Beam-beam correction in vdM scans

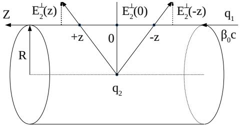
Vladik Balagura (LLR – Ecole polytechnique / IN2P3)

LHC Lumi Days, 4-5 Jun 2019

Beam-beam force

Two-dim. electrostatic force btw. 2 particles in q_2 rest frame ($\beta_0 \approx 1$ – velocity of q_1):

$$\Delta p_1 = \frac{E_2(x, y)}{\beta_0 c} = \frac{q_1 q_2}{2\pi R \epsilon_0 \beta_0 c} = \frac{2 Z_1 Z_2 \alpha \hbar}{R \beta_0}.$$



Kick from Gaussian round bunch:

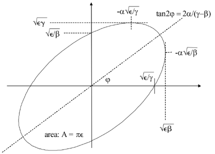
$$\Delta \vec{p}_1 = -\frac{2 Z_1 Z_2 \alpha \hbar N_2}{\beta_0} \frac{\vec{R}}{R^2} (1 - e^{-R^2/2\sigma_2^2}),$$

Bassetti-Erskine formula if $\sigma_x \neq \sigma_y$.
Modifies bunch shapes and their overlap integral, requires vdM correction.

Transverse accelerator beam dynamics

Bunch shape is created by transverse oscillations of p around stable orbit.

Along full LHC ring: $Q_{x,y} = 62.313, 60.317$ oscillations ("tunes", after 2017).
At any point, phase-space trajec. $=$ ellipse in (u, u'), where $u = X$ or Y, u' is corresp. angle wrt. beam.
One LHC turn $= Q$ cycles around ellipse.



At IP beams are maximally focused $\rightarrow$ Twiss parameter $\alpha = -\frac{1}{2}\frac{d\beta}{dz} = 0 \rightarrow$ ellipse not inclined.

$$u = \sqrt{\epsilon\beta^*}\cos(\phi), \quad u' = du/dz = -\sqrt{\epsilon/\beta^*}\sin(\phi),$$

$\beta^* = \beta_{min}$, ϵ – particle's emittance (or amplitude). Definition

$$z = u - i\beta^* u' = \sqrt{\epsilon\beta^*}e^{i\phi}$$

converts ellipse to circle, one turn map: $z_{n+1} = z_n e^{2\pi Qi}$.
Bunch = double Gaussian in (u, u').
Full phase space is 4-dimensional: (X, X', Y, Y').

Simulation

Per "macro"-particle = Gaussian bunch particles agglomerated at transverse grid points. Weight from two-dimensional Gaussian in (u, u'):

$$w_x^i \propto r_x/\sigma^2 \exp(-r_x^2/2\sigma^2) dr_x,$$

same for w_y, full $w^i = w_x^i w_y^i$ is normalized: $\sum_i w^i = 1$.
One turn of macro-particle in bunch 1

$$z_{n+1} = (z_n - i\beta^* \Delta u') e^{2\pi i Q},$$

$\Delta u' = \Delta\vec{p}_1/p$ = angular beam-beam kick, separately in x and y.
Round equal bunch profiles. Beam-beam moves particles by $O(1\mu\text{m}) \rightarrow$ neglect perturbation of source bunch, ie. use kick formula of Gaussian bunch.

$$\int ((\rho_1+\delta\rho_1)(\rho_2+\delta\rho_2)-\rho_1\rho_2)\, dxdy \approx \int 2\rho\delta = 2\int ((\rho_1+\delta\rho_1)\rho_2-\rho_1\rho_2)\, dxdy.$$

$\rightarrow$ determine effect in perturbed - pertubed as pert. - unpert. $\times 2$. Integral of perturbed macro-particles i with continuous unperturbed Gaussian "field" ρ_2:

$$\int (\rho_1 + \delta\rho_1)\rho_2\, dx\, dy = \sum_i w^i \cdot \rho_2(x_i, y_i).$$

Simulation optimisation

Aim: $\leq 0.1\%$ precision for 4D MC in reasonable CPU time.

