

# Outlook

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Cambridge

- Aims
- Parton shower issues
- Resummation issues
- Conclusions?

# What are we trying to achieve?

Plausible uncertainties for IRC-safe observables:

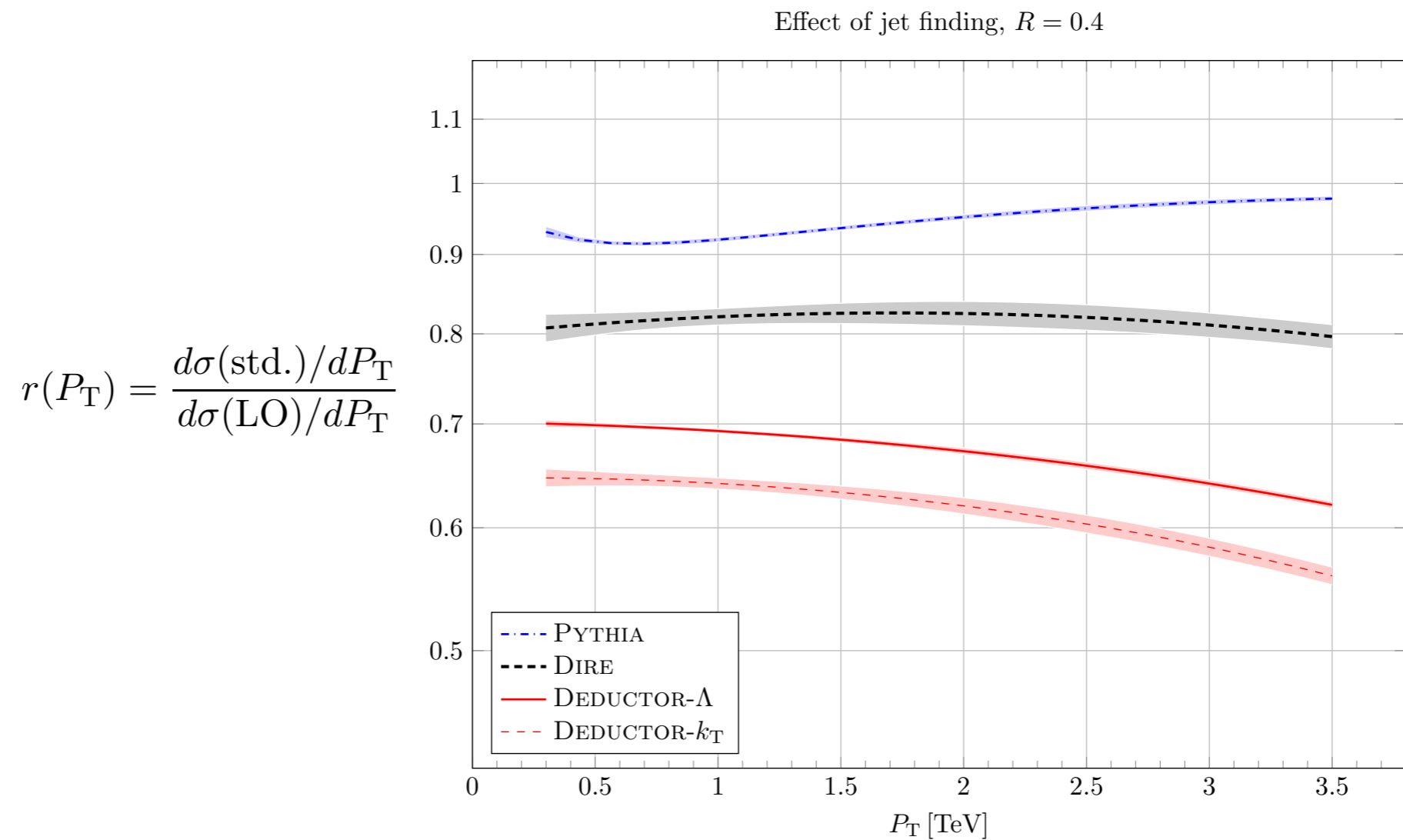
- Fixed order: few % over understood ranges
- Resummed: few times FO over extended ranges
- Parton shower: few times RS for most IRC-safe  
PLUS ~10% (tunable) for many IRC-unsafe

# Parton Shower Issues

- Generator dependence:
  - Evolution variables:  $q^2$ ,  $p_t$ ,  $\theta$ , “time”, ...
  - Partons vs dipoles
  - Recoil schemes
- NLO splitting
- Subleading colour
- Quantum correlations (spin, EW, ...)
- Hadronization

# PS Generator Dependence

- Dijet pt (parton level)

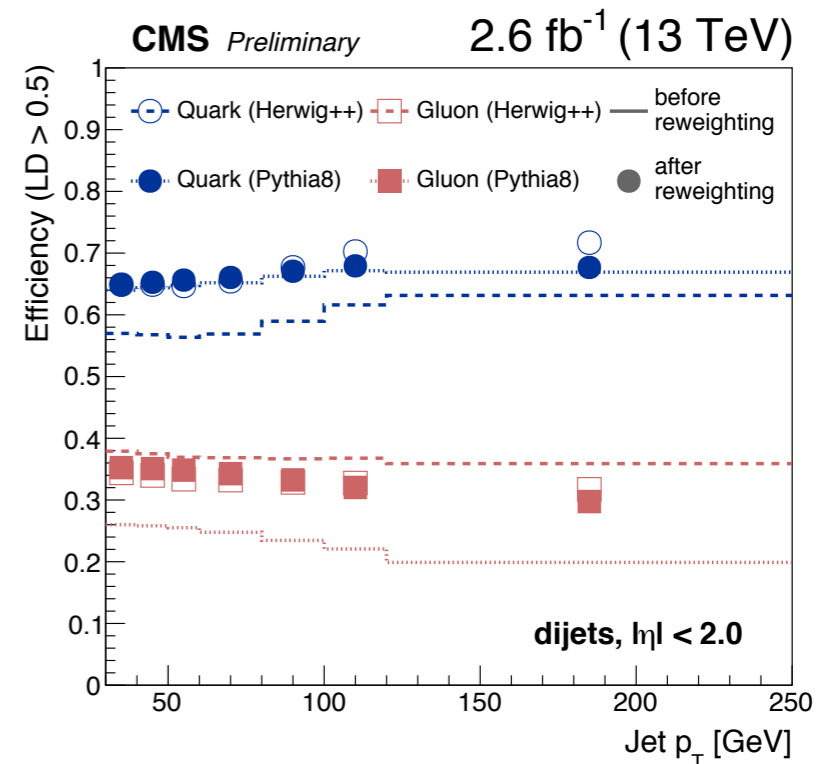
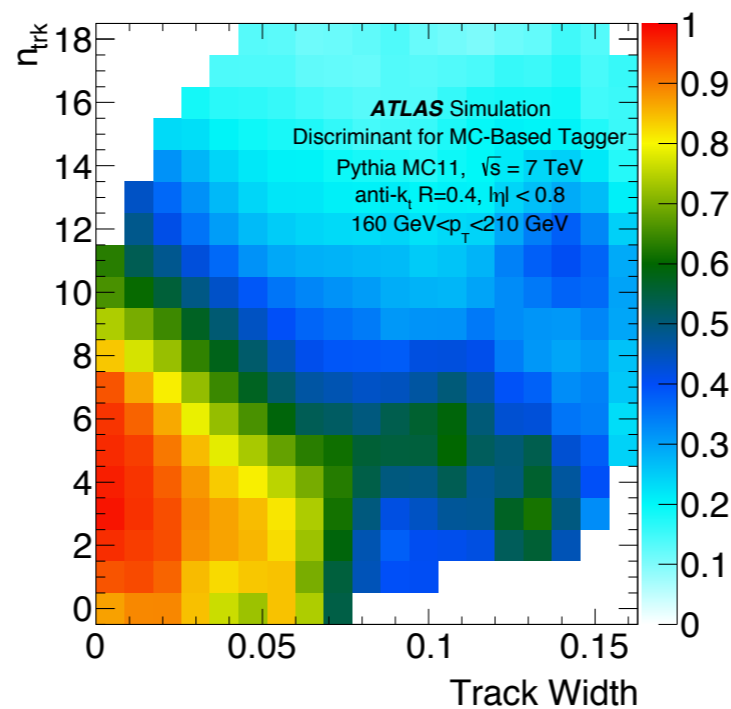
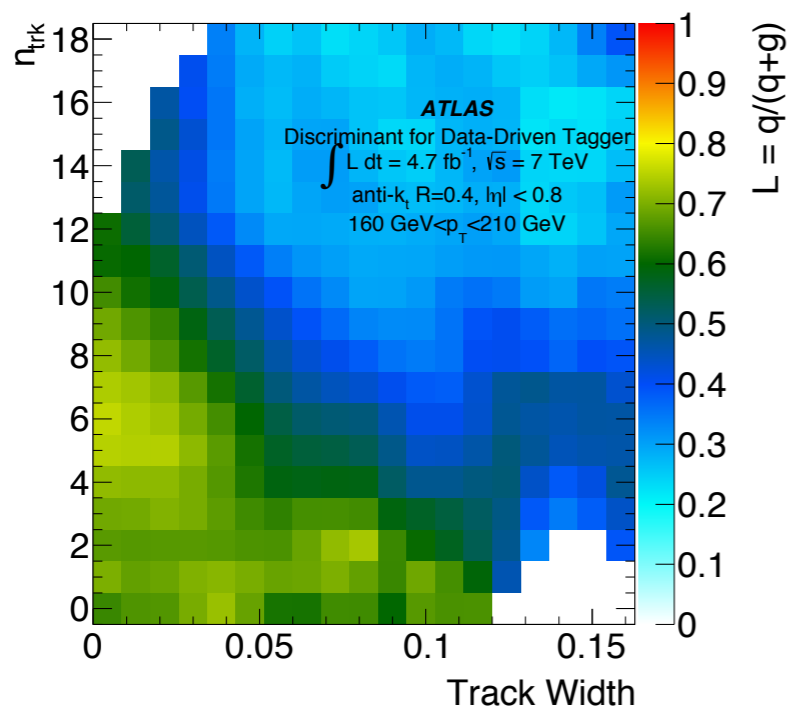


