# Predictions for the neutrino parameters in the minimal model extended by general lepton flavor-dependent U(1) gauge symmetries

Kento Asai (Univ. of Tokyo)

NDM-2020

Jan. 13, 2020 @ Hurghada

Based on K.A., Eur. Phys. J. **CXX** (2020), in press
arXiv : 1907.04042 [hep-ph]

# Extension of Standard Model

## Matter contents

SM fields + 3 right-handed neutrinos

$$N_\alpha$$

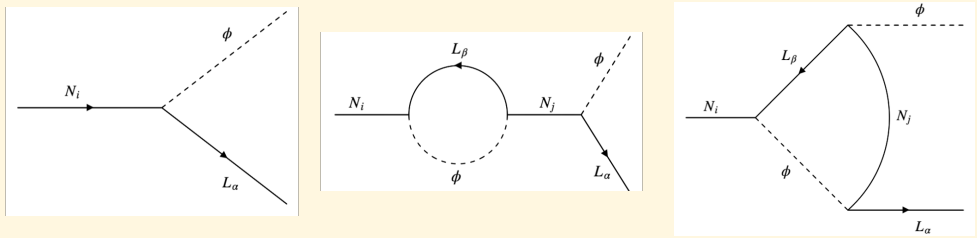
- Neutrino mass

$$\mathcal{L} \supset \lambda_\alpha N_\alpha^c \left(L_\alpha \cdot H\right) - \frac{1}{2} M_{\alpha\beta} N_\alpha^c N_\beta^c$$

Seesaw mechanism $(M_D \ll \mathcal{M}_R)$

$$\mathcal{M}_\nu \simeq -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

- Leptogenesis



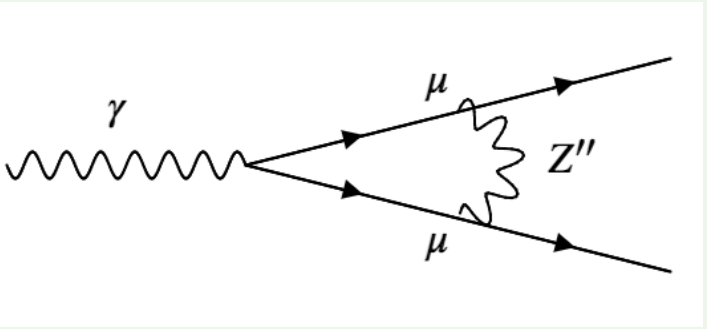

Some motivations

## Extension of Standard Model

Gauge sector

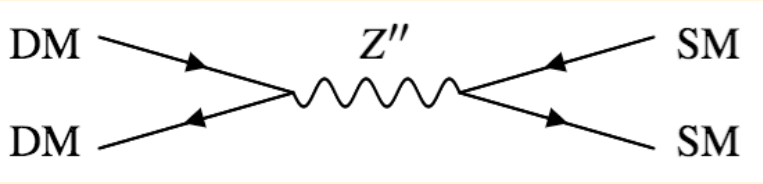
$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{L_\mu - L_\tau}$$

● Muon g-2




S. Baek, N. G. Deshpande, X. G. He, and P. Ko (2001); E. Ma, D. P. Roy, and S. Roy (2002); etc…

● Dark Matter




J.-C. Park, J. Kim, and S. C. Park (2016); S. Baek (2016); etc…

Some motivations

## Neutrino Mass Matrix Structure

**Matter contents**

SM fields + 3 right-handed neutrinos $N_\alpha$

**Gauge sector**

$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{L_\mu - L_\tau}$

Seesaw mechanism

$$\mathcal{M}_\nu \simeq -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

For $\mathrm{U}(1)_{L_\mu - L_\tau}$

$$\begin{pmatrix} * & & \\ & & * \\ & * & \end{pmatrix} \simeq - \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix} \begin{pmatrix} * & & \\ & & * \\ & * & \end{pmatrix} \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

$\mathcal{M}_\nu$ Block diagonal

⇒ ~~Neutrino mixing~~

# Neutrino Mass Matrix Structure

## Matter contents

SM fields + 3 right-handed neutrinos + U(1) breaking scalar

$N_\alpha$ $\sigma$ (singlet) or $\Phi$ (doublet)

Seesaw mechanism

$$\mathcal{M}_\nu^{-1} \simeq -(\mathcal{M}_D^T)^{-1}\mathcal{M}_R\mathcal{M}_D^{-1}$$

For $U(1)_{L_\mu - L_\tau}$

$$\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \simeq - \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix} \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$$

Neutrino mixing

&

two-zero minor structure

Places of vanishing component depend on $U(1)_{L_\alpha - L_\beta}$

# Neutrino Mass Matrix Structure

KA, K.Hamaguchi and N. Nagata, Eur. Phys. J. C77 (2017)

## Matter contents

SM fields + 3 right-handed neutrinos + U(1) breaking scalar

$N_\alpha$ $\sigma$ (singlet) or $\Phi$ (doublet)

Seesaw mechanism

$$\mathcal{M}_\nu^{-1} \simeq -(\mathcal{M}_D^T)^{-1}\mathcal{M}_R\mathcal{M}_D^{-1}$$

For $U(1)_{L_\mu - L_\tau}$

$$\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \simeq - \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix} \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix} = U_{\rm PMNS}\,\mathrm{diag}\left(m_1^{-1}, m_2^{-1}, m_3^{-1}\right) U_{\rm PMNS}^T$$

**Point**

two-zero minor structure

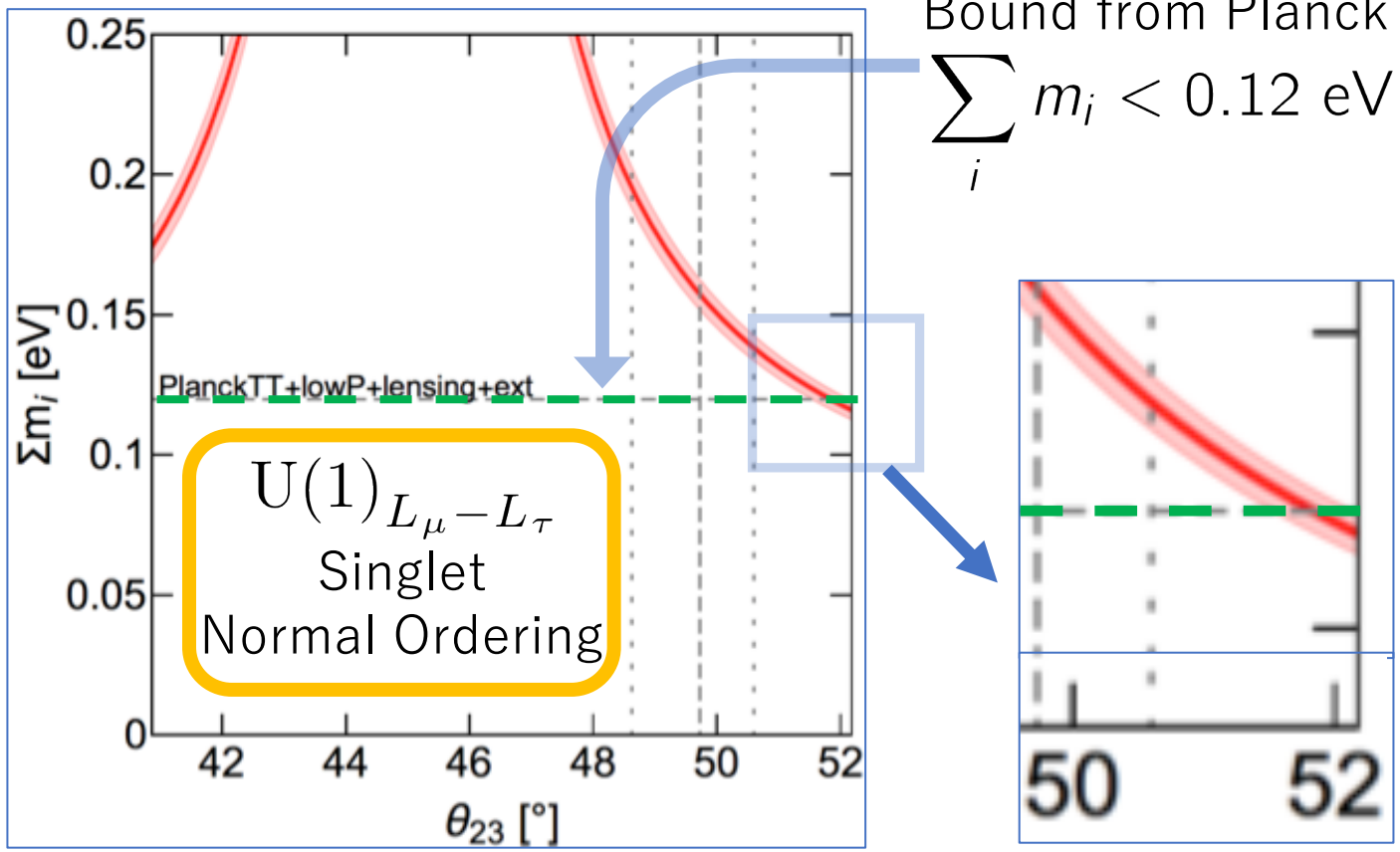
$$m_1, \underline{\delta}, \underline{\alpha_2, \alpha_3} = f(\underline{\theta_{12}, \theta_{13}, \theta_{23}}, \underline{\delta m^2, \Delta m^2})$$

