

Jens Erler
JGU & Helmholtz Institute Mainz
(on leave from IF-UNAM)

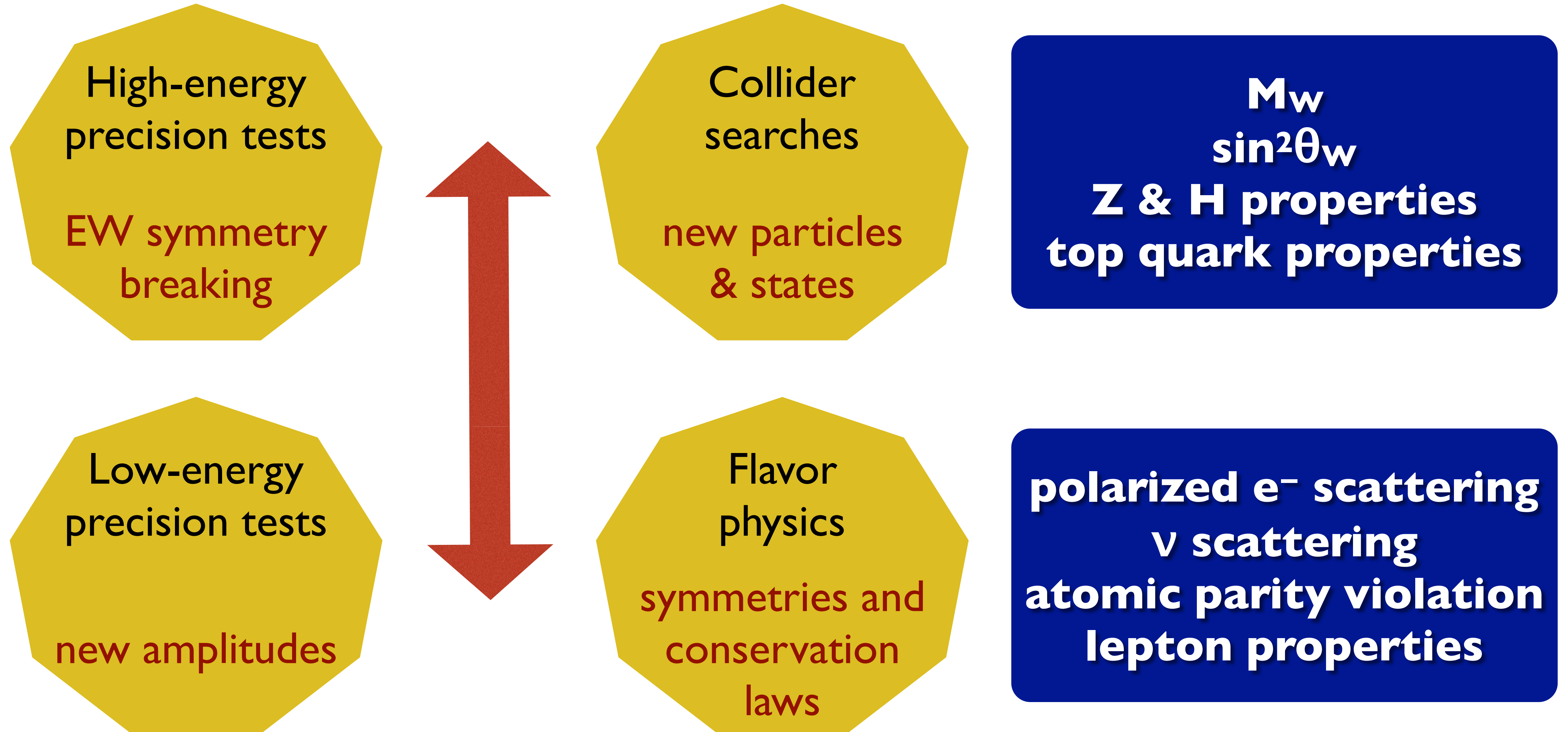
***Precision electroweak physics
with low energy measurements***



Cluster of Excellence
PRISMA+

Precision Physics, Fundamental Interactions
and Structure of Matter

Complementary physics



Outline

- * *Weak mixing angle:*
 - * global survey of $\sin^2\theta_W$ determinations
 - * Theoretical uncertainties: correlations in precision observables
- * *Vacuum polarization in global fits:*
 - * $\alpha(M_Z)$
 - * $\sin^2\theta_W(0)$
 - * $g_{\mu-2}$
 - * $m_{c,b}$
- * *Conclusions and outlook*

Weak Mixing Angle

Weak mixing angle at tree level

doubly over-constrained system

$$Z^\mu = \cos \theta_W W_3^\mu - \sin \theta_W B^\mu$$

$$A^\mu = \sin \theta_W W_3^\mu + \cos \theta_W B^\mu$$

$$M_W = \frac{g}{2}v \implies \sin^2 \theta_W = \frac{\pi\alpha(M_W)}{\sqrt{2}G_F M_W^2} \implies \theta_W = 28.68^\circ$$

$$M_Z^2 = \frac{g^2 + g'^2}{2}v^2 \implies \sin^2 2\theta_W = \frac{\sqrt{8}\pi\alpha(M_Z)}{G_F M_Z^2} \implies \theta_W = 28.90^\circ$$

$$\theta_W = \arccos \frac{M_W}{M_Z} = 28.18^\circ$$

only 1 relation if $\alpha(M_Z)$ would not be known (currently induces 3 MeV error)

Weak mixing angle approaches

- * tuning in on the Z resonance
- * leptonic and heavy quark FB asymmetries in e^+e^- annihilation near $s = M_Z^2$
- * leptonic FB asymmetries in pp ($p\bar{p}$) Drell-Yan in a window around $m_{||} = M_Z$
- * LR asymmetry (SLC) and final state τ polarization (LEP) and their FB asymmetries

	ν scattering	parity violating e^- scattering (PVES)
leptonic	$\nu_\mu - e^-$	$e^- - e^-$
DIS	heavy nuclei (NuTeV)	deuteron (E-122, PVDIS, SoLID)
elastic	CEvNS (COHERENT)	proton, ^{12}C (Q_{weak}, P2)
APV	heavy alkali atoms and ions	isotope ratios (Mainz)

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	ν scattering	recent first measurements	scattering (PVES)
leptonic	$\nu_{\mu} - e^-$		$e^- - e^-$
DIS	heavy nuclei (1-10 TeV)		deuteron (E-122, PVES, DIS, SoLID)
elastic	CEvNS (COHERENT)		proton, ^{12}C (Qweak, P2)
APV	heavy alkali atoms and ions		isotope ratios (Mainz)

Coherent Elastic ν Nucleus Scattering (CEvNS)

COHERENT @ SNS

CsI

$E_\nu \approx 16 - 53$ MeV

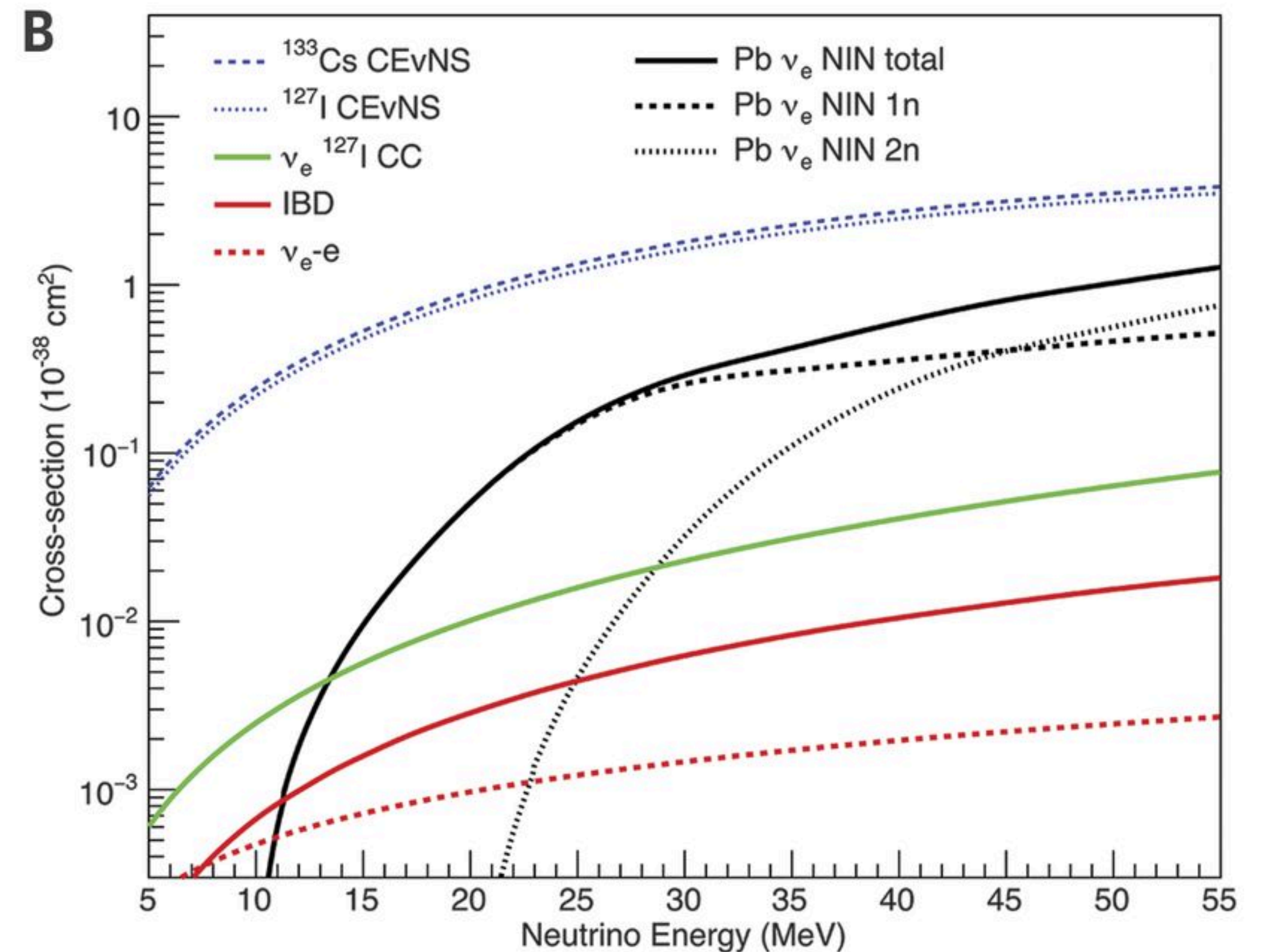
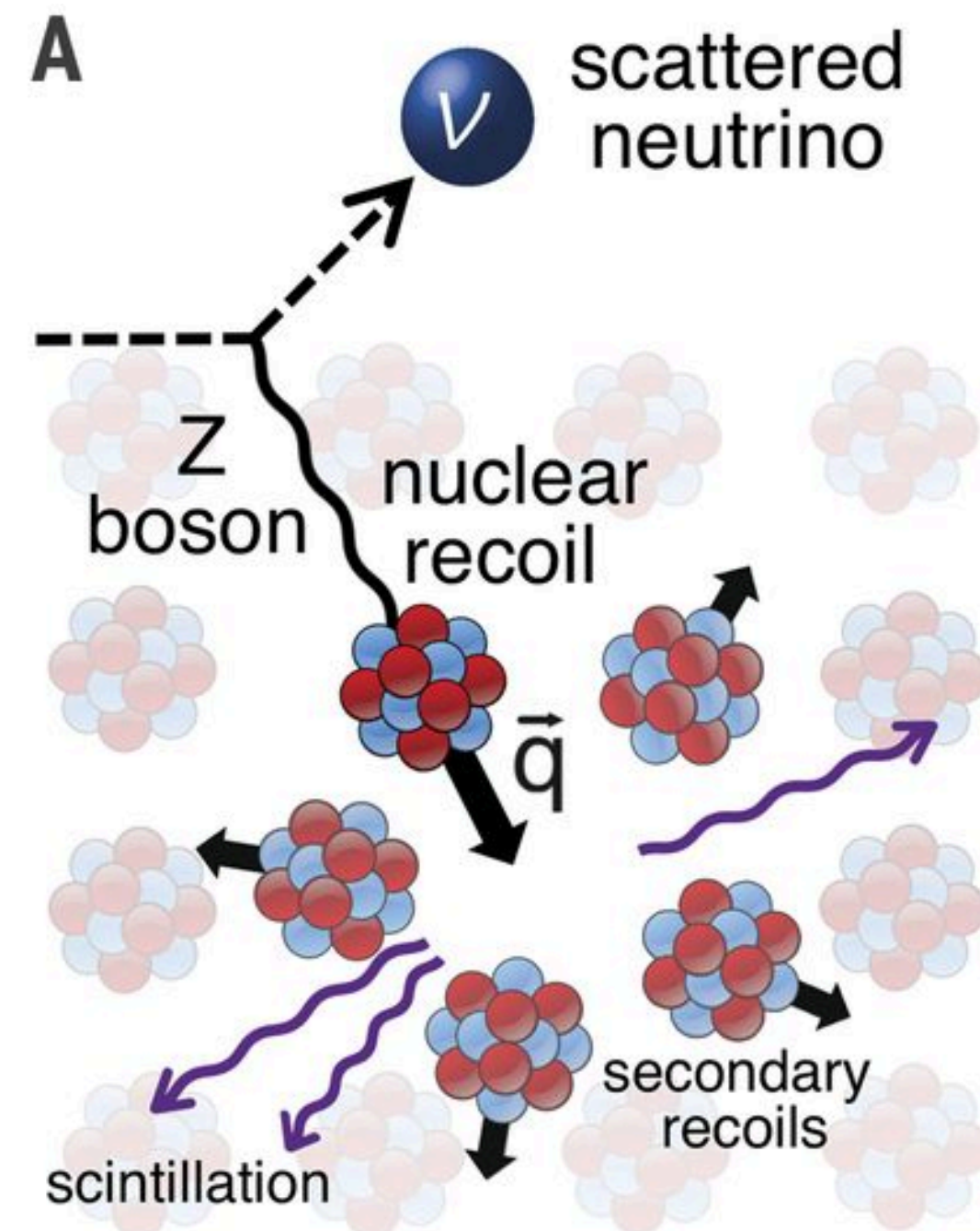
$\sigma \sim Q_W^2$

134 ± 22 events

constraints on NSI

neutron skin?

