Black holes in Modified Gravity

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PROGRAM

1. $f(T)$ Gravitational Theory

2. Asymptotically static AdS black holes

3. New charged AdS black hole solutions

I. $f(T)$ Theory

Teleparallel space

This space is denoted in the literature by many names teleparallel, distant parallelism, Weitzenöck, absolute parallelism, vielbein, parallelizable space. An AP-space is a pair $(M,\ h_a)$, where M is an 4-dimensional smooth manifold and h_a $(a = 1, \cdots, 4)$ are 4 independent vector fields defined globally on M. The vector fields h_a are called the parallelization vector fields.
Let $h_a{}^\mu$ $(\mu = 1, ..., 4)$ be the coordinate components of the a-th vector field h_a, where Greek and Latin indices are constrained by the Einstein summation convention. The covariant components $h_{a\mu}$ of h_a are given via the relations

$$h_a{}^\mu h^a{}_\nu = \delta^\mu_\nu \quad \text{and} \quad h_a{}^\mu h^b{}_\mu = \delta^b_a, \tag{1.1}$$

where δ is the Kronecker tensor. Because of the independence of h_a, the determinant $h \equiv \det(h_a{}^\mu)$ is nonzero. On a teleparallel space $(M,\ h_a)$, there exists a unique linear connection, namely Weitzenböck connection, with respect to which the parallelization vector fields h_a are parallel. This connection is given by

$$\Gamma^\alpha{}_{\mu\nu} \equiv h_a{}^\alpha \partial_\nu h^a{}_\mu = -h^a{}_\mu \partial_\nu h_a{}^\alpha, \tag{1.2}$$

and is characterized by the property that

$$\nabla^{(\Gamma)}_\nu h_a{}^\mu \equiv \partial_\nu h_a{}^\mu + \Gamma^\mu{}_{\lambda\nu} h_a{}^\lambda \equiv 0, \tag{1.3}$$

where the operator $\nabla_\nu^{(\Gamma)}$ is the covariant derivative with respect to the Weitzenböck connection. The connection (1.2) will also be referred to as the canonical connection. The relation (1.3) is known in the literature as the AP-condition.
The non-commutation of an arbitrary vector fields V_a is given by

$$\nabla_\nu^{(\Gamma)}\nabla_\mu^{(\Gamma)} V_a{}^\alpha - \nabla_\mu^{(\Gamma)}\nabla_\nu^{(\Gamma)} V_a{}^\alpha = R^\alpha{}_{\epsilon\mu\nu} V_a{}^\epsilon + T^\epsilon{}_{\nu\mu}\nabla_\epsilon^{(\Gamma)} V_a{}^\alpha,$$

where $R^\alpha{}_{\epsilon\mu\nu}$ and $T^\epsilon{}_{\nu\mu}$ are the curvature and the torsion tensors of the canonical connection, respectively. The AP-condition (1.3) together with the above non-commutation formula force the curvature tensor $R^\alpha{}_{\mu\nu\sigma}$ of the canonical connection $\Gamma^\alpha{}_{\mu\nu}$ to vanish identically. Moreover, the parallelization vector fields define a metric tensor on M by

$$g_{\mu\nu} \equiv \eta_{ab} h^a{}_\mu h^b{}_\nu, \tag{1.4}$$

with inverse metric

$$g^{\mu\nu} = \eta^{ab} h_a{}^\mu h_b{}^\nu. \tag{1.5}$$

The Levi-Civita connection associated with $g_{\mu\nu}$ is

$$\overset{\circ}{\Gamma}{}^\alpha{}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma}\left(\partial_\nu g_{\mu\sigma} + \partial_\mu g_{\nu\sigma} - \partial_\sigma g_{\mu\nu}\right). \tag{1.6}$$

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In view of (1.3), the canonical connection $\Gamma^{\alpha}{}_{\mu\nu}$ (1.2) is metric:

$$\nabla^{(\Gamma)}_{\sigma} g_{\mu\nu} \equiv 0.$$

The torsion tensor of the canonical connection (1.2) is defined as

$$T^{\alpha}{}_{\mu\nu} \equiv \Gamma^{\alpha}{}_{\nu\mu} - \Gamma^{\alpha}{}_{\mu\nu} = h_a{}^{\alpha} \left(\partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu\right). \tag{1.7}$$