Dirac CP, Majorana CP, Mixing angle, Squared mass difference

# Past work

KA, K.Hamaguchi, N. Nagata, Shih-Yen Tseng, and K. Tsumura, Phys. Rev. D99 (2019)

## Sum of neutrino masses




Bound from Planck

$$\sum_i m_i < 0.12 \text{ eV}$$

$\mathrm{U}(1)_{L_\mu - L_\tau}$
Singlet
Normal Ordering

## Past Works

Minimal gauged $\mathrm{U}(1)_{L_\alpha - L_\beta}$ models are driven into a corner

## This Work

Extension of gauge symm.
from $\mathrm{U}(1)_{L_\alpha - L_\beta}$
to $\mathrm{U}(1)_{B - x_e L_e - x_\mu L_\mu - x_\tau L_\tau}$

# Outline



| | |
|---|---|
| 1, Introduction | |
| 2, Model | Matter contents & charge assignment, Lagrangian |
| 3, Mass Matrix | Analysis : two-zero minor condition<br>Result : Dirac CP phase $\delta$ and sum of neutrino masses $\Sigma_i m_i$ |
| 4, Leptogenesis | Analysis : seesaw formula<br>Result : sign of the asymmetry parameter |
| 5, Conclusion | |

# Model

## Minimal Gauged $U(1)_{Y'}$ Models

**Singlet $\sigma$**

Charge assignment

| field | $U(1)_{B+L_e-3L_\mu-L_\tau}$ |
|---|---|
| quark | $+1/3$ |
| $e_{L,R}, \nu_e, N_e$ | $+1$ |
| $\mu_{L,R}, \nu_\mu, N_\mu$ | $-3$ |
| $\tau_{L,R}, \nu_\tau, N_\tau$ | $-1$ |
| $\sigma$ | $+2$ |
| others | $0$ |

**Doublet $\Phi_1$**

$$U(1)_{B-x_e L_e-x_\mu L_\mu-x_\tau L_\tau}$$

⇒ No U(1) realize two-zero structure

Lagrangian

$$\begin{aligned}\mathcal{L} \supset & -y_e(H^\dagger L_e)e_R^c - y_\mu(H^\dagger L_\mu)\mu_R^c - y_\tau(H^\dagger L_\tau)\tau_R^c \\ & - \lambda_e(H\cdot L_e)N_e^c - \lambda_\mu(H\cdot L_\mu)N_\mu^c - \lambda_\tau(H\cdot L_\tau)N_\tau^c \\ & - M_{e\tau}N_e^cN_\tau^c - \frac{1}{2}\lambda_{ee}\sigma N_e^cN_e^c - \lambda_{e\mu}\sigma^* N_e^cN_\mu^c - \frac{1}{2}\lambda_{\tau\tau}\sigma^* N_\tau^cN_\tau^c \\ & + h.c.\end{aligned}$$

# Model

## Minimal Gauged $U(1)_{Y'}$ Models

### Singlet $\sigma$

Mass matrix

・Dirac mass

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} \lambda_e & 0 & 0 \\ 0 & \lambda_\mu & 0 \\ 0 & 0 & \lambda_\tau \end{pmatrix}$$

・Majorana mass

$$\mathcal{M}_R = \begin{pmatrix} \lambda_{ee}\langle\sigma\rangle & \lambda_{e\mu}\langle\sigma\rangle & M_{e\tau} \\ \lambda_{e\mu}\langle\sigma\rangle & 0 & 0 \\ M_{e\tau} & 0 & \lambda_{\tau\tau}\langle\sigma\rangle \end{pmatrix}$$

### Doublet $\Phi_1$

$$U(1)_{B-x_e L_e - x_\mu L_\mu - x_\tau L_\tau}$$

→ No U(1) realize two-zero structure

・Charged lepton mass

$$\mathcal{M}_l = \frac{v}{\sqrt{2}} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$

# Mass matrix

## Two-zero minor condition

Singlet $\sigma$

Seesaw mechanism

$$\mathcal{M}_\nu^{-1} \simeq -(\mathcal{M}_D^T)^{-1}\mathcal{M}_R\mathcal{M}_D^{-1}$$

PMNS matirx

$$U_{\rm PMNS}^T\mathcal{M}_\nu U_{\rm PMNS} = {\rm diag}(m_1, m_2, m_3)$$

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix} = \mathcal{M}_\nu^{-1}$$

$$= U_{\rm PMNS}{\rm diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})U_{\rm PMNS}^T$$

Two complex equations in terms of $m_1, \delta, \alpha_2, \alpha_3$

# Mass matrix

## 4 Models

### Past Works

KA, K.Hamaguchi and N. Nagata, Eur. Phys. J. C77 (2017)

KA, K.Hamaguchi, N. Nagata, Shih-Yen Tseng, and K. Tsumura, Phys. Rev. D99 (2019)

### This Work

K.A., Eur. Phys. J. **CXX** (2020), in press

| $Y'$ | structural pattern: singlet | structural pattern: doublet | |
|---|---|---|---|
| $L_e - L_\mu$ | $\mathbf{E}_1^{\mathbf{R}}(+1)$ | $\mathbf{A}_2^{\nu}(+1)$ | $\mathbf{D}_1^{\nu}(-1)$ |
| $L_\mu - L_\tau$ | $\mathbf{C}^{\mathbf{R}}(+1)$ | $\mathbf{B}_3^{\nu}(+1)$ | $\mathbf{B}_4^{\nu}(-1)$ |
| $L_e - L_\tau$ | $\mathbf{E}_2^{\mathbf{R}}(+1)$ | $\mathbf{A}_1^{\nu}(+1)$ | $\mathbf{D}_2^{\nu}(-1)$ |
| $B - 3L_e - L_\mu + L_\tau$ | $\mathbf{A}_1^{\mathbf{R}}(+2)$ | | – |
| $B - 3L_e + L_\mu - L_\tau$ | $\mathbf{A}_2^{\mathbf{R}}(+2)$ | | – |
| $B - L_e - 3L_\mu + L_\tau$ | $\mathbf{B}_3^{\mathbf{R}}(+2)$ | | – |
| $B - L_e + L_\mu - 3L_\tau$ | $\mathbf{B}_4^{\mathbf{R}}(+2)$ | | – |
| $B + L_e - 3L_\mu - L_\tau$ | $\mathbf{D}_1^{\mathbf{R}}(+2)$ | | – |
| $B + L_e - L_\mu - 3L_\tau$ | $\mathbf{D}_2^{\mathbf{R}}(+2)$ | | – |
| $B - 3L_e$ | $\mathbf{F}_1^{\mathbf{R}}$ $(+6)$ | | – |
| $B - \frac{3}{2}L_\mu - \frac{3}{2}L_\tau$ | $(+3)$ | | |
| $B - 3L_\mu$ | $\mathbf{F}_2^{\mathbf{R}}$ $(+6)$ | | – |
| $B - \frac{3}{2}L_e - \frac{3}{2}L_\tau$ | $(+3)$ | | |
| $B - 3L_\tau$ | $\mathbf{F}_3^{\mathbf{R}}$ $(+6)$ | | – |
| $B - \frac{3}{2}L_e - \frac{3}{2}L_\mu$ | $(+3)$ | | |

Normal / Inverted

Two complex equations have no solution

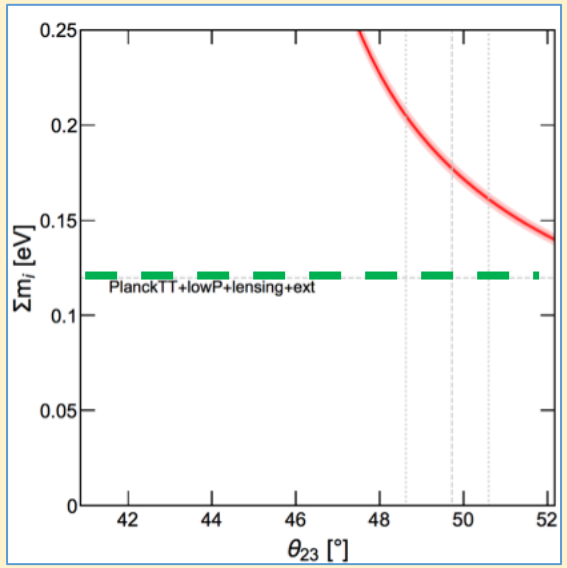
# Mass matrix

## Sum of neutrino masses

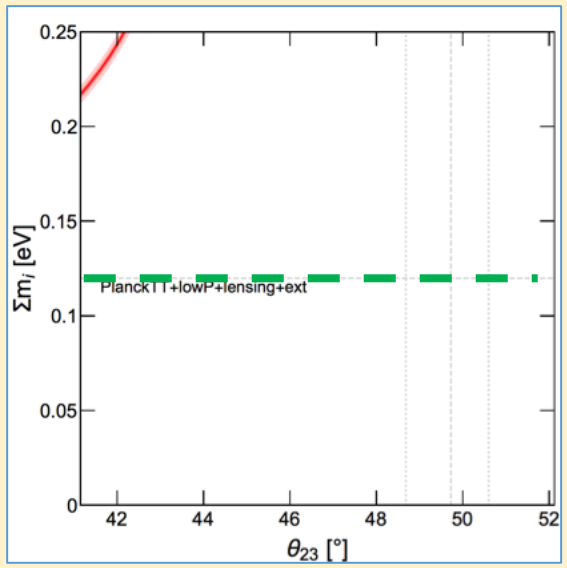
Planck limit
$$\sum m_i < 0.12 \text{ eV}$$

