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An S_4 model inspired from Self-complementary neutrino mixing

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An S_4 model inspired from Self-complementary neutrino mixing

What is self-complementarity?

What is self-complementary neutrino mixing?

Why an S_4 model for self-complementary neutrino mixing?

The story begins in 2012

self-complementarity

Parameter	NH	IH
$\sin^2\theta_{12}/10^{-1}$	$3.07^{+0.18}_{-0.16}$	$3.07^{+0.18}_{-0.16}$
$\sin^2\theta_{23}/10^{-1}$	$3.98^{+0.30}_{-0.26}$	$4.08^{+0.35}_{-0.30}$
$\sin^2\theta_{13}/10^{-2}$	$2.45^{+0.34}_{-0.31}$	$2.46^{+0.34}_{-0.31}$

Fogli, et. al., 2012

$$\theta_{12}+\theta_{13}=\theta_{23}=45^\circ$$
$$\theta_{12}+\theta_{13}=\theta_{23}$$
$$\theta_{12}+\theta_{13}=45^\circ$$

Zheng & Ma, 2012; Zhang & Ma 2012

Self-complementarity (SC) is a correlation of lepton mixing angles. It is observed from the data, proposed as a remnant of some unknown theory.

	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.310^{+0.013}_{-0.012}$	$0.275 \to 0.350$
$\theta_{12}/^\circ$	$33.82^{+0.78}_{-0.76}$	$31.61 \to 36.27$
$\sin^2\theta_{23}$	$0.558^{+0.020}_{-0.033}$	$0.427 \to 0.609$
$\theta_{23}/^\circ$	$48.3^{+1.1}_{-1.9}$	$40.8 \to 51.3$
$\sin^2\theta_{13}$	$0.02241^{+0.00066}_{-0.00065}$	$0.02046 \to 0.02440$
$\theta_{13}/^\circ$	$8.61^{+0.13}_{-0.13}$	$8.22 \to 8.99$
$\delta_{CP}/^\circ$	222^{+38}_{-28}	$141 \to 370$

NuFIT 4.1 (2019)

SC is not exact, possible reasons include:
RGE, HO operators, …

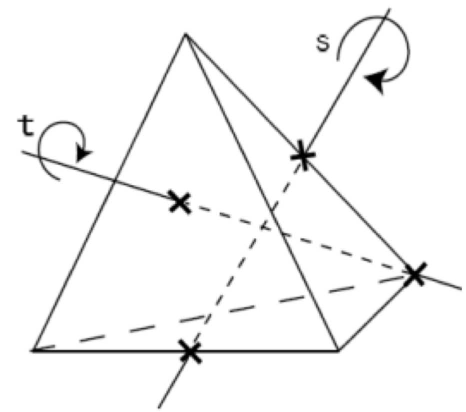
Stability: Singh et al, 2018; Haba et al, 2013
Sterile: Ke et al, 2014
Scheme-dependence: Zhang et al, 2012

Let's build a model for Self-complementarity!

Mixing is dictated by a symmetry!

Discrete flavor symmetry (DFS) model

assign three generations of leptons (quarks) to a triplet representation of a discrete flavor group -- subgroup of SO(3) or SU(3), the flavor sector exhibits a symmetry under this group



Ma & Rajasekaran, 2001; Babu, Ma, Valle, 2003; and many more

Pros of DFS

- definite meaning of generations, avoid Goldstone boson
- geometrical appealing & simple
- predictive
- embedded in other theoretical frameworks (SUSY-GUTs)
- extendable (seesaw, leptogenesis)

Cons of DFS

- flavons: number, vacuum alignments
- corrections from HO operators
- shaping symmetries needed

Modular symmetry?

the S_4 group

Contains permutations among four objects

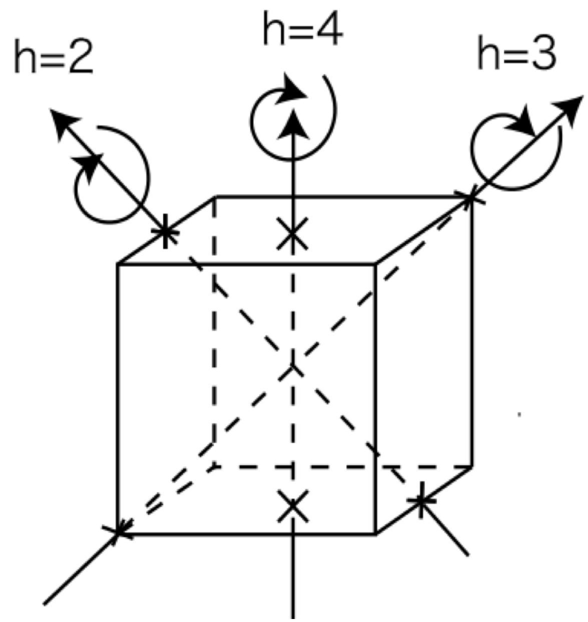
$$(x_1, x_3, x_2, x_4), \quad \rightarrow \quad (x_i, x_j, x_k, x_l)$$