10 000 macroparticles in uniform X-Y grid with $\sqrt{10000} = 100$ nodes along $\pm 5\sigma$ X and Y sides. All points at $> 5\sigma$ from bunch centers are removed.

How to sample X', Y' angular coordinates?
Every point (r_x, r_y) is rotated with random uniformly distrib. phases $\phi^i_{x,y}$:

$$z^i_x = r^i_x e^{i\phi^i_x}, \quad z^i_y = r^i_y e^{i\phi^i_y}.$$

Simulation runs 10 000 accelerator turns $\rightarrow$ every point well samples its perturbed circles $X - X'$ and $Y - Y'$. 10^4 particles $\times 10^4$ turns samples full 4D.

Beam-beam is OFF first 1000 turns $\rightarrow$ verify numeric integration with analytic formula (bias negligible). Next 1000 + 1000 turns – adiabatic switch ON and stabilization. Perturbed integral is accumulated during last 3000–10000 turns.

To increase randomness, tunes $Q_{x,y}$ are made irrational (eg. with 2 digits $Q * 100$ is integer, after 100 turns particles resample about the same points).

Notes on physical model 1

During beam-beam ($Z \sim O(1cm)$) $(\Delta X, \Delta Y)$ deflection negligible

Only angular kick $(\Delta X', \Delta Y') \neq 0$. $(\Delta X, \Delta Y)$ - at larger scale, eg. one turn. (kick, then turn) or (turn, then kick), ie. $z_{n+1} = (z_n - i\beta^* \Delta u')e^{2\pi iQ}$, or $z_n e^{2\pi iQ} - i\beta^* \Delta u'$: does not matter.

Assumption of Gaussian bunches if beam-beam at IP were OFF.

Whatever affects bunches: injection, beam-beam at other IPs etc, this only modifies (effective) initial Gaussian σ.

Notes on physical model 2

Best experim. sensitivity to beam-beam lumi change:

monitor L at IP #1 during vdM scan in IP #2 (nothing changes except beam-beam). Same $z_{n+1} = (z_n e^{2\pi i \Delta Q_{12}} - i\sqrt{\beta_1^* \beta_2^*} \Delta u_2') e^{2\pi i (Q - \Delta Q_{12})}$, except ΔQ_{12}, $Q - \Delta Q_{12}$ instead of Q and $\sqrt{\beta_1^* \beta_2^*}$.
Only fractional Q, ΔQ part matters $\rightarrow$ effect from ΔQ can be of the same order, $\sqrt{\beta_1^* \beta_2^*}$ can give extra enhancement: vdM at LHCb ($\beta_2^* = 24$ m), L measurement at ATLAS/CMS ($\beta_1^* = 1.5$ m), $\sqrt{24/1.5} = 4$.

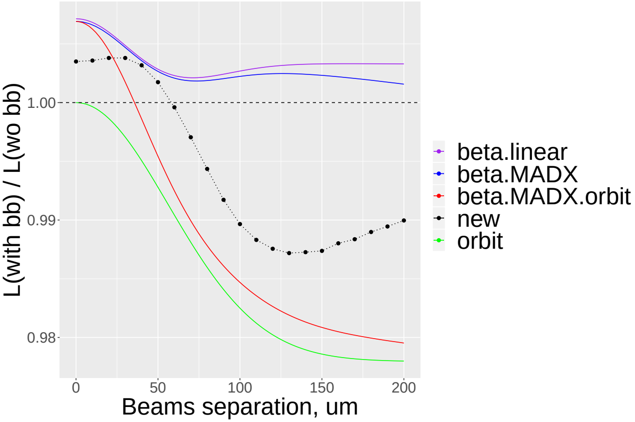
Beam-beam is simple

circular phase space trajectory = energy conserving harmonic oscillator, under influence of kicks - not difficult

Results

Beam parameters from previous'2012 simulation to compare

$E_p = 3500$ GeV, $Q_{x,y} = 0.31, 0.32$, $\beta = 1.5$ m, $\epsilon = 4\mu m \cdot rad \to \sigma = 40\mu$m, $N_p^{1,2} = 8.5 \cdot 10^{10}$.