The contortion tensor $K^{\alpha}_{\mu\nu}$ is defined by

$$K^{\alpha}{}_{\mu\nu} \equiv \Gamma^{\alpha}_{\mu\nu} - \overset{\circ}{\Gamma}{}^{\alpha}{}_{\mu\nu} = h_a{}^{\alpha} \nabla^{(\overset{\circ}{\Gamma})}_{\nu} h^a{}_\mu. \tag{1.8}$$

where the covariant derivative $\nabla^{(\overset{\circ}{\Gamma})}_{\sigma}$ is with respect to the Levi-Civita connection. Since $\overset{\circ}{\Gamma}{}^{\alpha}{}_{\mu\nu}$ is symmetric, it follows that (using (1.8)) one can also show the following useful relations:

$$T_{\alpha\mu\nu} = K_{\alpha\mu\nu} - K_{\alpha\nu\mu}, \tag{1.9}$$

$$K_{\alpha\mu\nu} = \frac{1}{2}\left(T_{\nu\alpha\mu} + T_{\alpha\mu\nu} - T_{\mu\alpha\nu}\right), \tag{1.10}$$

where $T_{\mu\nu\sigma} = g_{\epsilon\mu} T^{\epsilon}{}_{\nu\sigma}$ and $K_{\mu\nu\sigma} = g_{\epsilon\mu} K^{\epsilon}{}_{\nu\sigma}$. It is to be noted that $T_{\mu\nu\sigma}$ is skew-symmetric in the last pair of indices whereas $K_{\mu\nu\sigma}$ is skew-symmetric in the first

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pair of indices. Moreover, it follows from (1.9) and (1.10) that the torsion tensor vanishes if and only if the contortion tensor vanishes. In the teleparallel space there are three Weitzenböck invariants: $I_1 = T^{\alpha\mu\nu}T_{\alpha\mu\nu}$, $I_2 = T^{\alpha\mu\nu}T_{\mu\alpha\nu}$ and $I_3 = T^{\alpha}T_{\alpha}$, where $T^{\alpha} = T_{\rho}{}^{\alpha\rho}$. We next define the invariant $T = AI_1 + BI_2 + CI_3$, where A, B and C are arbitrary constants [Maluf, 2013]. For the values: $A = 1/4$, $B = 1/2$ and $C = -1$ the invariant T is just the Ricci scalar up to a total derivative term; then a teleparallel version of gravity equivalent to GR can be achieved. The teleparallel torsion scalar is given in the compact form

$$T \equiv T^{\alpha}{}_{\mu\nu}S_{\alpha}{}^{\mu\nu}, \tag{1.11}$$

where the superpotential tensor

$$S_{\alpha}{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_{\alpha} + \delta^{\mu}_{\alpha}T^{\beta\nu}{}_{\beta} - \delta^{\nu}_{\alpha}T^{\beta\mu}{}_{\beta}\right), \tag{1.12}$$

is skew symmetric in the last pair of indices.

There are different extensions of TEGR, e.g. Born-Infeld extension of the TEGR [Ferraro and Fiorini, 2008, Fiorini, 2013], another interesting variant is the modified teleparallel equivalent of Gauss-Bonnet gravity and its applications [Kofinas and Saridakis, 2014b, Kofinas and Saridakis, 2014a, Kofinas et al., 2014].
The extension to $f(T)$-gravity has been inspired by the $f(R)$-gravity by replacing the Ricci scalar in the Einstein-Hilbert action instead of the Ricci scalar. But the former is by replacing the teleparallel torsion scalar by an arbitrary function $f(T)$ [Bengochea and Ferraro, 2009, Linder, 2010].
We consider the action of the $f(T)$-gravity

$$\mathcal{S} = \frac{1}{2\kappa}\int |h|(f(T) - 2\Lambda)\, d^4x + \int |h|\mathcal{L}_{em}\, d^4x, \tag{1.13}$$

where κ is a dimensional constant and $|h| = \sqrt{-g} = \det\left(h^a{}_\mu\right)$ and $\mathcal{L}_{em} = -\frac{1}{2}F \wedge^\star F$ is the Maxwell Lagrangian, with $F = dA$, with $A = A_\mu dx^\mu$, is the electromagnetic potential 1-form. The variation of Eq. (1.13) with respect to the vielbein field ${h^i}_\mu$ and the vector potential A_μ gives the following field equations

