Effective Field Theories to All Orders

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Introduction

Motivation

- Effective field theories allow to simplify the physics when doing experiments at energy E small compared to the cutoff Λ

- The general structure is

$$\mathcal{L}_{\text{EFT}} = \sum_{n=0}^{\infty} \frac{1}{\Lambda^n} \sum_k c_{k,n} O_{k,n}$$

- $c_{k,n}$ are Wilson coefficients
- $O_{k,n}$ are EFT operators

- General Questions
- How to determine $c_{k,n}$?
- How to construct $O_{k,n}$?

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- How to construct $O_{k,n}$?

- Not just an academic question!

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- $|V_{cb}|$ extraction from inclusive B decays: OPE starts at dimension 3 current extractions use dimension 7 and 8 HQET operators [Gambino, Healey, Turczyk PLB **763**, 60 (2016)]

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Table 2
Default fit results: the second and third columns give the central values and standard deviations.

m_b^{kin}	4.546	0.021	r_1	0.032	0.024
$\overline{m}_c(3\text{ GeV})$	0.987	0.013	r_2	−0.063	0.037
μ_π^2	0.432	0.068	r_3	−0.017	0.025
μ_G^2	0.355	0.060	r_4	−0.002	0.025
ρ_D^3	0.145	0.061	r_5	0.001	0.025
ρ_{LS}^3	−0.169	0.097	r_6	0.016	0.025
$\overline{m}_1$	0.084	0.059	r_7	0.002	0.025
$\overline{m}_2$	−0.019	0.036	r_8	−0.026	0.025
$\overline{m}_3$	−0.011	0.045	r_9	0.072	0.044
$\overline{m}_4$	0.048	0.043	r_{10}	0.043	0.030
$\overline{m}_5$	0.072	0.045	r_{11}	0.003	0.025
$\overline{m}_6$	0.015	0.041	r_{12}	0.018	0.025
$\overline{m}_7$	−0.059	0.043	r_{13}	−0.052	0.031
$\overline{m}_8$	−0.178	0.073	r_{14}	0.003	0.025
$\overline{m}_9$	−0.035	0.044	r_{15}	0.001	0.025
χ^2/dof	0.46		r_{16}	0.001	0.025
$BR(\%)$	10.652	0.156	r_{17}	−0.028	0.025
$\mathbf{10^3\lvert V_{cb}\rvert}$	**42.11**	**0.74**	r_{18}	−0.001	0.025

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1) Current applications require higher dimensional operators:

- Applications of NRQED to proton structure effects in spectroscopy require Wilson coefficients of operators of dimension 5,6, and 7 [Hill, GP, PRL **107** 160402 (2011)]

$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \ldots$$

- Dimension 5 operator: $a_p = 1.793$
- Dimension 6 operator: $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm
- Dimension 7 operators: $r_M = 0.776(34)(17)$ fm, $\bar{\beta} = 2.5(4) \times 10^{-4}$ fm^3

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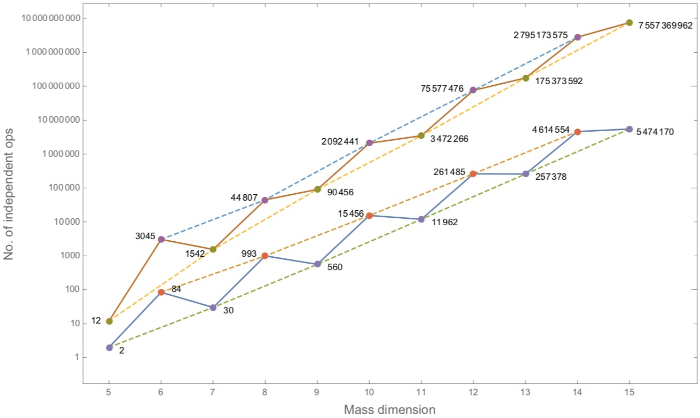
$$\mathcal{L}_{\text{EFT}} = \sum_{n=0}^{\infty} \frac{1}{\Lambda^n} \sum_k c_{k,n} O_{k,n}$$

2) The structure of the SMEFT is simpler than expected
[Henning, Lu, Melia, Murayama, JHEP **1708**, 016 (2017)]



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- Dimension 6:
  Buchmüller et al. '86 $\to$ Grzadkowski et al. '10

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- HQET/NRQCD
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    "*Two terms added to Lagrangian*"
  - Dimension 7,8: Paz et al. arXiv:1702.0890 v1 → v2
    "*discussion of operators with multiple color structures was added*"

Goal of this talk

- Describe the constructions of higher dimensional operators for the closely related EFTs
- Heavy Quark Effective Theory (HQET)
- Non Relativistic Quantum Electrodynamics (NRQED)
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    HQET for B decays : Perturbative c, Non-perturbative $\langle O \rangle$

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- Might have useful lessons to SMEFT

Topics not discussed here: Power Counting

- We construct operators based on their dimension

- Sometimes the dimensional counting is not be appropriate, e.g.