Nagy & Soper, 1711.02369



# PS Generator Dependence

- Quark-gluon tagging: track width & multiplicity



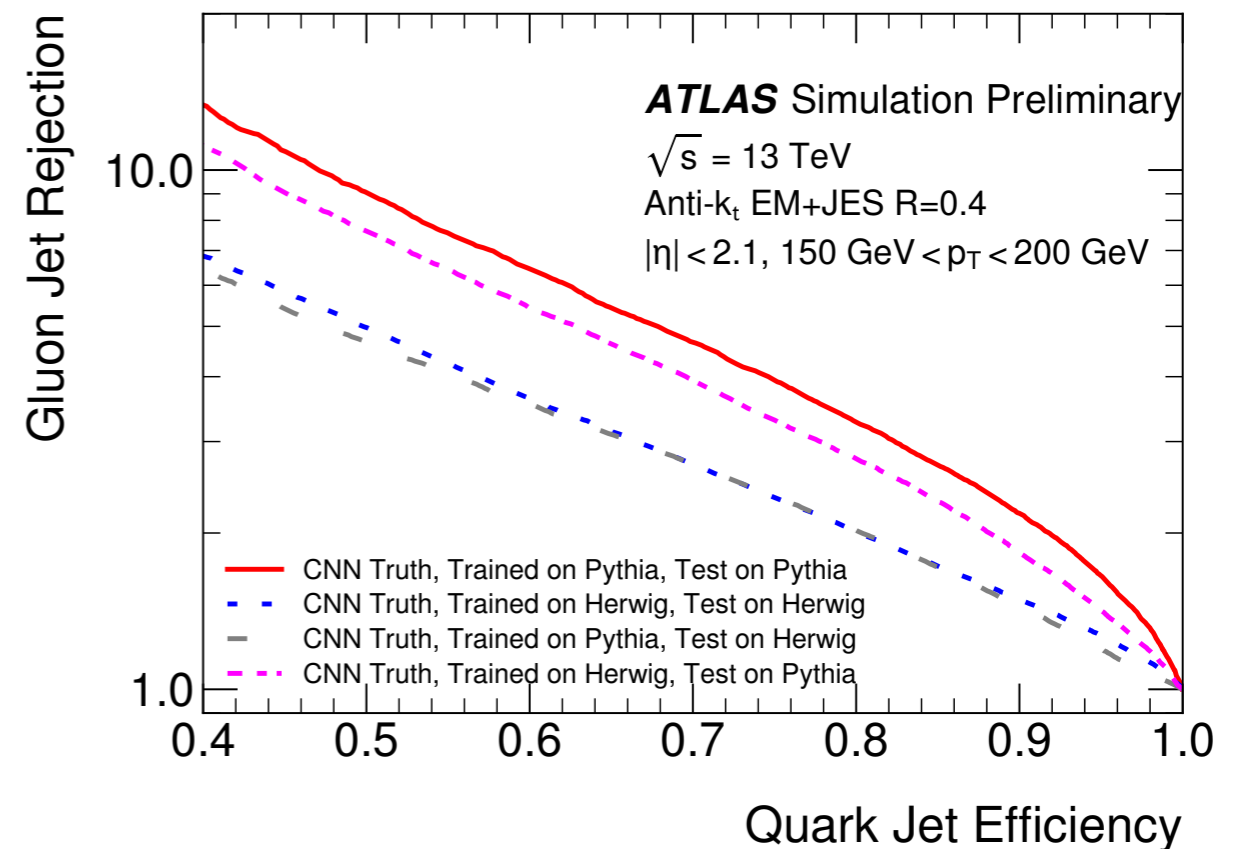
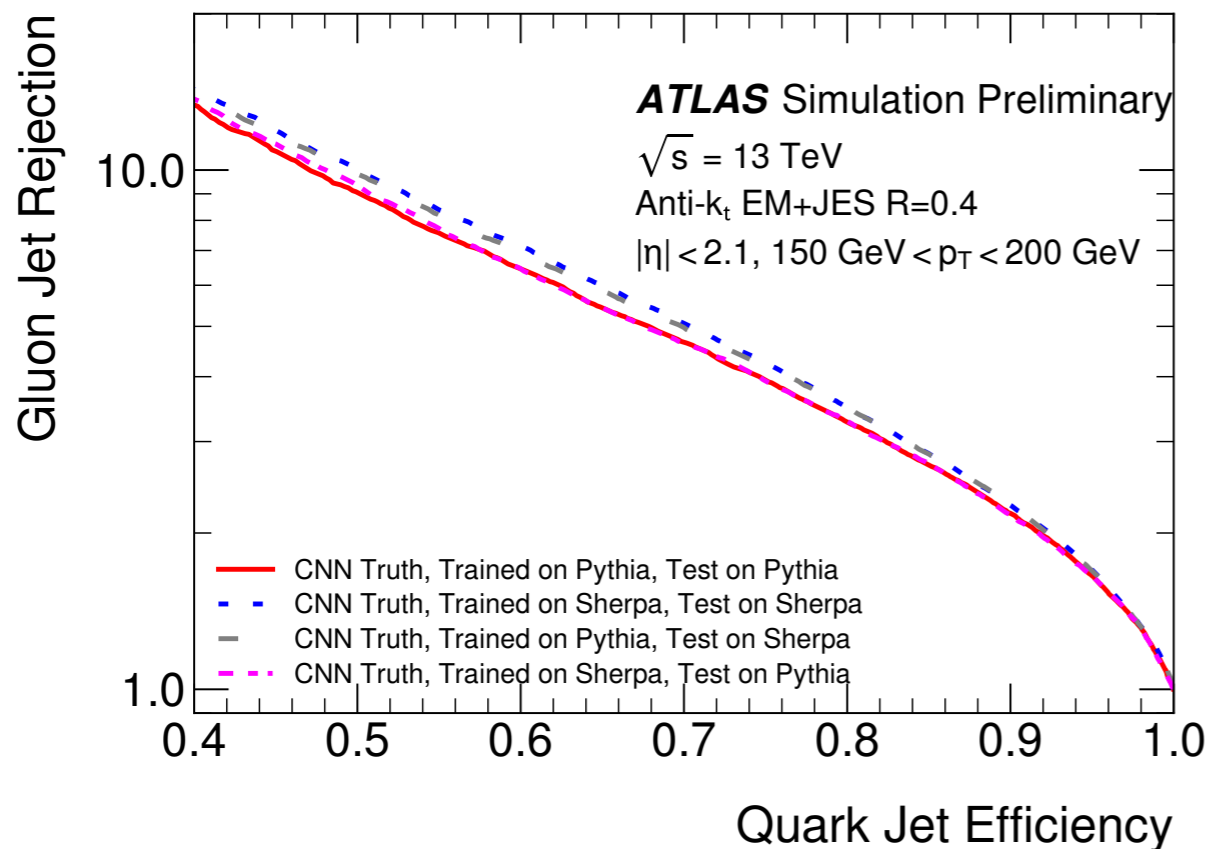
ATLAS, 1405.6583

