New Insights on Lepton Number and Dark Matter

Ernest Ma

Physics and Astronomy Department
University of California
Riverside, CA 92521, USA

Precis

Dark matter (DM) is usually assumed to be stabilized by a symmetry, which is mostly considered to be Z_2. For example, in supersymmetry it is R parity, i.e. $(-1)^{3B+L+2j}$. However, it may be Z_n or $U(1)_D$, and derivable from generalized lepton number. In this context, neutrino masses may be Majorana or Dirac, and owe their existence to dark matter, i.e. they are scotogenic.

Contents

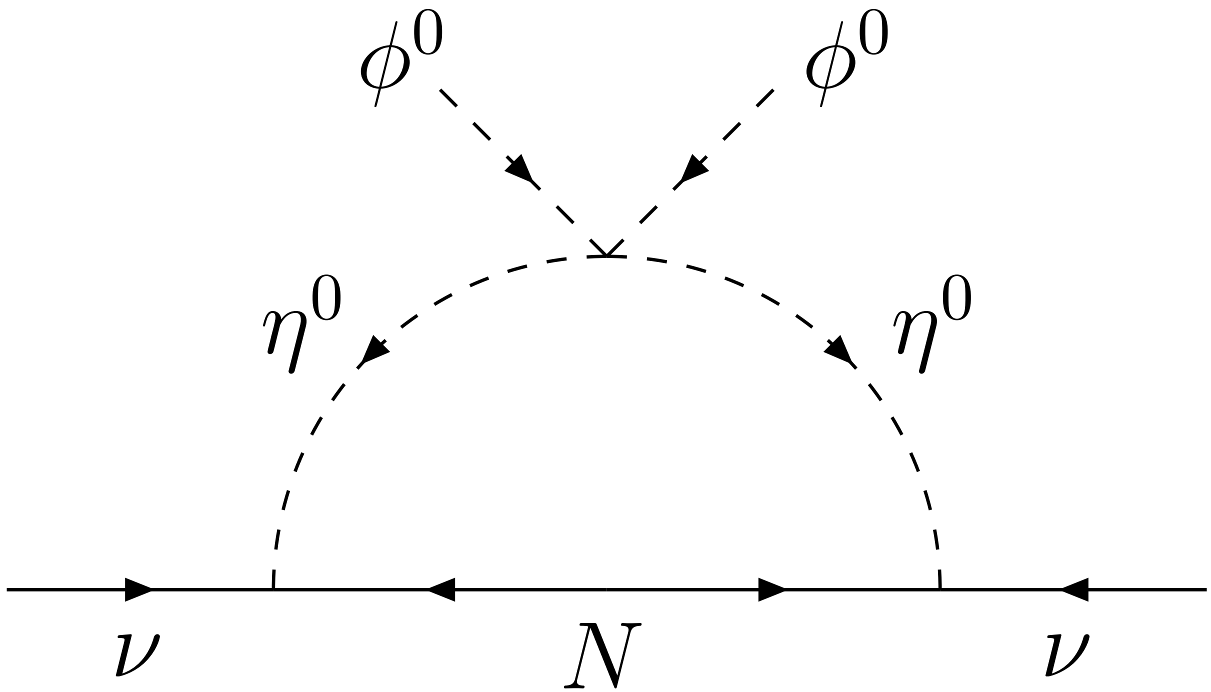
- Dark Matter Prototypes
- Dark Parity from Lepton Parity
- Lepton Parity with Dark $U(1)_D$
- Lepton Number Variants
- Scotogenic Dirac Neutrino Mass with Z_n^L and Z_n^D $(n \geq 5)$
- Scotogenic Dirac Neutrino Mass with Z_3^D
- Other Applications

Dark Matter Prototypes

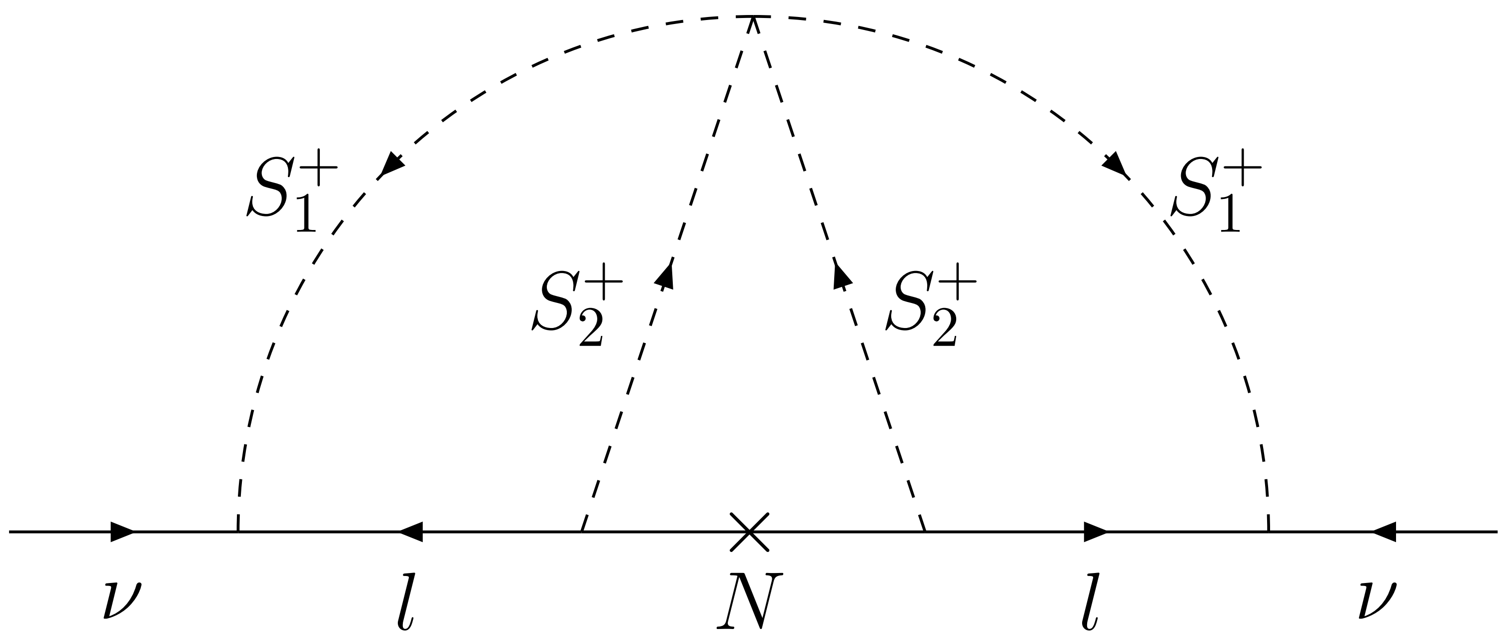
The simplest DM model [Silveira/Zee(1985)] is to add a real neutral singlet scalar S to the Standard Model (SM) with a new Z_2 symmetry under which S is odd and all other fields are even. This symmetry is necessary because the would-be allowed term $S\Phi^\dagger\Phi$ in the Lagrangian must be forbidden, Φ being the SM Higgs doublet. It also forbids the possible $S\nu_R\nu_R$ term if ν_R is added for ν_L to acquire a small seesaw Majorana mass. The next simplest model [Pospelov/Ritz/Voloshin(2008)] is to add a singlet Majorana fermion χ_L so that the term $S\bar{\chi}_L\nu_R$ is allowed.