$$\mathcal{L}_{NRQCD}^{kinetic} = \psi^\dagger iD_t\psi + \psi^\dagger \frac{\boldsymbol{D}^2}{2M}\psi$$

$$\mathcal{L}_{HQET}^{kinetic} = \bar{h}\, iv\cdot D\, h$$

- Lagrangians can be related by

  $h \to \psi$

  Choosing $v = (1,0,0,0)$

- But different kinetic term and power counting

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\- But different kinetic term and power counting

- Not a problem: construct $\mathcal{L}$ with arbitrary dimension and power count later

Topics not discussed here: Wilson coefficients

- Matching
- Perturbative:
  Matching from QCD gives NRQCD (HQET) Wilson coefficients
- Non-perturbative:
  For proton structure NRQED Wilson coefficients determined by proton magnetic moment, proton charge radius etc.
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- Power counting
  The Wilson coefficients can cause terms to be suppressed or enhanced

Outline

- Introduction
- A little bit of history
- HQET and NRQCD (NRQED) operators at dimension 8 and above: General Method
- HQET and NRQCD (NRQED) operators at dimension 8 and above: Applications
- Conclusions and Outlook

A little bit of history

A tale of two effective field theories

- HQET: Heavy Quark Effective Theory

$$\mathcal{L}_{HQET}^{kinetic} = \bar{h}\, iv \cdot D\, h$$

- NRQCD: Non Relativistic Quantum Chromodynamics
  (NRQED: Non Relativistic Quantum Electrodynamics)

$$\mathcal{L}_{NRQCD}^{kinetic} = \psi^\dagger iD_t \psi + \psi^\dagger \frac{\boldsymbol{D}^2}{2M}\psi$$

- Different kinetic term and power counting

- Lagrangians can be related by
  - $h \to \psi$
  - Choosing $v = (1, 0, 0, 0)$

- The relation is not as well known as it should be

Prehistory

$$D_t = \frac{\partial}{\partial t} + ieA^0, \quad \boldsymbol{D} = \boldsymbol{\nabla} - ie\boldsymbol{A}$$

- Schrödinger equation: $iD_t + \frac{\boldsymbol{D}^2}{2M}$ (1926)
- Hydrogen Fine Structure:

Spin-Orbit:	$\boldsymbol{\sigma} \cdot \boldsymbol{B}$	(1927)
Relativistic correction:	$\boldsymbol{D}^4$	(1905?)
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  Darwin term: $\boldsymbol{\partial} \cdot \boldsymbol{E}$ (1928)
- Organize operators in Lagrangian form
- The dim=5,6 were given in [Caswell, Lepage PLB **167**, 437 (1986)]

$$\mathcal{L}^{\text{dim}=5,6}_{\text{NRQED}} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2M} + \frac{\boldsymbol{D}^4}{8M^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8M^2} + ic_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8M^3} \Big\} \psi$$

Dimension 5 HQET operators

- Dimension 5 HQET operators were considered in

$$\mathcal{L}_{\text{HQET}}^{\text{dim}=5} = \bar{h}\, iv \cdot D\, h + \frac{\bar{h}\,(iD)^2\, h}{2M} + c_F g \frac{\bar{h}\sigma_{\mu\nu} G^{\mu\nu}\, h}{4M}$$

[Falk, Grinstein, Luke, NPB 357, 185 (1991)]

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- We can see the analogy between HQET and NRQED

	NRQED (1920's-1980's)	HQET(1990's)
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- What about higher dimensional operators?

Dimension 5 and 6 HQET operators

- First systematic discussion of HQET operators
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  Since $iv \cdot Dh = 0$

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- Dimension 3: $\bar{h}h$
- Dimension 4: $\bar{h}iD^\mu h \to 0$
- Dimension 5: Two operators $\bar{h}\, iD^{\mu_1} iD^{\mu_2} h, \quad \bar{h}iD^{\mu_1} iD^{\mu_2} s^\lambda h$
- Dimension 6: Two operators $\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} h, \quad \bar{h}iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} s^\lambda h$

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$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)h|B(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]\frac{1}{3}\lambda_1$$

$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\beta)s_\lambda h|B(v)\rangle = 2M_H d_H i\varepsilon_{\nu\alpha\beta\lambda}v^\nu \frac{1}{6}\lambda_2$$

- Dimension 6:

$$\langle B(v)|\bar{h}(iD_\alpha)(iD_\mu)(iD_\beta)h_v|B(v)\rangle = 2M_H[g_{\alpha\beta} - v_\alpha v_\beta]v_\mu \frac{1}{3}\rho_1$$

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- Same paper counted operators beyond dimension 6 but unfortunately it is wrong