Black: new, red: old'2012 – large difference

Cross-section correction

$$\sigma = \frac{\int \mu(\Delta x, \Delta y_0)\, d\Delta x \cdot \int \mu(\Delta x_0, \Delta y)\, d\Delta y}{\mu(\Delta x_0, \Delta y_0) N_1 N_2} \rightarrow 1 + \frac{\delta\sigma}{\sigma} = (1 + \frac{\delta \int \mu}{\int \mu})^2 (1 + \frac{\delta \mu_{0,0}}{\mu_{0,0}})^{-1}$$

Close beams (incl. head-on) – large weight, wide sep. – exponentially small.

	Old'2012	new Jan'2019
$\sigma(bb)/\sigma(no\ bb) - 1$	-1.2%	-0.3%

In real vdM analysis: divide each point by $L(bb)/L(no\ bb) \rightarrow$ eliminate beam-beam effect.

Old simulation'2012

Main approximation: beam-beam kicked bunch remains Gaussian
(does not work well)

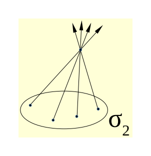
Two parts:

mean correction (jargon: "orbit shift")

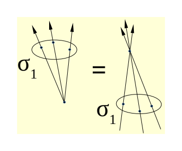
σ correction (jargon: "dynamic beta correction")

Correction of mean

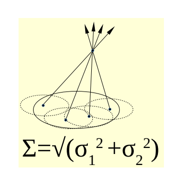
Bunch $\rightarrow$ particle



Particle $\rightarrow$ bunch,
flip to bunch $\rightarrow$ particle



Bunch $\rightarrow$ bunch



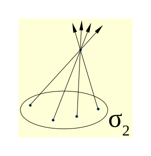
Average bunch 1 kick = single particle kick with $\sigma_2 \rightarrow \Sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$.

Good approximation even for non-Gaussian bunches: if round-bunch or Bassetti-Erskine formula can be applied to single particle.

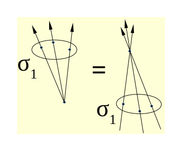
Fill free to use this proof in your future textbooks :)

Correction of mean

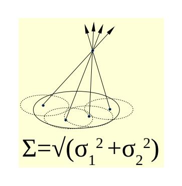
Bunch → particle



Particle → bunch,
flip to bunch → particle



Bunch → bunch



Average bunch 1 kick = single particle kick with $\sigma_2 \to \Sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$.

Good approximation even for non-Gaussian bunches: if round-bunch or Bassetti-Erskine formula can be applied to single particle.

Fill free to use this proof in your future textbooks :)

average angular kick → average X, Y-shift, textbook formula

$$< \Delta\vec{r}_{x,y} >= \frac{< \Delta\vec{p}_{x,y} >}{p} \frac{\beta^*_{x,y}}{2\tan(\pi Q_{x,y})}$$

Old: σ correction (jargon: "dynamic beta correction")

Bunch remains Gaussian if $\Delta x' \propto x$, $\Delta y' \propto y$ (= quadrupole magnet).
Beam-beam kick