five irreducible representations

$$1, 1', 2, 3, 3'$$

24 elements

$$S^2 = T^4 = (ST)^3 = (TS)^3 = 1$$



Many models on S_4:

TB: Lam, 2008; Bazzocchi et al., 2009; Ishimori 2010; Zhao, 2011;
BM: Altarelli et al., 2009; Merlo et al., 2010; Meloni 2011;
TM: King et al., 2011; Krishnan et al., 2013; Lavoura et al., 2013; Luhn, 2013;
CP: Feruglio et al., 2014; Ding et al., 2015
...

Self-complementary neutrino mixing

self-complementarity relation $\theta_{12} + \theta_{13} = \theta_{23} = 45^\circ$

maximal CP violation $\delta_{\rm CP} = -90^\circ$

$$U_{\rm SC} = \begin{pmatrix} \cos\left(\frac{\pi}{4} - \theta_{13}\right)\cos\theta_{13} & \sin\left(\frac{\pi}{4} - \theta_{13}\right)\cos\theta_{13} & i\sin\theta_{13} \\ \frac{i\cos\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13} - \sin\left(\frac{\pi}{4}-\theta_{13}\right)}{\sqrt{2}} & \frac{\cos\left(\frac{\pi}{4}-\theta_{13}\right) + i\sin\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13}}{\sqrt{2}} & \frac{\cos\theta_{13}}{\sqrt{2}} \\ \frac{\sin\left(\frac{\pi}{4}-\theta_{13}\right) + i\cos\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13}}{\sqrt{2}} & \frac{i\sin\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13} - \cos\left(\frac{\pi}{4}-\theta_{13}\right)}{\sqrt{2}} & \frac{\cos\theta_{13}}{\sqrt{2}} \end{pmatrix}$$

two observations:

- difficult to observe the mass matrix structure from direct construction
- $|U_{\mu i}| = |U_{\tau i}|$ gives a mass matrix of the form

$$m = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}$$

μ - τ symmetric

we actually only need

$$\theta_{23} = 45^\circ, \delta_{\rm CP} = \pm 90^\circ$$

to get the μ - τ symmetric mass matrix

Need to look more closely to see the mass matrix structure determined by Usc aim: to see the detailed structure … Expand it!

the perturbed SC mixing

set $\lambda = \sin\theta_{13} \simeq 0.15,$

$$
\begin{aligned}
U_{\rm SC} &\equiv U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + \dots \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & i \\ \frac{1}{2}+\frac{i}{2} & \frac{1}{2}+\frac{i}{2} & 0 \\ -\frac{1}{2}+\frac{i}{2} & -\frac{1}{2}+\frac{i}{2} & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{4}+\frac{i}{2} & -\frac{1}{4}-\frac{i}{2} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{4}+\frac{i}{2} & \frac{1}{4}-\frac{i}{2} & -\frac{1}{2\sqrt{2}} \end{pmatrix} \\
&+ \dots
\end{aligned}
$$

bimaximal mixing

$$
\begin{aligned}
\hat{m}_\nu &= U^*_{\rm SC}\hat{m}^d U^\dagger_{\rm SC} \\
&= (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + \dots)^* \hat{m}^d (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + \dots)^\dagger \\
&\equiv \hat{m}_0 + \lambda \hat{m}_1 + \lambda^2 \hat{m}_2 + \dots,
\end{aligned}
$$