CMA-PAS-JME-16-003

- Pythia good for quarks, not so good for gluons
- Herwig better for gluons

# PS Generator Dependence

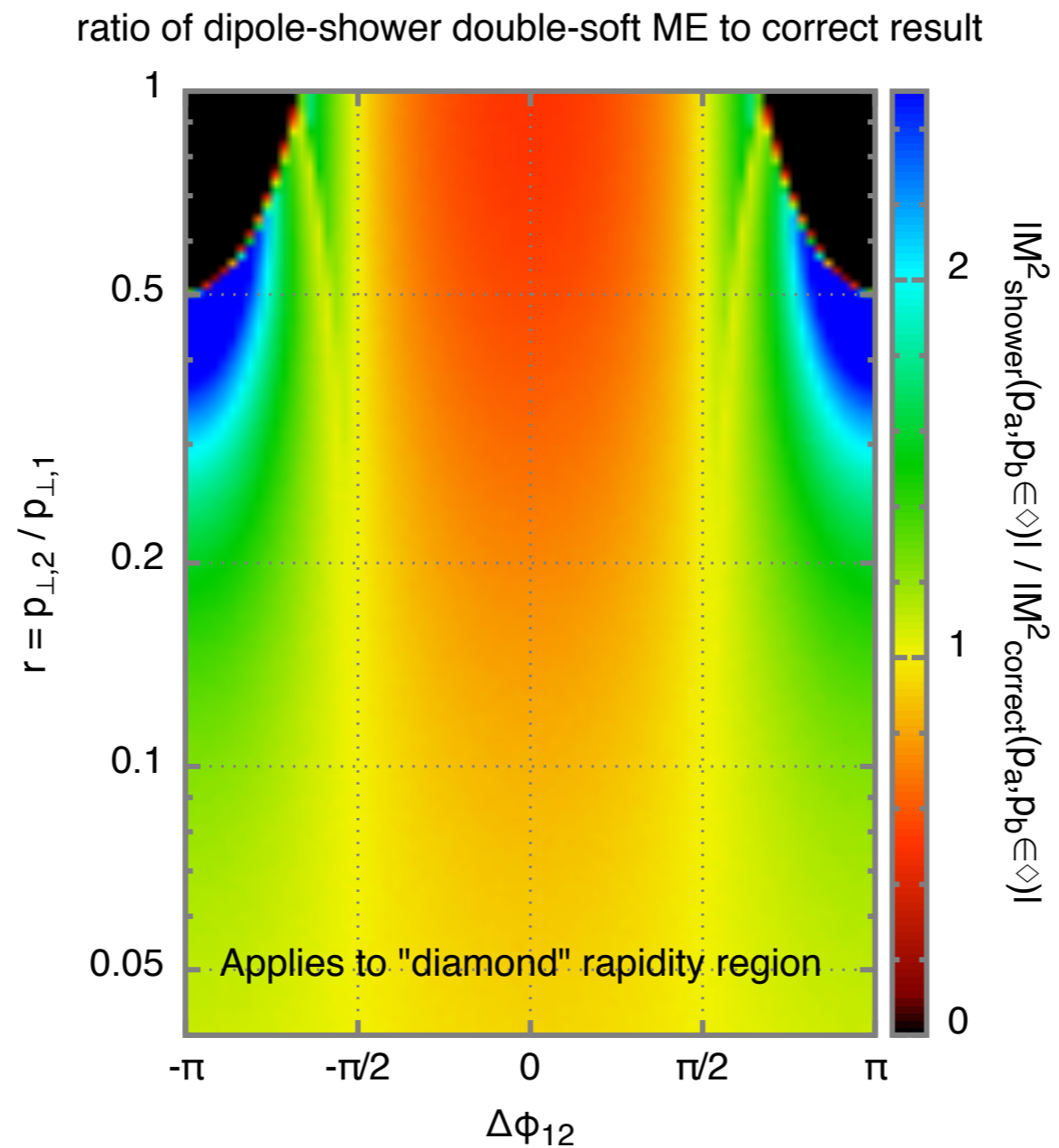
- Quark-gluon tagging: jet images



ATL-PHYS-PUB-2017-017

- Pythia, Sherpa similar, Herwig less

# Dire Shower vs ME

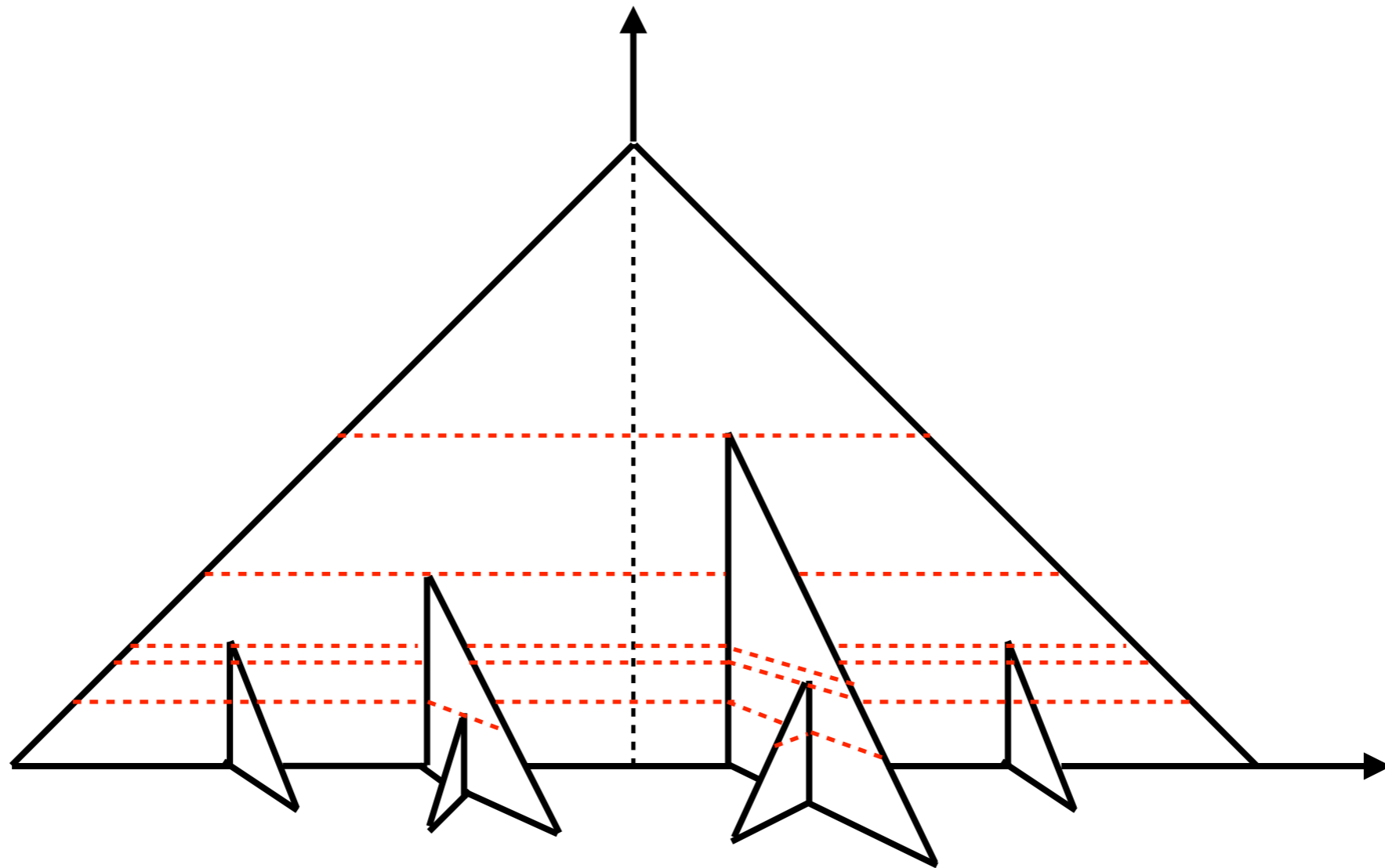


# Dipole vs Parton Showers

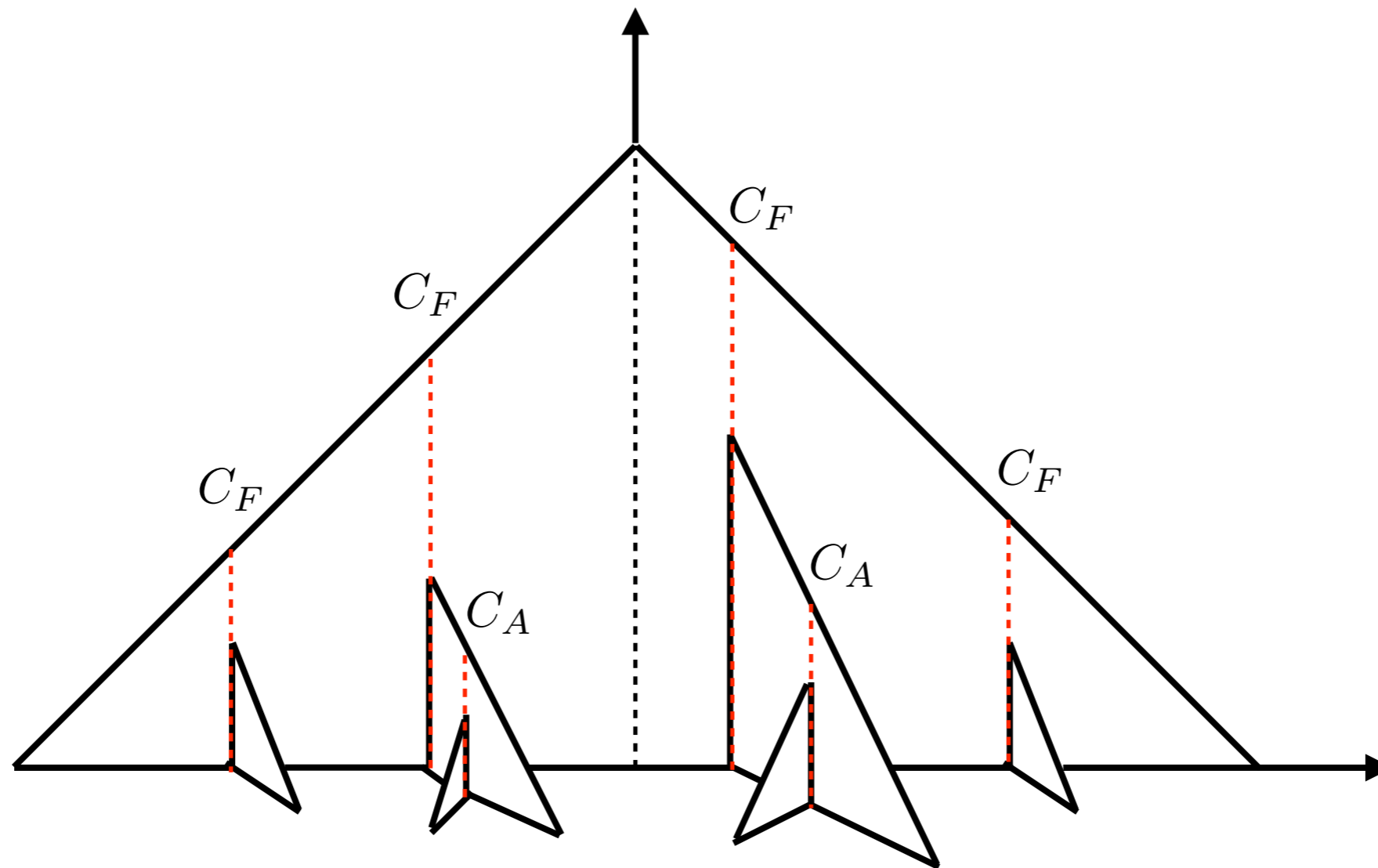
# Parton vs Dipole Showers

- Parton Shower
  - Simple 1-to-2 splittings: fewer recoil ambiguities
  - Colour structure simple at DL, NDL
  - Soft azimuthal correlations missing
- Dipole shower
  - 2-to-3 splittings mean more recoil ambiguities
  - Colour structure more difficult, even at DL
  - Azimuthal correlations included

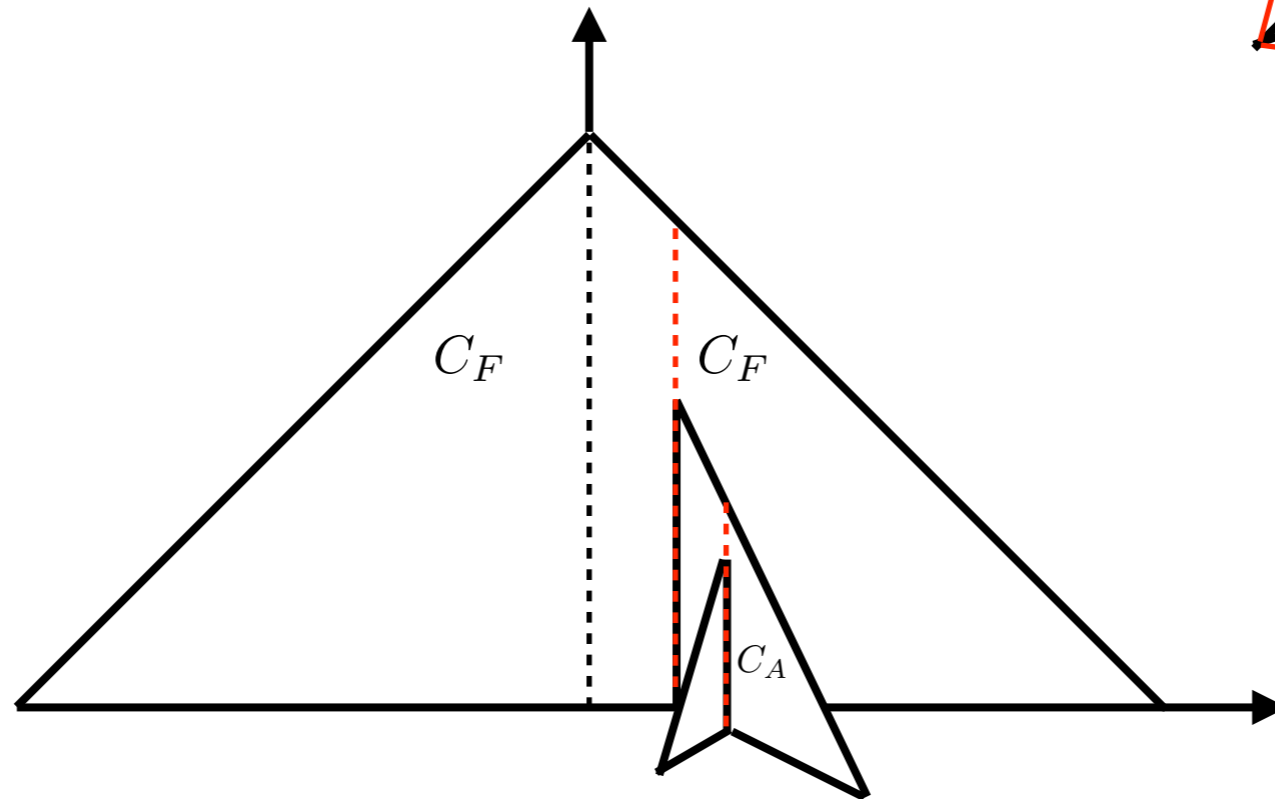
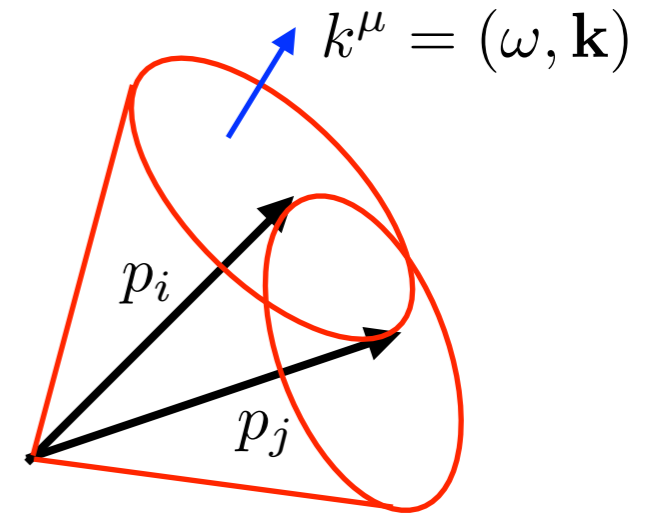
# $k_t$ -ordered dipole shower