$$\Delta \vec{p}_1 \propto \frac{\vec{R}}{R^2}(1 - e^{-R^2/2\sigma_2^2}),$$

was approximated as

$$\Delta p_{1x} = p_{1x}^0 + \frac{\partial \Delta p_{1x}}{\partial x} x,$$

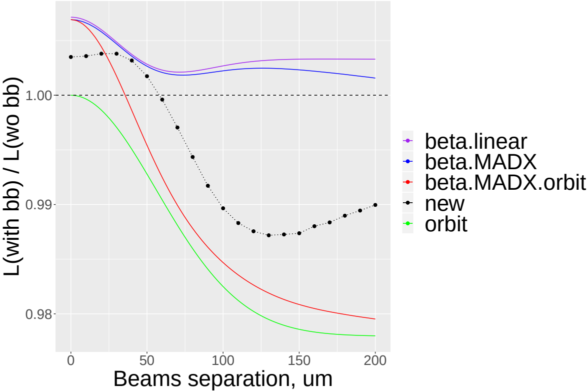
derivative was taken at bunch 2 center where $x = 0$. Same for y.
Cross-derivatives $\partial \Delta p_{1x}/\partial y = \partial \Delta p_{1y}/\partial x = 0$ except in offset scans where this approximation is not applicable.
p_{1x}^0 at bunch 2 center does not reproduce known "orbit shift" since kick model is not correct. Patch: p_{1x}^0 was substituted (ad hoc) to obtain correct mean.
Quadrupole magnet $(\frac{\partial \Delta p_{1x}}{\partial x})x$ changes only σ or β^* (jargon: "beta-beat") by:

$$\frac{\Delta \beta^*_{x,y}}{\beta^*_{x,y}} = \frac{\partial \Delta p_{1\ x,y}}{\partial x,y} \frac{\beta^*_{x,y}}{2\tan(2\pi Q_{x,y})}.$$

Old simulation'2012: résumé

Beam-beam modeled as dipole + quadrupole:

$$\Delta p_{1x} = p^0_{1x} + \frac{\partial \Delta p_{1x}}{\partial x} x.$$



MAD-X sim.'2012 (LHC soft, w/ non-linearities) $\approx$ linear "beta-beat" formula ($<$ 0.1% difference in x-section).

Old simulation: drawbacks

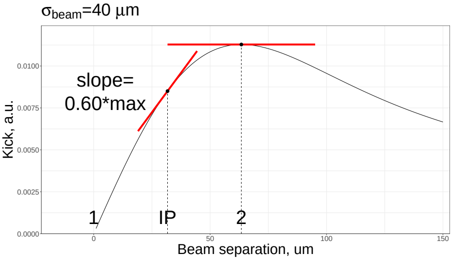
Precision $\lesssim$ 10% is sufficient for accelerator, no tools for L-integrals at $\leq$ 1%.
$\rightarrow$ Gaussian approximation, limited to quadrupole (linear) kick

$$\Delta p_{1x} = p^0_{1x} + \frac{\partial \Delta p_{1x}}{\partial x} x$$

instead of non-linear

$$\propto (1 - e^{-R^2/2\sigma_2^2})/R$$

(or Bassetti-Erskine).



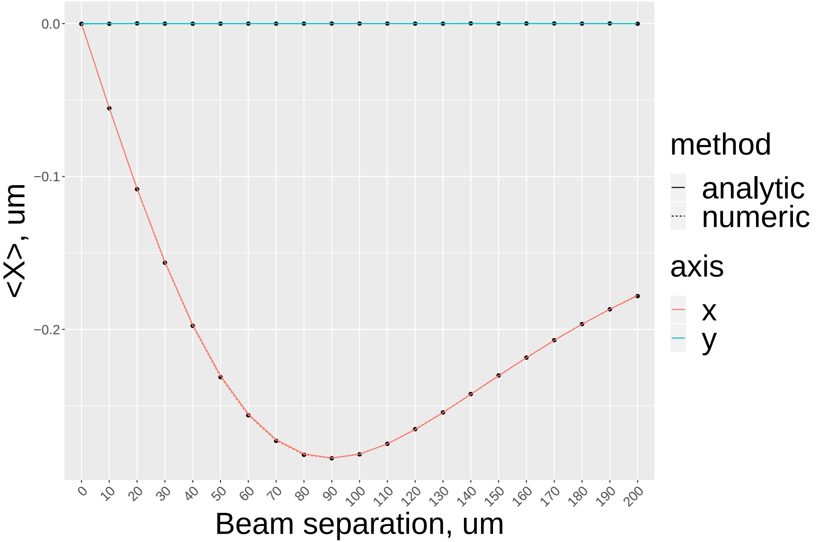
Eg. $\frac{\partial \Delta p_{1x}}{\partial x} = 0$ at beam separation = 1.5852σ, approximating quadrupole is absent, but slope at IP – 60% of maximal
$\rightarrow$ impossible to approximate kick linearly everywhere.

