

QED+QCD NNLO corrections to Drell Yan Production

Daniel de Florian
ICAS - UNSAM
Argentina

1st COFI Workshop on Precision Electroweak
July 18 2019



UNSAM
UNIVERSIDAD
NACIONAL DE
SAN MARTÍN

Drell Yan Production

- ▶ Vector Boson production measured with great precision
- ▶ Drell-Yan one of the best understood processes and most precise TH
- ▶ Standard Candle of particle physics
 - Luminosity monitor, detector calibration, PDF constrains
- ▶ Great test for BSM: new gauge interactions, susy, heavy resonances, etc...
- ▶ High resolution for SM : W mass, width and mixing angle
- ▶ QCD corrections can be rather large:
NNLO
Transverse momentum resummation

Drell Yan



TH precision

QCD NLO

'79 - Altarelli, Ellis & Martinelli

QCD NNLO

Inclusive: '91 - Haamberg, van Neerven & Matsuura

'92 - van Neerven & Zijlstra

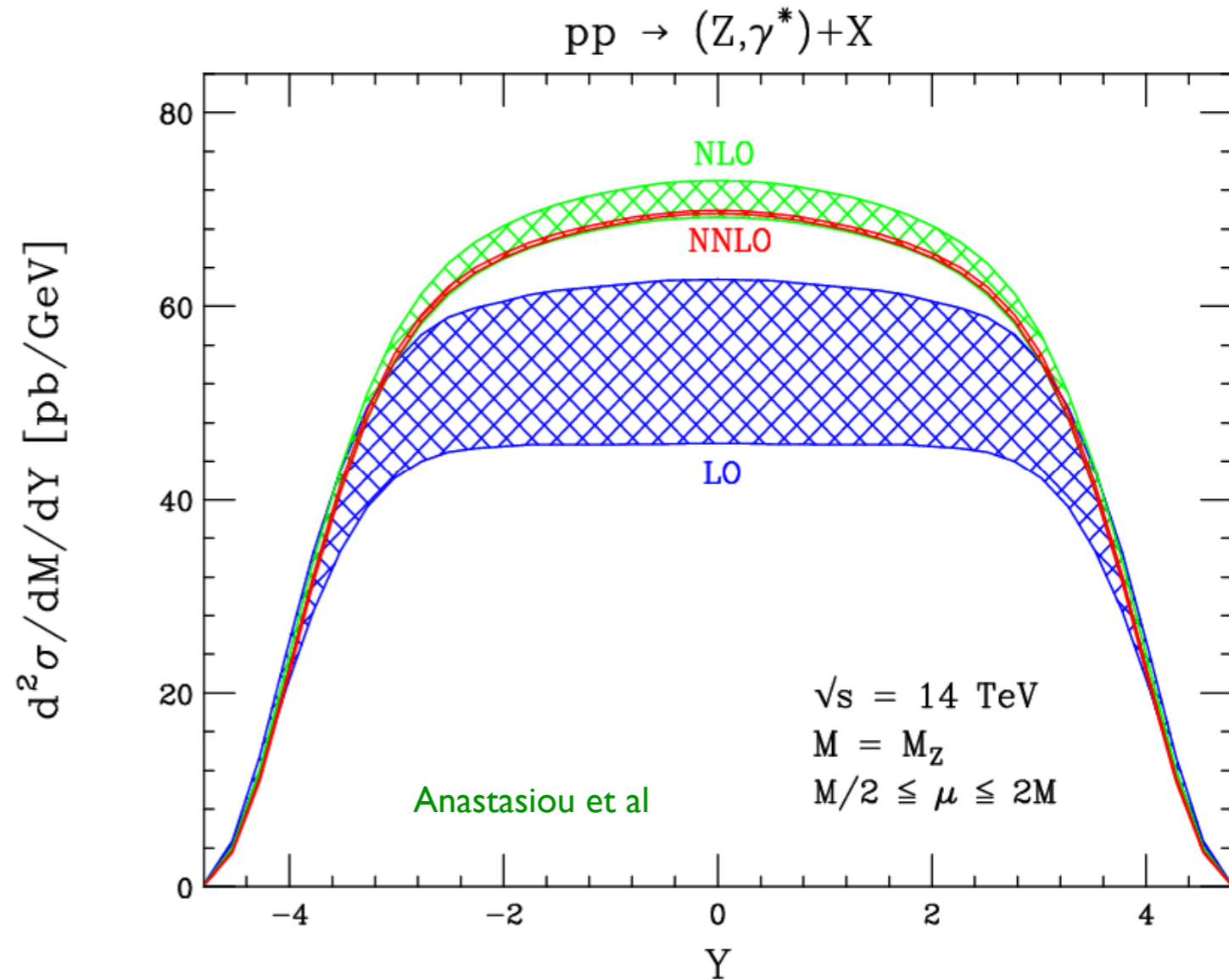
'02 - Haarlander & Kilgore

Exclusive:

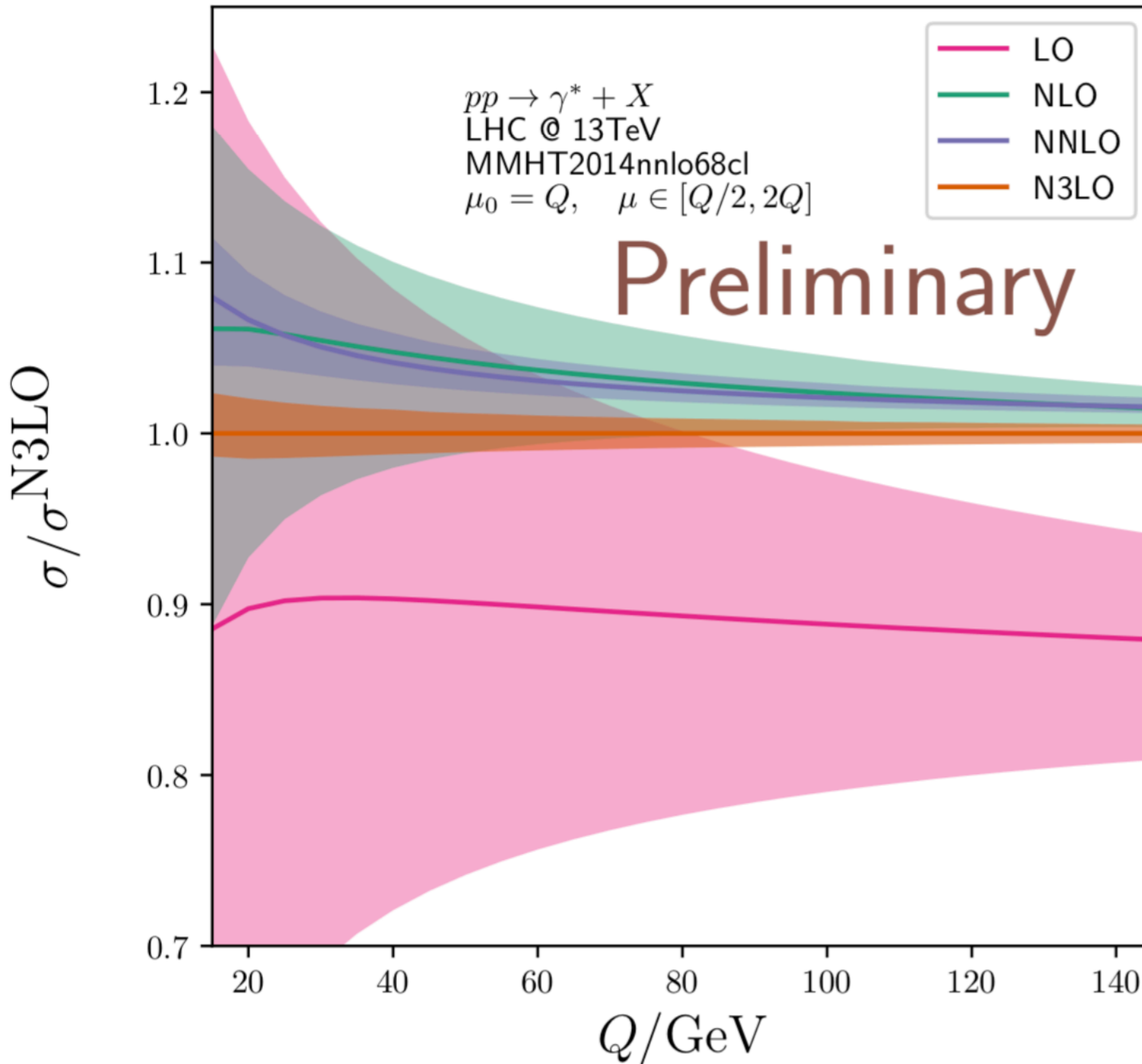
'06 - Melnikov & Petriello

'09 - Catani, Cieri, Ferrera,
de Florian & Grazzini

'17 - Boughezal, Campbell, Ellis,
Focke, Viele, Petriello & Williams



$$pp \rightarrow \gamma^* + X \rightarrow e^+ e^- + X$$



➡ Bands of of the same size and do not overlap!

➡ Central value shifts by a few %.

➡ Needs further study:

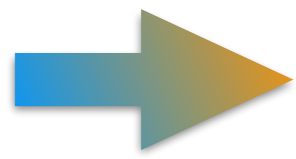
- Different scale/PDF choices?
- Missing N3LO PDFs?
- Z-boson contribution?

C. Duhr (EPS2019)

[CD, Dulat, Mistlberger (2019, to appear)]



$\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW \sim NNLO QCD

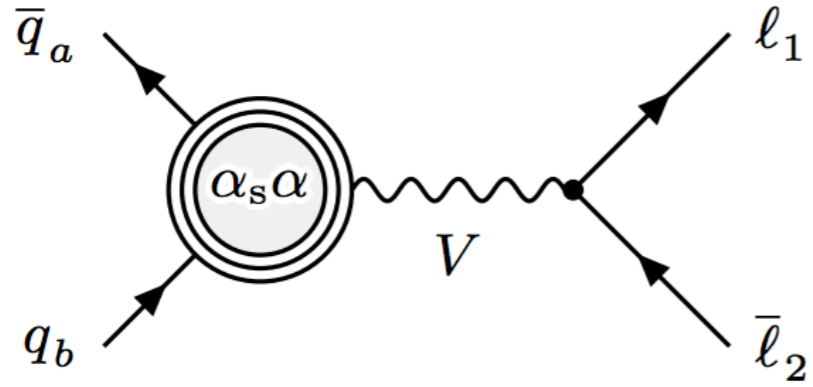


$\mathcal{O}(\alpha) \sim \mathcal{O}(\alpha_s^2)$ suggests NLO EW \sim NNLO QCD

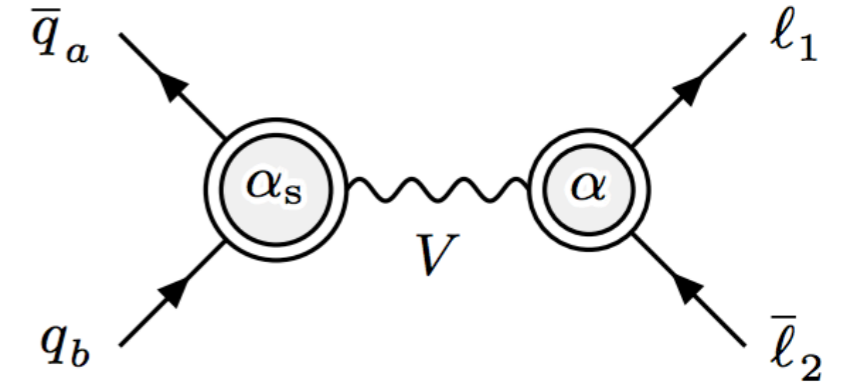
- ▶ **Enhanced**
 - **by photon emission**
kinematical effects, mass-singular log's $\propto \alpha \ln(m_\mu/Q)$ for bare muons, etc.
 - **at high energies**
EW Sudakov log's $\propto (\alpha/s_W^2) \ln^2(M_W/Q)$
- ▶ **Require perturbative and non-perturbative work**
 - **$\mathcal{O}(\alpha)$ corrections to all PDFs**
typical impact: $\Delta(\text{PDF}) \lesssim 0.3\%$ (1%) for $x \lesssim 0.1$ (0.4), $\mu_{\text{fact}} \sim M_W$
- ▶ **Big effort to obtain EW/QED perturbative corrections for DY**
- ▶ **Full results at NLO**
 - QED NLO: Baur, Keller, Sakumoto (1997)
 - EW NLO: Baur, Brein, Hollik, Schappacher, Wackerroth (2001)
- ▶ **Partial results at NNLO EWxQCD for inclusive cross section**
 - $\alpha \times \alpha_s$ mixed needed to reach below 1% accuracy
 - Real corrections : Bonciani, Buccioni, Mondini, Vicini (2017)
 - Master integrals for Virtual corrections : Bonciani, Di Vita, Mastrolia, Schubert (2016)

► Mixed EWxQCD corrections in the resonance region

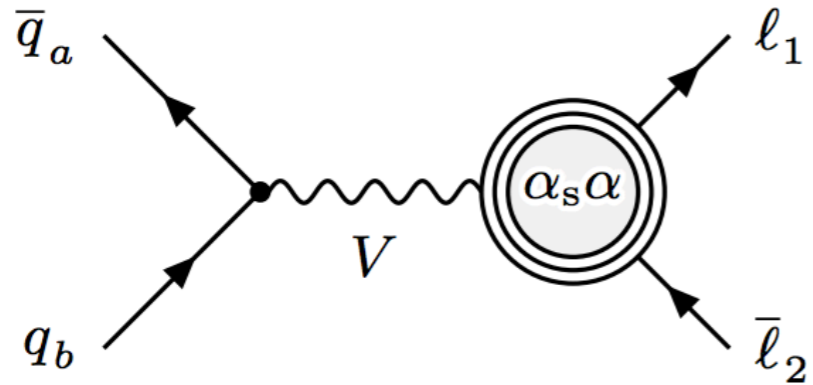
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



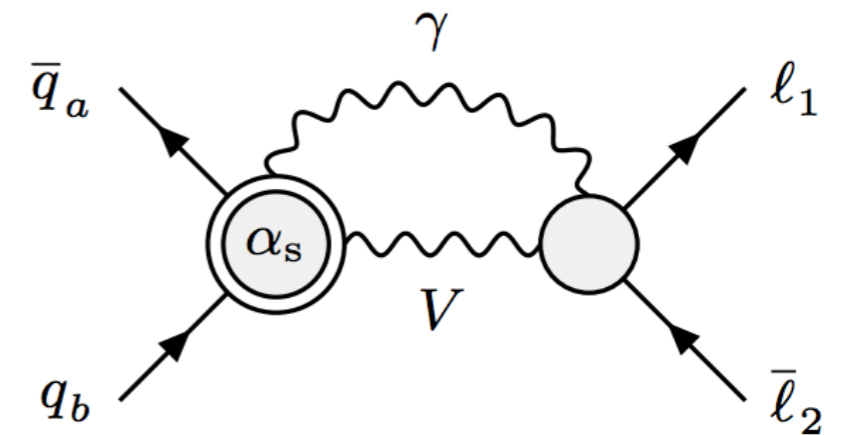
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



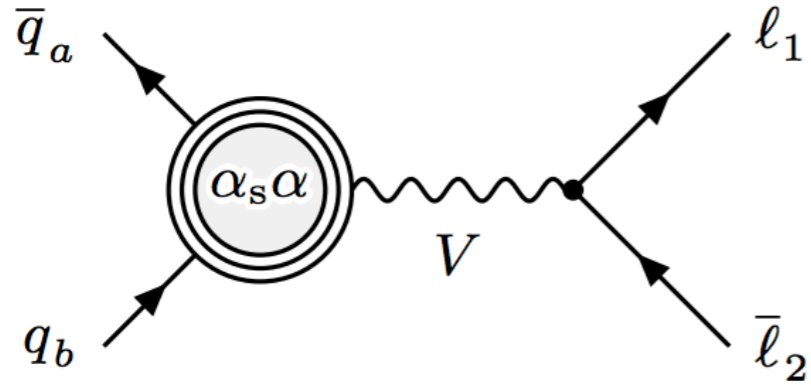
(c) Factorizable final–final corrections



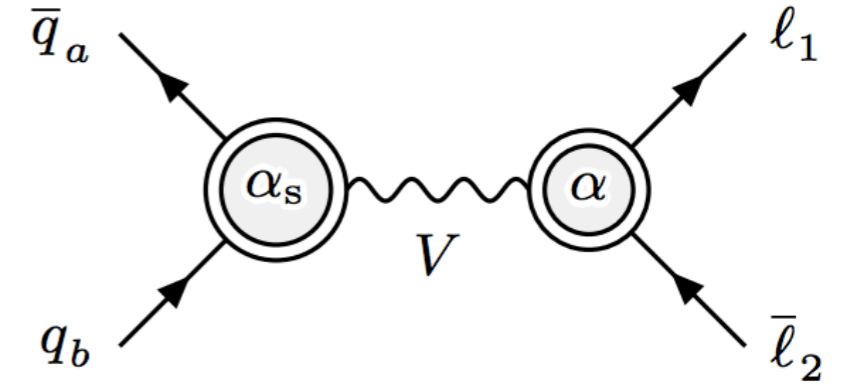
(d) Non-factorizable corrections

► Mixed EWxQCD corrections in the resonance region

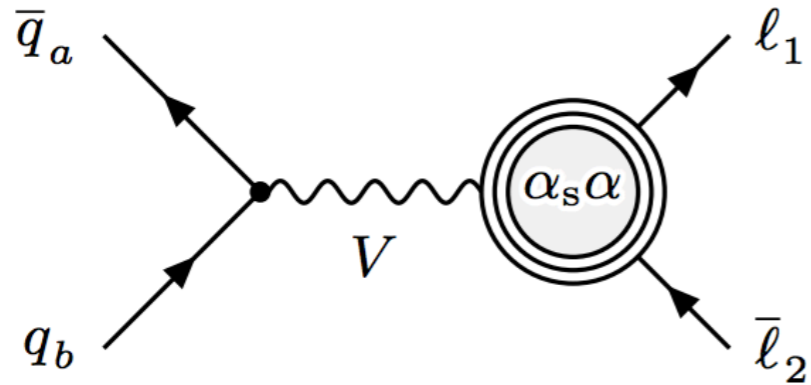
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



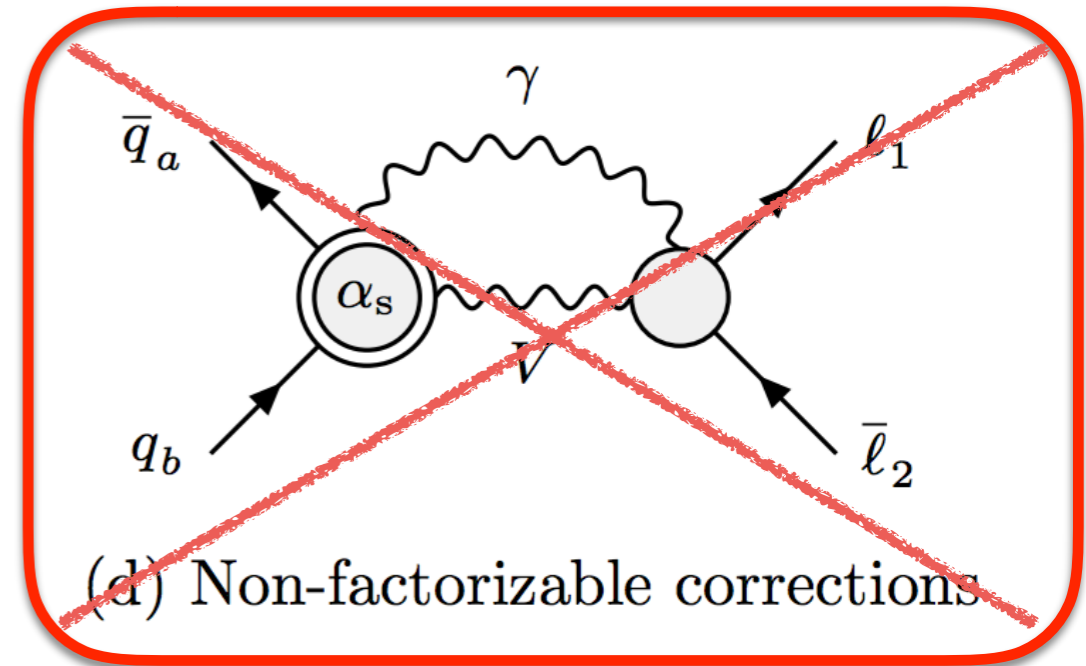
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



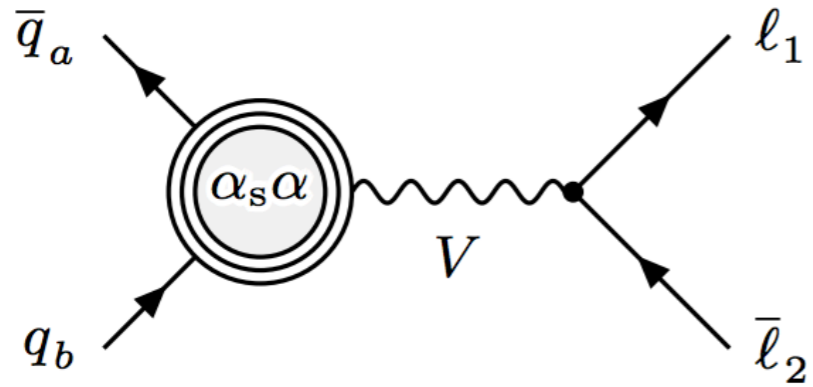
(c) Factorizable final–final corrections



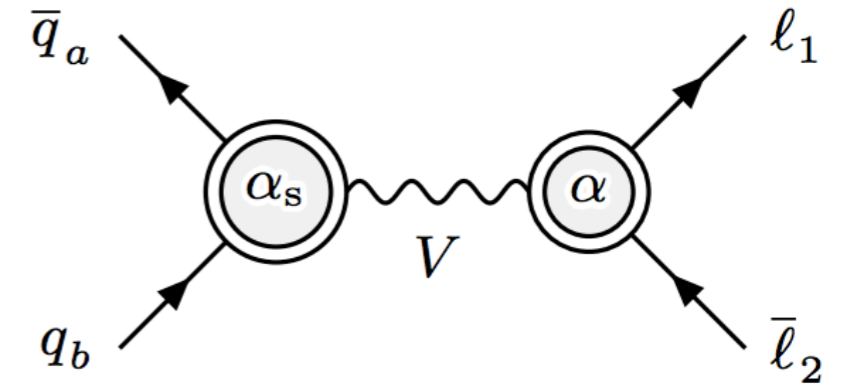
(d) Non-factorizable corrections

► Mixed EWxQCD corrections in the resonance region

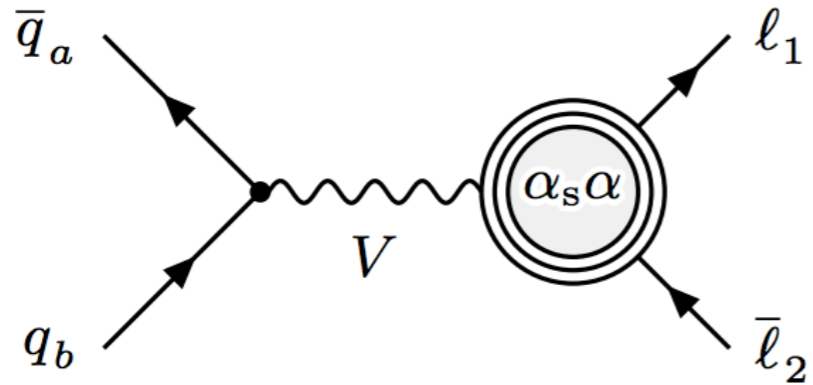
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



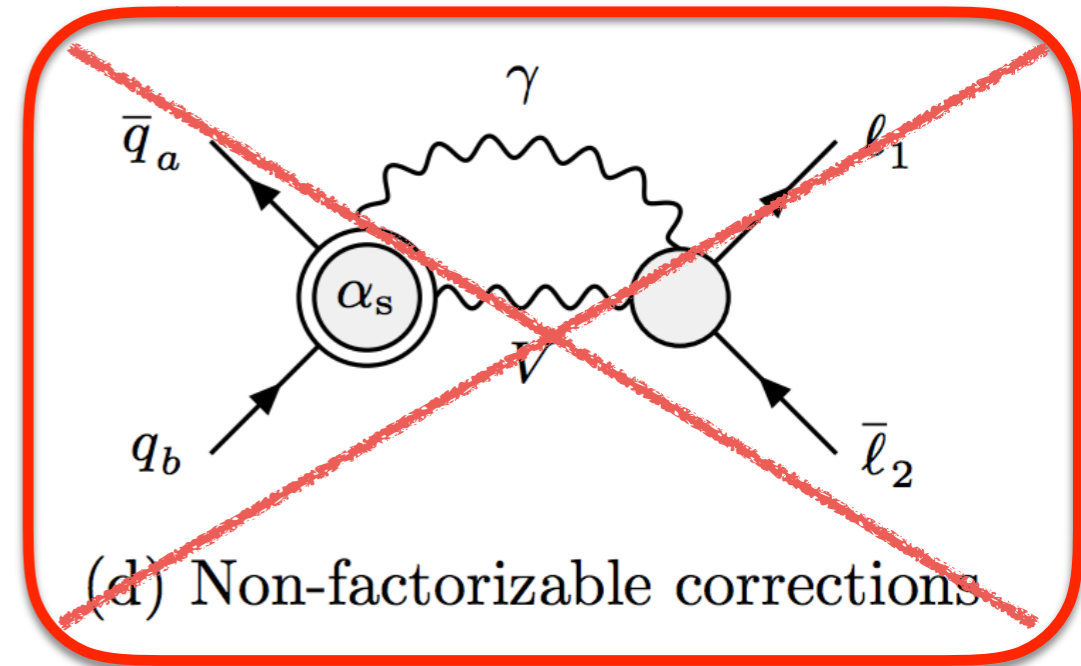
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

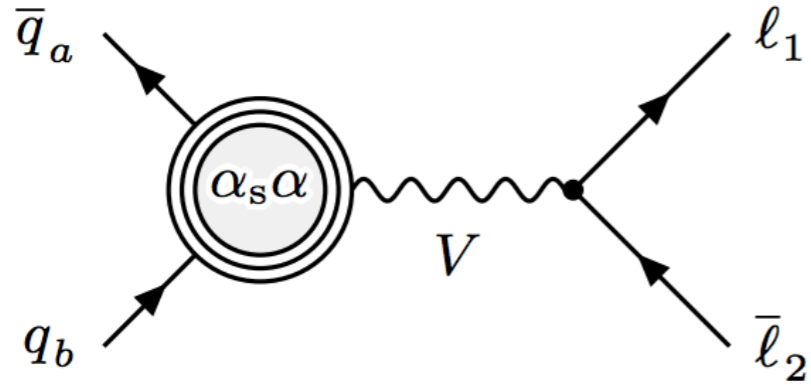


(d) Non-factorizable corrections

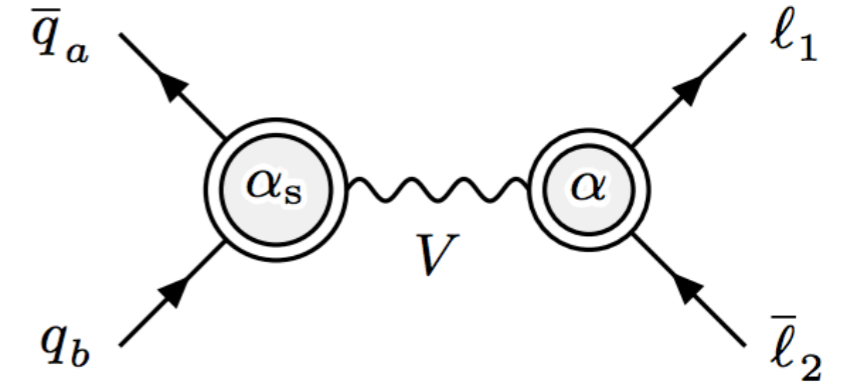
negligible (<0.1%)

► Mixed EWxQCD corrections in the resonance region

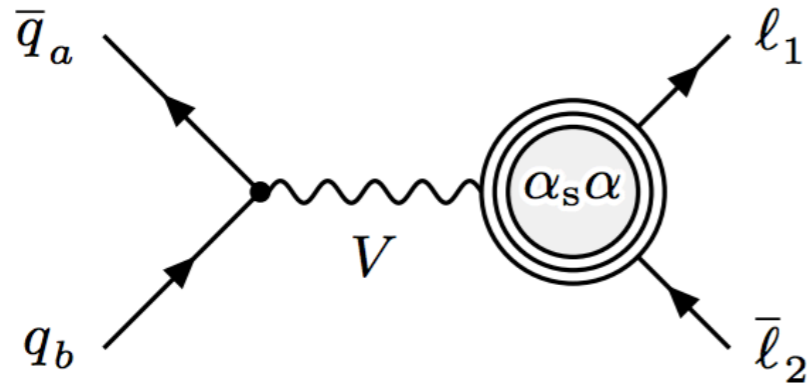
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