# Angular-ordered parton shower



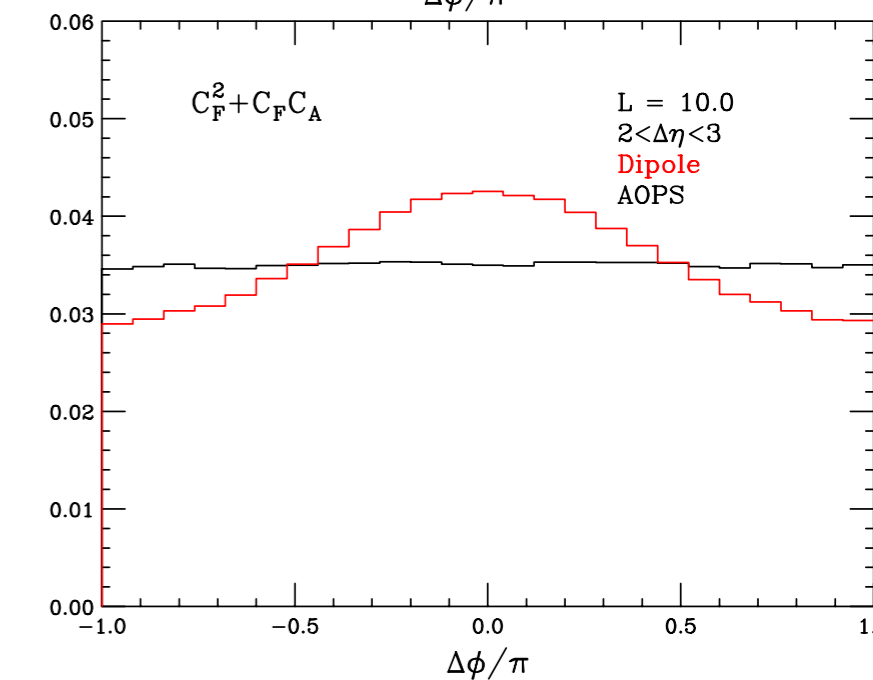
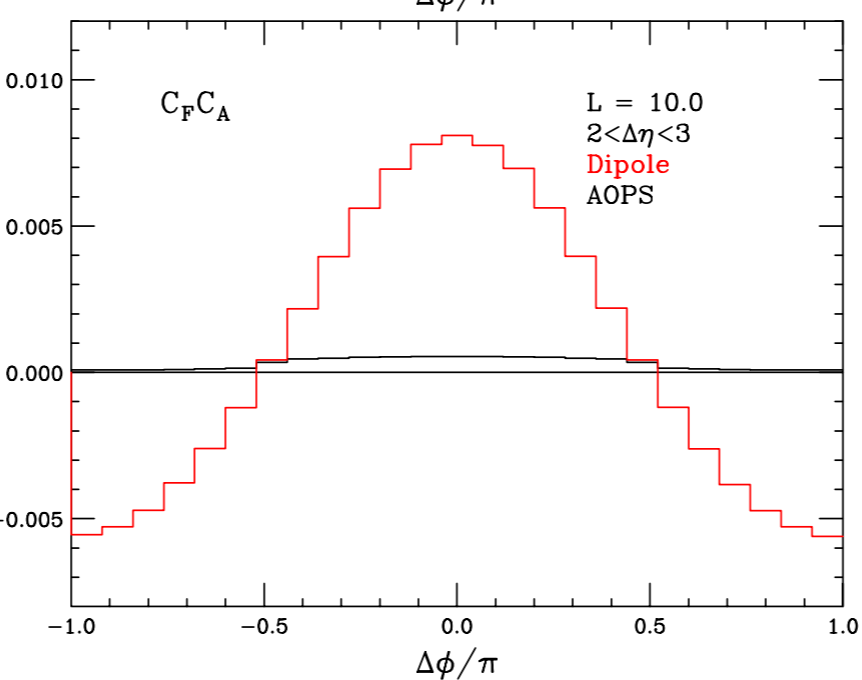
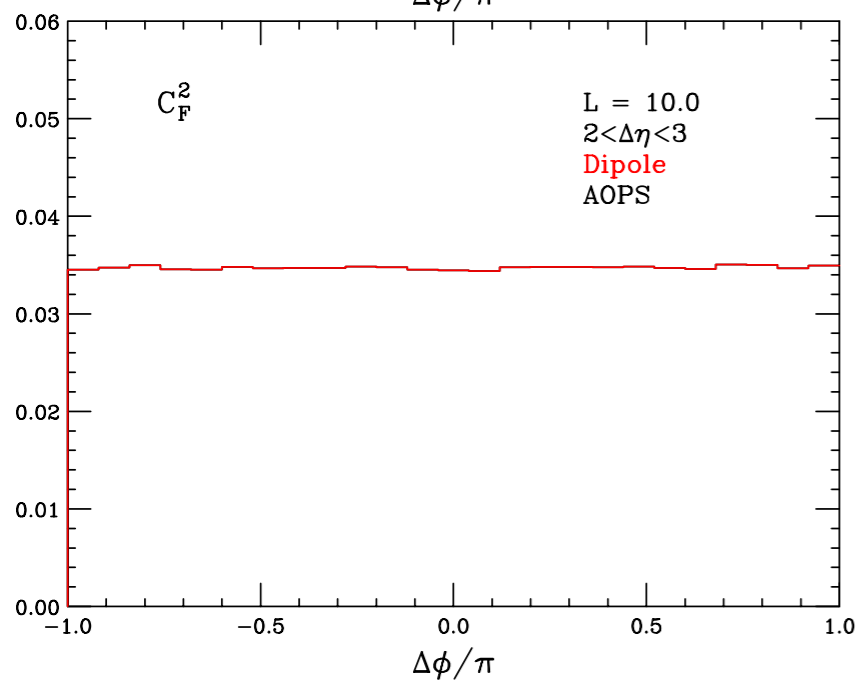
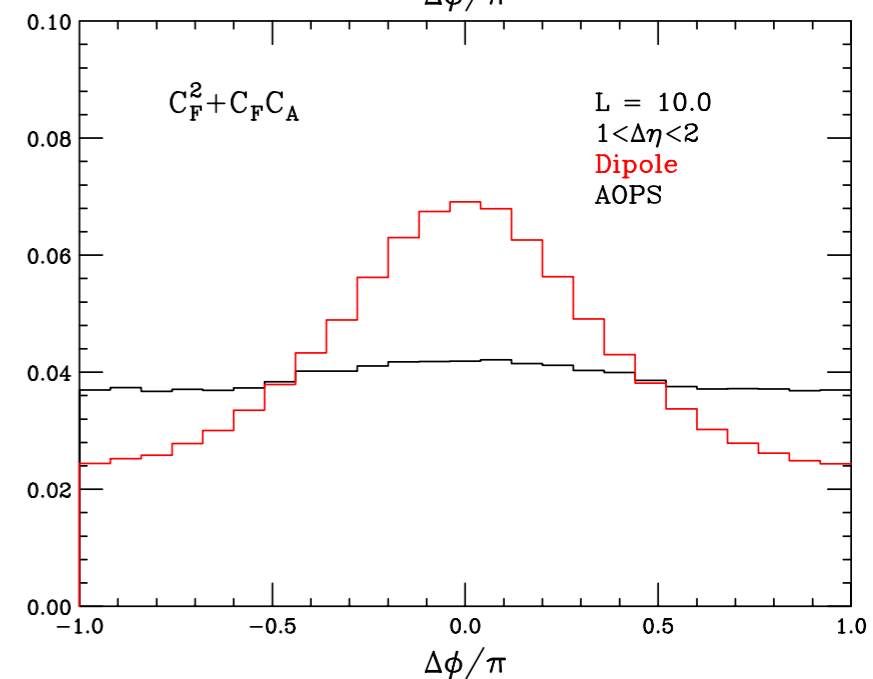
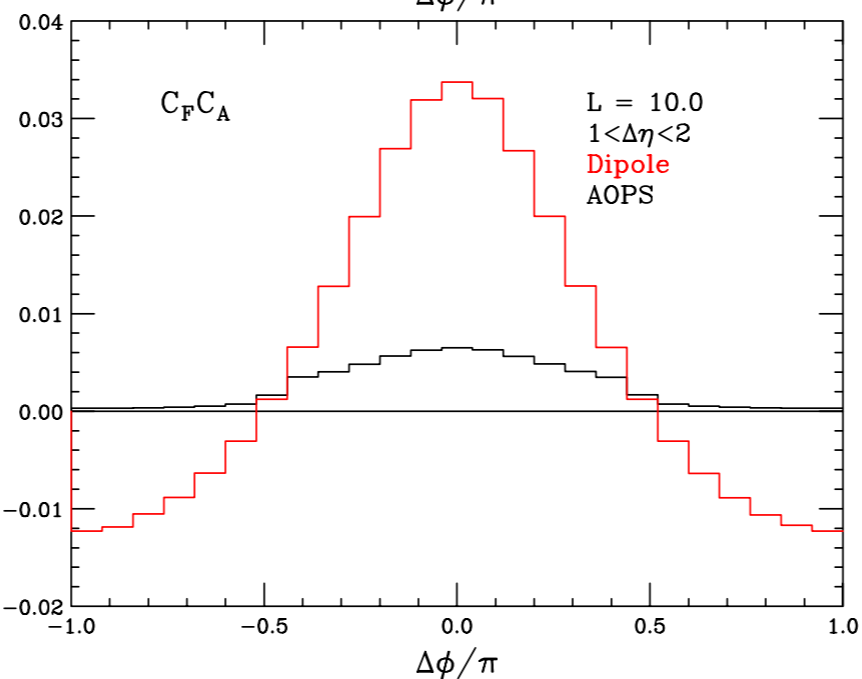
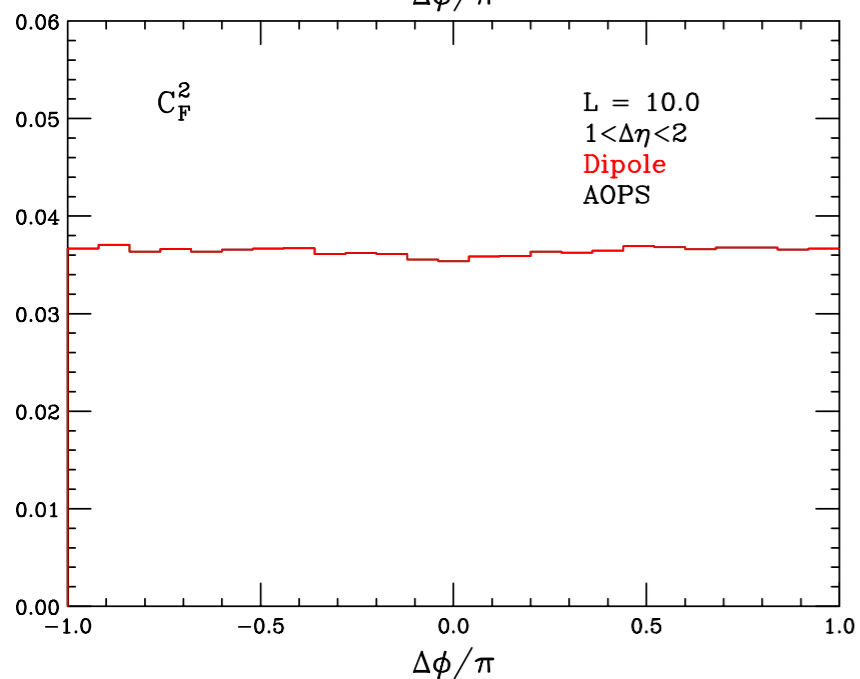
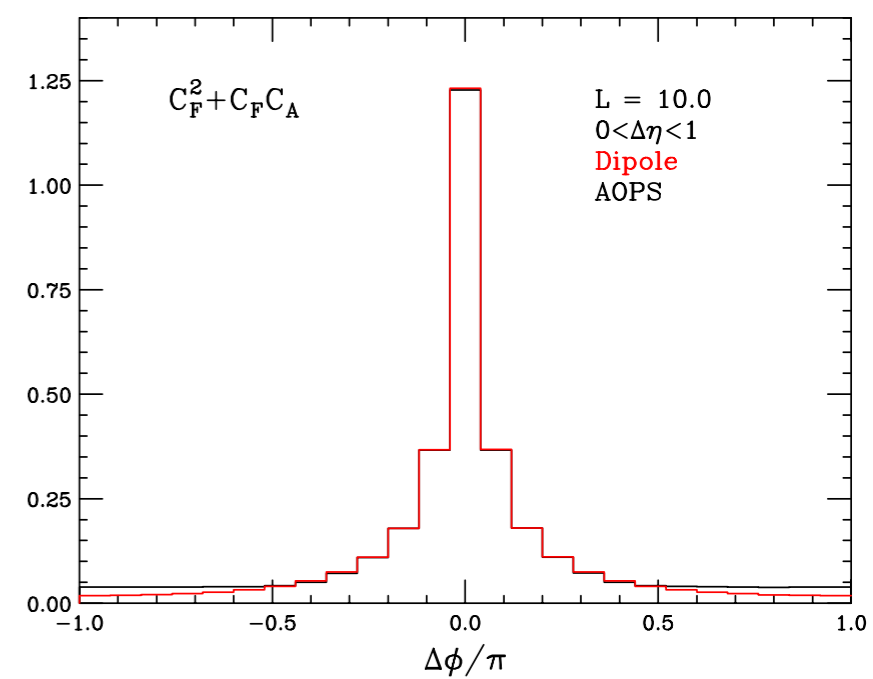
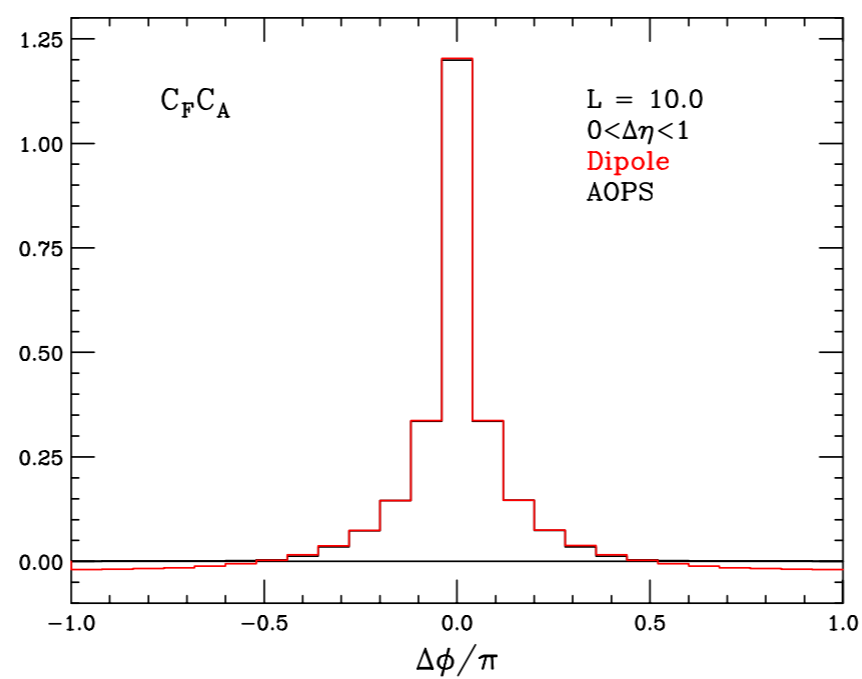
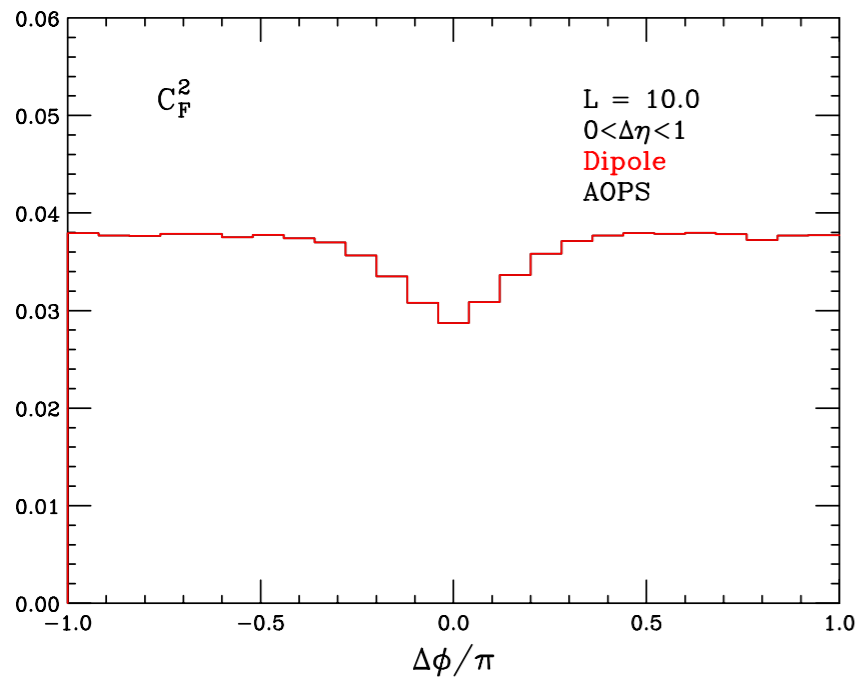
$$\int D_k^{ij} \frac{d\phi_k}{2\pi} = \frac{1}{\omega^2} \left[ \frac{\Theta(\theta_{ij} > \theta_{ik})}{1 - \cos \theta_{ik}} + \frac{\Theta(\theta_{ij} > \theta_{jk})}{1 - \cos \theta_{jk}} \right]$$