Dimension 7 NRQCD operators

- The dimension 7 operators listed in [Manohar PRD **56**, 230 (1997)]

$$\mathcal{L}^{\text{dim}=7}_{\text{NRQCD}} = \psi^\dagger \Big\{ \frac{\boldsymbol{D}^4}{8M^3} + ic_M g \frac{\{\boldsymbol{D}^i, [\boldsymbol{\partial} \times \boldsymbol{B}]^i\}}{8m_p^3}$$

$$+c_{A1} g^2 \frac{\left(\boldsymbol{B}^i_a \boldsymbol{B}^i_b - \boldsymbol{E}^i_a \boldsymbol{E}^i_b\right) \; T^a T^b}{8M^3} - c_{A2} g^2 \frac{\boldsymbol{E}^i_a \boldsymbol{E}^i_b \; T^a T^b}{16M^3}$$

$$+c_{A3} g^2 \frac{\left(\boldsymbol{B}^i_a \boldsymbol{B}^i_b - \boldsymbol{E}^i_a \boldsymbol{E}^i_b\right) \delta^{ab}}{8M^3} - c_{A4} g^2 \frac{\boldsymbol{E}^i_a \boldsymbol{E}^i_b \, \delta^{ab}}{16M^3}$$

$$+c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma} \cdot \boldsymbol{B}\}}{8M^3} - c_{W2} g \frac{D^i \boldsymbol{\sigma} \cdot \boldsymbol{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma} \cdot \boldsymbol{D} \boldsymbol{B} \cdot \boldsymbol{D} + \boldsymbol{D} \cdot \boldsymbol{B} \boldsymbol{\sigma} \cdot \boldsymbol{D}}{8m_p^3}$$

$$-c_{B1} g^2 \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{B}_a \times \boldsymbol{B}_b - \boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{E}_a \times \boldsymbol{E}_b) f^{abc} T^c}{16M^3} \Big\} \psi$$

Dimension 7 NRQCD operators

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- Comments:
- Explicit color structures are taken from [Gunawardna, GP JHEP **1707** 137 (2017)]
- Last line vanishes for NRQED but not for NRQCD

Dimension 7 HQET operators

- In 2010 Mannel, Turczyk, and Uraltsev calculated the contribution of dimension 7 & 8 HQET operators to inclusive semileptonic B decays [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

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- Dimension 7 inclusive semileptonic B decays need

- 4 Spin Independent (SI) operators
- 5 Spin Dependent (SD) operators
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- Dimension 7 NRQCD Lagrangian

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- Why the difference?

- No systematic derivation in either source

Dimension 8 HQET/NRQED operators

- Dimension 8 inclusive semileptonic B decays need
  7 SI operators and 11 SD operators
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]

Dimension 8 HQET/NRQED operators

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- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$
\begin{aligned}
\mathcal{L}^{\text{dim}=8}_{\text{NRQED}} = \psi^\dagger \Big\{ & c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{D}]}{M^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{M^4} \\
& + c_{X3} g \frac{[\boldsymbol{\partial}^2 \boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + i c_{X4} g^2 \frac{\{\boldsymbol{D}^i, [\boldsymbol{E}\times\boldsymbol{B}]^i\}}{M^4} \\
& + i c_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) D^i}{M^4} + i c_{X6} g \frac{\epsilon^{ijk} \sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{M^4} \\
& + c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{M^4} + c_{X8} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{M^4} + c_{X9} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{M^4} \\
& + c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{M^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{M^4} + c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t \boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{M^4} \Big\} \psi
\end{aligned}
$$

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- 4 SI operators and 8 SD operators
- Missing operators are presumably NRQCD operators

Dimension 8 HQET/NRQED operators

- The dim=8 NRQED Lagrangian was given in [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

- Lagrangian can be constructed by considering all possible combinations of iD_t, $i\boldsymbol{D}$, $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{\sigma}$ that are
- Rotationally invariant
- P and T even
- Hermitian

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- Such construction of $\mathcal{L}_{\text{NRQED}}^{\text{dim}\leq 6}$ was demonstrated in [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

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  - Are operators linearly independent?
  - How many linearly independent operators?

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- Are operators linearly independent?
- How many linearly independent operators?

- Is there an easier way?

HQET (and NRQCD) operators at dimension 8 and above: General Method

[Ayesh Gunawardna, GP JHEP **1707** 137 (2017)]

General method

- My graduate student Ayesh Gunawardana and I looked at this problem in [Gunawardna, GP JHEP **1707** 137 (2017)]
- Following [Mannel, PRD 50, 428 (1994)]
  we considered matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle$

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- We used the constraints
  - Orthogonality of v to μ_1, μ_n, λ [Mannel, PRD 50, 428 (1994)]
  - Parity and Time reversal symmetry
  - Hermitian conjugation
  - Four dimensions
  - Possible multiple color structures [Kobach, Pal PLB **772** 225 (2017)]

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- To decompose them in terms of the tensors
  - v^{μ_i}, $\Pi^{\mu\nu} = g^{\mu\nu} - v^{\mu}v^{\nu}$, $\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$

General method: Orthogonality

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- Since $iv\cdot Dh = 0$
- $v_{\mu_1}\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n}(s^{\lambda})h = 0$
- $v_{\mu_n}\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n}(s^{\lambda})h = 0$
- $v_{\lambda}\bar{h}\, iD^{\mu_1}\dots\, iD^{\mu_n}(s^{\lambda})h = 0$
  [Mannel, PRD 50, 428 (1994)]

- More accurately, the $1/M$ corrections to $iv\cdot Dh = 0$ give rise to higher dimensional operators. One can impose this order by order.

