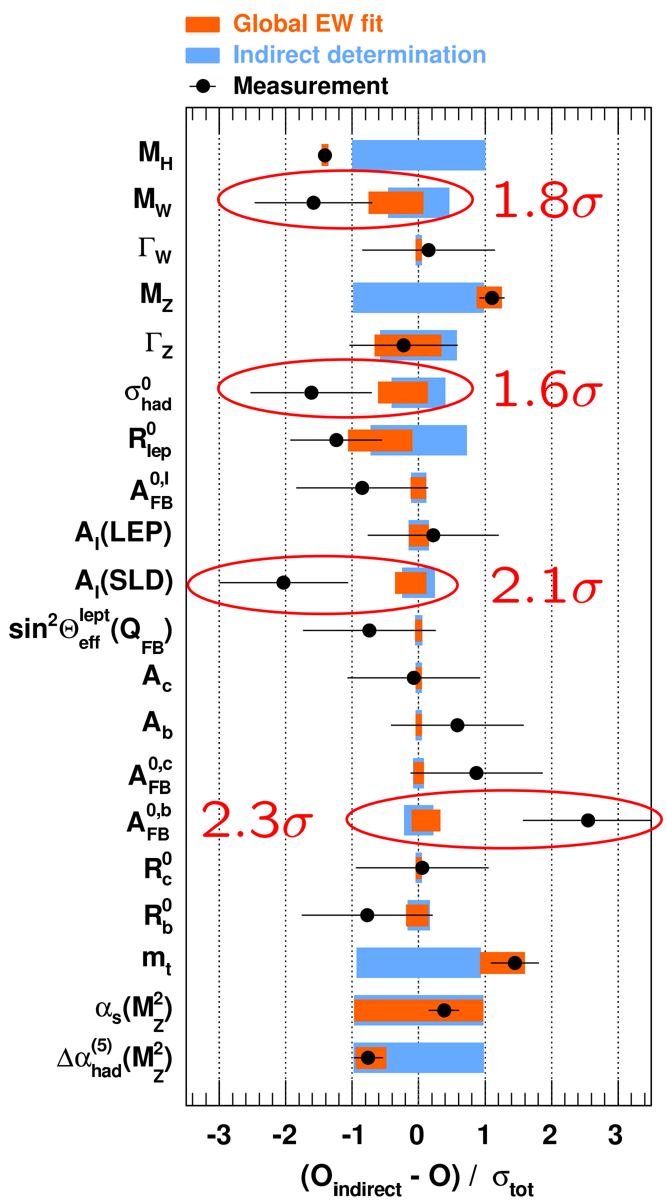
Most quantities measured with

1%–0.1% precision

RPP ’18

GFitter coll. ’14

Global electroweak fit



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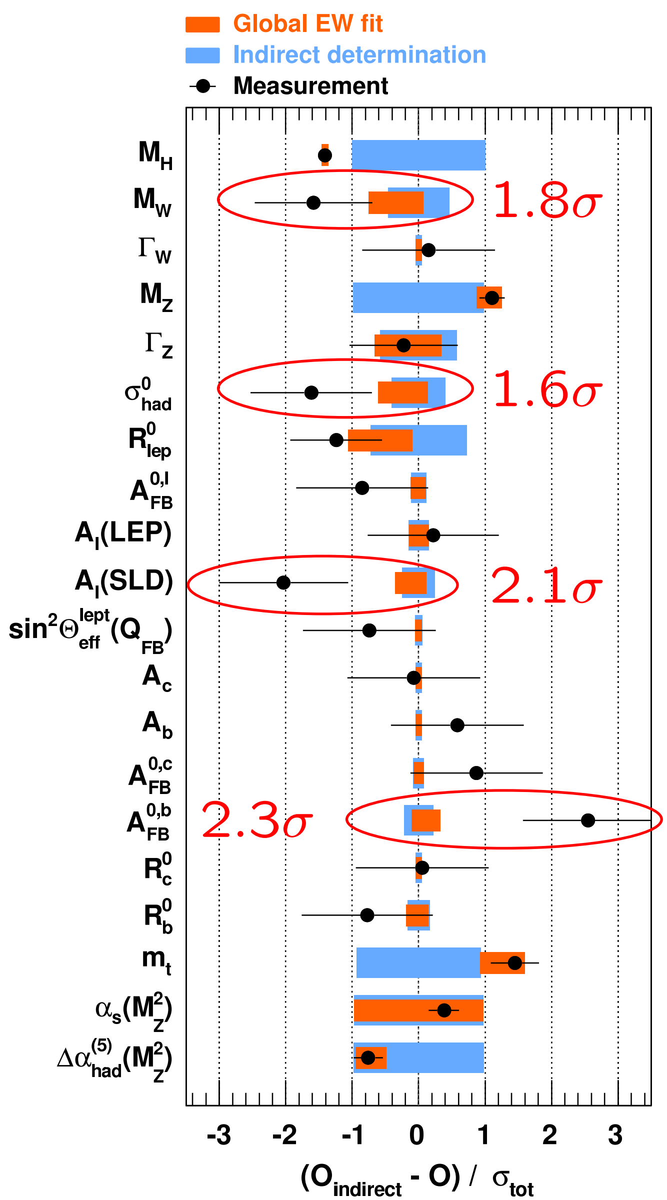
RPP '18

A few interesting deviations:

M_W	$(\sim 1.8\sigma)$
σ^0_{had}	$(\sim 1.6\sigma)$
A_ℓ(SLD)	$(\sim 2.1\sigma)$
A^b_{FB}	$(\sim 2.3\sigma)$
$(g_\mu - 2)$	$(> 3\sigma)$

GFitter coll. '14

Global electroweak fit



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A_ℓ(SLD) and A^b_{FB} together: $\sim 4\sigma$

GFitter coll. '14

SM input parameters

- m_t, M_H, M_W: Most precise measurements from TeVatron, LHC

  $\longrightarrow$ Camarda, Piccinini

- α_s:

  Most precise determination using Lattice QCD:

  $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10

  $\alpha_s = 0.1185 \pm 0.0008$ ALPHA '17

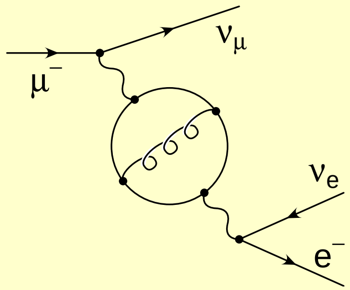
  $\alpha_s = 0.1179 \pm 0.0015$ Takaura et al. '18

  $\alpha_s = 0.1172 \pm 0.0011$ Zafeiropoulos et al. '19

  But e^+e^- event shapes and DIS prefer $\alpha_s \sim 0.114$

  Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13

- Impact on EWPOs:



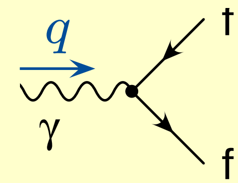
$$\delta\alpha_s = 0.005 \quad \Rightarrow \quad \delta M_W \approx 3.5 \text{ MeV}$$

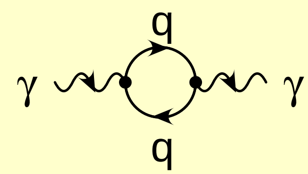
$$\delta \sin^2\theta^\ell_{\text{eff}} \approx 2 \times 10^{-5}$$

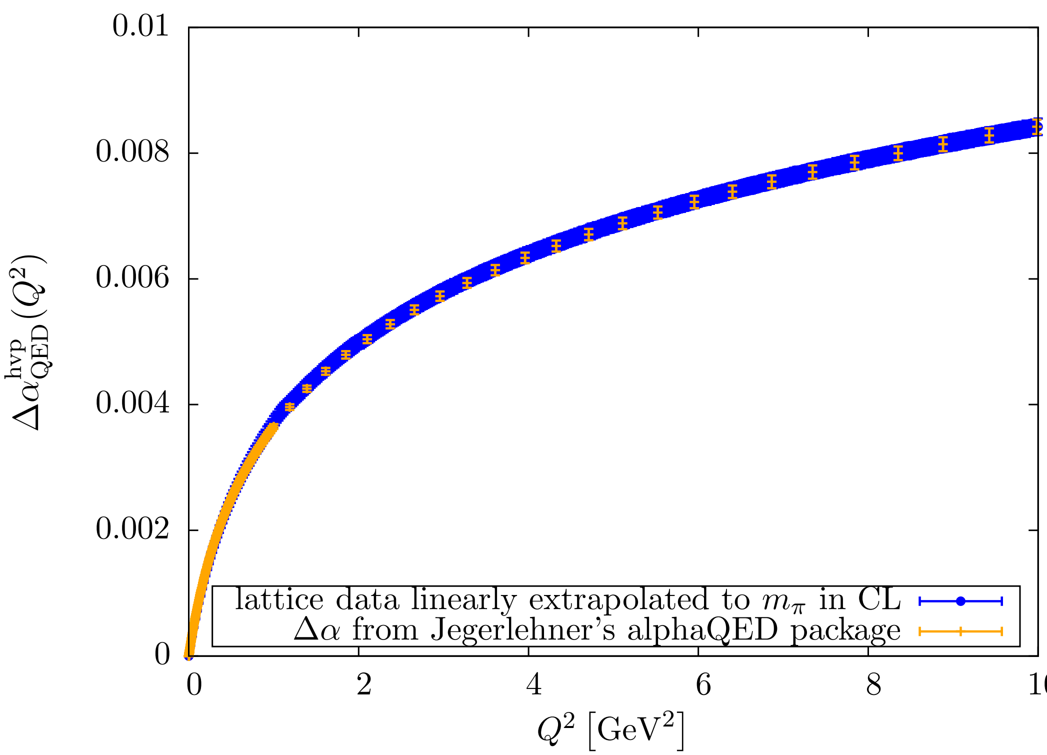
Currently not dominant, but similar magnitude as intrinsic theory error

Shift of finestructure constant

- α defined for $q^2 = 0$, but EWPOs need $\alpha(q^2 \sim M_Z^2)$
- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- One-loop: $\Delta\alpha = \sum_f \frac{\alpha}{3\pi}\left(\ln \frac{M_Z^2}{m_f^2} - \frac{5}{3}\right)$
- m_f not well-defined for $f = u, d, s, (c)$



