Electroweak Monopole: Topological Avatar of New Physics

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January 13, 2020

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Electroweak Monopole

January 13, 2020 1 / 50

• Why the electroweak monopole?

1. It is the only realistic topological elementary particle, the topological avatar of new physics, predicted by the standard model.

2. It is the electroweak generalization of the Dirac monopole. So it is this monopole, not the Dirac monopole, which exists in nature.

3. It is different from the Dirac monopole. The magnetic charge is twice bigger.

• What are the physical Implications?

1. The detection of the electroweak monopole, not the Higgs particle, becomes the final and topological test of the standard model.

2. It could play important roles in cosmology, on primordial black hole, large scale structure of the universe, intergalactic magnetic field.

3. It could generate the hithertoo unknown magnetic current which has unlimited applications.

4. If detected, it will be the first magnetically charged and stable topological elementary particle, the true God's particle, in human history.

Within the standard model!

Contents

- 1. Electroweak Monopole: A Review
- 2. Cosmic Production of Electroweak Monopole
- 3. Remnant Monopole Density
- 4. Physical Implications

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A. History

- Ever since Dirac predicted the Dirac monopole in 1931, the monopole has become an obsession, theoretically and experimentally.
- After Dirac we have had Wu-Yang (1969), 'tHooft-Polyakov (1974), and grand unification (Dokos-Tomaras; 1980) monopoles. But they are unrealistic or unphysical.
- Strangely, it has been asserted that the standard model has no monopole.

• No-Go theorem: The standard model has no monopole topology

$$\pi_2(G/H) = \pi_2\Big(\frac{SU(2) \times U(1)_Y}{U(1)_{(\text{em})}}\Big) = 0.$$

This topology, however, is not the only possible monopole topology.

- 1. With $U(1)_Y$ the Higgs doublet becomes $\mathbb{C}P^1$ which has S^2 topology.
- 2. Non-trivial $U(1)_{(em)}$ has Abelian monopole topology.
- In 1997 the existence of the electroweak monopole was established.

B. Abelian Decomposition of Standard Model

• Start from the Weinberg-Salam Lagrangian

$$\begin{aligned} \mathcal{L} &= -|\mathcal{D}_{\mu}\phi|^{2} - \frac{\lambda}{2} \left(|\phi|^{2} - \frac{\mu^{2}}{\lambda}\right)^{2} - \frac{1}{4}\vec{F}_{\mu\nu}^{2} - \frac{1}{4}G_{\mu\nu}^{2} \\ &= -\frac{1}{2}(\partial_{\mu}\rho)^{2} - \frac{\rho^{2}}{2}|\mathcal{D}_{\mu}\xi|^{2} - \frac{\lambda}{8}\left(\rho^{2} - \rho_{0}^{2}\right)^{2} - \frac{1}{4}\vec{F}_{\mu\nu}^{2} - \frac{1}{4}G_{\mu\nu}^{2}, \\ \mathcal{D}_{\mu}\phi &= \left(\partial_{\mu} - i\frac{g}{2}\vec{\tau}\cdot\vec{A}_{\mu} - i\frac{g'}{2}B_{\mu}\right)\phi = \left(D_{\mu} - i\frac{g'}{2}B_{\mu}\right)\phi, \\ &\phi &= \frac{1}{\sqrt{2}}\rho \ \xi \ (\xi^{\dagger}\xi = 1), \quad \rho_{0} = \sqrt{2\mu^{2}/\lambda}. \end{aligned}$$

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• Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3 = \hat{n})$ be an SU(2) orthonormal frame. Make the Abelian decomposition with $\hat{n} = -\xi^{\dagger} \vec{\tau} \xi$

$$ec{A_{\mu}} = \hat{A}_{\mu} + ec{W}_{\mu}, \ ec{F}_{\mu
u} = \hat{F}_{\mu
u} + \hat{D}_{\mu}ec{W}_{
u} - \hat{D}_{
u}ec{W}_{\mu} + gec{W}_{\mu} imes ec{W}_{
u}, \ \hat{A}_{\mu} = A_{\mu}\hat{n} - rac{1}{g}\hat{n} imes \partial_{\mu}\hat{n}, \quad ec{W}_{\mu} = W^{1}_{\mu}\hat{n}_{1} + W^{2}_{\mu}\hat{n}_{2}.$$