[arXiv:1708.01294](https://arxiv.org/abs/1708.01294)



$$Q_W(N, Z) = Z (1 - 4 \sin^2 \theta_w) - N$$

Atomic parity violation in an isotope chain

AG Budker @ JGU Mainz

Ytterbium

$^{170}\text{Yb} - ^{176}\text{Yb}$

$\pm 0.5\%$ per isotope

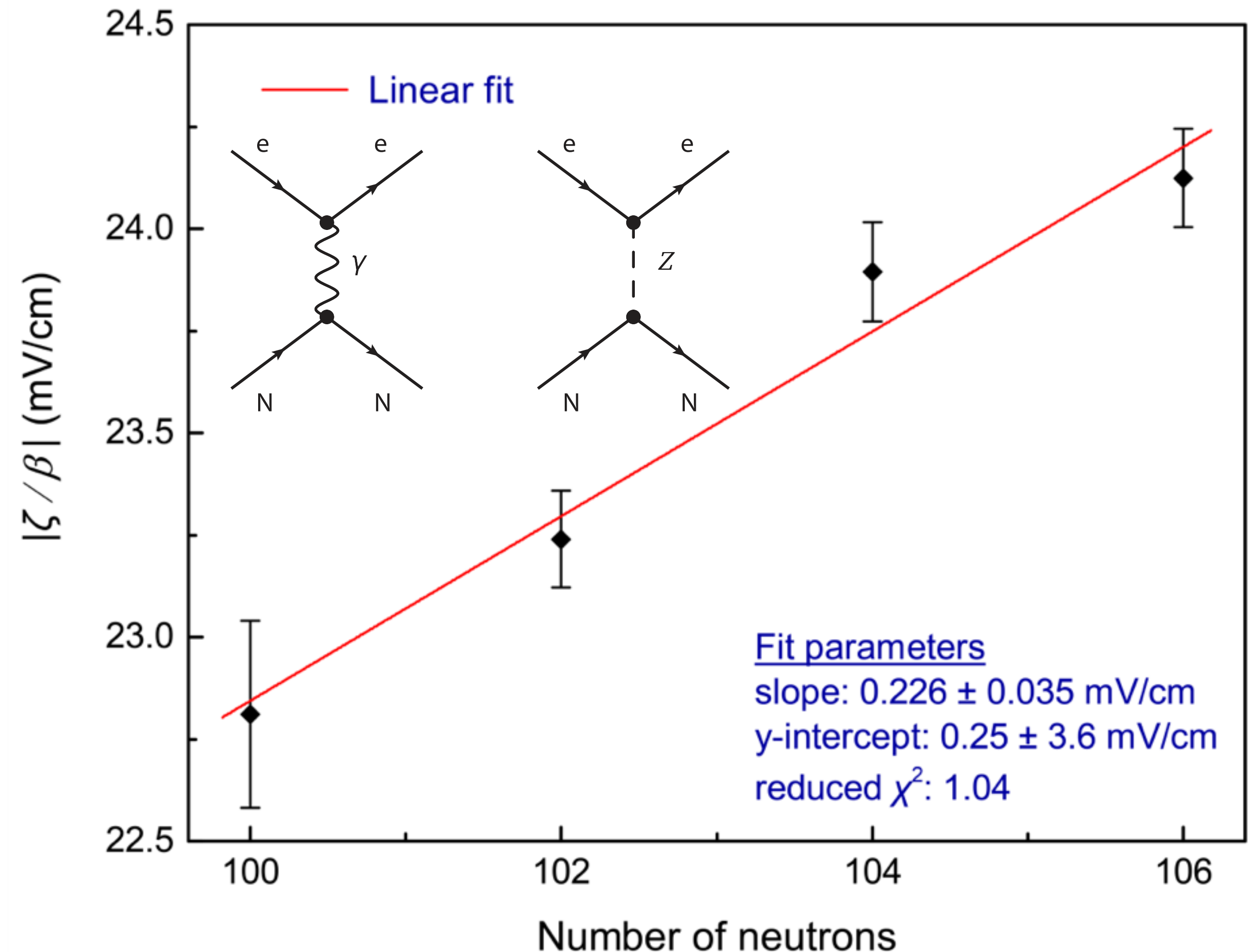
$\pm 100\%$ error in $\sin^2\theta_W$

constraints on Z' with $M < 100$ keV

$\Delta\sin^2\theta_W = \pm 0.2$

neutron skin?

[arXiv:1804.05747](https://arxiv.org/abs/1804.05747)



Parity Violating e^- Scattering (PVES) — Elastic

Qweak @ CEBAF (JLab)

hydrogen (completed)

$$E_e = 1165 \text{ MeV}$$

$$|Q| = 158 \text{ MeV}$$

$$A_{PV} = 2.3 \times 10^{-7}$$

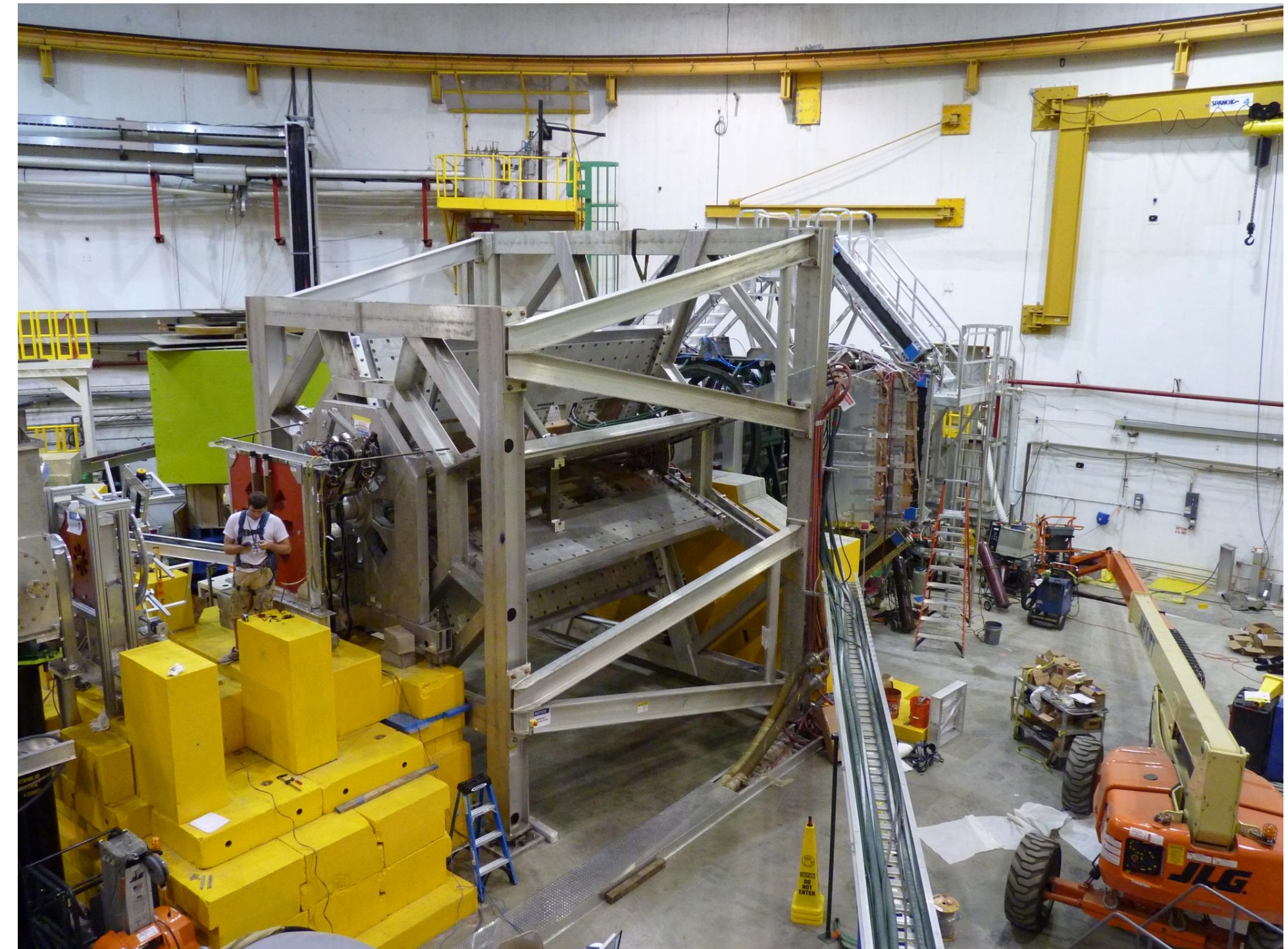
$$\Delta A_{PV} = \pm 4.1\%$$

$$\Delta Q_W(p) = \pm 6.25\%$$

$$\Delta \sin^2 \theta_W = \pm 0.0011$$

FFs from fit to ep asymmetries

[arXiv:1905.08283](https://arxiv.org/abs/1905.08283)



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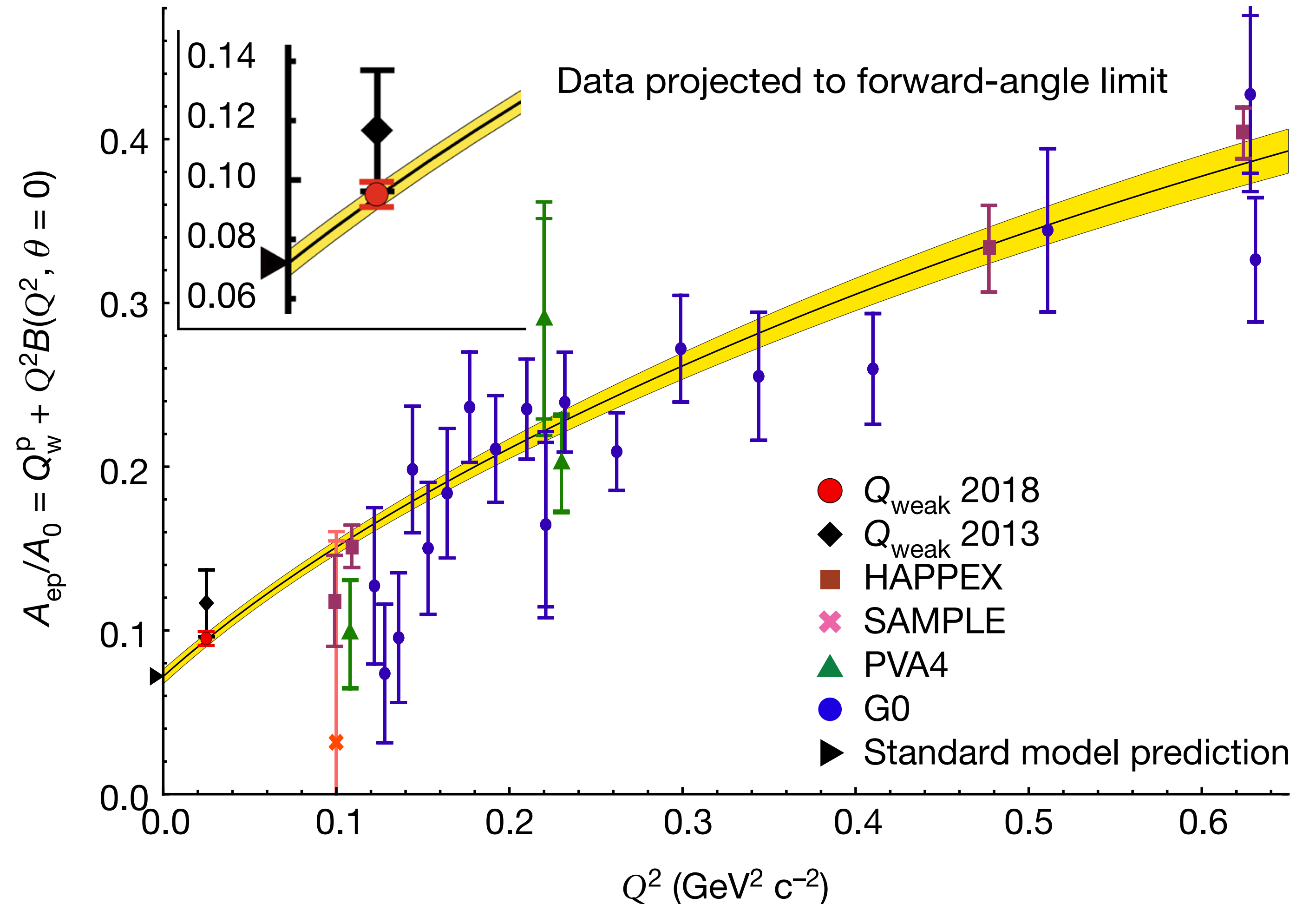
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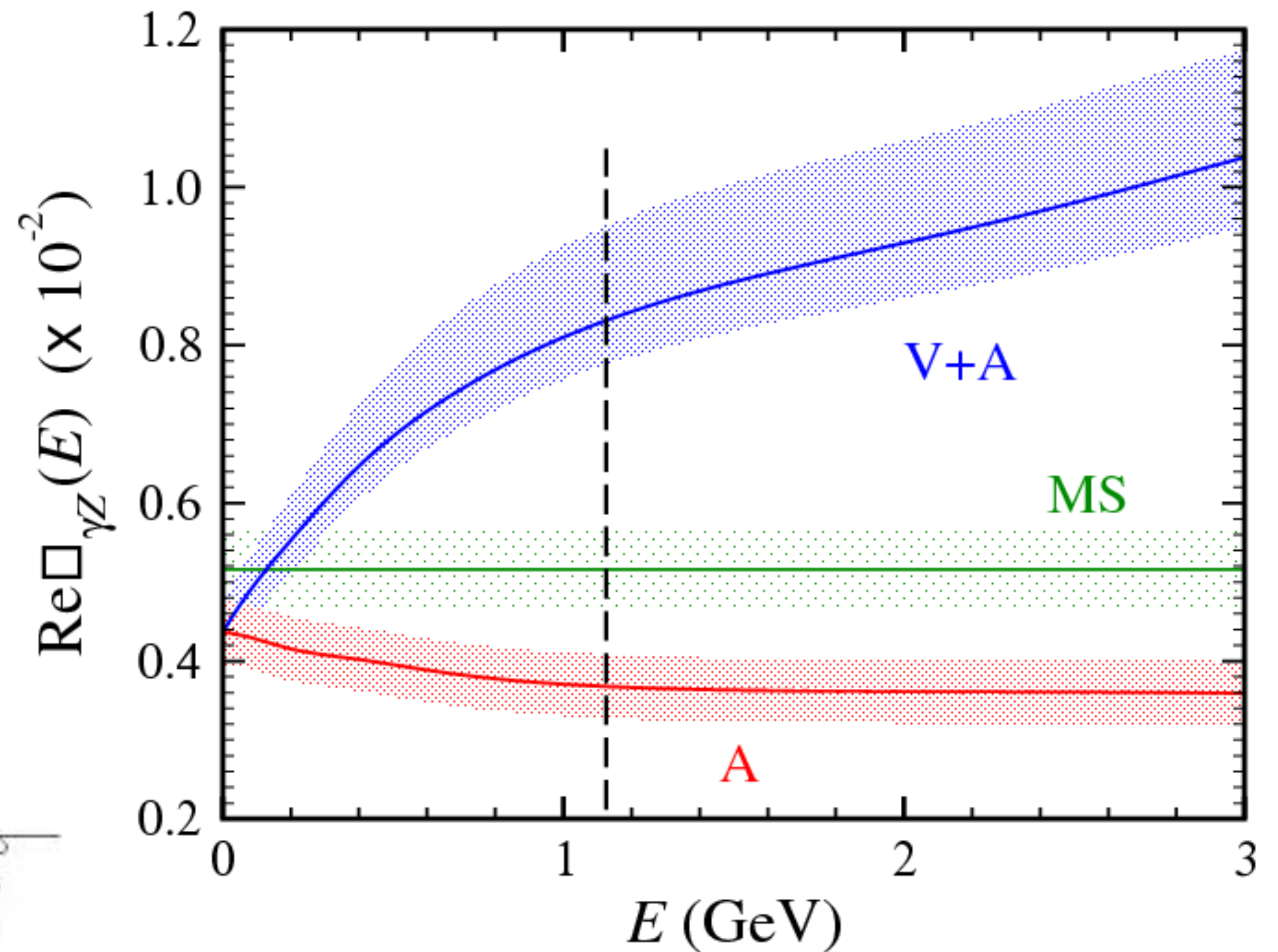
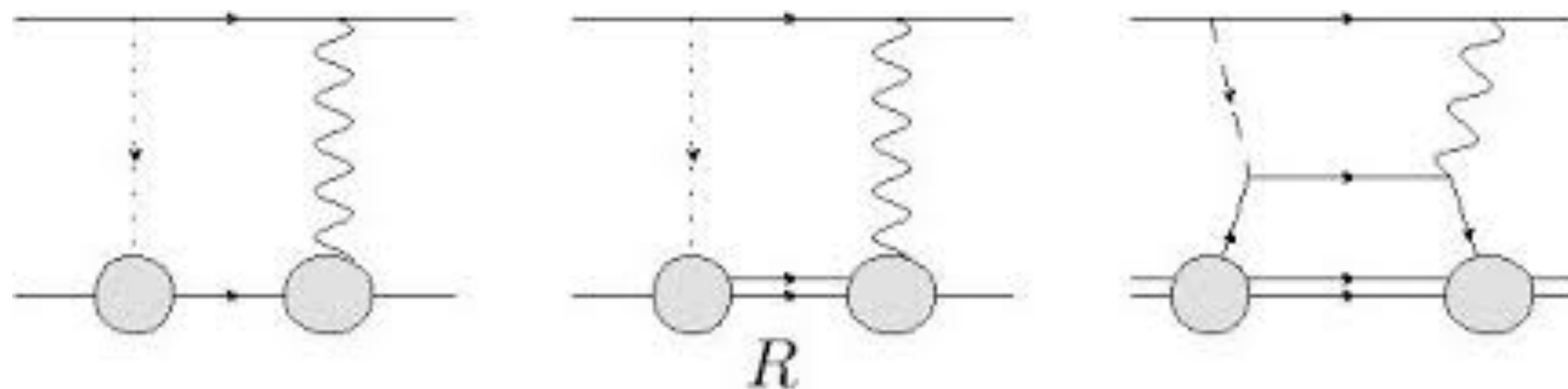
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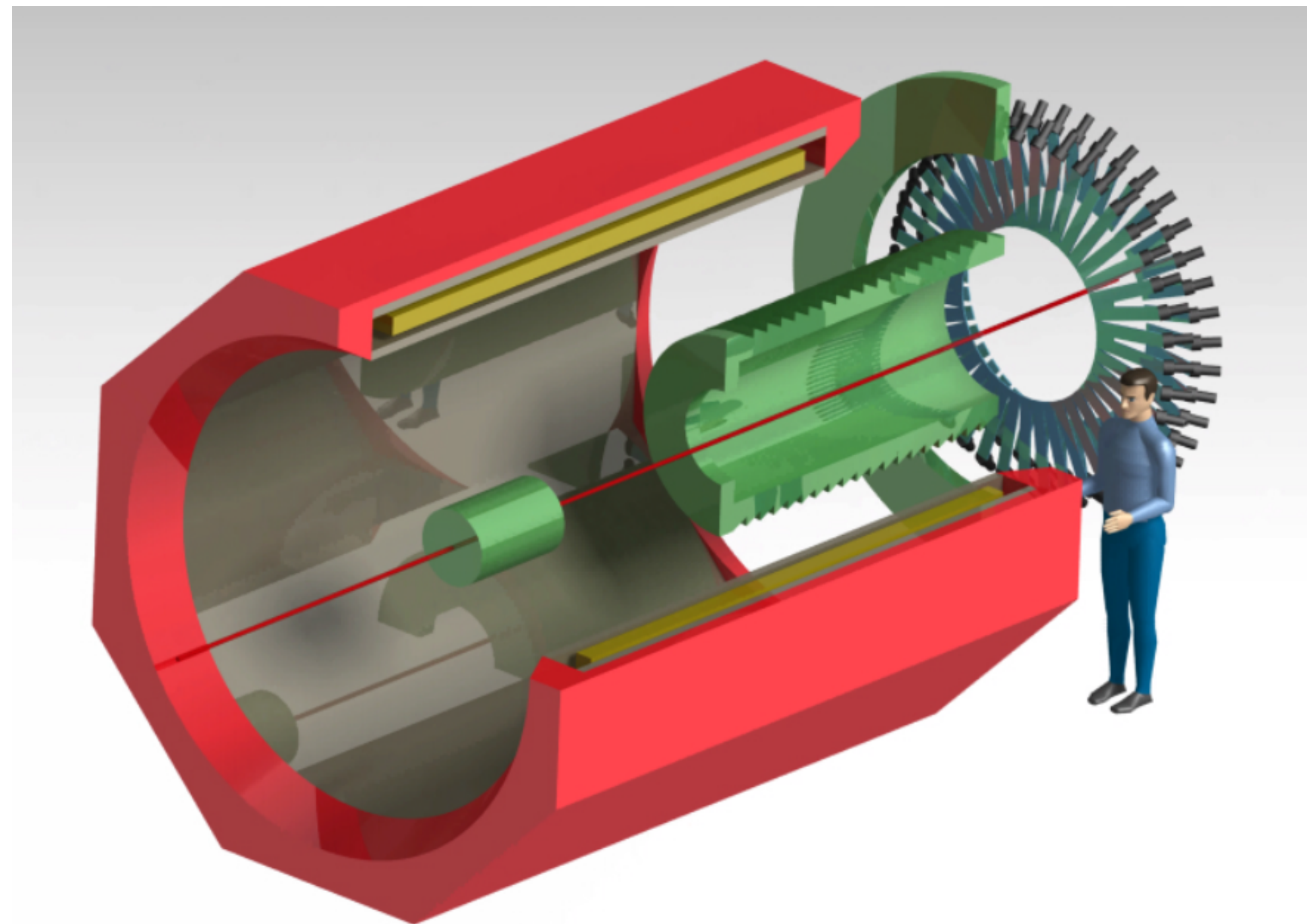
Theory issues in PVES