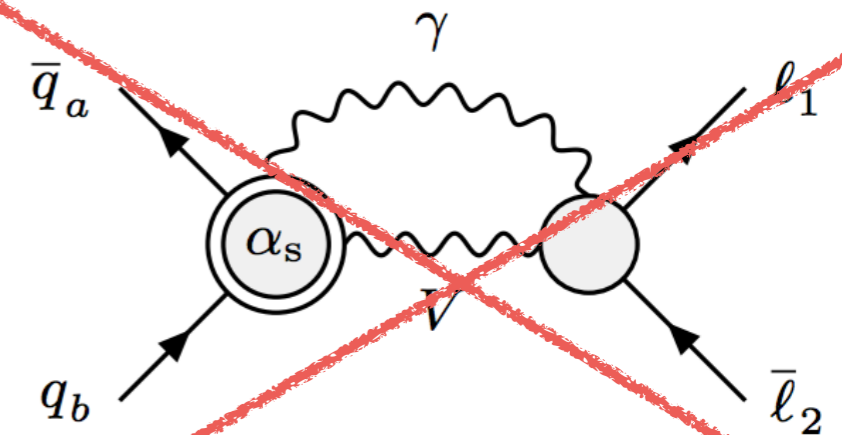
(a) Factorizable initial–initial corrections



(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

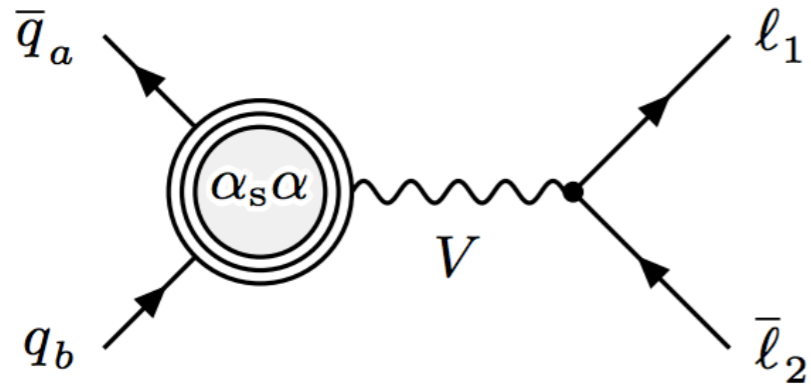


(d) Non-factorizable corrections

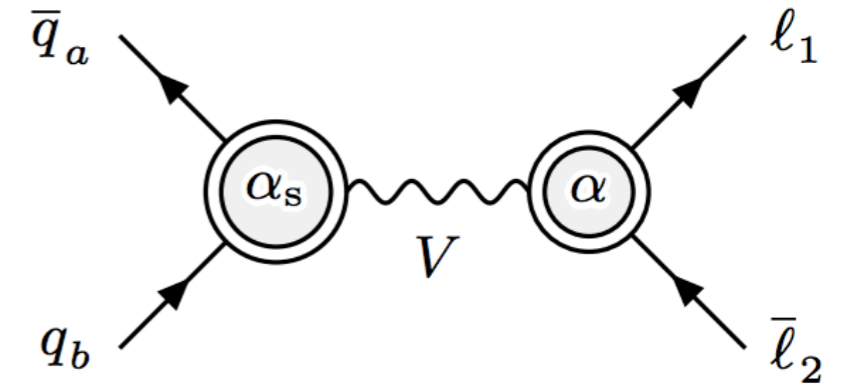
negligible (<0.1%)

► Mixed EWxQCD corrections in the resonance region

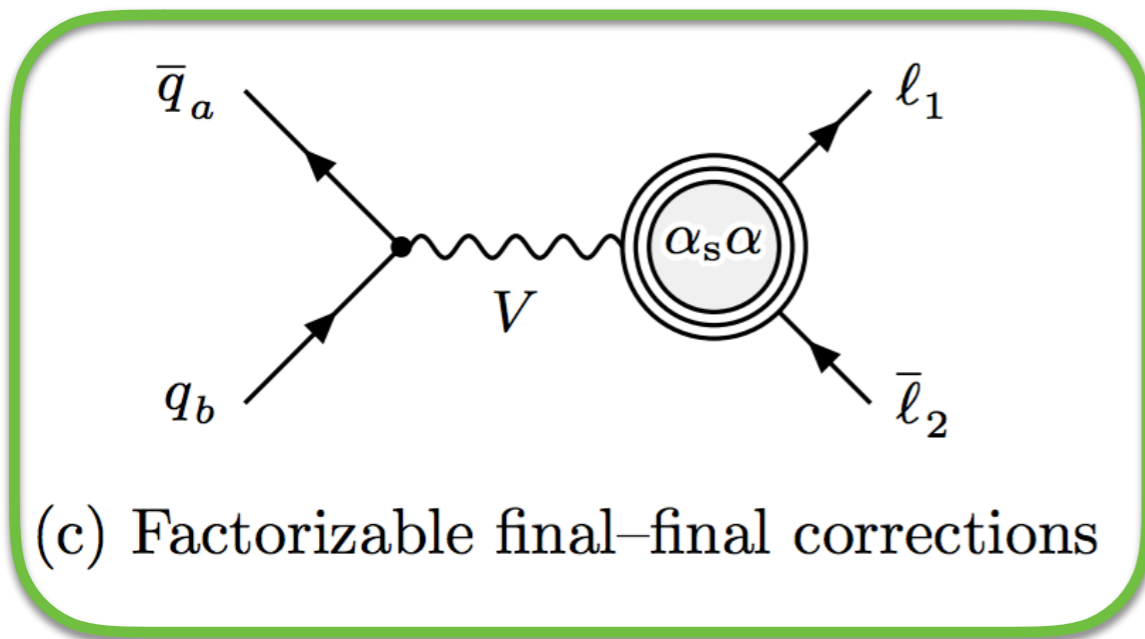
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



(a) Factorizable initial–initial corrections

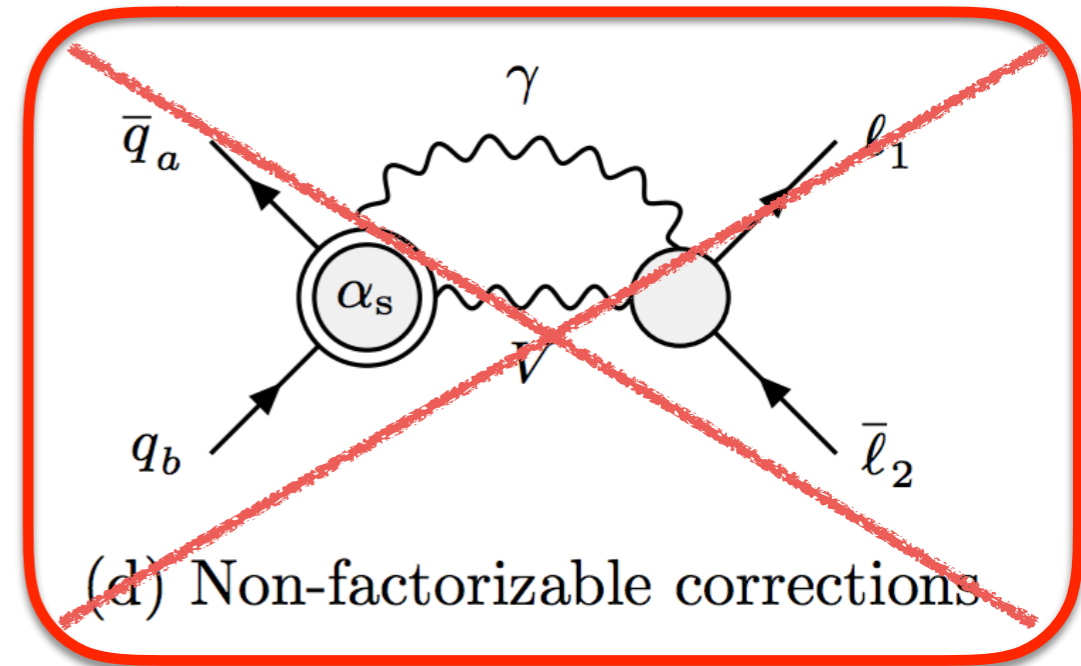


(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

negligible and known

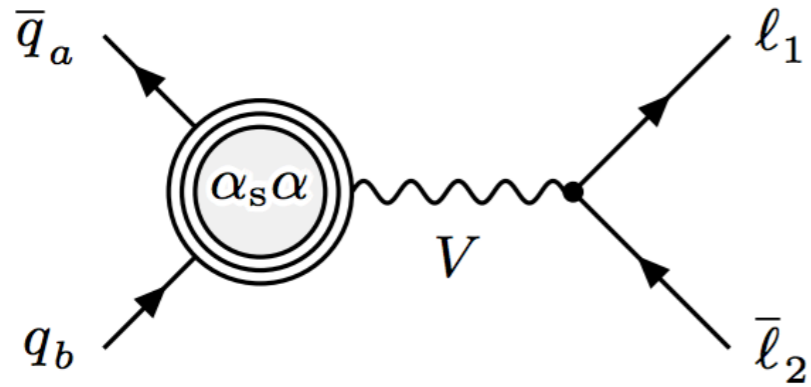


(d) Non-factorizable corrections

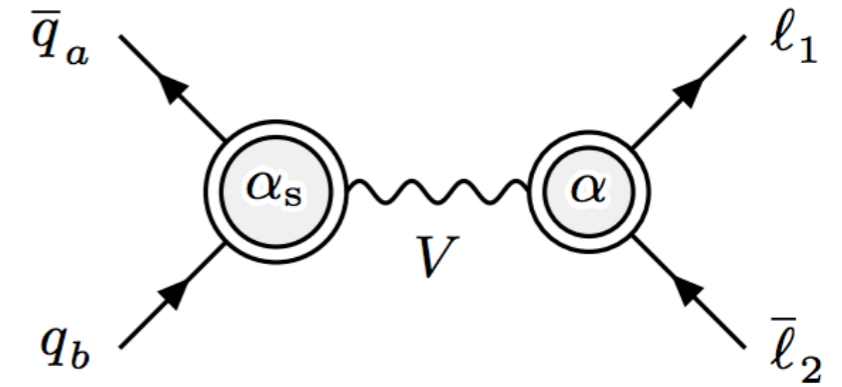
negligible (<0.1%)

► Mixed EWxQCD corrections in the resonance region

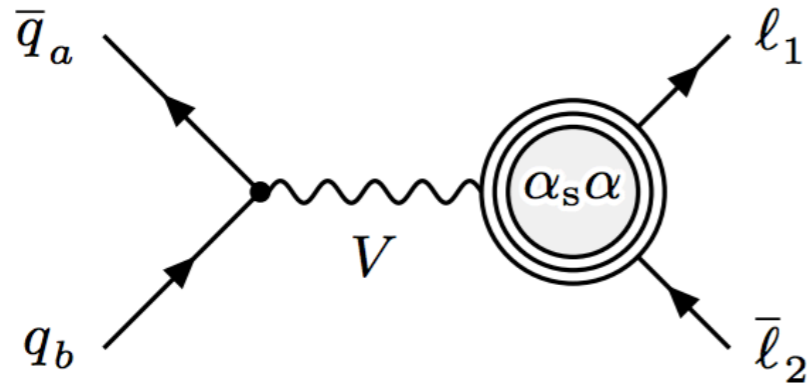
Dittmaier, Huss, Schwinn (2014, 2015, 2016)



(a) Factorizable initial–initial corrections

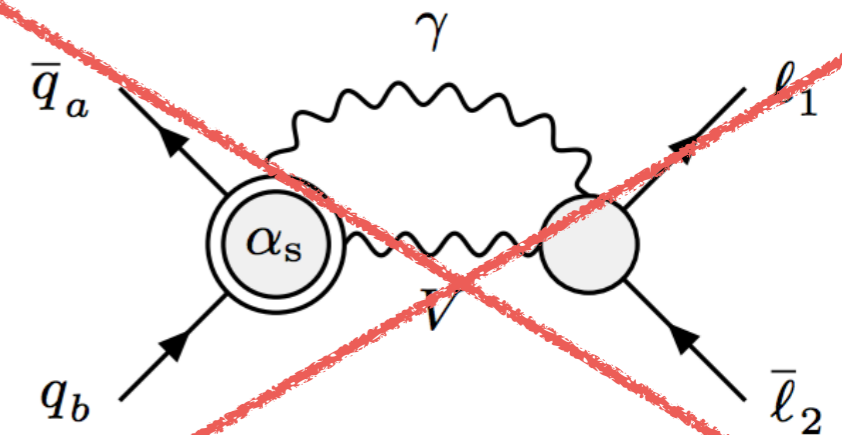


(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

negligible and known



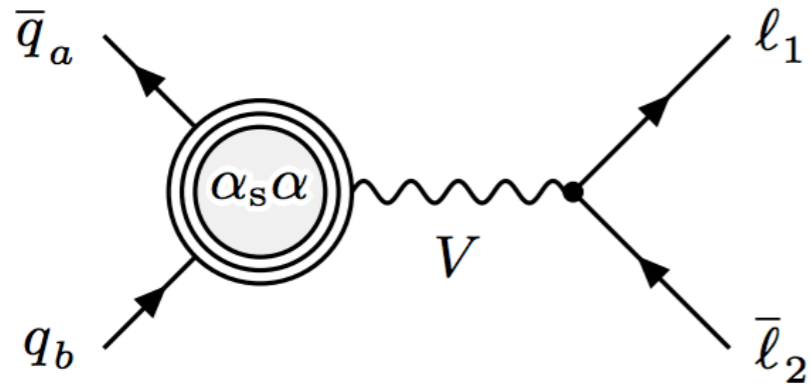
(d) Non-factorizable corrections

negligible (<0.1%)

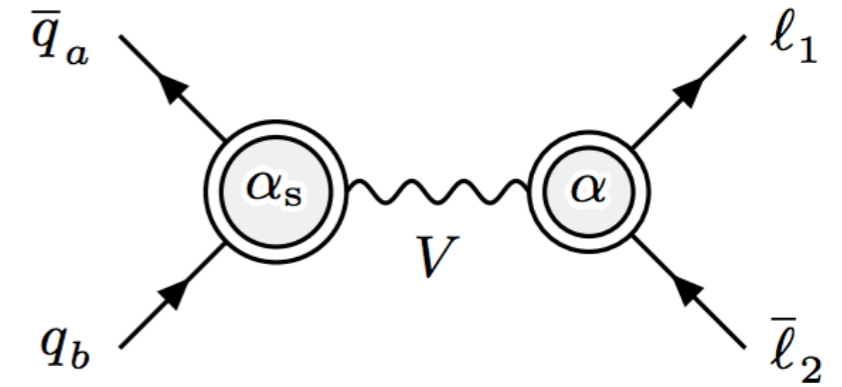
► Mixed EWxQCD corrections in the resonance region

Dittmaier, Huss, Schwinn (2014, 2015, 2016)

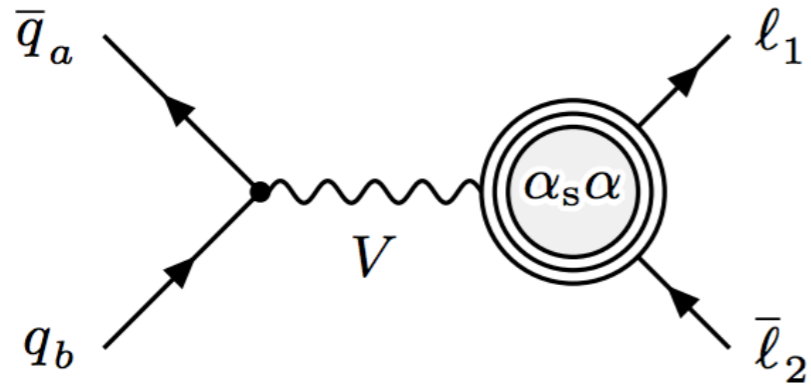
main contribution



(a) Factorizable initial–initial corrections

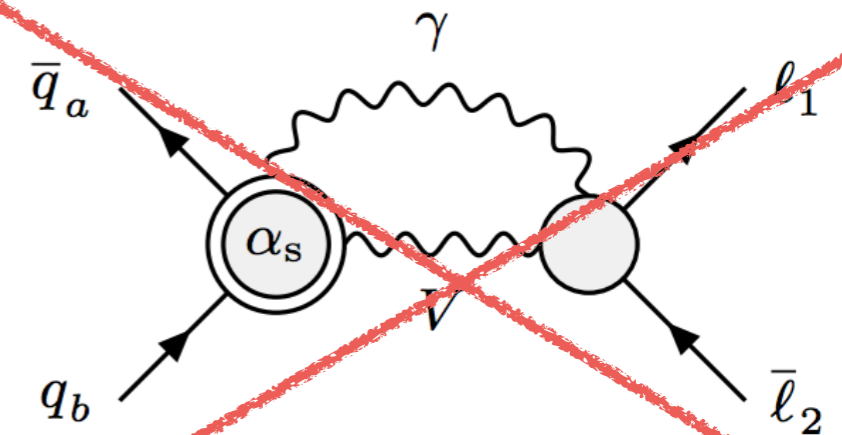


(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

negligible and known



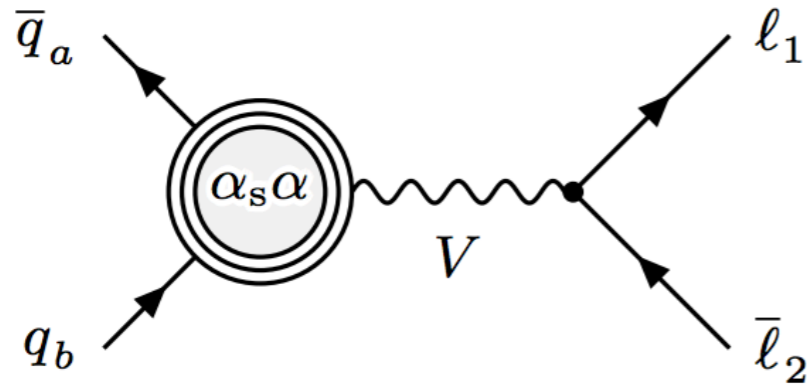
(d) Non-factorizable corrections

negligible (<0.1%)

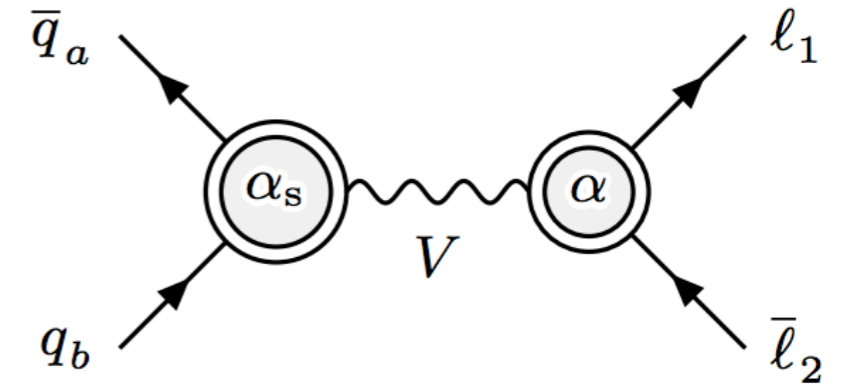
► Mixed EWxQCD corrections in the resonance region

Dittmaier, Huss, Schwinn (2014, 2015, 2016)

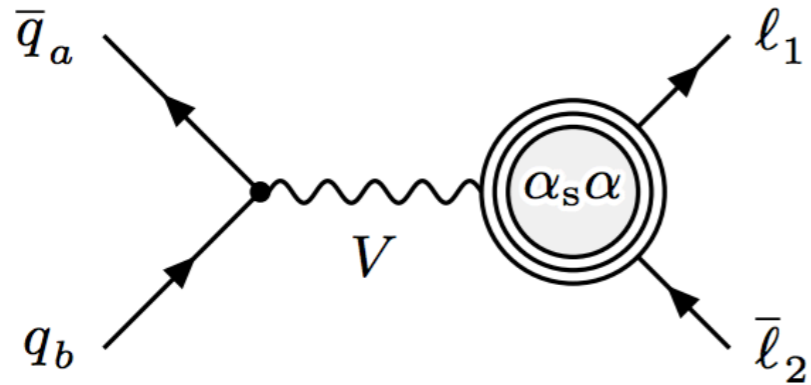
main contribution



(a) Factorizable initial–initial corrections

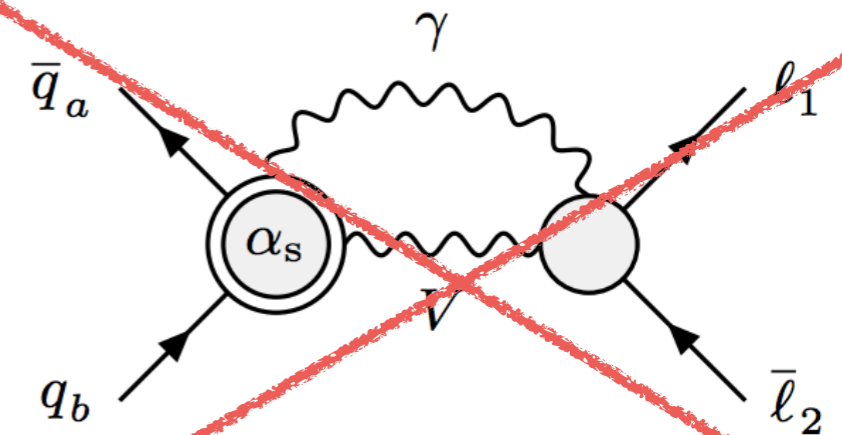


(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

negligible and known



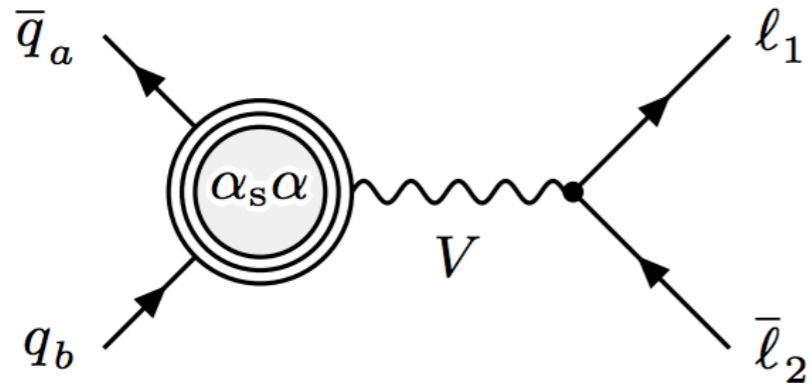
(d) Non-factorizable corrections

negligible (<0.1%)

► Mixed EWxQCD corrections in the resonance region

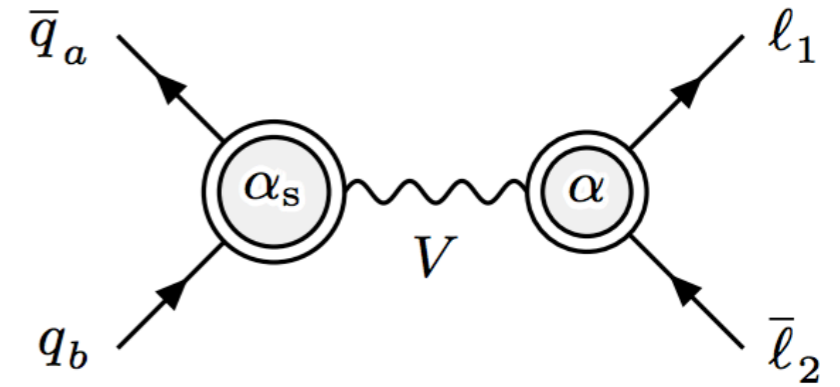
Dittmaier, Huss, Schwinn (2014, 2015, 2016)

unknown

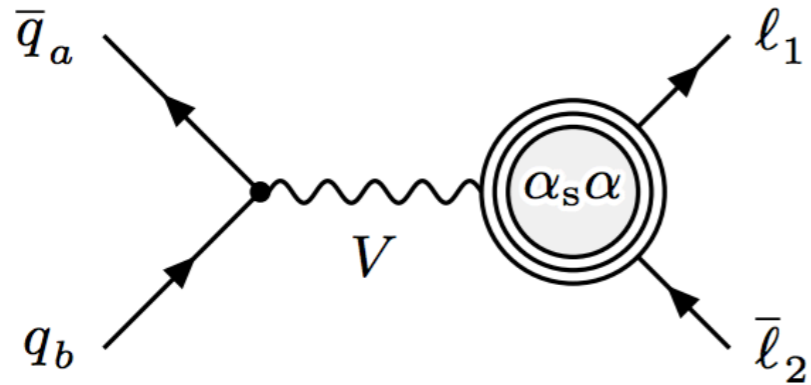


(a) Factorizable initial–initial corrections

main contribution

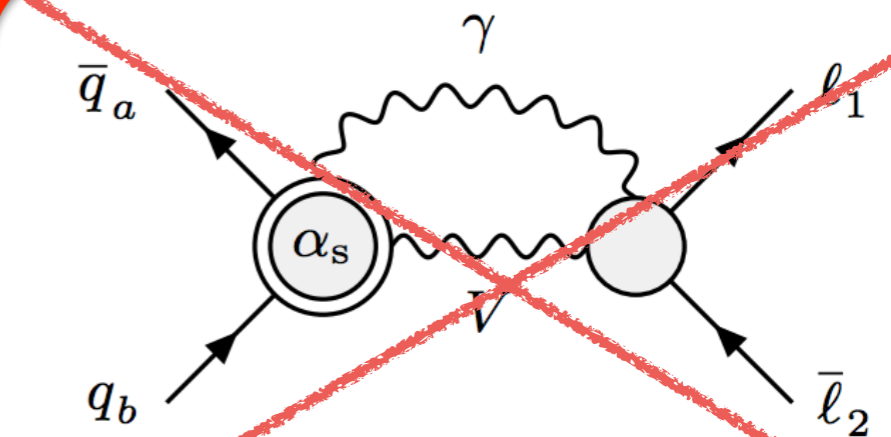


(b) Factorizable initial–final corrections



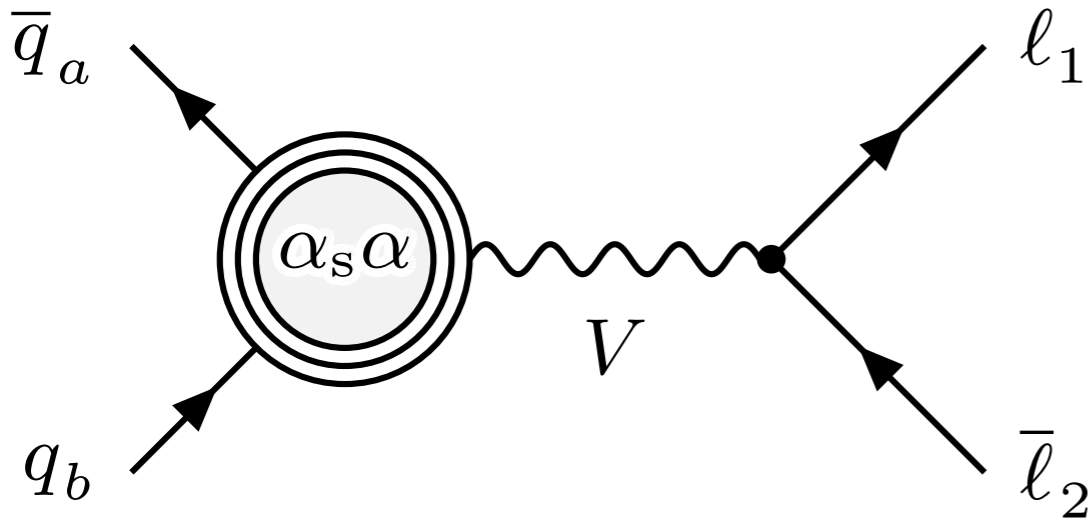
(c) Factorizable final–final corrections

negligible and known

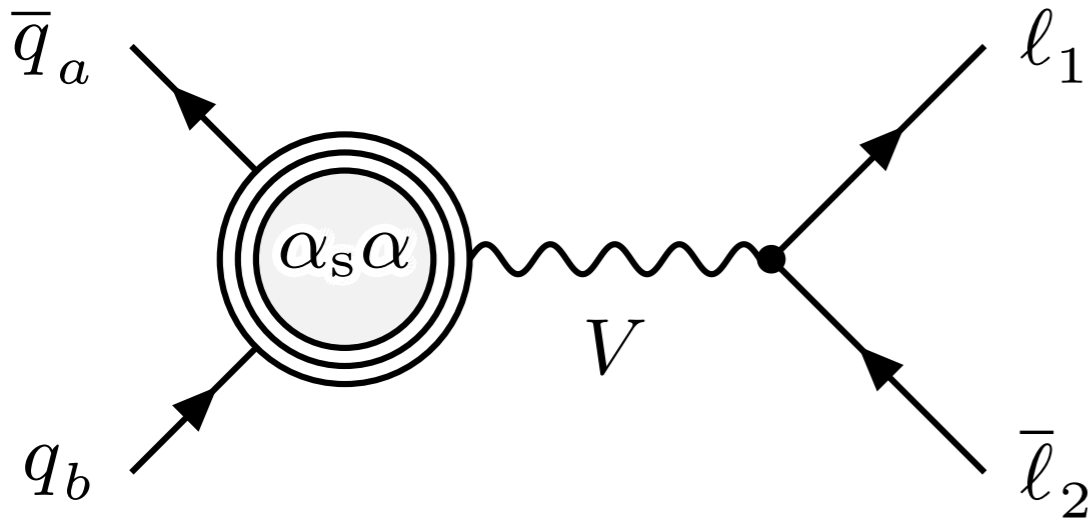


(d) Non-factorizable corrections

negligible (<0.1%)