• Find $\hat{F}_{\mu\nu}$ is made of two potentials, electric A_{μ} and magnetic C_{μ}

$$egin{aligned} &\mu
u
u &= (F_{\mu
u} + H_{\mu
u})\hat{n} = F'_{\mu
u}\hat{n} = (\partial_{\mu}A'_{
u} - \partial_{
u}A'_{\mu})\hat{n}, \ &F_{\mu
u} &= \partial_{\mu}A_{
u} - \partial_{
u}A_{\mu}, \ &H_{\mu
u} &= \partial_{\mu}C_{
u} - \partial_{
u}C_{\mu}, \quad &C_{\mu} = -rac{2i}{g}\xi^{\dagger}\partial_{\mu}\xi, \ &A'_{\mu} &= A_{\mu} + C_{\mu}. \end{aligned}$$

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January 13, 2020 8 / 50

Define the physical fields by

$$\begin{pmatrix} A_{\mu}^{(\mathrm{em})} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\mathrm{w}} & \sin \theta_{\mathrm{w}} \\ -\sin \theta_{\mathrm{w}} & \cos \theta_{\mathrm{w}} \end{pmatrix} \begin{pmatrix} B_{\mu} \\ A'_{\mu} \end{pmatrix},$$
$$W_{\mu} = \frac{W_{\mu}^{1} + iW_{\mu}^{2}}{\sqrt{2}}.$$

Notice that

1. This is a gauge independent definition. 2. $A_{\mu}^{(\rm em)}$ becomes non-trivial since A'_{μ} is non-trivial (unless the Weinberg angle is zero or $\pi/2$).)

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• With the identity

$$|\mathcal{D}_{\mu}\xi|^{2} = \frac{g^{2} + g^{\prime 2}}{4}Z_{\mu}^{2} + \frac{g^{2}}{2}|W_{\mu}|^{2},$$

we have the gauge independent Abelianization of the standard model

$$\begin{split} \mathcal{L} &= -\frac{1}{2} (\partial_{\mu} \rho)^2 - \frac{\lambda}{8} \big(\rho^2 - \rho_0^2 \big)^2 - \frac{1}{4} F_{\mu\nu}^{(\mathrm{em})^2} - \frac{1}{4} Z_{\mu\nu}^2 \\ &- \frac{1}{2} |(D_{\mu}^{(\mathrm{em})} W_{\nu} - D_{\nu}^{(\mathrm{em})} W_{\mu}) + i e \frac{g}{g'} (Z_{\mu} W_{\nu} - Z_{\nu} W_{\mu})|^2 \\ &+ i e F_{\mu\nu}^{(\mathrm{em})} W_{\mu}^* W_{\nu} - \frac{g^2}{4} \rho^2 W_{\mu}^* W_{\mu} - \frac{g^2 + g'^2}{8} \rho^2 Z_{\mu}^2 + i e \frac{g}{g'} Z_{\mu\nu} W_{\mu}^* W_{\nu} \\ &+ \frac{g^2}{4} (W_{\mu}^* W_{\nu} - W_{\nu}^* W_{\mu})^2, \\ D_{\mu}^{(\mathrm{em})} &= \partial_{\mu} + i e A_{\mu}^{(\mathrm{em})}, \quad e = \frac{gg'}{\sqrt{g^2 + g'^2}}. \end{split}$$

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C. Explicit Solution: A Review

• Choose the hybrid ansatz with non-trivial $U(1)_Y$

$$\begin{split} \phi &= \frac{1}{\sqrt{2}} \rho(r) \ \xi, \quad \xi = i \left(\begin{array}{c} \sin(\theta/2) \ e^{-i\varphi} \\ -\cos(\theta/2) \end{array} \right), \\ \vec{A}_{\mu} &= \frac{1}{g} A(r) \partial_{\mu} t \ \hat{r} + \frac{1}{g} (f(r) - 1) \ \hat{r} \times \partial_{\mu} \hat{r}, \quad (\hat{r} = -\xi^{\dagger} \vec{\tau} \ \xi) \\ B_{\mu} &= \frac{1}{g'} B(r) \partial_{\mu} t - \frac{1}{g'} (1 - \cos \theta) \partial_{\mu} \varphi. \end{split}$$

The string singularities in ξ and B_{μ} can be removed requiring $U(1)_Y$ non-trivial.

