Results on 3D Pixel Sensors for the CMS Inner Tracker Upgrade at the High Luminosity LHC

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on behalf of the CMS Tracker Group
"Trento" Workshop on Advanced Silicon Radiation Detectors Physics
17th-19th February 2020
Vienna - Austria
Introduction

I will not talk about details of CMS Inner Tracker: there are excellent talks on the subject at this workshop!

- Jordi Duarte Campderros talk (19th Feb 2020, 10:00): “Study of 3D pixel sensors after non-uniform proton irradiation” [https://indico.cern.ch/event/813597/contributions/3727824/](https://indico.cern.ch/event/813597/contributions/3727824/)
- Roberto Seidita talk (19th Feb 2020, 16:00): “Serial powering at CMS silicon tracker detector for High Luminosity Upgrade” [https://indico.cern.ch/event/813597/contributions/3727820/](https://indico.cern.ch/event/813597/contributions/3727820/)
3D pixel sensors

- **3D pixel R&D (FBK & CNM)**
- In 3D sensors the drift path is perpendicular to the active depth
  - Short drift distance → ~ 30 ÷ 50 μm (3D) vs 100 ÷ 150 μm (Planar)
- Many advantages with respect to planar sensors:
  - Smaller bias voltage needed to deplete the sensor (5 V before and ~ 150 V after irradiation)
  - Less trapping in irradiated sensors (shorter drift distance!)
  - Slim edges, i.e. smaller dead zones
  - Same charge produced
- Promising candidates for the high radiation environment of the inner layers and rings
RD53: the inner tracker readout chip

RD53A first prototype of ROC

- ½ total size (50×50 μm² cell, 65 CMOS nm technology)
- Used for R&D
- 3 analog Front-Ends (Synchronous, Linear, Differential)
- Time over Threshold counter to measure charge (4 bits); caveat ToT as charge unit
- CMS choice: Linear F.E.
- Low threshold and noise, radiation hard (proven up to at least 5 MGy)
- All results presented here obtained using Linear F.E. (central section)
Test beam setup @DESY in 2019

- **Beam**: 5.2 GeV electrons
- **Telescope**: Mimosa
  - 3 planes before DUT (Device Under Test)
  - 3 planes after DUT
  - Spatial resolution up to ~3.8 μm, depends on telescope and geometry configuration
- **Single Chip Modules on beam**:
  - Two FBK mask aligner fresh 3D sensors
    - 130 μm active thickness
    - One 50×50 μm², One 25×100 μm² pitch
    - Bump bonded at IZM (Germany)
  - One FBK stepper fresh 3D sensor
    - 150 μm active thickness
    - 50×50 μm² pitch
    - Indium bump bonded at Leonardo (Italy)
  - One FBK irradiated 3D sensor
    - 130 μm active thickness
    - 50×50 μm² pitch
    - Irradiated @CERN
    - Nominal fluence $1 \times 10^{16} \text{ n}_{\text{eq}} \text{cm}^{-2}$
Test beam setup @DESY in 2019

- Sensors bump-bonded to RD53A
- Mounted on RICE cards
- BDAQ (https://gitlab.cern.ch/silab/bdaq53)
- Irradiated sensor kept inside a cooling box:
  - Stable temperature of -27°C
  - Telescope resolution worsened (~ 10 μm)
Modules: data taking and results

**FRESH**

- Two HV bias point:
  - 6 V (minimum allowed by PS)
  - 30 V
- All modules tuned to ~900 e⁻ threshold

**IRRADIATED**

- Bias scan
  - > 20 V
  - < 150 V
- Tuned to 1100 e⁻ threshold

- Low voltage powering via RD53A LDO integrated circuit
- “Reference” hit efficiency measured for perpendicular tracks (effect of columnar electrodes)
- Cluster size studies and angle scan for spatial resolution measurements
- 24 V of bias voltage range with same performances guarantees a good behaviour with SP*

### Leakage Current @Bias Voltage

<table>
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<tr>
<th>Module</th>
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<th>Efficiency (Orthogonal Tracks)</th>
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*See Roberto Seidita’s talk (19th Feb 2020, 16:00): “Serial powering at CMS silicon tracker detector for High Luminosity Upgrade”
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Resolution vs Turning angle: 50×50 μm²