Another DM prototype is for it to generate a radiative Majorana neutrino mass, i.e. the scotogenic mechanism. The simplest one-loop example [Ma(2006)] adds three Majorana neutral singlet fermions N_R and one scalar doublet $\eta = (\eta^+, \eta^0)$ to the SM. A new Z_2 symmetry is again assumed under which they are odd and all other fields are even. Hence the tree-level terms $\bar{\nu}_L N_R \phi^0$ are forbidden, but $\bar{\nu}_L N_R \eta^0$ are allowed.

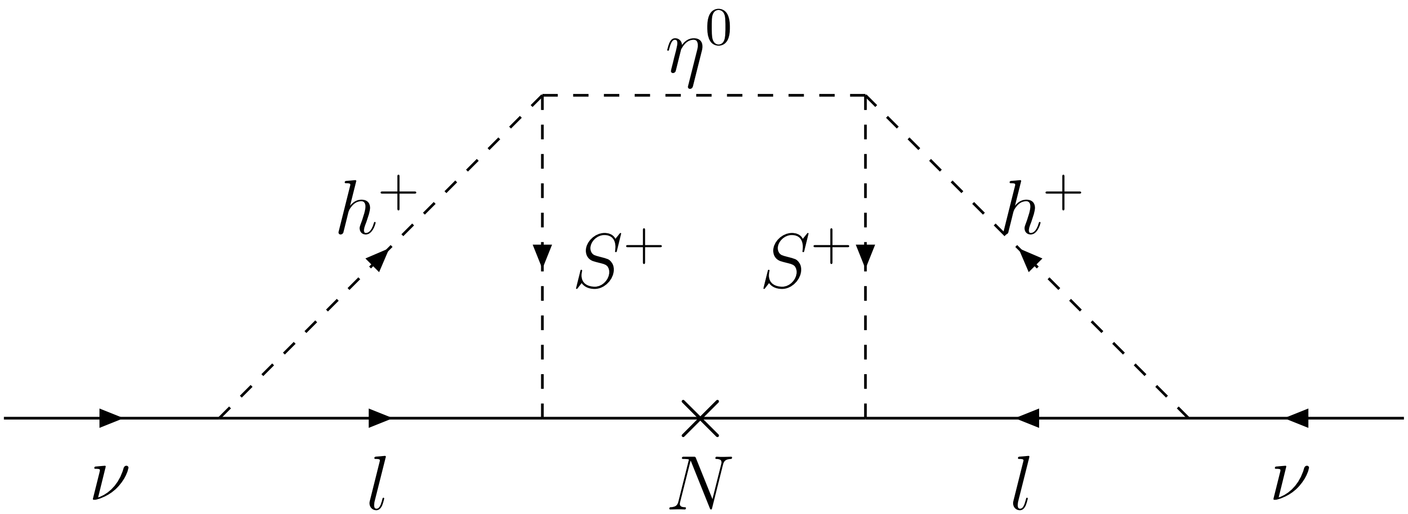
The same idea also works in some well-known three-loop models of neutrino mass: Krauss/Nasri/Trodden(2003), Aoki/Kanemura/Seto(2009), Gustafsson/No/Rivera(2013).