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• In the physical fields the ansatz becomes

$$\begin{aligned} A_{\mu}^{(\text{em})} &= \frac{e}{gg'} \Big(\frac{g'}{g} A(r) + \frac{g}{g'} B(r) \Big) \partial_{\mu} t - \frac{1}{e} (1 - \cos \theta) \partial_{\mu} \varphi, \\ W_{\mu} &= \frac{i}{g} \frac{f(r)}{\sqrt{2}} e^{i\varphi} (\partial_{\mu} \theta + i \sin \theta \partial_{\mu} \varphi), \\ Z_{\mu} &= \frac{e}{gg'} \Big(A(r) - B(r) \Big) \partial_{\mu} t. \end{aligned}$$

Dyon ansatz with W and Z dressing

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• With this we have the equations of motion

$$\begin{split} \ddot{\rho} &+ \frac{2}{r}\dot{\rho} - \frac{f^2}{2r^2}\rho + \frac{1}{4}(A-B)^2\rho = \frac{\lambda}{2}\big(\rho^2 - \rho_0^2\big)\rho, \\ & \ddot{f} - \frac{f^2 - 1}{r^2}f = \big(\frac{g^2}{4}\rho^2 - A^2\big)f, \\ & \ddot{A} + \frac{2}{r}\dot{A} - \frac{2f^2}{r^2}A = \frac{g^2}{4}\rho^2(A-B), \\ & \ddot{B} + \frac{2}{r}\dot{B} = -\frac{g'^2}{4}\rho^2(A-B). \end{split}$$

• This has the point monopole solution which has $q_m = 4\pi/e$

$$\begin{split} \rho &= \rho_0, \quad f = 0, \quad A = B = 0, \\ A_{\mu}^{(\text{em})} &= -\frac{1}{e} (1 - \cos \theta) \partial_{\mu} \varphi. \end{split}$$

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• With the boundary condition

$$\rho(0) = 0, \quad f(0) = 1, \quad A(0) = 0, \quad B(0) = b_0,$$

$$\rho(\infty) = \rho_0, \quad f(\infty) = 0, \quad A(\infty) = B(\infty) = A_0,$$

we have the dyon solution which has the asymptotic behavior,

$$\rho \simeq \rho_0 + \frac{\rho_1}{r} \exp(-M_H r),$$

$$f \simeq f_1 \exp(-\sqrt{1 - (A_0/M_W)^2} M_W r),$$

$$A \simeq A_0 + \frac{A_1}{r}, \quad B \simeq A + \frac{B_1}{r} \exp(-M_Z r),$$

$$M_H = \sqrt{\lambda}\rho_0, \quad M_W = g\rho_0/2, \quad M_Z = \sqrt{g^2 + g'^2}\rho_0/2.$$

So the monopole size is set by the weak boson masses.

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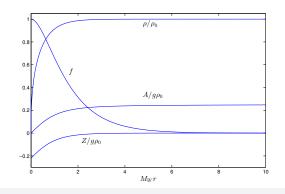


Figure: The electroweak dyon solution. Here Z = A - B and we have chosen $\sin^2 \theta_w = 0.2312$, $M_H/M_W = 1.56$, and $A(\infty) = M_W/2$.

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D. Mass of Electroweak Monopole

- The point singularity of the electroweak monopole makes the energy infinite. But we can estimate the mass.
- There are different ways to estimate the mass, the dimensional argument, the scaling argument, and the quantum correction. All of them predict the mass to be around 4 to 10 TeV.
- We can regularize the singularity with a non-trivial electromagnetic permittivity which mimics the charge screening.

Consider the effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{1}{2} (\partial_{\mu} \rho)^{2} - \frac{\lambda}{8} (\rho^{2} - \rho_{0}^{2})^{2} - \frac{1}{4} \epsilon(\rho) F_{\mu\nu}^{(em)^{2}} - \frac{1}{4} Z_{\mu\nu}^{2}$$
$$-\frac{1}{2} |(D_{\mu}^{(em)} W_{\nu} - D_{\nu}^{(em)} W_{\mu}) + i e \frac{g}{g'} (Z_{\mu} W_{\nu} - Z_{\nu} W_{\mu})|^{2} + \dots$$

1. It retains the $SU(2 \times U(1)_Y)$ gauge symmetry, and recover the standard model with $\epsilon \to 1$ asymptotically.