- Similarly for NRQCD (NRQED):
  $\psi^{\dagger}\left(iD_t O + OiD_t\right)\psi/M^n$ can be eliminated by $\psi\to\psi - O\psi/M^n$
  [GP, Mod. Phys. Lett. A 30, 1550128 (2015)]

General method: PT symmetry

- We consider matrix elements of the form
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- Parity and Time reversal are symmetries of HQET
  In particular under PT:

- $p = (p^0, \vec{p}) \stackrel{PT}{\rightarrow} (p^0, \vec{p}) = p \Rightarrow v = p/m \stackrel{PT}{\rightarrow} v$
- $iD^\mu \stackrel{PT}{\rightarrow} iD^\mu$
- $\bar{h}h \stackrel{PT}{\rightarrow} \bar{h}h$
- $\bar{h}s^\lambda h \stackrel{PT}{\rightarrow} -\bar{h}s^\lambda h$

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- Since T is anti-linear

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle \stackrel{PT}{=} \langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} h|H\rangle^*$$

$$\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle \stackrel{PT}{=} -\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} s^{\lambda} h|H\rangle^*$$

- SI matrix elements are real, SD matrix elements are imaginary

General method: Hermitian conjugation

- We consider matrix elements of the form
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- $\bar{h}h$, $\bar{h}s^\lambda h$, iD^μ are hermitian
  using Hermitian conjugation

$$\begin{aligned}\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h|H\rangle &= \langle H|\left(\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n}(s^\lambda)h\right)^\dagger|H\rangle^* \\ &= \langle H|\bar{h}\, iD^{\mu_n}\ldots\, iD^{\mu_1}(s^\lambda)h|H\rangle^*\end{aligned}$$

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- Combining with the PT constraints
  Under inversion of the indices:
- SI matrix elements are symmetric
- SD matrix elements are anti-symmetric

General method: Tensor decomposition

- We consider matrix elements of the form
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  Define $\Pi^{\mu\nu} = g^{\mu\nu} - v^{\mu} v^{\nu}$
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  For the standard choice of $v = (1, 0, 0, 0)$: $\epsilon^{\rho\sigma\alpha\beta}v_{\rho} \to \epsilon^{ijk}$

- Matrix element depend on $v^{\mu_i}, \Pi^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$

General method: Four dimensions

- We consider matrix elements of the form
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- Four dimensions $\Rightarrow$ only four independent directions

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- Four dimensions $\Rightarrow$ only four independent directions

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- Example: for dimension 7 SD HQET operators
  need $\Pi^{\mu\nu}\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$: three indices are the same
  Tensors obtained by permuting indices are not linearly independent

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- Example: for dimension 7 SD HQET operators
  need $\Pi^{\mu\nu}\epsilon^{\rho\sigma\alpha\beta}v_{\rho}$: three indices are the same
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- Example: for dimension 11 SI HQET operators
  need $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6}\Pi^{\mu_7\mu_8}$: four indices are the same

General method: Color factors

- We consider matrix elements of the form
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  $\langle H|\bar{h}\, iD^{\mu_1} \ldots iD^{\mu_n} s^\lambda h|H\rangle$

- Starting at dimension 7 we can have multiple color factors
  E.g. consider $\psi^\dagger E_a^i T^a E_b^i T^b \psi$ [Kobach, Pal PLB **772** 225 (2017)]
  $\left\{T^a, T^b\right\} = \frac{1}{3}\delta^{ab} + d^{abc} T^c \Rightarrow$ two color structures

General method: Color factors

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- Use basis $\left\{T^a, T^b\right\}$ and δ^{ab}:
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- Decomposition of $\langle H|\bar{h}\, iD^{\mu_1} \ldots\, iD^{\mu_n} (s^{\lambda}) h|H\rangle$ does not distinguish $\{T^a, T^b\}$ from δ^{ab}. Need to be put "by hand".

General method: Summary

- We consider matrix elements of the form
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} h|H\rangle$
  $\langle H|\bar{h}\, iD^{\mu_1}\ldots\, iD^{\mu_n} s^\lambda h|H\rangle$

- We express them in terms of $v^{\mu_i}, \Pi^{\mu_i\mu_j}$, and $\epsilon^{\rho\sigma\alpha\beta}v_\rho$ using
- Orthogonality: $v_{\mu_1} = v_{\mu_n} = v_\lambda = 0$
- P, T, and Hermitian conjugation:
  SI (SD) matrix elements are sym. (anti-sym.) under inversion
- Four dimensions:
  not all tensors are linearly independent
- Checking possible multiple color structures

HQET (and NRQCD) operators at dimension 8 and above: Applications

[Ayesh Gunawardna, GP JHEP **1707** 137 (2017)]

Results: SI Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4}\, h|H\rangle$
  It can depend on $v^{\mu_i}, \Pi^{\mu_i \mu_j}$,