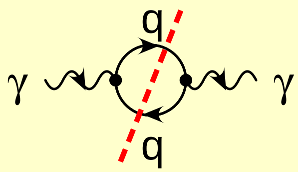




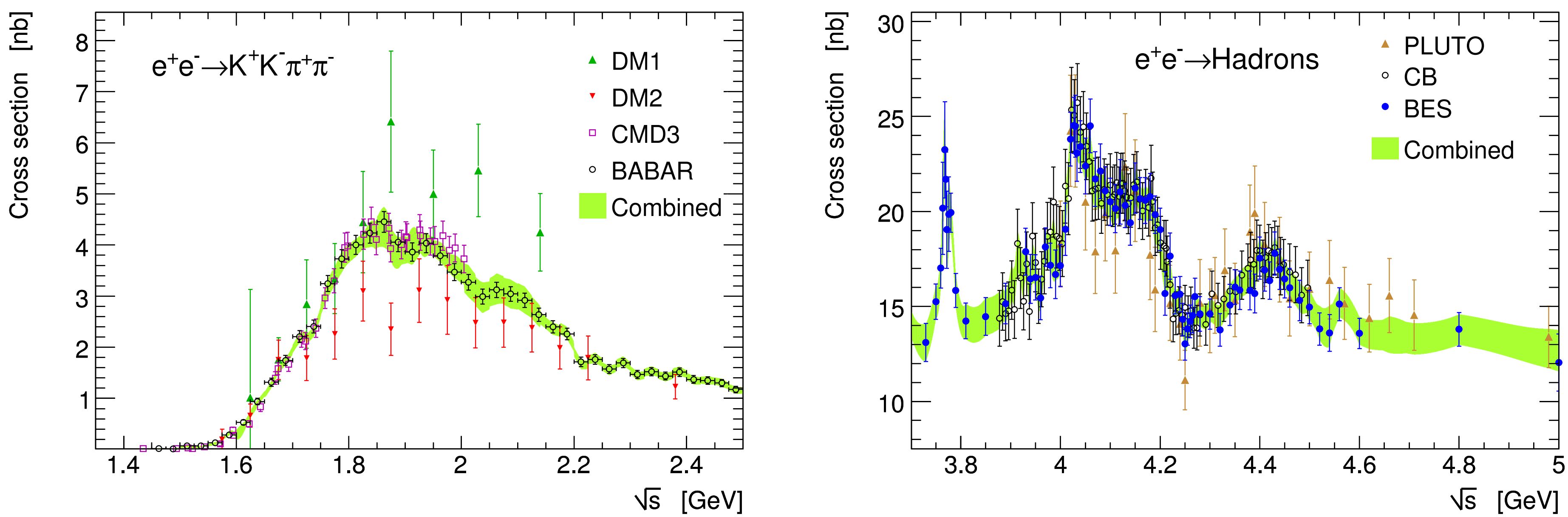
Burger, Jansen, Petschlies, Pientka '15

Shift of finestructure constant

- $\Delta\alpha \equiv 1 - \frac{\alpha(0)}{\alpha(M_Z)} \approx 0.059 = 0.0315_{\text{lept}} + 0.0276_{\text{had}}$
- Hadronic effects from $e^+e^- \to$ had. data
- Last 5 years: new data from BaBar, VEPP, BES
- No significant improvement from including τ data
- Robust precision $\sim 10^{-4}$



Davier et al. '17; Jegerlehner '17
Keshavarzi, Nomura, Teubner '18



Davier et al. '17

Observables and pseudo-observables

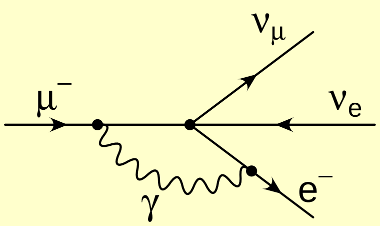
- **Observables** (cross-sections, decay rates) receive large corr. from QED/QCD
  - $\longrightarrow$ well understood, no room for new physics
  - $\longrightarrow$ play a role in exp. acceptance, particle ID

- **Pseudo-observables** are defined such that external QED/QCD is removed
  - $\longrightarrow$ convenient for comparison with new physics models

G_F from μ decay in Fermi Model

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \Delta q)$$

QED corrections (2-loop)



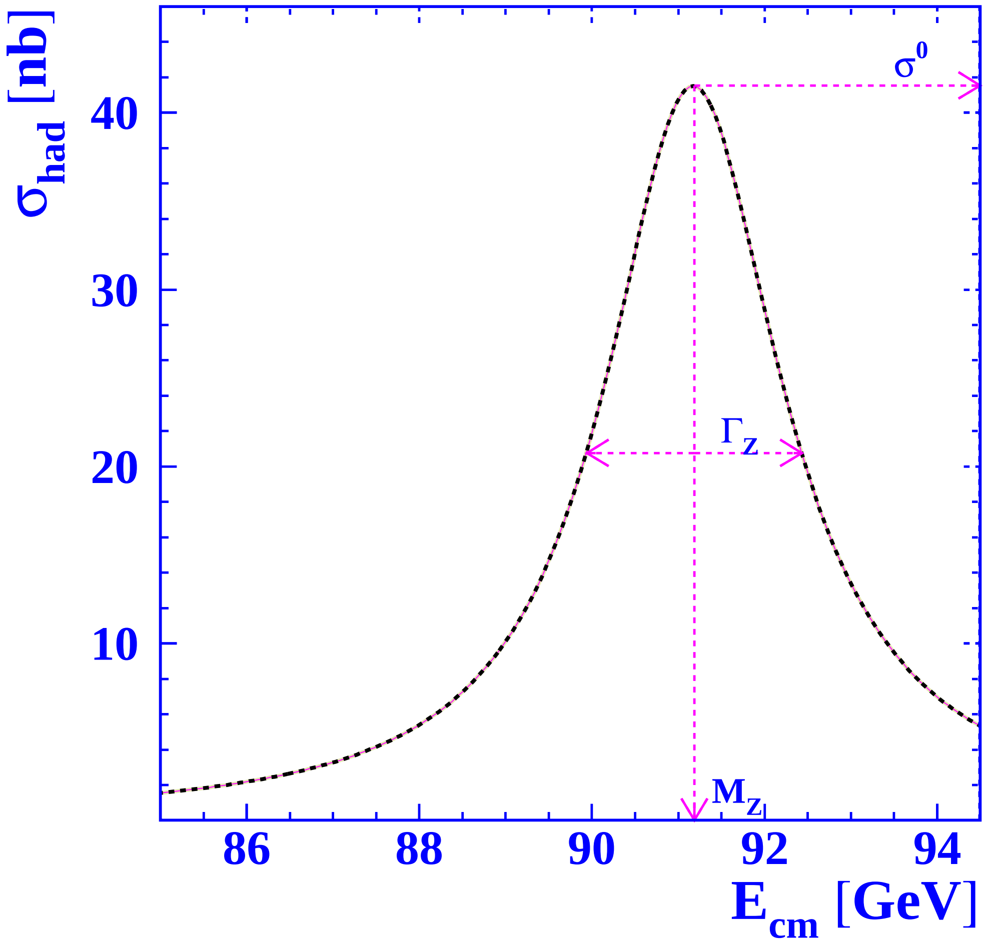
Ritbergen, Stuart '98
Pak, Czarnecki '08

G_F decay in Standard Model

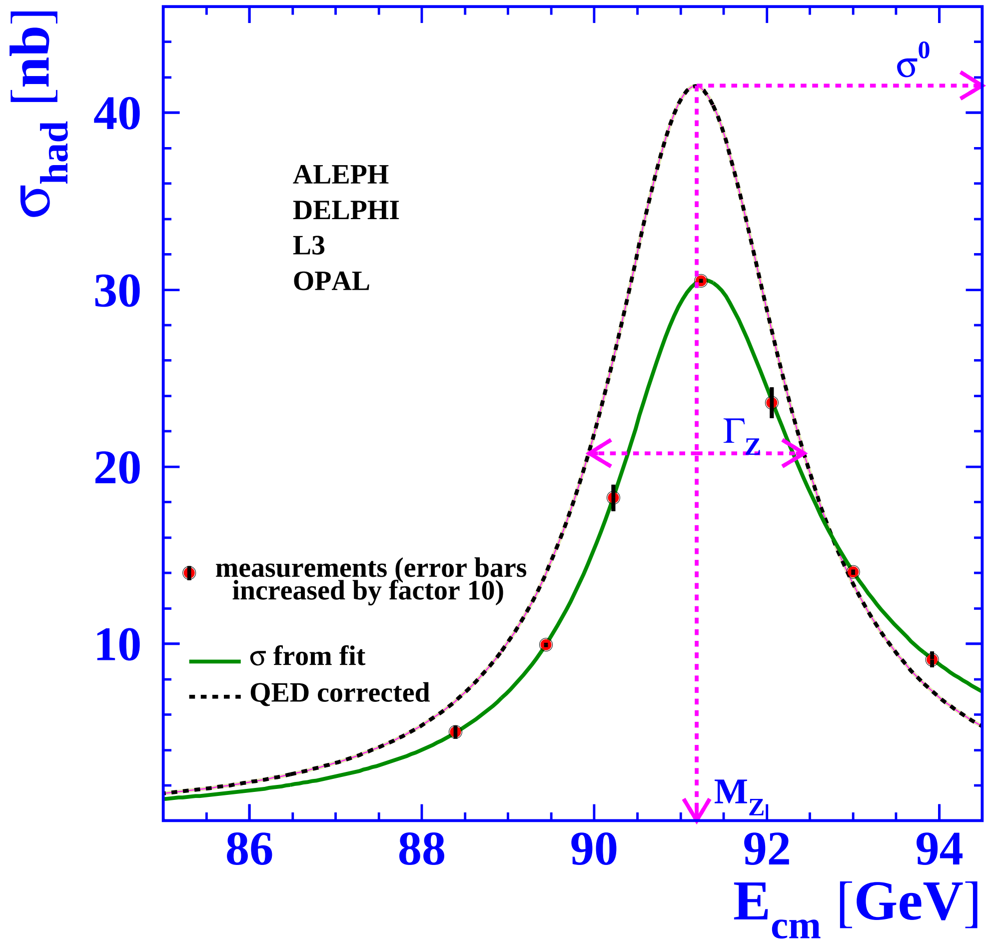
$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}(1 + \Delta r)$$

electroweak corrections

Z lineshape



Z lineshape



LEP EWWG '05

→ Large effects from initial-state QED radiation

→ Theory input necessary to extract relevant EWPOs ("pseudo-observables")