- Telescope resolution ($\sigma_{\text{tele}}$) subtracted in quadrature from DUT residual using both triplets
- $\sigma_{\text{tele}}$ goes from 3.8 μm to 6.2 μm (depends on telescope configuration, mainly distance between first and second MIMOSA triplets)
- Best $\sigma_{\text{DUT}}$~5 μm @ ~19° (expected value compatible)
- In most of the angular range $\sigma_{\text{DUT}}$ is 8 μm or better

\[
\tan^{-1}(\text{pitch/active Thickness}) = \tan^{-1}(50/130) \approx 21°
\]

Cluster size = 2

$V_{\text{bias}} = 30$ V

NOT TO SCALE
Resolution vs Turning angle: $25 \times 100 \, \mu m^2$

- Module rotated around short 25 \mu m pitch coordinate
- Best $\sigma_{DUT} \sim 3 \, \mu m$ @ $\sim 10^\circ$
- In most of the angular range $\sigma_{DUT}$ is 4.5 \mu m or better
- First measurement for a 3D $25 \times 100 \, \mu m^2$ along 25 \mu m direction

\[ \tan^{-1}\left( \frac{\text{pitch}}{\text{active thickness}} \right) = \tan^{-1}\left( \frac{25}{130} \right) \approx 11^\circ \]
Cluster size = 2

$V_{bias} = 30 \, V$
Hit efficiency and ToT: 50×50 μm² (MA)

- Efficiency map for 2×2 pixel grid
- Efficiency drop near p⁺ columns
- High efficiency near n⁺ columns
- Global efficiency > 99%

V_{bias} = 30 V
Orthogonal beam incidence

- Tot map for 2×2 pixel grid
- Low average Tot near n⁺ columns
  ○ Low collected charge (due to passive column), but high efficiency
Cluster size, ToT, Efficiency: 50×50 μm² (MA) @6°

- Column inefficiencies are no longer there @6°
- Charge sharing due to rotation visible from ToT maps for 2×2 pixel grid

\[ V_{\text{bias}} = 30 \text{ V} \]

Rotation of 6°
Hit efficiency and signal charge: 25×100 μm²

- Efficiency map for 4×1 pixel grid
- Efficiency drop near p⁺ columns
- High efficiency near n⁺ columns
- Global efficiency > 99%

V_{bias} = 30 V

Orthogonal beam incidence

- Tot map for 4×1 pixel grid
- Low average Tot near n⁺ columns
  - Low collected charge (due to passive column), but high efficiency
Cluster size, ToT, Efficiency: \(25 \times 100 \, \mu m^2 \, \@ 5^\circ\)

- Column inefficiencies are no longer there @5°
- Charge sharing due to rotation visible from ToT maps for \(4 \times 1\) pixel grid

\[ V_{\text{bias}} = 30 \, V \]

Rotation of 5°
Cluster size: 25×100 μm²

DUT Cluster Size

Cluster size maps for 4×1 pixel grid

- Higher cluster size at lower bias voltage
  - More diffusion → more charge sharing between pixels

V_{bias} = 6 V

Orthogonal beam incidence

V_{bias} = 30 V
Irradiated 50×50 μm²: bias scan

- Irradiated in Oct 2018 @ CERN
- Irradiated sensor kept inside a cooling box:
  - Stable temperature of -27°C
- Beam width smaller than chip
  → irradiation not uniform
- Fluence $1 \times 10^{16} n_{eq} cm^{-2}$, 6 MGy
- Orthogonal beam incidence
- Efficiency increases with effective bias voltage $V_{bias}$
  - Saturation @ $V_{bias} = 110$V
  - Max value 98.8% @ $V_{bias} = 146$V
- Efficiency > 99% with rotations > 6°
Irradiated $50 \times 50 \, \mu m^2$: hit efficiency studies

High fluence spot

Under depletion

$V_{bias} = 28V$ low efficiency $\rightarrow$ high fluence
Irradiated 50×50 μm²: hit efficiency studies

Full depletion

\( V_{bias} = 146V \) full efficiency recovered
Irradiated 50×50 μm²: hit efficiency studies

Irradiation FWHM
- 12 mm in Y
- ~20 mm in X (DUT angle w.r.t. the beam)

Sub-zones analysis to cross check efficiency results → Uniform!
~3×4 mm² sub-zones