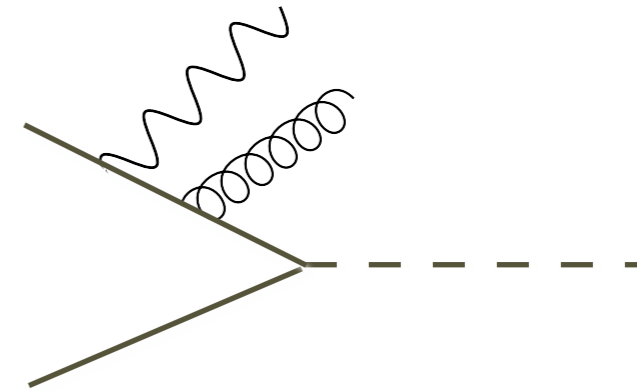
initial-initial corrections (on-shell Drell-Yan)
hard to compute (assumed to be small)

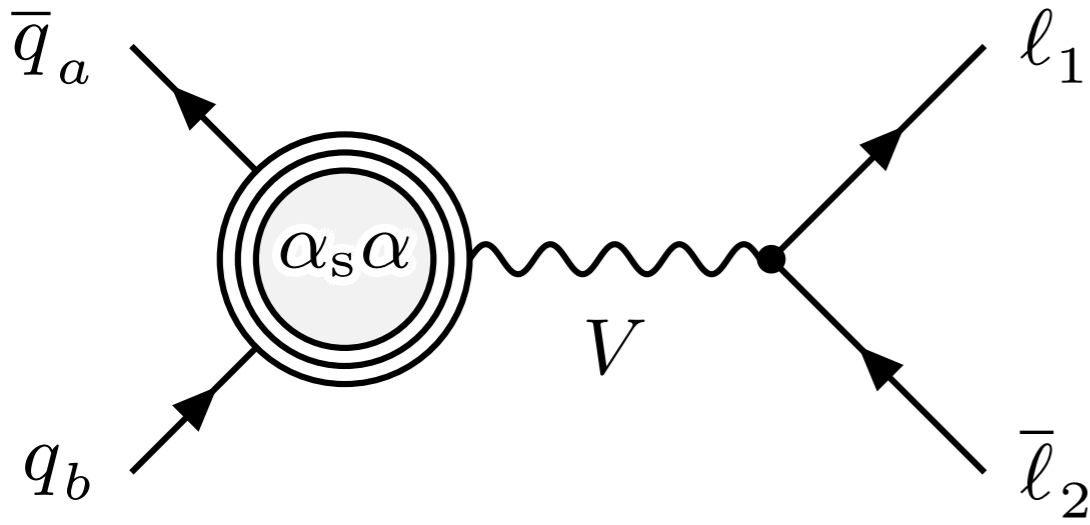


initial-initial corrections (on-shell Drell-Yan)
hard to compute (assumed to be small)

▶ genuine NNLO corrections (QEDxQCD)

- NNLO singularities
- requires NNLO factorization

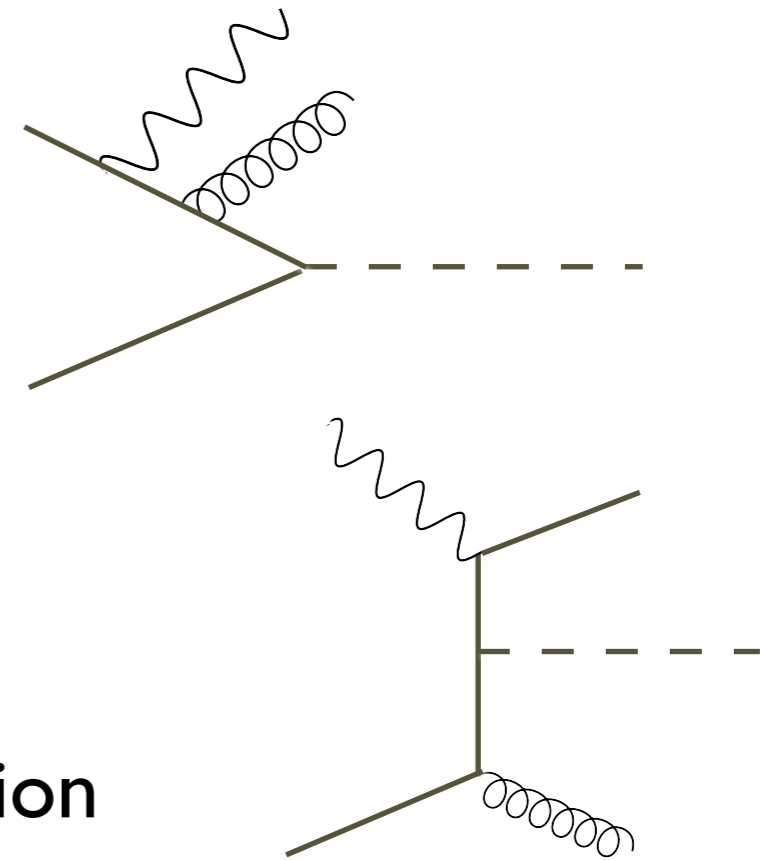


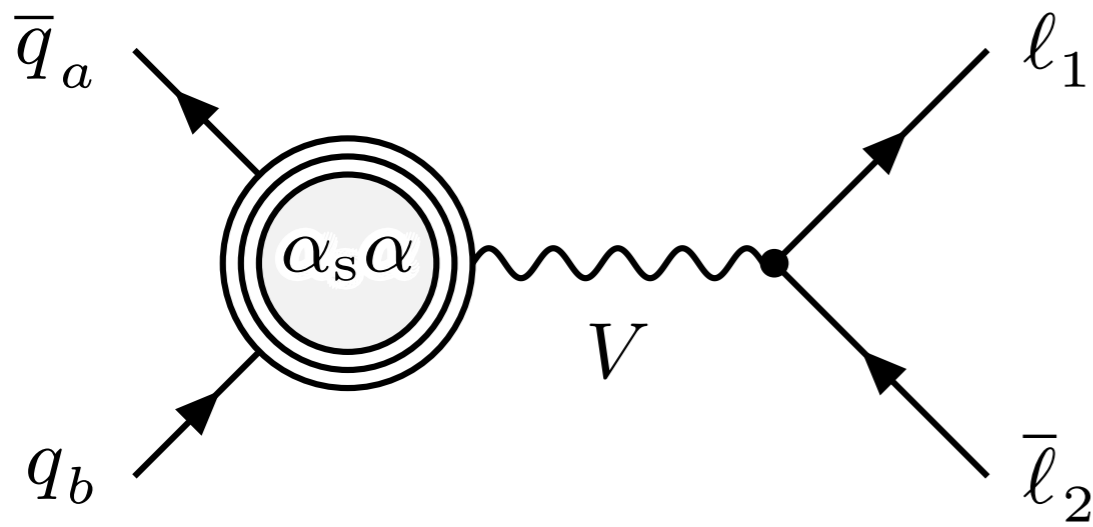


initial-initial corrections (on-shell Drell-Yan)
hard to compute (assumed to be small)

- ▶ genuine NNLO corrections (QEDxQCD)
 - NNLO singularities
 - requires NNLO factorization

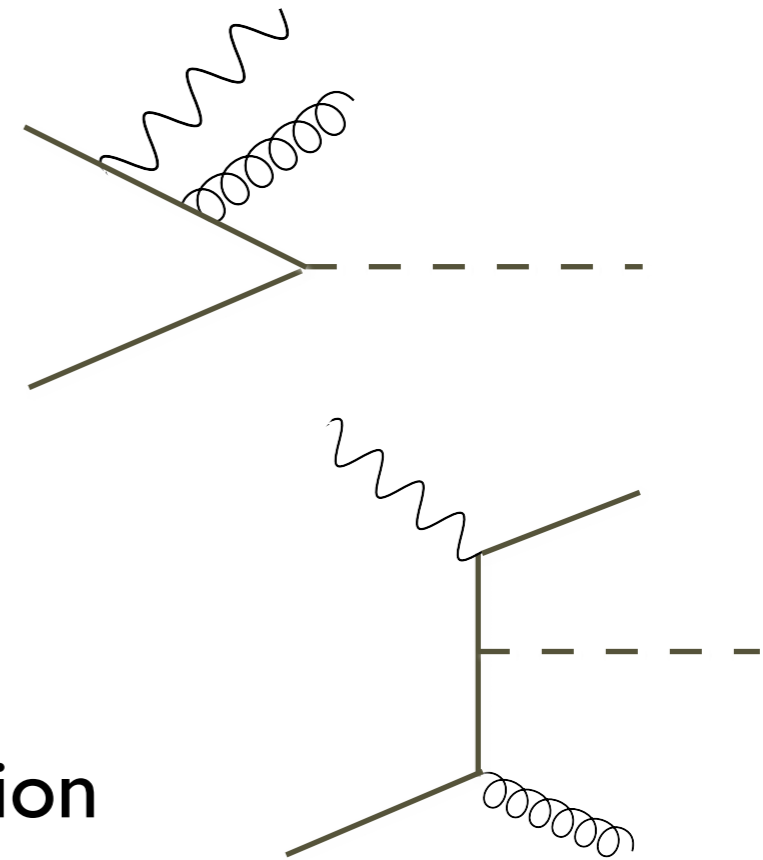
- ▶ Involve initial state photon distribution
 - precision and mixed QEDxQCD evolution





initial-initial corrections (on-shell Drell-Yan)
hard to compute (assumed to be small)

- ▶ genuine NNLO corrections (QEDxQCD)
 - NNLO singularities
 - requires NNLO factorization
- ▶ Involve initial state photon distribution
 - precision and mixed QEDxQCD evolution



this talk

- QEDxQCD splitting functions DdeF, Rodrigo, Sborlini (16)
- Full QED+QCD NNLO corrections to DY DdeF, M.Der, I.Fabre (18)

or how to do NNLO without
computing a single integral

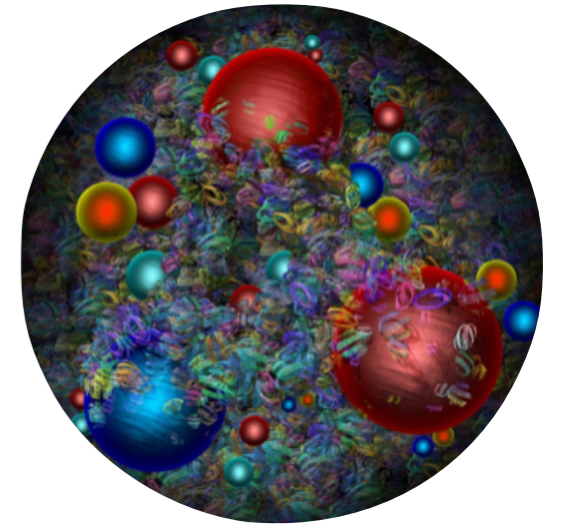
**QED+QCD NNLO corrections to
Splitting Functions
and photon PDFs**

- ▶ DGLAP very well known in QCD : quarks and gluons (**colored particles**)

$$\frac{dq_i}{dt} = \sum_{j=1}^{n_F} P_{q_i q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{q_i \bar{q}_j} \otimes \bar{q}_j + P_{q_i g} \otimes g$$

$$\frac{dg}{dt} = \sum_{j=1}^{n_F} P_{g q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{g \bar{q}_j} \otimes \bar{q}_j + P_{g g} \otimes g$$

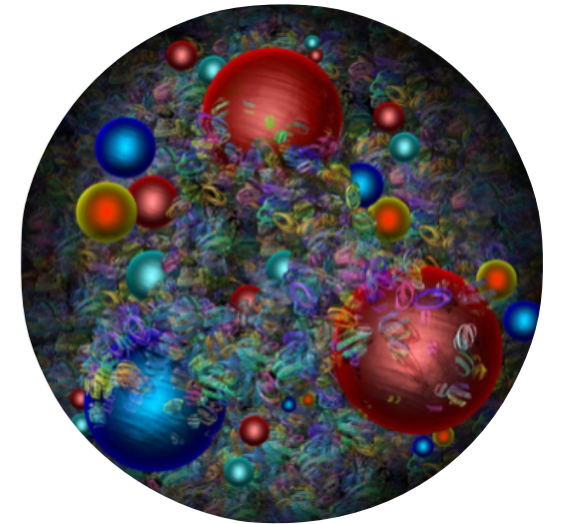
- ▶ Parton model content of proton quite more complicated than naive picture



- ▶ DGLAP very well known in QCD : quarks and gluons (**colored particles**)

$$\frac{dq_i}{dt} = \sum_{j=1}^{n_F} P_{q_i q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{q_i \bar{q}_j} \otimes \bar{q}_j + P_{q_i g} \otimes g$$

$$\frac{dg}{dt} = \sum_{j=1}^{n_F} P_{g q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{g \bar{q}_j} \otimes \bar{q}_j + P_{g g} \otimes g$$

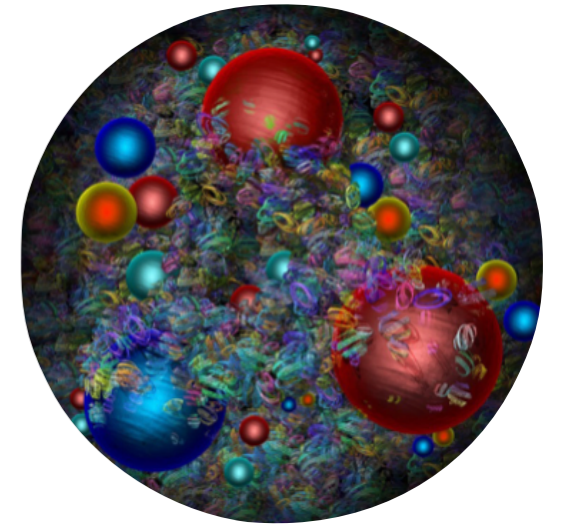


- ▶ Parton model content of proton quite more complicated than naive picture
- ▶ Partons could be quarks, gluons but also photons, leptons, Higgs, W,Z.
- ▶ Content for most of them negligible
- ▶ Also decoupled from q/g if only QCD considered

- ▶ DGLAP very well known in QCD : quarks and gluons (**colored particles**)

$$\frac{dq_i}{dt} = \sum_{j=1}^{n_F} P_{q_i q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{q_i \bar{q}_j} \otimes \bar{q}_j + P_{q_i g} \otimes g$$

$$\frac{dg}{dt} = \sum_{j=1}^{n_F} P_{g q_j} \otimes q_j + \sum_{j=1}^{n_F} P_{g \bar{q}_j} \otimes \bar{q}_j + P_{g g} \otimes g$$



- ▶ Parton model content of proton quite more complicated than naive picture
- ▶ Partons could be quarks, gluons but also photons, leptons, Higgs, W,Z.
- ▶ Content for most of them negligible
- ▶ Also decoupled from q/g if only QCD considered
- ▶ But when QED turned on, new distributions appear (and mix) in evolution

photon parton distribution function $\gamma(x, Q^2)$

lepton parton distribution function $l(x, Q^2)$

neglect heavy particles such as W,Z,H

QED+QCD set of DGLAP evolution equations

$$\frac{dq_i}{dt} = \sum_f P_{q_i f} \otimes f + \sum_f P_{q_i \bar{f}} \otimes \bar{f} + P_{q_i g} \otimes g + P_{q_i \gamma} \otimes \gamma$$

$$\frac{dg}{dt} = \sum_f P_{gf} \otimes f + \sum_f P_{g\bar{f}} \otimes \bar{f} + P_{gg} \otimes g + P_{g\gamma} \otimes \gamma$$

$$\frac{d\gamma}{dt} = \sum_f P_{\gamma f} \otimes f + \sum_f P_{\gamma \bar{f}} \otimes \bar{f} + P_{\gamma g} \otimes g + P_{\gamma\gamma} \otimes \gamma$$

$$\frac{dl_i}{dt} = \sum_f P_{l_i f} \otimes f + \sum_f P_{l_i \bar{f}} \otimes \bar{f} + P_{l_i g} \otimes g + P_{l_i \gamma} \otimes \gamma$$

f: quarks + leptons

$$a_S \equiv \frac{\alpha_S}{2\pi}$$

$$a \equiv \frac{\alpha}{2\pi}$$

Splitting functions expansion in QCD and QED couplings

$$P_{ij} = a_S P_{ij}^{(1,0)} + a P_{ij}^{(0,1)} + a_S^2 P_{ij}^{(2,0)} + a_S a P_{ij}^{(1,1)} + a^2 P_{ij}^{(0,2)} + \dots$$

LO QCD

LO QED

NLO QCD

NLO mixed
QCD+QED

NLO QED

(were) unknown

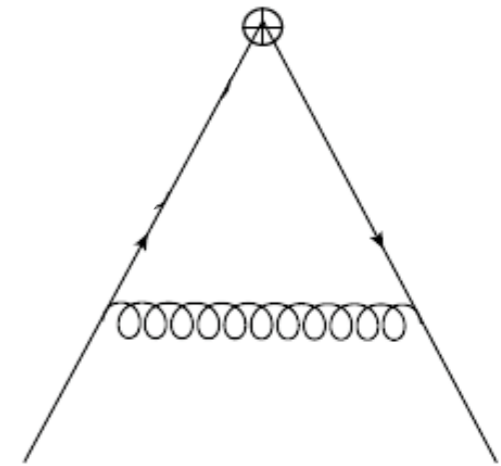
How to get them

- ▶ From QCD to QED (abelianization) simple LO example

$$P_{qq}^{(1,0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] :$$

$$\delta(1-x) \text{ from } \int_0^1 P_{qq}(x) dx = 0$$

$q \rightarrow q(g)$



Curci, Furmanski, Petronzio (1980)

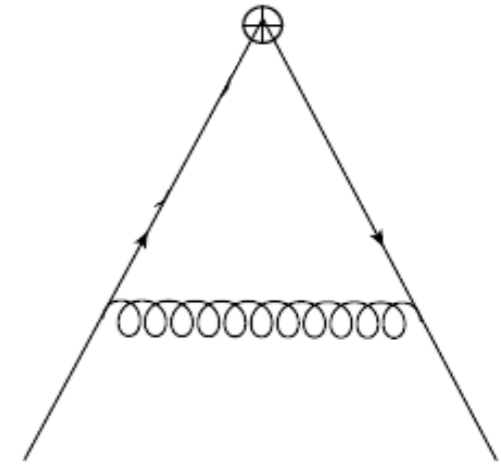
How to get them

- ▶ From QCD to QED (abelianization) simple LO example

$$P_{qq}^{(1,0)}(x) = C_F \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right] :$$

$$\delta(1-x) \text{ from } \int_0^1 P_{qq}(x) dx = 0$$

$q \rightarrow q(g)$

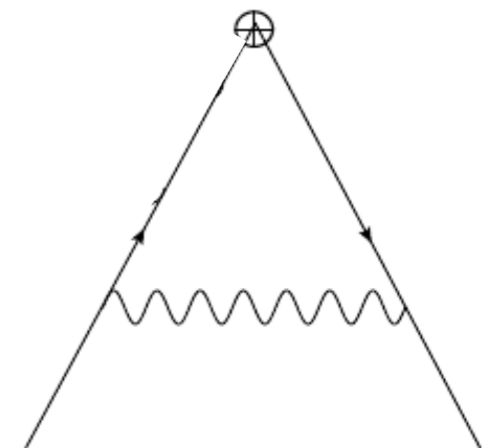


Curci, Furmanski, Petronzio (1980)

- ▶ to QED: change gluon into photon
= change QCD color factor into QED charge!

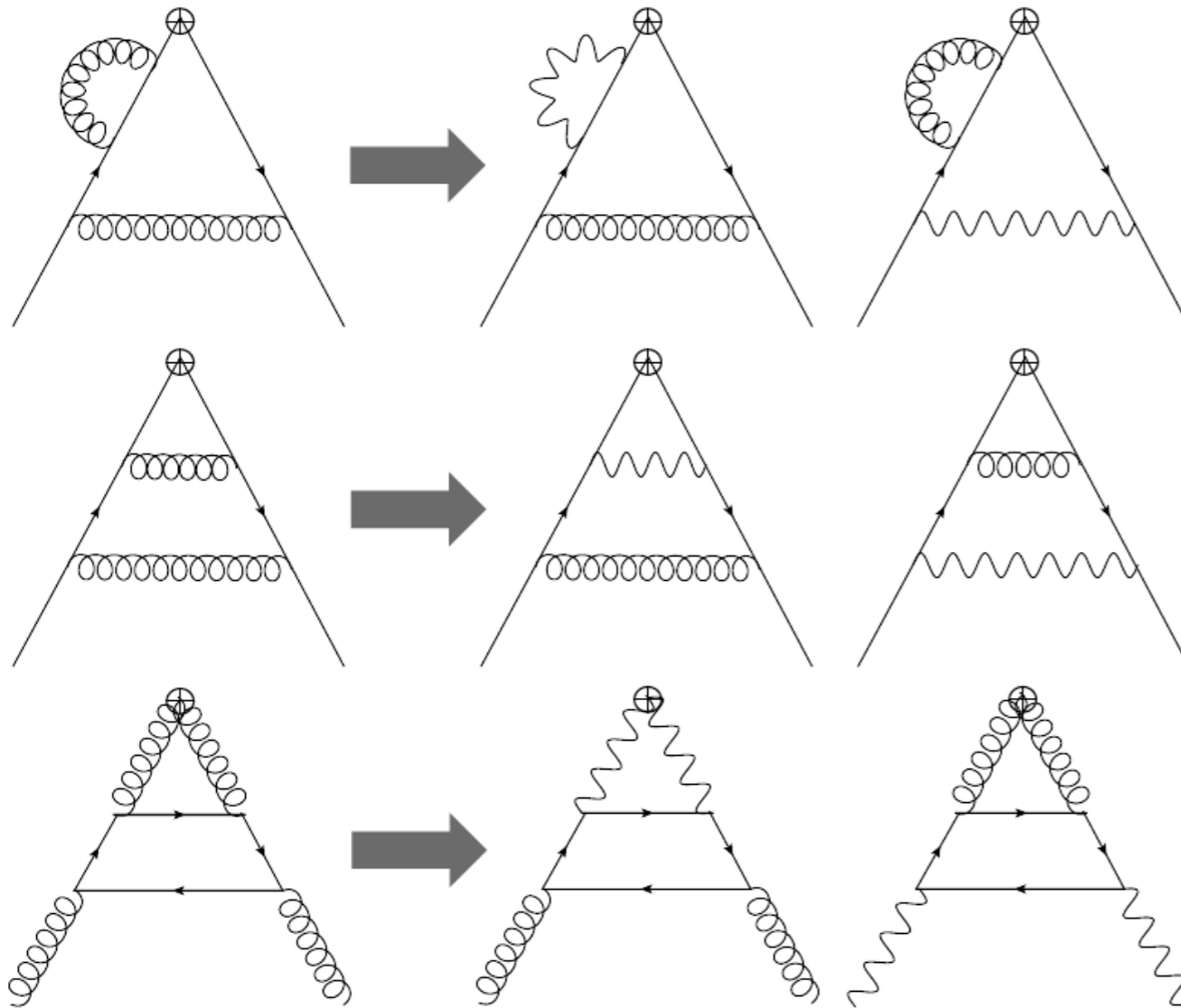
$$P_{ff}^{(0,1)}(x) = e_f^2 \left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2} \delta(1-x) \right]$$

$q \rightarrow q(\gamma)$



▶ Same approach can be applied at QCD²

- mixed QCD+QED
- QED²



- mixed QCD+QED

deF, Rodrigo, Sborlini (2016)

All two-loop splitting functions obtained

● QCD+QED

$$P_{q\gamma}^{(1,1)} = \frac{C_F C_A e_q^2}{2} \left\{ 4 - 9x - (1 - 4x)\ln(x) - (1 - 2x)\ln^2(x) + 4\ln(1 - x) \right. \\ \left. + p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \right\}, \\ P_{g\gamma}^{(1,1)} = C_F C_A \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \left\{ -16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6 + 10x)\ln(x) - 2(1 + x)\ln^2(x) \right\} \\ P_{\gamma\gamma}^{(1,1)} = -C_F C_A \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \delta(1 - x),$$

$$P_{qg}^{(1,1)} = \frac{T_R e_q^2}{2} \left\{ 4 - 9x - (1 - 4x)\ln(x) - (1 - 2x)\ln^2(x) + 4\ln(1 - x) \right. \\ \left. + p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \right\},$$

$$P_{gq}^{(1,1)} = T_R \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \left\{ -16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6 + 10x)\ln(x) - 2(1 + x)\ln^2(x) \right\}$$

$$P_{gg}^{(1,1)} = -T_R \left(\sum_{j=1}^{n_F} e_{q_j}^2 \right) \delta(1 - x),$$

$$P_{qq}^{S(1,1)} = P_{q\bar{q}}^{S(1,1)} = 0,$$

$$P_{qq}^{V(1,1)} = -2C_F e_q^2 \left[\left(2\ln(1 - x) + \frac{3}{2} \right) \ln(x) p_{qq}(x) + \frac{3 + 7x}{2} \ln(x) + \frac{1 + x}{2} \ln^2(x) \right. \\ \left. + 5(1 - x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3 \right) \delta(1 - x) \right],$$

$$P_{q\bar{q}}^{V(1,1)} = 2C_F e_q^2 [4(1 - x) + 2(1 + x)\ln(x) + 2p_{qq}(-x)S_2(x)],$$

$$P_{gq}^{(1,1)} = C_F e_q^2 \left[-(3\ln(1 - x) + \ln^2(1 - x))p_{gq}(x) + \left(2 + \frac{7}{2}x \right) \ln(x) \right. \\ \left. - \left(1 - \frac{x}{2} \right) \ln^2(x) - 2x\ln(1 - x) - \frac{7}{2}x - \frac{5}{2} \right],$$

$$P_{\gamma q}^{(1,1)} = P_{gq}^{(1,1)},$$

● QED²

$$P_{q\gamma}^{(0,2)} = \frac{C_A e_q^4}{2} \left\{ 4 - 9x - (1 - 4x)\ln(x) - (1 - 2x)\ln^2(x) + 4\ln(1 - x) \right. \\ \left. + p_{qg}(x) \left[2\ln^2\left(\frac{1-x}{x}\right) - 4\ln\left(\frac{1-x}{x}\right) - \frac{2\pi^2}{3} + 10 \right] \right\},$$

$$P_{\gamma q}^{(0,2)} = e_q^4 \left[-(3\ln(1 - x) + \ln^2(1 - x))p_{gq}(x) + \left(2 + \frac{7}{2}x \right) \ln(x) - \left(1 - \frac{x}{2} \right) \ln^2(x) \right. \\ \left. - 2x\ln(1 - x) - \frac{7}{2}x - \frac{5}{2} \right] - e_q^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}x + p_{gq}(x) \left(\frac{20}{9} + \frac{4}{3}\ln(1 - x) \right) \right]$$

$$P_{qq}^{V(0,2)} = -e_q^4 \left[\left(2\ln(x)\ln(1 - x) + \frac{3}{2}\ln(x) \right) p_{qq}(x) + \frac{3 + 7x}{2} \ln(x) \right. \\ \left. + \frac{1 + x}{2} \ln^2(x) + 5(1 - x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3 \right) \delta(1 - x) \right] \\ - e_q^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}(1 - x) + p_{qq}(x) \left(\frac{2}{3}\ln(x) + \frac{10}{9} \right) + \left(\frac{2\pi^2}{9} + \frac{1}{6} \right) \delta(1 - x) \right],$$

$$P_{q\bar{q}}^{V(0,2)} = e_q^4 [4(1 - x) + 2(1 + x)\ln(x) + 2p_{qq}(-x)S_2(x)],$$

$$P_{qQ}^{S(0,2)} = P_{q\bar{Q}}^{S(0,2)} = C_A e_q^2 e_Q^2 p_s(x),$$

$$P_{l\gamma}^{(0,2)} = \frac{e_l^4}{C_A e_q^4} P_{q\gamma}^{(0,2)},$$

$$P_{\gamma l}^{(0,2)} = e_l^4 \left[-(3\ln(1 - x) + \ln^2(1 - x))p_{gq}(x) + \left(2 + \frac{7}{2}x \right) \ln(x) - \left(1 - \frac{x}{2} \right) \ln^2(x) \right. \\ \left. - 2x\ln(1 - x) - \frac{7}{2}x - \frac{5}{2} \right] - e_l^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}x + p_{gq}(x) \left(\frac{20}{9} + \frac{4}{3}\ln(1 - x) \right) \right]$$

$$P_{ll}^{V(0,2)} = -e_l^4 \left[\left(2\ln(x)\ln(1 - x) + \frac{3}{2}\ln(x) \right) p_{qq}(x) + \frac{3 + 7x}{2} \ln(x) \right. \\ \left. + \frac{1 + x}{2} \ln^2(x) + 5(1 - x) + \left(\frac{\pi^2}{2} - \frac{3}{8} - 6\zeta_3 \right) \delta(1 - x) \right] \\ - e_l^2 \left(\sum_f e_f^2 \right) \left[\frac{4}{3}(1 - x) + p_{qq}(x) \left(\frac{2}{3}\ln(x) + \frac{10}{9} \right) + \left(\frac{2\pi^2}{9} + \frac{1}{6} \right) \delta(1 - x) \right],$$

$$P_{ll}^{V(0,2)} = \frac{e_l^4}{e_q^4} P_{q\bar{q}}^{V(0,2)},$$

$$P_{lL}^{S(0,2)} = P_{l\bar{L}}^{S(0,2)} = e_l^2 e_L^2 p_s(x).$$

leptons

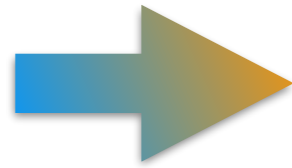
$$P_{\gamma\gamma}^{(0,2)} = \left(\sum_f e_f^4 \right) \left[-16 + 8x + \frac{20}{3}x^2 + \frac{4}{3x} - (6 + 10x)\ln(x) \right. \\ \left. - 2(1 + x)\ln^2(x) - \delta(1 - x) \right],$$

more combinations of pdfs
dependence on electric charge

Several sources of QED+QCD effects in fit of parton distributions

► New distributions

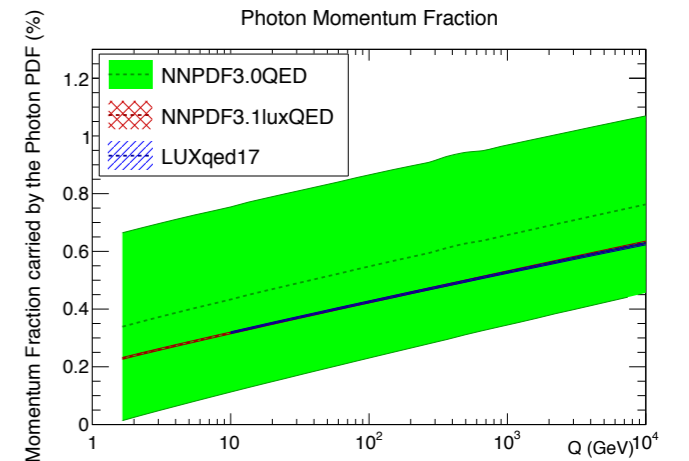
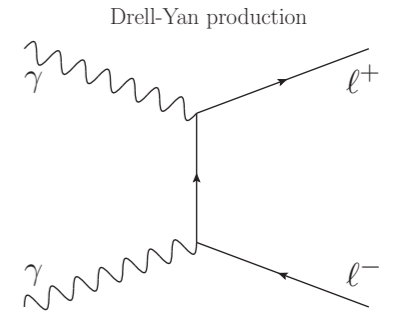
γ , lepton