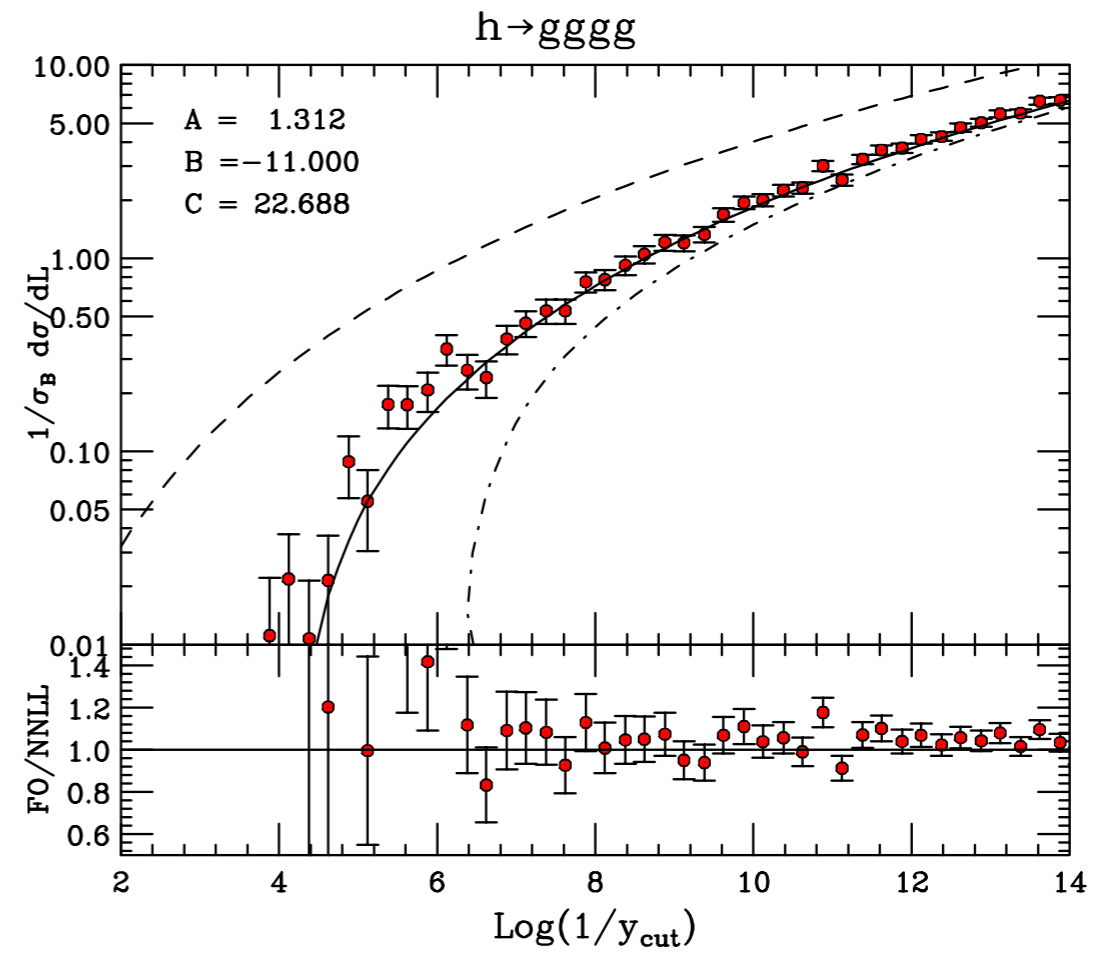
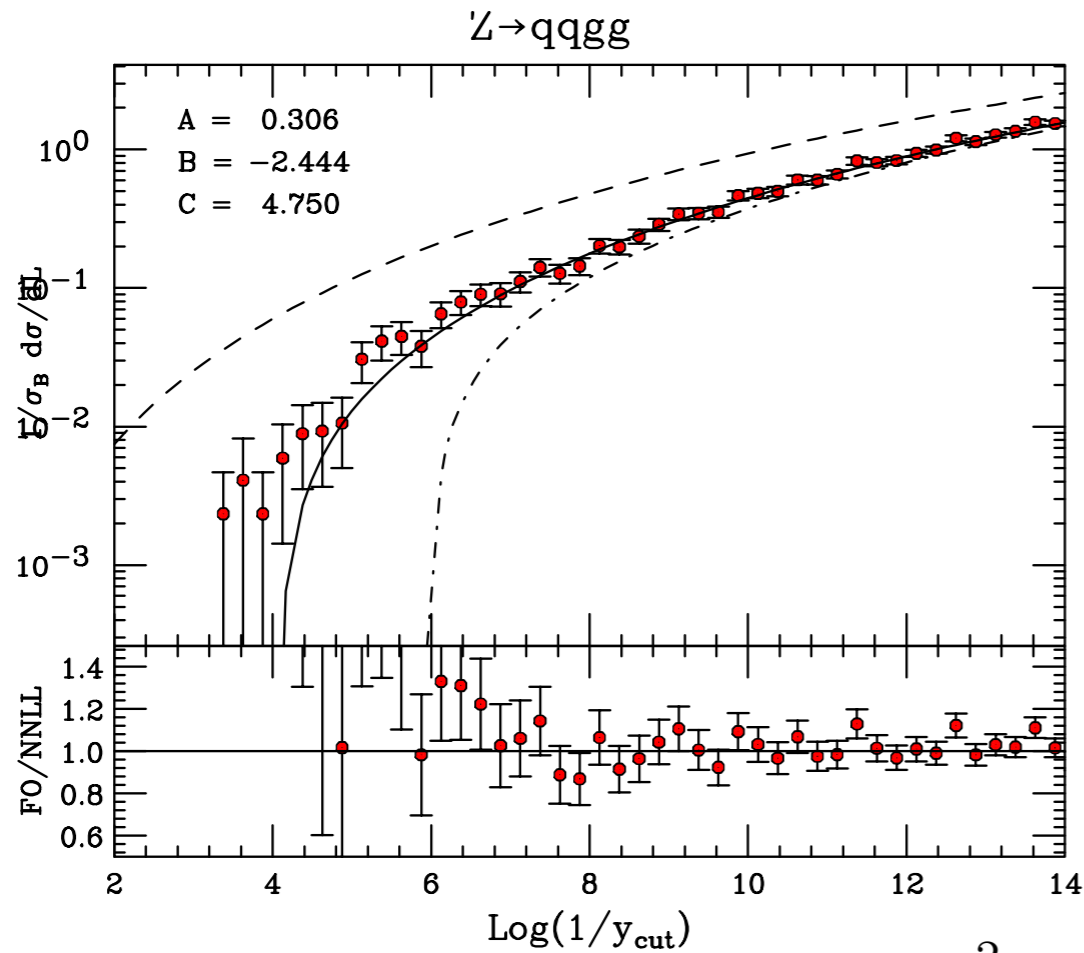


$$dP(q\bar{q}gg) \simeq C_F C_A \left( \frac{\alpha_s}{\pi} \right)^2 \frac{\Theta(\theta_{q\bar{q}} > \theta_{q1})}{1 - \cos \theta_{q1}} \frac{\Theta(\theta_{q1} > \theta_{12})}{1 - \cos \theta_{12}} \frac{d\omega_1}{\omega_1} \frac{d\omega_2}{\omega_2} d \cos \theta_1 d \cos \theta_2$$