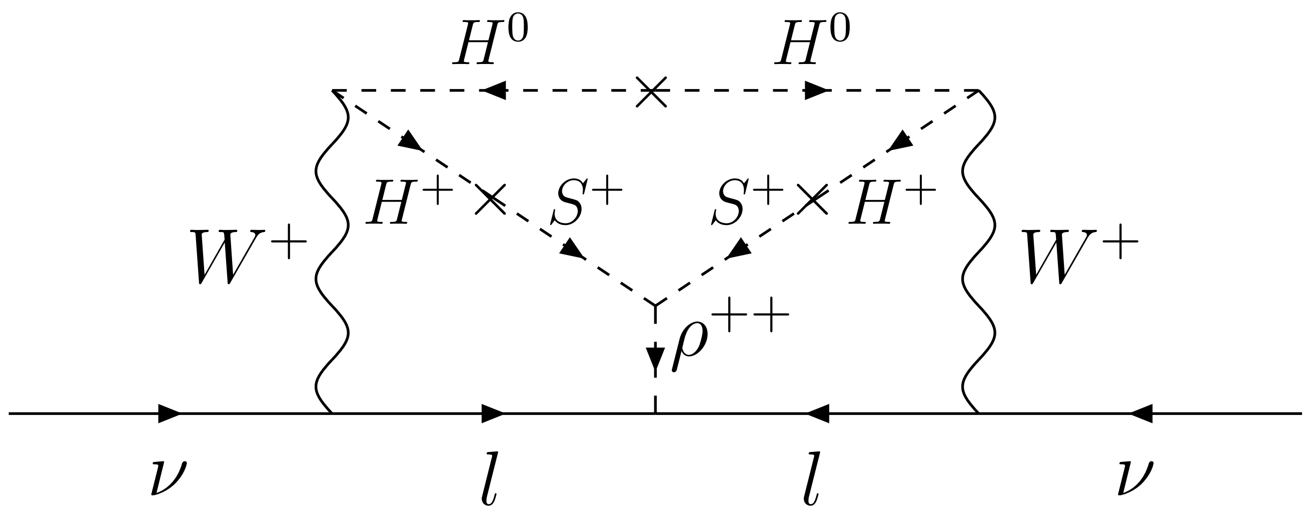


ϕ^0
ϕ^0
η^0
η^0
ν
N
ν



S_1^+
S_1^+
S_2^+
S_2^+
ν
l
N
l
ν


η^0
h^+
S^+
S^+
h^+
ν
l
N
l
ν


H^0
H^0
H^+
S^+
S^+
H^+
W^+
W^+
ρ^{++}
ν
l
l
ν

Dark Parity from Lepton Parity

Even without supersymmetry, the factor $(-1)^{2j}$ may be used to obtain dark parity π_D from lepton parity $\pi_L = (-1)^L$. This simple observation [Ma, PRL 115, 011801 (2015)] shows that the assignment of lepton parity to new particles added to the SM would also determine the dark sector, i.e. no new Z_2 symmetry is required to obtain exactly the same Lagrangian.

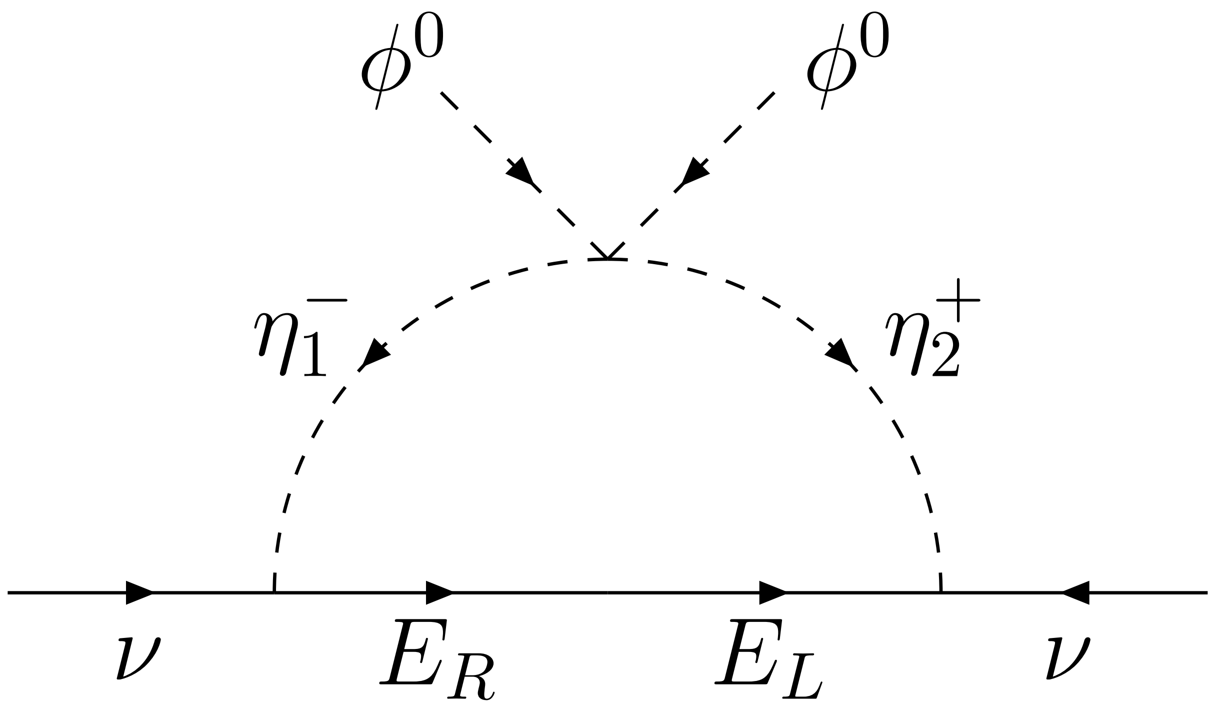
In the SM, under π_L, leptons (which are all fermions) are odd and other fields are even. In the DM prototypes, S should be assigned odd and χ_L even, so that $S\Phi^\dagger\Phi$ and

$S\nu_R\nu_R$ are forbidden, whereas $S\bar{\chi}_L\nu_R$ is allowed. It is clear that the previously imposed Z_2 dark parity π_D is just $(-1)^{2j}\pi_L$.

In the scotogenic model, N and η should be assigned even and odd respectively. In the KNT model, N, S_1^+ are even and S_2^+ is odd. In the AKS model, N, h^+ are even and S^+, η^0 are odd. In the GNR model, the scalars H^+, H^0, S^+ are odd and ρ^{++} is even, but there is no new fermion. Note however π_L is still involved because it forbids the term $S^+(\nu_i l_j - l_i \nu_j)$ as required.