- leading order bimaximal mixing -> S4 group
- $\mathcal{O}(\lambda^2)$ -> percent level accuracy

neutrino mass matrix constructed from the perturbed SC mixing

$$\begin{aligned}\hat{m}_\nu &= U_{\rm SC}^* \hat{m}^d U_{\rm SC}^\dagger \\ &= (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + \ldots)^* \hat{m}^d (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + \ldots)^\dagger \\ &\equiv \hat{m}_0 + \lambda \hat{m}_1 + \lambda^2 \hat{m}_2 + \ldots,\end{aligned}$$

$$\hat{m}_0 = \begin{pmatrix} \frac{1}{2}(m_1+m_2) & \frac{1}{2\sqrt{2}}(m_2-m_1) & \frac{1}{2\sqrt{2}}(m_1-m_2) \\ \frac{1}{2\sqrt{2}}(m_2-m_1) & \frac{1}{4}(m_1+m_2+2m_3) & \frac{1}{4}(-m_1-m_2+2m_3) \\ \frac{1}{2\sqrt{2}}(m_1-m_2) & \frac{1}{4}(-m_1-m_2+2m_3) & \frac{1}{4}(m_1+m_2+2m_3) \end{pmatrix}$$

$$\hat{m}_1 = a\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1+i & 1 \\ 0 & 1 & -1-i \end{pmatrix} + b\begin{pmatrix} 0 & i & i \\ i & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$a = \frac{1}{2}(m_1-m_2), \quad b = -\frac{1}{2\sqrt{2}}(m_1+m_2+2m_3),$$

$$\hat{m}_2 = c\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + d\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} + e\begin{pmatrix} 0 & i & i \\ i & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$

$$c = -\frac{1}{4}(m_1+m_2+2m_3), \quad d = \frac{5}{4\sqrt{2}}(m_1-m_2), \quad e = -\frac{1}{\sqrt{2}}(m_1-m_2)\ ,$$

To get this structure in a model!

the model

Table 2: Field representations in S_4 and charges under additional symmetries of the model

	L	e_R	μ_R	τ_R	N	ϕ_e	ϕ_μ	ϕ_τ	θ	ξ_1	ϕ_1	ψ_1	ϕ_{21}	ϕ_{22}	ϕ_{23}	ϕ_{31}	ϕ_{32}	ψ_3	ξ_3
S_4	3	1	1	1	3	3	3	3	1	1	3	2	3	3	3	3	3	2	1
U(1)	-x	z	m	n	x	$\frac{1}{2}(x-z)$	x-m	x-n	0	-2x	-2x	-2x	-x	-x	-x	$-\frac{2}{3}x$	$-\frac{2}{3}x$	y	-2x-2y
$U(1)_{FN}$	0	2	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
$\mathbb{Z}_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0

$$
\begin{aligned}
-\mathcal{L} = {} & y_e \bar{L}\tilde{H} e_{\mathrm{R}} \left(\frac{\phi_e}{\Lambda}\right)^2 \left(\frac{\theta}{\Lambda}\right)^2 + y_\mu \bar{L}\tilde{H}\mu_{\mathrm{R}} \left(\frac{\phi_\mu}{\Lambda}\right)\left(\frac{\theta}{\Lambda}\right) + y_\tau \bar{L}\tilde{H}\tau_{\mathrm{R}}\left(\frac{\phi_\tau}{\Lambda}\right) + y_\nu \bar{L} H N \\
& + \frac{y_{11}}{\Lambda}(NN)_1\xi_1 + \frac{y_{12}}{\Lambda}(NN)_2\psi_1 + \frac{y_{13}}{\Lambda}(NN)_3\phi_1 \\
& + \frac{y_{21}}{\Lambda^2}(N\phi_{21})_3(N\phi_{21})_3 + \frac{y_{22}}{\Lambda^2}(NN)_3(\phi_{22}\phi_{22})_3 + \frac{y_{23}}{\Lambda^2}(NN)_3(\phi_{23}\phi_{23})_3 \\
& + \frac{y_{31}}{\Lambda^3}(N\psi_3)_3(N\psi_3)_3\xi_3 + \frac{y_{32}}{\Lambda^3}\left((NN)_3\phi_{31}\right)_1(\phi_{31}\phi_{31})_1 + \frac{y_{33}}{\Lambda^3}\left((NN)_3\phi_{32}\right)_1(\phi_{32}\phi_{32})_1,
\end{aligned}
$$

no high order corrections up to dim 10 operators

the charged lepton sector

$$\langle\phi_e\rangle = v_{\phi_e}(0,0,1), \quad \langle\phi_\mu\rangle = v_{\phi_\mu}(0,0,1), \quad \langle\phi_\tau\rangle = v_{\phi_\tau}(0,1,0)$$

$$\hat{m}_l = \begin{pmatrix} \frac{y_e}{\Lambda^4} v v_\theta^2 v_{\phi_e}^2 (b^2 - a^2) & 0 & 0 \\ 0 & \frac{y_\mu}{\Lambda^2} v v_\theta v_{\phi_\mu} & 0 \\ 0 & 0 & \frac{y_\tau}{\Lambda} v v_{\phi_\tau} \end{pmatrix}$$

assume $y_e \sim y_\mu \sim y_\tau$, $v_{\phi_e} \sim v_{\phi_\mu} \sim v_{\phi_\tau} \equiv v_{\phi_l}$ with $\frac{m_e}{m_\mu}|_{expt} \simeq 0.005, \quad \frac{m_\mu}{m_\tau}|_{expt} \simeq 0.06$

we get $\frac{v_{\phi_l}}{\Lambda} \sim 0.08, \quad \frac{v_\theta}{\Lambda} \sim 0.06$

