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Finite temperature effects on particle decays

PARTIKELDAGARNA 2019 STUDENT TALK BY TORBJÖRN LUNDBERG

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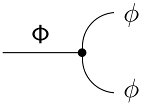
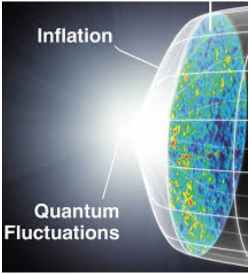
Who am I and what will I present?

- Why *thermal* quantum field theory (TQFT)?
- How to incorporate temperature?
    - Formalism dependence?
- 

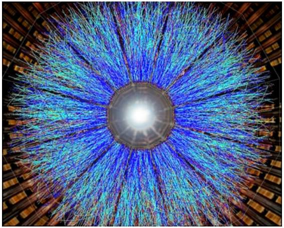
- What have I done?
- Where are we going?

Why TQFT?



https://lecospa.ntu.edu.tw

https://en.wikipedia.org/wiki/Supernova



http://news.mit.edu/2010/exp-quark-gluon-0609

Defining thermal observables

- Marry statistical mechanics with QFT!

Thermal observable

$$\langle\hat{\mathcal{O}}\rangle = \operatorname{Tr}\hat{\rho}\hat{\mathcal{O}}, \qquad \hat{\rho}_{T=0} = \mathbb{1}.$$

- Expressions of interest are Green's functions

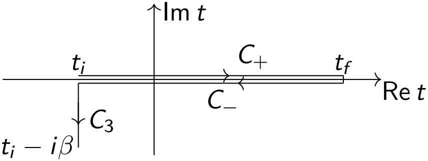
$$G_C(x_1, x_2, \ldots, x_n) = \langle T_C\hat{\phi}(x_1)\hat{\phi}(x_2)\cdots\hat{\phi}(x_n)\rangle$$

with

$$\hat{\phi}(x) = e^{it\hat{H}}\hat{\phi}(0,\mathbf{x})e^{-it\hat{H}}.$$

- The density operator often comes as $\hat{\rho} \propto e^{b\hat{\mathcal{B}}}$.
- Note the formal equivalence between $\hat{\rho}$ and the time-evolution operator if $\hat{\mathcal{B}}$ contains $\hat{H}$. (Bloch, 1932)

The contour propagator



- Due to the doubling of the d.o.f. on the Keldysh contour the propagator acquires a matrix structure. (Keldysh, 1964)

Matrix propagator

$$\tilde{\mathbf{D}}(k) = \begin{pmatrix} \tilde{D}^{++}(k) & \tilde{D}^{+-}(k) \\ \tilde{D}^{-+}(k) & \tilde{D}^{--}(k) \end{pmatrix}.$$

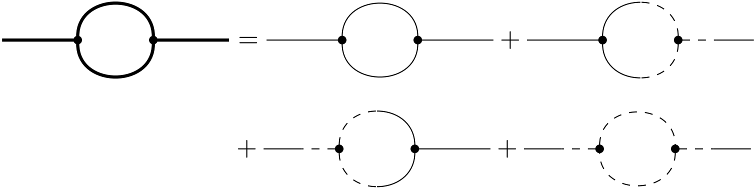
- Example of propagator: $i\tilde{D}^{++}(k) = \frac{i}{k^2 - m^2 + i\epsilon} + \eta 2\pi n(|k_0|)\delta\left(k^2 - m^2\right).$

Thermal decay rates

- At $T = 0$ the decay rate given by the optical theorem reads $\gamma_D = -\frac{\mathrm{Im}\,\Pi_{T=0}(E_0)}{E_0}$.
- In the thermal bath this must be modified to $\Gamma = -\frac{\mathrm{Im}\,\Pi(E)}{E}$.

Distribution approach rate

$$\Gamma = -\frac{\mathrm{Im}\,\Pi(E)}{E}. \qquad \text{(Weldon, 1983)}$$

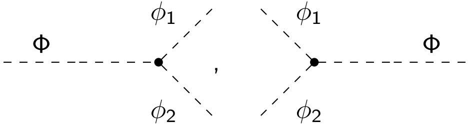


What is Γ_D?

- $\Gamma = \Gamma_D - \eta\Gamma_I$.

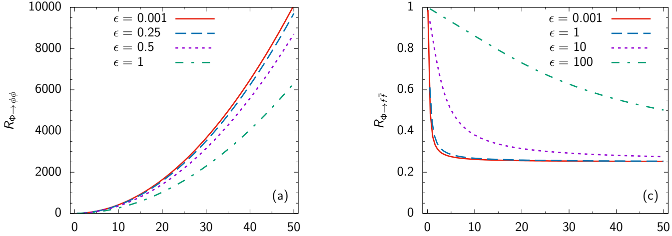
Thermal decay rate

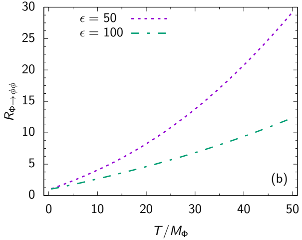
$$\Gamma_D = \frac{1}{1-\eta e^{-\beta E}}\left(-\frac{\text{Im}\,\Pi(E)}{E}\right) \qquad \text{(Weldon, 1983)}$$



- $R = \Gamma_D/\gamma_D$ characterises the missing factor between zero-temperature and $T \neq 0$-theory. (Ho&Scherrer, 2015)

Comparison to $T = 0$





(Reproduced results of Ho&Scherrer, 2015)

- $R_{\Phi\to ij} = \Gamma_D/\gamma_D$, $\epsilon = |\mathbf{p}|/M_\Phi$.
- Results plotted for $s \gg (m_1 + m_2)^2$.
- Temperature effects kick in $\sim$ a few-10 MeV. (Zheng et al. 2014)

What is to be done?