- photon initial state

- share proton momentum

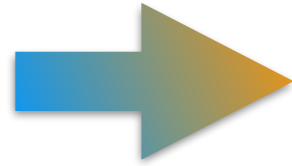
$$\int_0^1 dx x (\Sigma(x, Q) + g(x, Q) + \gamma(x, Q) + \Sigma_L(x, Q)) = 1$$



Several sources of QED+QCD effects in fit of parton distributions

► New distributions

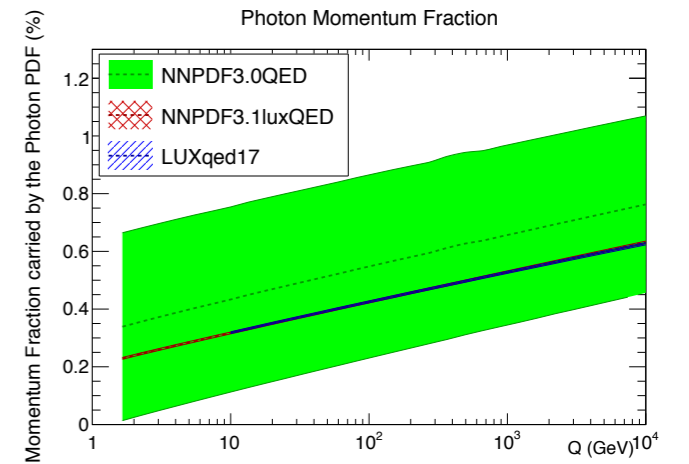
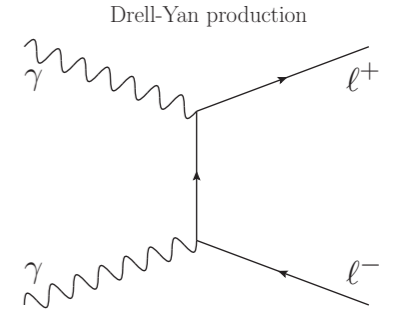
γ , lepton



- photon initial state

- share proton momentum

$$\int_0^1 dx x (\Sigma(x, Q) + g(x, Q) + \gamma(x, Q) + \Sigma_L(x, Q)) = 1$$



► New splitting functions



mixing and charge separation

$$\begin{aligned} \frac{d\Sigma}{dt} = & \frac{P_u^+ + P_d^+}{2} \otimes \Sigma + \frac{P_u^+ - P_d^+}{2} \otimes \Delta_{UD} + \frac{n_u \bar{P}_{uu}^S + n_d \bar{P}_{dd}^S + (n_u + n_d) \bar{P}_{ud}^S}{2} \otimes \Sigma \\ & + \frac{n_u \bar{P}_{uu}^S - n_d \bar{P}_{dd}^S - (n_u - n_d) \bar{P}_{ud}^S}{2} \otimes \Delta_{UD} + (n_u \bar{P}_{ul}^S + n_d \bar{P}_{dl}^S) \otimes \Sigma^l \\ & + 2(n_u P_{ug} + n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma, \end{aligned}$$

Several sources of QED+QCD effects in fit of parton distributions

▶ New distributions

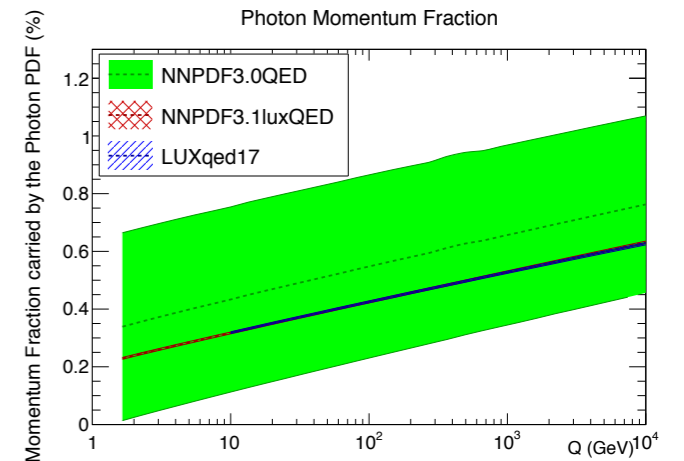
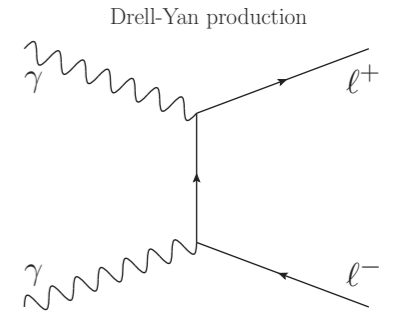
γ , lepton



- photon initial state

- share proton momentum

$$\int_0^1 dx x (\Sigma(x, Q) + g(x, Q) + \gamma(x, Q) + \Sigma_L(x, Q)) = 1$$



▶ New splitting functions



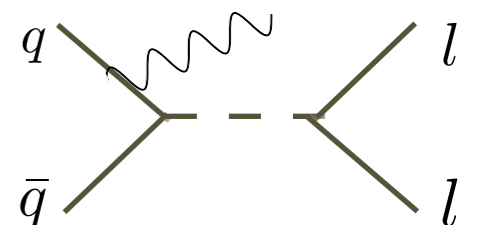
mixing and charge separation

$$\begin{aligned} \frac{d\Sigma}{dt} = & \frac{P_u^+ + P_d^+}{2} \otimes \Sigma + \frac{P_u^+ - P_d^+}{2} \otimes \Delta_{UD} + \frac{n_u \bar{P}_{uu}^S + n_d \bar{P}_{dd}^S + (n_u + n_d) \bar{P}_{ud}^S}{2} \otimes \Sigma \\ & + \frac{n_u \bar{P}_{uu}^S - n_d \bar{P}_{dd}^S - (n_u - n_d) \bar{P}_{ud}^S}{2} \otimes \Delta_{UD} + (n_u \bar{P}_{ul}^S + n_d \bar{P}_{dl}^S) \otimes \Sigma^l \\ & + 2(n_u P_{ug} + n_d P_{dg}) \otimes g + 2(n_u P_{u\gamma} + n_d P_{d\gamma}) \otimes \gamma, \end{aligned}$$

▶ QED (+QCD) corrections



effect on observables



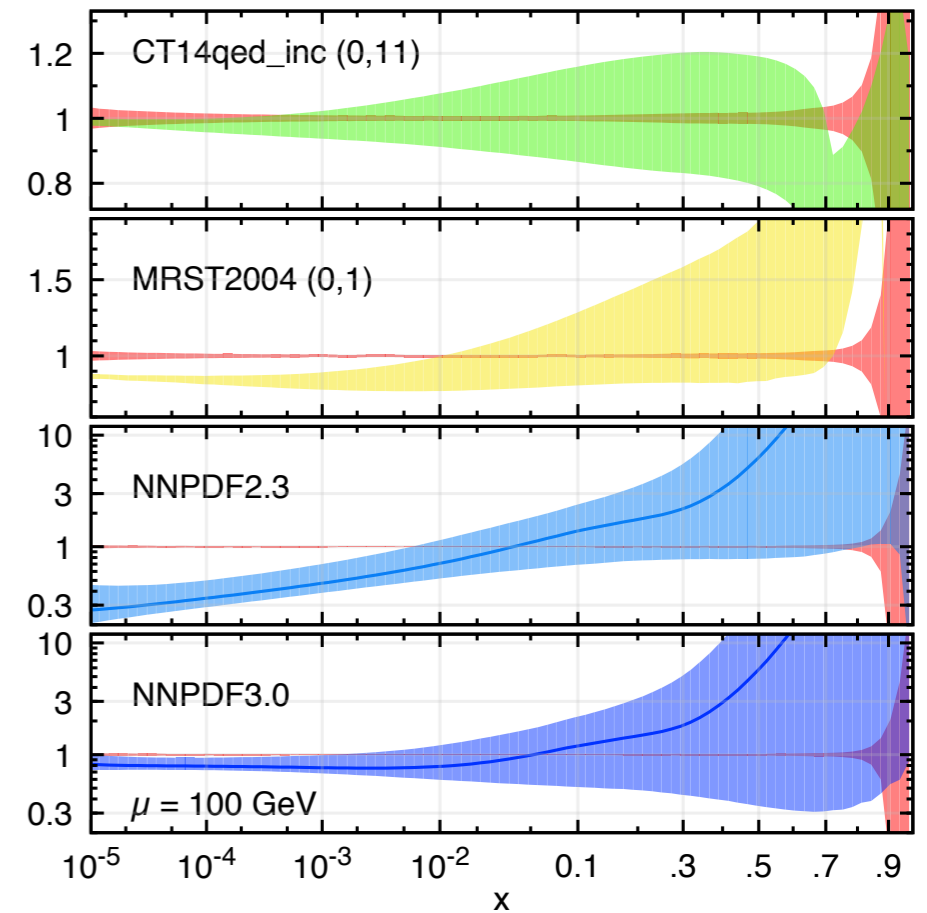
► Large uncertainty on (QED dominant) photon distribution

- Until LUXqed

Manohar, Nason, Salam, Zanderighi (2016,2017)

photon PDF can be expressed in terms of the structure functions F_2 and F_L by means of an exact QED calculation

$$x\gamma(x, \mu) = \frac{1}{2\pi\alpha(\mu)} \int_x^1 \frac{dz}{z} \left\{ \int_{Q_{\min}^2}^{\mu^2/(1-z)} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[-z^2 F_L(x/z, Q^2) + \left(zP_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) \right] - \alpha^2(\mu) z^2 F_2(x/z, \mu^2) \right\} + \mathcal{O}(\alpha\alpha_s, \alpha^2)$$



involve low Q^2
elastic/resonance region

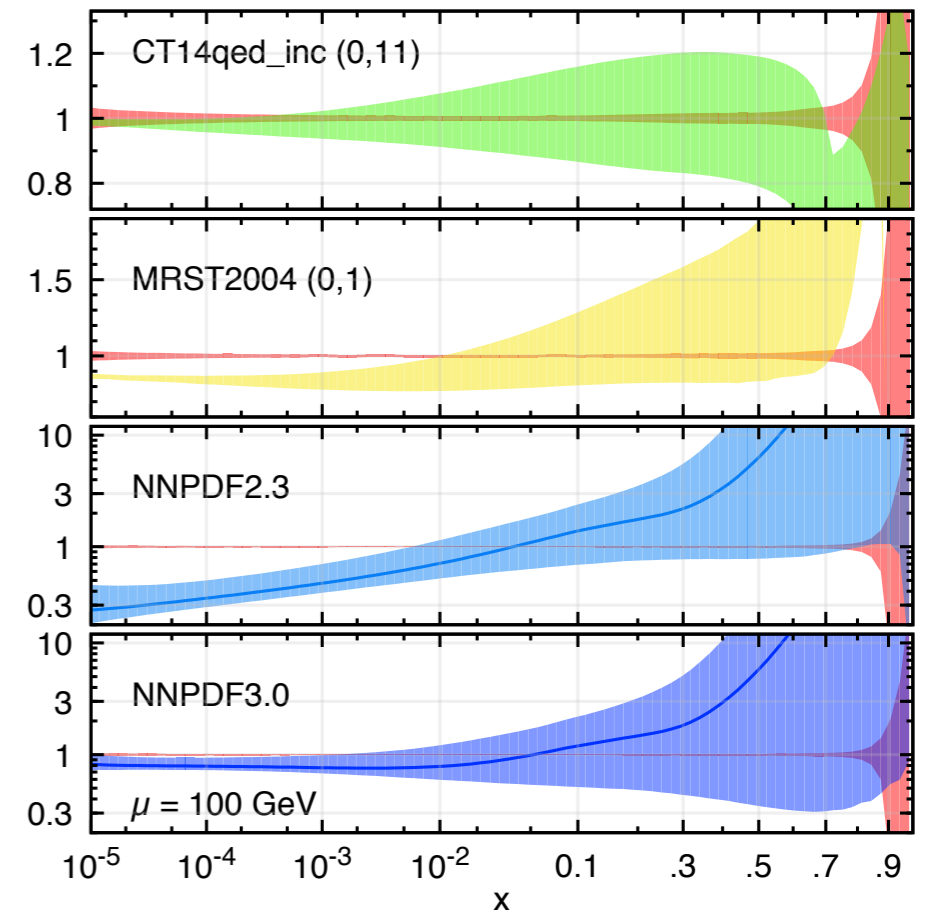
► Large uncertainty on (QED dominant) photon distribution

- Until LUXqed

Manohar, Nason, Salam, Zanderighi (2016,2017)

photon PDF can be expressed in terms of the structure functions F_2 and F_L by means of an exact QED calculation

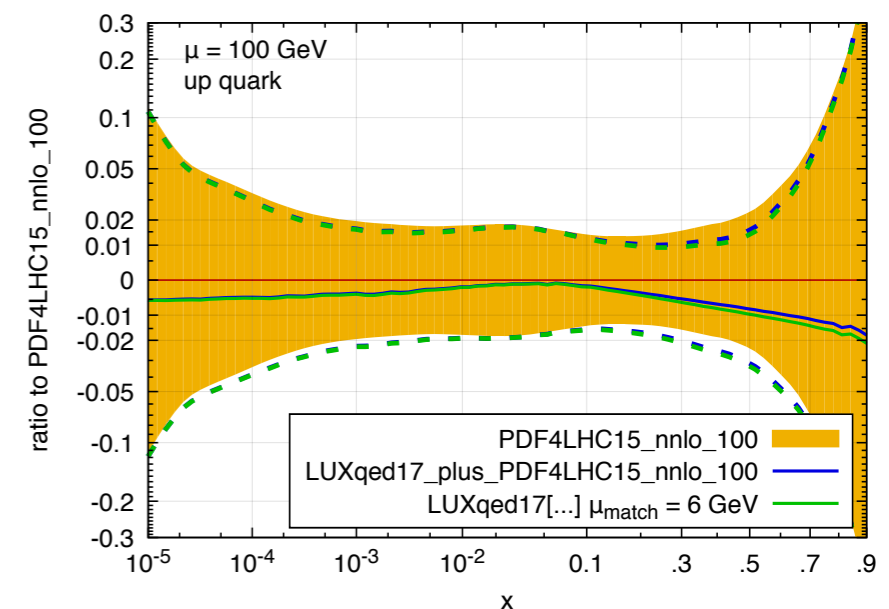
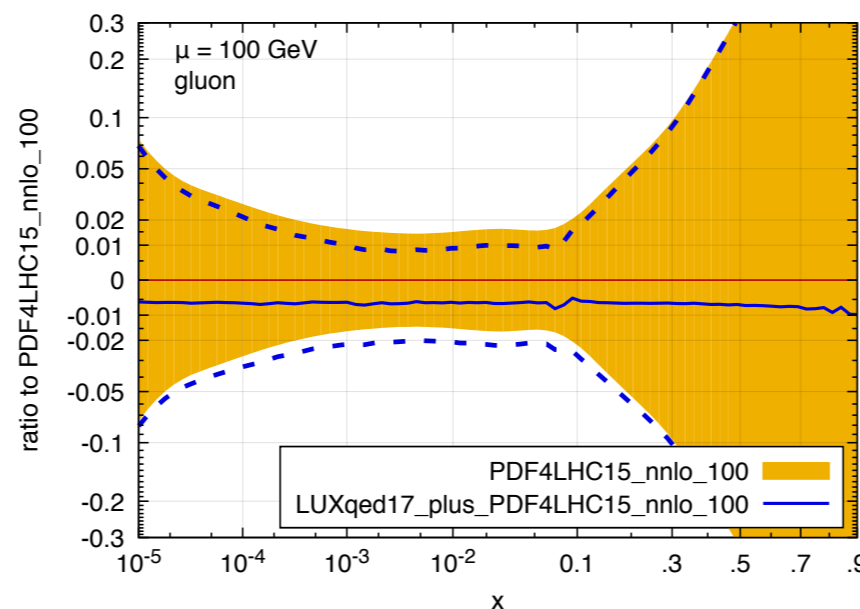
$$x\gamma(x, \mu) = \frac{1}{2\pi\alpha(\mu)} \int_x^1 \frac{dz}{z} \left\{ \int_{Q_{\min}^2}^{\mu^2/(1-z)} \frac{dQ^2}{Q^2} \alpha^2(Q^2) \left[-z^2 F_L(x/z, Q^2) + \left(zP_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2(x/z, Q^2) \right] - \alpha^2(\mu) z^2 F_2(x/z, \mu^2) \right\} + \mathcal{O}(\alpha\alpha_s, \alpha^2)$$



involve low Q^2 elastic/resonance region

“iterative” extraction of QED modified pdfs

LUXqed17 plus PDF4LHC15



- ▶ Full fit including QED+QCD corrections

NNPDF collaboration
Bertone, Carraza, Hartland, Rojo (2018)

NNPDF3.1luxQED

QCD + $\alpha_s \alpha$ + α^2 splitting functions plus α corrections to DIS

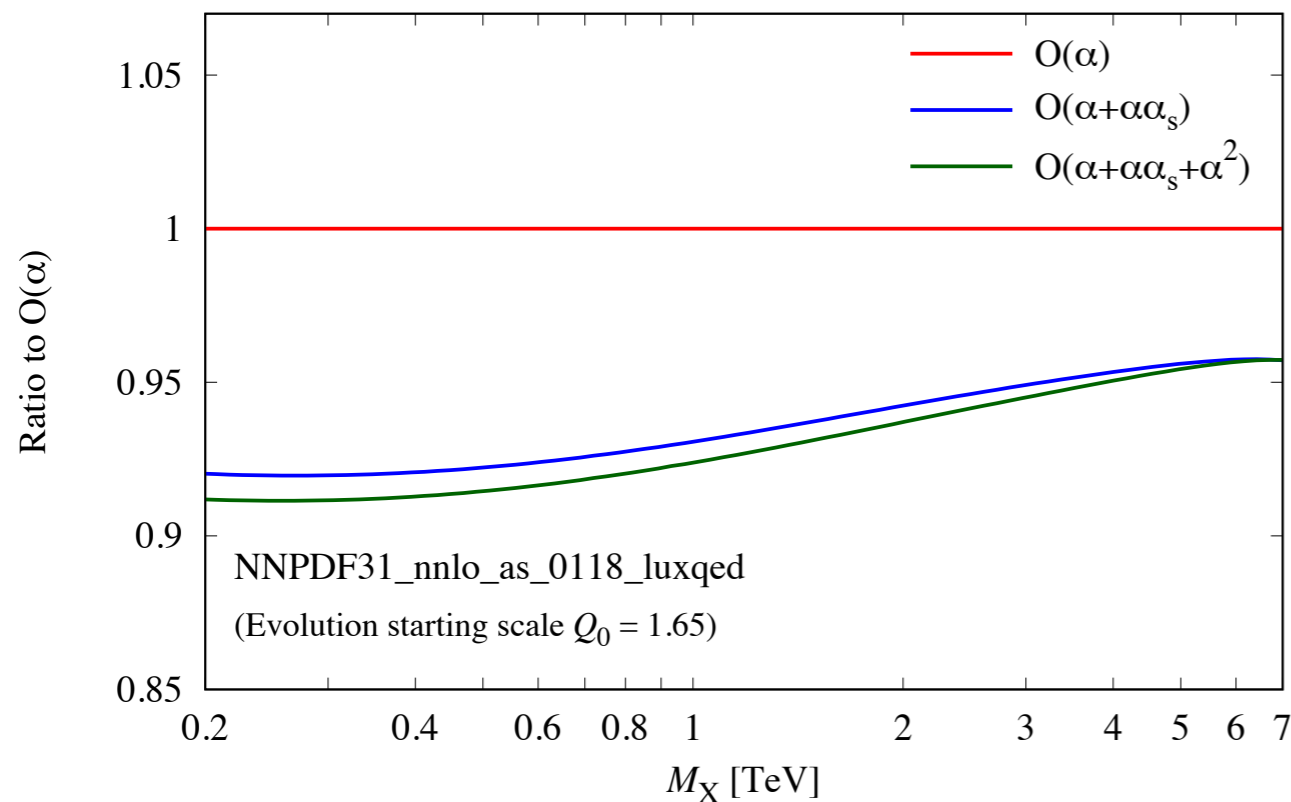
- ▶ NNPDF global analysis with photon pdf based on LUXqed

- QED+QCD effects in photon distribution ($\gamma\gamma$ Luminosity)

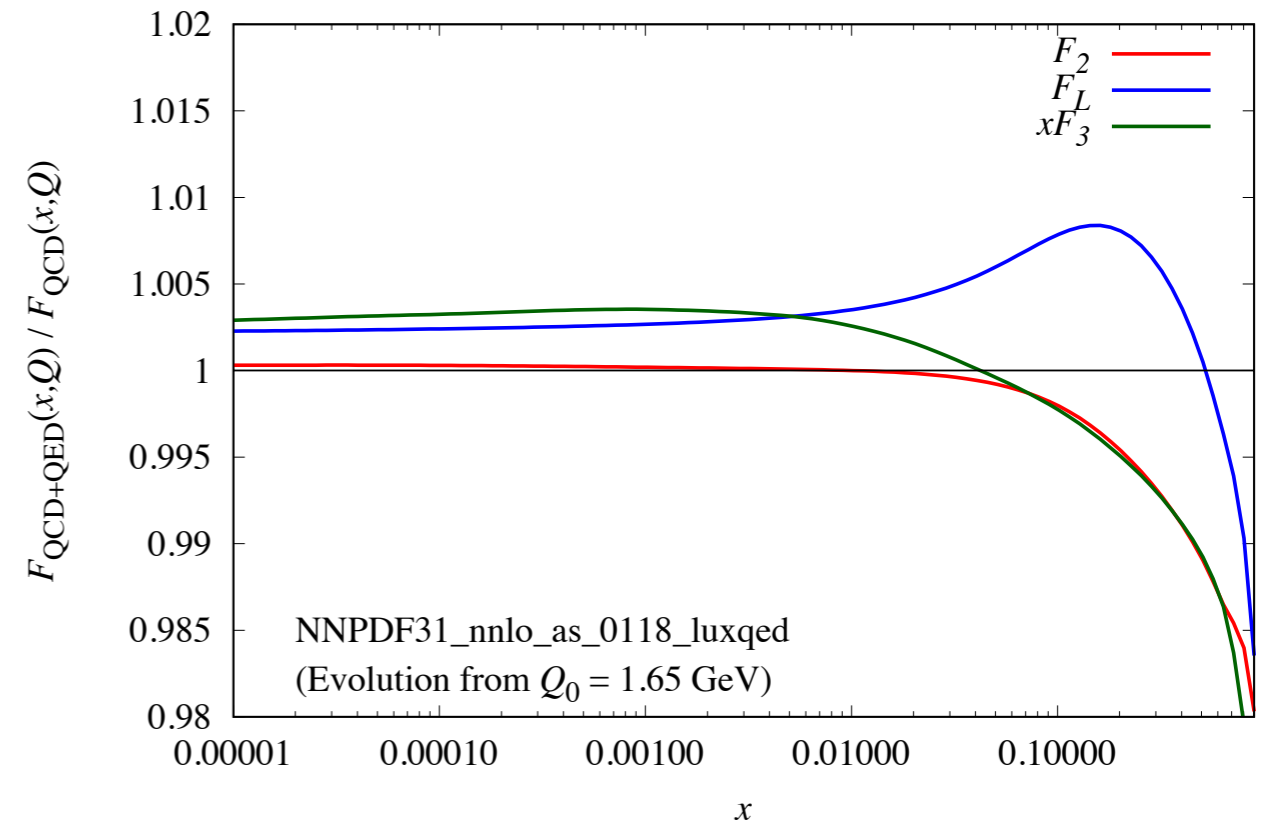
$\alpha_s \alpha \sim 10\%$ $\alpha^2 \sim 1\%$