$$
\begin{aligned}
&{S_\mu}^{\rho\nu}\partial_\rho T f_{TT} + \left[h^{-1}{h^i}_\mu \partial_\rho\left(h{h_i}^\alpha {S_\alpha}^{\rho\nu}\right) - {T^\alpha}_{\lambda\mu}{S_\alpha}^{\nu\lambda}\right] f_T - \frac{f-2\Lambda}{4}\delta^\nu_\mu + \frac{1}{2}\kappa \overset{em}{\mathcal{T}}{}^{\nu}{}_{\mu} \\
&= {H^\nu}_\mu \equiv 0, \\
&\partial_\nu\left(\sqrt{-g}F^{\mu\nu}\right) = 0\,, \qquad (1.14)
\end{aligned}
$$

where $f = f(T)$, $f_T = \frac{\partial f(T)}{\partial T}$, $f_{TT} = \frac{\partial^2 f(T)}{\partial T^2}$ such that the TEGR theory is recovered by setting $f(T) = T$, $\overset{em}{\mathfrak{T}}{}^{\nu}{}_{\mu}$ the energy-momentum tensor of the electromagnetic field defined as

$$
\overset{em}{\mathcal{T}}{}^{\nu}{}_{\mu} = F_{\mu\alpha}F^{\nu\alpha} - \frac{1}{4}{\delta_\mu}^\nu F_{\alpha\beta}F^{\alpha\beta}\,, \qquad (1.15)
$$

where $F = dA$ and $A = A_\mu dx^\mu$ the electromagnetic potential 1-form [Capozziello et al., 2013].

$f(T)$ gravity

For more details of $f(T)$-gravity, see the recent review [Cai et al., 2015]. In the next section we are going to find a charged AdS/dS black hole for a cubic form of $f(T)$, i.e.,

$$f(T) = T + \beta T^2 + \gamma T^3 - 2\Lambda\,, \tag{1.16}$$

where Λ is the cosmological constant.

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We apply the field equations of extended teleparallel gravity $f(T)$, Eq. (1.14), to the flat 4-dimensional spacetime horizon, which directly gives rise to the tetrad written in cylindrical coordinate $(t,\, r,\, \phi,\, \phi_1)$ as follows:

$$h_\mu{}^a = \text{diag}\left(\sqrt{B(r)},\ \frac{1}{\sqrt{B_1(r)}},\ r,\ r\right), \tag{2.1}$$

where $B(r)$ and $B_1(r)$ are two unknown functions of r. Substituting from Eq. (2.1) into Eq. (1.11), we evaluate the torsion scalar as

$$T = 4\frac{B' B_1}{rB} + 2\frac{B_1}{r^2}. \tag{2.2}$$

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Using the 4-dimensional spacetime of Eq. (2.1) with a vector potential $A = q(r)dt$, the field equations have the following non-vanishing components: The neutral form of the field equations (1.14) of $f(T)$ take the form

$$
\begin{aligned}
& H^r{}_r = 2Tf_T + 2\Lambda - f = 0, \\
& H^\phi{}_\phi = H^{\phi_1}{}_{\phi_1} = \frac{f_{TT}[r^2 T + 2B_1]T'}{2r} + \frac{f_T}{2r^2 B^2}\Bigg\{2r^2 BB_1 B'' - r^2 B_1 B'^2 + 6rBB_1 B' \\
& \qquad + r^2 BB' B_1' + 2B^2[2B_1 + rB_1']\Bigg\} - f + 2\Lambda = 0, \\
& H^t{}_t = \frac{4B_1 f_{TT} T'}{r} + \frac{2f_T}{r^2 B}\Bigg\{2BB_1 + rB_1 B' + rBB_1'\Bigg\} - f + 2\Lambda = 0.
\end{aligned}
\tag{2.3}
$$