2. With the rescaling of $A_{\mu}^{(\rm em)}$ to $A_{\mu}^{(\rm em)}/e$, the $U(1)_{(\rm em)}$ gauge coupling e changes to the running coupling $\bar{e} = e/\sqrt{\epsilon}$.

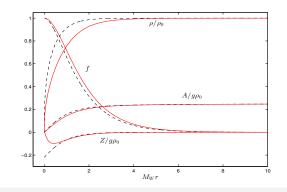


Figure: The finite energy electroweak dyon. The solid line (red) represents the regularized dyon and the dotted (blue) line represents the singular dyon.

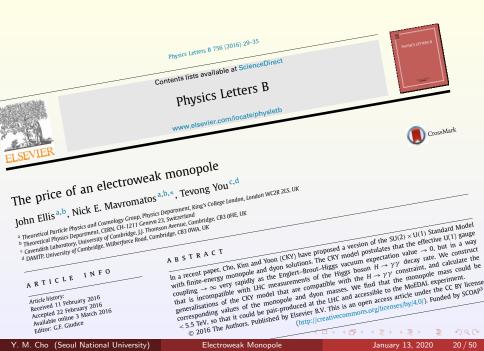
• With $\epsilon = (\frac{\rho}{\rho_0})^6,$ the regularized monopole has the energy

$$E \simeq 0.65 \times \frac{4\pi}{e^2} M_W \simeq 7.19 \text{ TeV}.$$

The monopole mass has been constrained further:

1. Choosing a realistic $\epsilon(\rho)$ which explains the experimental data on $H \rightarrow \gamma + \gamma$, people argued that the mass may not exceed 5.5 TeV. 2. With a BPS extension, Blaschke and Benes showed that the mass can not be smaller than 2.37 TeV.

The BPS bound is improved to 3.75 TeV.



PTEP

Prog. Theor. Exp. Phys. 2018, 073B03 (18 pages) DOI: 10.1093/ptep/pty071

BPS Cho-Maison monopole

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Received March 27, 2018; Accepted May 19, 2018; Published July 27, 2018

We present exact solutions to the Cho–Maison magnetic monopole in a family of effective electroweak models that have a Bogomol'nyi–Prasad–Sommerfield limit. We find that the lower bound to the mass of the magnetic monopole is $M \ge 2\pi v/g \approx 2.37$ TeV. We argue that this bound holds universally, not just in theories with a BPS limit.

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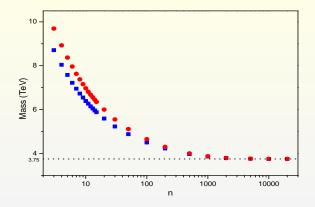


Figure: The monopole energy with $\epsilon = (\rho/\rho_0)^n$ in terms of n in log scale. The red dots represent the energy regularized by the electromgnetic permittivity. For comparison we plot the monopole energy regularized by the hypercharge permittivity in blue dots. Here the dotted line represents the asymptotic energy when n goes to infinity.

- This strongly implies that the standard model has the electroweak monopole whose mass could be 4 to 10 TeV. But ultimately this has to be confirmed by experiment.
- If the monopole mass is less than 7 TeV, the present LHC should be able to produce the monopole pair, and the MoEDAL could actually detect it.
- If the mass is larger, however, we may have to look for the remnant monopole in the present universe.



66 physicists from 14 countries & 24 institutes. on 4 continents:

U. Alberta, UBC, INFN Bologna, U. Bologna, CAAG-Algeria, U. Cincinatti, Concordia U., CSIC Valencia, Gangneung-Wonju Nat. U., U. Geneva, U. Helsinki, IEAP/CTU Prague, IFIC Valencia, Imperial College London, ISS Bucharest, King's College London, Konkuk U., U. Montréal, MISIS Moscow, Muenster U., National Inst. Tec. (india),Northeastern U., Simon Langton School UK, Stanford University [is the latest (associate) member of MoEDAL], Tuft's.