$\sim 1\%$ level in DIS SF

$\gamma\gamma$ Luminosity at $\sqrt{s} = 13$ TeV



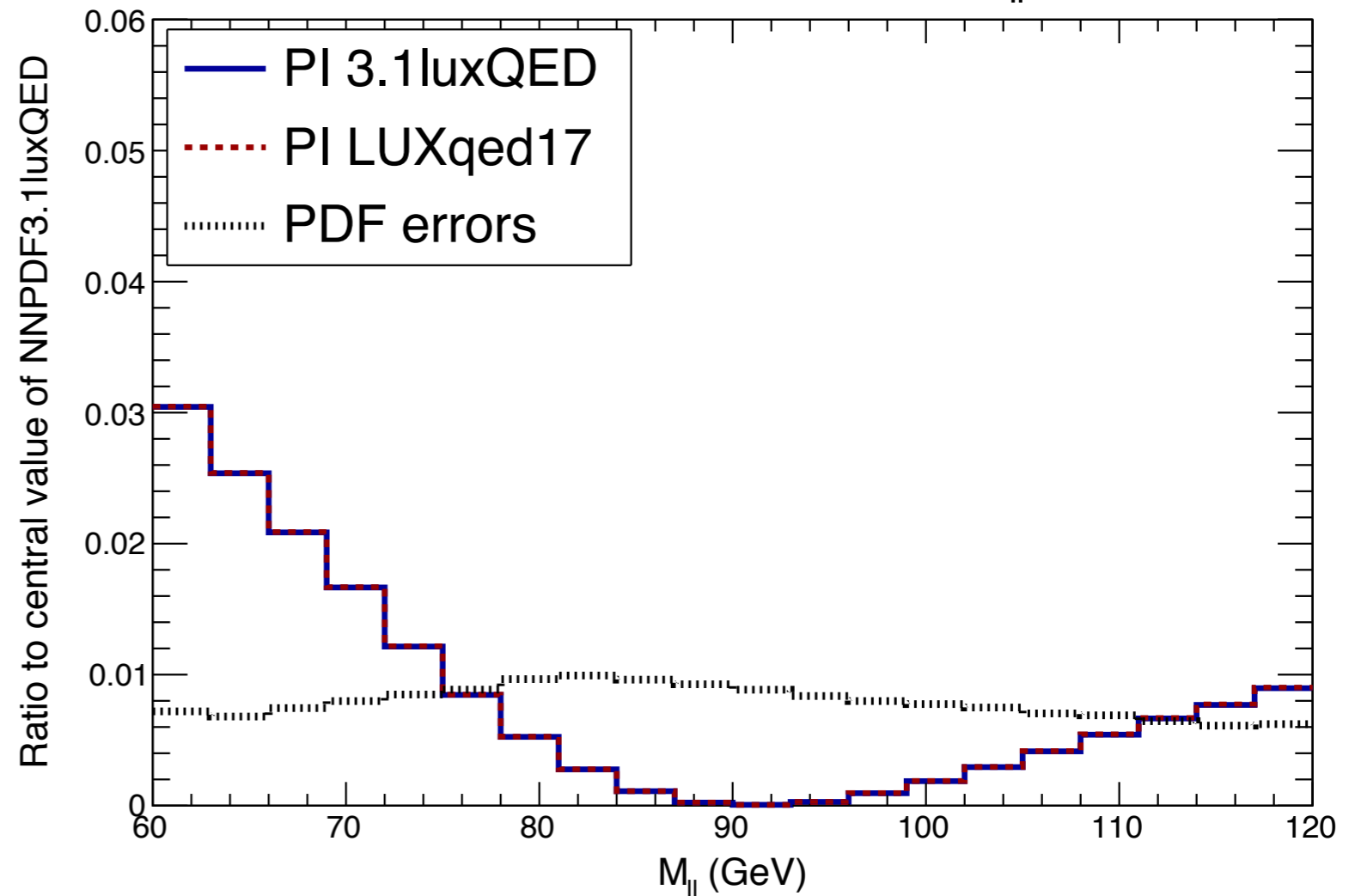
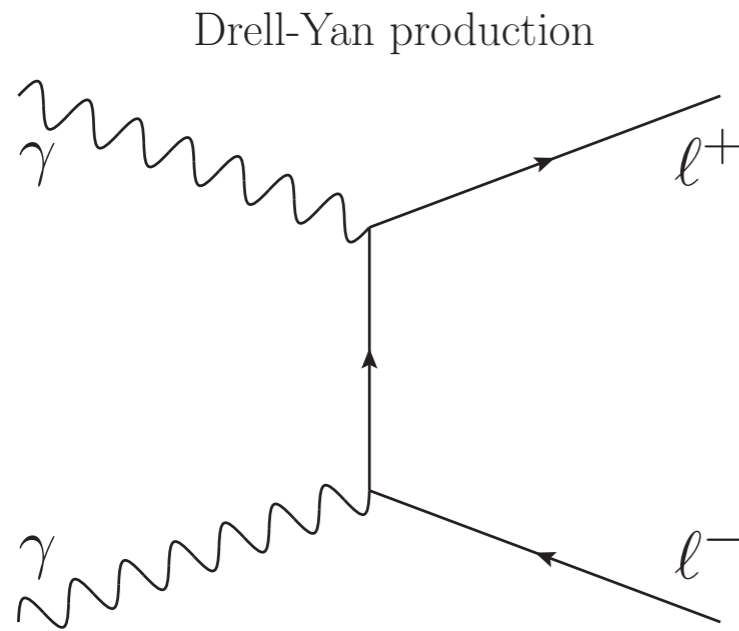
Neutral current structure functions in the FONLL-C scheme ($Q = 100$ GeV)



► One example of photon initiated processes : Drell-Yan

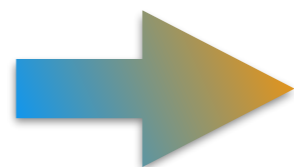
NNPDF collaboration Bertone, Carraza, Hartland, Rojo (2018)

$$p p \rightarrow l^+ l^- @ \sqrt{s} = 13 \text{ TeV}, 0 < |y_{ll}| < 2.5$$



► Photon Initiated effects very small at Z, but larger away from peak

3% at $M_{ll}=60$ GeV

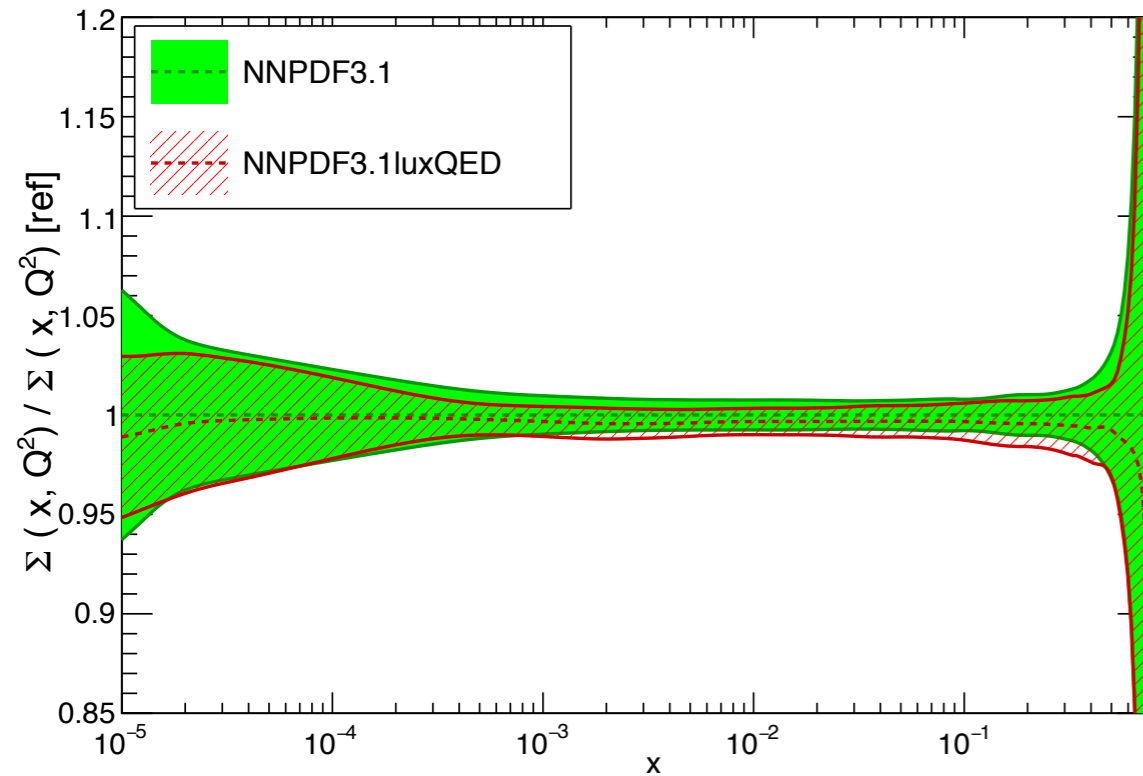


larger than pdf DY uncertainty

Impact of QED in quark and gluon pdfs

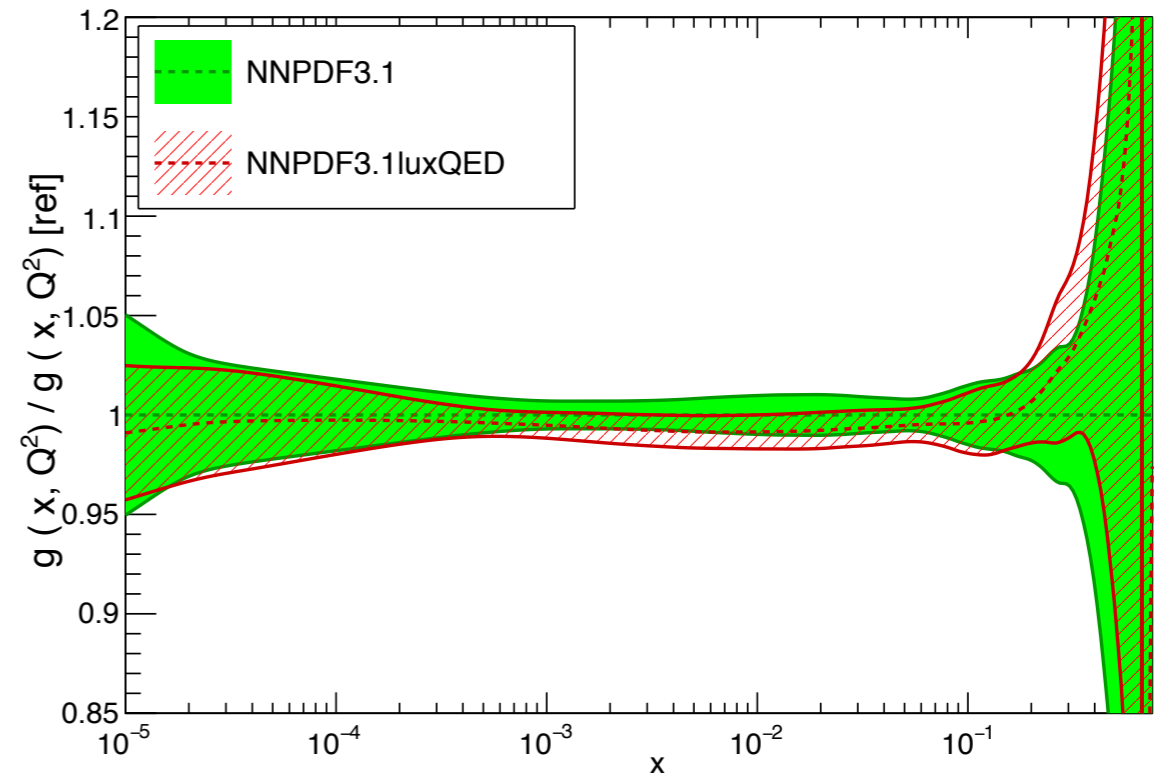
NNPDF collaboration
Bertone, Carraza, Hartland, Rojo (2018)

NNLO, $Q^2=10^4 \text{ GeV}^2$



small for the quark singlet
below 1%

NNLO, $Q^2=10^4 \text{ GeV}^2$



larger for the gluon (within band)

-1% around $x=0.01$
+5% at $x=0.5$

- ▶ Effect explained by photon PDF carrying $\sim 0.5\%$ of proton momentum
- ▶ Mostly for gluon since quark singlet more constrained by DIS

Very recent ▶ MMHT global analysis

MMHT collaboration

Harland-Lang, Martin, Nathvani, Thorne(2019)

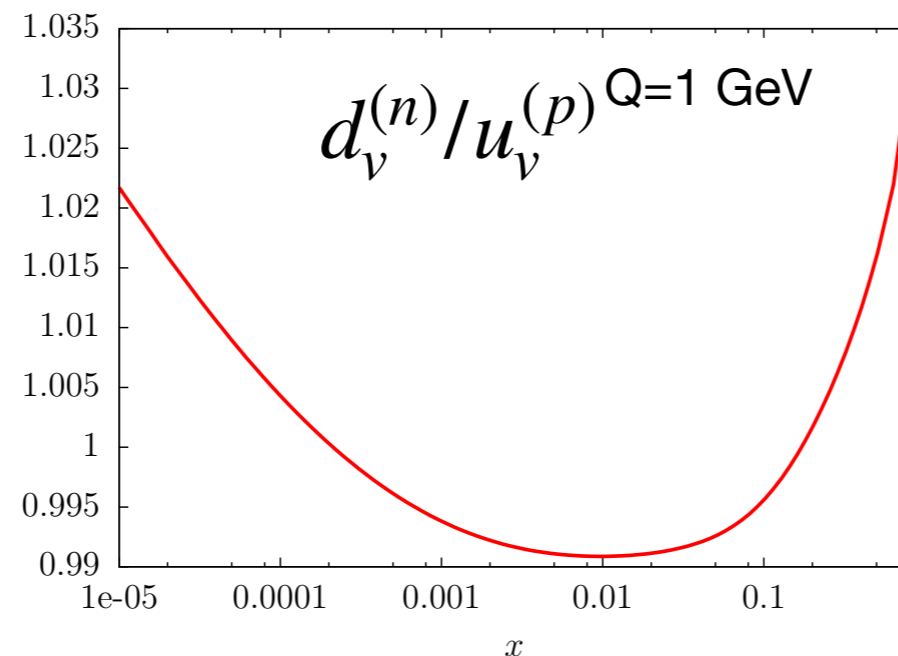
▶ Full fit including QED+QCD corrections with photon pdf based on LUXqed

QCD + $\alpha_s \alpha$ + α^2 splitting functions plus α corrections to DIS

▶ QED introduces isospin breaking (proton ↔ neutron)

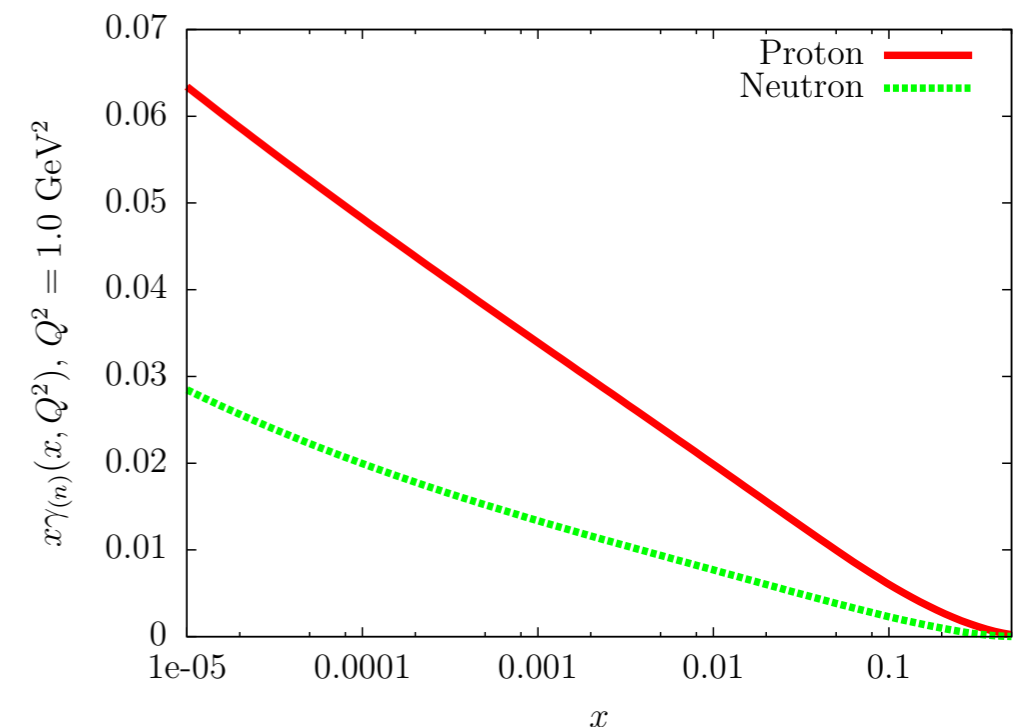
$$u_v^{(p)} \neq d_v^{(n)}$$

$$d_v^{(p)} \neq u_v^{(n)}$$



$$\Delta u_{V,(n)}(x, Q_0^2) = \epsilon \left(1 - \frac{e_u^2}{e_d^2} \right) d_{V,(p)}^{(QED)}(x, Q_0^2)$$

▶ also photon pdf larger for proton than for neutron $\sim 2x$ momentum



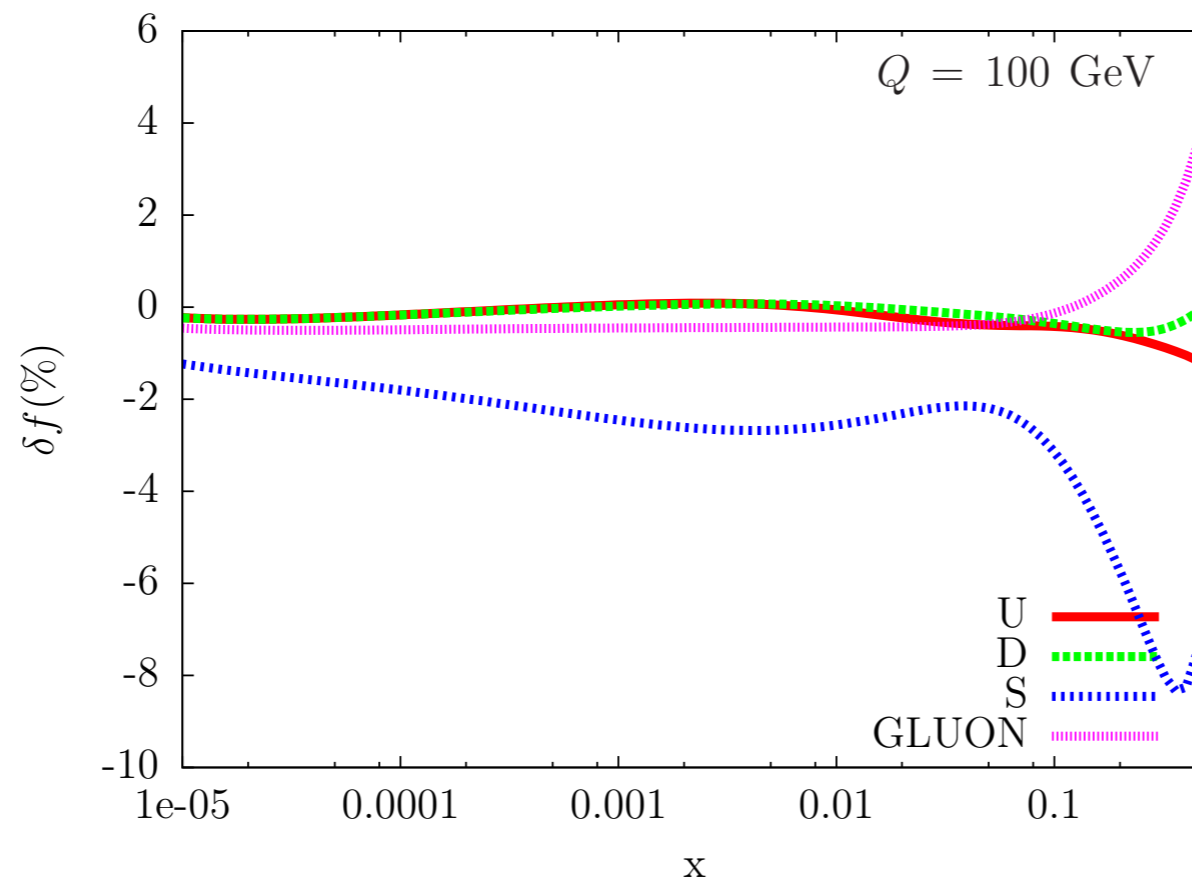
► Change in α_s

Leading QED effect $\alpha_s \rightarrow \alpha' = \left(\alpha_s + \frac{e_q^2 \alpha}{C_F} \right)$ expect per mil reduction in α_s

- but gluon momentum loss by photon requires larger α_s and compensate

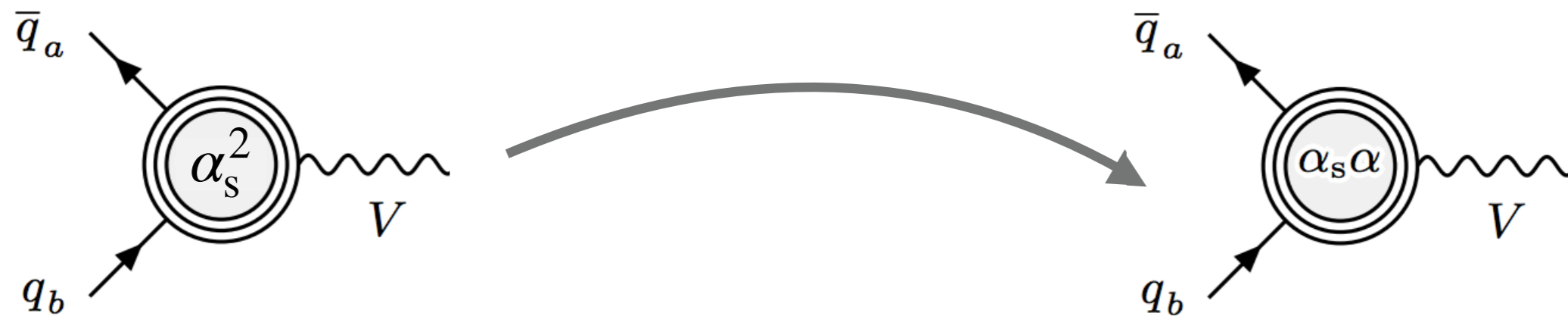
central value almost unchanged $0.1181 \rightarrow 0.1180$

► Again, gluon (and s) most affected distribution in MMHT QED



- In summary, some QED effects from PDFs might exceed the 1% level

QED+QCD NNLO corrections to Drell Yan Production

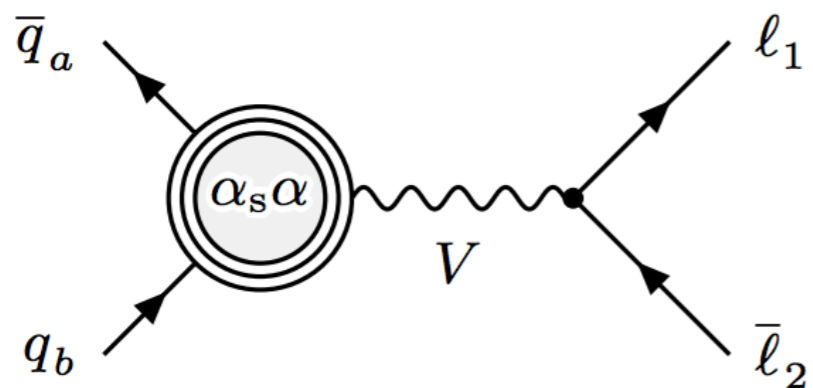


► Mixed EWxQCD corrections in the resonance region (pole approx.)

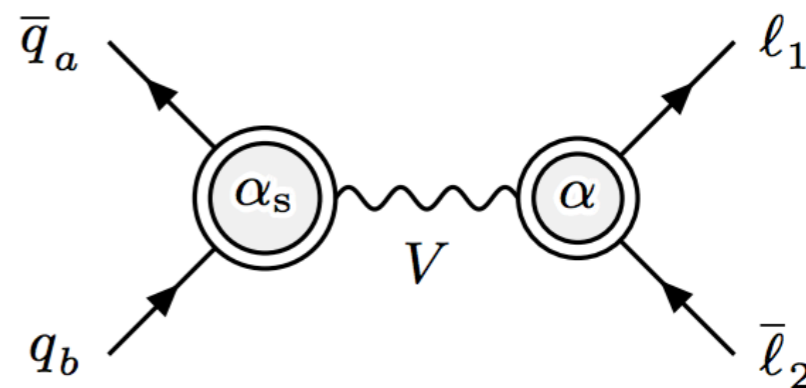
Dittmaier, Huss, Schwinn (2014, 2015, 2016)

Inclusive Drell-Yan

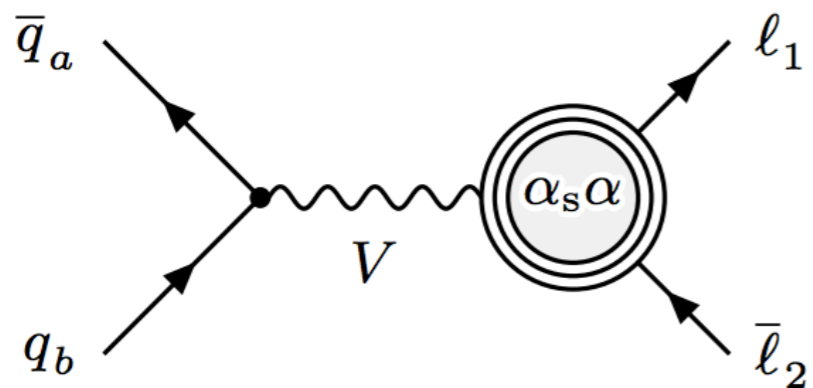
main contribution



(a) Factorizable initial–initial corrections

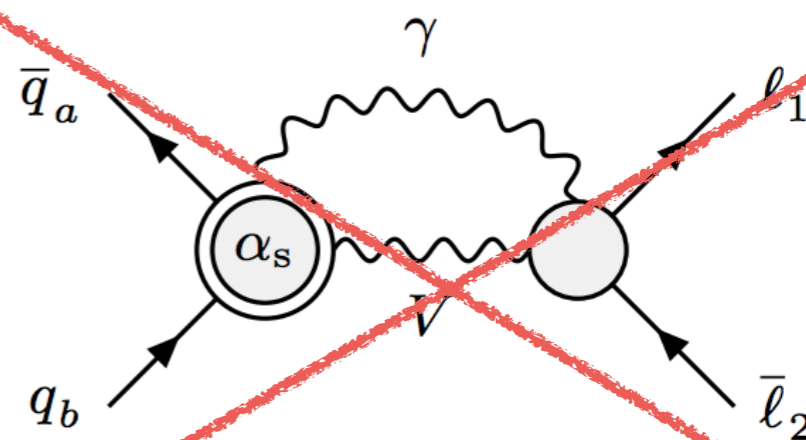


(b) Factorizable initial–final corrections



(c) Factorizable final–final corrections

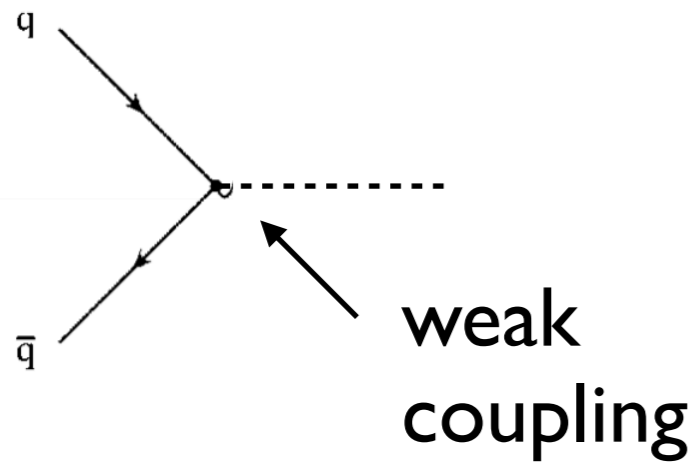
negligible and known



(d) Non-factorizable corrections

negligible (<0.1%)

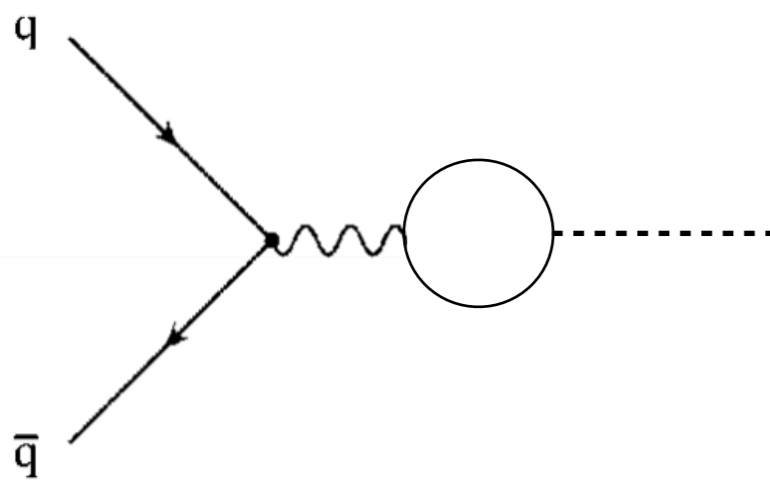
QED corrections to inclusive Drell-Yan (on-shell Z production)



$$\sigma_Z(M_Z^2) = \frac{\pi^2 \alpha}{4M_Z^2 N_C \sin^2 \theta_W \cos^2 \theta_W} \equiv \frac{G_F \pi}{4\sqrt{2} N_C}$$

Considered as effective
“Weak” coupling (“not QED”)