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In the case of the $f(T)$ form (1.16) these equations reduce to

$$H^r{}_r = T + 3\beta T^2 + 5\gamma T^3 + 2\Lambda = 0, \tag{2.4}$$

$$H^\phi{}_\phi = \xi^{\phi_1}{}_{\phi_1} = \frac{2(\beta+3\gamma T)[r^2 T + 2B_1]T'}{2r} + \frac{(1+2\beta T+3\gamma T^2)}{2r^2 B^2}\Big\{2r^2 BB_1 B'' - r^2 B_1 B'^2$$
$$+6rBB_1 B' T^2 - \gamma + r^2 BB' B_1' + 2B^2[2B_1 + rB_1']\Big\} - T - \beta T^3 + 2\Lambda = 0, \tag{2.5}$$

$$H^t{}_t = \frac{8(\beta+3\gamma T)B_1 T'}{r} + \frac{(1+2\beta T+3\gamma T^2)2}{r^4}\Big\{2BB_1 + rB_1 B' + rBB_1'\Big\}$$
$$-T - \beta T^2 - \gamma T^3 + 2\Lambda = 0, \tag{2.6}$$

where $T' \equiv dT(r)/dr$ is calculated through (2.2).

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A first observation is that Eq. (2.4) is a third-order algebraic equation and hence it implies that $T = T_0 = const.$. Hence, the differential equation (2.2) for $T = const.$ leads easily to the general solution

$$B(r) = \Lambda_{eff} r^2 - \frac{m}{r},$$
$$B_1(r) = B(r)\mathbb{B}\,, \tag{2.7}$$

where m is an integration constant related to the mass parameter, and the function $\mathbb{B}$ is calculated by inserting (2.7) into (2.5),(2.6), giving

$$\mathbb{B} = \frac{1}{6} T_0 \gamma = const., \tag{2.8}$$

in the case where we set the explicit cosmological constant Λ to zero (in which case $T_0 = \frac{-3\beta \pm \sqrt{9\beta^2 - 20\gamma}}{10\gamma}$). In the above expressions the constant Λ_{eff} is given by

$$\Lambda_{eff} = \frac{1}{\gamma}, \tag{2.9}$$

and we can clearly see that it plays the role of an effective cosmological constant. The important observation here is that we obtain an effective cosmological constant that

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arises solely from the $f(T)$ modification, even if the initial explicit cosmological constant is absent. Hence, interestingly enough, the structure of the $f(T)$ gravity leads to an effective cosmological constant and in the case where it is negative the solution is an AdS one. This feature, namely the induction of an effective cosmological constant due to the $f(T)$ structure, was indicated to happen in $f(T)$ gravity [Iorio and Saridakis, 2012, Kofinas et al., 2015], however in the present work we show robustly that it does appear and moreover in general dimensions.
We mention that the above solution exists only for $\gamma \neq 0$, namely it is a result of the higher-power correction to standard TEGR, i.e to general relativity, and it reveals the effect of such corrections. In the case where $\beta = \gamma = 0$ and $\Lambda \neq 0$ then $\Lambda_{eff} \propto \Lambda$, that is we recover standard TEGR with a cosmological constant, and its Schwarzschild-(A)dS solution. Lastly, note that although $B(r)$ and $B_1(r)$ differ by a constant, the g_{tt} and g_{rr} components of the metric have the same Killing and event horizons. The solution has a singularity at $r = 0$, while it possesses a horizon at $m = \Lambda_{eff} r^3$.

We continue our analysis for the special choice where

$$\Lambda = \frac{1}{18\beta} \qquad \text{and} \qquad \gamma = \frac{3\beta^2}{5}, \tag{2.10}$$

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with $\beta \neq 0$, since in this case even for $\Lambda \neq 0$ the solution has $B(r) = B_1(r)$. The investigation of solutions with $g_{tt} = g_{rr}^{-1}$ has an increased interest in the literature since it exhibits an extra symmetry. In particular, following the above steps we extract the solution

$$B(r) = \Lambda_{eff} r^2 - \frac{m}{r},$$
$$B_1(r) = B(r), \tag{2.11}$$

where m is an integration constant related to the mass parameter and where

$$\Lambda_{eff} = -\frac{1}{18\beta}. \tag{2.12}$$

Similarly to the previous case we obtain an effective cosmological constant which now depends on the parameter β. In the case where $\beta > 0$ we obtain an AdS solution. The horizon of the solution (2.12) is again at $m = \Lambda_{eff} r^{N-1}$.