Figure: The MoEDAL (The Magnificent Seventh) Collaboration at CERN, now 70 physicists from 30 institutes.

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Cosmic Production of Electroweak Monopole

A. Electroweak Phase Transition

• The electroweak phase transition is controlled by the effective potential of the standard model

$$V_T(\rho) \simeq \frac{\lambda}{8} (\rho^2 - \rho_0^2)^2 - \frac{C_1}{12\pi} \rho^3 T + \frac{C_2}{2} \rho^2 T^2 - \frac{\pi^2}{90} NT^4,$$

$$C_1 = \frac{6M_W^3 + 3M_Z^3}{\rho_0^3} \simeq 0.36,$$

$$C_2 = \frac{4M_W^2 + 2M_Z^2 + M_H^2 + 4m_t^2}{8\rho_0^2} \simeq 0.36,$$

which is charactrized by three temperatures, T_2 , T_c , and T_1 .

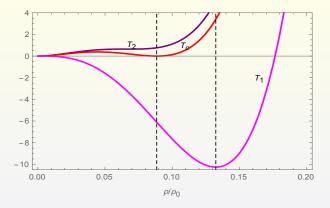


Figure: The temperature-dependent effective potential of the standard model with $T_2 = 146.7$ GeV, $T_c = 146.6$ GeV, and $T_1 = 146.4$ GeV.

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• The phase transition is first order:

1. Above $T_1\simeq 146.7~{\rm GeV}$ the standard model is in the symmetric state. But it develops a second unstable vacuum ρ_+ at T_1 , while $\rho_0=0$ remains the true vacuum.

2. At $T_c \simeq 146.6$ GeV we have $\rho_+ \simeq 21.8$ GeV, and the two vacua become degenerate, and ρ_+ becomes the true vacuum below T_c . 3. Below $T_1 \simeq 146.4$ GeV ρ_+ becomes the only vacuum, which approaches to the Higgs vacuum at zero temperature.

• However, since T_1 , T_c , T_2 are very close the phase transition is very mildly first order ("cross-over"), and effectively becomes second order.

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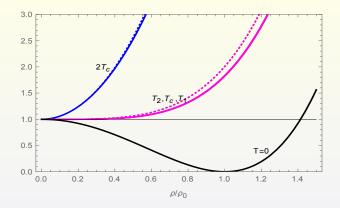


Figure: The macroscopic view of the effective potential of the standard model for $2T_c$, T_c , and T = 0.

• There are two popular cosmic monopole production mechanisms.

1. In the first order phase transition, the monopoles are produced during the phase transition through the vacuum bubble collisions.

2. In the second order phase transition, however, the monopoles produced by the Kibble-Zurek mechanism around the critical temperature.

• However, none of the above mechanism may apply for the electroweak monopole production. For the monopole production, the important thing is when the monopole formation takes place.

- To produce the monopole we need the change of topology induced by the zero points of the Higgs vacuum $\langle \rho \rangle = 0$, which become the seed of the monopole.
- These zero points are induced by thermal fluctuations even after the phase transition, not just during the phase transition.
- This starts below $T_c \simeq 146.6$ GeV but become insignificant below the Ginzburg temperature $T_G \simeq 57.6$ GeV.

B. Initial Monopole Density

• The initial monopole density is determined by the correlation length $\xi\simeq 1/\bar{M}_H$ fixed by the Higgs mass,

$$\bar{M}_{H}^{2} = \begin{cases} [(T/T_{1})^{2} - 1]M_{H}^{2}/2, & T \geq T_{c}, \\ [(\rho_{+}/\rho_{0})^{2} + 1 - (T/T_{1})^{2}]M_{H}^{2}/2, & T < T_{c}. \end{cases}$$

which acquires the minimum value 5.53 GeV at $T = T_c$ and becomes 11.7 GeV at $T = T_1$.

• Similarly the W-boson starts to become massive toward the value $g\rho_+(T_c)/2 \simeq 7.0$ GeV at T_c , and acquires the mass 10.5 GeV at $T = T_1$.