Full depletion

V_{bias} = 146V full efficiency recovered

98.8% 99.0%
99.0% 98.8%
Irradiated 50×50 μm²: hit efficiency studies

- Irradiation FWHM:
  - 12 mm in Y
  - ~20 mm in X (DUT angle w.r.t. the beam)

- Sub-zones analysis to cross check efficiency results → Uniform!
  ~3×4 mm² sub-zones

- Full depletion
  
- V_{bias} = 146V full efficiency recovered

- 98.9% hit efficiency
Irradiated 50×50 μm²: Resolution vs Turning angle

- Residuals fit (Student’s t-distribution) on rotated (rot) and unrotated (no-rot) coordinates
- Using σ from fit, telescope resolution \( (\text{res}_{\text{tele}}) \) estimated with
  \[
  \text{res}_{\text{tele}}^2 = \sigma_{\text{no-rot}}^2 - \text{res}_{\text{trivial}}^2, \text{ with } \text{res}_{\text{trivial}} = (50/\sqrt{12}) \, \mu\text{m}
  \]
- The resolution of the rotated coordinate \( (\text{res}_{\text{rot}}) \) has been estimated with (conservative):
  \[
  \text{res}_{\text{rot}}^2 = \sigma_{\text{rot}}^2 - \text{res}_{\text{tele}}^2
  \]
- Irradiated 3D sensor resolution is 5.7 μm at about 20° turn angle

\[
\tan^{-1}(\text{pitch/active\_thickness}) = \tan^{-1}(50/130) \approx 21°
\]
Cluster size = 2

\[V_{\text{bias}} = 146 \, \text{V}\]
Fresh 50×50 μm²: recent studies on ToT estimation

Mean

Different scales, but same range “shifted”. Takes into account the different “mean value” of the plot

- High statistics plot with 100×100 bins, low statistics with 50×50 bins, contributions from the whole sensor moduled 100 μm (2×2 pixels for 50×50 μm² pitch)
- Mean value of ToT inside the bins tend to over-estimate the MPV and is sensible to cut_off
- Moyal distribution, on the other hand, needs post processing → different possible approaches studied
- All the methods give very similar and reliable results

Moyal
Fresh 25×100 \( \mu m^2 \): recent studies on ToT estimation

- High statistics plot with 100×100 bins, low statistics with 50×50 bins, contributions from the whole sensor moduled 100 \( \mu m \) (2×2 pixels for 50×50 \( \mu m^2 \) pitch)
- Mean value of ToT inside the bins tend to over-estimate the MPV and is sensible to cut_off
- Moyal distribution, on the other hand, needs post processing → different possible approaches studied
- All the methods give very similar and reliable results
Conclusions

➲ CMS Inner Tracker Upgrade is extremely challenging
➲ Good results with fresh FBK 3D modules, both 50×50 μm\(^2\) and 25×100 μm\(^2\):
  ✓ Fully efficient (>99%) at 6 V of bias voltage for perpendicular tracks
  ✓ Resolution up to ~5(3) μm for 50×50 (25×100) μm\(^2\)
➲ Irradiated (1×10\(^{16}\)n\(_{eq}\)cm\(^{-2}\)) FBK 3D module is also very good in performances:
  ✓ Fully efficient at ~100 V
  ✓ Resolution up to ~5.7 μm for 50×50 μm\(^2\)
  ✓ 98.8% hit efficiency
➲ 3D pixels are confirmed to be strong candidates for the inner layers of the silicon trackers to be built for the HL-LHC
Thank you for your attention
Thank you for your attention

Many thanks to:
Backup
CMS Phase-2 tracker @ HL-LHC

LHC / HL-LHC Plan

Run 1 | Run 2 | Run 3 | Run 4 - 5...
--- | --- | --- | ---
**LS1** | **LS2** | **LS3**
splice consolidation button collimators R2E project | Diode Consolidation LIU installation 11 T dipole coll. Civil Eng. P1 PS | HL-LHC installation


7 TeV 8 TeV 13 TeV

experiment beam pipes cryostat interaction regions

nominal Lumi 2 x nominal Lumi

30 fb−1 190 fb−1 350 fb−1

75% nominal Lumi

14 TeV

energy

5 to 7.5 x nominal Lumi

3000 fb−1 4000 (ultimate)

HL-LHC TECHNICAL EQUIPMENT:

**DESIGN STUDY** | **PROTOTYPES** | **CONSTRUCTION** | **INSTALLATION & COMM.** | **PHYSICS**
--- | --- | --- | --- | ---