Lepton Parity with Dark $U(1)_D$

Instead of assuming lepton parity to begin with, a more general approach is to use global $U(1)_L$ and break it softly by two units, but with a particle content such that a dark $U(1)_D$ symmetry remains. Add to the SM three pairs of charged fermions $E_L \sim 0$, $E_R \sim 2$, and two scalar doublets $(\eta_1^0, \eta_1^-) \sim 1$, $(\eta_2^{++}, \eta_2^+) \sim -1$, plus one scalar singlet $\chi^0 \sim -1$, then use the soft term $\bar{E}_L E_R$ to break $U(1)_L$ by two units. A scotogenic Majorana neutrino mass is obtained, but $U(1)_D$ remains. Here χ^0 (mixing slightly with $\bar{\eta}^0$) is DM.


ϕ^0
ϕ^0
η_1^-
η_2^+
ν
E_R
E_L
ν

Lepton Number Variants

The usual theoretical thinking on neutrinos is that they should be Majorana. Given that there is still no experimental proof, i.e. no evidence of neutrinoless double beta decay, it is time that this idea is re-examined. The usual argument goes like this. For ν_L to acquire mass, ν_R should be added to the SM, but then ν_R is allowed to have a large Majorana mass, hence ν_L gets a small seesaw mass and everyone is happy. However, ν_R is a trivial singlet in the SM and its existence is not required.

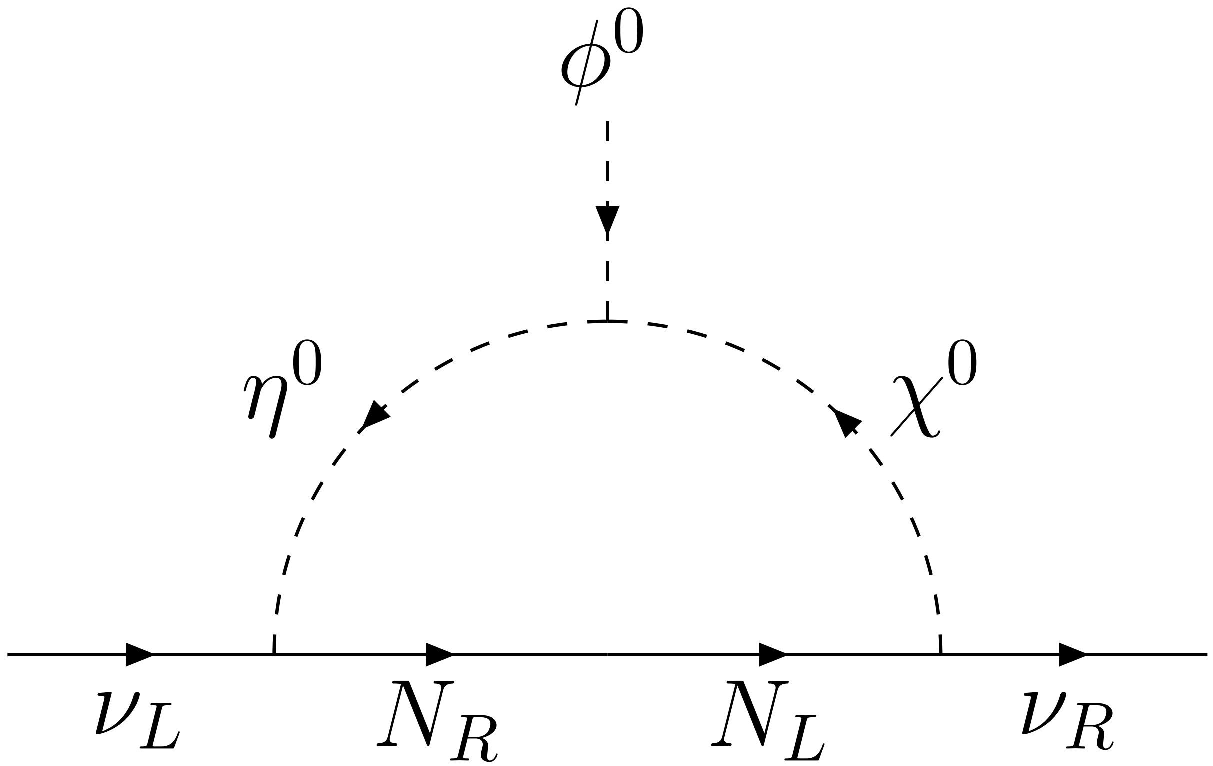
To enforce its existence, the SM should be extended, including gauge $B-L$ for example. In that case, the breaking of $B-L$ by two units would allow ν_R to have a Majorana mass as usual, but breaking it by three units would not. This means that a residual global U(1) remains which protects the neutrino as a Dirac fermion [Ma/Picek/Radovcic(2013)].

Depending on the details of the new particle content, the new lepton symmetry may be
Z_3 [Ma/Pollard/Srivastava/Zakeri(2015)] or
Z_4 [Heeck/Rodejohann(2013)] or Z_n $(n \geq 5)$.

Combining this recent insight with that on DM, new models of Dirac neutrinos and dark matter are possible. Using gauge $B-L$, instead of having three $\nu_R \sim 1$, the theory is also anomaly-free with three right-handed neutral singlet fermions transforming as $4, 4, -5$ [Montero/Pleitez(2009)]. In that case, tree-level Dirac neutrino masses are forbidden, but they may be generated radiatively by adding a suitable set of new fermions and scalars. Three recent studies are [Bonilla/Centelles Chulia/Cepedello/Peinado/Srivastava(2018), Calle/Restrepo/Yaguna/Zapata(2019), Jana/Vishnu/Saad(2019)].