- * need full 1-loop QED under experiment-specific conditions
- * box diagrams (γZ -box)
- * enhanced 2-loop electroweak (γWW -double box)
- * running weak mixing angle (see later)
- * unknown neutron distribution (neutron skin for heavier nuclei)



Blunden et al., arXiv:1102.5334

Parity Violating e^- Scattering (PVES) — Elastic



P2 @ MESA (JGU Mainz)

hydrogen (CDR)

$$E_e = 155 \text{ MeV}$$

$$|Q| = 67 \text{ MeV}$$

$$A_{PV} = 4 \times 10^{-8}$$

$$\Delta A_{PV} = \pm 1.4\%$$

$$\Delta Q_W(p) = \pm 1.83\%$$

$$\Delta \sin^2 \theta_W = \pm 0.00033$$

FFs from backward angle data

[arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

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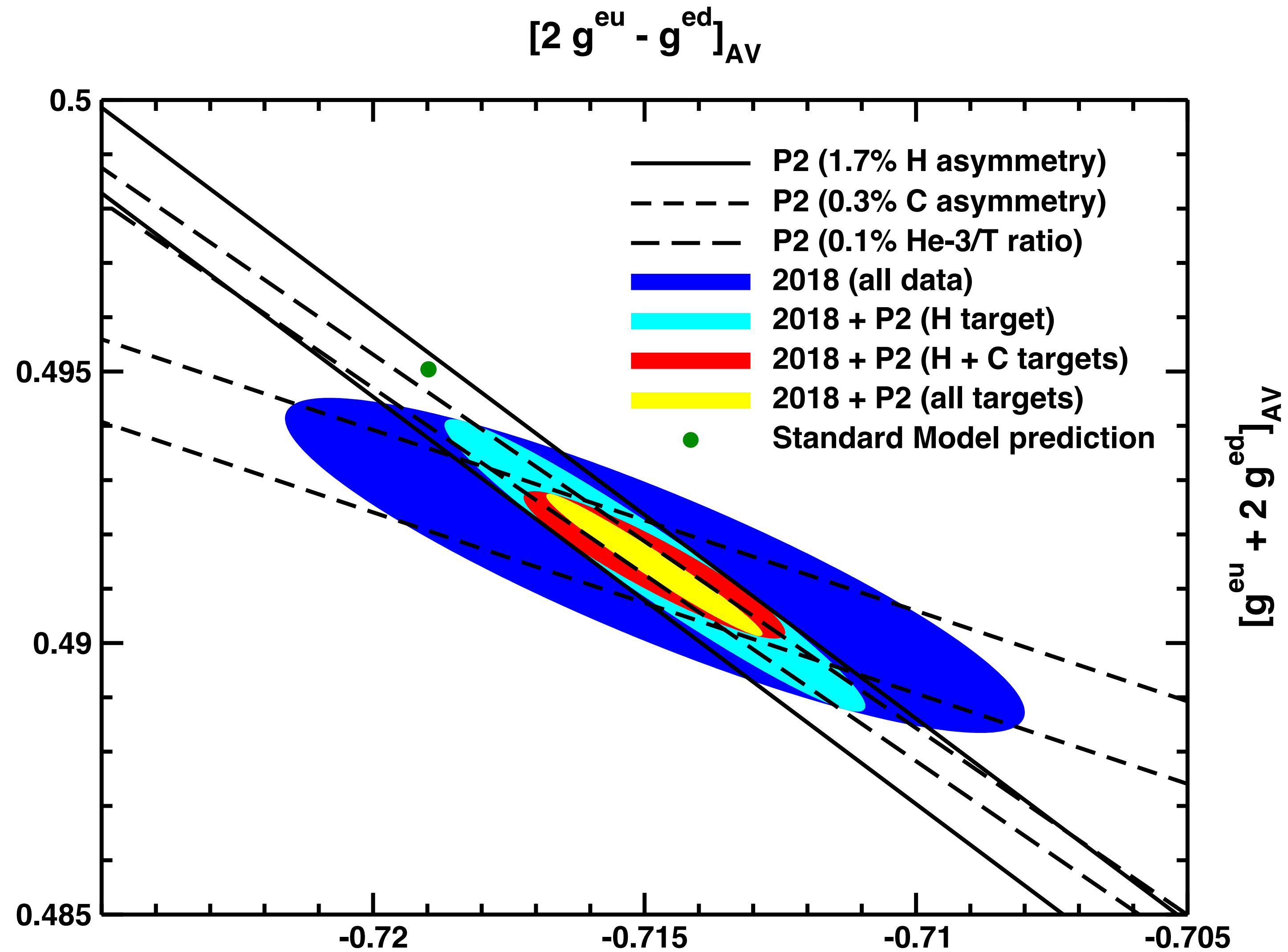
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FFs from backward angle data

[arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

Effective couplings (Wilson coefficients)



Parity Violating e⁻ Scattering (PVES) — Elastic

Qweak @ CEBAF

H (completed)

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FFs from backward angles

[arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

P2 @ MESA

¹²C (CDR)

$$E_e = 150 \text{ MeV}$$

$$A_{PV} = 6 \times 10^{-7}$$

$$\Delta A_{PV} = \pm 0.3\%$$

$$\Delta Q_W(^{12}\text{C}) = \pm 0.3\%$$

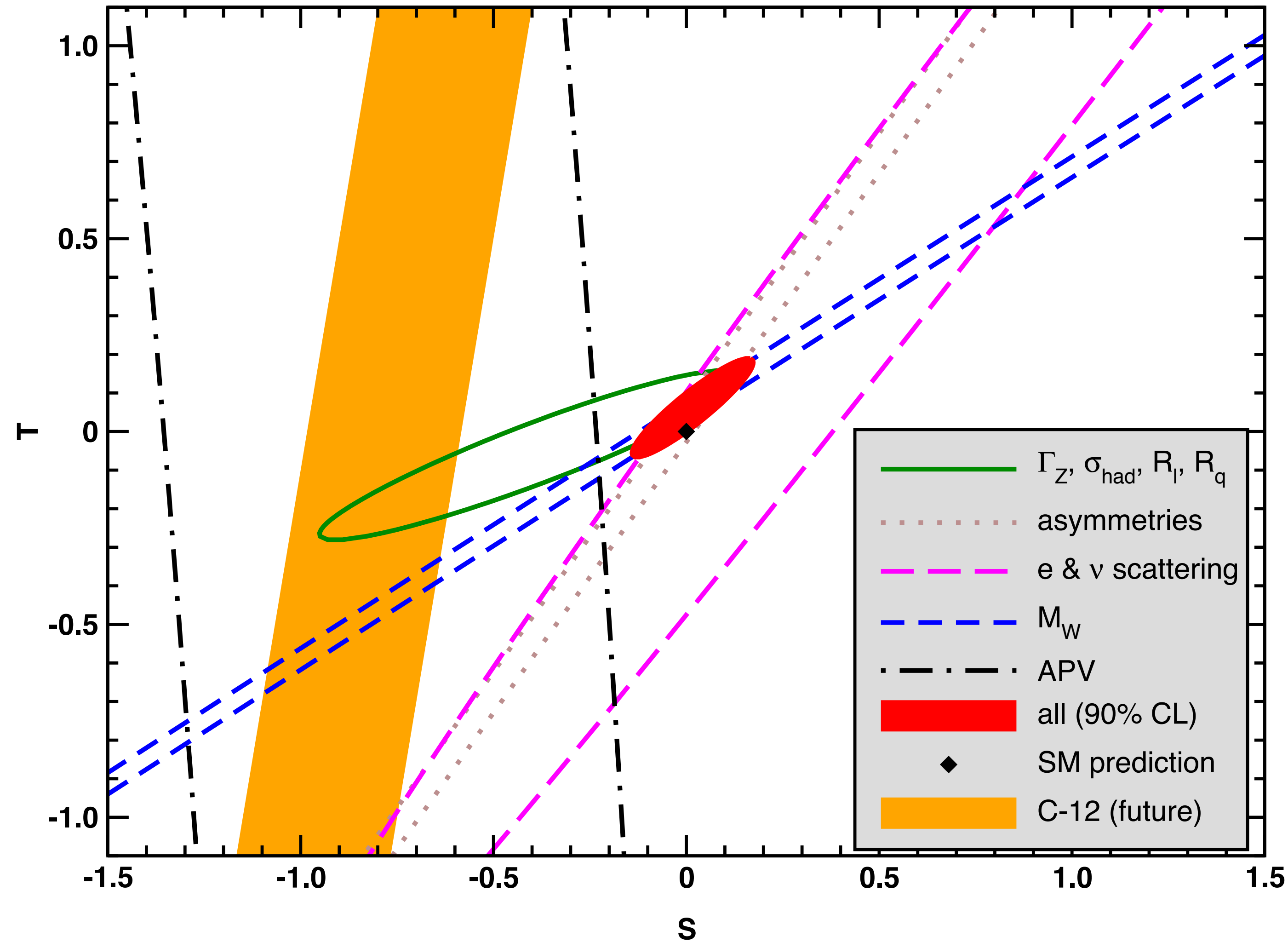
$$\Delta \sin^2 \theta_W = \pm 0.0007$$

neutron skin?