- ▶ Distinguish “pure” QED corrections from “EW” ones

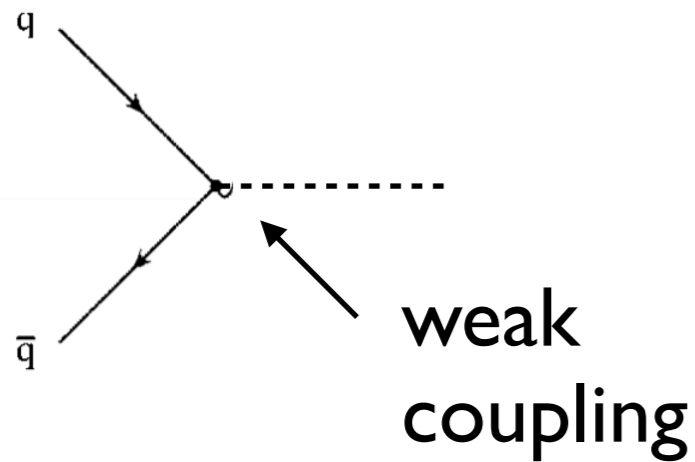


discard self-energy insertions
in Z propagator



Renormalization of EW couplings

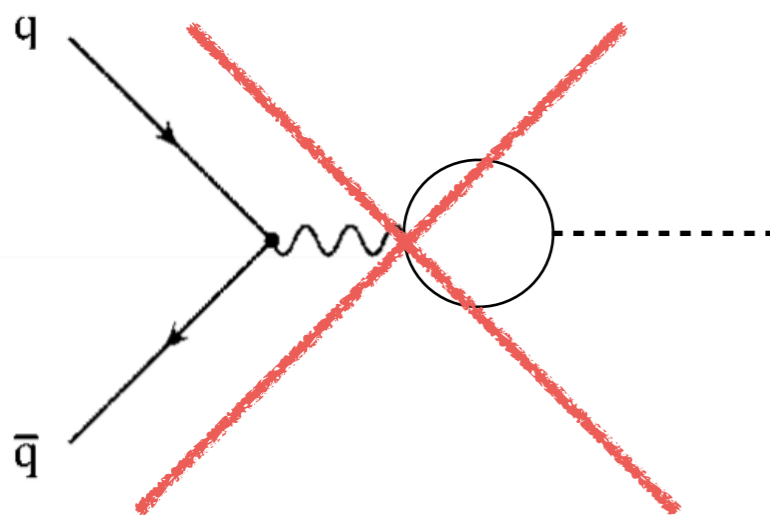
QED corrections to inclusive Drell-Yan (on-shell Z production)



$$\sigma_Z(M_Z^2) = \frac{\pi^2 \alpha}{4M_Z^2 N_C \sin^2 \theta_W \cos^2 \theta_W} \equiv \frac{G_F \pi}{4\sqrt{2} N_C}$$

Considered as effective
“Weak” coupling (“not QED”)

- Distinguish “pure” QED corrections from “EW” ones



discard self-energy insertions
in Z propagator



Renormalization of EW couplings

► QCD NNLO for (inclusive) DY has been available for quite some time

A COMPLETE CALCULATION OF THE ORDER α_s^2 CORRECTION TO THE DRELL-YAN K-FACTOR

R. HAMBERG and W.L. van NEERVEN*

Instituut-Lorentz, University of Leiden, P.O.B. 9506, 2300 RA Leiden, The Netherlands

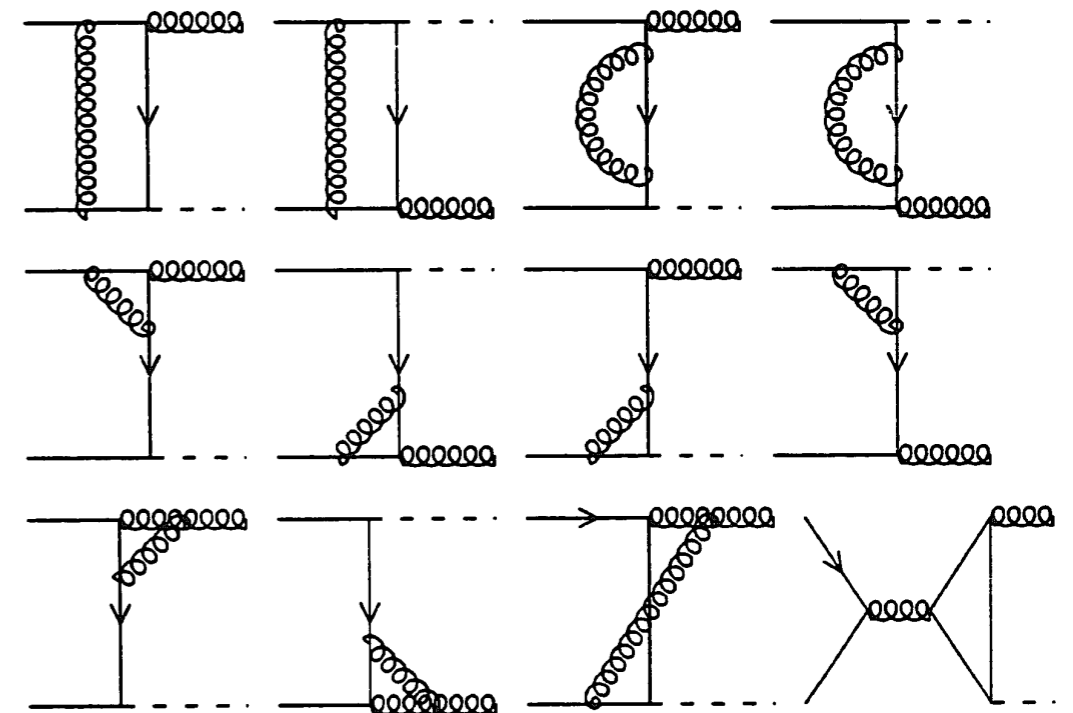
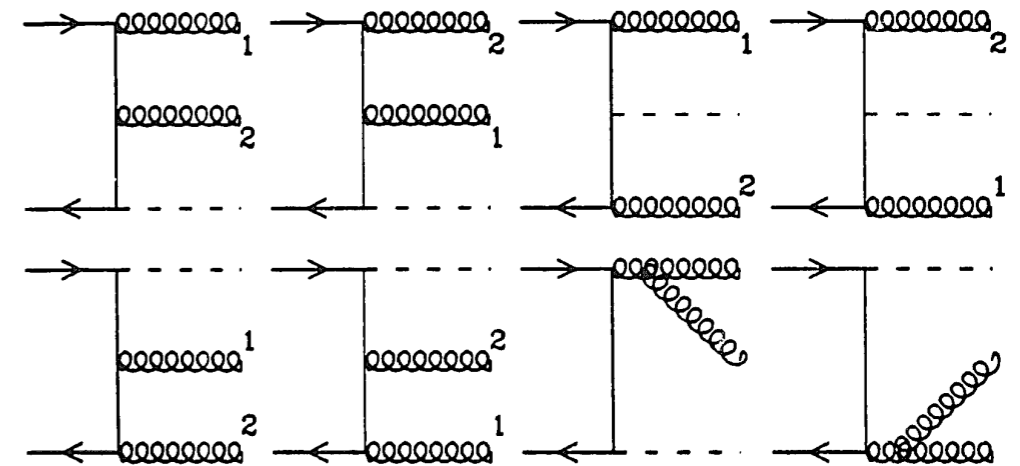
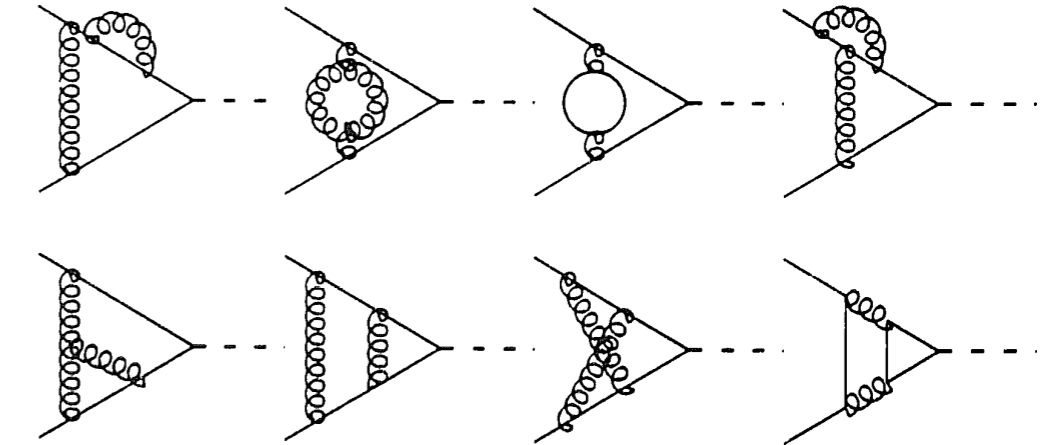
T. MATSUURA**

II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50, Germany

Received 16 November 1990

(Revised 13 February 1991)

small corrections by Harlander, Kilgore (2002)



- ▶ It is possible to use the NNLO QCD result to obtain the QEDxQCD mixed terms and the QED²

general expansion in both couplings $d\sigma = \sum_{i,j} \alpha_s^i \alpha^j d\sigma^{(i,j)}$


“Full NNLO” means $i + j = 2$

(2,0)	QCD ²
(1,1)	QEDxQCD
(0,2)	QED ²

- ▶ Abelianization procedure

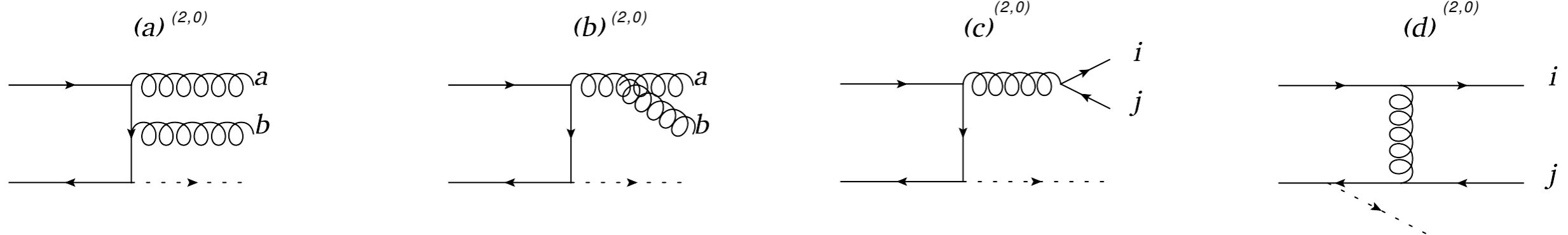
QCD  QED



Same kinematical structure for Abelian contributions  change of color factors

example: qqbar channel

► Identify Topologies and compute Color factors



$$|(a)^{(2,0)}|^2 \sim \frac{1}{2N_c^2} \text{Tr}[T^b T^a T^a T^b] = \frac{1}{2N_c} C_F^2$$

$$[(a)^{(2,0)}(a'^*)^{(2,0)}] \sim \frac{1}{2N_c^2} \text{Tr}[T^b T^a T^b T^a] = \frac{1}{2N_c} C_F \left(C_F - \frac{C_A}{2} \right)$$

$$[(b)^{(2,0)}(a^*)^{(2,0)}] \sim \frac{1}{2N_c^2} f^{abc} \text{Tr}[T^c T^a T^b] = -\frac{1}{2N_c} C_F \frac{C_A}{2}$$

$$|(c)^{(2,0)}|^2 \sim n_F T_R$$

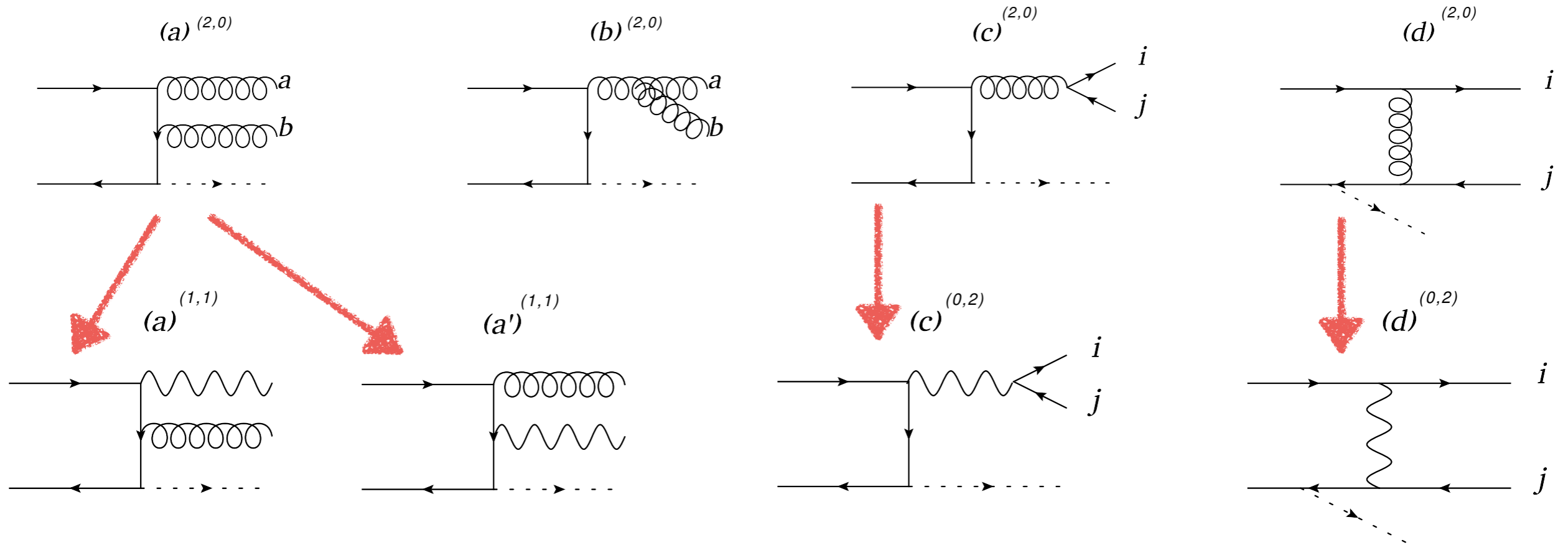
$$C_F^2$$

$$C_F C_A$$

$$n_F T_R$$

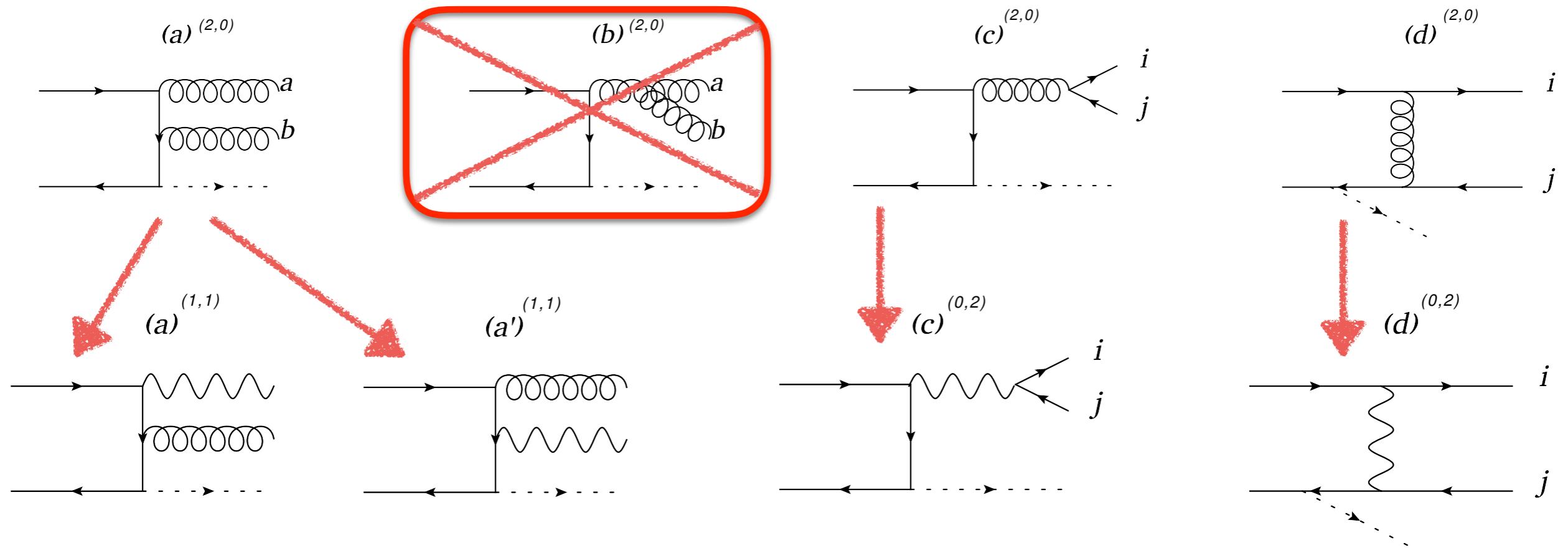
► Replace one gluon by a photon

$\alpha_s \alpha$



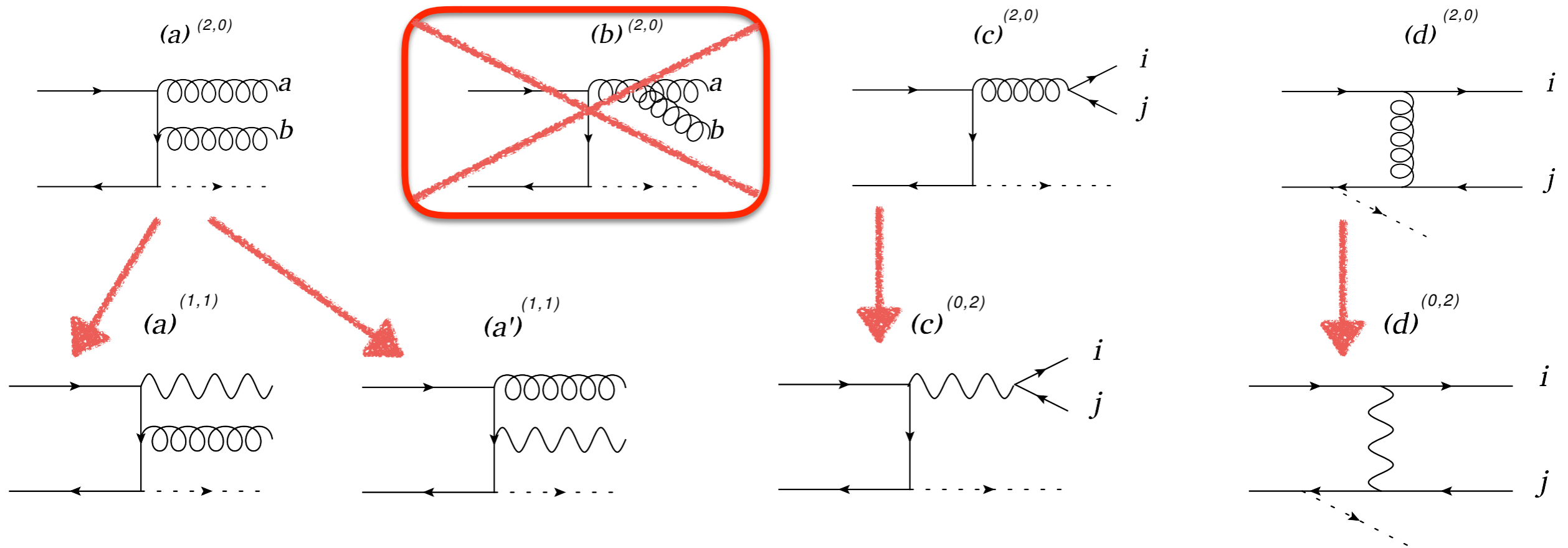
► Replace one gluon by a photon

$\alpha_s \alpha$



► Replace one gluon by a photon

$\alpha_s \alpha$

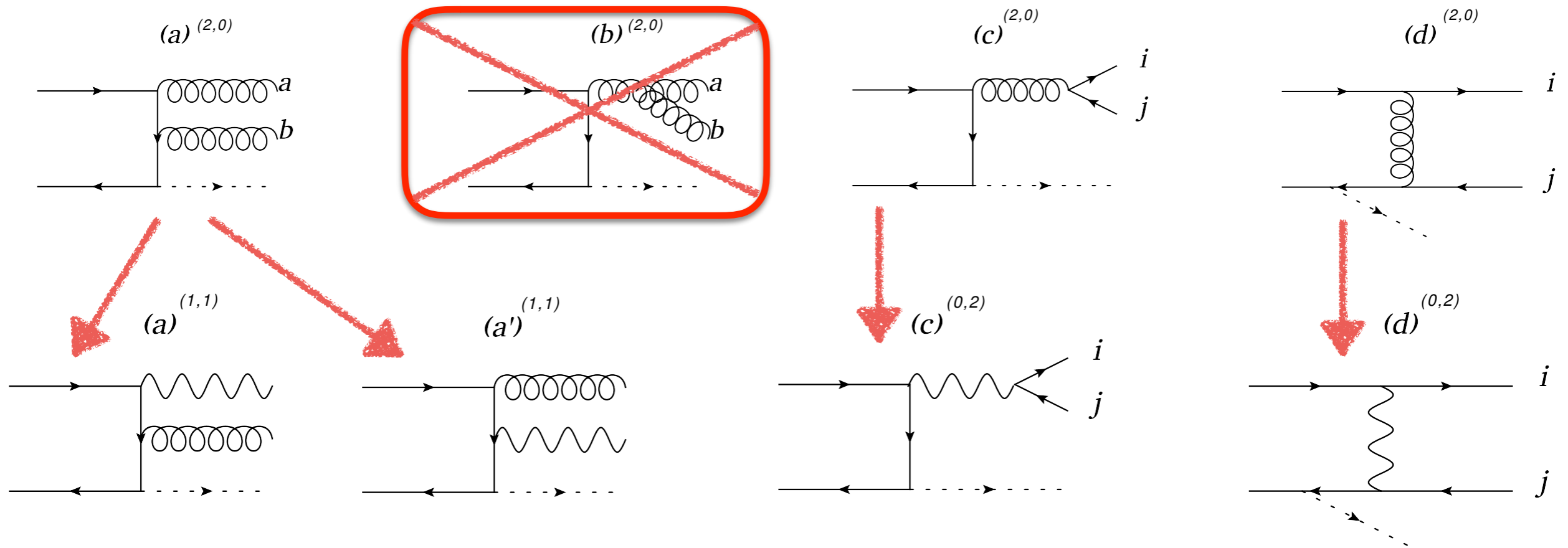


$$|(a)^{(1,1)}|^2 \sim [(a)^{(1,1)}(a'^*)^{(1,1)}] \sim \frac{e_q^2}{N_c^2} \text{Tr}[T^a T^a] = \frac{e_q^2}{N_c} C_F$$

$$-\frac{1}{2N_c} C_F \frac{C_A}{2} \rightarrow 0$$

► Replace one gluon by a photon

$\alpha_s \alpha$



$$|(a)^{(1,1)}|^2 \sim [(a)^{(1,1)}(a'^*)^{(1,1)}] \sim \frac{e_q^2}{N_c^2} \text{Tr}[T^a T^a] = \frac{e_q^2}{N_c} C_F$$

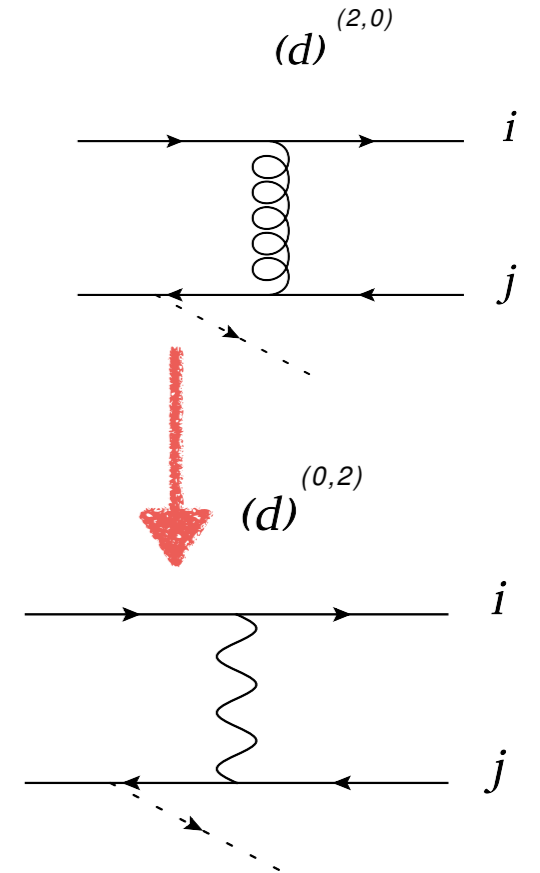
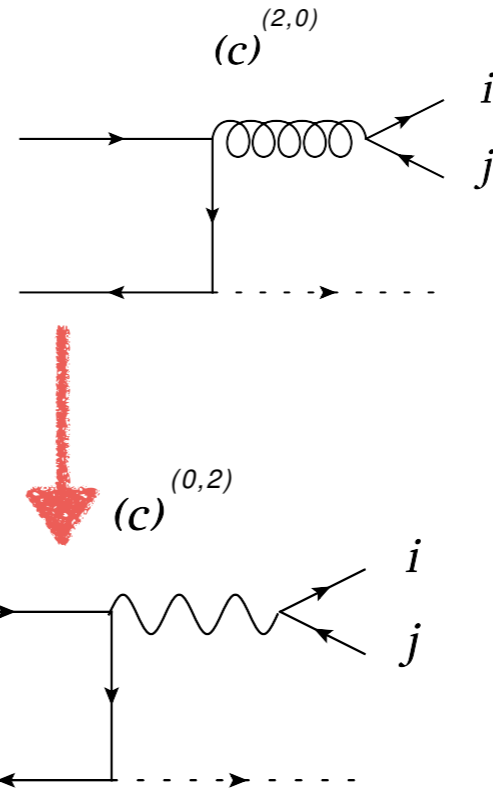
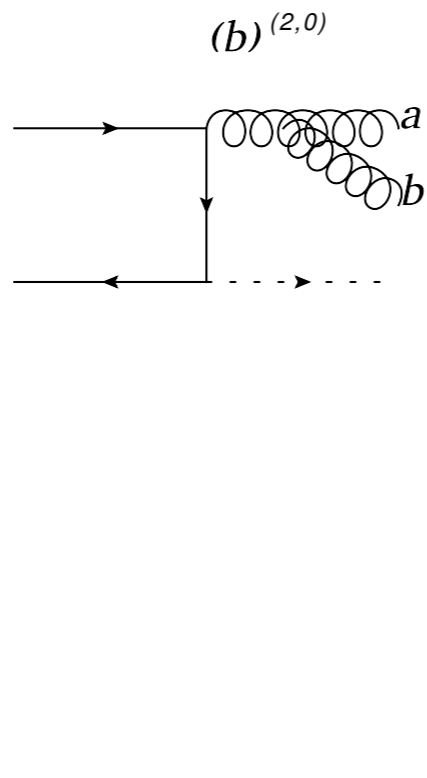
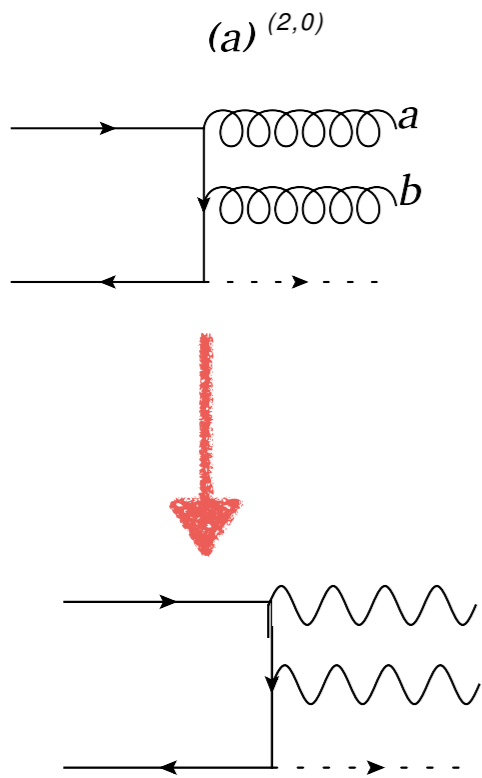
$$-\frac{1}{2N_c} C_F \frac{C_A}{2} \rightarrow 0$$

$$C_F^2 \rightarrow 2 e_q^2 C_F$$

$$C_A \rightarrow 0$$

$$T_R \rightarrow 0$$

► Replace two gluon by photons α^2



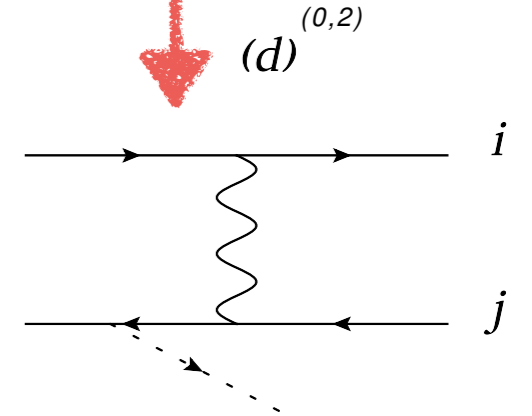
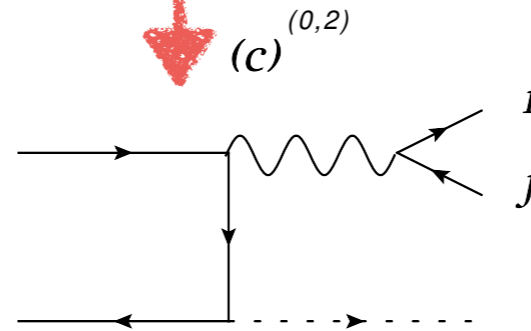
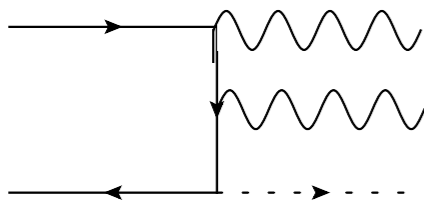
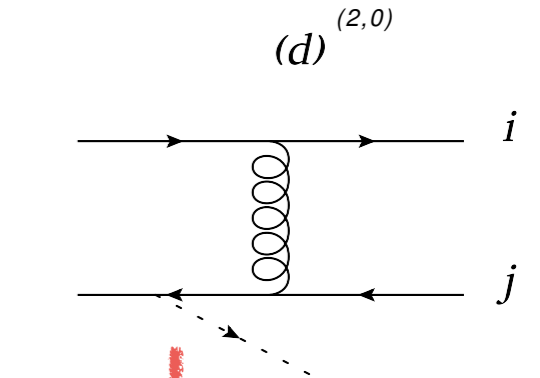
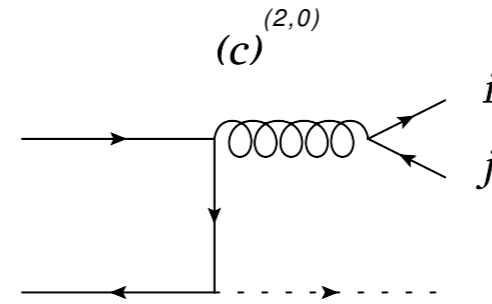
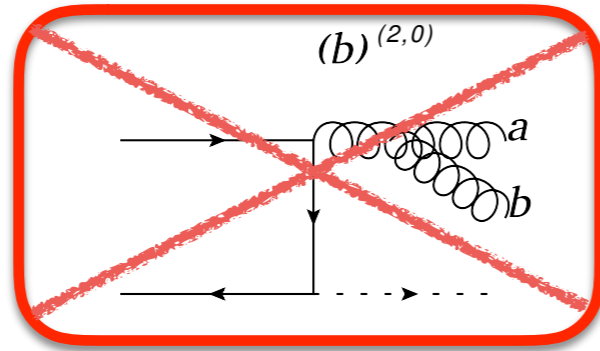
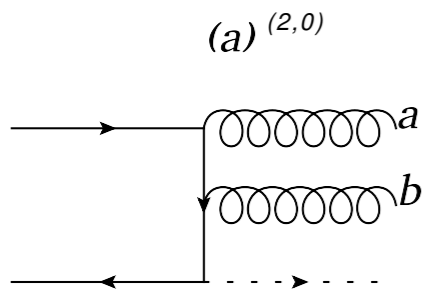
$$C_F^2 \longrightarrow e_q^4$$

$$n_F C_F T_R \longrightarrow e_q^2 \left[N_C \sum_{k \in Q} e_k^2 + \sum_{k \in L} e_k^2 \right]$$

$$C_F^2 \longrightarrow e_q^4$$

$$\beta_0^{\text{QCD}} = \frac{11C_A - 4T_R n_f}{3} \rightarrow \beta_0^{\text{QED}}$$