Scotogenic Dirac Neutrino Mass with Z_n^L and Z_n^D $(n \geq 5)$

To obtain a radiative Dirac neutrino mass induced by dark matter (scotogenic), three symmetries are usually assumed [Gu/Sarkar(2008),Farzan/Ma(2012)]: (A) conventional lepton number, where $\nu_{L,R}, N_{L,R}$ have $L = 1$, and Φ, η, χ have $L = 0$, which is strictly conserved; (B) dark Z_2 symmetry, under which $N_{L,R}, \eta, \chi$ are odd and others even, which is strictly conserved; and (C) an *ad hoc* Z_2 symmetry under which ν_R, χ are odd and all others even, which is softly broken by $\eta^\dagger \Phi \chi$.


ϕ^0
η^0
χ^0
ν_L
N_R
N_L
ν_R

To obtain exactly the same one-loop diagram, it has been shown recently [Ma,arXiv:1912.11950] that a softly broken $U(1)_L$ by itself will do the job. Consider the following particle content:

fermion/scalar	$SU(2)$	$U(1)_Y$	$U(1)_L$	$*$	Z_n^L	Z_n^D
$(\nu, e)_L$	2	$-1/2$	1	1	ω	1
e_R	1	-1	1	1	ω	1
ν_R	1	0	x	$-n+1$	ω	1
N_L	1	0	y	$2-n$	ω^2	ω
N_R	1	0	y	$2-n$	ω^2	ω
$\Phi = (\phi^+, \phi^0)$	2	$1/2$	0	0	1	1
$\eta = (\eta^+, \eta^0)$	2	$1/2$	$y-1$	$1-n$	ω	ω
χ^0	1	0	$y-x$	1	ω	ω

Here $x \neq 1$ is imposed so that ν_R does not couple to ν_L at tree level. To connect them in one loop, the trilinear $\bar{\eta}^0 \phi^0 \chi^0$ term must break $U(1)_L$ softly by $x - 1$. The y charge of $N_{L,R}$ must not be ± 1 or $\pm x$ to avoid undesirable couplings to $\nu_{L,R}$. The soft terms $N_L N_L$ or $N_R N_R$ would break $U(1)_L$ by $2y$, $\nu_R \nu_R$ by $2x$, $N_R \nu_R$ by $x + y$, $\bar{N}_L \nu_R$ by $x - y$, and $\chi^0 \chi^0$ by $2(y - x)$. They should be absent, hence they must not be zero or divisible by $x - 1$. The column denoted by $*$ shows a class of solutions where $U(1)_L$ breaks to Z_n, i.e. $x = -n + 1$ and $y = 2 - n$.

If $n = 3$, then $x + y = -3$. If $n = 4$, then $2y = -4$. Hence $n = 3, 4$ are ruled out. Any $n \geq 5$ works.

This results in two related symmetries:

(I) Z_n^L lepton symmetry under which $\nu_{L,R}, e_{L,R}, \eta, \chi \sim \omega$ and $N_{L,R} \sim \omega^2$, where $\omega^n = 1$;

(II) Z_n^D dark symmetry, derivable from Z_n^L by multiplying it by ω^{-2j} where j is the particle's spin.

As a result, $\nu_{L,R}, e_{L,R} \sim 1$ and $N_{L,R}, \eta, \chi \sim \omega$.

This is the Dirac generalization of $\pi_D = (-1)^{2j}\pi_L$ for Majorana neutrinos.

In a renormalizable theory, the Z_n symmetry is not simply realizable. For $n \geq 5$, $(\chi^0)^n$ is not admissible. Hence the Lagrangian actually has a redefined $U(1)_L$ symmetry under which $\nu_{L,R}, e_{L,R}, \eta, \chi \sim 1$ and $N_{L,R} \sim 2$. The dark symmetry is then $U(1)_D$ where it is derived from $U(1)_L$ by subtracting $2j$, i.e. $\nu_{L,R}, e_{L,R} \sim 0$ and $N_{L,R}, \eta, \chi \sim 1$.

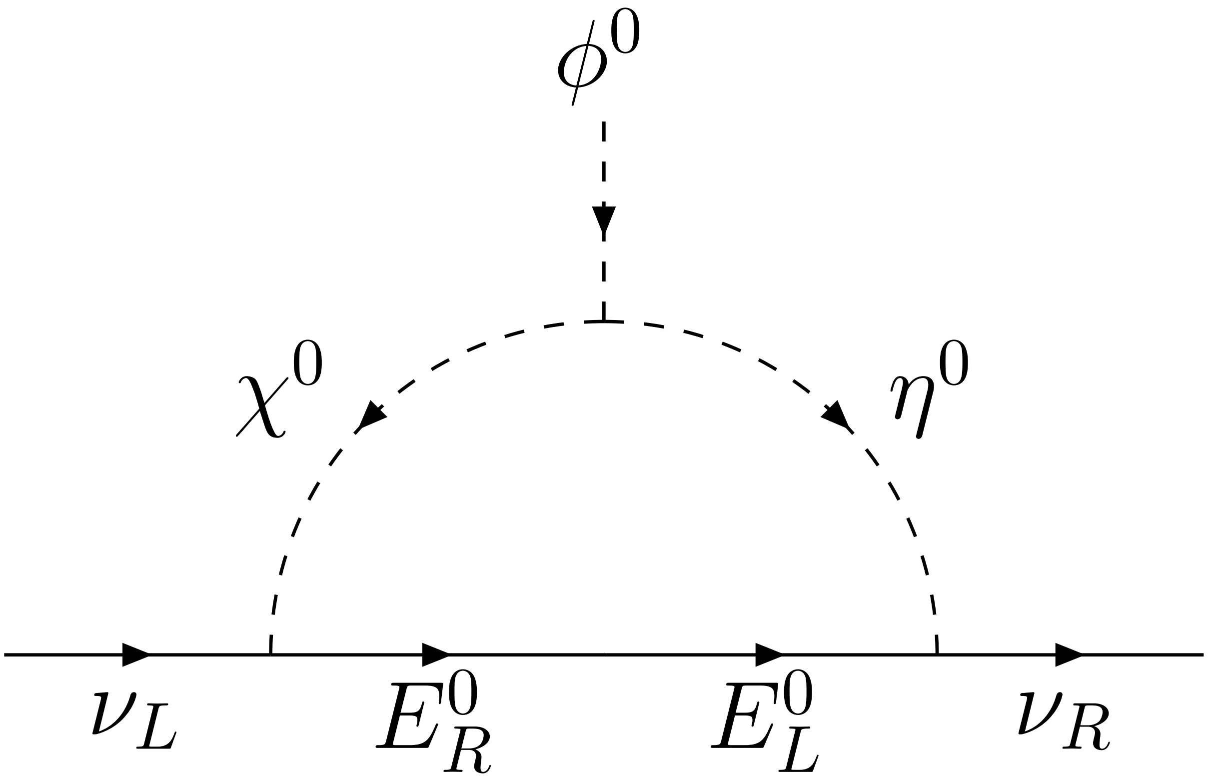
If Z_n symmetry is desired, the scalar sector must be expanded. If $n = 5$, let $\sigma \sim 3$ and $\kappa \sim 7$ be added. Then the terms $\chi^3\sigma^*$, $\chi^2\sigma$, $\chi\sigma^2\kappa^*$, and $\kappa N_R \nu_R$ are allowed. Together they would enforce Z_5^L and Z_5^D.