Orbit shift formula describes center-of-mass of even perturbed non-Gaussian.
But: in luminosity correction it was used as a Gaussian center – not correct.

Beam-beam induces $X - Y$ coupling absent in old model.

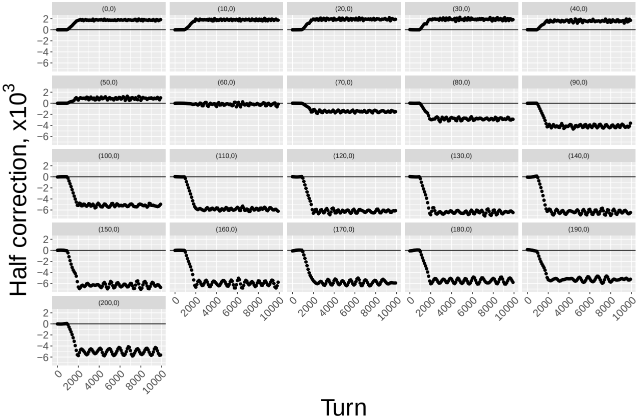
Cross-checks of new simulation, center

Bunch center-of-mass agrees with analytic formula (dashed == solid)



Cross-checks of new simulation, convergence

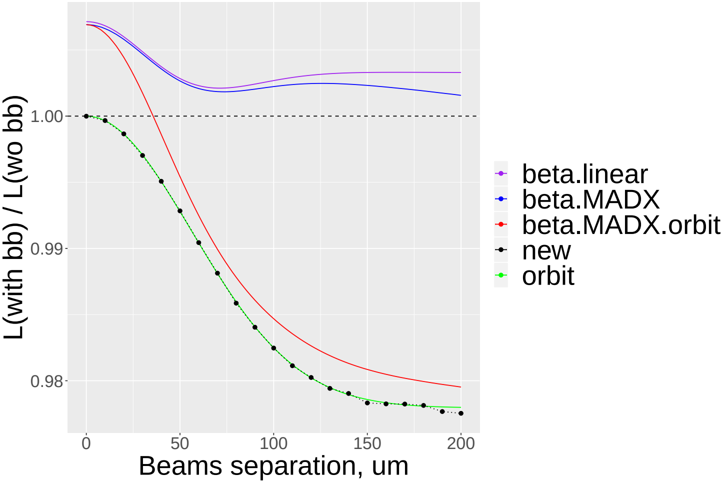
First 1000 turns – beam-beam OFF, next 1000 – adiabatic switch ON, 1000 – wait, last 7000 – final averaging.



100 turn averages. Perturbed - unpert. integral $\rightarrow$ $\times 2$ for pertub. - perturb.

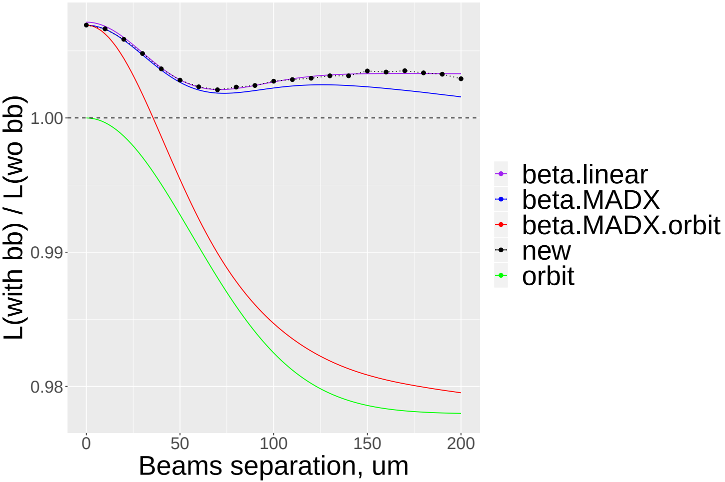
Cross-checks of new simulation, average const kick