- We can have $\Pi\,\Pi$:

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- or $\Pi\, vv$: $\quad \Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} h|H\rangle = a_{12}^{(7)}\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4} + a_{13}^{(7)}\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4} + $$
$$+ a_{14}^{(7)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3} + b^{(7)}\Pi^{\mu_1\mu_4} v^{\mu_2} v^{\mu_3}$$

Notice that the tensors are symmetric under inversion of indices

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Results: SI Dimension 7 HQET operators

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  - We can have $\Pi\,\Pi$: $\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}$ $\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}$ $\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}$
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  Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$
  $\Rightarrow$ 2 op. with 2 color structures: 6 in total but only 4 at tree level

Results: SI Dimension 7 HQET operators

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  Contracting with tensors above: $a_{13}^{(7)} - a_{14}^{(7)}$ and $b^{(7)}$
  $\Rightarrow$ 2 op. with 2 color structures: 6 in total but only 4 at tree level

- Explains 4 HQET SI op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 6 NRQCD SI in [Manohar PRD **56**, 230 (1997)]

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, s^{\lambda}\, h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda}v_\rho$
  The 2 other indices can be

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, s^{\lambda}\, h|H\rangle$

  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda}v_\rho$

  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda}v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i}v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle = i\tilde{a}^{(7)}_{12}\left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right)$$
$$+ \quad i\tilde{a}^{(7)}_{13}\left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$
$$+ \quad i\tilde{a}^{(7)}_{14}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}^{(7)}_{23}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho$$

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  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle = i\tilde{a}_{12}^{(7)}\left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right)$$
$$+ \quad i\tilde{a}_{13}^{(7)}\left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$
$$+ \quad i\tilde{a}_{14}^{(7)}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda}v_\rho + i\tilde{a}_{23}^{(7)}\Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho + i\tilde{b}^{(7)}v^{\mu_2}v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho$$

- Multiple color structure arise from $\bar{h}\left\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\right\} h$
  Contractions with tensors above give *no* contribution

Results: SD Dimension 7 HQET operators

- We look at $\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle$
  By parity it must contain $\epsilon^{\rho\mu_k\mu_l\lambda} v_\rho$
  The 2 other indices can be $\Pi^{\mu_i\mu_j}$ or $v^{\mu_i} v^{\mu_j}$
  The tensors must also be anti-symmetric under inversion of indices

$$
\begin{aligned}
\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} s^\lambda h|H\rangle &= i\tilde{a}_{12}^{(7)} \left(\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_3\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda} v_\rho\right) \\
&+ \quad i\tilde{a}_{13}^{(7)} \left(\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_2\mu_4\lambda} v_\rho - \Pi^{\mu_4\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda} v_\rho\right) + \\
&+ \quad i\tilde{a}_{14}^{(7)} \Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_3\lambda} v_\rho + i\tilde{a}_{23}^{(7)} \Pi^{\mu_2\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda} v_\rho + i\tilde{b}^{(7)} v^{\mu_2} v^{\mu_3}\epsilon^{\rho\mu_1\mu_4\lambda} v_\rho
\end{aligned}
$$

- Multiple color structure arise from $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\, h$
  Contractions with tensors above give *no* contribution

- Explains 5 HQET SD op. in [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)] and 5 NRQCD SD in [Manohar PRD **56**, 230 (1997)]

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

- Multiple color structures arise from
  - $\bar{h}\,\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\}\, h$: 20 possibilities
  - $\bar{h}\,\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}\, h$: 15 possibilities

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
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- Multiple color structures arise from
  - $\bar{h}\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\}\, h$: 20 possibilities
  - $\bar{h}\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\}\, h$: 15 possibilities

  Contractions with tensors above give 1 contribution

  $\Rightarrow$ 1 op. with 2 color structures: 8 in total but only 7 at tree level

Results: SI Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} h|H\rangle = a_{12}^{(8)}\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}v^{\mu_4} + \Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_5}v^{\mu_2}\right) +$$
$$a_{13}^{(8)}\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}v^{\mu_4} + \Pi^{\mu_3\mu_5}\Pi^{\mu_1\mu_4}v^{\mu_2}\right) + a_{15}^{(8)}\left(\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_4}v^{\mu_2} + \Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}v^{\mu_4}\right) +$$
$$b_{12}^{(8)}\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_5}v^{\mu_3} + b_{14}^{(8)}\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}v^{\mu_3} + b_{15}^{(8)}\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}v^{\mu_3} +$$
$$c^{(8)}\Pi^{\mu_1\mu_5}v^{\mu_2}v^{\mu_3}v^{\mu_4}$$

- Multiple color structures arise from
  - $\bar{h}\left\{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, [iD^{\mu_l}, iD^{\mu_m}]]\right\} h$: 20 possibilities
  - $\bar{h}\left\{iD^{\mu_m}, \{[iD^{\mu_i}, iD^{\mu_j}], [iD^{\mu_k}, iD^{\mu_l}]\}\right\} h$: 15 possibilities