A possible variation is to add $\zeta \sim n$ and require $U(1)_L$ to be spontaneously broken in the $\zeta^* \eta^\dagger \Phi \chi$ term, thereby yielding a massless Goldstone boson, i.e. the diracon [Bonilla/Valle(2016)], as the analog of the majoron for Majorana neutrinos. A further application is to allow ζ to couple anomalously to exotic color fermion triplets or a color fermion octet [Demir/Ma(2000)]. The diracon becomes the QCD axion and $U(1)_L$ is extended Peccei-Quinn symmetry, as proposed long ago [Shin(1988)] for Majorana neutrinos, and very recently for Dirac neutrinos [Peinado/Reig/Srivastava/Valle(2019),Baek(2019)].

Scotogenic Dirac Neutrino Mass with Z_3^D

The $N_{L,R}$ fermion singlets may be replaced by $(E^0, E^-)_{L,R}$ fermion doublets.

fermion/scalar	$SU(2)$	$U(1)_Y$	$U(1)_L$	$**$	L	Z_3^D
$(\nu, e)_L$	2	$-1/2$	1	1	1	1
e_R	1	-1	1	1	1	1
ν_R	1	0	x	-2	1	1
$(E^0, E^-)_L$	2	$-1/2$	y	2	1	ω
$(E^0, E^-)_R$	2	$-1/2$	y	2	1	ω
$\Phi = (\phi^+, \phi^0)$	2	$1/2$	0	0	0	1
$\eta = (\eta^+, \eta^0)$	2	$1/2$	$x - y$	-4	0	ω^{-1}
χ^0	1	0	$y - 1$	1	0	ω



ϕ^0
χ^0
η^0
ν_L
E_R^0
E_L^0
ν_R

This construction eliminates the existence of many fermion bilinears except $\nu_R\nu_R$ and $\bar{\nu}_L E_R^0 + e_L^+ E_R^-$. Hence only $2x$ and $y-1$ must not be zero or divisible by $x-1$. Also $y \neq x$ is required. Now Z_3^D is possible as shown in the column denoted by $**$. Here $U(1)_L$ is broken by the soft $\Phi^\dagger \eta\chi$ and χ^3 terms. However the dimension-four term $\chi^0\nu_R\nu_R$ is not allowed by the original $U(1)_L$ even though it is allowed by Z_3^D. Hence the usual lepton assignment holds: $L=1$ for $\nu_{L,R}, E^0_{L,R}$ and $L=0$ for all scalars. [Note that in Z_3, if $\nu_R \sim \omega$, then $\chi \sim \omega$ or ω^2. Hence either $\chi\nu_R\nu_R$ or $\chi^*\nu_R\nu_R$ must exist and χ cannot be stable.]