only one FF

[arXiv:1802.04759](https://arxiv.org/abs/1802.04759)

S and T



S	0.02 ± 0.07
T	0.06 ± 0.06
$\Delta\chi^2$	-4.2

* $M_{KK} \gtrsim 3.2$ TeV in warped extra dimension models

* $M_V \gtrsim 4$ TeV in minimal composite Higgs models

**Freitas & JE
PDG (2018)**

Parity Violating e⁻ Scattering (PVES) — DIS

E122 @ SLAC

D (completed)

$$|Q| = 0.96 - 1.40 \text{ GeV}$$

$$A_{PV} = 1.2 \times 10^{-4}$$

$$\Delta A_{PV} = \pm 8\%$$

$$\Delta \sin^2 \theta_W = \pm 0.011$$

PLB 84, 524 (1979)

PVDIS @ CEBAF

D (completed)

$$|Q| = 1.04 \text{ \& } 1.38 \text{ GeV}$$

$$A_{PV} = 1.6 \times 10^{-4}$$

$$\Delta A_{PV} = \pm 4.4\%$$

$$\Delta \sin^2 \theta_W = \pm 0.0051$$

arXiv:1411.3200

SoLID @ CEBAF

D (pre-CDR)

$$|Q| = 2.1 - 3.1 \text{ GeV}$$

$$A_{PV} = 8 \times 10^{-4}$$

$$\Delta A_{PV} = \pm 0.6\%$$

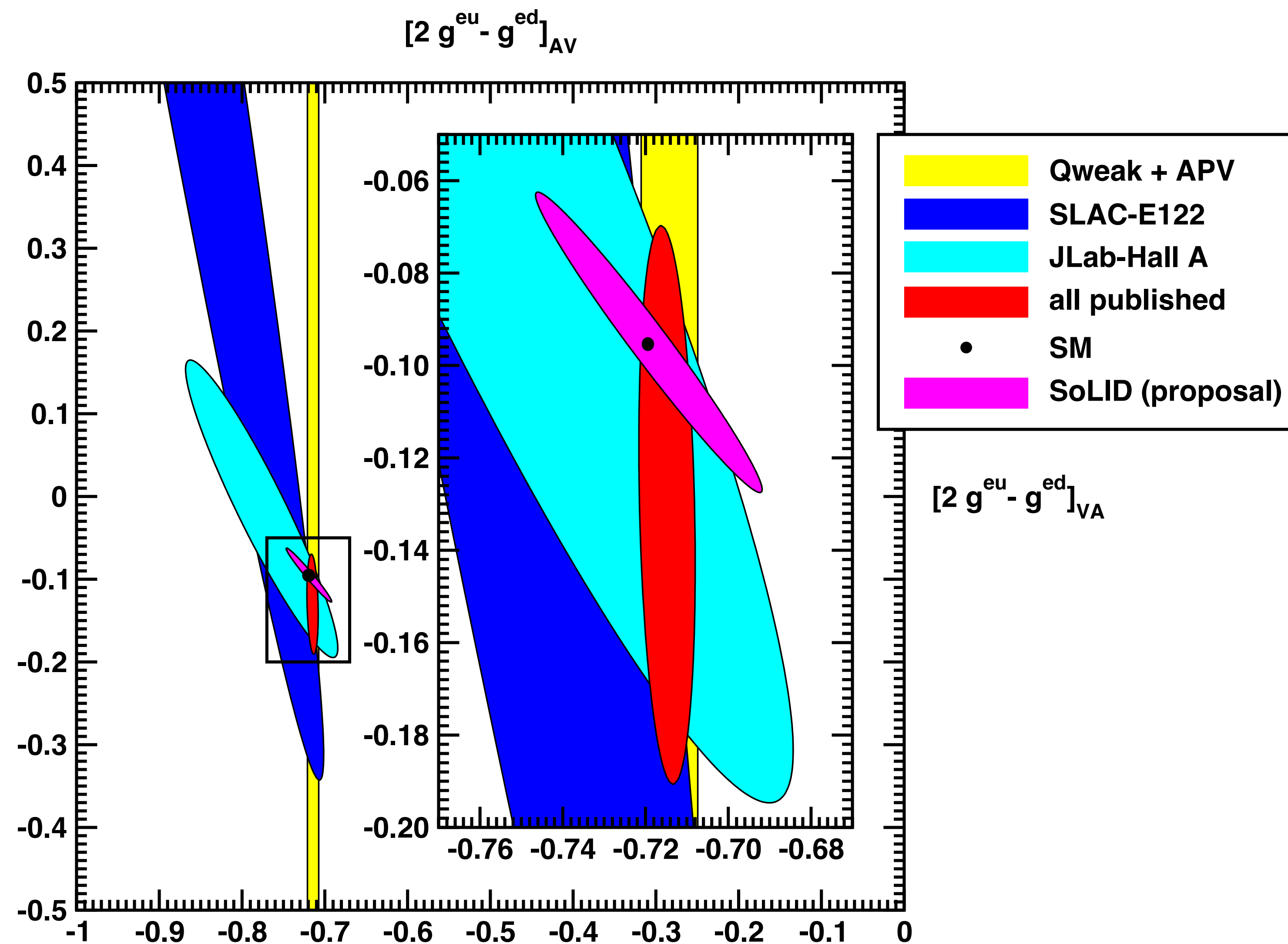
$$\Delta \sin^2 \theta_W = \pm 0.00057$$

Higher twist?

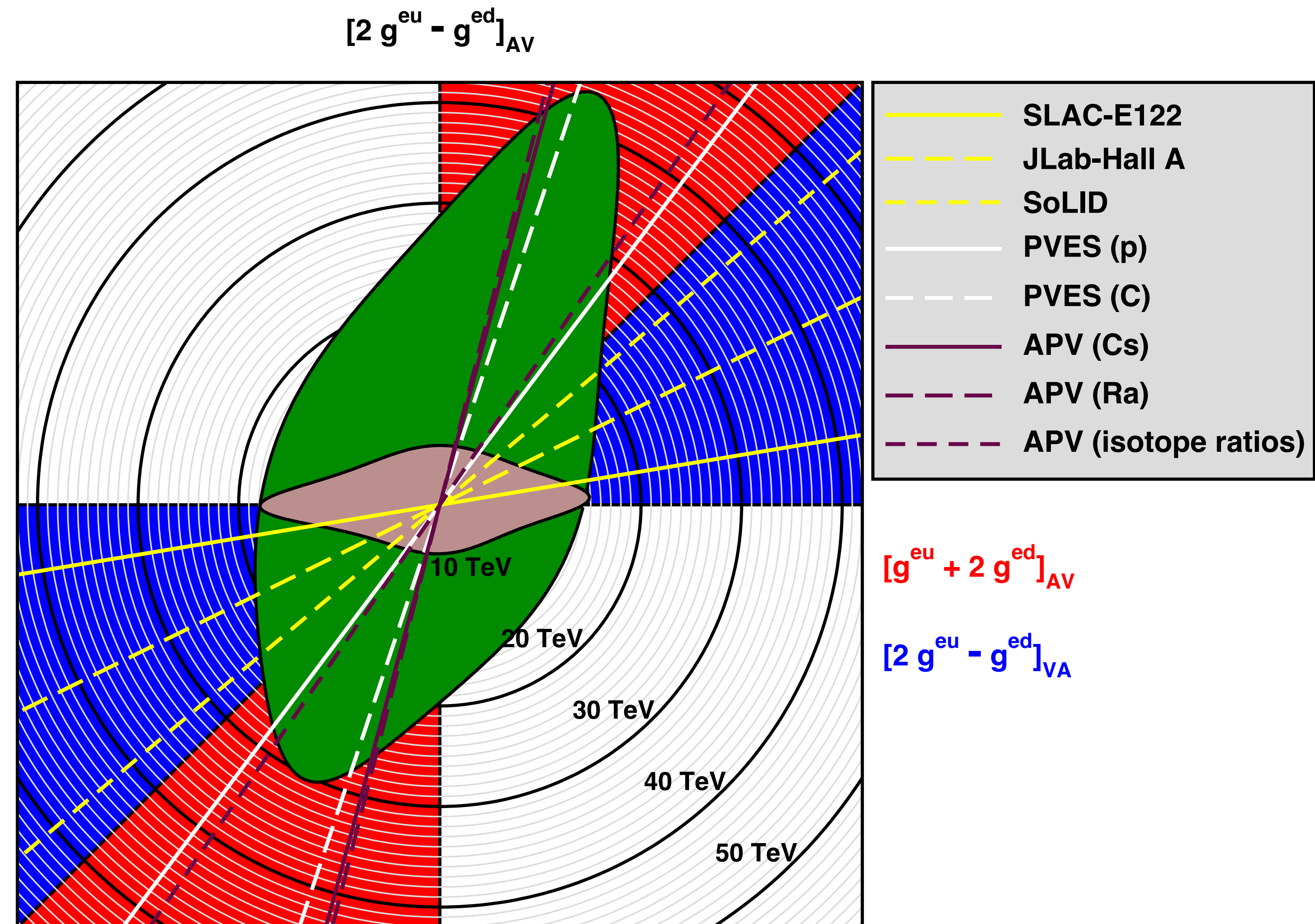
Isospin violation?

arXiv:1810.00989

Effective couplings (Wilson coefficients)



Scale exclusions post Qweak



Parity Violating e^- Scattering (PVES) — Møller

E158 @ SLC (SLAC)

hydrogen (completed)

$$E_e = 45 \text{ \& } 48 \text{ GeV}$$

$$|Q| = 161 \text{ MeV}$$

$$A_{PV} = 1.31 \times 10^{-7}$$

$$\Delta A_{PV} = \pm 13\%$$

$$\Delta Q_W(e) = \pm 13\%$$

$$\Delta \sin^2 \theta_W = \pm 0.0013$$

[hep-ex/0504049](https://arxiv.org/abs/hep-ex/0504049)

MOLLER @ CEBAF (JLab)

hydrogen (proposal)

$$E_e = 11.0 \text{ GeV}$$

$$|Q| = 76 \text{ MeV}$$

$$A_{PV} = 3.3 \times 10^{-8}$$

$$\Delta A_{PV} = \pm 2.4\%$$

$$\Delta Q_W(e) = \pm 2.4\%$$

$$\Delta \sin^2 \theta_W = \pm 0.00027$$

[arXiv:1411.4088](https://arxiv.org/abs/1411.4088)

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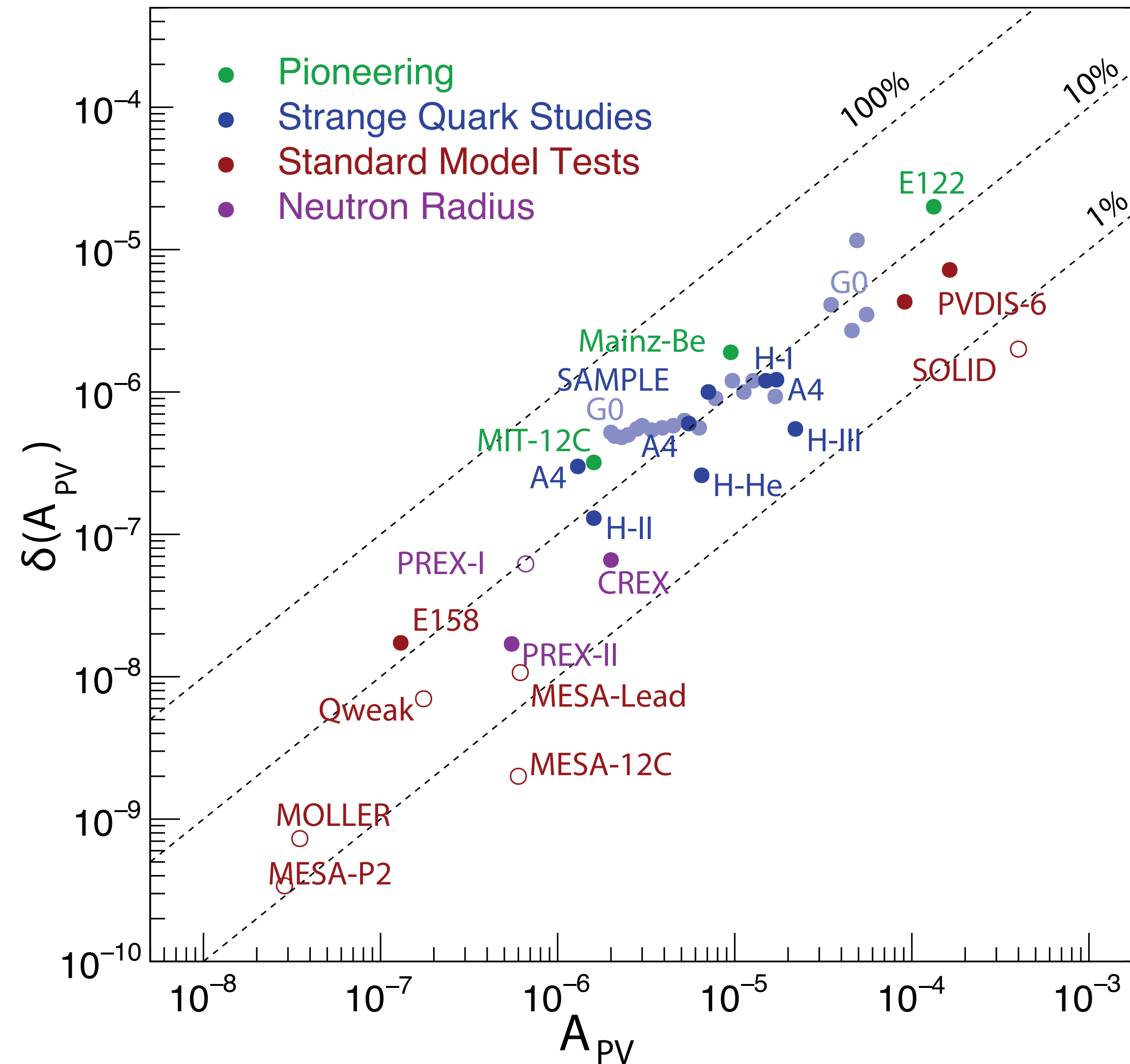
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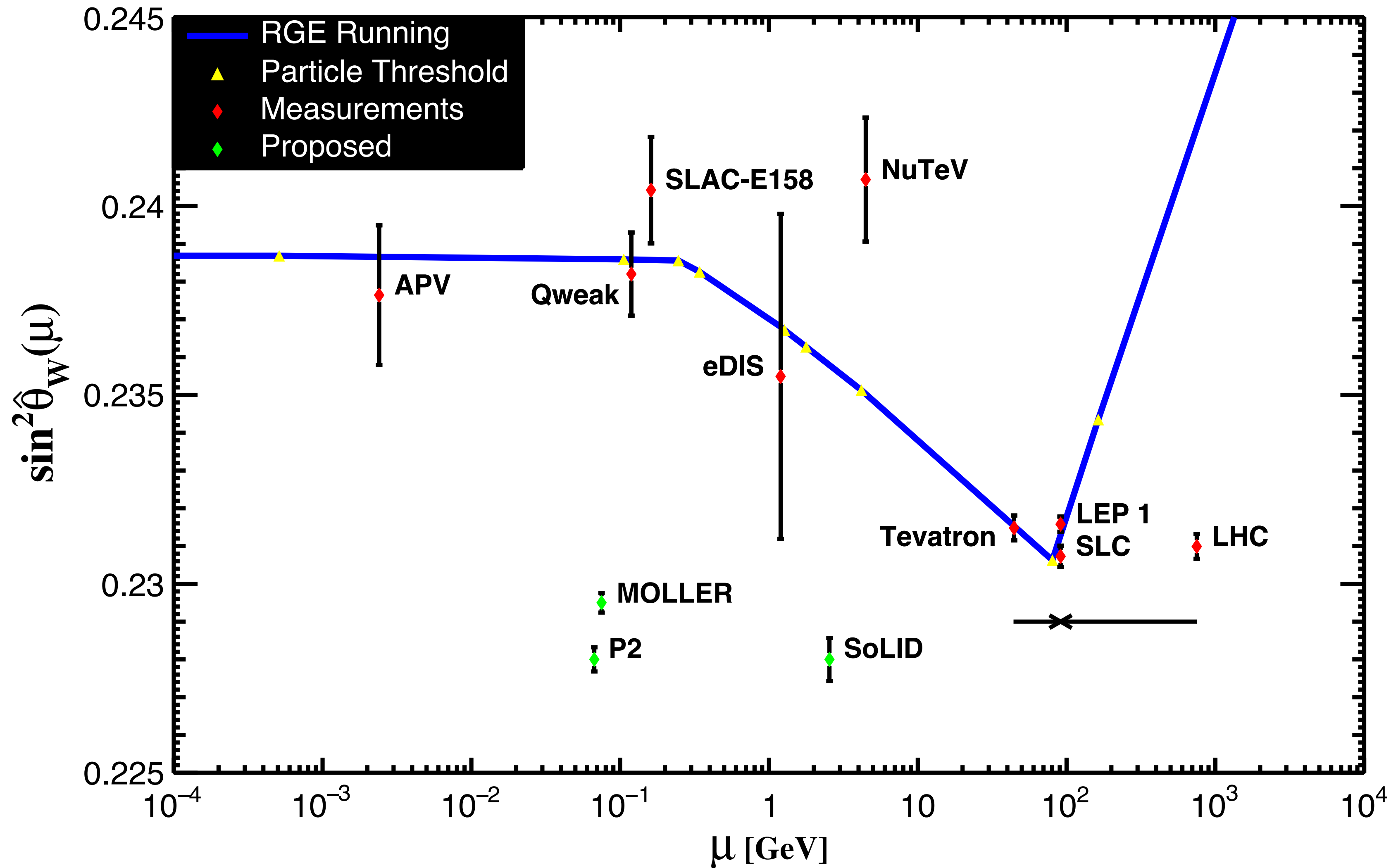
[arXiv:1411.4088](https://arxiv.org/abs/1411.4088)