Altarelli et al., 2009

- diagonal even with HO operator;
- separated from the neutrino sector;
- U(1)FN → hierarchical mass.

We will focus on neutrino sector from now on.

the neutrino sector

$$\hat{m}_{\rm D} = y_\nu v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{m}_{\rm LO} = \frac{y_{11}}{\Lambda} v_{\xi_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y_{12}}{\Lambda} v_{\psi_1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{6} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{6} \end{pmatrix} \frac{y_{13}}{\Lambda} v_{\phi_1} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\hat{m}_{\rm NLO} = \frac{y_{21}}{\Lambda^2} v^2_{\phi_{21}} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{y_{22}}{\Lambda^2} v^2_{\phi_{22}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{y_{23}}{\Lambda^2} v^2_{\phi_{23}} \begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$\hat{m}_{\rm NNLO} = \frac{y_{31}}{\Lambda^3} v^2_{\psi_3} v_{\xi_3} \begin{pmatrix} 3 & 0 & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} \end{pmatrix} + \frac{y_{32}}{\Lambda^3} v^3_{\phi_{31}} \begin{pmatrix} 0 & -2 & 2 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} + \frac{y_{33}}{\Lambda^3} v^3_{\phi_{32}} \begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}$$

$$\hat{m}_{\nu_{\rm model}} = \hat{m}_{\rm D} (\hat{m}_{\rm LO} + \hat{m}_{\rm NLO} + \hat{m}_{\rm NNLO} + \ldots)^{-1} \hat{m}_{\rm D}^T \equiv \hat{m}_{\rm D} \hat{m}_{\rm R}^{-1} \hat{m}_{\rm D}^T$$

compared with the desired structure, we are left with five independent real parameters:

$\tilde{v}_{\xi_1}, \tilde{v}_{\psi_1}, \gamma, \tilde{v}_{\phi_1}, \rho$

flavor symmetry breaking

all the invariant terms allowed by the model

$$(\xi^\dagger\xi)_1,\quad (\psi^{a\dagger}\psi^a)_1,\quad (\phi^{a\dagger}\phi^b)_1,$$
$$(\xi^\dagger\xi\xi^\dagger\xi)_1,\quad (\xi^\dagger\xi)_1(\psi^{a\dagger}\psi^a)_1,\quad (\xi^\dagger\xi)_1(\phi^{a\dagger}\phi^b)_1,\quad (\psi^{a\dagger}\psi^a)_1(\psi^{b\dagger}\psi^b)_1,\quad (\psi^{a\dagger}\psi^a)_2(\psi^{b\dagger}\psi^b)_2,$$
$$(\psi^{a\dagger}\psi^a)_1(\phi^{b\dagger}\phi^c)_1,\quad (\psi^{a\dagger}\psi^a)_2(\phi^{b\dagger}\phi^c)_2,$$
$$(\phi^{a\dagger}\phi^b)_1(\phi^{a\dagger}\phi^c)_1,\quad (\phi^{a\dagger}\phi^b)_2(\phi^{a\dagger}\phi^c)_2,\quad (\phi^{a\dagger}\phi^b)_3(\phi^{a\dagger}\phi^c)_3,\quad (\phi^{a\dagger}\phi^b)_{3'}(\phi^{a\dagger}\phi^c)_{3'}.$$

the most general scalar potential contains all the invariant terms for each flavon and for all their possible mixing