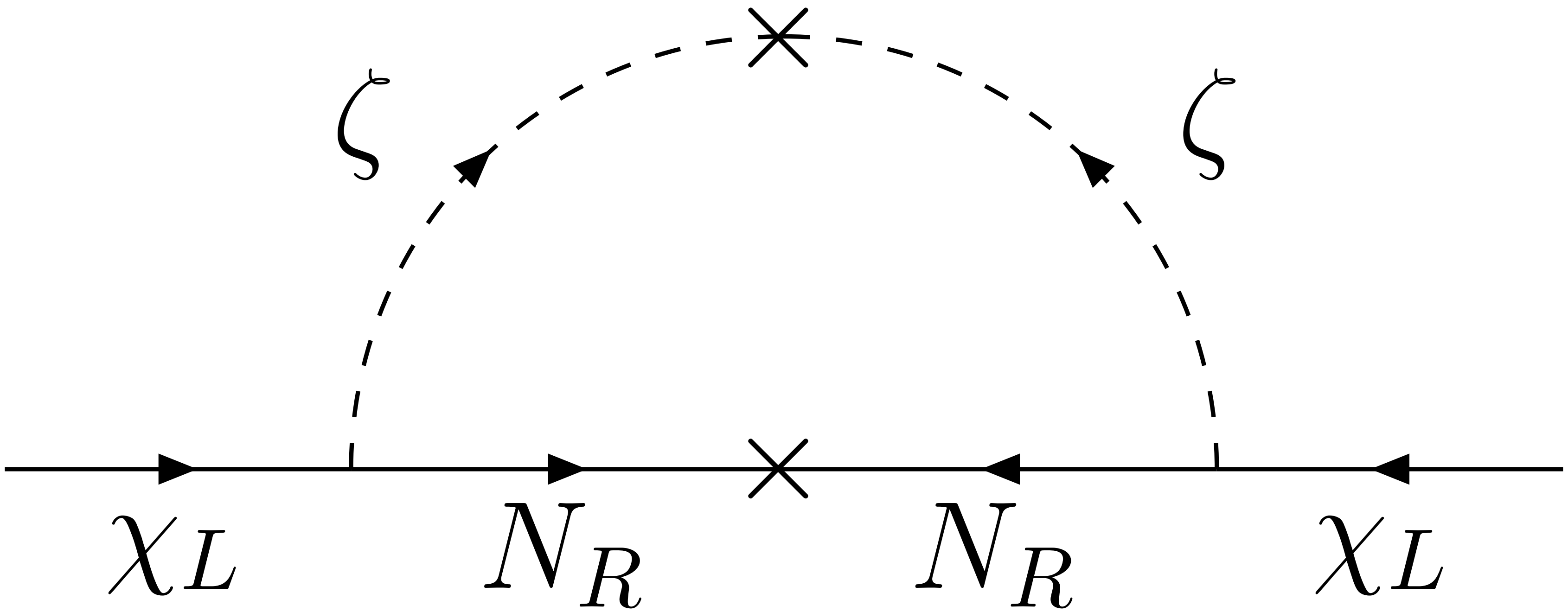
In this example, $U(1)_L$ is anomalous. To make it anomaly-free, the three copies of ν_R with charge -2 should be changed to 1. The difference is then $3[1-(-2)] = 3[3] = 9$ for the sum of the charges, and $3[1-(-8)] = 3[9] = 27$ for the sum of the cubes of the charges. This may be accomplished with singlet right-handed fermions $\psi_{2,3,4}$ with charges $-2, 3, -4$. For 9 copies of ψ_3 and 3 copies each of $\psi_{2,4}$, $3[3(3)+(-2)+(-4)] = 3[3] = 9$ and $3[3(27)+(-8)+(-64)] = 3[9] = 27$.
Add $\zeta_{3,6}$ to break $U(1)_L$ to Z_3^D. Then $\psi_{2,3,4}$ have $L = 1, 0, -1$.

Other Applications

In models of Dirac neutrinos where lepton number L is conserved, a scalar χ with $L = 1$ is a dark-matter candidate. Suppose another scalar ζ exists with $L = 2$. It then has two allowed interactions: $\zeta^*\chi\chi$ and $\zeta^*\nu_R\nu_R$. Suppose this mediator (ζ) is light (10 to 100 MeV), then the elastic scattering of $\chi\chi^* \to \chi^*\chi$ through ζ exchange may be large enough [$\sigma \sim 1\ \mathrm{cm}^2/\mathrm{g}$] to affect the density profile of dark matter in dwarf galaxies near its center, thus resolving the cusp-core discrepancy. [χ is called self-interacting DM.]

The annihilation of dark matter to its mediator is also enhanced through the exchange of its light mediator and becomes larger at late times as it slows down. The subsequent decay of the mediator to electrons would disrupt the Cosmic Microwave Background (CMB) [Bringmann/Kahlhoefer/Schmidt-Hoberg/Walia(2017)]. Whereas a scalar mediator would usually mix with the SM Higgs boson, and a vector mediator kinetically with the photon, thereby decaying to electrons, the light scalar mediator ζ here decays only to two Dirac neutrinos because it has $L = 2$. [Ma(2018)]