PVES history

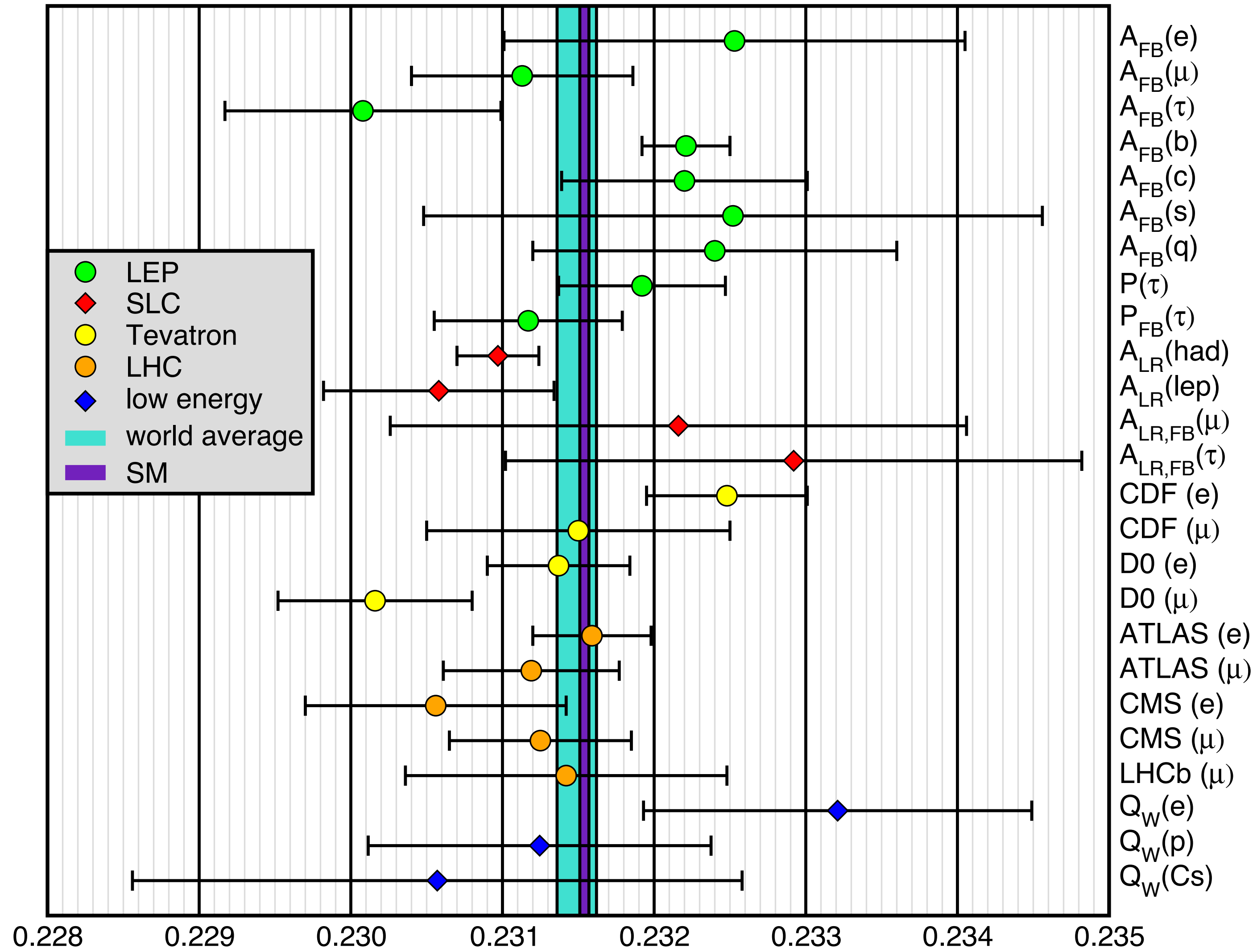


Running weak mixing angle

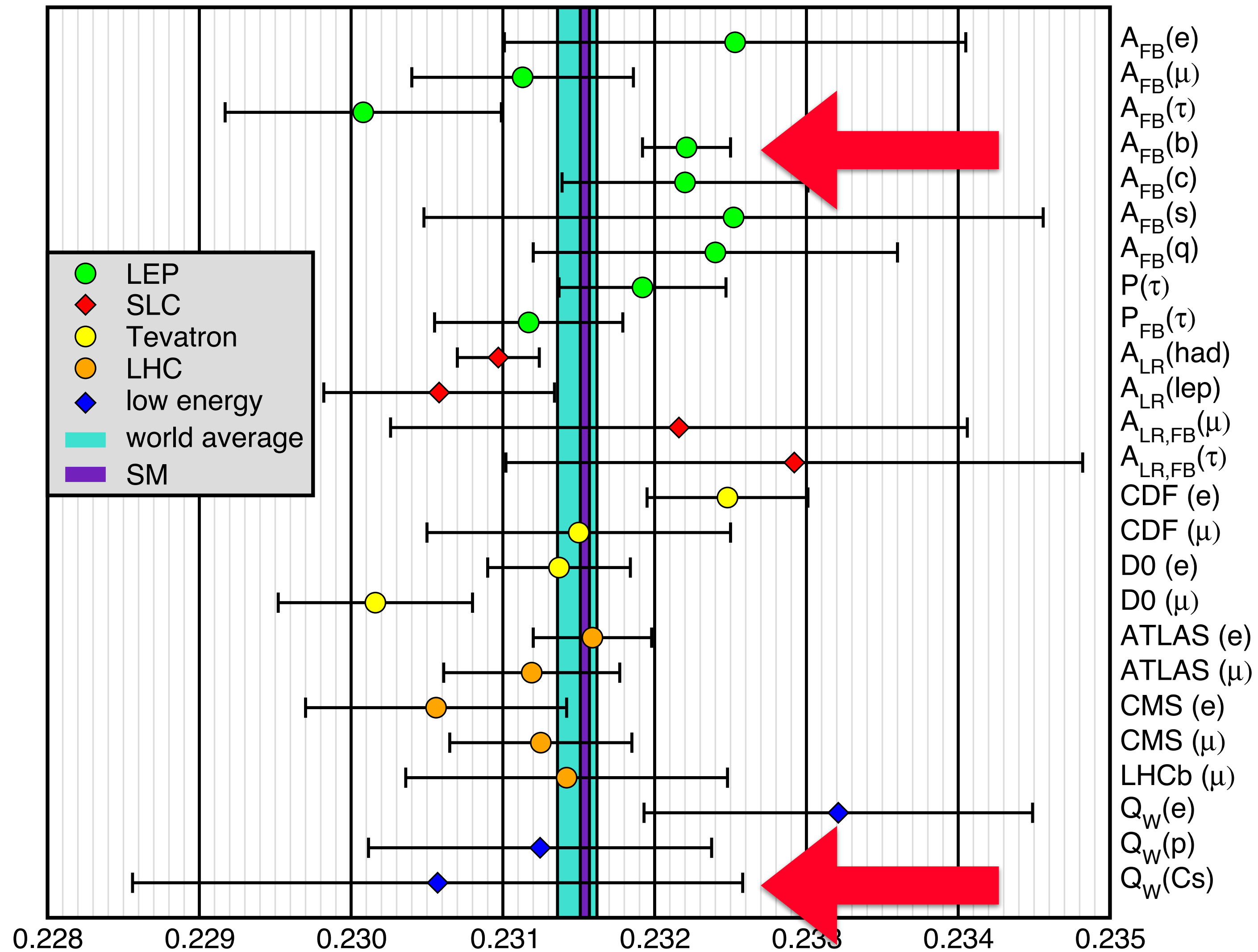


**Ferro-Hernández
& JE
arXiv:1712.09146**

Weak mixing angle measurements



Weak mixing angle measurements



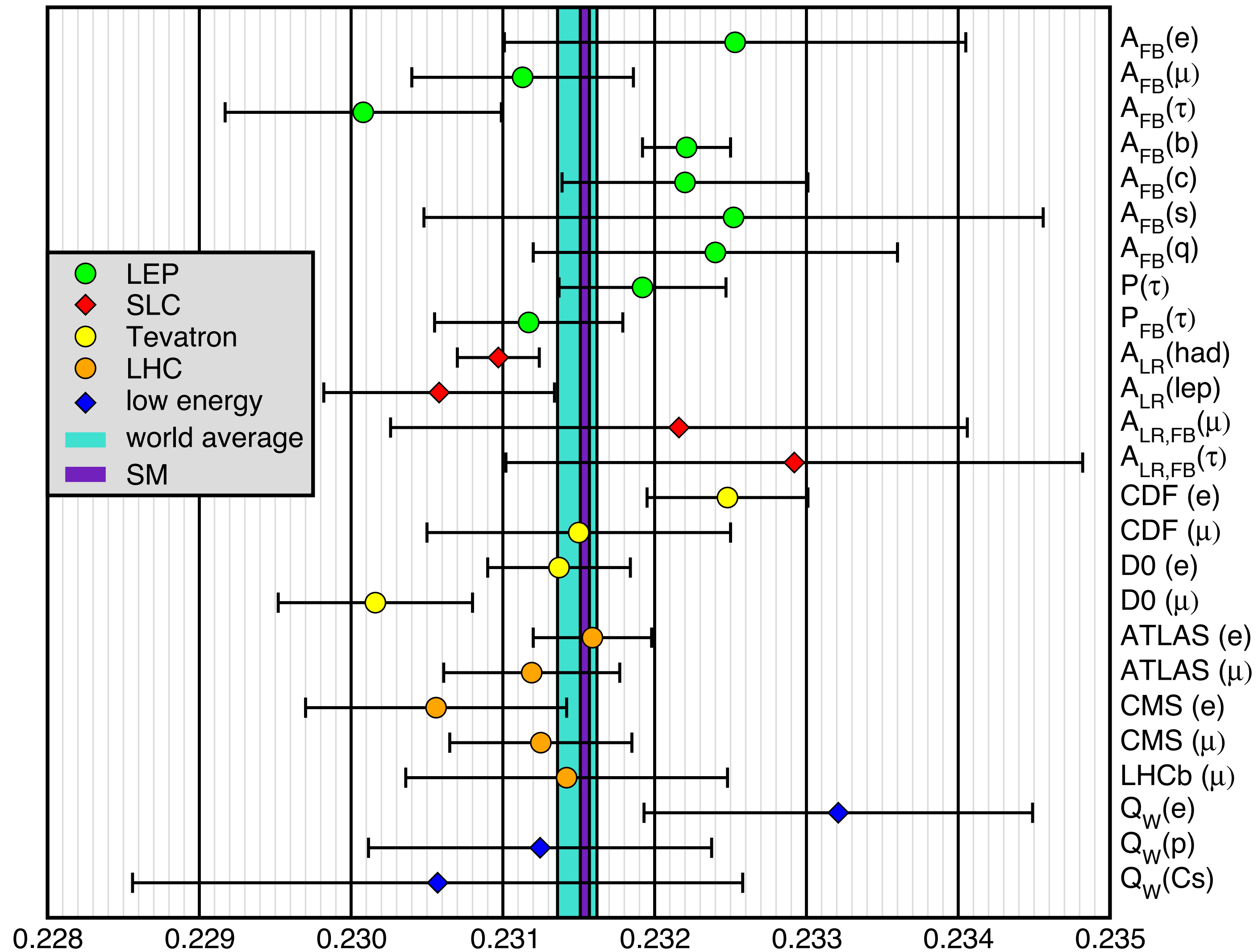
2-loop QCD correction
with $m_b \neq 0$

Bernreuther et al.
arXiv:1611.07942

new measured
transition vector polarizability

Tho et al.
arXiv:1905.02768

Weak mixing angle measurements



LEP & SLC:
 0.23153 ± 0.00016

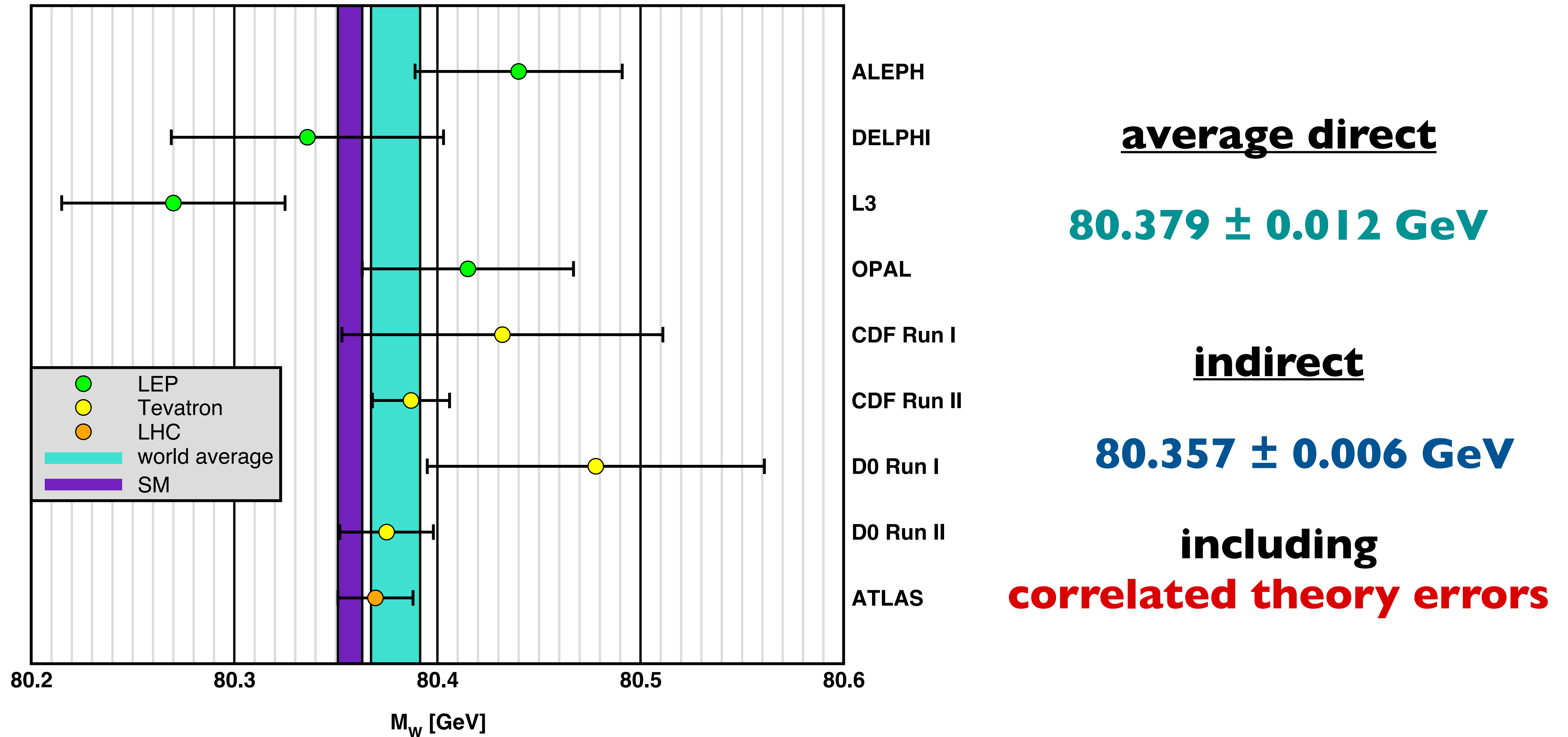
Tevatron:
 0.23148 ± 0.00033

LHC:
 0.23131 ± 0.00033

average direct
 0.23149 ± 0.00013

global fit
 0.23153 ± 0.00004

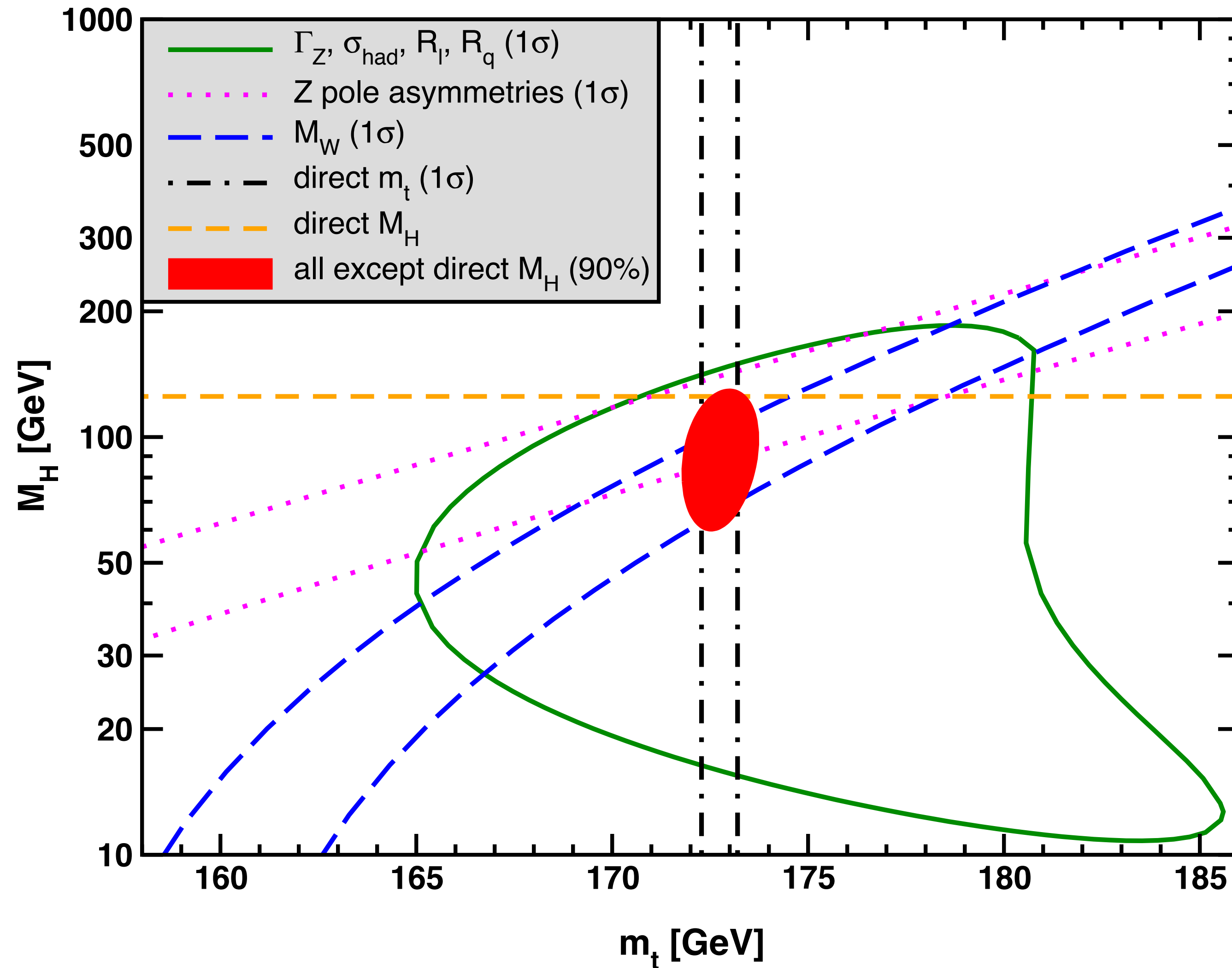
W boson mass measurements



Theoretical uncertainties and correlations

- * loop factors including enhancement factors $N_C = N_F = 3$ or $\sin^{-2}\theta_W \approx m_t^2/M_W^2 \approx 4$:
 - * $8 \alpha(M_W)/\pi = 0.020$ (QED)
 - * $3 \alpha_s(M_W)/\pi = 0.116$ (QCD)
 - * $3 \alpha(M_W)/\pi \sin^2\theta_W(M_W) = 0.032$ (CC)
 - * $(3 - 6 s_W^2 + 8 s_W^4)/\pi s_W^2 c_W^2 = 0.029$ (NC)