► Replace two gluon by photons α^2

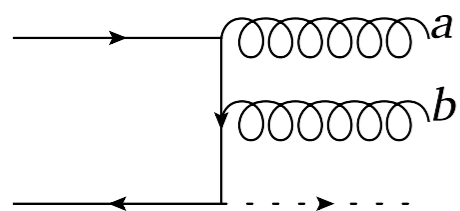


$$C_F^2 \longrightarrow e_q^4$$

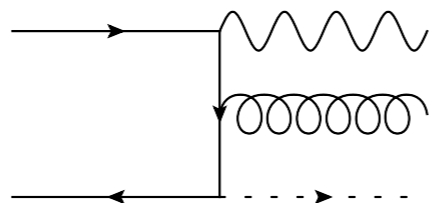
$$n_F C_F T_R \longrightarrow e_q^2 \left[N_C \sum_{k \in Q} e_k^2 + \sum_{k \in L} e_k^2 \right]$$

$$C_F^2 \longrightarrow e_q^4$$

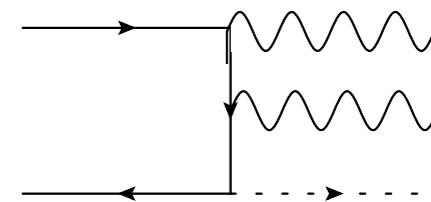
$$\beta_0^{\text{QCD}} = \frac{11C_A - 4T_R n_f}{3} \rightarrow \beta_0^{\text{QED}}$$

α_s^2 

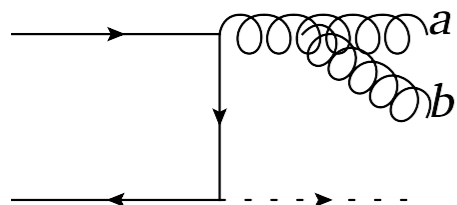
C_F^2

 $\alpha_s \alpha$ 

$2e_q^2 C_F$

 α^2 

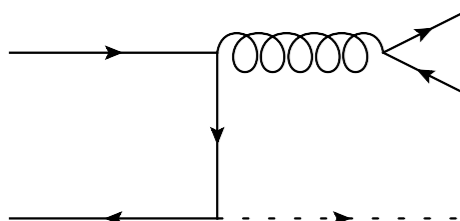
e_q^4



$-\frac{C_F C_A}{2}$

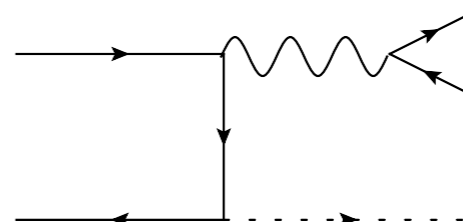
0

0



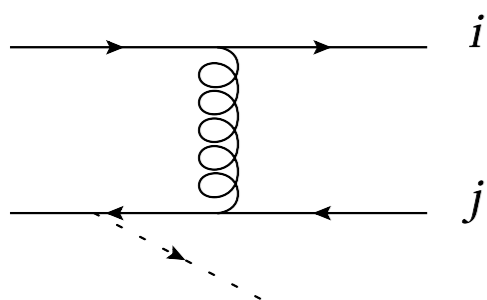
$n_F C_F T_R$

0



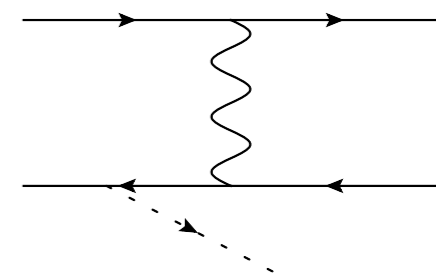
$$e_q^2 \left[N_C \sum_{k \in Q} e_k^2 + \sum_{k \in L} e_k^2 \right]$$

Interferences



$C_F^2 - \frac{C_F C_A}{2}$

$2e_q^2 C_F$



e_q^4

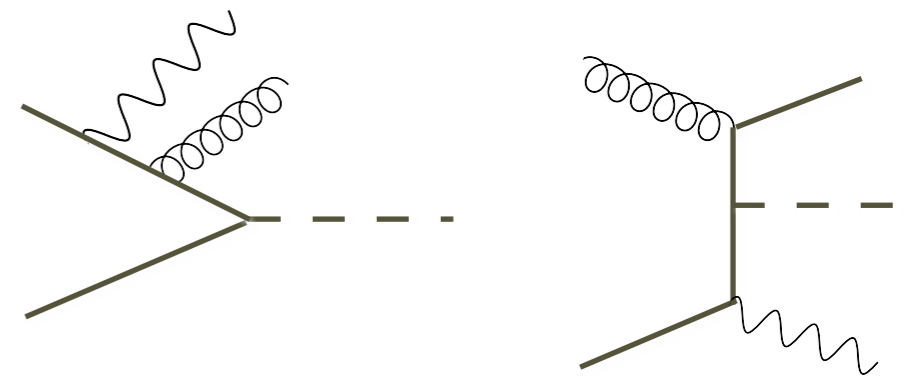
QED+QCD corrections to DY: phenomenology

$$\sigma = \tau \sigma_Z(M_Z^2) W_Z(\tau, M_Z^2)$$

$$w_Z^{(1,1)} = \sum_{i \in Q, \bar{Q}} q_i(x_1) \bar{q}_i(x_2) c_i 2e_i^2 C_F \Delta_{q\bar{q}}^{(2)C_F}(x) + \sum_{i \in Q, \bar{Q}} q_i(x_1) q_i(x_2) c_i 2e_i^2 C_F \Delta_{qq}^{(2)\text{id}}(x)$$

$$+ \sum_{i \in Q, \bar{Q}} [2C_A C_F (q_i(x_1) \gamma(x_2) + \gamma(x_1) q_i(x_2)) + (q_i(x_1) g(x_2) + g(x_1) q_i(x_2))] \times c_i e_i^2 \Delta_{qg}^{(2)C_F}(x)$$

$$+ (g(x_1) \gamma(x_2) + \gamma(x_1) g(x_2)) 2C_A \left(\sum_{k \in Q} c_k e_k^2 \right) \Delta_{gg}^{(2)}(x)$$

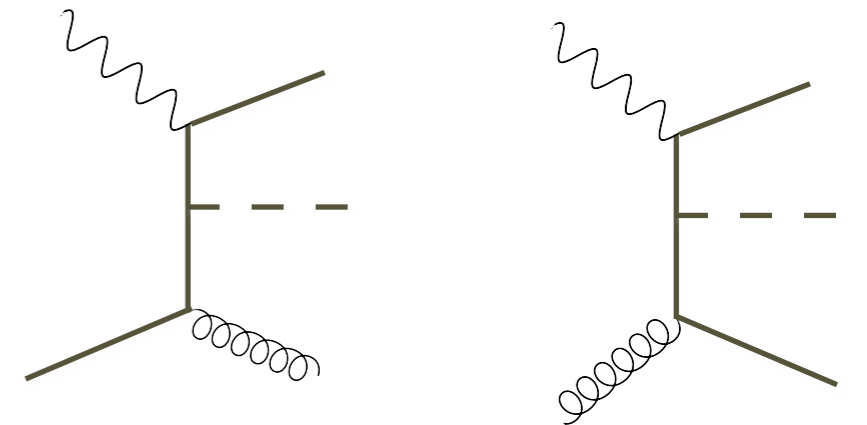


Parameters set up

$$M_Z = 91.187 \text{ GeV} \quad \sin^2 \theta_W = 0.23$$

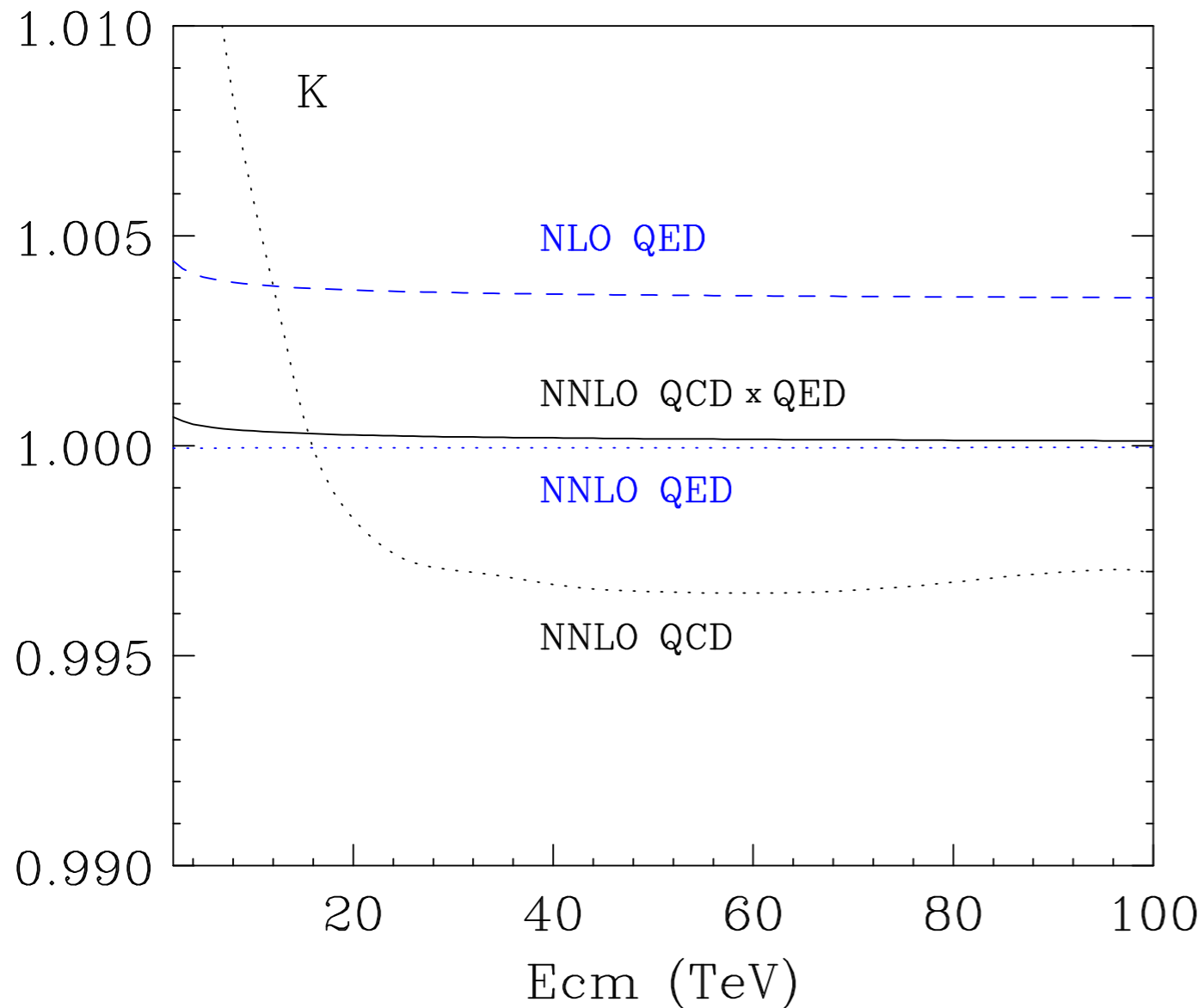
$$\text{Default scales choice} \quad \mu_R = \mu_F = M_Z$$

$$\text{Running couplings} \quad \alpha(M_Z) \sim \frac{1}{128}$$



PDF: LUXqed NNLO set (LHAPDF)

Manohar, Nason, Salam, Zanderighi(2016)



$$K_{QED}^{NLO} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)}}{\sigma^{(0,0)}}$$

$$K_{QCD}^{NNLO} = \frac{\sigma^{(0,0)} + \alpha_s \sigma^{(1,0)} + \alpha_s^2 \sigma^{(2,0)}}{\sigma^{(0,0)} + \alpha_s \sigma^{(1,0)}}$$

$$K_{QED}^{NNLO} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha^2 \sigma^{(0,2)}}{\sigma^{(0,0)} + \alpha \sigma^{(0,1)}}$$

$$K_{QCD \times QED}^{NNLO} = \frac{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)} + \alpha \alpha_s \sigma^{(1,1)}}{\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)}}$$

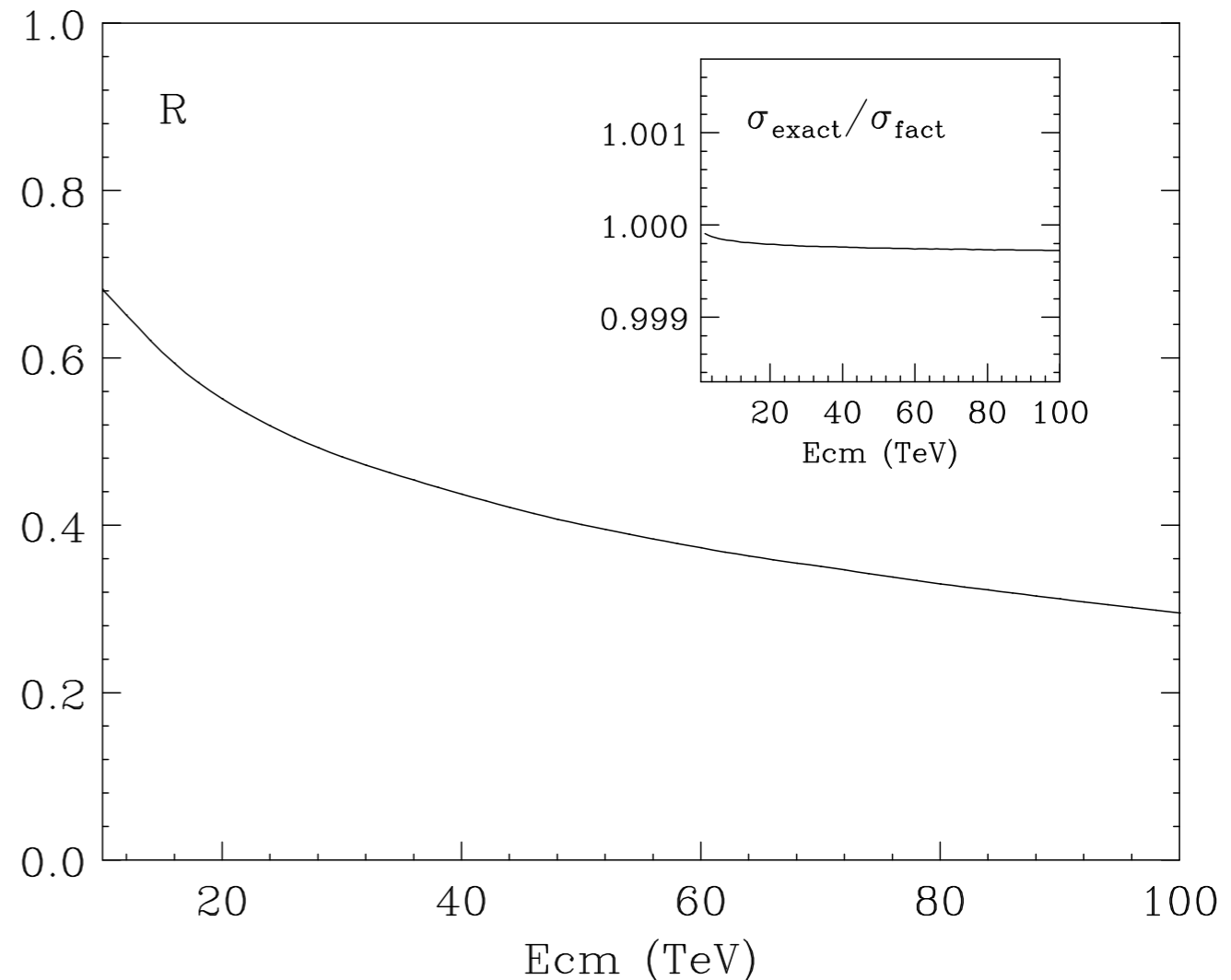
- ▶ $\alpha_s^2 \sim \alpha$ QED NLO \sim QCD NNLO (opposite sign) around 5 per-mille
- ▶ Mixed QEDxQCD below the per-mille level (max. \sim 2 TeV)
- ▶ At 14 TeV QCD NNLO \sim 3.5 mixed QEDxQCD
- ▶ $QED^2 \sim \mathcal{O}(10^{-5})$

▶ Previous work based on “factorization” of mixed effects $K \approx [K_{QED} \times K_{QCD}]$

$$\kappa_{\text{fact}} = \left[K_{QED}^{NLO} \times K_{QCD}^{NLO} \right]_{\mathcal{O}(\alpha\alpha_s)} = \alpha\alpha_s \frac{\sigma^{(0,1)}\sigma^{(1,0)}}{\sigma^{(0,0)}\sigma^{(0,0)}}$$

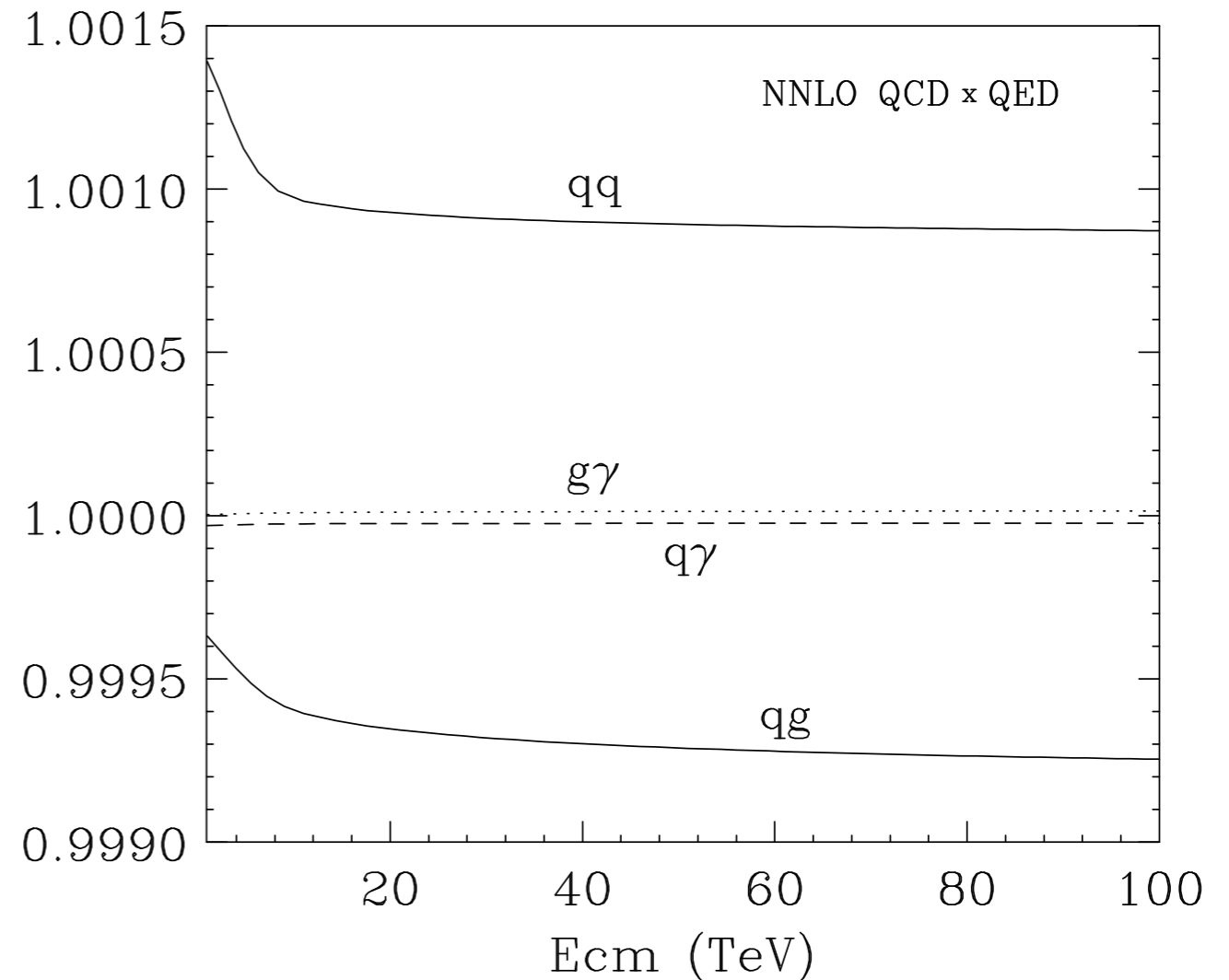
$$\kappa_{\text{mixed}} = \alpha\alpha_s \frac{\sigma^{(1,1)}}{\sigma^{(0,0)}}$$

$$R = \frac{\kappa_{\text{mixed}}}{\kappa_{\text{fact}}} = \frac{\sigma^{(0,0)}\sigma^{(1,1)}}{\sigma^{(0,1)}\sigma^{(1,0)}}$$



- ▶ Factorization approach fails by more than a factor of 2
- ▶ Effect in cross section small (because QED small)
- ▶ Might be worse for some distributions

- Mixed QEDxQCD contribution from different channels



- ▶ Tiny photon initiated contribution
- ▶ Dominated by qq and qg
- ▶ qg and qq with different sign : 50% cancellation
- ▶ qg contribution might be suppressed in exclusive distributions (cuts)

Enhance QEDxQCD

0.05 % effect



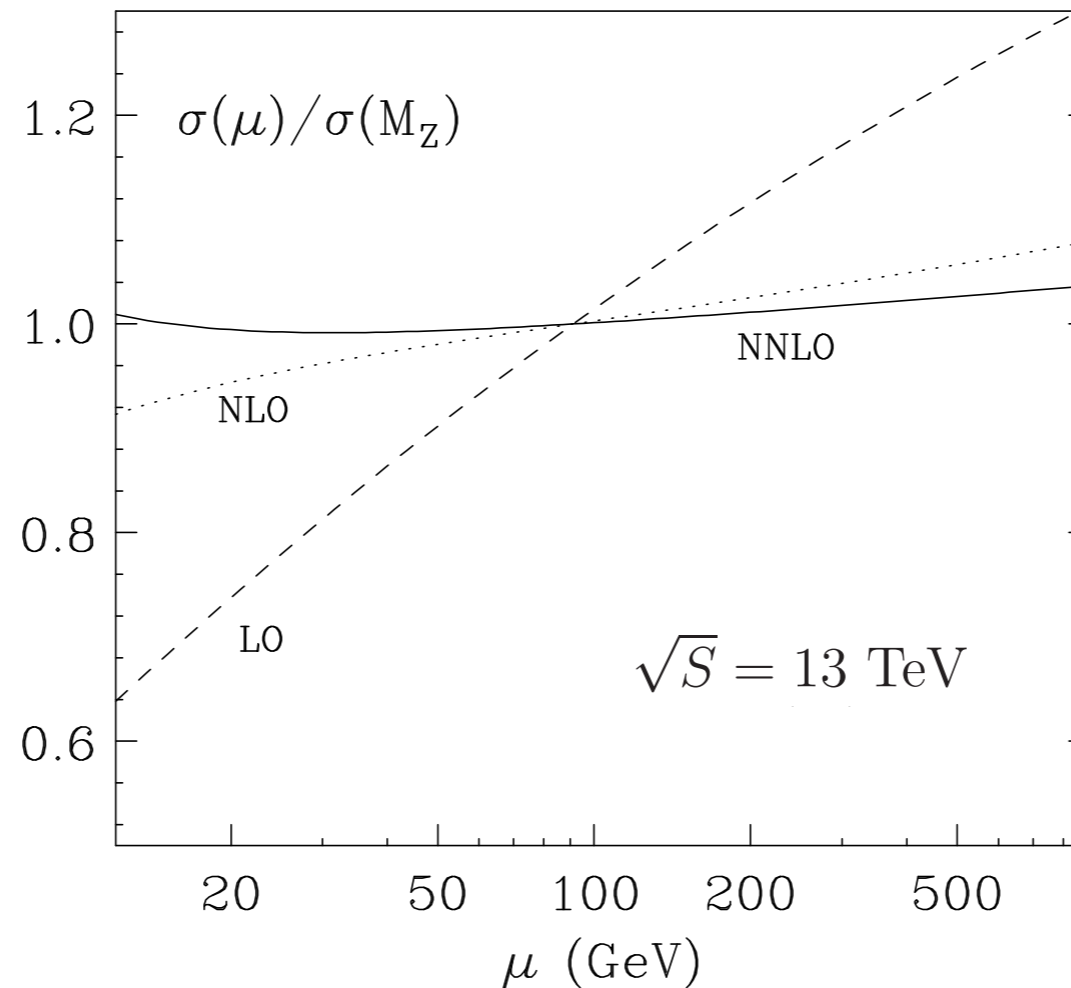
0.1% effect

Scale dependence

$$\text{LO } (\sigma^{(0,0)})$$

$$\text{NLO } (\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)})$$

$$\text{NNLO } (\sigma^{(0,0)} + \alpha \sigma^{(0,1)} + \alpha_s \sigma^{(1,0)} + \alpha\alpha_s \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)} + \alpha_s^2 \sigma^{(2,0)})$$



- ▶ Clear improvement in stabilization at higher orders
- ▶ Mostly QCD dominated but small QED effect

Conclusions

- ▶ QED+QCD NNLO DGLAP kernels
- ▶ Full QED+QCD NNLO corrections to DY (on-shell Z production)
- ▶ QED NLO \sim QCD NNLO (opposite sign) around 5 per-mille
- ▶ Mixed QEDxQCD below the per-mille level

Cancellation between qq and qg channels

- ▶ At 14 TeV QCD NNLO \sim 3.5 mixed QEDxQCD (QCD cancellation)
- ▶ Factorization approach for mixed QEDxQCD fails by factor of 2
- ▶ Very stable under scale variations at NNLO

Future

- ▶ Fully differential NNLO QCD+QED DY calculation
- ▶ Final state (photon) radiation from leptonic decays

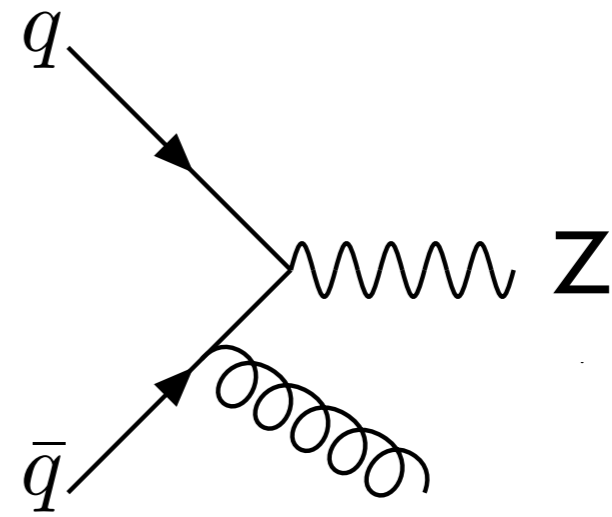
QED+QCD corrections to transverse momentum resummation

L.Cieri, G.Ferrera, G.Sborlini (2018)

why q_T resummation?