Z lineshape

- Deconvolution of initial-state QED radiation:

$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s, s') \otimes \sigma_{\text{hard}}(s')$$

Kureav, Fadin '85
Berends, Burgers, v. Neerven '88
Kniehl, Krawczyk, Kühn, Stuart '88
Beenakker, Berends, v. Neerven '89
Skrzypek '92
Montagna, Nicrosini, Piccinini '97

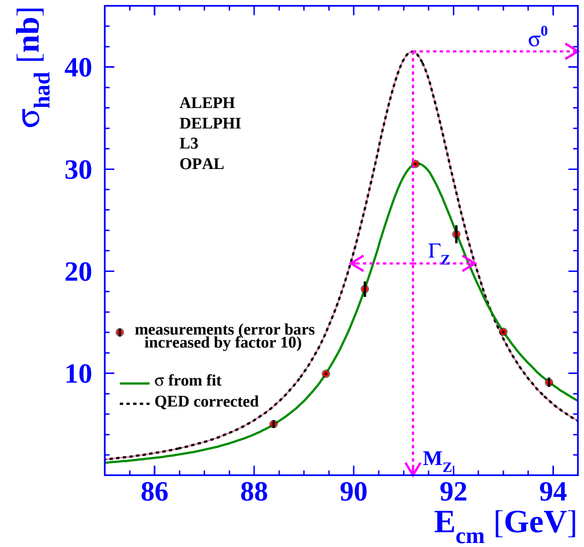
Soft photons (resummed) + collinear photons

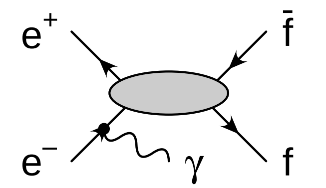
$$\mathcal{R}_{\text{ini}} = \frac{\zeta(1 - s'/s)^{\zeta-1}}{\Gamma(1-\zeta)} e^{-\gamma_E \zeta + 3\alpha L/2\pi} - \frac{\alpha}{\pi} L \left(1 + \frac{s'}{s}\right) + \alpha^2 L^2 \ldots + \alpha^3 L^3 \ldots$$

$$\zeta = \frac{2\alpha}{\pi}(L-1)$$

$$L = \log \frac{s}{m_e^2}$$

LEP EWWG '05





Z lineshape

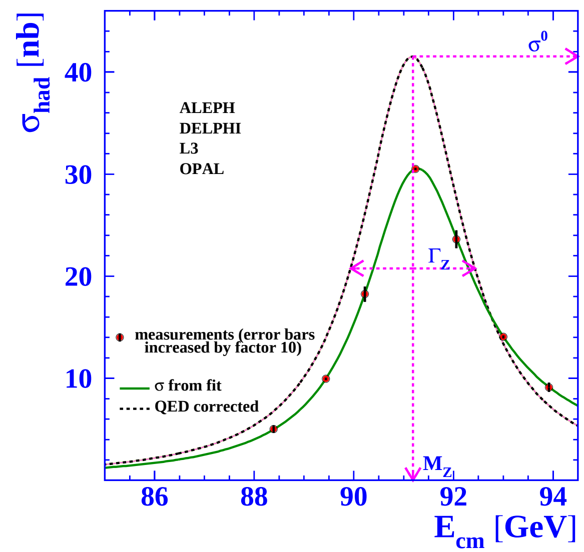
- Deconvolution of initial-state QED radiation:

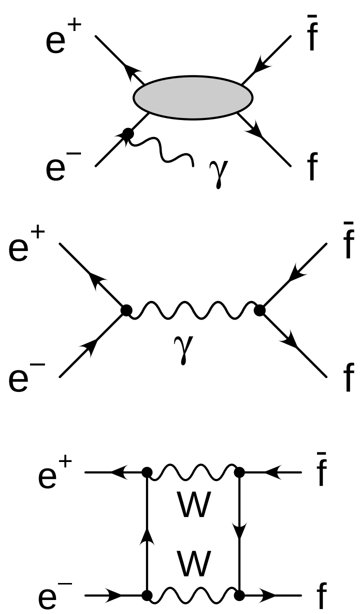
$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

- Subtraction of γ-exchange, γ–Z interference, box contributions:

$$\sigma_{\text{hard}} = \sigma_Z + \sigma_\gamma + \sigma_{\gamma Z} + \sigma_{\text{box}}$$

LEP EWWG '05





Z lineshape

- Deconvolution of initial-state QED radiation:

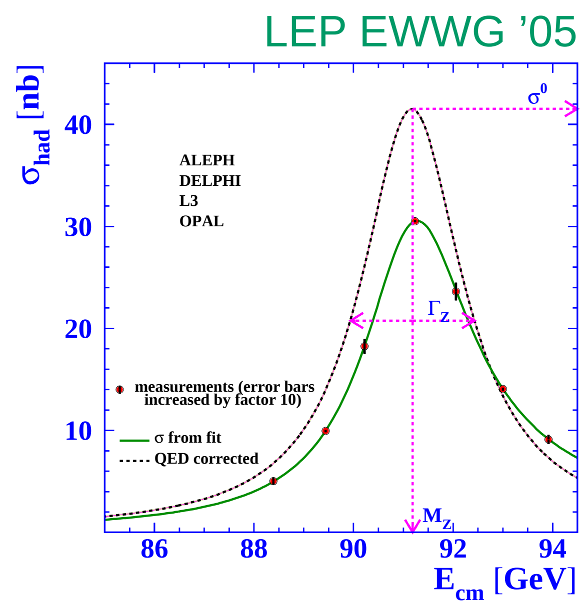
$$\sigma[e^+e^- \to f\bar{f}] = \mathcal{R}_{\text{ini}}(s,s') \otimes \sigma_{\text{hard}}(s')$$

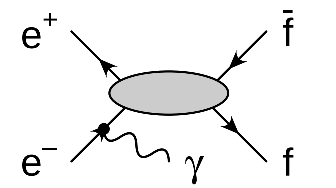
- Subtraction of γ-exchange, γ–Z interference, box contributions:

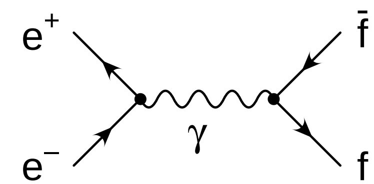
$$\sigma_{\text{hard}} = \sigma_{\text{Z}} + \sigma_{\gamma} + \sigma_{\gamma\text{Z}} + \sigma_{\text{box}}$$

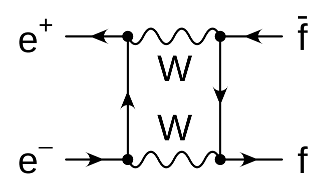
- Z-pole contribution:

$$\sigma_{\text{Z}} = \frac{R}{(s - \overline{M}_{\text{Z}}^2)^2 + \overline{M}_{\text{Z}}^2 \overline{\Gamma}_{\text{Z}}^2} + \sigma_{\text{non-res}}$$









Z lineshape

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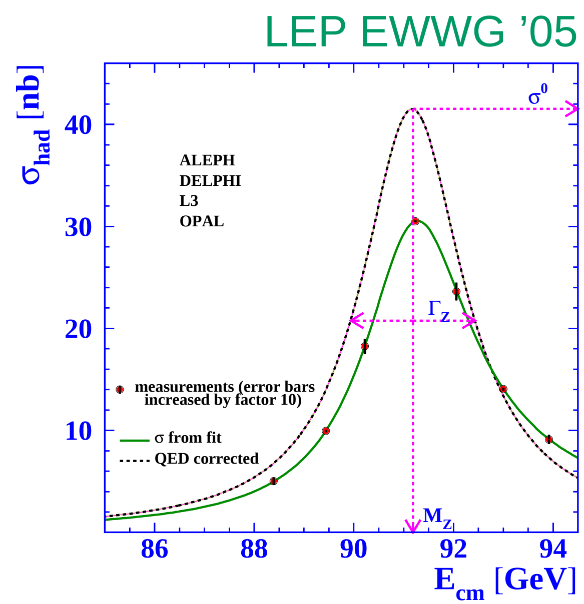
$$\sigma_{\mathrm{Z}} = \frac{R}{(s - \overline{M}_{\mathrm{Z}}^2)^2 + \overline{M}_{\mathrm{Z}}^2 \overline{\Gamma}_{\mathrm{Z}}^2} + \sigma_{\mathrm{non-res}}$$

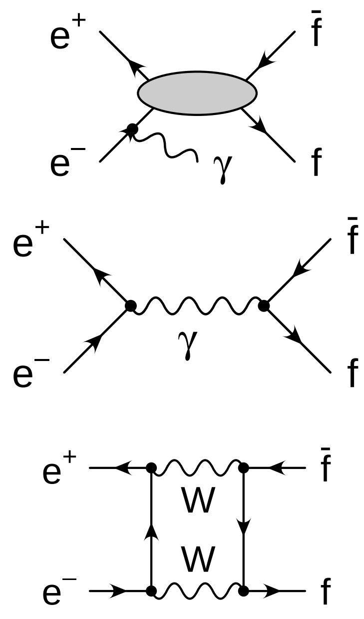
- In experimental analyses:

$$\sigma \sim \frac{1}{(s - M_{\mathrm{Z}}^2)^2 + s^2 \Gamma_{\mathrm{Z}}^2 / M_{\mathrm{Z}}^2}$$

$$\overline{M}_{\mathrm{Z}} = M_{\mathrm{Z}} \Big/ \sqrt{1 + \Gamma_{\mathrm{Z}}^2 / M_{\mathrm{Z}}^2} \approx M_{\mathrm{Z}} - 34\ \mathrm{MeV}$$

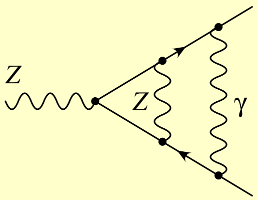
$$\overline{\Gamma}_{\mathrm{Z}} = \Gamma_{\mathrm{Z}} \Big/ \sqrt{1 + \Gamma_{\mathrm{Z}}^2 / M_{\mathrm{Z}}^2} \approx \Gamma_{\mathrm{Z}} - 0.9\ \mathrm{MeV}$$





Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_\mathrm{Z}}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1 + \mathrm{Re}\,\Sigma'_\mathrm{Z}}\right]_{s=\overline{M}_\mathrm{Z}^2}$$



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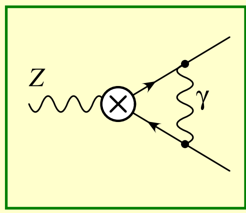
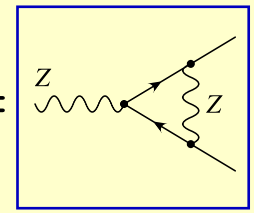


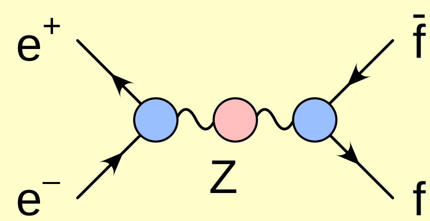
$+$ finite, with $\otimes =$


$\mathcal{R}_V^f$, $\mathcal{R}_A^f$: Final-state QED/QCD radiation;
known to $\mathcal{O}(\alpha_\mathrm{s}^4)$, $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_\mathrm{s})$ Kataev '92

Chetyrkin, Kühn, Kwiatkowski '96

Baikov, Chetyrkin, Kühn, Rittinger '12

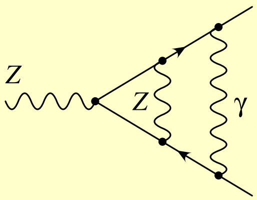
g_V^f, g_A^f, Σ'_Z: Electroweak corrections



Z decay

Factorization of massive and QED/QCD FSR:

$$\overline{\Gamma}_f \approx \frac{N_c \overline{M}_Z}{12\pi}\left[\left(\mathcal{R}_V^f |g_V^f|^2 + \mathcal{R}_A^f |g_A^f|^2\right)\frac{1}{1+\mathrm{Re}\,\Sigma_Z'}\right]_{s=\overline{M}_Z^2}$$



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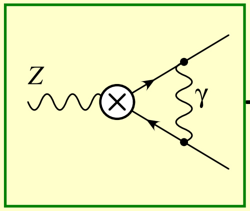
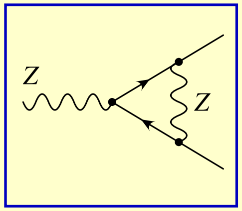
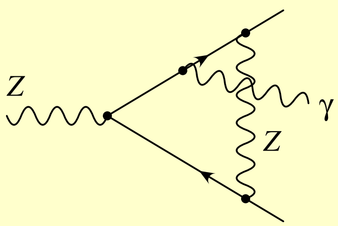


$+$ finite, with $\otimes =$


Additional non-factorizable contributions, e.g.



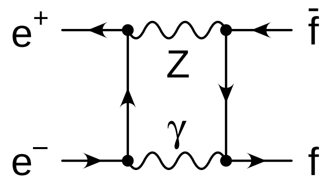
$\longrightarrow$ Known at $\mathcal{O}(\alpha\alpha_s)$ Czarnecki, Kühn '96
Harlander, Seidensticker, Steinhauser '98

$\longrightarrow$ Currently not known at $\mathcal{O}(\alpha^2)$ and beyond

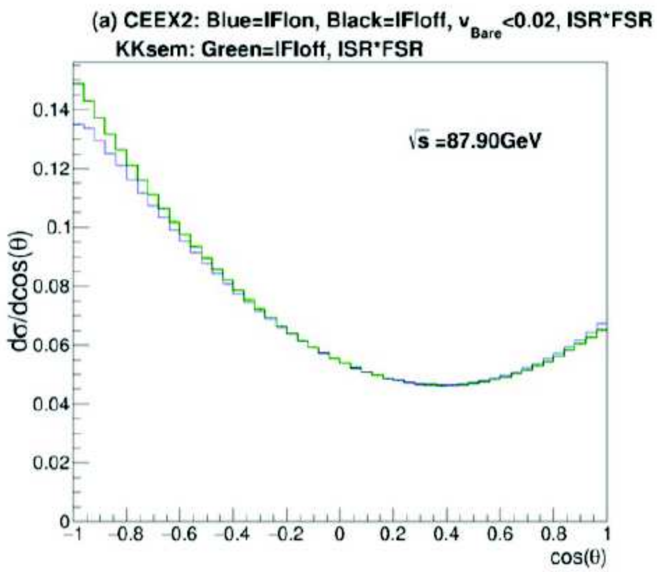
$\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty on Γ_Z, σ_Z, maybe larger for A_b

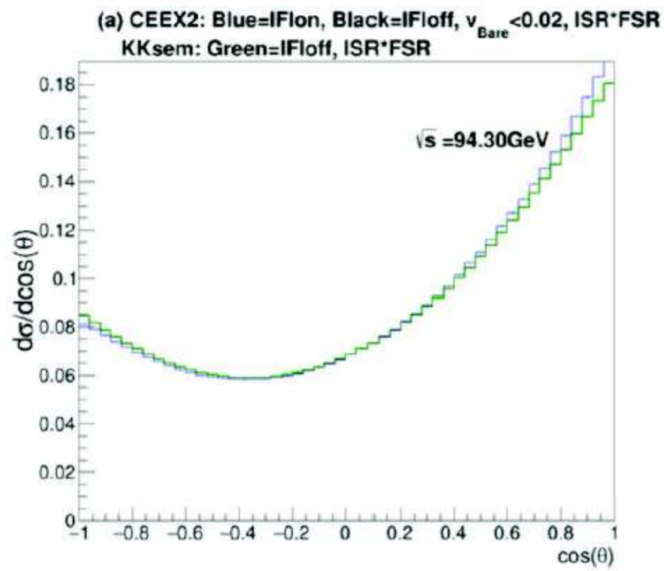
Z-boson initial-final inference

- Interference between ISR and FSR suppressed by Γ_Z/M_Z on Z resonance



- Still relevant for high precision an off-resonance





Jadach, Yost '18

- Factorization from hard matrix element requires 4-variable convolution
- Soft-photon resummation can be included

Jadach, Yost '18

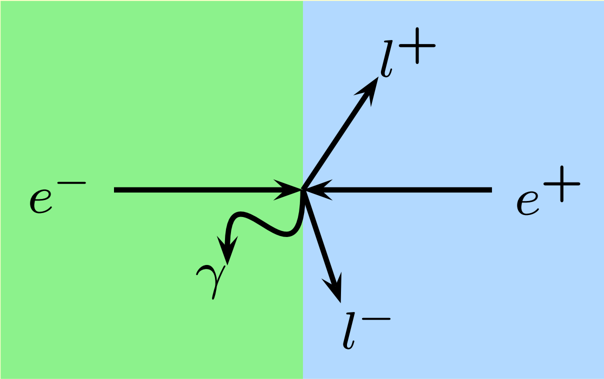
Greco, Pancheri-Srivastava, Srivastava '75

QED radiation in Z asymmetries

QED radiation in principle cancels in asymmetries, e.g. $A_{\mathrm{FB}} = \frac{\sigma_{\mathrm{F}} - \sigma_{\mathrm{B}}}{\sigma_{\mathrm{F}} + \sigma_{\mathrm{B}}}$

Some effects from detector acceptance and cuts

Typical influence $< 10^{-3}$



Implementation of QED effects:

a) Analytical formulae, e.g. `ZFITTER`

Arbuzov, Bardin, Christova, Kalinovskaya, Riemann, Riemann, ...

$\longrightarrow$ exact $\mathcal{O}(\alpha^2)$ ISR/FSR corrections

b) Monte Carlo event generator, e.g. `KORALZ, KKMC` Jadach, Ward, ...

$\longrightarrow$ only $\mathcal{O}(\alpha^2 L)$ accuracy, but more flexible

$L = \log(s/m_{\mathrm{e}}^2)$

“Hard” matrix element

Consistent (gauge-invariant) theory setup:

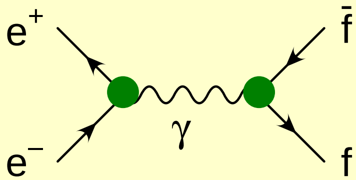
Expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

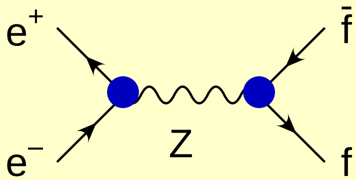
$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \ldots$$

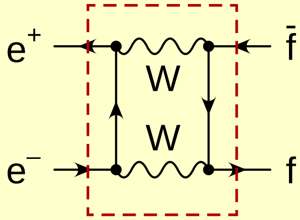
$$R = g_Z^e(s_0)\, g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings







At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Multi-loop calculations

	M_W [GeV]	$\sin\theta^{\text{lept}}_{W,\text{eff}}$
exp.	± 0.012	$\pm$ 16
1-loop	± 0.450	± 1000
2-/3-loop QCD	± 0.070	$\pm$ 45
ferm. 2-loop EW	± 0.050	$\pm$ 90
bos. 2-loop EW	± 0.002	$\pm$ 1
leading 3-loop	± 0.005	$\pm$ 25

Experimental precision sensitive to 2-/3-loop effects

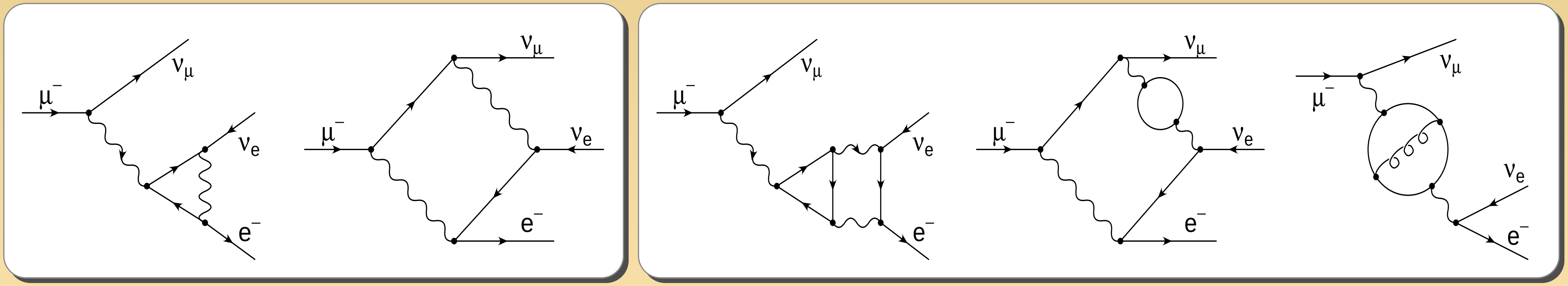
Marciano, Sirlin '80

Djouadi et al. '88
Chetyrkin, Kühn, Steinhauser '95

Freitas et al. '00 Awramik, Czakon '03
Awramik, Czakon, Freitas, Weiglein '04

Awramik, Czakon, Freitas '06

Faisst, Kühn, Seidensticker, Veretin '03



Theory calculations: Status

- Many seminal works on 1-loop and leading 2-loop corrections

  Veltman, Passarino, Sirlin, Marciano, Bardin, Hollik, Riemann, Degrassi, Kniehl, ...