$B - L_e - 3L_\mu + L_\tau [\mathbf{B_3^R}]$

Normal

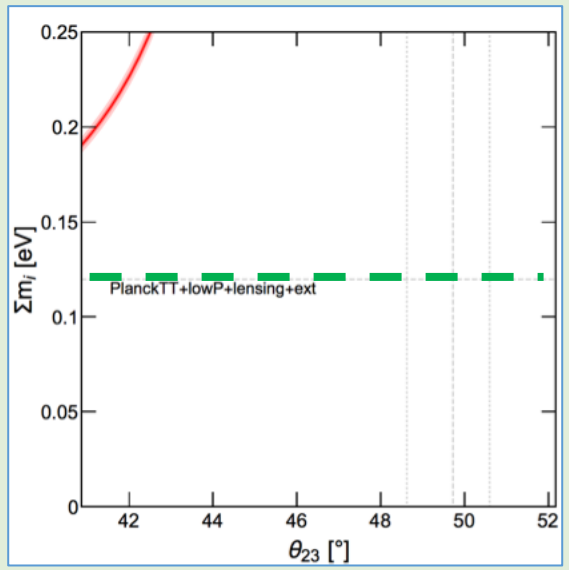



Inverted

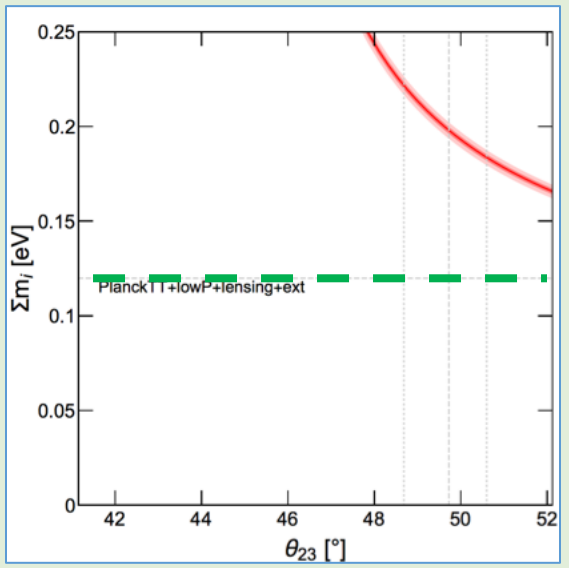



$B - L_e + L_\mu - 3L_\tau [\mathbf{B_4^R}]$

Normal




Inverted



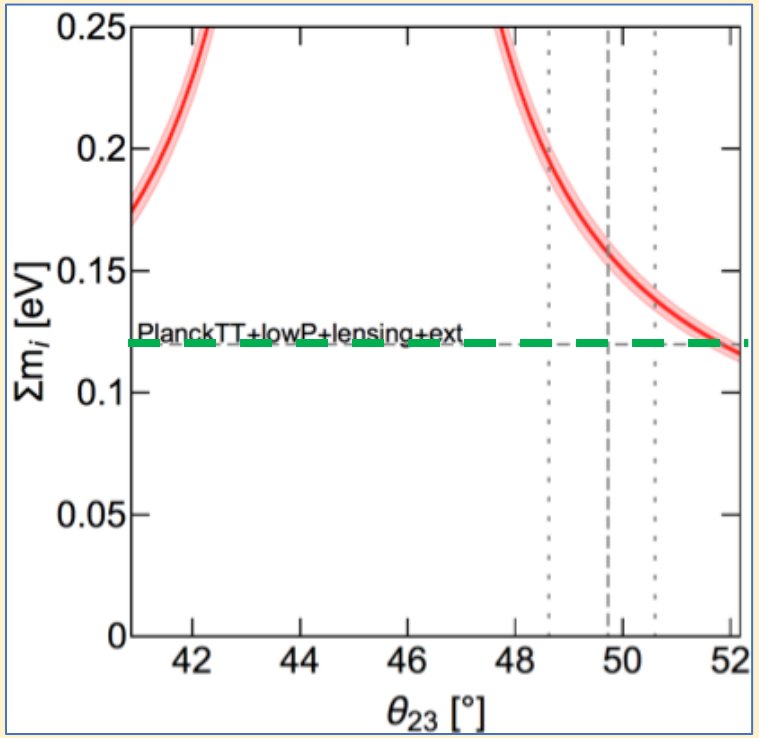

These cases are excluded by Planck limit

# Mass matrix

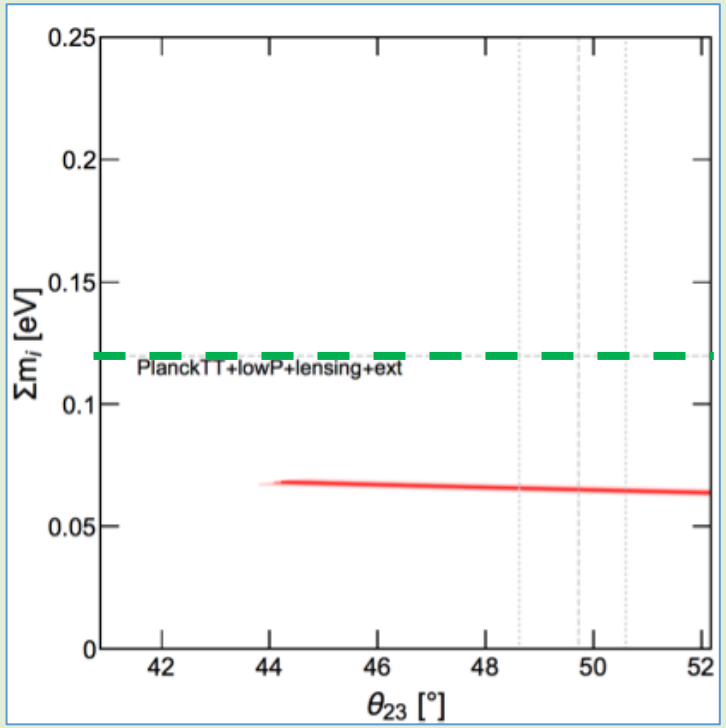
## Sum of neutrino masses

Planck limit
$$\sum m_i < 0.12 \text{ eV}$$

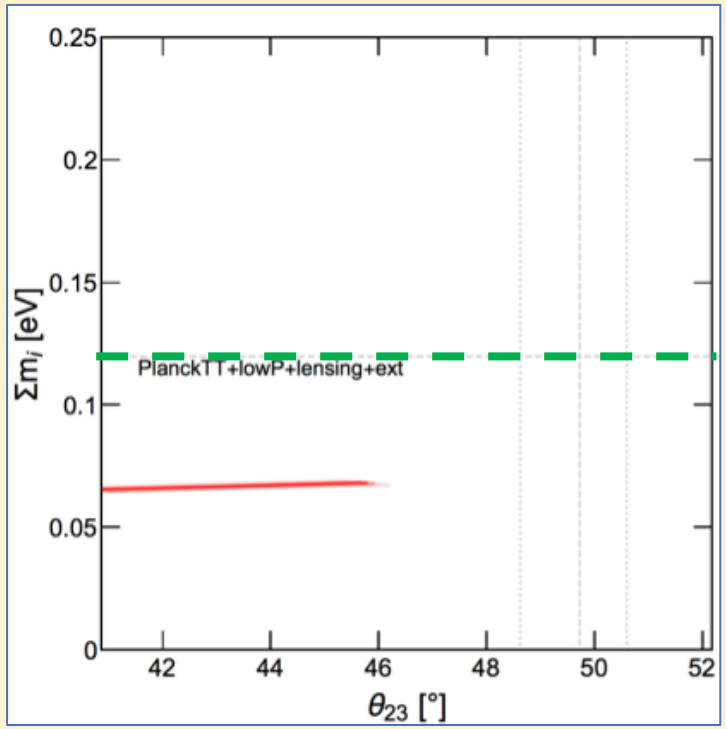
$L_\mu - L_\tau[\mathbf{C^R}]$




$B + L_e - 3L_\mu - L_\tau[\mathbf{D_1^R}]$




$B + L_e - L_\mu - 3L_\tau[\mathbf{D_2^R}]$




Past Work

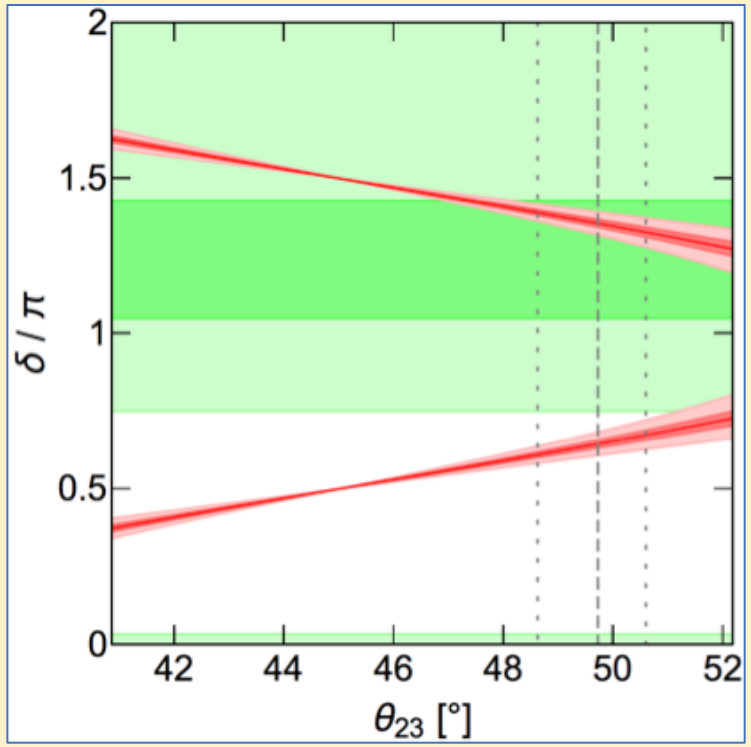
This Work

# Mass matrix

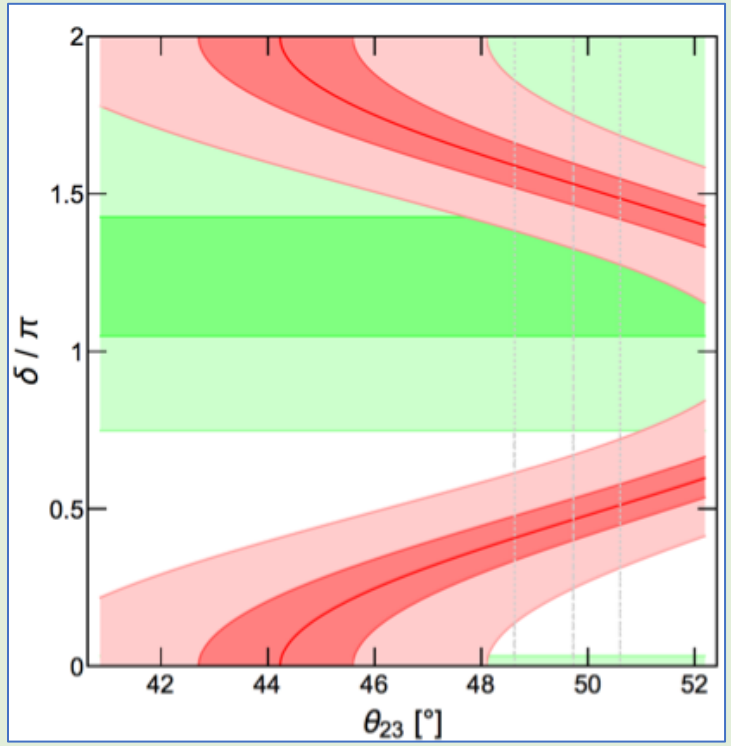
## Dirac CP phase

dark (light) green band
: 1 $\sigma$ (3 $\sigma$) favored region of $\delta$

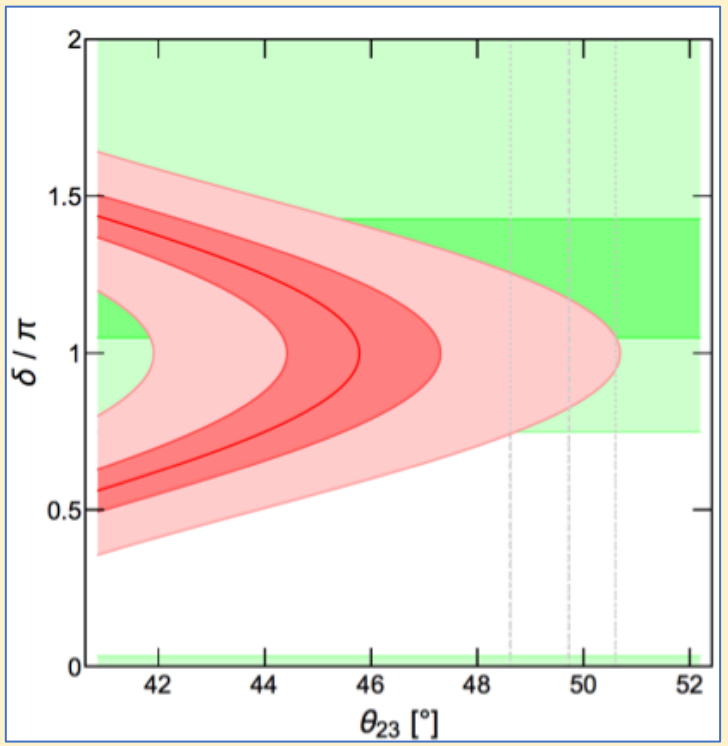
$L_\mu - L_\tau[\mathbf{C^R}]$