HL-LHC CIVIL ENGINEERING:

**DEFINITION** | **EXCAVATION / BUILDINGS**
--- | ---
Results on 3D Pixel Sensors for the CMS IT at HL-LHC

CMS Phase-2 tracker @ HL-LHC

- $\bar{\mathcal{L}}$ Luminosity = 3000÷4500 fb$^{-1}$
- Luminosity = $5\div7.5\times10^{34}$ cm$^{-2}$ s$^{-1}$
- $<\text{PU}> \sim 140\div200$
- Dose & fluence 10x higher
- 750 kHz L1 rate
- 12.5 μs L1 latency

LHC / HL-LHC Plan

- LHC
  - Run 1
  - Run 2
  - 7 TeV
  - 8 TeV
  - experiment beam pipes
  - nominal Lumi
  - 30 fb$^{-1}$
  - 13 TeV
  - 2 x nominal Lumi
  - 190 fb$^{-1}$

- HL-LHC
  - 14 TeV
  - 2019 2020 2021 2022 2023 2024 2025 2026 2027 2040
  - Diode Consolidation
  - L1 Installation
  - HL-LHC Installation
  - ATLAS - CMS
  - upgrade phase 1
  - ALICE - LHCb
  - upgrade
  - ATLAS - CMS
  - HL upgrade

- HL-LHC TECHNICAL EQUIPMENT:
  - DESIGN STUDY
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- Dose & fluence 10x higher
- 750 kHz L1 rate
- 12.5 μs L1 latency
The present CMS tracker cannot sustain the foreseen radiation levels and data rates and has to be completely replaced.
Inner Tracker upgrade for High Luminosity
Inner Tracker upgrade for High Luminosity
Main requests and innovations

- **High radiation tolerance:**
  - $2.3 \times 10^{16}$ n$_{eq}$/cm$^2$, fluence
  - 1.2 Grad, TID

- **Improve tracks separation:**
  - High granularity
  - High bandwidth (up to 3.5 GHz/cm$^2$ occupancy)
  - Low material budget

- **Extend tracking coverage:**
  - $|\eta| \leq 4$

- **Stringent space constraints**

- **TBPX 4 layers, TFPX 2x8 disks, TEPX 2x4 disks**
- **3900 modules**
- **$2 \times 10^9$ pixels** (124$x10^6$ in Phase-1)
- 4.9 m$^2$
- Hybrid modules with 2 ($1 \times 2$) or 4 ($2 \times 2$) readout chips
- **1156 modules, 2736 modules**
- Simple mechanics:
  - Can be removed for maintenance
  - Barrel splits in half at $z \sim 0$
  - Disks with planar geometry
Inner Tracker upgrade for High Luminosity

A lot of talks on HL-LHC CMS Inner Tracker (and not only):

- Corrinne Mills talk (17th Feb 2020, 15:15): “Performance of highly irradiated pixel sensors for the CMS HL-LHC upgrade” [https://indico.cern.ch/event/813597/contributions/3727828/]
- Finn Feindt talk (17th Feb 2020, 15:55): “Test Beam Characterization of Planar Pixel Sensors for the CMS Phase 2 Upgrade” [https://indico.cern.ch/event/813597/contributions/3727834/]
- Jordi Duarte Campderros talk (19th Feb 2020, 10:00): “Study of 3D pixel sensors after non-uniform proton irradiation” [https://indico.cern.ch/event/813597/contributions/3727824/]
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3D Pixel Sensors R&D

- 3D sensors at FBK Foundry (Trento, Italy): a common R&D program with FBK shared between Italian researchers of CMS and ATLAS and funded by INFN (Italy)
- 6” Si-Si DWB (Direct Wafer Bonding). Active Device Float Zone, P type, resistivity>3kOhm cm, 130µm (or 150µm) thick. CZ Handle wafer, 0.1-1 Ohm cm resistivity, 500µm thick.
- FBK Fully Top-side process: DRIE (Deep Reactive Ion Etching) technique
- Most of the sensors shown in this talk have been thinned down to a total thickness of 200µm; the thickness foreseen for HL-LHC CMS Inner Tracker is 250-275µm
RD53: the inner tracker readout chip

RD53 ROC

RD53 collaboration is developing an ROC with:

- Dead time $\leq 1\%$ @3.2 GHz/cm$^2$
- 0.5 (possibly 1) Grad TID resistant
- 65 nm technology
- 50x50 $\mu$m$^2$ cell
- Low threshold ($\leq 1000$ e$^-$)
- High hit and trigger rate (up to 4x1.28 Gb/s output links)
- Serial powering capabilities
- CMS chip size (16.8x21.6 mm$^2$, 336x432 cells)
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RD53A first prototype
- ½ total size (50×50 μm² cell, 65 nm technology)
- Used for R&D
- Radiation hard up to 0.5 Grad
- Low threshold and high hit and rate capabilities (160 Mbps input and 1.28 Gbps output links)
- 3 analog Front-Ends (Synchronous, Linear, Differential)
- Time over Threshold counter to measure charge (4 bits); caveat ToT as charge unit
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FBK 3D Pixel Batches: Mask Aligner and Stepper processes

“RD53A type” Sensors have $50 \times 50 \mu \text{m}^2$ or $25 \times 100 \mu \text{m}^2$ unit pixel cell size

Sensor corner with guard columns (photo)

3D Mask Aligner Oct 2017
18x RD53A sensors

I-V curves done with Temporary Metal at FBK on all sensors

47x RD53A sensors

3D Stepper_1 July 2019

Readout Cards

RICE irradiation card (before irradiation)

RICE adapter card

Bonn Card (after irradiation)
25×100 μm² setup

Telescope 4\textsuperscript{th} plane

DUT (on beam, turned)

Telescope 3\textsuperscript{rd} plane
Rotation wrt bias voltage

![Graph showing rotation with respect to bias voltage]

- Blue line represents 30 V bias voltage.
- Red line represents 6 V bias voltage.

Y-axis: Resolution [μm]
X-axis: Turn Angle [deg]
Cluster size: 50×50 μm² (MA)

- Cluster size maps for 2×2 pixel grid
- Higher cluster size at lower bias voltage
  - More diffusion → more charge sharing between pixels

- $V_{\text{bias}} = 6$ V
- $V_{\text{bias}} = 30$ V
Cluster size, ToT, Efficiency: $50 \times 50 \, \mu m^2$ (MA) @6°

- Column inefficiencies are no longer there @6°
- Charge sharing due to rotation visible from cluster size and ToT maps for 2×2 pixel grid

V\text{bias} = 30 \, V
Rotation of 6°
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V_{bias} = 30 V
Orthogonal beam incidence
25M events

- Tot map for 2×2 pixel grid
- Low average Tot near n⁺ columns
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\[ V_{bias} = 30 \, V \]
Cluster size: 50×50 μm² (Stepper)

Orthogonal beam incidence

- Cluster size maps for 2×2 pixel grid
- Higher cluster size at lower bias voltage
  - More diffusion → more charge sharing between pixels

\( V_{\text{bias}} = 6 \text{ V} \)
\( 1 \text{M events} \)

\( V_{\text{bias}} = 30 \text{ V} \)
\( 25 \text{M events} \)
Cluster size, ToT, Efficiency: $25 \times 100 \, \mu m^2$

- Cluster size
- ToT
- Efficiency

- $V_{\text{bias}} = 30 \, V$
- Rotation of $5^\circ$

- Column inefficiencies are no longer there @$5^\circ$
- Charge sharing due to rotation visible from cluster size and ToT maps for $4 \times 1$ pixel grid
Estimation methods

- We use the full sensor ToT distributions
  - Using clusters associated to a reconstructed track
  - We fit the distribution in several ways and we use the width to feed into the Moyal approx

- We then compare the estimated MPV
  - In some cases, some tuning is needed to get a better fit
What should we be using?

- But, an analytical distribution could approximate the Landau-Vavilov distribution → **Moyal** distribution

  \[ f(x) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{1}{2} \left( \frac{x - \mu}{\sigma} + e^{-\frac{x - \mu}{\sigma}} \right) \right\} \]

- Assuming N-measurements, following the Moyal distribution, the maximization of the Likelihood function gives the MPV (mode) estimator as,

  \[ \mu = -\sigma \ln \left( \frac{1}{N} \sum_{i=1}^{N} e^{-\frac{x_i}{\sigma}} \right) = -\sigma \ln \left( e^{-\frac{x}{\sigma}} \right) \]

  → This is what it’s used in Daniel Pitzl’s code

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