  Contractions with tensors above give 1 contribution

  $\Rightarrow$ 1 op. with 2 color structures: 8 in total but only 7 at tree level

- Explains 7 HQET dimesion 8 SI operators in
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  The new operator will be listed below

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\,iD^{\mu_1}\,iD^{\mu_2}\,iD^{\mu_3}\,iD^{\mu_4}\,iD^{\mu_5}\,s^{\lambda}\,h|H\rangle =$$

$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{a}_{15}^{(8)}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho .$$

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, s^{\lambda}\, h|H\rangle =$$

$$i\tilde{a}^{(8)}_{12}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}^{(8)}_{14}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{a}^{(8)}_{15}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}^{(8)}_{24}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+ i\tilde{b}^{(8)}_{13}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{14}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}^{(8)}_{15}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{34}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+ i\tilde{b}^{(8)}_{35}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}^{(8)}_{45}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+ i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

- Checking for multiple color structures as before
  Contractions with tensors above give 6 contributions
  $\Rightarrow$ 6 op. with 2 color structures: 17 in total but only 11 at tree level

Results: SD Dimension 8 HQET operators

- Using the general method

$$\frac{1}{2M_H}\langle H|\bar{h}\,iD^{\mu_1}\,iD^{\mu_2}\,iD^{\mu_3}\,iD^{\mu_4}\,iD^{\mu_5}\,s^{\lambda}\,h|H\rangle =$$

$$i\tilde{a}_{12}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_2}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{a}_{14}^{(8)}\left(v^{\mu_3}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_2\mu_5\lambda}v_\rho - v^{\mu_3}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_4\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{a}_{15}^{(8)}v^{\mu_3}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_2\mu_4\lambda}v_\rho + i\tilde{a}_{24}^{(8)}v^{\mu_3}\Pi^{\mu_2\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho +$$

$$+i\tilde{b}_{13}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_3}\epsilon^{\rho\mu_4\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_3}\epsilon^{\rho\mu_2\mu_1\lambda}v_\rho\right) + i\tilde{b}_{14}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_4}\epsilon^{\rho\mu_3\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_5\mu_2}\epsilon^{\rho\mu_3\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{15}^{(8)}\left(v^{\mu_2}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_1\mu_5}\epsilon^{\rho\mu_3\mu_2\lambda}v_\rho\right) + i\tilde{b}_{34}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_2}\epsilon^{\rho\mu_5\mu_1\lambda}v_\rho\right) +$$

$$+i\tilde{b}_{35}^{(8)}\left(v^{\mu_2}\Pi^{\mu_3\mu_5}\epsilon^{\rho\mu_1\mu_4\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_3\mu_1}\epsilon^{\rho\mu_5\mu_2\lambda}v_\rho\right) + i\tilde{b}_{45}^{(8)}\left(v^{\mu_2}\Pi^{\mu_4\mu_5}\epsilon^{\rho\mu_1\mu_3\lambda}v_\rho - v^{\mu_4}\Pi^{\mu_2\mu_1}\epsilon^{\rho\mu_5\mu_3\lambda}v_\rho\right) +$$

$$+i\tilde{c}^{(8)}v^{\mu_2}v^{\mu_3}v^{\mu_4}\epsilon^{\rho\mu_1\mu_5\lambda}v_\rho.$$

- Checking for multiple color structures as before
  Contractions with tensors above give 6 contributions
  $\Rightarrow$ 6 op. with 2 color structures: 17 in total but only 11 at tree level

- Explains 11 HQET dimesion 8 SD operators in
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]
  The new operators will be listed below

New Result: Dimension 8 NRQCD Lagrangian

- We can now list the dimension 8 NRQCD Lagrangian
  [Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

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- We can now list the dimension 8 NRQCD Lagrangian
  [Gunawardna, GP JHEP **1707** 137 (2017), Kobach, Pal PLB **772** 225 (2017)]