$B + L_e - 3L_\mu - L_\tau[\mathbf{D_1^R}]$




$B + L_e - L_\mu - 3L_\tau[\mathbf{D_2^R}]$




Past Work

This Work

# Leptogenesis

## Parameters

Majorana mass matrix

Dirac Yukawa coupling
$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$

$$\mathcal{M}_R = -\mathcal{M}_D U_{\rm PMNS} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U_{\rm PMNS}^T \mathcal{M}_D$$

Functions of
$\theta_{12},\ \theta_{23},\ \theta_{13}$
$\Delta m^2_{21},\ \Delta m^2_{31}$

# Leptogenesis

## Baryon asymmetry

### Sign of baryon asymmetry

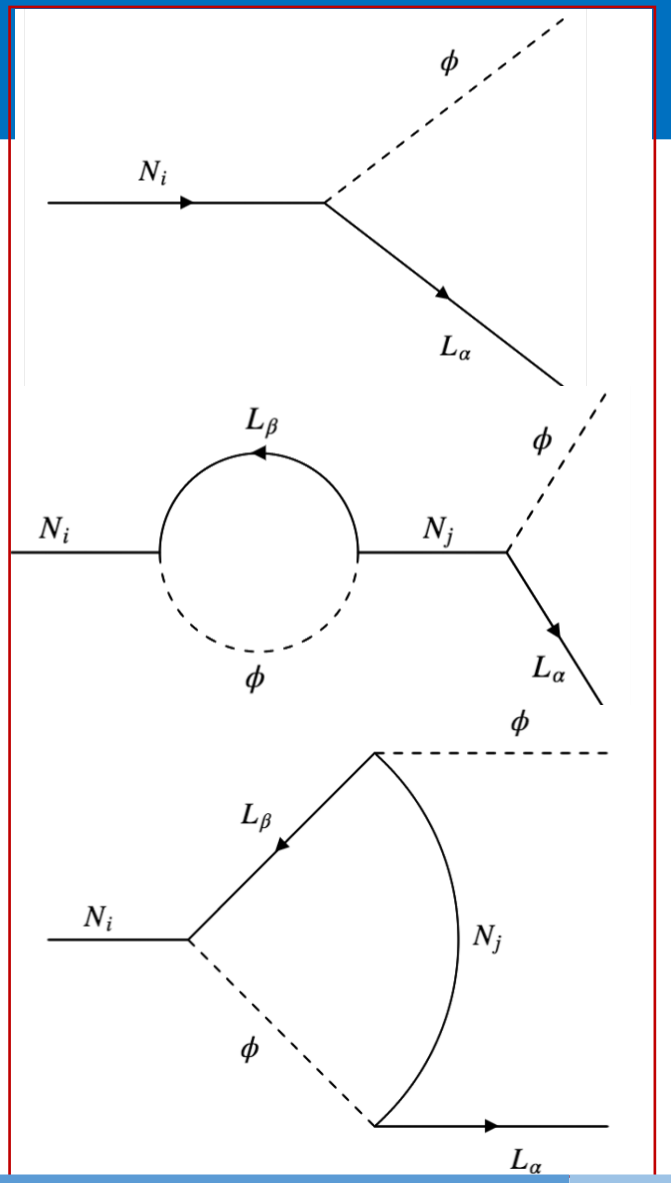
Yukawa couplings

$$\mathrm{diag}(\lambda_e, \lambda_\mu, \lambda_\tau) = \lambda \mathrm{diag}(\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$$
$$\equiv \lambda n$$

| flavor | | $N_R$-diagonal |
|---|---|---|
| $n_\alpha$ | $\to$ | $\hat{n}_\alpha$ |
| $\lambda_\alpha N_\alpha (L_\alpha \cdot H)$ | $\to$ | $\hat{\lambda} N_i (L_\alpha \cdot H)$ |