- Some azimuthal correlations lost through averaging







$$P(q\bar{q}gg) \simeq \left(\frac{\alpha_s}{\pi}\right)^2 (AL^4 + BL^3 + CL^2)$$

$$\frac{dP}{dL} \simeq \left(\frac{\alpha_s}{\pi}\right)^2 (4AL^3 + 3BL^2 + 2CL)$$

- AOPS vs Exact LOME (Madgraph)
  - A=collinear-soft, B=collinear-nonsoft
  - C not reliable (but improves agreement)

# NLO Showers

- Höche, & Prestel, 1705.00742, & Krauss 1705.00982
  - Include NLO terms in  $1 \rightarrow 2$  ( $q \rightarrow q'$  differential)
- Dulat, Höche & Prestel, 1805.03757
  - Differential double soft vs CMW

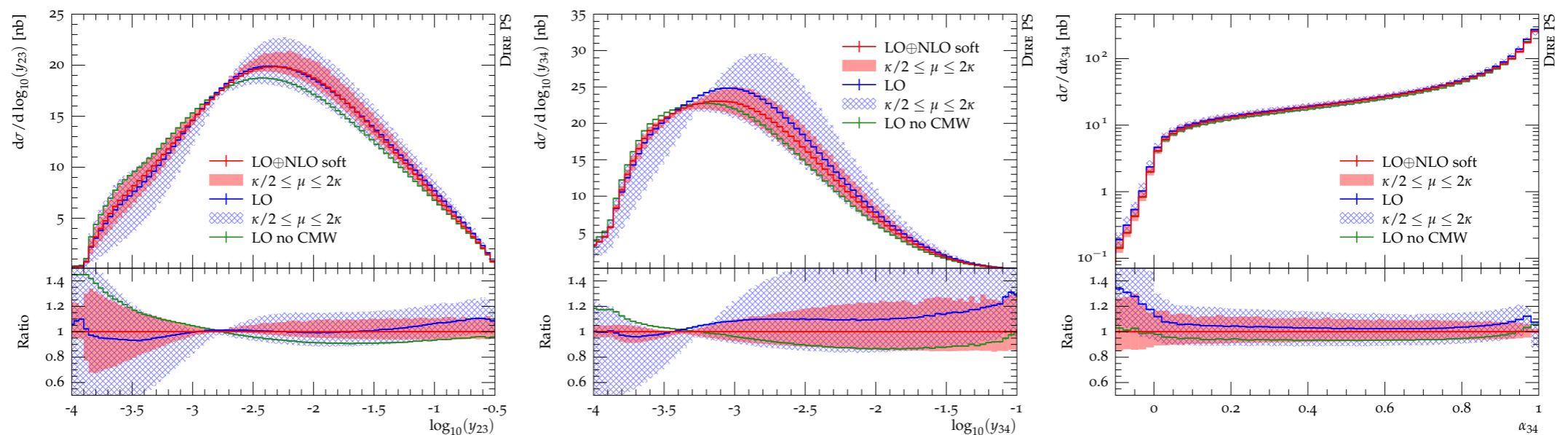
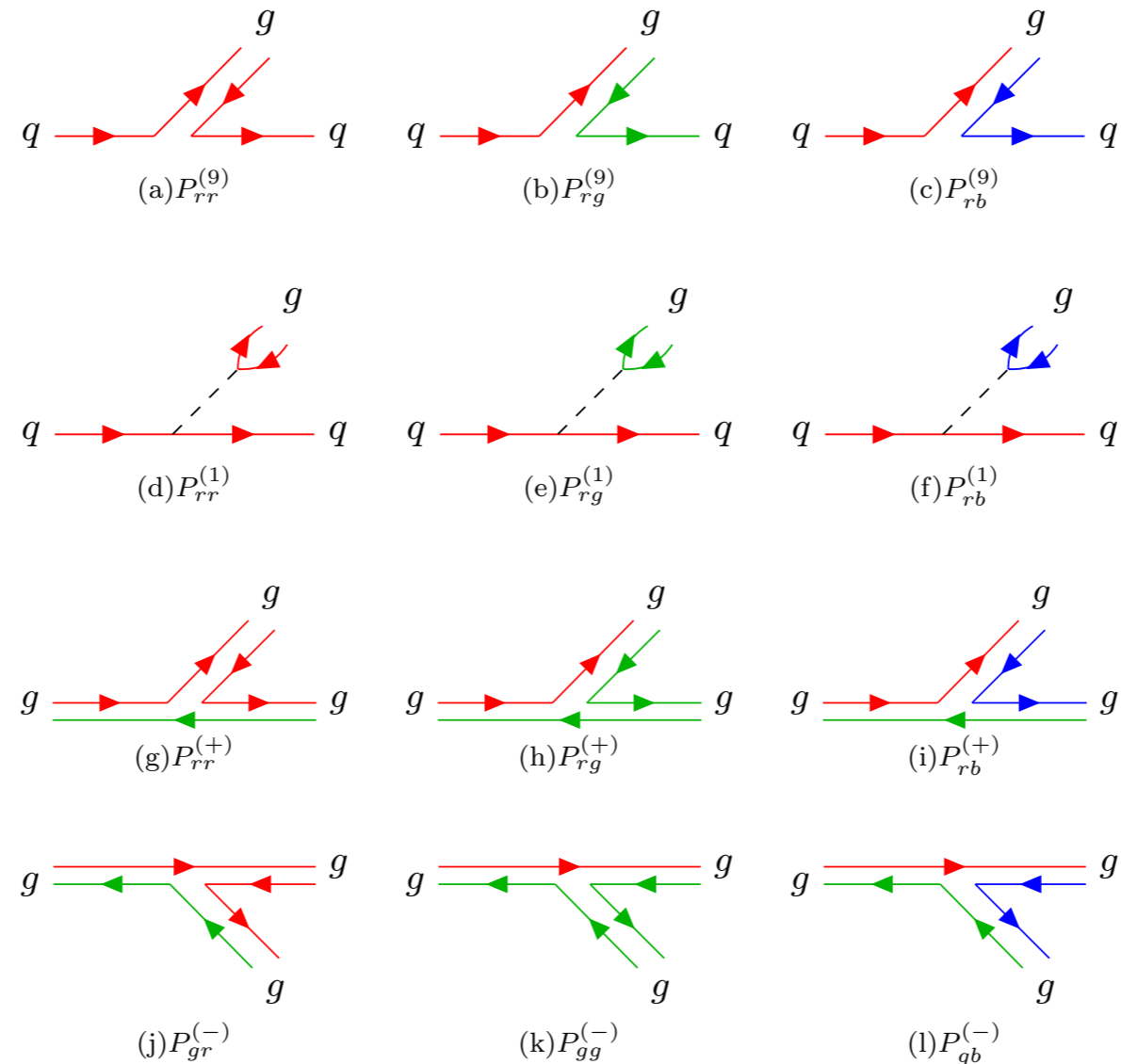


FIG. 8. Scale variations in the leading-order and next-to-leading order (soft) parton shower simulation of  $e^+e^- \rightarrow \text{hadrons}$  at LEP I energies at parton level. We compare to both the plain leading-order predictions (green) and the result in the CMW scheme (blue).

# Subleading Colour

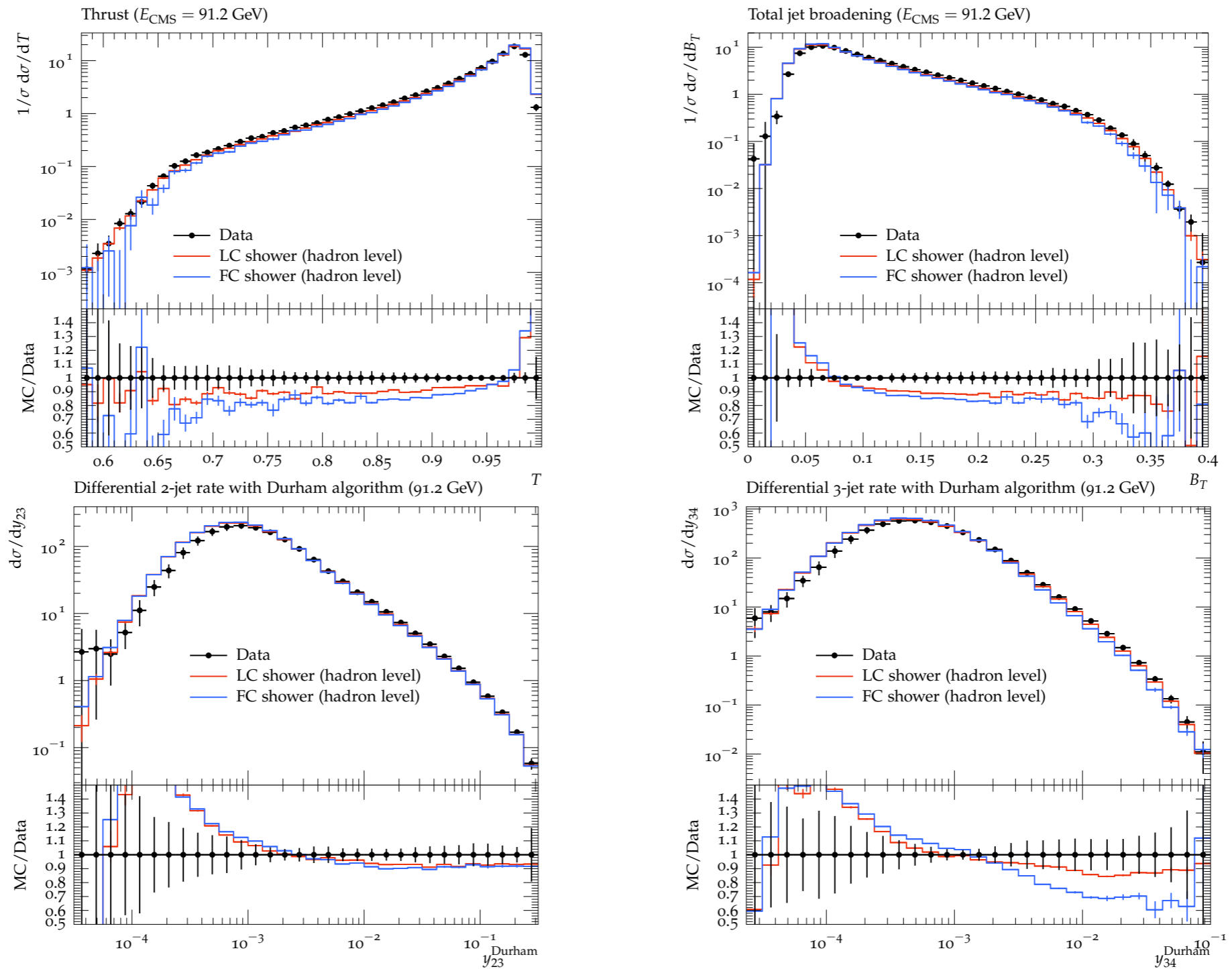
- Isaacson & Prestel, 1806.10102 use colour-flow basis



- Negative weights: MC efficiency? Looks OK ...