$$\mathcal{L}_{\text{NRQCD}}^{\text{dim}=8} = \psi^\dagger \Big\{ \ldots c_{X1} g \frac{[\boldsymbol{D}^2, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{D}^i, \boldsymbol{E}^i]\}}{m_p^4} + c_{X3} g \frac{[\boldsymbol{D}^i, [\boldsymbol{D}^i, [\boldsymbol{D}^j, \boldsymbol{E}^j]]]}{m_p^4}$$
$$+ i c_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \,\{T^a, T^b\}\}}{2M^4} + i c_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \,\delta^{ab}\}}{m_p^4} + i c_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{m_p^4}$$
$$+ i c_{X6} g \frac{\epsilon^{ijk} \sigma^i \boldsymbol{D}^j [\boldsymbol{D}^l, \boldsymbol{E}^l] \boldsymbol{D}^k}{m_p^4} + c_{X7a}\, g^2 \frac{\{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a T^a, [\boldsymbol{D}^i, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X7b}\, g^2 \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}_a [\boldsymbol{D}^i, \boldsymbol{E}^i]_a}{m_p^4}$$
$$+ c_{X8a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{m_p^4} + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4}$$
$$+ c_{X9b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_a}{m_p^4} + c_{X10a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_b T^b\}}{2M^4} + c_{X10b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i]_a}{m_p^4}$$
$$+ c_{X11a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_b T^b\}}{2M^4} + c_{X11b}\, g^2 \frac{\boldsymbol{B}^i_a [\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{E}^i]_a}{m_p^4} + \tilde{c}_{X12a}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a \,[D_t, \boldsymbol{E}^k]_b \,\{T^a, T^b\}}{2M^4}$$
$$+ \tilde{c}_{X12b}\, g^2 \frac{\epsilon^{ijk} \boldsymbol{\sigma}^i \boldsymbol{E}^j_a \,[D_t, \boldsymbol{E}^k]_a}{m_p^4} + i c_{X13} g^2 \frac{[\boldsymbol{E}^i, [D_t, \boldsymbol{E}^i]]}{m_p^4} + i c_{X14} g^2 \frac{[\boldsymbol{B}^i, (\boldsymbol{D} \times \boldsymbol{E} + \boldsymbol{E} \times \boldsymbol{D})^i]}{m_p^4}$$
$$+ i c_{X15} g^2 \frac{[\boldsymbol{E}^i, (\boldsymbol{D} \times \boldsymbol{B} + \boldsymbol{B} \times \boldsymbol{D})^i]}{m_p^4} + c_{X16} g^2 \frac{[\boldsymbol{\sigma} \cdot \boldsymbol{B}, \{\boldsymbol{D}^i, \boldsymbol{E}^i\}]}{m_p^4} + c_{X17} g^2 \frac{[\boldsymbol{B}^i, \{\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}\}]}{m_p^4} + c_{X18} g^2 \frac{[\boldsymbol{E}^i, \{\boldsymbol{\sigma} \cdot \boldsymbol{D}, \boldsymbol{B}^i\}]}{m_p^4} \Big\} \psi$$

  - 25 operators
  - c_{Xib} start at $\mathcal{O}(\alpha_s)$

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1}\, iD^{\mu_2}\, iD^{\mu_3}\, iD^{\mu_4}\, iD^{\mu_5}\, iD^{\mu_6}\, h|H\rangle = a^{(9)}_{12,34}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} +$$

$$+a^{(9)}_{12,35}\,\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6}+\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}\right)+a^{(9)}_{12,36}\,\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}\right)+$$

$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6}+a^{(9)}_{13,26}\,\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}\right)+a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6}+$$

$$+a^{(9)}_{14,26}\,\left(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}\right)+a^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4}+a^{(9)}_{16,23}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5}+$$

$$+a^{(9)}_{16,24}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5}+a^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4}+b^{(9)}_{12,36}\,\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}\right)+$$

$$+b^{(9)}_{12,46}\,\left(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}\right)+b^{(9)}_{12,56}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4}+$$

$$+b^{(9)}_{13,26}\,\left(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_3}\right)+b^{(9)}_{13,46}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_5}+$$

$$+b^{(9)}_{14,26}\,\left(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_4}\right)+b^{(9)}_{14,36}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_5}+b^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_4}+$$

$$+b^{(9)}_{16,23}\,\left(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5}v^{\mu_2}v^{\mu_3}\right)+b^{(9)}_{16,24}\,\left(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5}v^{\mu_2}v^{\mu_4}\right)+$$

$$+b^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}v^{\mu_3}v^{\mu_4}+b^{(9)}_{16,34}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4}v^{\mu_2}v^{\mu_5}+c^{(9)}\,\Pi^{\mu_1\mu_6}v^{\mu_2}v^{\mu_3}v^{\mu_4}v^{\mu_5}$$