Asymmetry parameter

$$\epsilon_1 \simeq \frac{1}{8\pi} \frac{\lambda^2}{(\hat{n}\hat{n}^\dagger)_{11}} \sum_{i=2,3} \mathrm{Im}\left[\left\{(\hat{n}\hat{n}^\dagger)_{1j}\right\}^2\right] f\left(\frac{M_j^2}{M_1^2}\right)$$




Sign of the asymmetry parameter depends on $(\theta, \phi)$

# Leptogenesis

## Baryon asymmetry

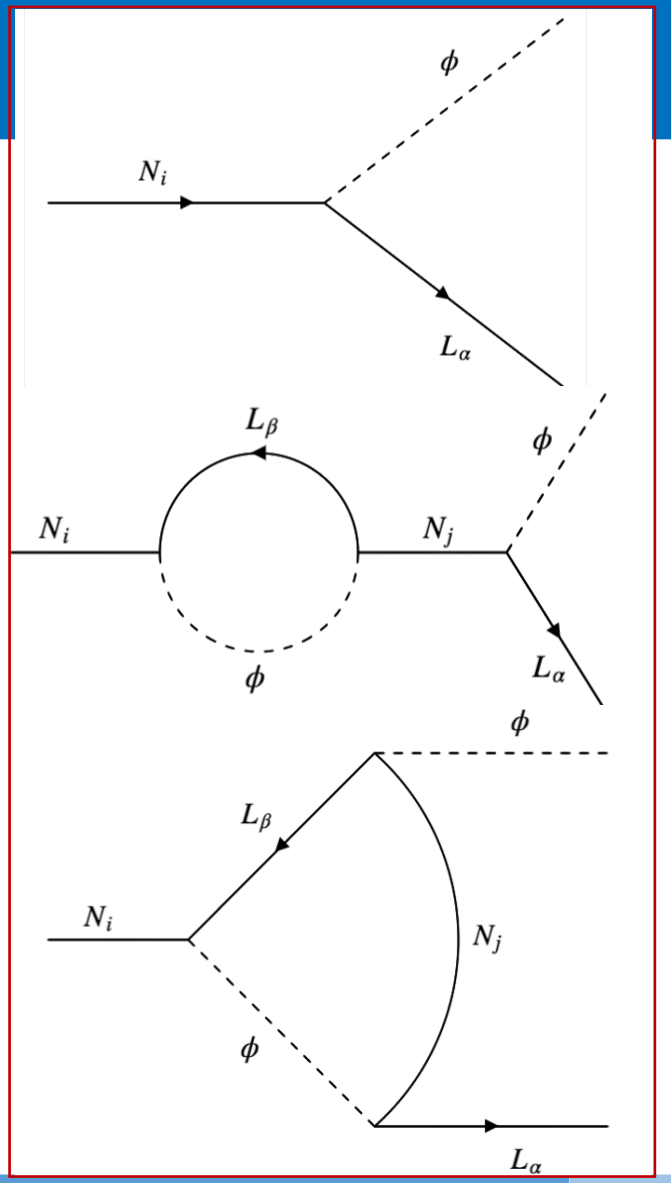
### Sign of baryon asymmetry

Relation between baryon and lepton generated

$$\frac{n_B/s}{n_L/s} = -\frac{28}{79} < 0$$

Observed baryon asymmetry of the universe

$$Y_{\Delta B} \equiv \frac{n_B}{s} \simeq 8.7 \times 10^{-11} > 0$$




Sign of asymmetry parameter is negative

$$\epsilon_1 < 0$$

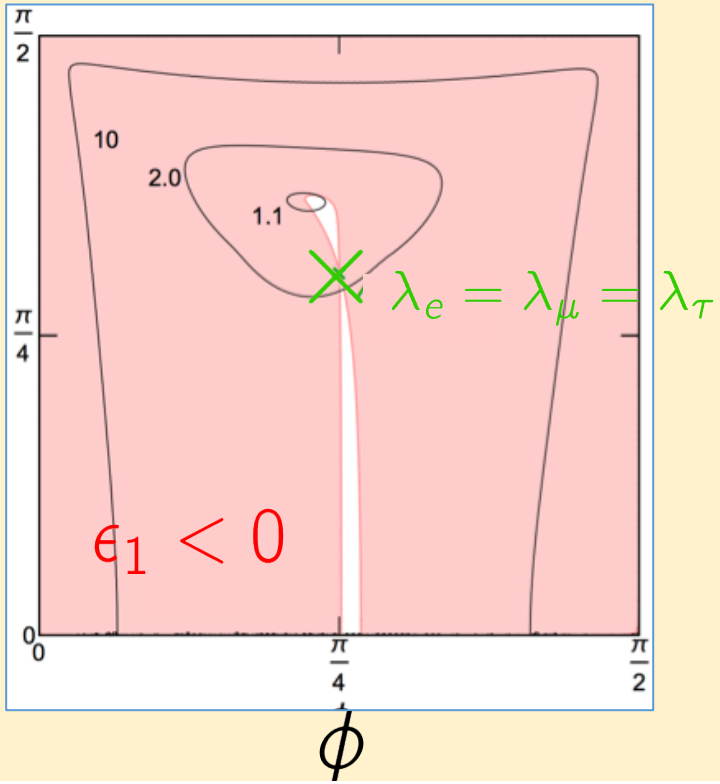
# Leptogenesis

## Results

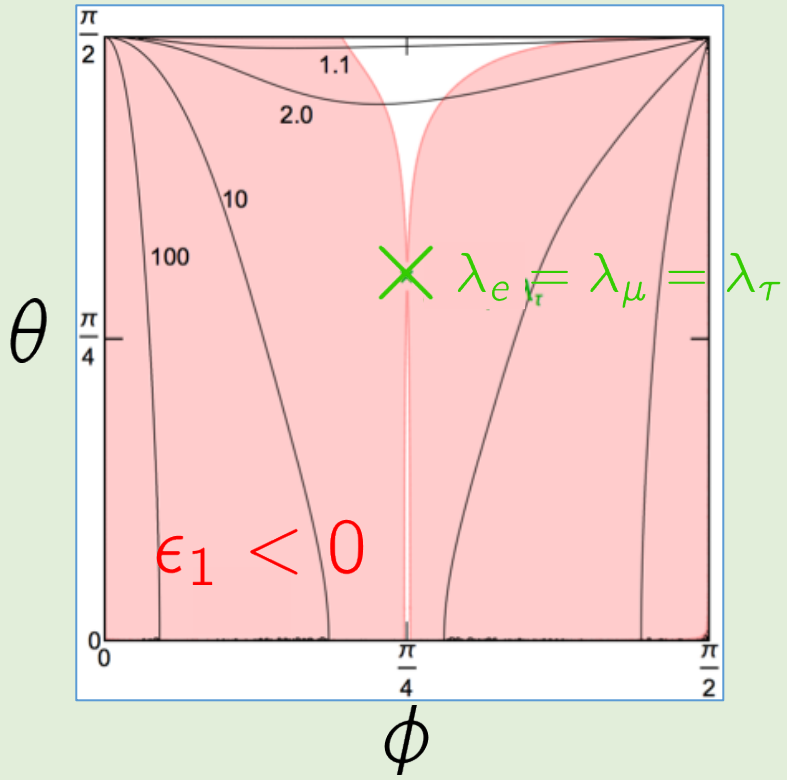
$(\lambda_e, \lambda_\mu, \lambda_\tau)$
$= \lambda(\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$