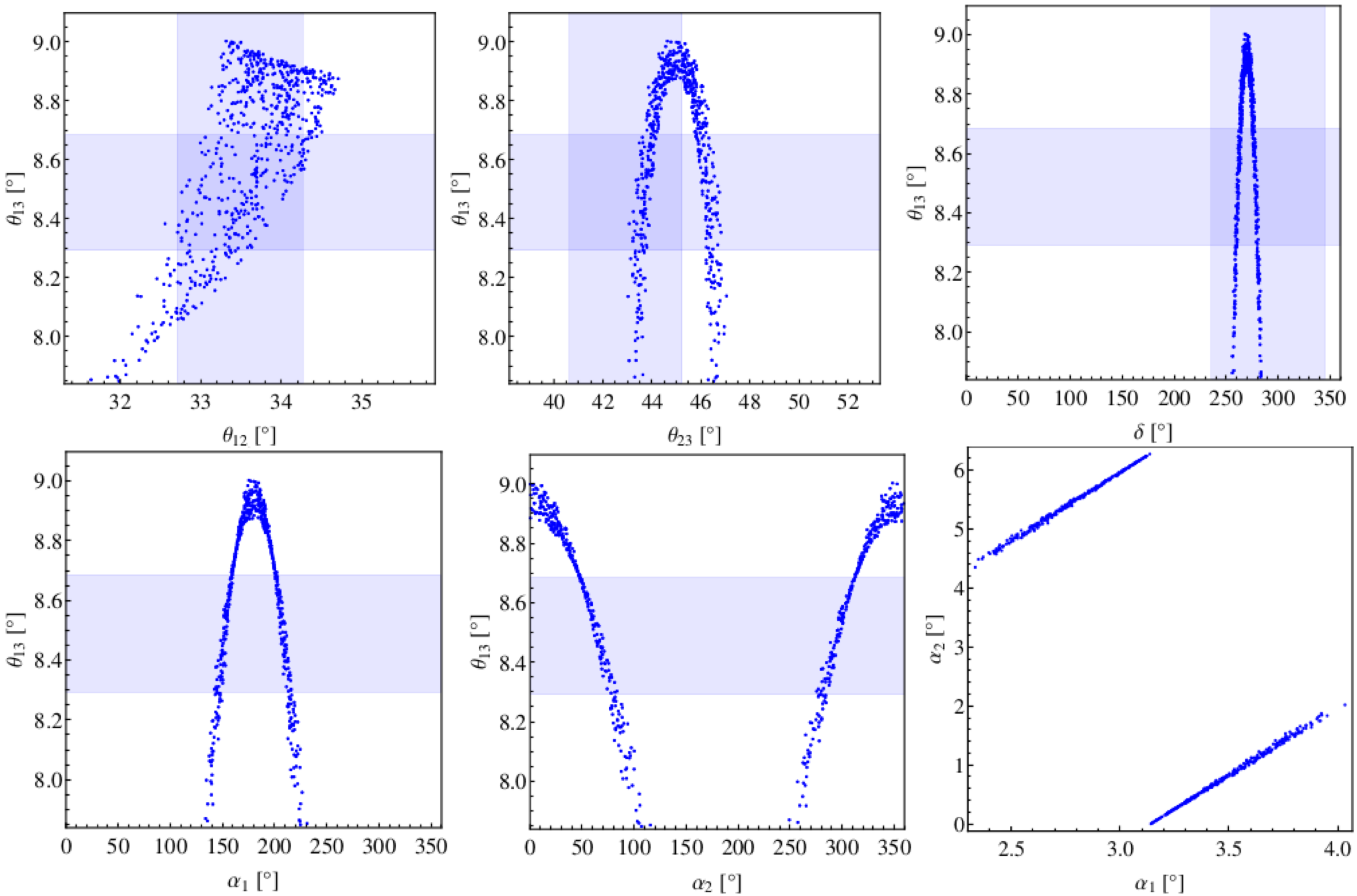
very demanding!

$$\begin{aligned}
V \supset\ & c_1\left((\psi_1^a)^2+(\psi_2^a)^2\right)+c_2\left((\psi_1^b)^2+(\psi_2^b)^2\right) \\
&+c_3\left((\psi_1^a)^2+(\psi_2^a)^2\right)^2+c_4\left(4(\psi_1^a)^2(\psi_2^a)^2+\left(-(\psi_1^a)^2+(\psi_2^a)^2\right)^2\right) \\
&+c_5\left((\psi_1^b)^2+(\psi_2^b)^2\right)^2+c_6\left(4(\psi_1^b)^2(\psi_2^b)^2+\left(-(\psi_1^b)^2+(\psi_2^b)^2\right)^2\right) \\
&+c_7\left((\psi_1^a)^2+(\psi_2^a)^2\right)\left((\psi_1^b)^2+(\psi_2^b)^2\right),
\end{aligned}$$