New Result: Dimension 9 HQET operators

- Using the general method: SI Dimension 9 HQET operators

$$\frac{1}{2M_H}\langle H|\bar{h}\, iD^{\mu_1} iD^{\mu_2} iD^{\mu_3} iD^{\mu_4} iD^{\mu_5} iD^{\mu_6}\, h|H\rangle = a^{(9)}_{12,34}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_4}\Pi^{\mu_5\mu_6} +$$
$$+a^{(9)}_{12,35}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_5}\Pi^{\mu_4\mu_6}+\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_4}\Pi^{\mu_5\mu_6}) + a^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_3}\Pi^{\mu_5\mu_6}) +$$
$$+a^{(9)}_{13,25}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_5}\Pi^{\mu_4\mu_6} + a^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}\Pi^{\mu_4\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_6}) + a^{(9)}_{14,25}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_6} +$$
$$+a^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_6}) + a^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}\Pi^{\mu_3\mu_4} + a^{(9)}_{16,23}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}\Pi^{\mu_4\mu_5} +$$
$$+a^{(9)}_{16,24}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}\Pi^{\mu_3\mu_5} + a^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}\Pi^{\mu_3\mu_4} + b^{(9)}_{12,36}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_3\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_4}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_3}) +$$
$$+b^{(9)}_{12,46}\,(\Pi^{\mu_1\mu_2}\Pi^{\mu_4\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_3}\Pi^{\mu_5\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{12,56}\,\Pi^{\mu_1\mu_2}\Pi^{\mu_5\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{13,26}\,(\Pi^{\mu_1\mu_3}\Pi^{\mu_2\mu_6}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{13,46}\,\Pi^{\mu_1\mu_3}\Pi^{\mu_4\mu_6}v^{\mu_2}v^{\mu_5} +$$
$$+b^{(9)}_{14,26}\,(\Pi^{\mu_1\mu_4}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_5}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_4}) + b^{(9)}_{14,36}\,\Pi^{\mu_1\mu_4}\Pi^{\mu_3\mu_6}v^{\mu_2}v^{\mu_5} + b^{(9)}_{15,26}\,\Pi^{\mu_1\mu_5}\Pi^{\mu_2\mu_6}v^{\mu_3}v^{\mu_4} +$$
$$+b^{(9)}_{16,23}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_3}v^{\mu_4}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_4\mu_5}v^{\mu_2}v^{\mu_3}) + b^{(9)}_{16,24}\,(\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_4}v^{\mu_3}v^{\mu_5}+\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_5}v^{\mu_2}v^{\mu_4}) +$$
$$+b^{(9)}_{16,25}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_2\mu_5}v^{\mu_3}v^{\mu_4} + b^{(9)}_{16,34}\,\Pi^{\mu_1\mu_6}\Pi^{\mu_3\mu_4}v^{\mu_2}v^{\mu_5} + c^{(9)}\,\Pi^{\mu_1\mu_6}v^{\mu_2}v^{\mu_3}v^{\mu_4}v^{\mu_5}$$

- There are also multiple color structures
  Arise from combining pure color octets:
  $[iD^{\mu_i}, iD^{\mu_j}],\quad [iD^{\mu_i},[iD^{\mu_j}, iD^{\mu_k}]],\quad [iD^{\mu_i},[iD^{\mu_j},[iD^{\mu_k}, iD^{\mu_l}]]]$
  For phenomenological applications at the current level of precision only T^aT^b is needed

Results: Relating different bases

- The method allows to easily relate different bases
- Dimension 7: Manohar '97 to Mannel-Turczyk-Uraltsev '10
- Dimension 8: Mannel-Turczyk-Uraltsev '10 to Hill, Lee, GP, Solon '12

(See also [Heinonen, Mannel, arXiv:1609.01334])

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(See also [Heinonen, Mannel, arXiv:1609.01334])

- Useful since even simple quantities can depend on multiple operators e.g., B meson PDF, $S(\omega)$:

$$2M_B \int d\omega\, \omega^k\, S(\omega) = n_{\mu_1}...n_{\mu_k} \langle \bar{B}(v)|\bar{h}\, iD^{\mu_1}...iD^{\mu_k}\, h|\bar{B}(v)\rangle$$

Its fifth moment

$$\int d\omega\, \omega^5\, S(\omega) = \left(-8r_1 + 2r_2 + 2r_3 + 2r_4 + r_5 + r_6 + r_7\right)/15$$

Comparison to Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators with dim≤ 8

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- Still, a useful check

Conclusions and Outlook

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- The question of higher dimensional NRQED/HQET/NRQCD operators has roots in the early days of quantum mechanics
- The first discussion of number and identity of such operators was in [Mannel PRD **50**, 428 (1994)]
- In 2017 the problem was solved in
  - [Gunawardna, GP JHEP **1707** 137 (2017)]
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- An explicit construction [Gunawardna, GP JHEP **1707** 137 (2017)]
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Thank you!

Backup

Non perturbative Wilson Coefficients

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)|J^\mu|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2M}F_2(q^2)q_\nu\right]u(p_i)$$

- Define $D_t = \frac{\partial}{\partial t} + ieZA^0, \quad \boldsymbol{D} = \boldsymbol{\nabla} - ieZ\boldsymbol{A}$

  NRQED Lagrangian up to order $1/M^2$:

$$\mathcal{L} = \psi^\dagger\left\{iD_t + \frac{\boldsymbol{D}^2}{2M} + c_F e\frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D e\frac{[\boldsymbol{\nabla}\cdot\boldsymbol{E}]}{8M^2} + ic_S e\frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8M^2}\right\}\psi + \cdots$$

- Non perturbative matching
  - Order $1/M^0$: $Z = F_1(0)$
  - Order $1/M$: $c_F = F_1(0) + F_2(0)$,
  - Order $1/M^2$: $c_D = F_1(0) + 2F_2(0) + 8M^2F_1'(0)$, $c_S = F_1(0) + 2F_2(0)$

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  - For inclusive B decays

    Perturbative Wilson coefficients, Non-perturbative matrix elements:
    $\langle \bar{h} \sigma_{\mu\nu} G^{\mu\nu}\, h \rangle$ related to $B - B^*$ mass difference

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- Convergence:
  "..it has been argued that the OPE results in an asymptotic series with limitations paralleling those for the perturbative series."
  [Mannel, Turczyk, Uraltsev JHEP **1011**, 109 (2010)]