- * $\Delta S_Z = \pm 0.0034$ (may be combined with $\Delta\alpha_{\text{had}}$),
- * $\Delta T = \pm 0.0073$ (t-b doublet)
- * $\Delta U = S_W - S_Z = \pm 0.0051$
- * assuming ΔS_Z , ΔT and ΔU to be sufficiently different (uncorrelated) induces **theory correlations** between different observables **Schott & JE, arXiv:1902.05142**

$M_H - m_t$



indirect m_t

176.4 ± 1.8 GeV (2.0σ high)

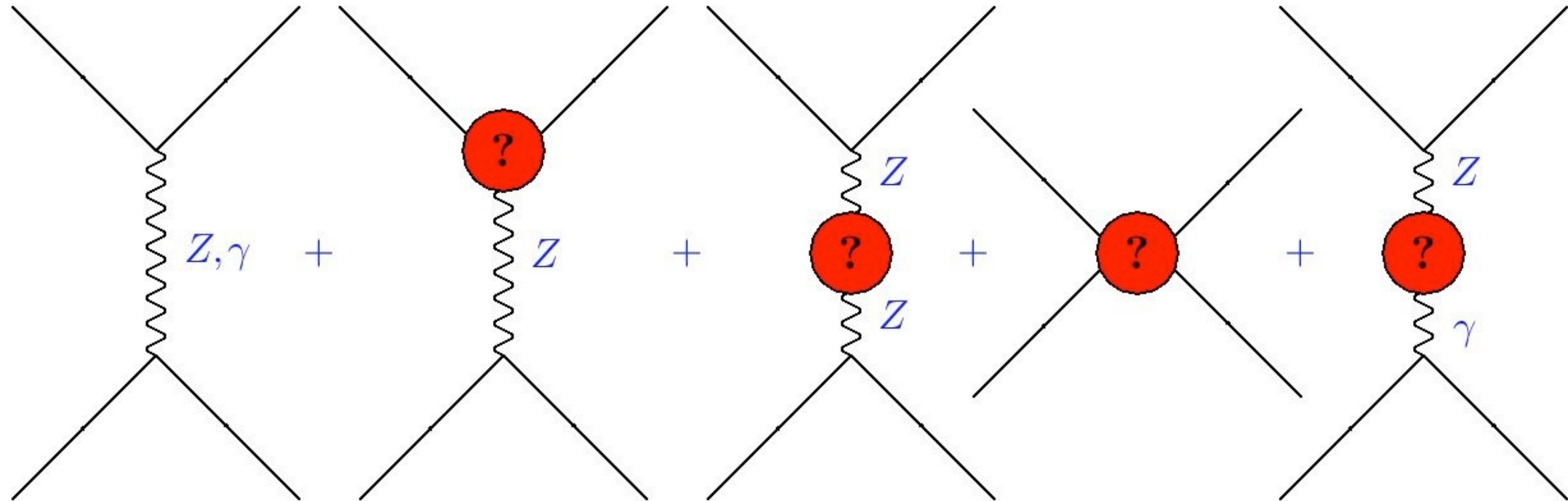
indirect M_H

90^{+17}_{-15} GeV (1.9σ low)

including theory error

91^{+18}_{-16} GeV (1.8σ low)

Beyond the SM

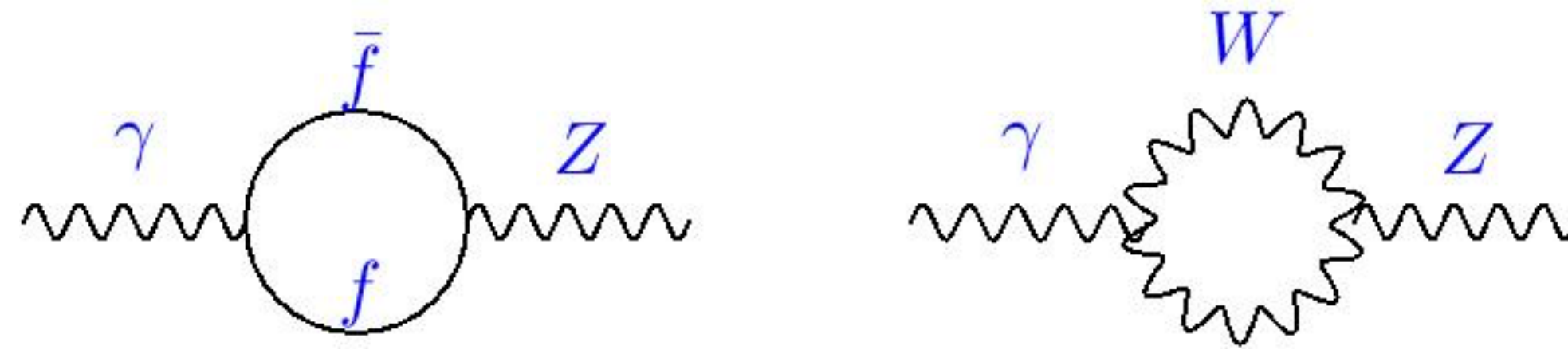


- * **Z-Z'** mixing: modification of Z vector coupling
- * **oblique parameters:** STU (also need M_W and Γ_Z)
- * **new amplitudes:** off- versus on-Z pole measurements (e.g. Z')
- * **dark Z:** renormalization group evolution (running)

Vacuum Polarization in Global Fits

$\alpha(M_Z)$ $\sin^2\theta_W(0)$ $g_{\mu-2}$ $m_{b,c}$

sin²θ_w(0) and Δα(M_Z)



$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

$$\mu^2 \frac{d\hat{\alpha}}{d\mu^2} = \frac{\hat{\alpha}^2}{\pi} \left[\frac{1}{24} \sum_i K_i \gamma_i Q_i^2 + \sigma \left(\sum_q Q_q \right)^2 \right]$$