- Thermal decays involving gauge particles.
- Study thermal effects on cosmological models.
- Work towards non-equilibrium.

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Thank you for listening!

Bonus frames!

Lorentz covariance

- Feynman rules break Lorentz invariance.
- Consider a local observer $U^\mu(x)$, $U^2 = 1$.
- Define Hamiltonian and charges as

$$\begin{aligned} \hat{H} &= U_\mu \hat{P}^\mu, \qquad \text{and} \qquad \beta_\mu = \beta U_\mu. \\ \hat{Q}_a &= U_\mu \hat{J}^\mu_a, \end{aligned}$$

- A *covariant* formulation of the Gibbs ensemble is feasible.

$$n^\pm_\eta = \frac{1}{\exp\{\beta(|P \cdot U| \pm \mu)\} - \eta}. \qquad \text{(Niemi\&Semenoff, 1984).}$$

Path-integral formulation

Matrix element in trace:

$$\langle \varphi(\mathbf{x}); t_i | e^{-\beta \hat{H}} \hat{\mathcal{O}} | \varphi(\mathbf{x}); t_i \rangle = \langle \varphi(\mathbf{x}); t_i - i\beta | \hat{\mathcal{O}} | \varphi(\mathbf{x}); t_i \rangle.$$

- The FSM-formula allows this matrix element to be written as a path-integral over the action. (Matthews&Salam, 1955 and Feynman&Hibbs, 1965)
- The path integral must now be evaluated along C, a contour from t_i to $t_i - i\beta$.
  $$\Rightarrow \quad G_{0,C}(x_1, x_2, \ldots, x_n) = \frac{1}{Z_0[0]} \frac{\delta^n Z_0[j]}{i\delta j_+(x_1) \cdots i\delta j_+(x_n)} \bigg|_{j=0}$$
  with $Z_0[j] = Z_0[0] \exp\left\{-\frac{i}{2} \int_C \mathrm{d}x \int_C \mathrm{d}x' \, j(x) D_C(x - x') j(x')\right\}$.
- Note: the propagator explicitly depends on the contour C!

Multi-component multi-spin fields

Very generally it has been shown (Takahashi, 1969) that for a field carrying both a Lorentz- and an internal spin-structure $\Phi^{ij}_{\alpha\beta}$ the propagator of the free field is

Free field propagator

$$D^{ij}_{\alpha\beta,C}(x-x') = d^{ij}_{\alpha\beta}(i\partial)D_C(x-x').$$

The *Klein-Gordon divisor* is constructed so that

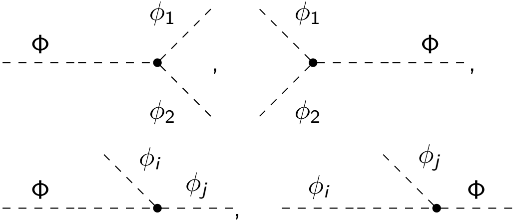
$$d^{ij}_{\alpha\beta}(i\partial)\Lambda^{jk}_{\beta\gamma}(i\partial) = \delta_{\alpha\gamma}\prod_{l}\big(-\partial^2-m_l^2\big).$$

The periodicity condition of the trace imposes

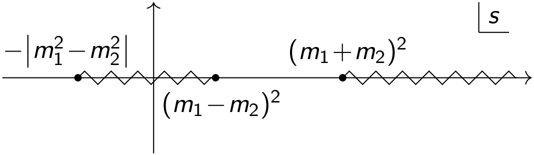
$$D^{ij}_{\alpha\beta,C}(t_i-i\beta-t') = \eta e^{-\beta\mu}D^{ij}_{\alpha\beta,C}(t_i-t').$$

Interpretation of Γ

- $\Gamma = \Gamma_D - \eta\Gamma_I$.
- If $s \geq (m_1 + m_2)^2$
  - Φ will decay to ϕ_1 and ϕ_2 contributing to Γ_D.
  - Also, real ϕ_1 and ϕ_2 in the medium will produce Φ thereby contributing to Γ_I.
- If $-\left|m_1^2 - m_2^2\right| < s < (m_1 - m_2)^2$
  - Φ will decay through absorption of ϕ_1 or ϕ_2 from the medium. This contributes to Γ_D.
  - A real ϕ_1 or ϕ_2 in the medium will produce Φ thereby contributing to Γ_I.



Interesting properties of the decay



- The non-thermal part of $\mathrm{Im}\,\Pi^{++}(p; m_1, m_2)$ vanishes below threshold $s = (m_1 + m_2)^2$.
- The thermal part of $\mathrm{Im}\,\Pi^{++}(p; m_1, m_2) \neq 0$ in interesting regions.
  - $s \geq (m_2 + m_1)^2$
  - $(m_2 - m_1)^2 \leq s < (m_2 + m_1)^2$
  - $0 \leq s < (m_2 - m_1)^2$
  - $\Big(s < 0\Big)$