### Sign of baryon asymmetry

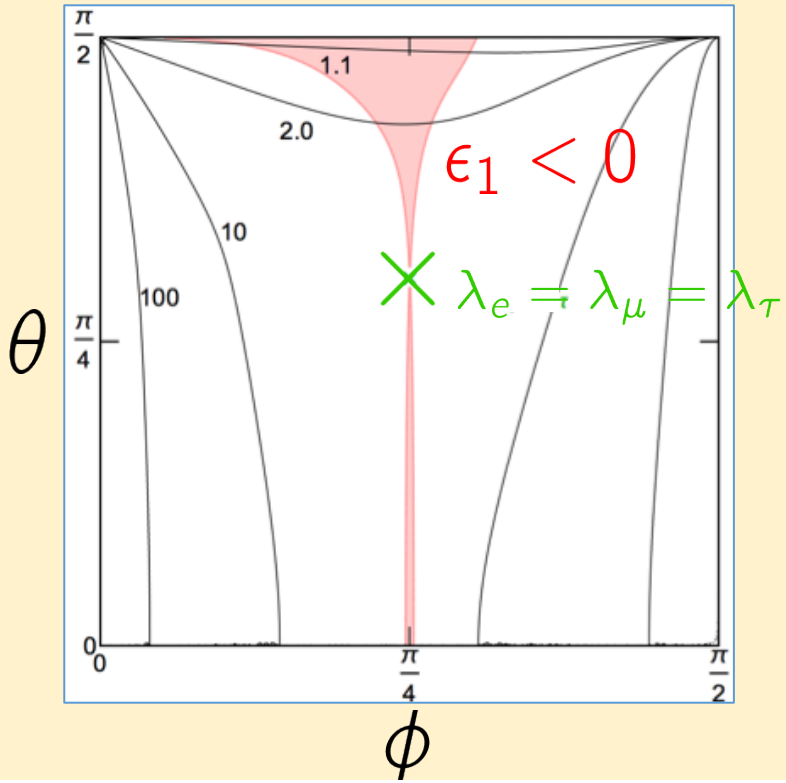
$L_\mu - L_\tau[\mathbf{C^R}]$




$B + L_e - 3L_\mu - L_\tau[\mathbf{D_1^R}]$




$B + L_e - L_\mu - 3L_\tau[\mathbf{D_2^R}]$



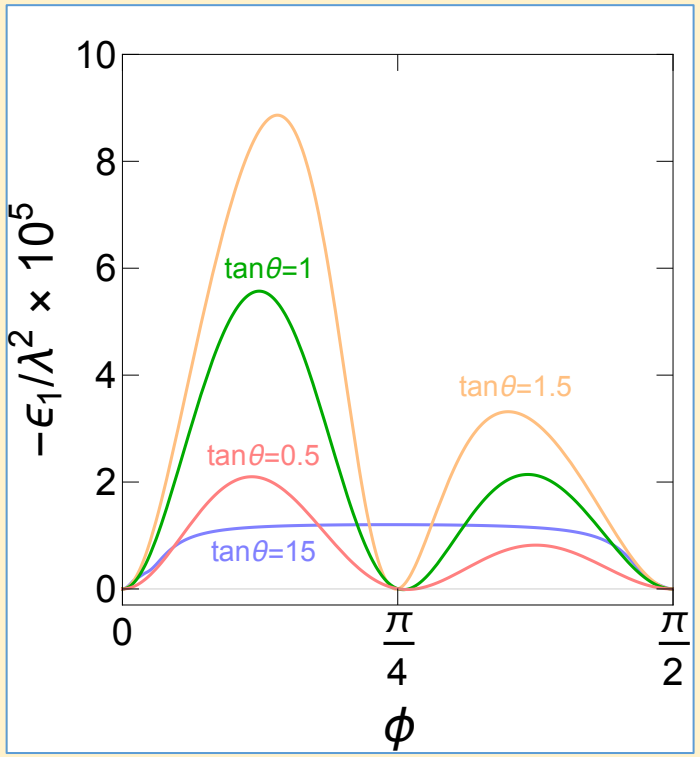
# Leptogenesis

## Results

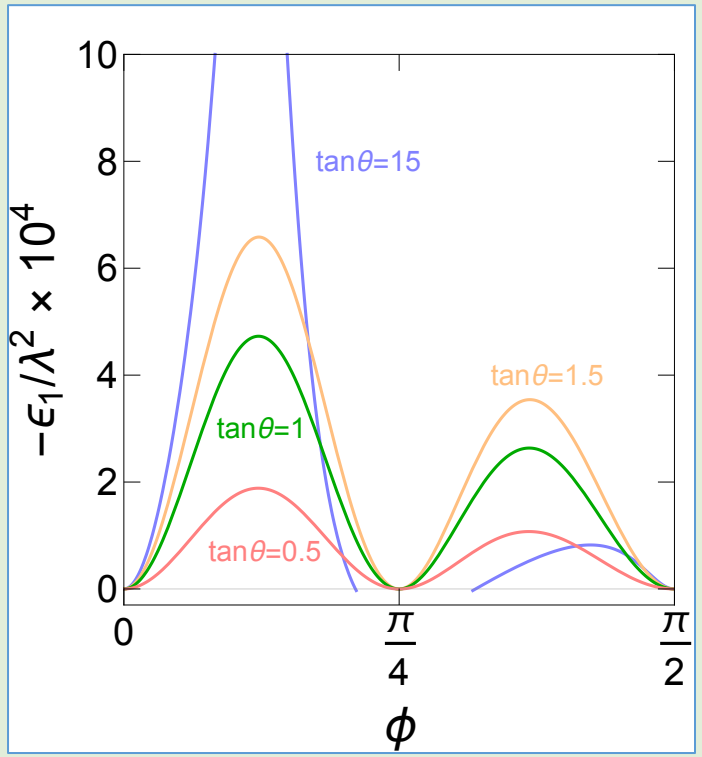
$$(\lambda_e, \lambda_\mu, \lambda_\tau) = \lambda(\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$$

### Size of baryon asymmetry

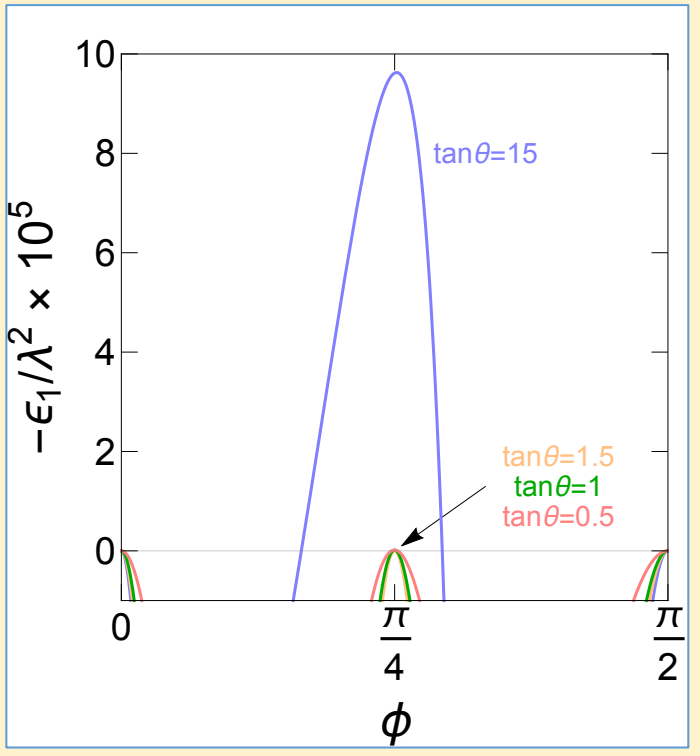
$L_\mu - L_\tau[\mathbf{C^R}]$




$B + L_e - 3L_\mu - L_\tau[\mathbf{D_1^R}]$




$B + L_e - L_\mu - 3L_\tau[\mathbf{D_2^R}]$



# Conclusion



- By analyzing two-zero minor condition, we found the prediction, which is independent of the $\mathrm{U}(1)_{Y'}$ breaking scale, for the CP phases, the neutrino masses.

- Only $\mathrm{U}(1)_{L_\mu - L_\tau}$ and the other two cases with a singlet scalar satisfy two-zero minor condition and are consistent with the recent experiments.

- We found that, in the $\mathrm{U}(1)_{L_\mu - L_\tau}$ and $\mathrm{U}(1)_{B+L_e-3L_\mu-L_\tau}$ cases, the correct sign of the baryon asymmetry can be obtained, and it is possible that enough large baryon asymmetry can be realized.

# Conclusion

## Abstract of this work

Extra U(1) gauge symmetry

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{U}(1)_{B-x_e L_e - x_\mu L_\mu - x_\tau L_\tau}$$

1 zero minor　3 zero minor　4 zero minor　• • •

2 zero minor structure

neutrino mass $m_1, m_2, m_3$

CP phase $\delta, \alpha_2, \alpha_3$

Effective Majorana neutrino mass $\langle m_{\beta\beta} \rangle$

as functions of $\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2$

Leptogenesis

Dirac Yukawa couplings

$$\mathcal{L} \supset \lambda_\alpha N_\alpha^c (L_\alpha \cdot H)$$

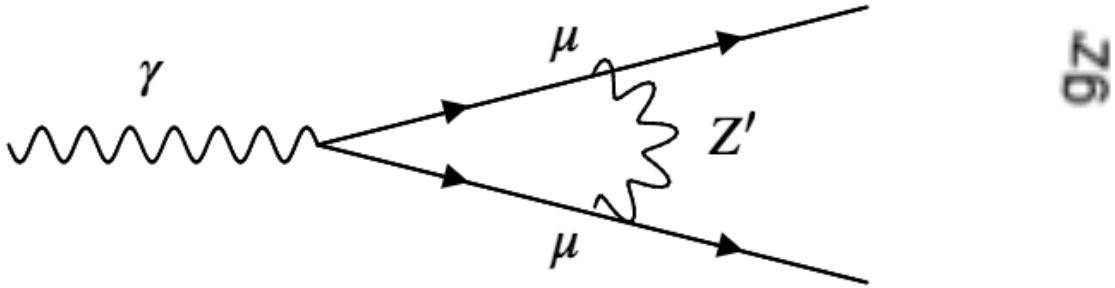
sign of baryon asymmetry

# Appendix

## Muon g-2

$$\delta a_\mu = a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM})$$
$$= (26.1 \pm 8.0) \times 10^{-10}$$