Two scales: q_T , M_Z appear as

$$\mathcal{A}^{(i)} \sim \log^{2i} \frac{q_T^2}{M_Z^2}$$

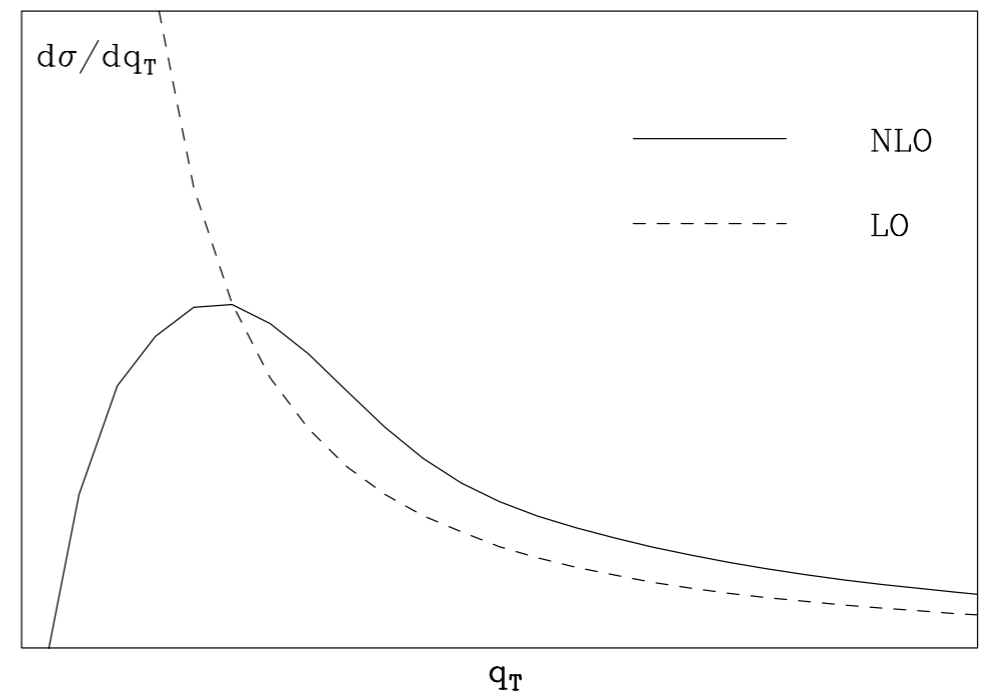


- ▶ If transverse momentum large $\mathcal{O}(M_Z)$ expansion is safe
- ▶ But for very small transverse momentum convergence is spoiled

LO $\frac{d\sigma}{dq_T} \rightarrow +\infty$

as

NLO $\frac{d\sigma}{dq_T} \rightarrow -\infty$

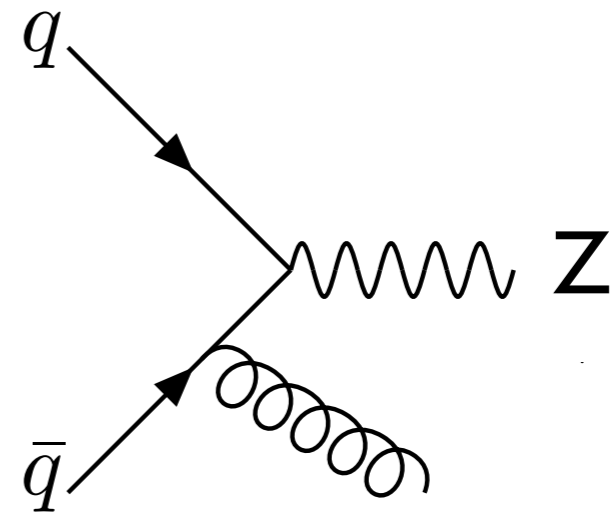


The recoiling gluon is forced to be either soft or collinear to one of the incoming partons

why q_T resummation?

Two scales: q_T , M_Z appear as

$$\mathcal{A}^{(i)} \sim \log^{2i} \frac{q_T^2}{M_Z^2}$$

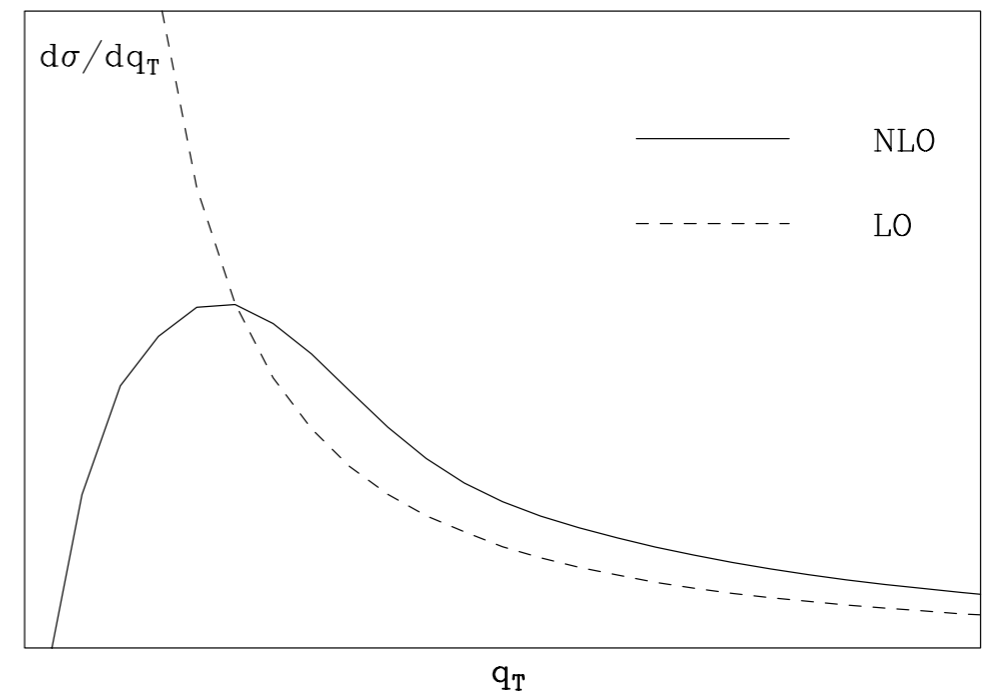


- ▶ If transverse momentum large $\mathcal{O}(M_Z)$ expansion is safe
- ▶ But for very small transverse momentum convergence is spoiled

LO $\frac{d\sigma}{dq_T} \rightarrow +\infty$

as

NLO $\frac{d\sigma}{dq_T} \rightarrow -\infty$



➔ The recoiling gluon is forced to be either soft or collinear to one of the incoming partons

- ▶ No matter how small the coupling constant, perturbative expansion fails in the kinematical region where the bulk of the data appears! **Resummation needed**

▶ partonic cross-section decomposed as $\frac{d\hat{\sigma}_{ab}}{dq_T^2} = \frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_T^2} + \frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_T^2}$

with $\frac{d\hat{\sigma}_{ab}^{(\text{fin.})}}{dq_T^2} = \left[\frac{d\hat{\sigma}_{ab}}{dq_T^2} \right]_{\text{f.o.}} - \left[\frac{d\hat{\sigma}_{ab}^{(\text{res.})}}{dq_T^2} \right]_{\text{f.o.}}$

fix. order contr. exp. of res. component

▶ resummation achieved after Fourier transform

$$\frac{d\hat{\sigma}_{a_1 a_2 \rightarrow F}^{(\text{res.})}}{dq_T^2}(q_T, M, \hat{s}; \mu_F) = \frac{M^2}{\hat{s}} \int_0^\infty db \frac{b}{2} J_0(b q_T) \mathcal{W}_{a_1 a_2}^F(b, M, \hat{s}; \mu_F)$$

▶ where the large Log becomes $L \equiv \log(M^2 b^2)$

Hard factor

Sudakov form factor

$$\mathcal{W}_N^F(b, M; \mu_F) = \hat{\sigma}_F^{(0)}(M) \mathcal{H}_N^F(\alpha_S; M^2/\mu_R^2, M^2/\mu_F^2, M^2/Q^2) \times \exp \left\{ \mathcal{G}_N(\alpha_S, L; M^2/\mu_R^2, M^2/Q^2) \right\}$$

$$\mathcal{G}_N(\alpha_S, L) = L g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \sum_{n=4}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^{n-2} g_N^{(n)}(\alpha_S L)$$

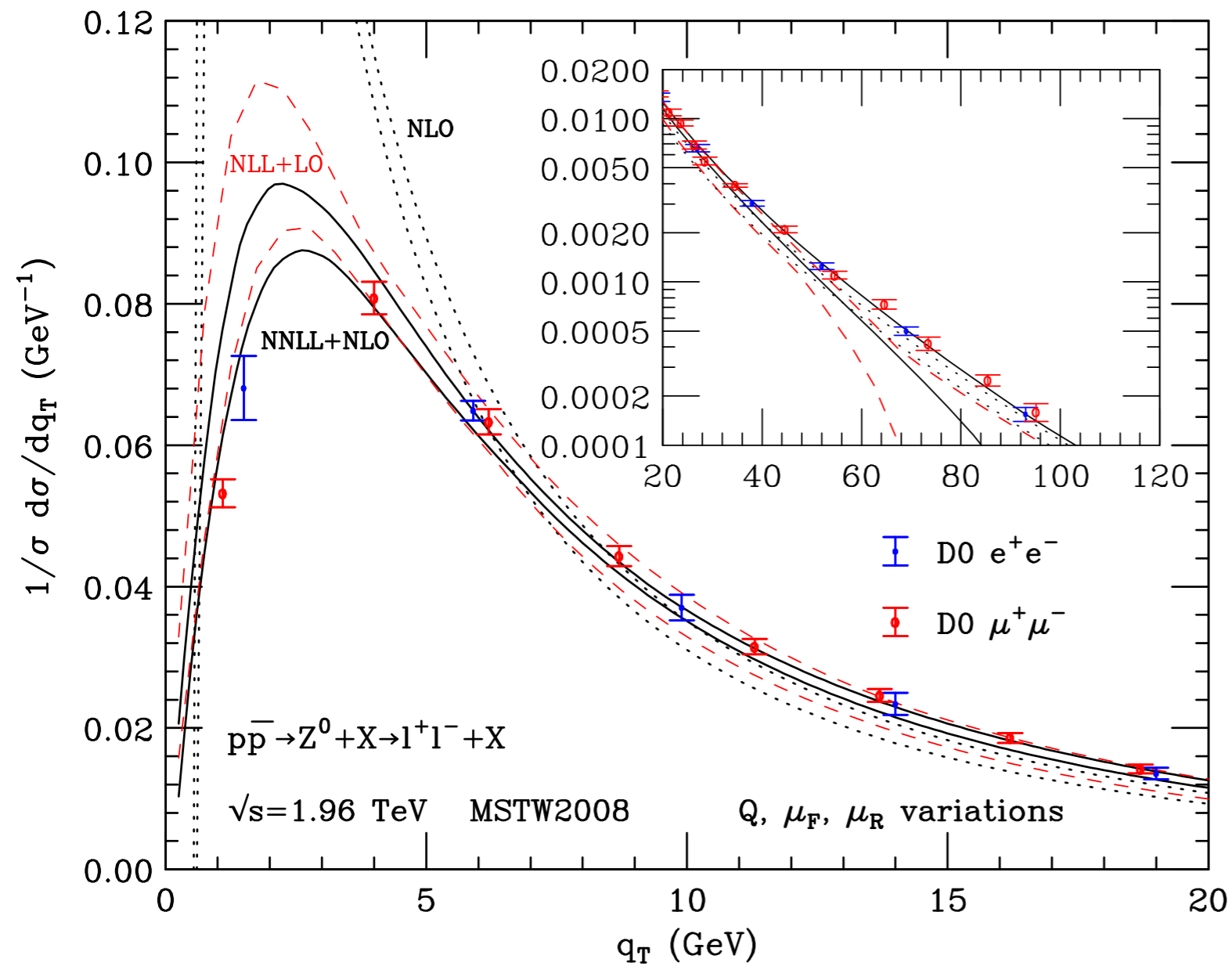
LL

NLL

NNLL

N...NLL

$$\mathcal{H}_N^F(\alpha_S) = 1 + \frac{\alpha_S}{\pi} \mathcal{H}_N^{F(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}_N^{F(2)} + \sum_{n=3}^{\infty} \left(\frac{\alpha_S}{\pi} \right)^n \mathcal{H}_N^{F(n)}$$



NNLO+NNLL QCD
implemented

DYqT

DYRes

S. Catani, D.deF, G.Ferrera, M.Grazzini
G. Bozzi

DYTurbo

(+) S. Camarda, J.Cuth, M.Schott,
M.Greta Vincter, A. Glazov,
M.Boonekamp

D0 data for the $Z \, q_T$ spectrum compared
with perturbative results.

QED corrections

- ▶ include QED corrections in Sudakov form factor

$$\begin{aligned}
 \mathcal{G}'_N(\alpha_S, \alpha, L) = & \mathcal{G}_N(\alpha_S, L) + \underbrace{L g'^{(1)}(\alpha L)}_{\text{LL QED}} + \underbrace{g'_N{}^{(2)}(\alpha L)}_{\text{NLL QED}} + \sum_{n=3}^{\infty} \left(\frac{\alpha}{\pi}\right)^{n-2} g'_N{}^{(n)}(\alpha L) \\
 & + \underbrace{g'^{(1,1)}(\alpha_S L, \alpha L)}_{\text{NLL mixed}} + \sum_{\substack{n,m=1 \\ n+m \neq 2}}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^{n-1} \left(\frac{\alpha}{\pi}\right)^{m-1} g'_N{}^{(n,m)}(\alpha_S L, \alpha L) \ .
 \end{aligned}$$

- ▶ include QED corrections in Hard factor

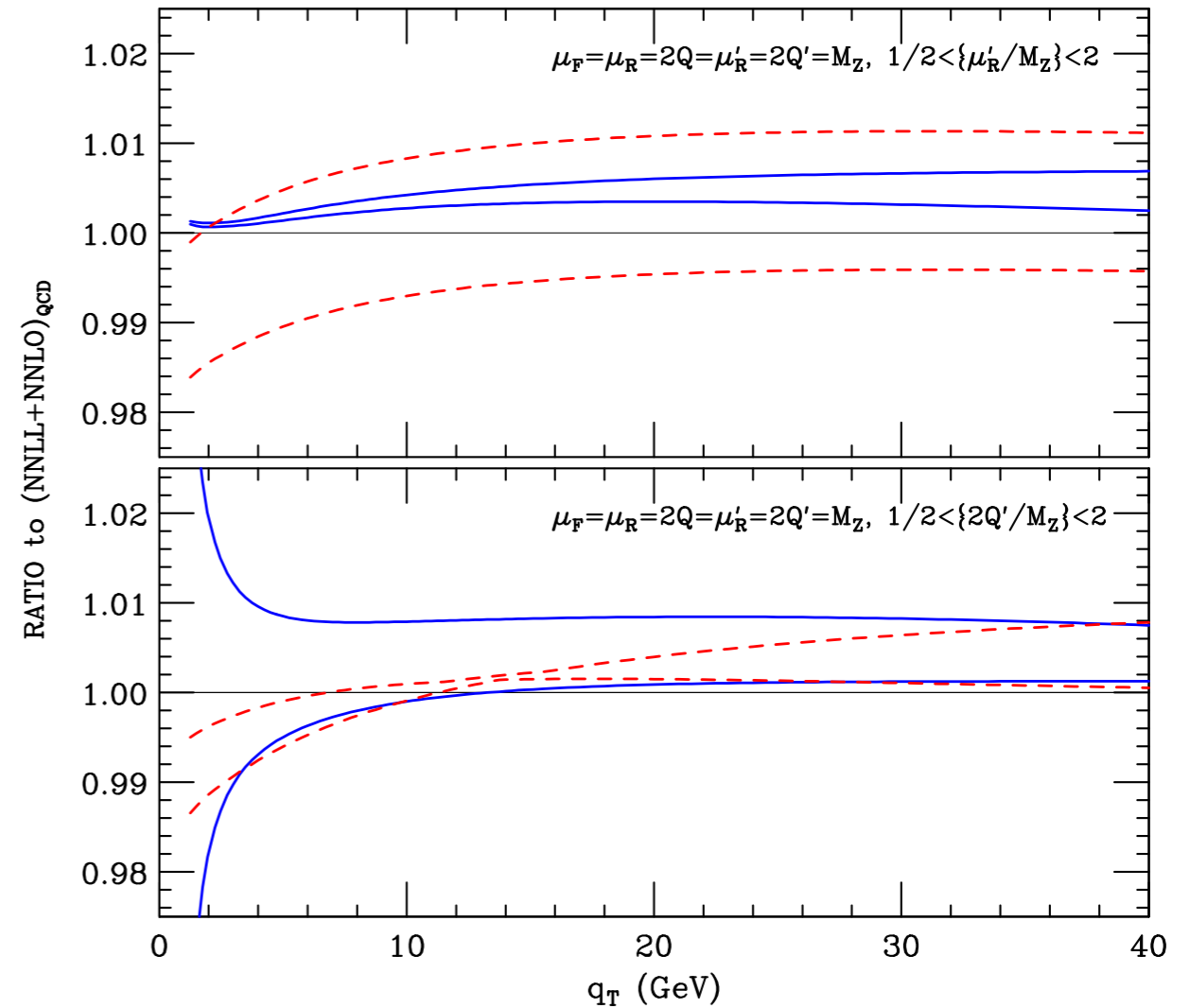
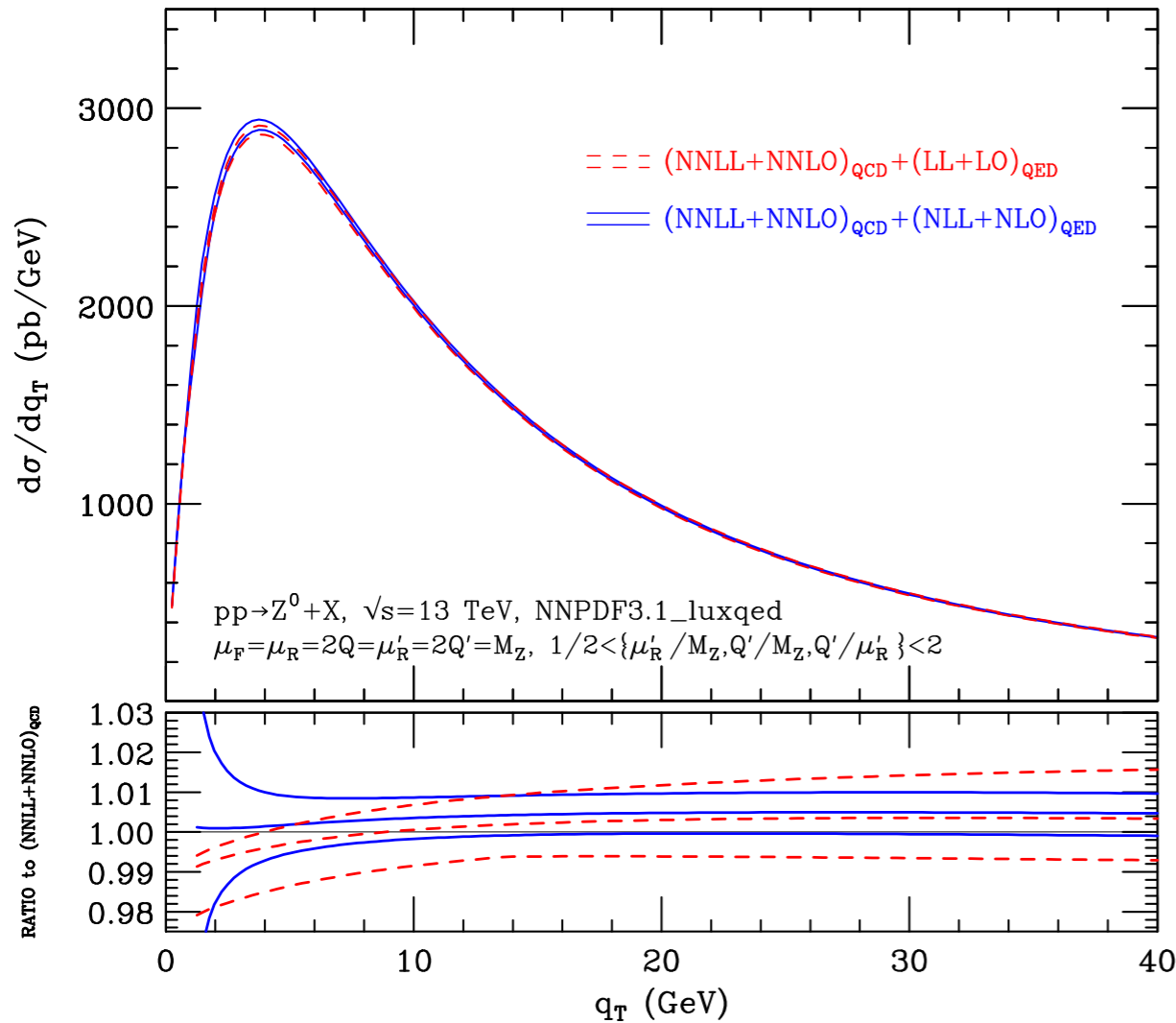
$$\mathcal{H}'_N{}^F(\alpha_S, \alpha) = \mathcal{H}_N^F(\alpha_S) + \underbrace{\frac{\alpha}{\pi} \mathcal{H}'_N{}^F{}^{(1)}}_{\text{NLL QED}} + \sum_{n=2}^{\infty} \left(\frac{\alpha}{\pi}\right)^n \mathcal{H}'_N{}^F{}^{(n)} + \sum_{n,m=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \left(\frac{\alpha}{\pi}\right)^m \underbrace{\mathcal{H}'_N{}^F{}^{(n,m)}}_{\text{mixed (not included)}}$$

- ▶ g' and H' also obtained by abelianization of QCD results

involve
beta functions

$$\frac{d \ln \alpha_S(\mu^2)}{d \ln \mu^2} = \beta(\alpha_S(\mu^2), \alpha(\mu^2)) = - \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_S}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty} \beta_{n,m} \left(\frac{\alpha_S}{\pi}\right)^{n+1} \left(\frac{\alpha}{\pi}\right)^m$$

$$\frac{d \ln \alpha(\mu^2)}{d \ln \mu^2} = \beta'(\alpha(\mu^2), \alpha_S(\mu^2)) = - \sum_{n=0}^{\infty} \beta'_n \left(\frac{\alpha}{\pi}\right)^{n+1} - \sum_{n,m+1=0}^{\infty} \beta'_{n,m} \left(\frac{\alpha}{\pi}\right)^{n+1} \left(\frac{\alpha_S}{\pi}\right)^m$$

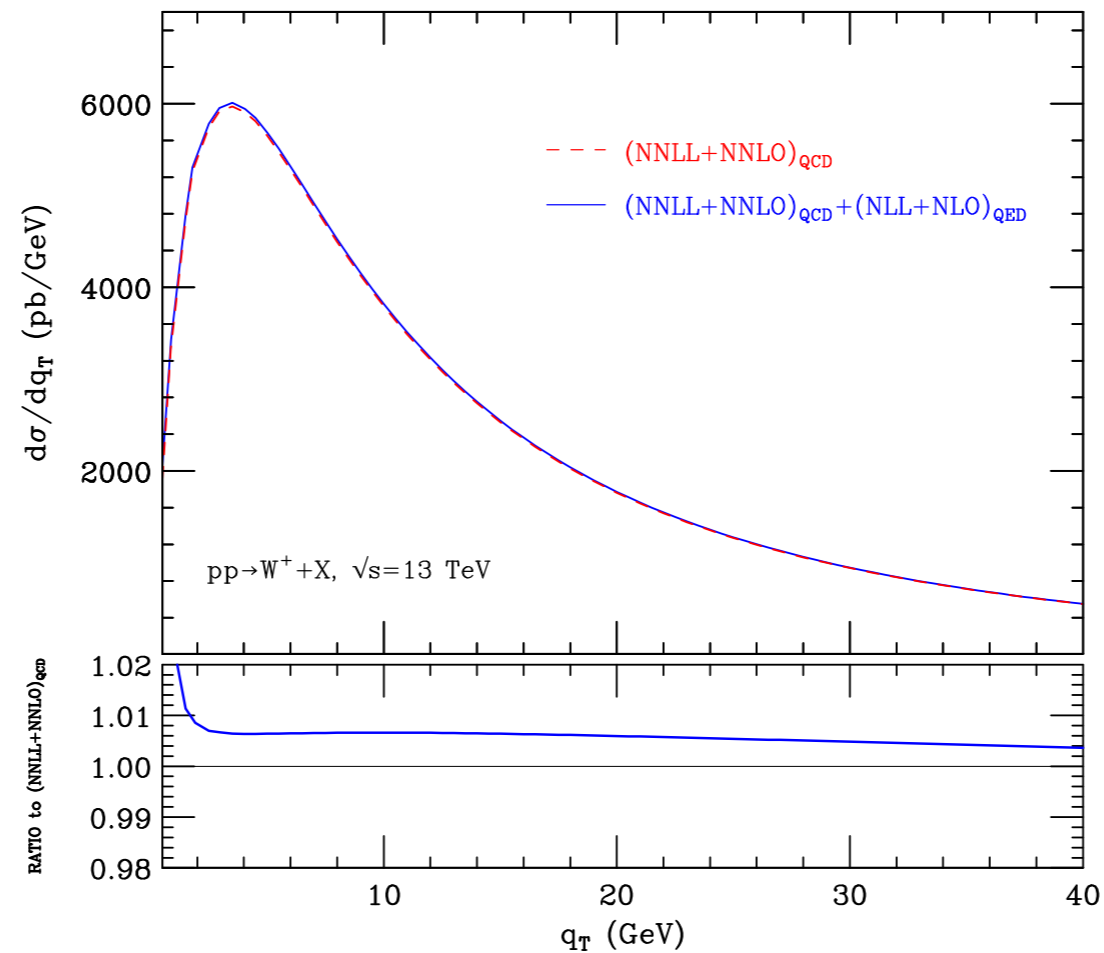


$$1/2 < \{ \mu'_R / M_Z, Q' / M_Z, Q' / \mu'_R \} < 2$$

- ▶ LL QED effects uncertainty 2-3%
- ▶ reduced by ~factor of 1.5/2 by NLL QED
- ▶ NLL+NLO QED effects not negligible and around +1% (rather flat)

Combined QED and QCD q_T resummation for W production at the LHC (preliminary)

[S.Rota (degree thesis '18)]



W q_T spectrum at the LHC. NNLL QCD results combined with the NLL QED effects.