- Full 2-loop results for M_{W}, Z-pole observables

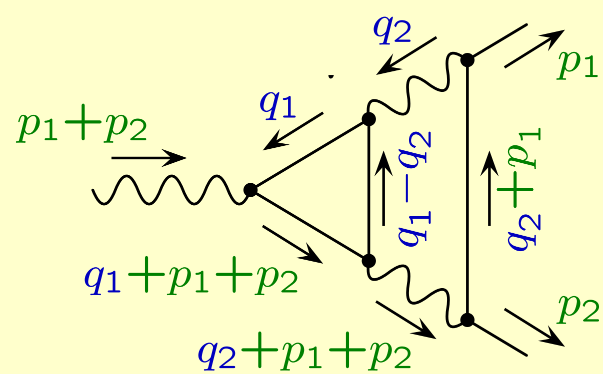
  Freitas, Hollik, Walter, Weiglein '00
  Awramik, Czakon '02
  Onishchenko, Veretin '02
  Awramik, Czakon, Freitas, Weiglein '04
  Awramik, Czakon, Freitas '06
  Hollik, Meier, Uccirati '05,07
  Awramik, Czakon, Freitas, Kniehl '08
  Freitas '13,14
  Dubovyk,Freitas,Gluza,Riemann,Usovitsch '16,18

- Approximate 3- and 4-loop results (to ρ parameter)

  Chetyrkin, Kühn, Steinhauser '95
  Faisst, Kühn, Seidensticker, Veretin '03
  Boughezal, Tausk, v. d. Bij '05
  Schröder, Steinhauser '05
  Chetyrkin et al. '06
  Boughezal, Czakon '06

Theory calculations: Challenges

Experimental precision requires inclusion of **radiative corrections** in theory (1-loop, 2-loop, and partial 3-loop)



Integrals over loop momenta:

$$\int d^4q_1 d^4q_2\ f(q_1,\, q_2,\, p_1,\, p_2,\, \ldots,\, m_1,\, m_2,\, \ldots)$$

Computer algebra tools:

- Generation of diagrams, $\mathcal{O}(100) - \mathcal{O}(10000)$
- Lorentz and Dirac algebra
- Integral simplification (and expansion)

Evaluation of loop integrals:

- In general not possible analytically
- Numerical methods must be automizable, stable, fastly converging
- Need procedure for isolating divergent pieces

Analytic calculations

- Useful for diagrams with up to two scales
  ($e.\,g.$ M_W & m_t or M_W & M_Z)
- Reduce to **master integrals** with integration-by-parts and other identities
  Chetyrkin, Tkachov '81; Gehrmann, Remiddi '00; Laporta '00; ...

  Public programs:

Public programs:	`Reduze`	von Manteuffel, Studerus '12
	`FIRE`	Smirnov '13,14
	`LiteRed`	Lee '13
	`KIRA`	Maierhoefer, Usovitsch, Uwer '17

- Evaluate master integrals with differential equations or Mellin-Barnes rep.
  Kotikov '91; Remiddi '97; Smirnov '00,01; Henn '13; ...

  $\longrightarrow$ Result in terms of Goncharov polylogs / multiple polylogs

  $\longrightarrow$ Some problems need iterated elliptic integrals / elliptic multiple polylogs
  Broedel, Duhr, Dulat, Trancredi '17,18
  Ablinger er al. '17

  $\longrightarrow$ Even more classes of functions needed in future?

Asymptotic expansions

- Exploit large mass ratios,
  e. g. $M_Z^2/m_t^2 \approx 1/4$
- Evaluate coeff. integrals analytically
- Fast numerical evaluation

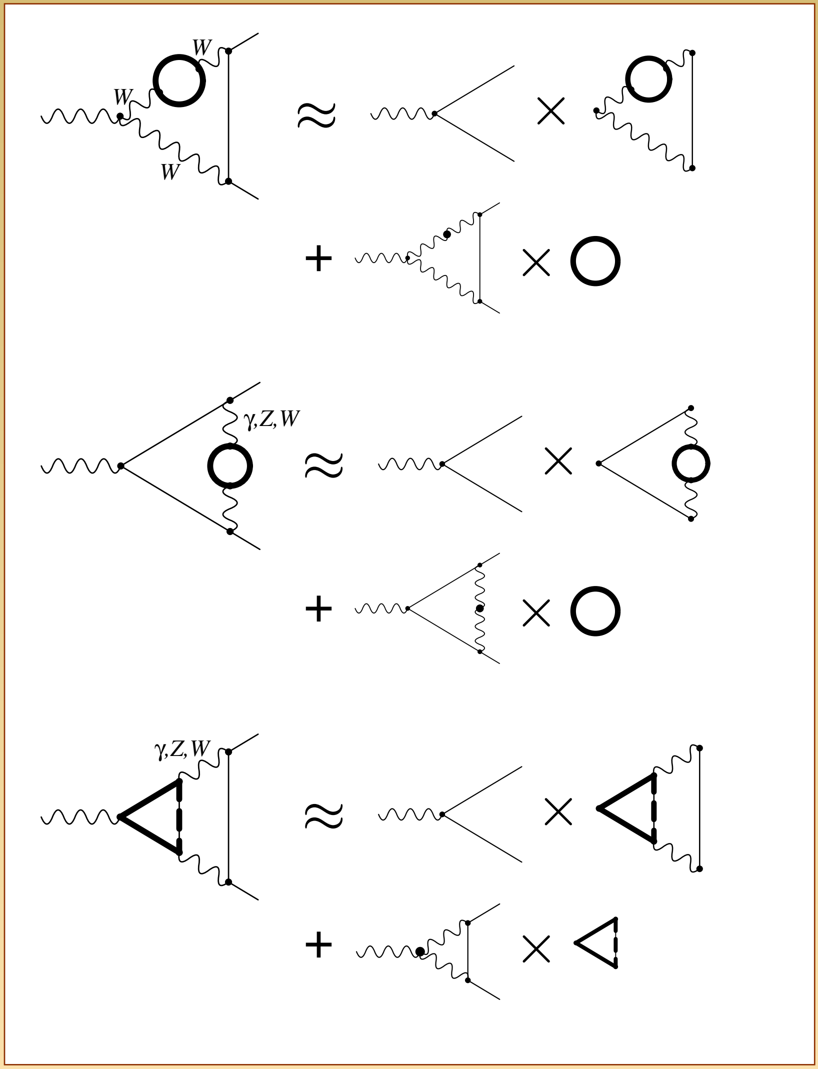
⟶ Used in some 2/3-scale problems

⟶ Public programs:

`exp` Harlander, Seidensticker, Steinhauser '97
`asy` Pak, Smirnov '10

⟶ Possible limitations:

- Difficult coefficient integrals
- bad convergence



Direct numerical integration

- Can be applied to any number of scales and loop order
- Need to extract UV and IR divergencies
- Diagrams with internal thresholds can cause numerical instabilities

Two general (automizable) approches:

- **Sector decomposition:** Binoth, Heinrich '00,03

  Advantageous for diagrams with many massive propagators

  Public programs: `SecDec` Carter, Heinrich '10; Borowka et al. '12,15,17
  `FIESTA` Smirnov, Tentyukov '08; Smirnov '13,15

- **Mellin-Barnes representations:** Smirnov '99; Tausk '99

  ... with fewer independent parameters Czakon '06 ; Anastasiou, Daleo '06

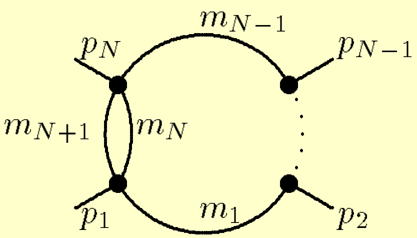
  Public programs: `MB/MBresolve` Czakon '06; Smirnov, Smirnov '09
  `AMBRE/MBnumerics` Gluza, Kajda, Riemann '07
  Dubovyk, Gluza, Riemann '15
  Usovitsch, Dubovyk, Riemann '18

Semi-numerical techniques

Numerical integrals of low dimensionality for special integral classes

e.g. Ghinculov '99; Bauberger et al. '95; Awramik, Czakon, Freitas '06; Freitas '16, ...