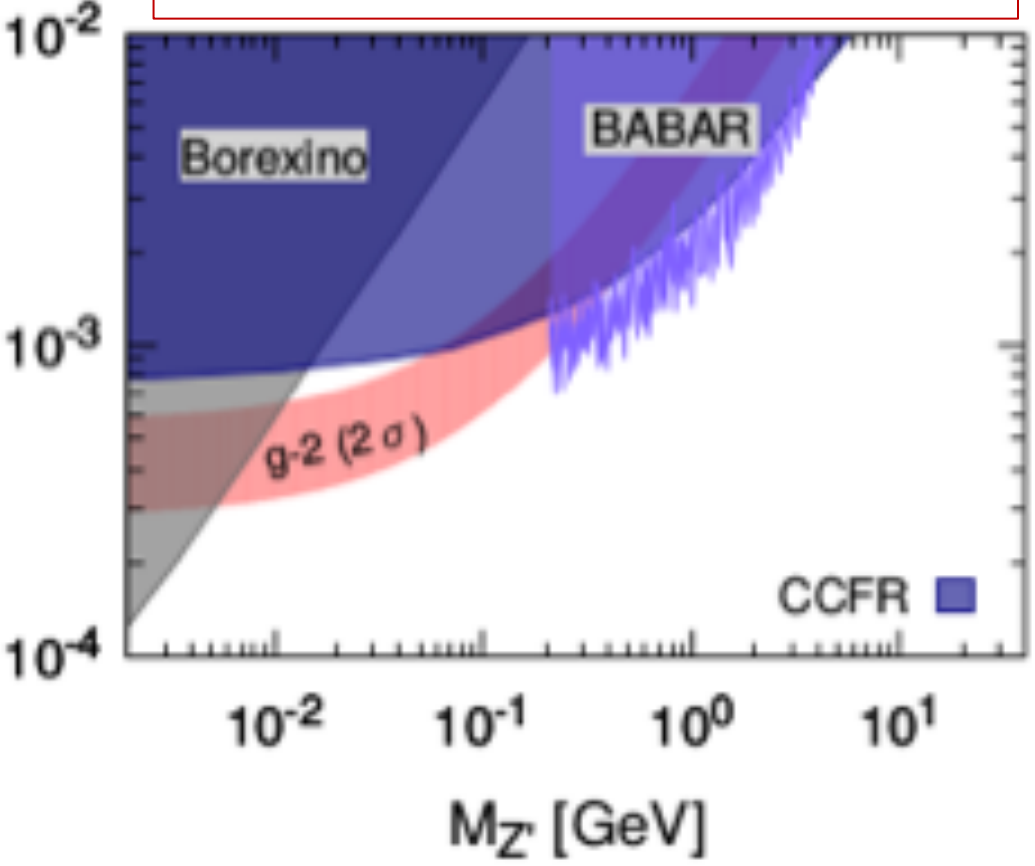
K. Hagiwara, R. Liao, A. D. Martin, D. Nomura, and T. Teubner (2011)




Z'boson contributes to muon g-2

$$\delta a_\mu = \frac{g_{Z'}^2}{8\pi^2} \int_0^1 dx \frac{2m_\mu^2 x^2(1-x)}{x^2 m_\mu^2 + (1-x)m_{Z'}^2}$$

T. Araki, S. Hosino, T. Ota, J. Sato, and T.Shimomura (2017)



# Appendix

## Neutrino Mass Matrix Structure

KA, K.Hamaguchi, N. Nagata, Shih-Yen Tseng, and K. Tsumura, Phys. Rev. D99 (2019)

### Matter contents

SM fields + 3 right-handed neutrinos + U(1) breaking scalar

$N_\alpha$ $\sigma$ (singlet) or $\Phi$ (doublet)

Seesaw mechanism

$$\mathcal{M}_\nu \simeq -\mathcal{M}_D \mathcal{M}_R^{-1} \mathcal{M}_D^T$$

For $U(1)_{L_\mu - L_\tau}$

$$\begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix} \simeq \begin{pmatrix} * & * & 0 \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix} \begin{pmatrix} * & 0 & 0 \\ 0 & 0 & * \\ 0 & * & 0 \end{pmatrix} \begin{pmatrix} * & 0 & * \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix} = U_{\rm PMNS}^* \, \mathrm{diag}\,(m_1, m_2, m_3)\, U_{\rm PMNS}^\dagger$$

**Point**

two-zero texture structure

$$m_1, \underline{\delta}, \underline{\alpha_2, \alpha_3} = f(\underline{\theta_{12}, \theta_{13}, \theta_{23}}, \underline{\delta m^2, \Delta m^2})$$

Dirac CP, Majorana CP, Mixing angle, Squared mass difference

# Appendix

## Two zero minor condition

$$\begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix} = \mathcal{M}_\nu^{-1} = U_{\rm PMNS}{\rm diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})U_{\rm PMNS}^T$$

Two complex equations

$$\begin{cases} \frac{1}{m_1}V_{\mu 1}^2 + \frac{1}{m_2}V_{\mu 2}^2 e^{i\alpha_2} + \frac{1}{m_3}V_{\mu 3}^2 e^{i\alpha_3} = 0 \\ \frac{1}{m_1}V_{\mu 1}V_{\tau 1} + \frac{1}{m_2}V_{\mu 2}V_{\tau 2} e^{i\alpha_2} + \frac{1}{m_3}V_{\mu 3}V_{\tau 3} e^{i\alpha_3} = 0 \end{cases}$$

$$\Longrightarrow \quad e^{i\alpha_2} = \frac{m_2}{m_1}R_2(\delta), \quad e^{i\alpha_3} = \frac{m_3}{m_1}R_3(\delta)$$

$$\Longrightarrow \quad \frac{m_2}{m_1} = \frac{1}{|R_2(\delta)|}, \quad \frac{m_3}{m_1} = \frac{1}{|R_3(\delta)|}$$

Neutrino mass ratios can be obtained as functions of the Dirac CP phase

# Appendix

## Two zero minor condition

$$\frac{m_2}{m_1} = \frac{1}{|R_2(\delta)|}, \quad \frac{m_3}{m_1} = \frac{1}{|R_3(\delta)|}$$

$$\Longrightarrow \begin{cases} \delta m^2 = m_1^2 \left( \dfrac{1}{|R_2(\delta)|} - 1 \right) \\ \Delta m^2 + \dfrac{\delta m^2}{2} = m_1^2 \left( \dfrac{1}{|R_3(\delta)|} - 1 \right) \end{cases}$$

$$\Longrightarrow |R_3(\delta)| \left(1 - |R_2(\delta)|\right) - \epsilon \, |R_2(\delta)| \left(1 - |R_3(\delta)|\right) = 0$$

$$\epsilon \equiv \frac{\delta m^2}{\Delta m^2 + \delta m^2/2} \ll 1$$

$$\Longrightarrow \boxed{\delta = f(\theta_{12}, \theta_{23}, \theta_{13}, \epsilon)}$$

$$\boxed{m_1 = f(\theta_{12}, \theta_{23}, \theta_{13}, \delta m^2, \Delta m^2)}$$

CP phase and neutrino masses can be obtained as functuions of the neutrino oscillation parameters

# Appendix

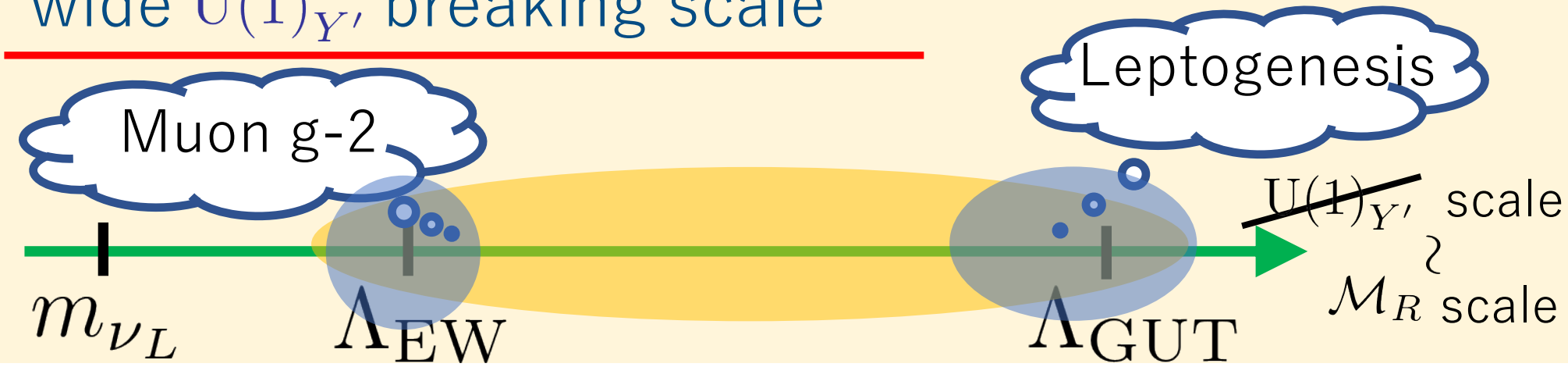
Notice !