New simulation with precise kick substituted by constant (change of $\sim$ 5 lines of code): $\Delta p_{1x} = p^0_{1x}$ = old dipole $\rightarrow$ reproduces analytic formula



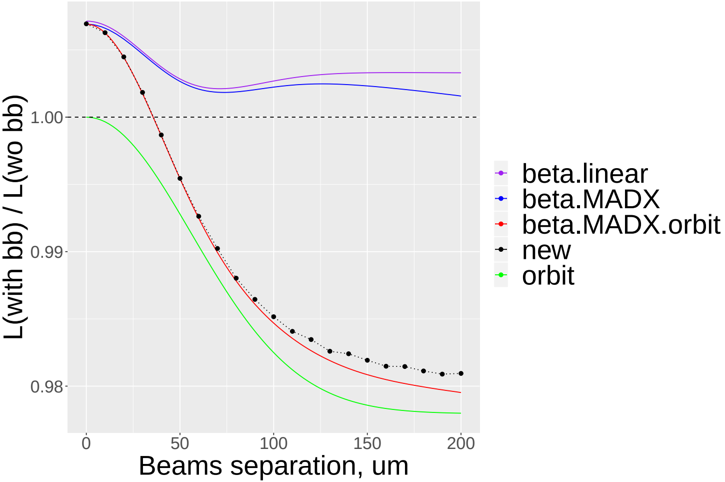
Cross-checks of new simulation, old quadrupole model

New simulation with same quadrupole as in MAD-X'2012 $\rightarrow$ reproduces analytic formula. Non-linear effects in MAD-X gave $< 0.1\%$ in x-section.



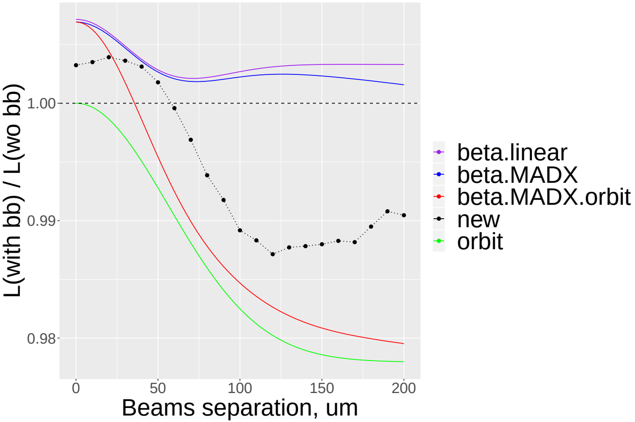
Cross-checks of new simulation, old di- + quadrupole

It does not matter whether dipole is included in kick or orbit shift is corrected analytically. Here: kick = dipole + quadrupole
$\rightarrow$ old model up to MAD-X non-linearities.



Cross-checks, beam-beam switch ON

Adiabatic or abrupt? $\rightarrow$ it does not matter:



Adiabatic: gradually transforms circle points to $\approx$ same perturbed phase trajectory.
Abrupt: point = circle $\cap$ some perturbed trajectory. In one step it switches from one to another $\rightarrow$ points are transported to different perturbed trajectories intersecting initial circle.
Abrupt gives spread around adiabatic trajectory, but $\ll$ circle's R $\rightarrow$ negligible in overlap integral.

Conclusions

New simulation in GitHub

- `https://github.com/balagura/beam-beam-simulation-for-vdM-scans-at-LHC`
- Main code in C++ without dependencies
- One vdM step takes a few seconds on my laptop
- Initially written in Jan'19 with x-checks added in Feb-Mar, 800 C++ lines

Best sensitivity to detect beam-beam or set upper limits

observe L changes at CMS/ATLAS during vdM scan in LHCb (with large β^*)

All LHC cross-sections using old beam-beam corrections (from 2012 on) are affected

- New simulation predicts much smaller corrections
- Disagreement of order 1%