Example: Sub-loop dispersion relations



$$T_{N+1}(p_i; m_i^2) = -\int_{s_0}^{\infty} \mathrm{d}s \; \Delta B_0(s, m_N^2, m_{N+1}^2)$$

$$\times \int \mathrm{d}^4 q \, \frac{1}{q^2 - s} \, \frac{1}{(q+p_1)^2 - m_1^2} \cdots \frac{1}{(q+p_1+\cdots+p_{N-1})^2 - m_{N-1}^2}$$

- High-precision results (8+ digits) in sec/min
- Typically works for limited class of diagrams
- No algorithmic procedure for extracting UV/IR divergencies

Parametrization formulae

Problem for dissemination of numerical result(s):

Large # of numerical integrals, slow evaluation

Solution: Fit formula with sufficient # of (motivated) terms to data grid spanning at least 2σ of each input parameter

$$\sin^2\theta^{\mathrm{b}}_{\mathrm{eff}} = \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right)(1 + \Delta\kappa_{\mathrm{b}}),$$

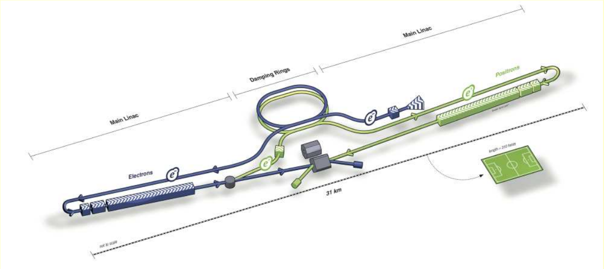
$$\Delta\kappa_{\mathrm{b}}^{(\alpha^2,\mathrm{bos})} = k_0 + k_1 c_{\mathrm{H}} + k_2 c_{\mathrm{t}} + k_3 c_{\mathrm{t}}^2 + k_4 c_{\mathrm{H}} c_{\mathrm{t}} + k_5 c_{\mathrm{W}},$$

$$c_{\mathrm{H}} = \log\left(\frac{M_{\mathrm{H}}}{M_{\mathrm{Z}}} \times \frac{91.1876\ \mathrm{GeV}}{125.1\ \mathrm{GeV}}\right), \quad c_{\mathrm{t}} = \left(\frac{m_{\mathrm{t}}}{M_{\mathrm{Z}}} \times \frac{91.1876\ \mathrm{GeV}}{173.2\ \mathrm{GeV}}\right)^2 - 1,$$

$$c_{\mathrm{W}} = \left(\frac{M_{\mathrm{W}}}{M_{\mathrm{Z}}} \times \frac{91.1876\ \mathrm{GeV}}{80.385\ \mathrm{GeV}}\right)^2 - 1.$$

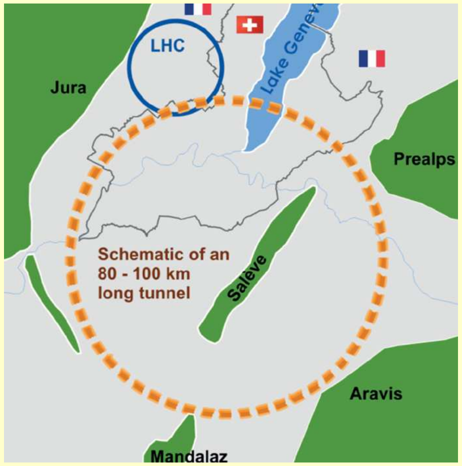
$$k_0 = -0.98605 \times 10^{-4},\ k_1 = 0.3342 \times 10^{-4},\quad k_2 = 1.3882 \times 10^{-4},$$
$$k_3 = -1.7497 \times 10^{-4},\quad k_4 = -0.4934 \times 10^{-4},\ k_5 = -9.930 \times 10^{-4}.$$

- International Linear Collider (ILC)
  Int. lumi at $\sqrt{s} \sim M_Z$: 100 $\times$ LEP



- Circular Electron-Positron Collider (CEPC)
  Int. lumi at $\sqrt{s} \sim M_Z$: 1,000-10,000 $\times$ LEP

- Future Circular Collider (FCC-ee)
  Int. lumi at $\sqrt{s} \sim M_Z$: $> 10^5 \times$ LEP



Electroweak precision tests at future colliders

	Current exp.	ILC/GigaZ	CEPC	FCC-ee
M_{W} [MeV]	15	3–4	1	0.5–1
M_{Z} [MeV]	2.1	–	0.5	0.1
Γ_Z [MeV]	2.3	0.8	0.5	0.1
$R_\ell = \Gamma_{\mathrm{Z}}^{\mathrm{had}}/\Gamma_{\mathrm{Z}}^{\ell}$ $[10^{-3}]$	25	10	2	1
$R_b = \Gamma_{\mathrm{Z}}^{b}/\Gamma_{\mathrm{Z}}^{\mathrm{had}}$ $[10^{-5}]$	66	14	4.3	6
$\sin^2\theta^\ell_{\mathrm{eff}}$ $[10^{-5}]$	16	1.3	$<$1	0.5

$\longrightarrow$ Existing theoretical calculations adequate for LEP/SLC/LHC,
but not ILC/CEPC/FCC-ee!

Theory calculations: Uncertainties

	Experiment	Theory error	Main source
M_W	80.379 ± 0.012 MeV	4 MeV	α^3, $\alpha^2\alpha_\text{s}$
Γ_Z	2495.2 ± 2.3 MeV	0.4 MeV	α^3, $\alpha^2\alpha_\text{s}$, $\alpha\alpha_\text{s}^2$
R_ℓ	20.767 ± 0.025	0.005	α^3, $\alpha^2\alpha_\text{s}$
R_b	0.21629 ± 0.00066	0.0001	α^3, $\alpha^2\alpha_\text{s}$
$\sin^2\theta^\ell_\text{eff}$	0.23153 ± 0.00016	4.5×10^{-5}	α^3, $\alpha^2\alpha_\text{s}$

- Theory error estimate is not well defined, ideally $\Delta_\text{th} \ll \Delta_\text{exp}$
- Common methods:
  - Count prefactors (α, N_c, N_f, ...)
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence

Future theory projections

	FCC-ee	CEPC	perturb. error with 3-loop†	Param. error FCC-ee*	Param. error CEPC**
M_{W} [MeV]	0.5–1	3	1	1	2.1
Γ_Z [MeV]	0.1	0.5	$\lesssim 0.2$	0.06	0.15
R_b [10^{-5}]	6	17	5–10	< 1	< 1
$\sin^2\theta^{\ell}_{\text{eff}}$ [10^{-5}]	0.5	2.3	1.5	2	2

† **Theory scenario:** $\mathcal{O}(\alpha\alpha_{\text{s}}^2)$, $\mathcal{O}(N_f\alpha^2\alpha_{\text{s}})$, $\mathcal{O}(N_f^2\alpha^2\alpha_{\text{s}})$

(N_f^n = at least n closed fermion loops)

Parametric inputs:

***FCC-ee:** $\delta m_t = 50$ MeV, $\delta\alpha_{\text{s}} = 0.0001$, $\delta M_{\text{Z}} = 0.1$ MeV

****CEPC:** $\delta m_t = 600$ MeV, $\delta\alpha_{\text{s}} = 0.0002$, $\delta M_{\text{Z}} = 0.5$ MeV

also: $\delta(\Delta\alpha) = 5 \times 10^{-5}$

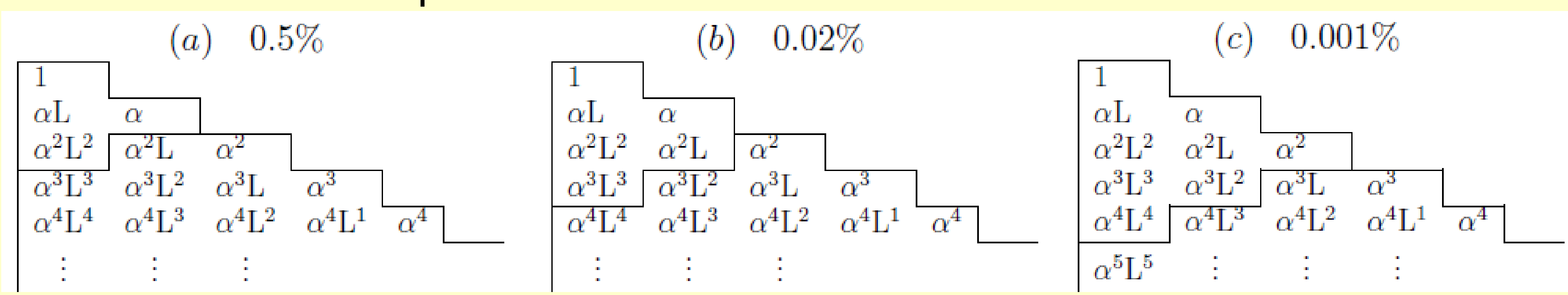
QED effects

- Important QED effects from ISR / FSR / interference, enhanced by $L = \log \frac{s}{m_e^2}$

Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$
M_Z [MeV]	Z linesh. [28]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [28]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$ [33]	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12
σ^0_{had}[nb]	σ^0_{had} [28]	$41.541 \pm 0.037\{0.25\}$	$0.1 \cdot 10^{-3}$	$4 \cdot 10^{-3}$	6
N_ν	$\sigma(M_Z)$ [28]	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6
$\sin^2\theta_W^{eff} \times 10^5$	$A_{FB}^{lept.}$ [33]	$23099 \pm 53\{28\}$	0.3	0.5	55
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [28]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100

Jadach, Skrzypek '19

- One to two orders improvement needed:



- Implementation in MC program for experimental efficiency and particle ID

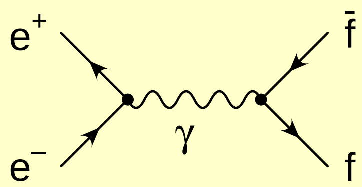
Background subtraction

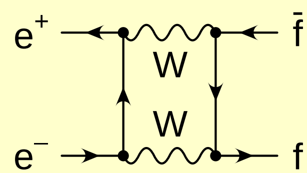
- Subtaction of non-resonant γ-exchange, γ–Z interf., box contributions, Bhabha scattering

  see, e.g., Bardin, Grünewald, Passarino '99

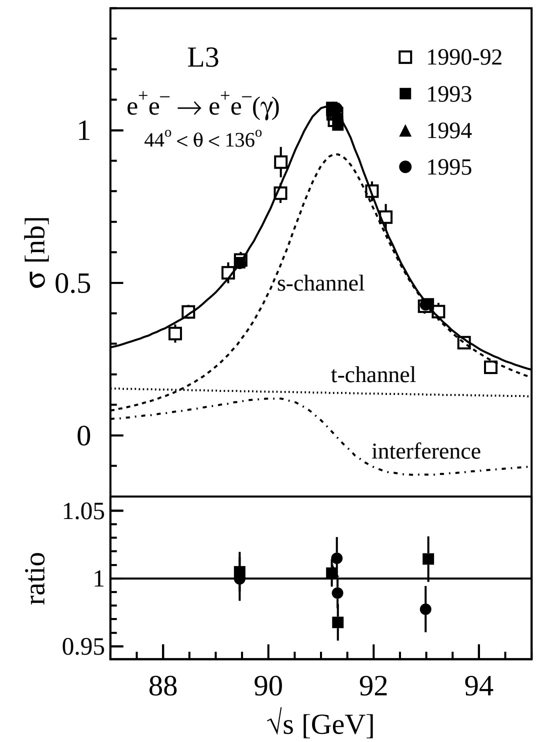
  $\longrightarrow$ $\mathcal{O}(0.01\%)$ uncertainty within SM (improvements may be needed)