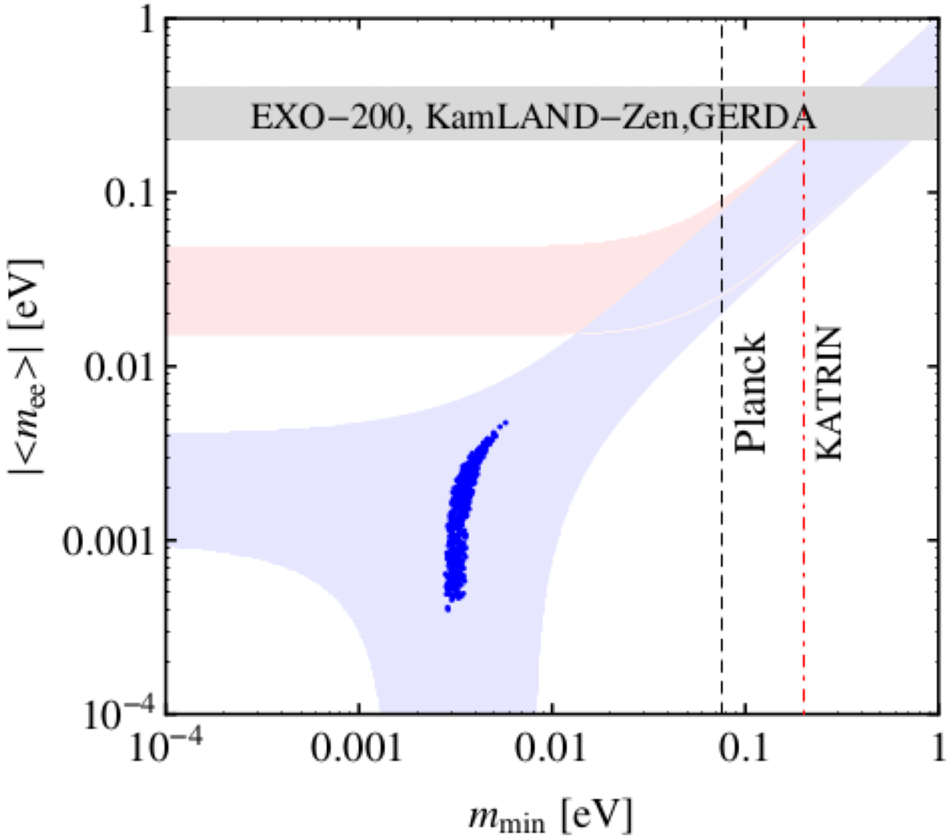
$$\begin{aligned}
c_1+20c_3+20c_4+4c_7&=0, \\
c_2+8c_5+8c_6+10c_7&=0, \\
c_5+c_6&>0, \\
c_3+c_4&>0.
\end{aligned}$$

$$\psi^a\sim(3,1),\quad \psi^b\sim(-\sqrt{3},1)$$

numerical results



compatible with low energy observables



$$J_{CP} = \frac{1}{8}\cos\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\sin\delta \in [0.029, 0.035]$$

$$m_\beta = \sqrt{m_1^2 c_{12}^2 c_{13}^2 + m_2^2 s_{12}^2 c_{13}^2 + m_3^2 s_{13}^2} \in [0.009, 0.010]\ \text{eV}$$

$$|\langle m_{ee}\rangle| = |m_1 c_{12}^2 c_{13}^2 e^{-i\alpha_1} + m_2 s_{12}^2 c_{13}^2 e^{-i\alpha_2} + m_3 s_{13}^2 e^{-2i\delta}| \in [0.0004, 0.006]\ \text{eV}$$