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Let us now proceed with the analysis of the charged solutions, that is we consider also the electromagnetic Lagrangian $\mathcal{L}_{em}$ in (1.13), choosing additionally without loss of generality the vector potential to have the general form $A = q(r)dt$. Imposing again the 4-dimensional tetrad of (2.1), the field equations (1.14) have the following

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non-vanishing components:

$$
\begin{aligned}
&H^r{}_r = 2Tf_T + 2\Lambda - f + \frac{2q'^2(r)B_1}{B} = 0,\\
&H^\phi{}_\phi = H^{\phi_1}{}_{\phi_1} = \frac{f_{TT}[r^2T + 2B_1]T'}{2r}\\
&+\frac{f_T}{2r^2B^2}\Bigg\{2r^2BB_1B'' - r^2B_1B'^2 + 4B^2B_1\\
&+6rBB_1B' + r^2BB'B_1' + 2rB^2B_1'\Bigg\} - f\\
&+2\Lambda - \frac{2q'^2(r)B_1}{B} = 0,\\
&H^t{}_t = \frac{4B_1f_{TT}T'}{r} + \frac{2f_T[2BB_1 + rB_1B' + rBB_1']}{r^2B}\\
&-f + 2\Lambda + \frac{2q'^2(r)B_1}{B} = 0,
\end{aligned}
\tag{3.1}
$$

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where $q' = \frac{dq(r)}{dr}$. In the case of the $f(T)$ form (1.16) these equations reduce to

$$
\begin{aligned}
&H^r{}_r = T + 3\beta T^2 + 5\gamma T^3 + 2\Lambda + \frac{2q'^2(r)B_1}{B} = 0, \\
&H^\phi{}_\phi = H^{\phi_1}{}_{\phi_1} = \frac{2(\beta+3\gamma T)[r^2T + 2B_1]T'}{2r} \\
&+\frac{(1+2\beta T+3\gamma T^2)}{2r^2B^2}\Big\{2r^2BB_1B'' - r^2B_1B'^2 + 2(2N-5)rBB_1B' \\
&\quad + r^2BB'B_1' + 2B^2[2B_1 + rB_1']\Big\} - T - \beta T^2 - \gamma T^3 + 2\Lambda - \frac{2q'^2(r)B_1}{B} = 0, \\
&H^t{}_t = \frac{4(N-2)(\beta+3\gamma T)B_1T'}{r} + \frac{2(1+2\beta T+3\gamma T^2)}{r^8}\Big\{2BB_1 + rB_1B' + rBB_1'\Big\} \\
&\quad - T - \beta T^2 - \gamma T^3 + 2\Lambda + \frac{2q'^2(r)B_1}{B} = 0.
\end{aligned}
\tag{3.2}
$$

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Although the above equations in the case of a general Λ can be solved only numerically, analytical solutions can still be extracted when Λ is given by (2.10). In this case the general 4-dimensional solution takes the form

$$B(r) = r^2\Lambda_{eff} - \frac{m}{r} + \frac{15q^2}{8r^2} + \frac{45q^3}{16\times 14\sqrt[3]{r^{10}}} + \frac{9q^4}{16\times 11\sqrt[3]{r^{14}}},$$
$$B_1(r) = h(r)B(r), \tag{3.3}$$

with

$$h(r) = \left[1 + \frac{2q}{3\sqrt[3]{r^4}} + \frac{4q^2}{9\sqrt[3]{r^8}} + \frac{q^3}{9r^4} + \frac{q^4}{36\sqrt[3]{r^{16}}}\right]^{-1},$$
$$q(r) = \frac{q}{r} + \frac{q^2}{7\sqrt[3]{r^7}} + \frac{q^3}{22\sqrt[3]{r^{11}}}, \tag{3.4}$$

where

$$\Lambda_{eff} = \frac{81}{24q}, \tag{3.5}$$

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with $m, q \neq 0$ being the constants of integration related to mass and electric charge respectively. We stress that the above solution exists only for $q \neq 0$, since for $q = 0$ we have the solutions of the previous subsection, thus it arises from the structure of the electromagnetic sector.
Let us now discuss on the properties of the above solution. First of all, inserting (3.3) into (2.1) and then into (1.4) we obtain the metric as