  $\longrightarrow$ Sensitivity to some NP beyond EWPO





LEP EWWG '05



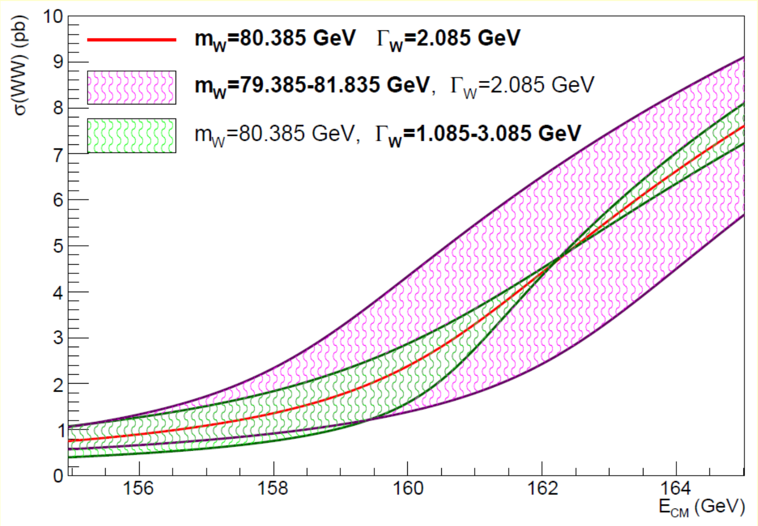
WW threshold

- High-precision measurement of M_W from $e^+e^- \to W^+W^-$ at threshold

- a) Corrections near threshold enhanced by $1/\beta$ and $\ln\beta$

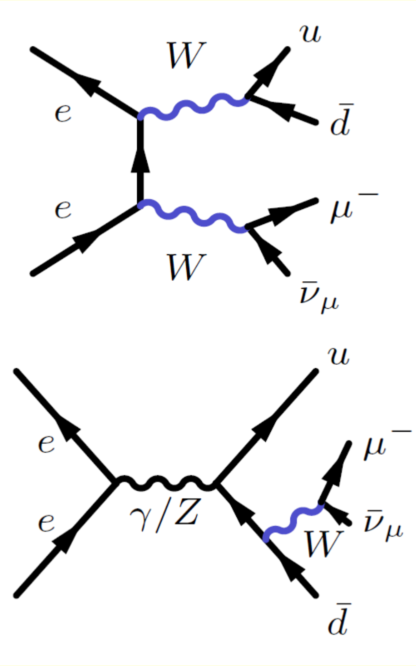
  $$\beta \sim \sqrt{1 - 4\frac{M_W^2 - iM_W\Gamma_W}{s}} \sim \sqrt{\Gamma_W/M_W}$$

  b) Non-resonant contributions are important



- Full $\mathcal{O}(\alpha)$ calculation of $e^+e^- \to 4f$
  Denner, Dittmaier, Roth, Wieders ’05

- EFT expansion in $\alpha \sim \Gamma_W/M_W \sim \beta^2$
  Beneke, Falgari, Schwinn, Signer, Zanderighi ’07
  - NLO corrections with NNLO Coulomb correction ($\propto 1/\beta^n$): $\delta_{th}M_W \sim 3$ MeV
    Actis, Beneke, Falgari, Schwinn ’08
  - Adding NNLO corrections to $ee \to WW$ and $W \to f\bar{f}$ and NNLO ISR: $\delta_{th}M_W \lesssim 0.6$ MeV



Future colliders: Summary

- Planned e^+e^- colliders increase sensitivity for EWPOs by factor 10–30

- Needed improvements of SM theory calculations by 1–2 orders
  $\longrightarrow$ Analytical calculations for QED matrix elements
  $\longrightarrow$ Numerical loop integration techniques for electroweak corrections

- Development of new MC tools and SM form factor libraries

See Freitas, Heinemeyer et al., arXiv:1906.05379

Summary

- **Electroweak precision tests** have played an important role in testing the Standard Model
  $\longrightarrow$ Constaints on m_t and M_H before their discovery

- Today they probe physics beyond the Standard Model at **TeV scale**

- **Electroweak fits** rely on detailed theory calculations for QED effects and backgrounds, SM predictions, etc.

- **FCC-ee/CEPC/GigaZ** with $\sqrt{s} \sim M_\text{Z}$ will reduce exp. error by $\mathcal{O}(10)$
  $\longrightarrow$ 3-loop (and maybe some 4-loop) corrections needed!

- **Numerical techniques** are promising but need to be improved substantially

Backup slides

Other electroweak precision parameters

- M_Z, Γ_Z: From $\sigma(\sqrt{s})$ lineshape
  $\longrightarrow$ Main uncertainties: B-field calibration, QED
  $\longrightarrow$ δM_Z, $\delta\Gamma_Z \sim 0.1$ MeV could be achievable

- m_t: Current status $\delta m_t \sim 0.4$ GeV at LHC — PDG '18
  $\longrightarrow$ Additional theory uncertainties? — Butenschoen et al. '16; Ferrario Ravasio, Nason, Oleari '18

From $e^+e^- \to t\bar{t}$ at $\sqrt{s} \sim 350$ GeV

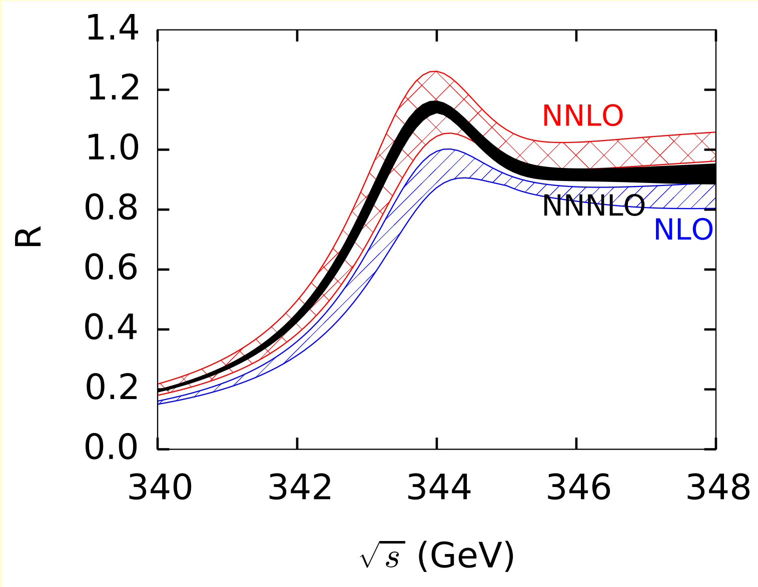
$\oplus\ [< 40\text{ MeV}]_{\text{exp. sys.}}$

$\oplus\ [\ll 50\text{ MeV}]_{\text{QCD}}$

$\oplus\ [10\text{ MeV}]_{\text{mass def.}}$

$\oplus\ [10\ldots20\text{ MeV}]_{\alpha_s}$

~ 50 MeV



Beneke et al. '15

Other electroweak precision parameters

- α_s: d'Enterria, Skands, et al. '15
  - Most precise determination using Lattice QCD from Υ spectroscopy:
    $\alpha_s = 0.1184 \pm 0.0006$ HPQCD '10
    $\longrightarrow$ Difficulty in evaluating systematics
  - e^+e^- event shapes and DIS: $\alpha_s \sim 0.114$
    Alekhin, Blümlein, Moch '12; Abbate et al. '11; Gehrmann et al. '13
    $\longrightarrow$ Subject to sizeable non-pertubative power corrections
    $\longrightarrow$ Systematic uncertainties in power corrections?
  - Hadronic τ decays: $\alpha_s = 0.119 \pm 0.002$ PDG '18
    $\longrightarrow$ Non-perturbative uncertainties in OPE and from duality violation
    Pich '14; Boito et al. '15,18

Other electroweak precision parameters

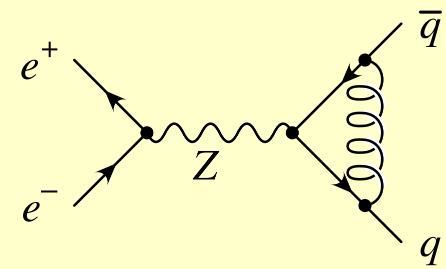
- α_s: d'Enterria, Skands, et al. '15
  - Electroweak precision ($R_\ell = \Gamma_Z^{had}/\Gamma_Z^\ell$):
    $\alpha_s = 0.120 \pm 0.003$ PDG '18

    $\longrightarrow$ No (negligible) non-perturbative QCD effects

    FCC: $\delta R_\ell \sim 0.001$

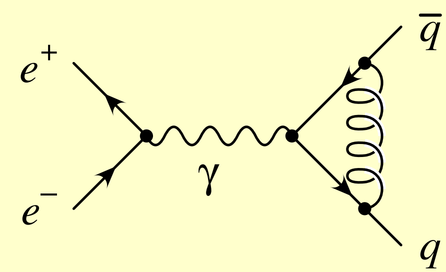
    $\Rightarrow \quad \delta\alpha_s < 0.0002$ (subj. to theory error)

    Caviat: R_ℓ could be affected by new physics



  - $R = \frac{\sigma[ee\to\text{had.}]}{\sigma[ee\to\mu\mu]}$ at lower $\sqrt{s}$

    e.g. CLEO ($\sqrt{s} \sim 9$ GeV): $\alpha_s = 0.110 \pm 0.015$
    Kühn, Steinhauser, Teubner '07