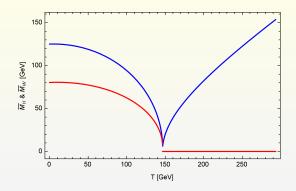


Figure: The temperature-dependent Higgs and W-boson masses. The blue and red curves represent the Higgs and W-boson masses.

- So the monopoles are produced between T_c and T_G , around $T_i \simeq (T_c + T_G)/2 \simeq 102.0$ GeV, when the correlation length becomes $\xi_i \simeq (\xi_c + \xi_G)/2 \simeq 9.3 \times 10^{-16}$ cm.
- From this we have the initial monopole density n_i,

$$\left(\frac{n_m}{T^3}\right)_i \simeq \frac{g_P}{\xi_i^3 T_i^3} \simeq 9.1 \times 10^{-4},$$

where $g_P \simeq 0.1$ is the probability that one monopole is produced in one correlation volume.

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• From this we have the energy density of the monopoles

$$\begin{split} \varrho_{mo}(T_i) &= M_m(n_m)_i \simeq 9.0 \times 10^{-3} \ T_i^4 \ \left(\frac{M_m}{1 \ \text{TeV}}\right), \\ &\frac{\varrho_{mo}(T_i)}{\varrho(T_i)} \simeq 2.5 \times 10^{-4} \Big(\frac{M_m}{1 \ \text{TeV}}\Big). \end{split}$$

So the universe need to consume only a tiny fraction (about 0.025 %) of the total energy to produce the monopoles.

• This assures that, unlike the grand unification monopole, the electroweak monopole does not alter the standard cosmology.

A. Evolution of Monopole

• The cosmic evolution of the initial monopoles are described by the Boltzmann equation

$$\frac{dn_m}{dt} + 3Hn_m = -\sigma n_m^2,$$

where H and σ are the Hubble expansion parameter and the monopole annihilation cross section.

• The annihilation cross section is affected by two factors, the mean free length l_{free} of the Brownian motion of the monopole and the capture radius r_{capt} of the monopole-antimonopole attraction.

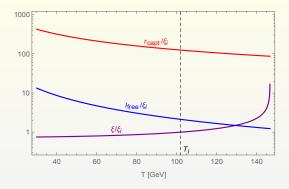


Figure: The relevant scales, ξ in purple, l_{free} in blue, and r_{capt} in red, against T. They are normalized by the correlation length ξ_i at T_i . Here we set $M_m = 5$ TeV. • Solving the Boltzmann equation we have

$$\frac{n_m}{T^3} = \frac{1}{A(M_m/T - M_m/T_i) + B},$$
$$A \simeq \frac{0.02}{\alpha} \times \frac{m_p}{M_m}, \quad B = \left(\frac{n_m}{T^3}\right)_i^{-1}.$$

The solution remains valid till the annihilation stops at T_f .

• When $T_f \ll T_i$, the final monopole density becomes independent of the initial value, and approaches to

$$\frac{n_m}{T^3} \simeq \frac{\alpha}{0.02} \times \frac{T}{m_p}, \quad (T \ll T_i).$$

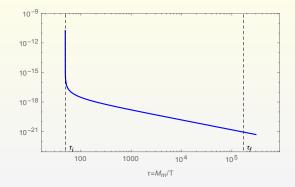


Figure: The evolution of the monopole density n_m/T^3 against $\tau = M_m/T$. The final value of the monopole density is independent of the initial value.

- Most of the initial monopoles are quickly annihilated since r_{capt} becomes much bigger (by the factor 10^2) than l_{free} .
- The annihilation lasts very long, and stops around $T_f \simeq 29.5$ MeV when $l_{\rm free}$ becomes bigger than $r_{\rm capt}$. The terminal density at T_f becomes

$$\left(\frac{n_m}{T^3}\right)_f \simeq 1.8 \times 10^{-22} \ \left(\frac{M_m}{1 \ {\rm TeV}}\right). \label{eq:mm}$$

• The number of monopole within the comoving volume is conserved thereafter. But they interact with the electron pairs before decouple around $T_d \simeq 0.5$ MeV, when the interaction rate becomes less than the expansion rate.