$$\begin{aligned} ds^2 = & \left[r^2\Lambda_{eff} - \frac{m}{r} + \frac{15q^2}{8r^2} + \frac{45q^3}{224\sqrt[3]{r^{10}}} + \frac{9q^4}{16 \times 11\sqrt[3]{r^{14}}} \right] dt^2 \\ & - \left[1 + \frac{2q}{3\sqrt[3]{r^4}} + \frac{4q^2}{9\sqrt[3]{r^8}} + \frac{q^3}{9r^4} + \frac{q^4}{36\sqrt[3]{r^{16}}} \right] \\ & \times \left[r^2\Lambda_{eff} + \frac{m}{r} + \frac{15q^2}{8r^2} + \frac{45q^3}{224\sqrt[3]{r^{10}}} + \frac{9q^4}{16 \times 11\sqrt[3]{r^{14}}} \right]^{-1} dr^2 \\ & - r^2 \left(d\phi^2 + d{\phi_1}^2 \right) . \end{aligned} \tag{3.6}$$

As we observe, in this case the solution is more complicated, however it is still asymptotically AdS or dS according to the sign of q. We mention the interesting

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feature that the effective cosmological constant is a result of the electric charge. Nevertheless, note that both β and γ are important for the solution structure, and hence this solution does not have a TEGR, i.e. general relativity, limit, nor an uncharged one. Moreover, this solution subclass is also outside the ones obtained in [Capozziello et al., 2013, Awad et al., 2017], due to the use of more general $f(T)$ forms in the present work. Therefore, solution (3.3) corresponds to a new charged AdS black hole in power-law $f(T)$ gravity.

Additionally, we mention here that apart from the difference in the metric part, the above solution has also a difference in the charge structure too, comparing to those obtained in [González et al., 2012, Capozziello et al., 2013, Awad et al., 2017], since the potential $q(r)$ depends on a monopole and higher-order electromagnetic potential. This electromagnetic potential will have a vanishing value only when the constant $q = 0$, which is not allowed in the above solution, and thus this implies that within the framework of $f(T)$ gravity we cannot find a charged solution with monopole only.

We now proceed to the investigation of the singularity structure of the solution, by calculating curvature and torsion invariants. The curvature scalars are calculated from the metric (1.4) while the torsion scalar is calculated through the vielbeins (2.1).

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Calculating the Ricci scalar, the Ricci tensor square, and the Kretschmann scalar, we respectively find:

$$R = F_1(r)\left(\frac{1}{\sqrt[3]{r^4}}\right), \qquad R^{\mu\nu}R_{\mu\nu} = F_2(r)\left(\frac{1}{\sqrt[3]{r^8}}\right),$$
$$K \equiv R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} = F_3(r)\left(\frac{1}{\sqrt[3]{r^8}}\right), \tag{3.7}$$

while calculating the torsion scalar we respectively obtain:

$$T(r) = \frac{12\sqrt[3]{r^4} + 2\beta[3]^5}{36\,|\beta|\,\sqrt[3]{r^4}}, \tag{3.8}$$

where $F_i(r)$ are polynomial functions of r.
The above invariants first of all show the singularity at $r = 0$. Close to $r = 0$ the behavior of these invariants are given by $(K, R_{\mu\nu}R^{\mu\nu}) \sim \sqrt[3]{r^{-8}}$ and $(R, T) \sim \sqrt[3]{r^{-4}}$, in contrast to the solutions of the Einstein-Maxwell theory in both general relativity and TEGR formulations which have $(K, R_{\mu\nu}R^{\mu\nu}) \sim r^{-8}$ and $(R, T) \sim r^{-4}$. This shows clearly that the singularity of our charged solution is softer than the one obtained in GR and TEGR for the charged case. Finally, notice that although in the solution the g_{tt}

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and g_{rr} components of the metric are different, they have the same Killing and event horizons.
As we observe, as q increases and m decreases, and in particular for $q > m$, we enter in a parameter region where there is no horizon, and thus the central singularity is a naked singularity. This is an interesting result of Maxwell-$f(T)$ gravity (we mention that we see the issue from the mathematical point of view and we do not examine whether such a solution can indeed be formed physically through gravitational collapse), which does not appear in the absence of the electromagnetic sector (indications of this feature had also been discussed in [González et al., 2012, Capozziello et al., 2013]).
Moreover, note that for suitable m and q the two horizons coincide and become degenerate, namely we obtain $r_- = r_+ \equiv r_{dg}$.

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Thank You