    $\longrightarrow$ dominated by s-channel photon, less room for new physics
    $\longrightarrow$ QCD still perturbative

    naive scaling to 50 ab^{-1} (BELLE-II): $\delta\alpha_s \sim 0.0001$

Shift of finestructure constant

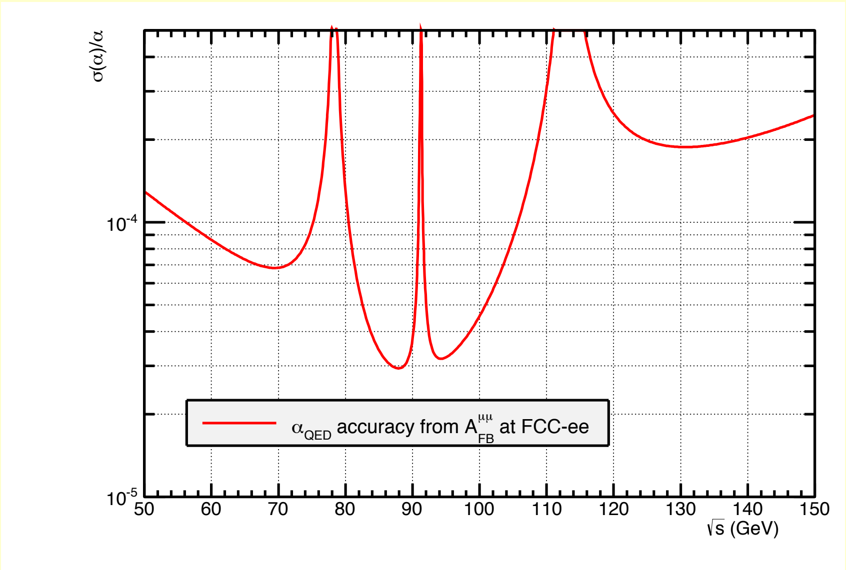
- $\Delta\alpha_{\text{had}}$: Could be limiting factor

  a) From $e^+e^- \to$ had. using dispersion relation
  Current: $\delta(\Delta\alpha_{\text{had}}) \sim 10^{-4}$
  Improvement to $\delta(\Delta\alpha_{\text{had}}) \sim 5 \times 10^{-5}$ likely

  b) Direct determination at FCC-ee from $e^+e^- \to \mu^+\mu^-$ off the Z peak
  (i.e. $A_{\text{FB}}^{\mu\mu}$ at $\sqrt{s} \sim 88$ GeV and $\sqrt{s} \sim 95$ GeV)
  $\to \delta_{\text{th}}(\Delta\alpha_{\text{had}}) \sim 3 \times 10^{-5}$

Janot '15

Requires high-precision theory prediction for $e^+e^- \to \mu^+\mu^-$ including 2/3-loop corrections for γ-exchange and box contributions



Example: Error estimation for Γ_Z

- Goemetric perturbative series

$\alpha_t = \alpha m_t^2$

$$\mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.26 \text{ MeV}$$

$$\mathcal{O}(\alpha^2\alpha_s) - \mathcal{O}(\alpha_t^2\alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.30 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s) \sim 0.23 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^3) - \mathcal{O}(\alpha_t\alpha_s^3) \sim \frac{\mathcal{O}(\alpha\alpha_s) - \mathcal{O}(\alpha_t\alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha\alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim 0.1 \text{ MeV}$$

- Parametric prefactors:

$$\mathcal{O}(\alpha_{bos}^2) \sim \Gamma_Z \alpha^2 \sim 0.1 \text{ MeV}$$

$$\mathcal{O}(\alpha\alpha_s^2) - \mathcal{O}(\alpha_t\alpha_s^2) \sim \frac{\alpha n_{lq}}{\pi}\alpha_s^2 \sim 0.29 \text{ MeV}$$

Total: $\delta\Gamma_Z \approx 0.5$ MeV

Example: Error estimation for M_{W}

- Renormalization scheme dependence:

a) Uncertainty of $\mathcal{O}(\alpha^2)$ corrections beyond leading $\alpha^2 m_t^4$ and $\alpha^2 m_t^2$ from comparison of $\overline{\text{MS}}$ and OS schemes: Degrassi, Gambino, Sirlin '96

$\delta M_{\mathrm{W}} \sim 2$ MeV (for $M_{\mathrm{H}} \sim 100$ GeV)

Actual remaining $\mathcal{O}(\alpha^2)$ corrections: Freitas, Hollik, Walter, Weiglein '00

$\delta M_{\mathrm{W}} \sim 3$ MeV (for $M_{\mathrm{H}} \sim 100$ GeV)

b) Estimate of missing $\mathcal{O}(\alpha^3)$ corrections from comparison of $\overline{\text{MS}}$ and OS results: Awramik, Czakon, Freitas, Weiglein '03
Degrassi, Gambino, Giardino '14

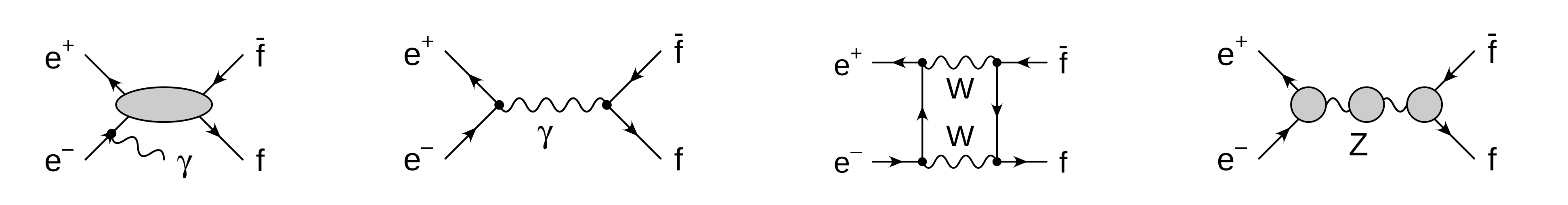
$\delta M_{\mathrm{W}} \sim 4...5$ MeV (after accounting for $\mathcal{O}(\alpha_t \alpha_s^3)$ corrections)

$\longrightarrow$ Saturates previous δM_{W} estimate!

Note: Differences in (implicitly) resummed higher-order contributions

“Analytical” tools for $e^+e^- \to f\bar{f}$

- State of the art: `Zfitter 6.42` Bardin et al. ’99, Arbuzov et al. ’05
  Older code: `TOPAZ0` Montagna, Nicrosini, Passarino, Piccinini ’98,01
- Describes true observables ($\sigma_{e^+e^-\to f\bar{f}}(s)$, etc.)
  and pseudo-observables (Γ_Z, σ^0_{had}, $\mathcal{A}_f$, etc.)
- Final-state QED and QCD corrections at $\mathcal{O}(\alpha^2)$, $\mathcal{O}(\alpha\alpha_s)$, $\mathcal{O}(\alpha_s^3)$
- Deconvolution of initial-state and initial-final QED radiation
  at $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha^2 L)$ and $\mathcal{O}(\alpha^3 L^2)$ ($L \equiv \log(s/m_e^2)$)
- Full NLO electroweak corrections for $e^+e^- \to f\bar{f}$
- Partial $\mathcal{O}(\alpha^2)$ and higher-order electroweak corrections



“Analytical” tools for $e^+e^- \to f\bar{f}$

Drawbacks:

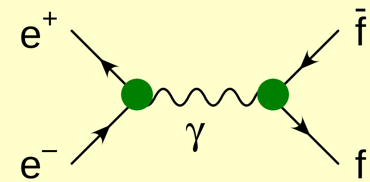
- Not all available NNLO and higher-order corrections implemented
  (code structure makes implementation difficult)
- For consistent treatment beyond NLO, need expansion of $\mathcal{A}[e^+e^- \to \mu^+\mu^-]$ about $s_0 = M_Z^2 - iM_Z\Gamma_Z$:

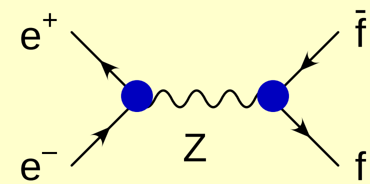
$$\mathcal{A}[e^+e^- \to f\bar{f}] = \frac{R}{s - s_0} + S + (s - s_0)T + \dots$$

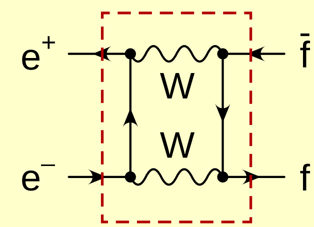
$$R = g_Z^e(s_0)\, g_Z^f(s_0)$$

$$S = \left[\frac{1}{M_Z^2} g_\gamma^e g_\gamma^f + g_Z^e g_Z^{f\,\prime} + g_Z^{e\,\prime} g_Z^f + S_{\text{box}}\right]_{s=s_0}$$

$g_V^f(s)$: effective $Vf\bar{f}$ couplings







At NNLO: Need R at $\mathcal{O}(\alpha^2)$, S at $\mathcal{O}(\alpha)$, etc.

Monte-Carlo tools for $e^+e^- \to f\bar{f}$

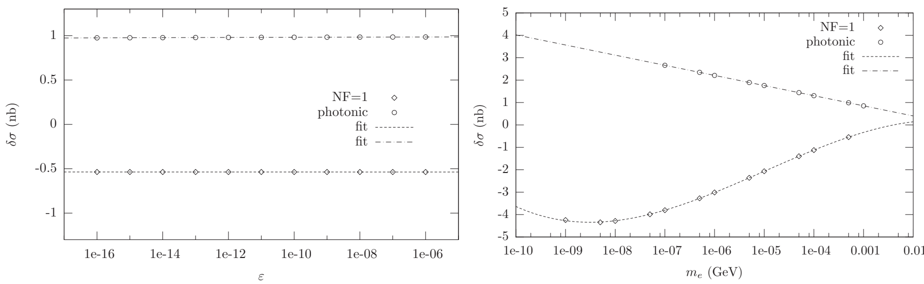
- State of the art: `KKMC, BabaYaga` — Jadach, Ward, Wąs ’13; Carloni Calame et al. ’12
- YFS exponentiation for QED radiation, approximate NNLO QED
- currently $\mathcal{O}(0.1\%)$ precision, $\mathcal{O}(0.01\%)$ feasible in (near) future, but more may be needed for FCC-ee

Comparison with (a subset of) NNLO

G. Balossini et al., NPB758 (2006) 227

Comparison of $\sigma_{\rm SV}^{\alpha^2}$ calculation of BabaYaga@NLO with

- Penin (photonic): switching off the vacuum polarisation contribution in BabaYaga@NLO, as a function of the logarithm of the soft photon cut–off (left plot) and of a fictitious electron mass (right plot)



★ differences are infrared safe, as expected

★ $\delta\sigma(\text{photonic})/\sigma_0 \propto \alpha^2 L$, as expected

- Numerically, for various selection criteria at the Φ and B factories

$$\sigma_{\rm SV}^{\alpha^2}(\text{Penin}) - \sigma_{\rm SV}^{\alpha^2}(\text{BabaYaga@NLO}) < 0.02\% \times \sigma_0$$

F. Piccinini