• Assuming the adiabatic expansion we have the relic monopole density

$$\Omega_{mo} \ h^2 = \frac{\rho_{mo,0} \ h^2}{\rho_{\rm c,0}} \simeq 1.2 \times 10^{-12} \ \left(\frac{M_m}{1 \ {\rm TeV}}\right)^2,$$

where $\rho_{c,0}$ is the critical density and h is the scaled Hubble parameter. But the actual monopole density could be much lower.

• When decoupled, the monopoles were non-relativistic. But the inter-galactic magnetic field makes them highly relativistic in the present universe.

B. Parker Bound

- The universe has inter-galactic magnetic field $B \simeq 1.2 \times 10^{-9} T$, and the monopole traveling through the magnetic field drains the energy from the magnetic field.
- Requiring that the monopoles do not drain too much energy to sustain the magnetic field, we have the Parker bound

$$\Omega_{mo}h^2 \lesssim 4.3 \times 10^{-17} \left(\frac{M_m}{1 \text{ TeV}}\right).$$

But our estimate of Ω_{mo} is too big, by the factor 10^5 .

• Notice, however, that not all the monopoles are free streaming.

1. As the heaviest stable particles in the universe, they can easily generate the density perturbation and become the seed of the primordial magnetic black holes (PMBH) which could explain the dark matter.

2. Exactly for the same reason they could become the seed of large scale structures of the universe. The strong radial magnetic field near the galactic center could be an evidence.

- The relativistic monopoles can penetrate less than 10 m in the Aluminum before trapped. So many of them could have been filtered out by the stellar objects.
- This strongly implies that most of the monopoles produced in the early universe may have been buried in the galactic centers or filtered out by the stellar objects.
- In fact, these buried/trapped monopoles could be the source of strong magnetic field near the galactic center and inter-galactic magnetic field.

C. Monopole Production Mechanism at LHC

- In high energy physics the monopoles are thought to be produced by the Drell-Yan and two photon production process described by Feynman diagrams.
- In this case the monopole production is induced by the intermediate photon, and the production rate is determined by the electromagnetic coupling constant.
- This monopole production mechanism, however, completely ignores the fact that the monopoles are produced by the change of topology.

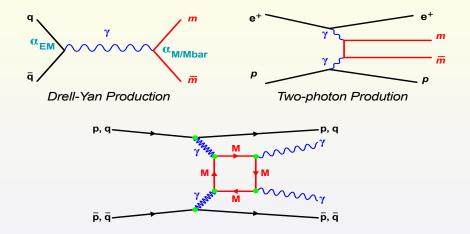


Figure: The conventional (non-topological) monopole production mechanism.

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- In contrast, our monopole production is made by the change of topology induced by the thermal fluctuation of the Higgs vacuum, which can not be described by any Feynman diagram or fundamental constant.
- In this view LHC should satisfy the following extra conditions to produce the monopole:

The fireball size should be bigger than the correlation length.
 The collision should last long enough for the Higgs vacuum to fluctuate.

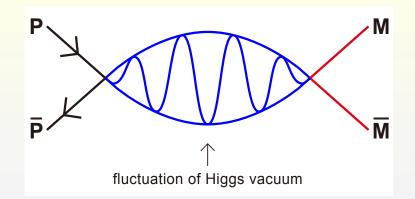


Figure: The topological monopole production mechanism by thermal fluctuation of Higgs vacuum which can not be expressed by Feynman diagram.

A. Experimental

- In spite of the huge efforts the search for the monopole has not been successful. Most were the blind searches in the dark room. Focusing on the electroweak monopole, we could detect the monopole.
- A major concern at LHC has been whether it can satisfy conditions to produce the monopole. It can, and MoEDAL has the best chance.
- The existing remnant monopole detectors (IceCube, ANTARES, Auger, etc.) have a serious trouble: The monopoles can hardly pass though the earth atmosphere and reach the detectors. So we need "Cosmic" MoEDAL installed in high mountains.

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B. Theoretical Challenges

- Can the monopoles indeed be the seed of the primordial black holes and large scale structures? Do they generate the inter-galactic magnetic field? Could they trigger the electroweak baryogenesis?
- How can we justify the perturbative expansion in the presence of monopole? Can we construct the quantum field theory of monopole which generalizes QED?
- If detected, it will open the new era in physics.

New Physics!

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