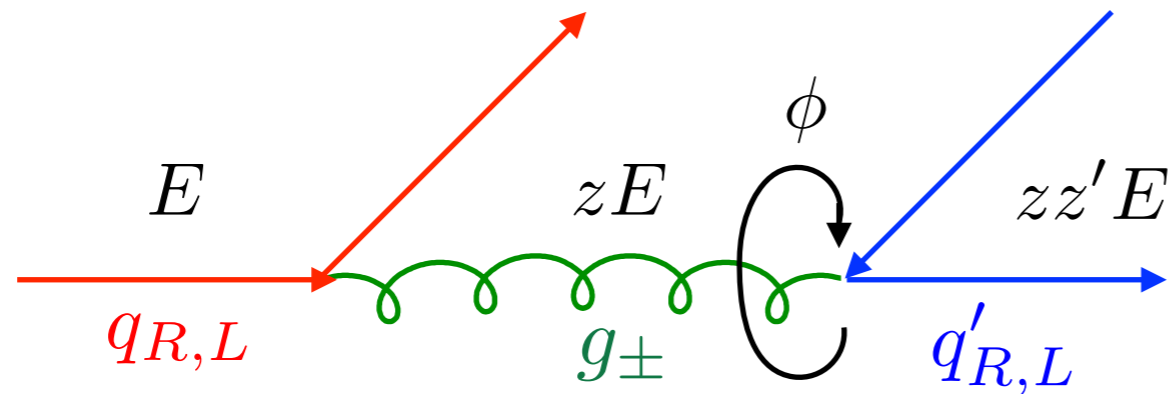
# Subleading Colour

- Isaacson & Prestel, 1806.10102



# Spin Effects in Showers

# Azimuthal Correlations



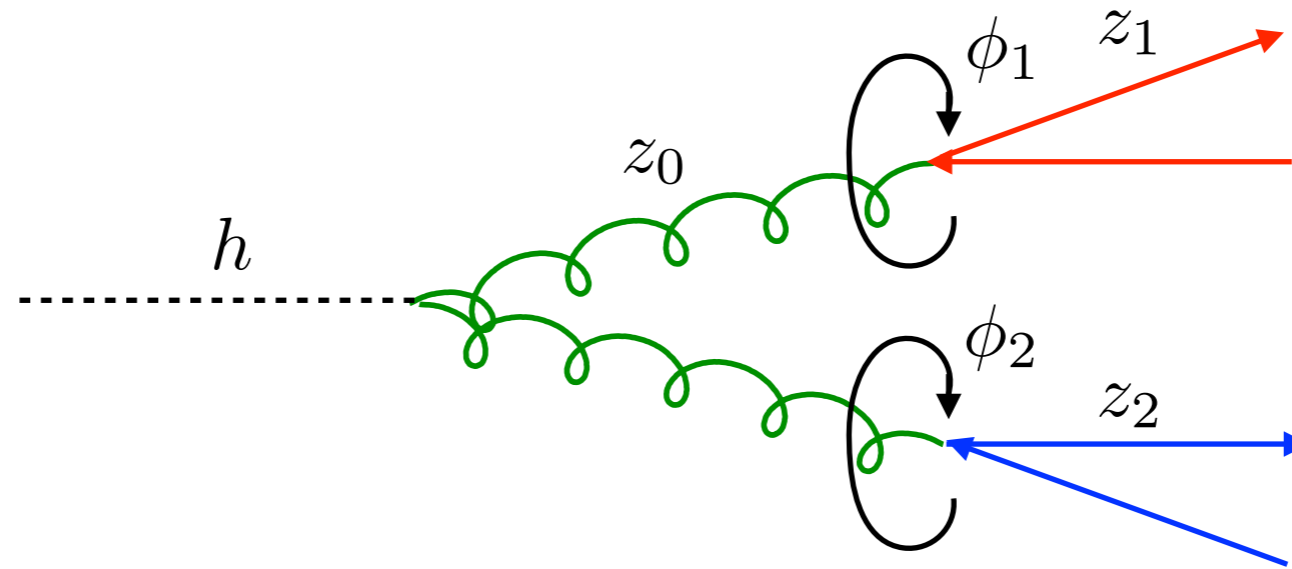
			$\mathcal{M}_{h_1 h_2 h_3}$
R	+	R	$z^{-\frac{1}{2}} e^{i\phi} z'$
R	+	L	$z^{-\frac{1}{2}} e^{i\phi} (1 - z')$
R	-	R	$-(1 - z) z^{-\frac{1}{2}} e^{-i\phi} (1 - z')$
R	-	L	$-(1 - z) z^{-\frac{1}{2}} e^{-i\phi} z'$

$$|\mathcal{M}_{R+R} + \mathcal{M}_{R-R}|^2 = \frac{1}{z} [z'^2 + (1 - z)^2 (1 - z')^2 - 2(1 - z) z' (1 - z') \cos 2\phi]$$

$$|\mathcal{M}_{R+L} + \mathcal{M}_{R-L}|^2 = \frac{1}{z} [(1 - z')^2 + (1 - z)^2 z'^2 - 2(1 - z) z' (1 - z') \cos 2\phi]$$

$$\sum_{h_3} \left| \sum_{h_2} \mathcal{M}_{h_1 h_2 h_3} \right|^2 = \frac{1 + (1 - z)^2}{z} [z'^2 + (1 - z')^2] - 4 \frac{(1 - z)}{z} z' (1 - z') \cos 2\phi$$

# EPR Correlations



$$P(h \rightarrow q\bar{q}q\bar{q}) \propto 1 + a(z_1)a(z_2) \cos 2(\phi_1 - \phi_2)$$

- where  $a(z) = \frac{2z(1-z)}{1-2z(1-z)}$
- Fully included in Herwig (CKR method)

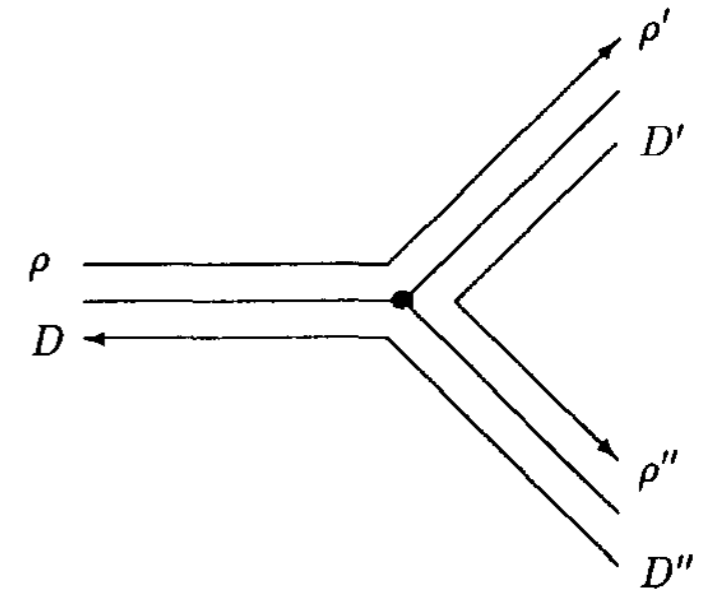
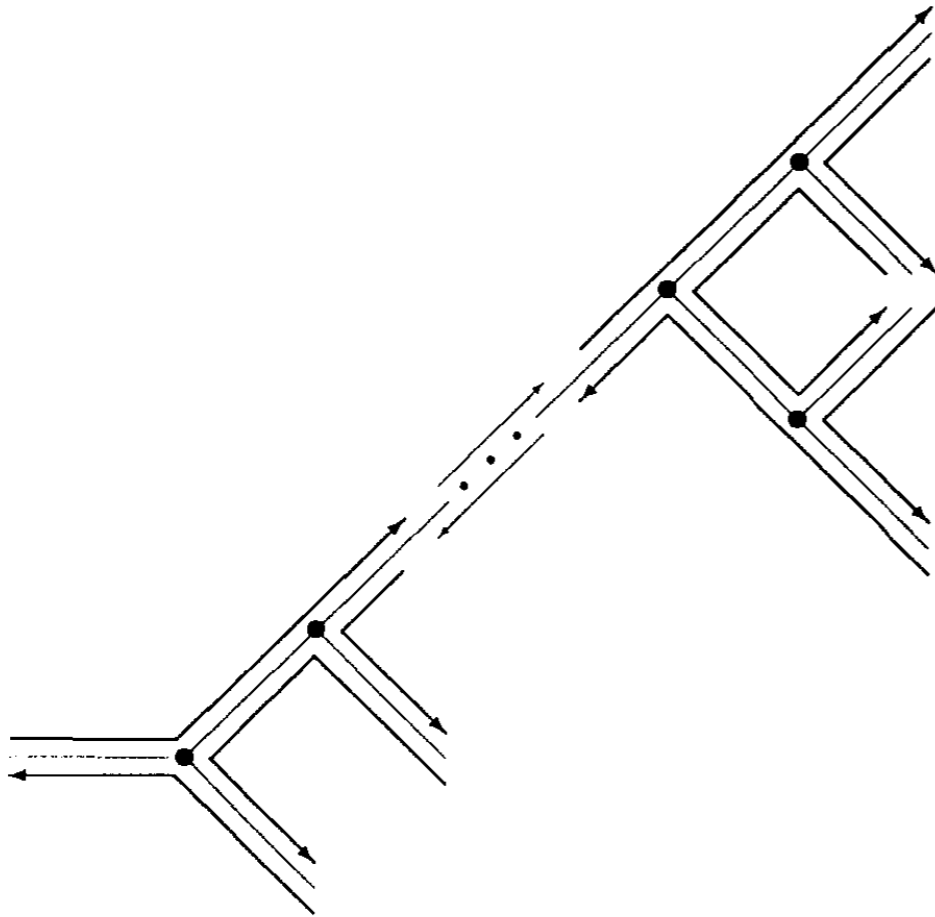
Collins, NPB304(1988)794