* coupled system of equations

Ramsey-Musolf & JE, hep-ph/0409169

* Δα(M_Z)_{had} errors in sin²θ_w(0) = κ(0) sin²θ_w(M_Z) **add** because

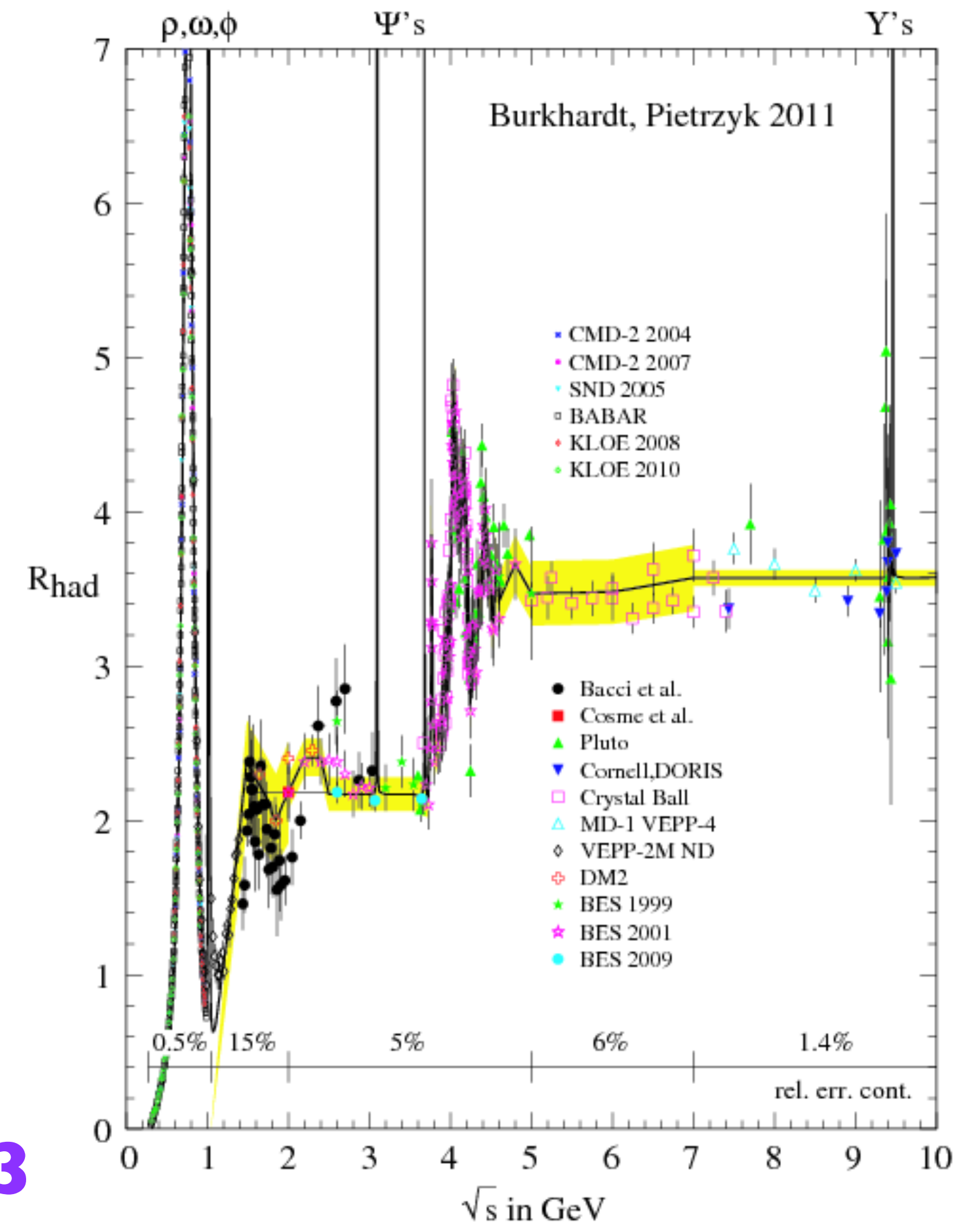
$$M_Z^2 \sim g_Z^2(M_Z) v^2 \sim [\alpha / s_w^2 c_w^2](M_Z) G_F^{-1}$$

$\alpha(M_Z)$

- * Dispersive approach: integral over $\sigma(e^+e^- \rightarrow \text{hadrons})$ and τ -decay data
 - * $\alpha^{-1}(M_Z) = 128.947 \pm 0.012$ **Davier et al., arXiv:1706.09436**
 - * $\alpha^{-1}(M_Z) = 128.958 \pm 0.016$ **Jegerlehner, arXiv:1711.06089**
 - * $\alpha^{-1}(M_Z) = 128.946 \pm 0.015$ **Keshavarzi et al., arXiv:1802.02995**
- * **$\alpha^{-1}(M_Z) = 128.949 \pm 0.010$ **Ferro-Hernández & JE, arXiv:1712.09146****
- * converted from the \overline{MS} scheme and uses e^+e^- annihilation and τ spectral functions
- * PQCD for $\sqrt{s} > 2 \text{ GeV}$ (using \bar{m}_c & \bar{m}_b)
- * (anti)correlation with $g_\mu - 2$ at two (three) loop order and with $\sin^2\theta_W(0)$

$m_c(m_c)$

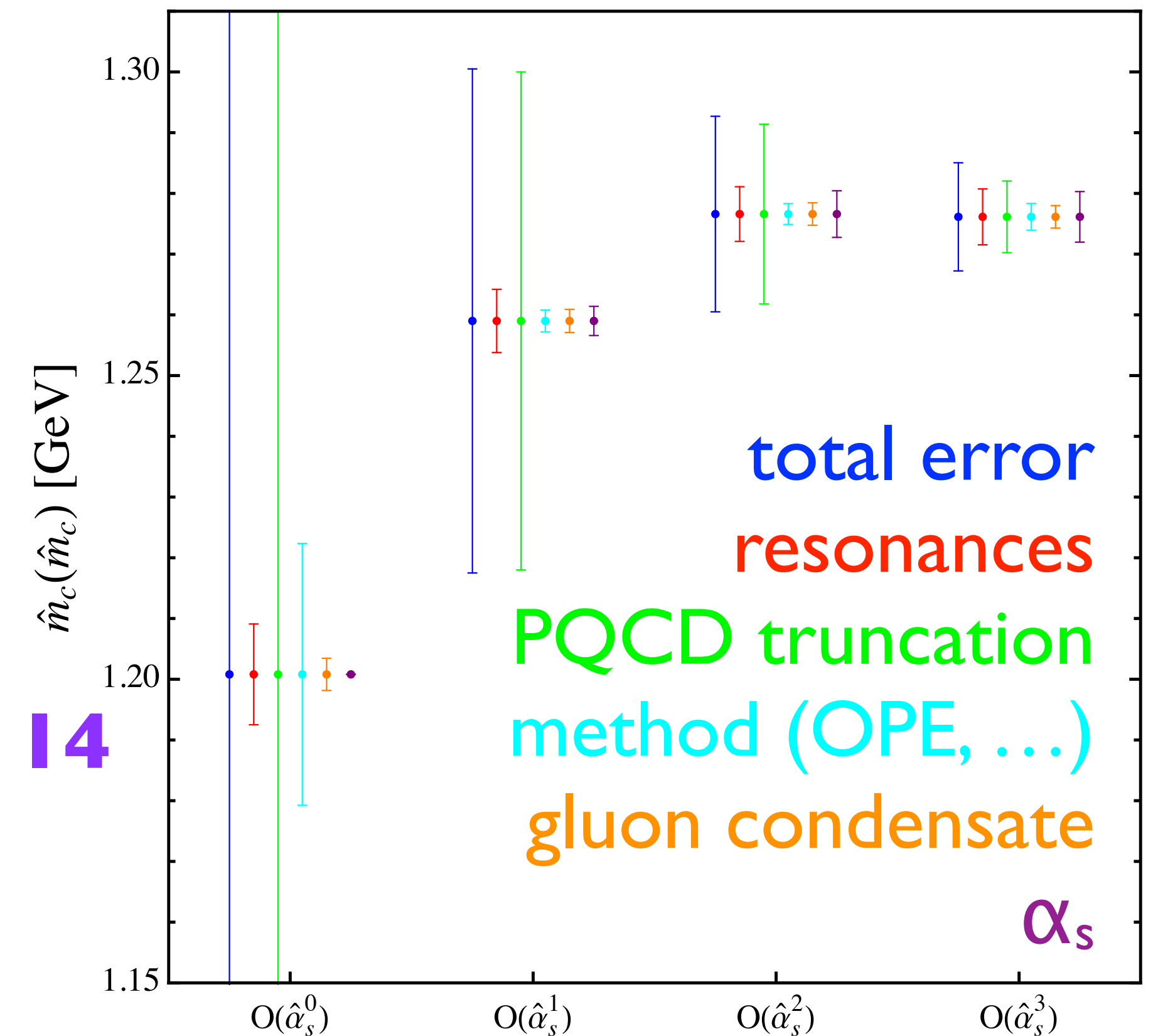
- * only experimental input:
electronic widths of J/ψ and $\psi(2S)$
- * continuum contribution from
self-consistency between sum rules
→ continuum over-constrained
- * include M_0 → stronger (milder) sensitivity
to continuum (m_c) **Luo & JE, hep-ph/0207114**
- * quark-hadron duality needed
only in finite region (**not locally**)
- * $\bar{m}_c(\bar{m}_c) = 1272 \pm 8 + 2616 [\bar{\alpha}_s(M_Z) - 0.1182]$ **MeV**
Masjuan, Spiesberger & JE, arXiv:1610.0853



$m_c(m_c)$

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Masjuan, Spiesberger & JE, arXiv:1610.08531



$\sin^2\theta_W(0)$

source	uncertainty in $\sin^2\theta_W(0)$
$\Delta\alpha^{(3)}(2 \text{ GeV})$	1.2×10^{-5}
flavor separation	1.0×10^{-5}
isospin breaking	0.7×10^{-5}
singlet contribution	0.3×10^{-5}
PQCD	0.6×10^{-5}
Total	1.8×10^{-5}

Ferro-Hernández
& JE
arXiv:1712.09146

Freitas & JE
PDG (2018)

➔ $\sin^2\theta_W(0) = 0.23861 \pm 0.00005_{\text{Z-pole}} \pm 0.00002_{\text{theory}} \pm 0.00001_{\alpha_s}$

(errors from m_c and m_b negligible)

$g_\mu - 2$

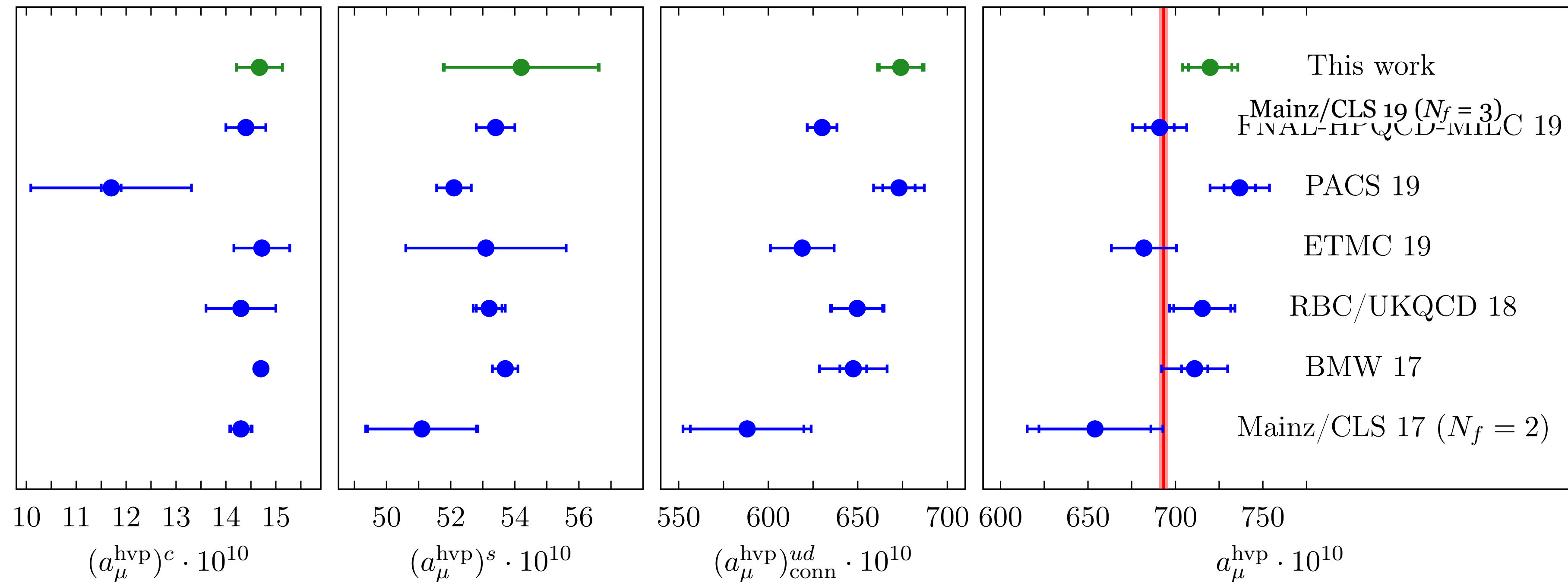
PQCD: $(a_\mu^{\text{hvp}})^c = (14.6 \pm 0.5_{\text{theory}} \pm 0.2_{\text{mc}} \pm 0.1_{\alpha_s}) \times 10^{-10}$

$(a_\mu^{\text{hvp}})^b = 0.3 \times 10^{-10}$

Luo & JE, hep-ph/0101010

Lattice gauge theory:

A. Gérardin et al., arXiv:1904.03120



Conclusions and outlook

- * new players:
 - * coherent V-scattering
 - * ultra-high precision PVES
 - * APV isotope ratios
- * ultra-high precision frontier \implies fields merge (incl. **theory communities**):
 - * collider physics
 - * V-physics
 - * nuclear physics (anapole moments)
 - * astrophysics (neutron skins)
 - * atomic physics (APV, proton radius)
 - * lattice gauge theory (vacuum polarization, ...)