The following discussion is applicable to

1, the other U(1) lepton flavor symmetries

→ The methods of analysis are same but the structures of the neutrino mass matrices are different

2, wide $\mathrm{U}(1)_{Y'}$ breaking scale



# Appendix

## Neutrino oscillation parameters

NuFIT v4.0 result with the Super-Kamiokande atmospheric data

| Parameter | Normal Ordering | | Inverted Ordering | |
|---|---|---|---|---|
| | Best fit $\pm 1\sigma$ | $3\sigma$ range | Best fit $\pm 1\sigma$ | $3\sigma$ range |
| $\sin^2\theta_{12}$ | $0.310^{+0.013}_{-0.012}$ | 0.275–0.350 | $0.310^{+0.013}_{-0.012}$ | 0.275–0.350 |
| $\sin^2\theta_{23}$ | $0.582^{+0.015}_{-0.019}$ | 0.428–0.624 | $0.582^{+0.015}_{-0.018}$ | 0.433–0.623 |
| $\sin^2\theta_{13}$ | $0.02240^{+0.00065}_{-0.00066}$ | 0.02044–0.02437 | $0.02263^{+0.00065}_{-0.00066}$ | 0.02067–0.02461 |
| $\Delta m^2_{21}/10^{-5}$ eV$^2$ | $7.39^{+0.21}_{-0.20}$ | 6.79–8.01 | $7.39^{+0.21}_{-0.20}$ | 6.79–8.01 |
| $\Delta m^2_{3\ell}/10^{-3}$ eV$^2$ | $2.525^{+0.033}_{-0.031}$ | 2.431–2.622 | $-2.512^{+0.034}_{-0.031}$ | $-(2.606$–$2.413)$ |
| $\delta$ [°] | $217^{+40}_{-28}$ | 135–366 | $280^{+25}_{-28}$ | 196–351 |

# Appendix

## Quantum correction to neutrino mass matrix

$U(1)_{L_\mu - L_\tau}$ symmetry breaking scale $\gg$ electroweak scale

➡ Large quantum corrections break two zero minor structure of $\mathcal{M}_{\nu_L}$ ?

Result

The two-zero minor neutrino-mass structure in our model is robust against quantum corrections

# Appendix

## Quantum correction to neutrino mass matrix

The right-handed neutrinos are integrated out to give the following dimension-five effective operator:

$$\mathcal{L}_{eff} = \frac{1}{2} C_{\alpha\beta} (L_\alpha \cdot H)(L_\beta \cdot H) + \text{h.c.}$$

$$C_{\alpha\beta} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

@ right-handed neutrino mass scale

# Appendix

## Quantum correction to neutrino mass matrix

The right-handed neutrinos are integrated out to give the following dimension-five effe

$$\mathcal{L}_{eff} = \frac{1}{2}C_{\alpha\beta}(L_\alpha \cdot H)(L_\beta \cdot H) +$$

The renormalization group equation of the Wilson coe t le

$$= \frac{}{32\pi^2} \qquad \frac{}{16\pi^2} C$$

with

$$K = -3g_2^2 + 2\mathrm{Tr}\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\right) + 2\lambda$$

Higgs quartic coupling

$$\mathcal{L}_{\mathrm{quart}} = -\frac{1}{2}\lambda(H^\dagger H)^2$$

$\mathrm{SU}(2)_L$ gauge coupling

up-type, down-type, and charged-lepton Yukawa matrices

# Appendix

## Quantum correction to neutrino mass matrix

$$\mu\frac{dC}{d\mu} = -\frac{3}{32\pi^2}\left[(Y_e^\dagger Y_e)^T C + C(Y_e^\dagger Y_e)\right] + \frac{K}{16\pi^2}C$$

$$K = -3g_2^2 + 2\mathrm{Tr}\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\right) + 2\lambda$$

$$C(t) = I_K(t)\mathcal{I}(t)C(0)\mathcal{I}(t)$$

$Y_e$ : diagonal
$\Longrightarrow \mathcal{I}(t)$ : diagonal

where

$$t \equiv \ln(\mu/\mu_0) \quad \mu_0 : \text{initial scale}$$

$$I_K(t) = \exp\left[\frac{1}{16\pi^2}\int_0^t K(t')dt'\right],\ \mathcal{I}(t) = \exp\left[-\frac{3}{32\pi^2}\int_0^t Y_e^\dagger Y_e(t')dt'\right]$$

# Appendix

## Quantum correction to neutrino mass matrix

$$\mu\frac{dC}{d\mu} = -\frac{3}{32\pi^2}\left[(Y_e^\dagger Y_e)^T C + C(Y_e^\dagger Y_e)\right] + \frac{K}{16\pi^2}C$$

$$K = -3g_2^2 + 2\mathrm{Tr}\left(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e\right) + 2\lambda$$

$$C(t) = I_K(t)\mathcal{I}(t)C(0)\mathcal{I}(t)$$

$$C_{\mu\mu}^{-1}(0) = C_{\tau\tau}^{-1}(0) = 0 \Longrightarrow C_{\mu\mu}^{-1}(t) = C_{\tau\tau}^{-1}(t) = 0$$

# Appendix

## Parameters

Parameters in this model

in Lagranginan

Dirac Yukawa coupling

$$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$$

Majorana mass

$$M_{e\tau},\ \lambda_{ee},\ \lambda_{e\mu},\ \lambda_{\tau\tau}$$

$\varphi$ : phase

from experiments

Mixing angle

$$\theta_{12},\ \theta_{23},\ \theta_{13}$$

Squared mass difference

$$\Delta m^2_{21},\ \Delta m^2_{31}$$

Lightest neutrino mass

$$m_1$$

CP phase

$$\delta,\ \alpha_2,\ \alpha_3$$

Right-handed neutrino mass

$$M_1, M_2, M_3$$

# Appendix

## Parameters

Parameters in this model

in Lagranginan

Dirac Yukawa coupling

$$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$$

Majorana mass

$$M_{e\tau},\ \lambda_{ee},\ \lambda_{e\mu},\ \lambda_{\tau\tau}$$

$$\varphi:\ \text{phase}$$

from experiments

Mixing angle

$$\theta_{12},\ \theta_{23},\ \theta_{13}$$

Squared mass difference

$$\Delta m^2_{21},\ \Delta m^2_{31}$$

Lightest neutrino mass

$$m_1$$

CP phase

$$\delta,\ \alpha_2,\ \alpha_3$$

Two zero minor condition

Right-handed neutrino mass

$$M_1, M_2, M_3$$

# Appendix

## Parameters

Parameters in this model

Dirac Yukawa coupling

$$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$$

Majorana mass

$$M_{e\tau},\ \lambda_{ee},\ \lambda_{e\mu},\ \lambda_{\tau\tau}$$

$\varphi$ : phase

from experiments

Mixing angle

$$\theta_{12},\ \theta_{23},\ \theta_{13}$$

Squared mass difference

$$\Delta m^2_{21},\ \Delta m^2_{31}$$

$$\mathcal{M}_R = -\mathcal{M}_D U_{\rm PMNS} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} U^T_{\rm PMNS} \mathcal{M}_D$$

Right-handed neutrino mass

$$M_1, M_2, M_3$$

# Appendix

## Parameters

Parameters in this model

Dirac Yukawa coupling

$$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$$

Majorana mass

$$M_{e\tau},\ \lambda_{ee},\ \lambda_{e\mu},\ \lambda_{\tau\tau}$$

$\varphi$ : phase

from experiments

Mixing angle

$$\theta_{12},\ \theta_{23},\ \theta_{13}$$

Squared mass difference

$$\Delta m^2_{21},\ \Delta m^2_{31}$$

Lightest neutrino mass

$$m_1$$

CP phase

$$\delta,\ \alpha_2,\ \alpha_3$$

Right-handed neutrino mass

$$M_1, M_2, M_3$$

# Appendix

## Parameters

### Parameters in this model

in Lagranginan

Dirac Yukawa coupling

$$\lambda_e,\ \lambda_\mu,\ \lambda_\tau$$

➡ Free parameters

Majorana mass

$$M_{e\tau},\ \lambda_{ee},\ \lambda_{e\mu},\ \lambda_{\tau\tau}$$

$\varphi$ : phase

from experiments

Mixing angle

$$\theta_{12},\ \theta_{23},\ \theta_{13}$$

Squared mass difference

$$\Delta m^2_{21},\ \Delta m^2_{31}$$

Lightest neutrino mass

$$m_1$$

CP phase

$$\delta,\ \alpha_2,\ \alpha_3$$

Right-handed neutrino mass

$$M_1, M_2, M_3$$