summary

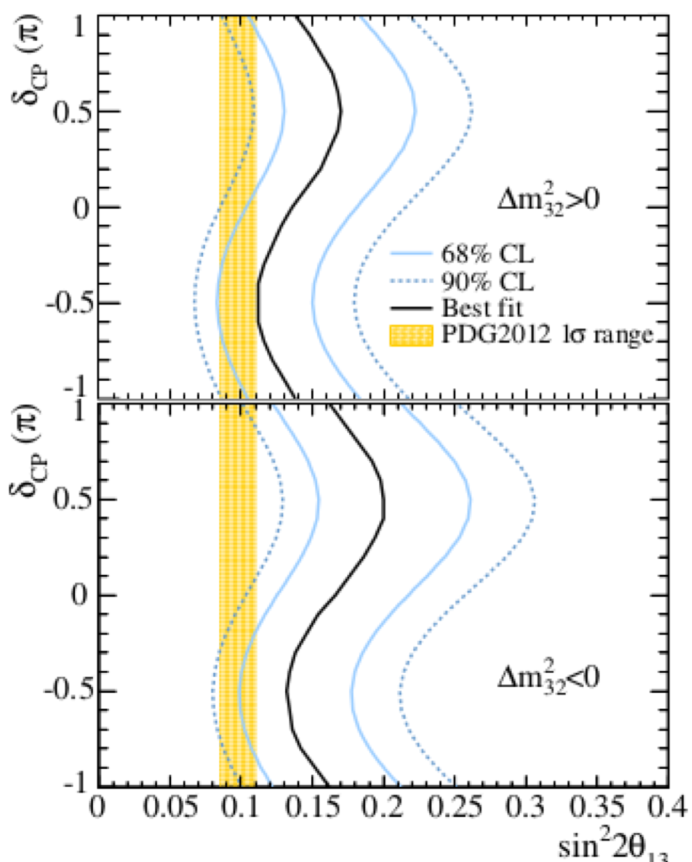
1. The SC mixing has to be perturbatively realized in a model
2. From the perturbed SC mixing we construct the mass matrix
3. We build an S4 flavor model to reproduce the structure of the Majorana mass matrix at the high energy
4. After evolving down to low energy, we find parameter space that is compatible with all the low energy observables only in the case of normal ordering
5. The model also gives predictions on the not-yet observed quantities.

Thank you !

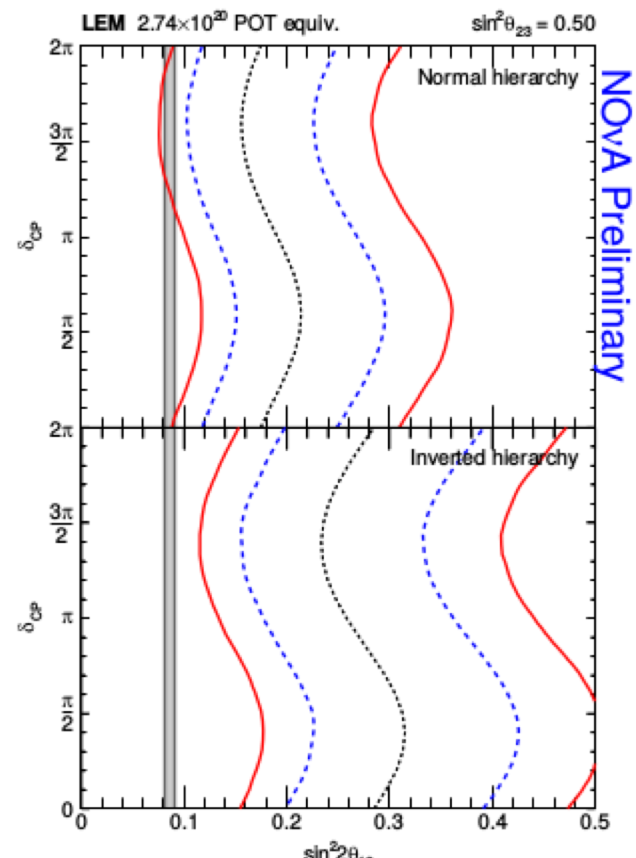
a new mixing pattern?

	bfp$\pm 1\sigma$	3σ range
θ_{12} [°]	$33.48^{+0.78}_{-0.75}$	$31.29 - 35.91$
θ_{23} [°]	$42.3^{+3.0}_{-1.6}$ (N), $49.5^{+1.5}_{-2.2}$ (I)	$38.2 - 53.3$ (N), $38.6 - 53.3$ (I)
θ_{13} [°]	$8.50^{+0.20}_{-0.21}$ (N), $8.21^{+0.20}_{-0.21}$ (I)	$7.85 - 9.10$ (N), $7.87 - 9.11$ (I)
δ_{CP} [°]	306^{+39}_{-70} (N), 254^{+63}_{-62} (I)	$0 - 360$

$\theta_{12} + \theta_{13} \sim \theta_{23} \sim 45°$



T2K, 2013



NOvA, 2015

$\delta_{CP} \sim -90°$

neutrino model building

things we know

- massive
- 2 squared mass differences

- 3 mixing angles

things we don't know

- magnitude of absolute mass
- mass ordering
- why so light

- why so large mixing
- whether there is CP violation

we are unware of the origin of flavor mixing and mass, and more basically, we do not know neutrinos are Dirac particles or Majorana particles

neutrino mass models

explain the smallness of neutrino mass, e.g., the seesaw models, FN mechanism

neutrino mixing models

explain neutrino mixing parameters, e.g., flavor symmetry models, anarchy models

"Large mixing angles would result from a random, structureless matrix,"