Backups

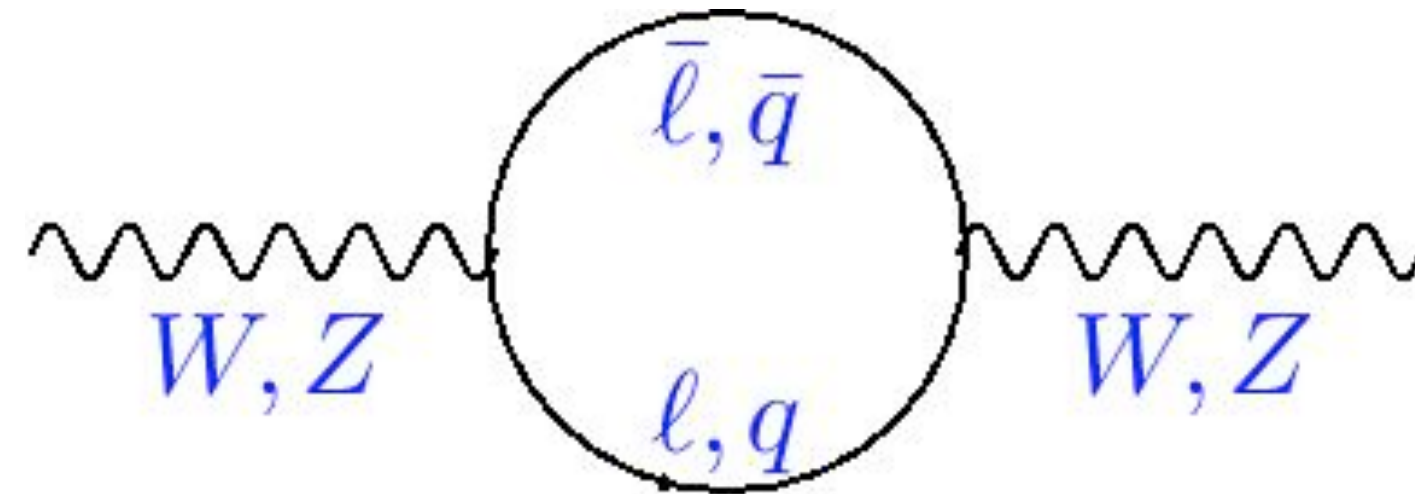
Standard global fit

M_H		$125.14 \pm 0.15 \text{ GeV}$		
M_Z		$91.1884 \pm 0.0020 \text{ GeV}$		
$\bar{m}_b(\bar{m}_b)$		$4.180 \pm 0.021 \text{ GeV}$		
$\Delta\alpha_{\text{had}}^{(3)}(2 \text{ GeV})$		$(59.0 \pm 0.5) \times 10^{-4}$		
$\bar{m}_t(\bar{m}_t)$	$163.28 \pm 0.44 \text{ GeV}$	1.00	-0.13	-0.28
$\bar{m}_c(\bar{m}_c)$	$1.275 \pm 0.009 \text{ GeV}$	-0.13	1.00	0.45
$\alpha_s(M_Z)$	0.1187 ± 0.0016	-0.28	0.45	1.00

other correlations small

Freitas & JE, PDG 2018

Oblique physics beyond the SM



- STU describe corrections to gauge-boson self-energies
- T breaks custodial $SO(4)$
- a multiplet of heavy **degenerate** chiral fermions contributes $\Delta S = N_C/3\pi \sum_i [t_{3L}^i - t_{3R}^i]^2$
- extra **degenerate** fermion family yields $\Delta S = 2/3\pi \approx 0.21$
- S and T (U) correspond to dimension 6 (8) operators

ρ_0 fit

- $\Delta\rho_0 = G_F \sum_i C_i / (8\sqrt{2}\pi^2) \Delta m_i^2$
- where $\Delta m_i^2 \geq (m_1 - m_2)^2$
- despite appearance there is decoupling (see-saw type suppression of Δm_i^2)
- $\rho_0 = 1.00039 \pm 0.00019$ (2.0 σ)
- $(16 \text{ GeV})^2 \leq \sum_i C_i / 3 \Delta m_i^2 \leq (48 \text{ GeV})^2$ @ 90% CL
- $Y = 0$ Higgs triplet VEVs v_3 strongly disfavored ($\rho_0 < 1$)
- consistent with $|Y| = 1$ Higgs triplets if $v_3 \sim 0.01 v_2$

S fit

- S parameter rules out QCD-like technicolor models
- S also constrains extra degenerate fermion families:
 - ➔ $N_F = 2.75 \pm 0.14$ (assuming $T = U = 0$)
- compare with $N_V = 2.991 \pm 0.007$ from Γ_Z

STU fit

$\sin^2\theta_W(M_Z)$		0.23113 ± 0.00014		
$\alpha_s(M_Z)$		0.1189 ± 0.0016		
U	0.00 ± 0.09	-0.66	-0.86	1.00

- $M_{KK} \gtrsim 3.2$ TeV in warped extra dimension models
- $M_V \gtrsim 4$ TeV in minimal composite Higgs models *Freitas & JE (PDG 2018)*

m_t measurements

	central	statistical	systematic	total
Tevatron	174.30	0.35	0.54	0.64
ATLAS	172.51	0.27	0.42	0.50
CMS	172.43	0.13	0.46	0.48
CMS Run 2	172.25	0.08	0.62	0.63
grand average	172.74	0.11	0.31	0.33

JE, EPJC 75 (2015)

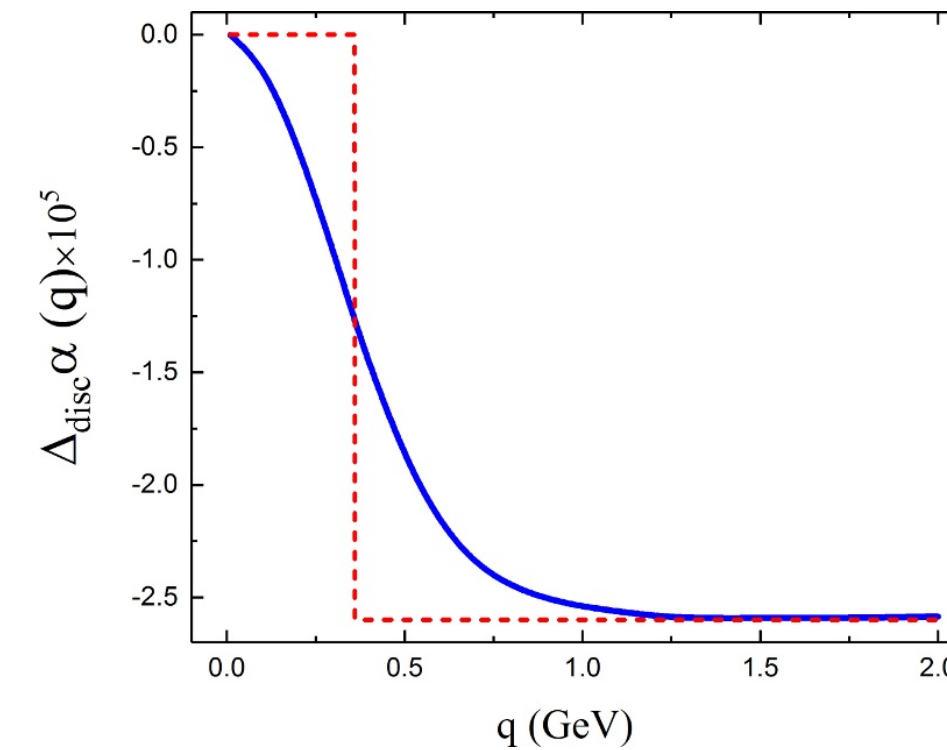
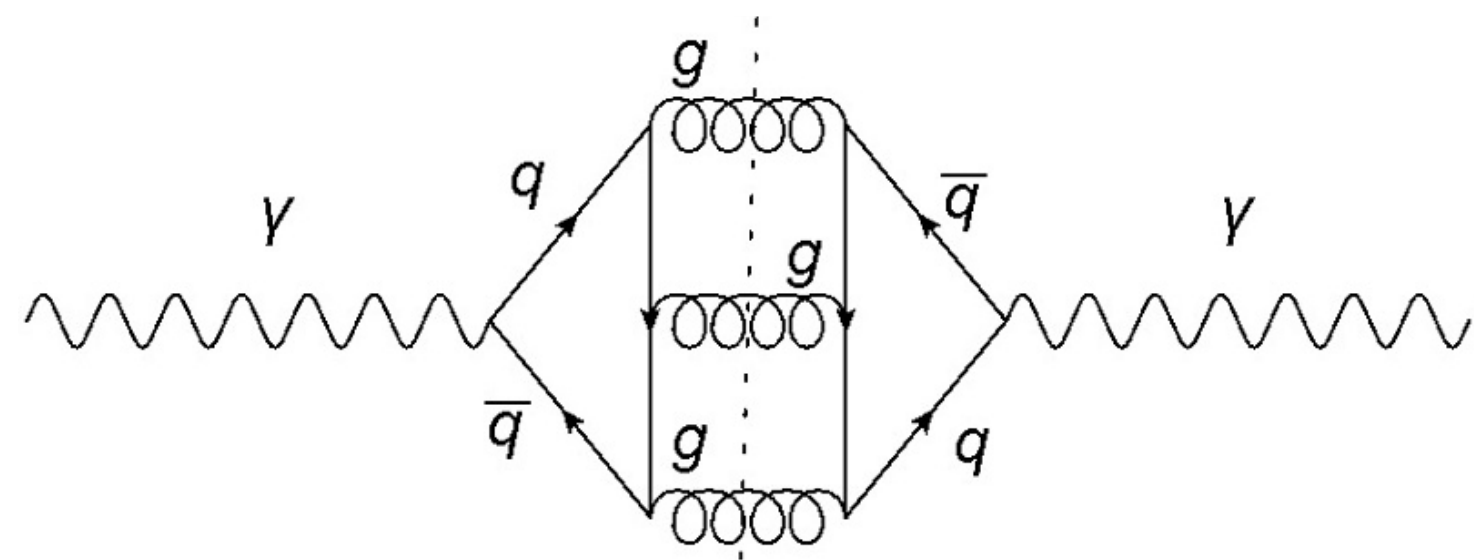
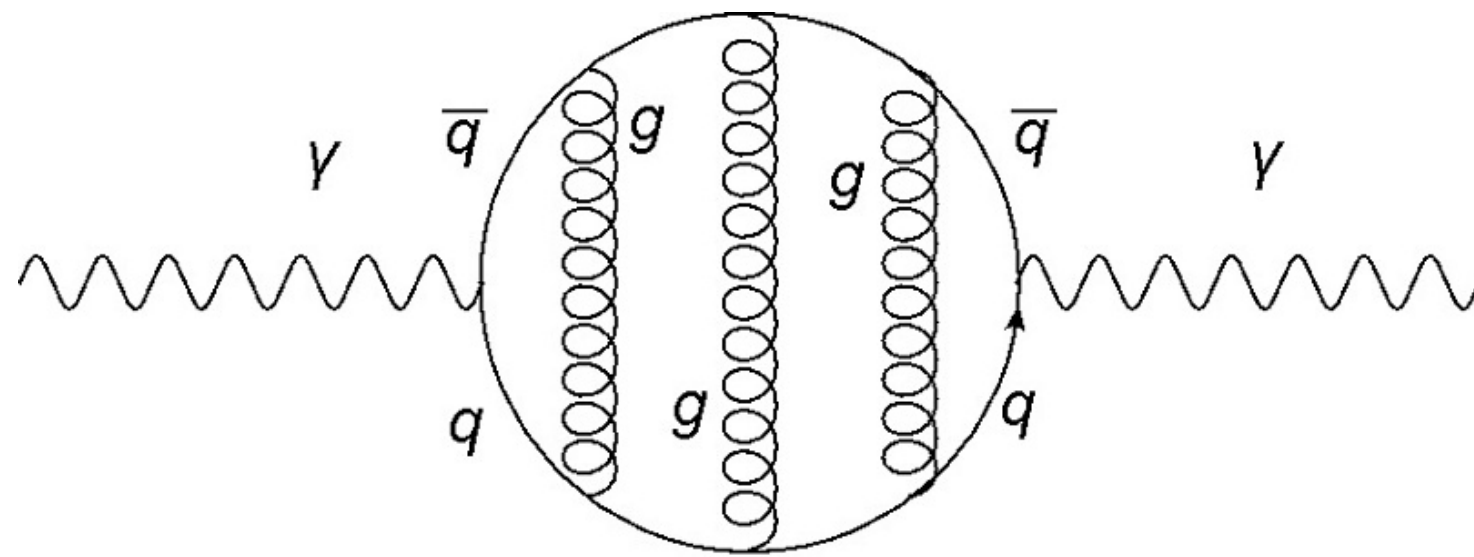
- $m_t = 172.74 \pm 0.25_{\text{uncorr.}} \pm 0.21_{\text{corr.}} \pm 0.32_{\text{QCD}} \text{ GeV} = 172.74 \pm 0.46 \text{ GeV}$
- somewhat larger shifts and smaller errors conceivable in the future
Butenschoen et al., PRL 117 (2016); Andreassen & Schwartz, JHEP 10 (2017)
- 2.8 σ discrepancy between lepton + jet channels from $D\bar{D}$ and CMS Run 2
- indirectly from EW fit: $m_t = 176.4 \pm 1.8 \text{ GeV} (2 \sigma)$ *Freitas & JE (PDG 2018)*

$\sin^2\theta_W(0)$: flavor separation

strange quark external current	ambiguous external current
ϕ	$K\bar{K}$ (non- ϕ)
$K\bar{K}\pi$ [almost saturated by $\phi(1680)$]	$K\bar{K}2\pi, K\bar{K}3\pi$
$\eta\phi$	$K\bar{K}\eta, K\bar{K}\omega$

- use of result for $\alpha(2 \text{ GeV})$ also needs isolation of strange contribution $\Delta_s\alpha$
- left column assignment assumes OZI rule
- expect right column to originate mostly from strange current ($m_s > m_{u,d}$)
- quantify expectation using averaged $\Delta_s(g_\mu-2)$ from lattices as Bayesian prior
RBC/UKQCD, JHEP 04 (2016); HPQCD, PRD 89 (2014)
- $\Delta_s\alpha(1.8 \text{ GeV}) = (7.09 \pm 0.32) \times 10^{-4}$ (threshold mass $\bar{m}_s = 342 \text{ MeV} \approx \bar{m}_s^{\text{disc}}$)

$\sin^2\theta_W(0)$: singlet separation



Ferro-Hernández & JE, JHEP 03 (2018)
 adapted from lattice $g_\mu-2$ calculation
RBC/UKQCD, PRL 116 (2016)

- use of result for $\alpha(2 \text{ GeV})$ needs singlet piece isolation $\Delta_{\text{disc}} \alpha(2 \text{ GeV})$
- then $\Delta_{\text{disc}} \bar{S}^2 = (\bar{S}^2 \pm 1/20) \Delta_{\text{disc}} \alpha(2 \text{ GeV}) = (-6 \pm 3) \times 10^{-6}$
- **step function** \Rightarrow singlet threshold mass $\bar{m}_s^{\text{disc}} \approx 350 \text{ MeV}$

α_s from τ decays

$$\tau_\tau = \hbar \frac{1 - \mathcal{B}_\tau^s}{\Gamma_\tau^e + \Gamma_\tau^\mu + \Gamma_\tau^{ud}} = 290.75 \pm 0.36 \text{ fs},$$

$$\Gamma_\tau^{ud} = \frac{G_F^2 m_\tau^5 |V_{ud}|^2}{64\pi^3} S(m_\tau, M_Z) \left(1 + \frac{3}{5} \frac{m_\tau^2 - m_\mu^2}{M_W^2} \right) \times$$

$$\left[1 + \frac{\alpha_s(m_\tau)}{\pi} + 5.202 \frac{\alpha_s^2}{\pi^2} + 26.37 \frac{\alpha_s^3}{\pi^3} + 127.1 \frac{\alpha_s^4}{\pi^4} + \frac{\hat{\alpha}}{\pi} \left(\frac{85}{24} - \frac{\pi^2}{2} \right) + \delta_{\text{NP}} \right]$$

- τ_τ result includes leptonic branching ratios
- $\mathcal{B}_{\tau^s} = 0.0292 \pm 0.0004$ ($\Delta S = -1$) *PDG 2018*
- $S(m_\tau, M_Z) = 1.01907 \pm 0.0003$ *JE, Rev. Mex. Fis. 50 (2004)*
- $\delta_{\text{NP}} = 0.003 \pm 0.009$ (within OPE & OPE breaking) based on (controversial)
Boito et al., PRD 85 (2012) & PRD 91 (2015); Davier et al., EPJC 74 (2014); Pich & Rodríguez-Sánchez, PRD 94 (2016)

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- dominant uncertainty from PQCD truncation (FOPT vs. CIPT vs. geometric continuation)
- $\alpha_s^{(4)}(m_\tau) = 0.323^{+0.018}_{-0.014}$
- $\alpha_s^{(5)}(M_Z) = 0.1184^{+0.0020}_{-0.0018}$
- updated from [Luo & JE, PLB 558 \(2003\)](#) in [Freitas & JE \(PDG 2018\)](#)