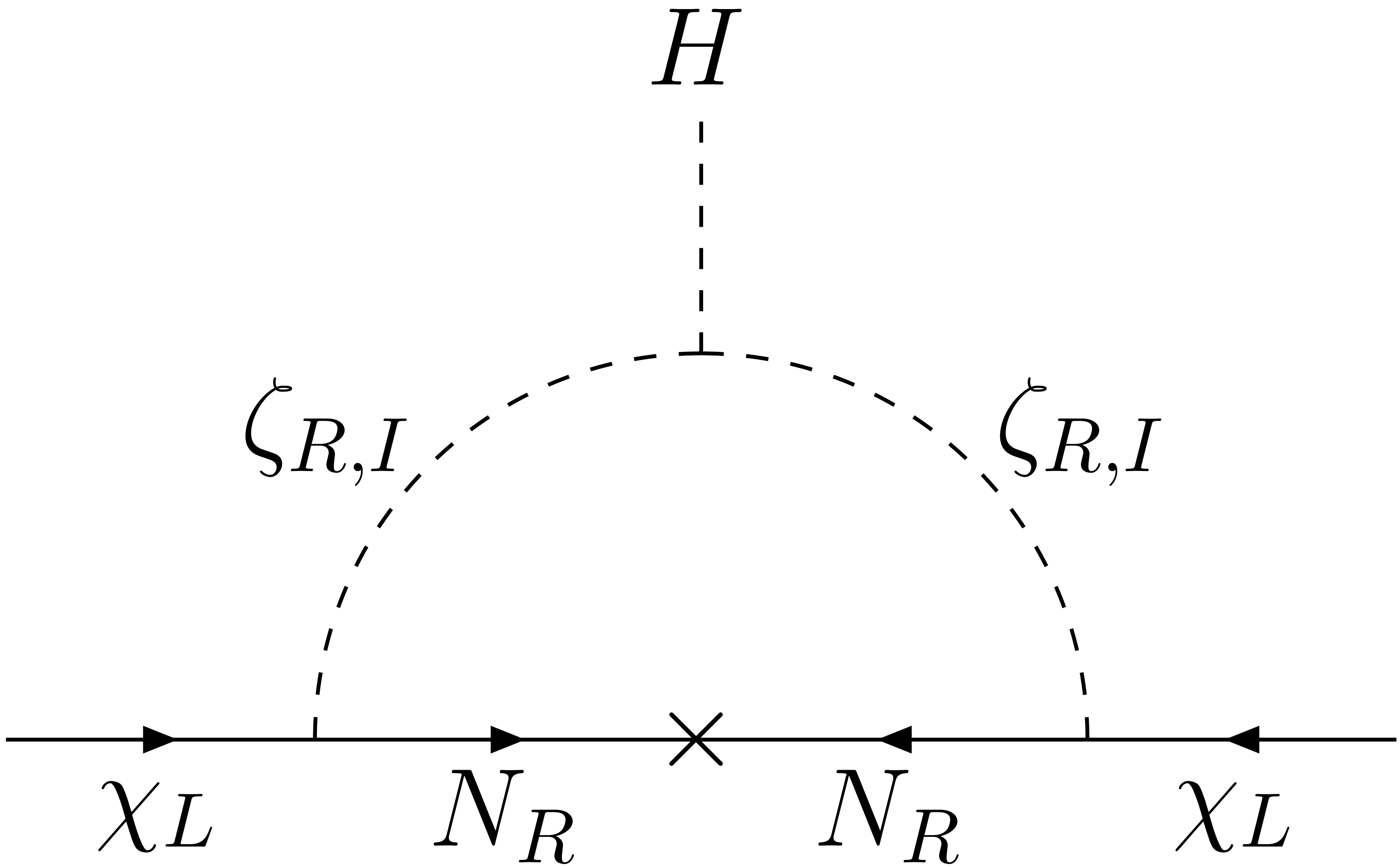
Another application of lepton number for dark matter is a variation of the S/χ model. Under L, let the scalar $\zeta \sim 1$ and the fermion $\chi_L \sim 2$ in the presence of N_R. All dimension-4 terms of the Lagrangian including $f\bar{\chi}_L N_R \zeta$ obey L, whereas the dimension-2 term $\mu^2[\zeta^2 + (\zeta^*)^2]/2$ and the dimension-3 term $(m_N/2)N_R N_R + H.c.$ break it softly by 2 units. Whereas neutrinos obtain Majorana masses through the conventional seesaw mechanism, i.e. $m_\nu \simeq m_D^2/m_N$, the dark fermion χ obtains a radiative mass in one loop, also anchored by m_N. The resulting dark symmetry is $\pi_D = (-1)^{2j}\pi_L$ as noted previously.


ζ
ζ
χ_L
N_R
N_R
χ_L

The light dark fermion χ interacts only through the very heavy ζ and N particles. If the reheat temperature of the Universe is much below m_N, χ may be produced only by the freeze-in mechanism through Higgs decay. The effective one-loop coupling f_H of H to $\chi\chi$ is

$$\frac{\lambda_3 v f^2 m_N}{32\pi^2}\left[\frac{1}{m_R^2 - m_N^2} - \frac{m_N^2 \ln(m_R^2/m_N^2)}{(m_R^2 - m_N^2)^2} - (m_R^2 \to m_I^2)\right],$$

where λ_3 is the $(\Phi^\dagger\Phi)(\zeta^*\zeta)$ coupling and $v/\sqrt{2}$ the vacuum expectation value of ϕ^0. It is proportional to m_χ with the factor $\lambda_3 v/m_0^2(\ln(m_N^2/m_0^2) - 1)$.


H
$\zeta_{R,I}$
$\zeta_{R,I}$
χ_L
N_R
N_R
χ_L

The decay rate of the SM Higgs boson to $\chi\chi$ is

$$\Gamma_H = \frac{f_H^2 m_H}{32\pi}\sqrt{1-4x^2}(1-2x^2),$$

where $x = m_\chi/m_H$. The production of χ is through H decay before the latter decouples from the thermal bath.

For $x << 1$, the correct relic abundance from freeze-in through Higgs decay is obtained for $f_H \sim 10^{-12}x^{-1/2}$. In this example, this is satisfied for $\lambda_3 = 1.16$, $m_\chi = 0.1$ MeV, $m_0 = 1$ TeV, and $m_N = 10^6$ GeV. Thus χ is a possible feebly interacting light dark fermion (FIMP).

Concluding Remarks

The notion of generalized $U(1)_L$ is useful in connecting leptons to dark matter. If it breaks softly to Z_2, then many DM prototype models may be understood in terms of lepton parity π_L (conserved for Majorana neutrinos) alone, with dark parity $\pi_D = (-1)^{2j}\pi_D$.

In scotogenic models, π_L and $U(1)_D$ are possible together. For Dirac neutrinos, softly broken $U(1)_L$ may also lead to Z_n^L and Z_n^D with $n \geq 5$, or redefined $U(1)_L$ and $U(1)_D$. An example of Z_3^D and conventional L is also possible.

A light $L = 2$ scalar ζ may act as the light mediator of self-interacting dark matter to solve the cusp-core discrepancy in the central density profile of dwarf galaxies. Its coupling $\zeta^* \nu_R \nu_R$ makes it decay only to two Dirac neutrinos, thereby not disrupting the CMB at late times in the early Universe when it is copiously produced.

Using a heavy scalar ζ with $L = 1$ and the usual very heavy N_R for canonical seesaw Majorana neutrino masses, the $L = 2$ Majorana fermion χ may acquire a small radiative mass through soft breaking of L and becomes FIMP through rare Higgs decay.