Hall, Murayama, Weiner

flavor symmetry

continuous or discrete?

continuous: free rotations among generations

discrete: definite meaning of generations, avoid Goldstone boson from spontaneously symmetry breaking

non-Abelian discrete flavor symmetry

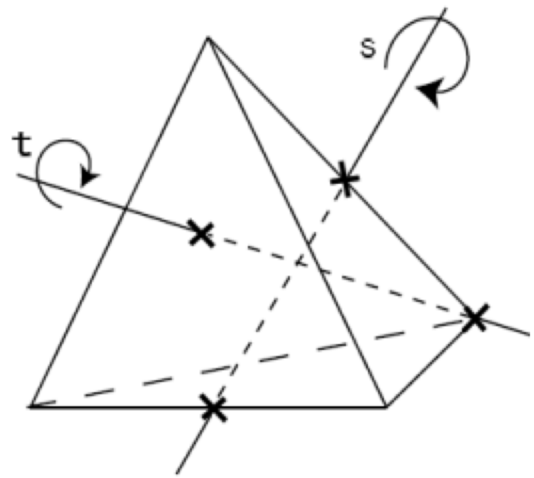
Abelian or non-Abelian?

Abelian: discriminate generations

non-Abelian: connect different generations

what is a discrete flavor symmetry?

assign three generations of leptons (quarks) to a triplet representation of a discrete flavor group -- subgroup of SO(3) or SU(3), the flavor sector exhibits a symmetry under this group



Ma & Rajasekaran, 2001; Babu, Ma, Valle, 2003 largely inspired from the special mixing pattern

Harrison, Perkins, Scott 2002

tribimaximal mixing

$$U = \begin{matrix} \\ e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\nu_2 = \frac{1}{\sqrt{3}}(\nu_e + \nu_\mu + \nu_\tau)$$
$$\nu_3 = \frac{1}{\sqrt{2}}(-\nu_\mu + \nu_\tau)$$

facts: residual symmetry

$$S_\nu^T M_\nu S_\nu = M_\nu ,$$
$$T M_\ell M_\ell^\dagger T^\dagger = M_\ell M_\ell^\dagger .$$

$$G_f \to \text{breaking} \to \begin{cases} G_\nu \\ G_l \end{cases}$$

why discrete flavor symmetry?

- geometrical appealing & simple
- predictive
- embedded in other theoretical frameworks (SUSY-GUTs)
- extendable (seesaw, leptogenesis)

flavor symmetry breaking: spontaneously - scalar potential,
explicitly - boundary conditions in extra dimension theories
origin of flavor symmetry: continuous group, orbifolds, string, brane, ...

for reviews, see: Altarelli & Feruglio, 2010; Morisi & Valle, 2013; King & Luhn, 2013 neutrino mixing parameters after Daya Bay & RENO

Parameter	NH	IH
$\sin^2\theta_{12}/10^{-1}$	$3.07^{+0.18}_{-0.16}$	$3.07^{+0.18}_{-0.16}$
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Fogli, et. al., 2012

there are in general three ways to account for the non-zero reactor angle:

•corrections: high order operators, charged lepton sectors, renormalization group running, ..

•new mixing pattern: e.g. tri-bimaximal-Cabibbo mixing

•new symmetry: e.g. Generalized CP symmetry

$$s_{13} = \frac{\lambda}{\sqrt{2}}, \qquad s_{12} = \frac{1}{\sqrt{3}}, \qquad s_{23} = \frac{1}{\sqrt{2}}$$

$$U_{\mathrm{TBC}} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{3}}(1-\frac{1}{4}\lambda^2) & \frac{1}{\sqrt{2}}\lambda e^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+\lambda e^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \\ \frac{1}{\sqrt{6}}(1-\lambda e^{i\delta}) & -\frac{1}{\sqrt{3}}(1+\frac{1}{2}\lambda e^{i\delta}) & \frac{1}{\sqrt{2}}(1-\frac{1}{4}\lambda^2) \end{pmatrix} P_{12}$$

King, 2012

$$\nu_L(x) \xrightarrow{CP} iX_\nu \gamma^0 C \bar{\nu}_L^T(x_P)$$

$$X_{\mathbf{r}}\rho_{\mathbf{r}}^*(g)X_{\mathbf{r}}^\dagger = \rho_{\mathbf{r}}(g'), \quad g, g' \in G_f$$

Holthausen, Lindner, Schmidt, 2012; Feruglio, Hagedorn, Ziegler, 2013; Ding, King, 2013; ...