Knowles, CPC58(1990)271

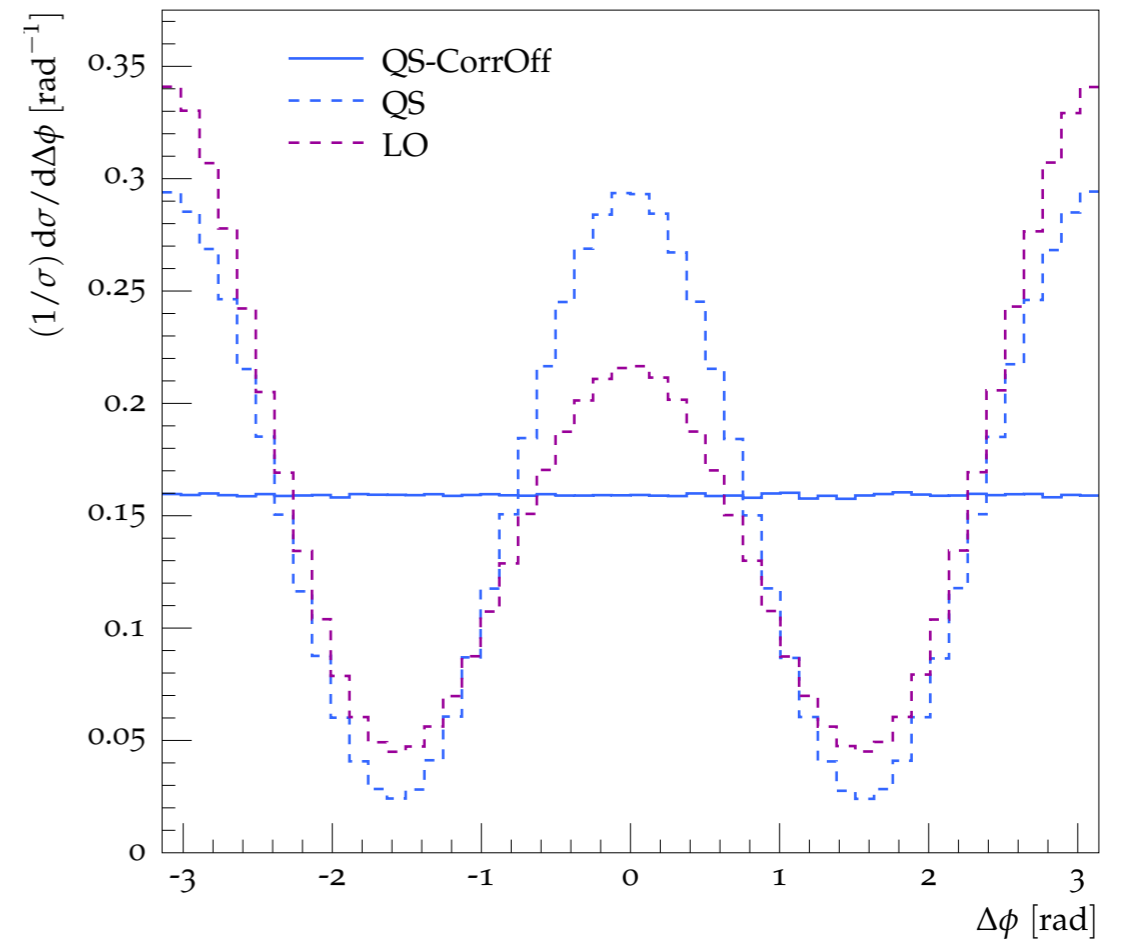
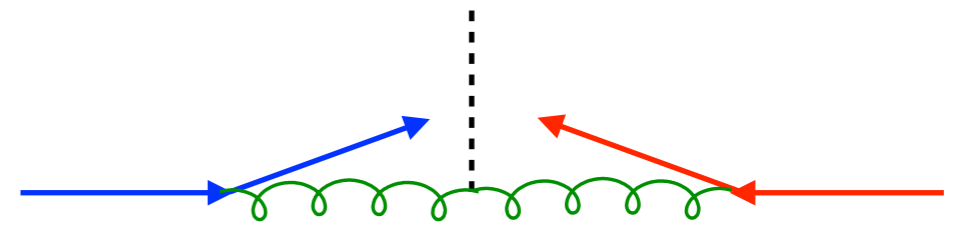
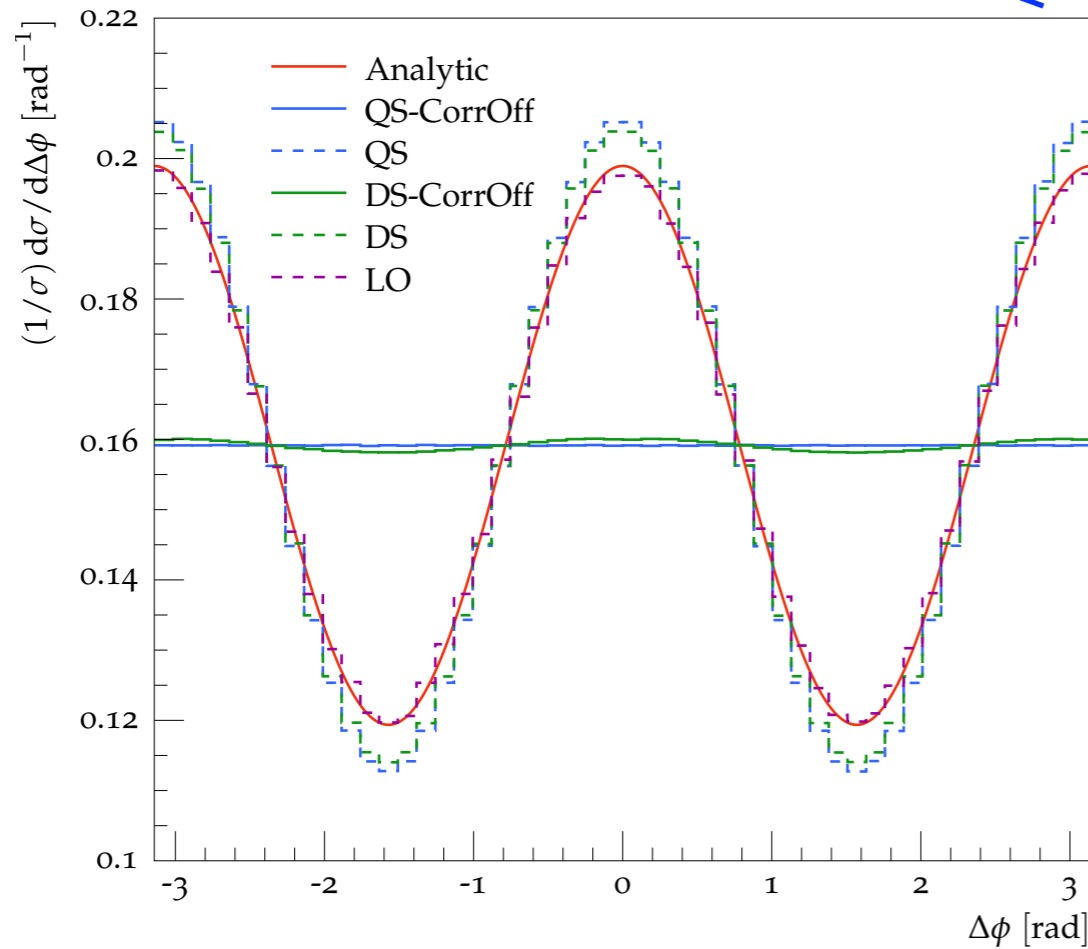
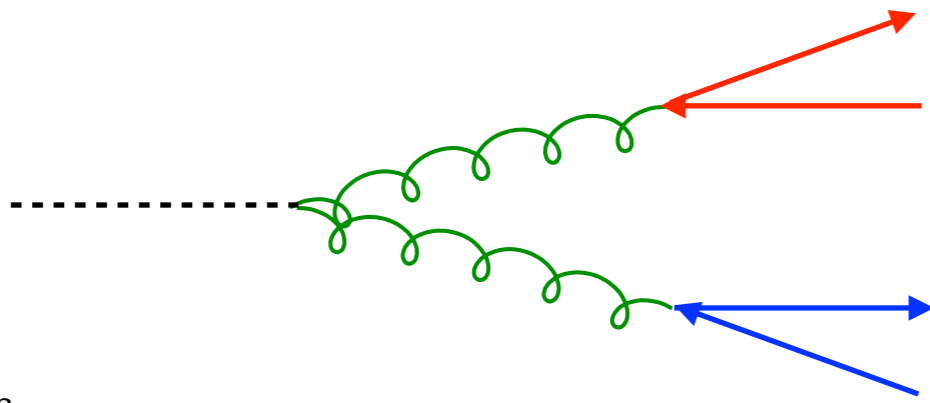
Richardson, JHEP111(2001)029



# CKR Method



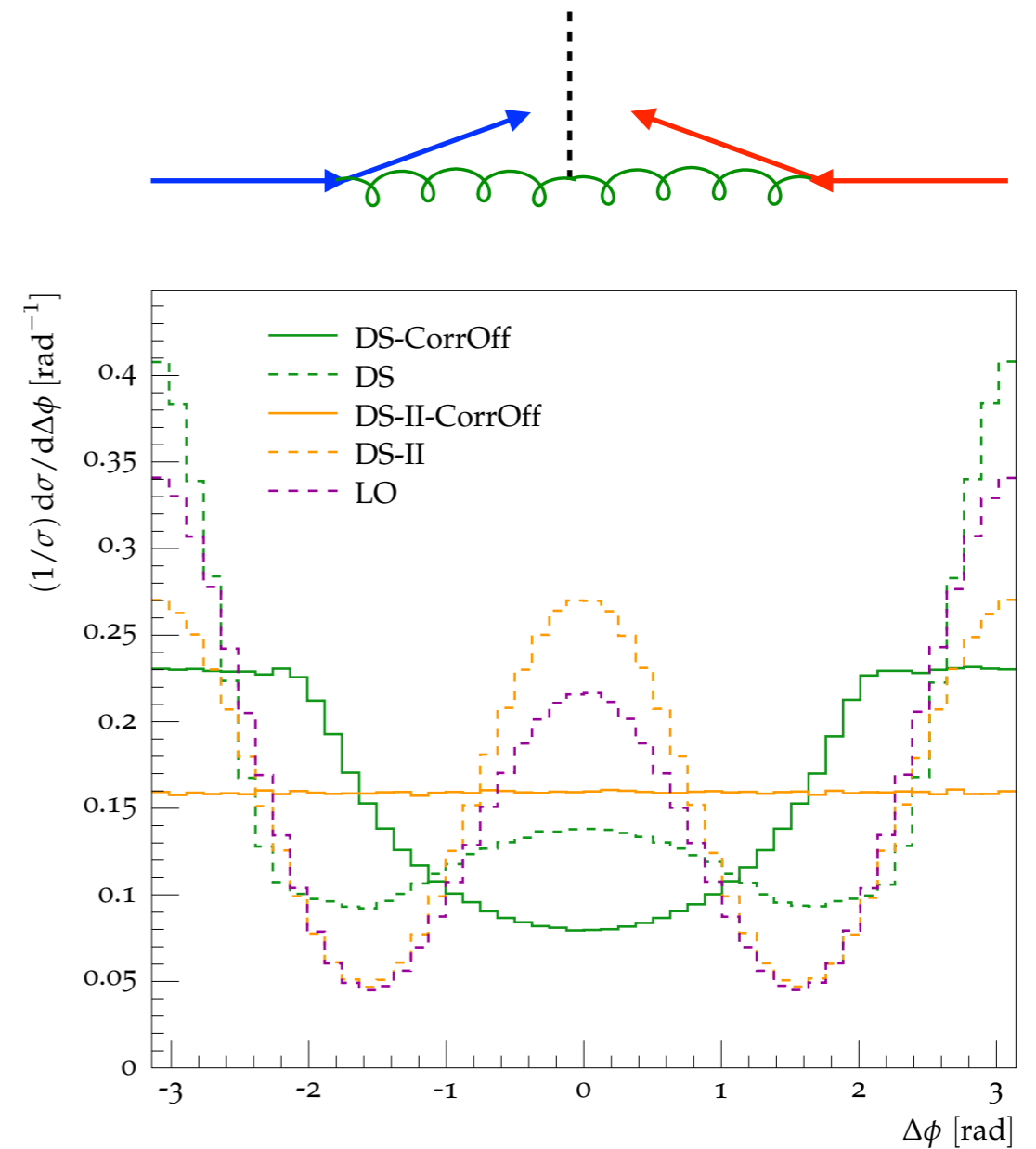
- Backtracking essential for linear algorithm



- LO=MadGraph5, QS=Herwig7AO, DS=Herwig7DS

Richardson & Webster, 1807.01955

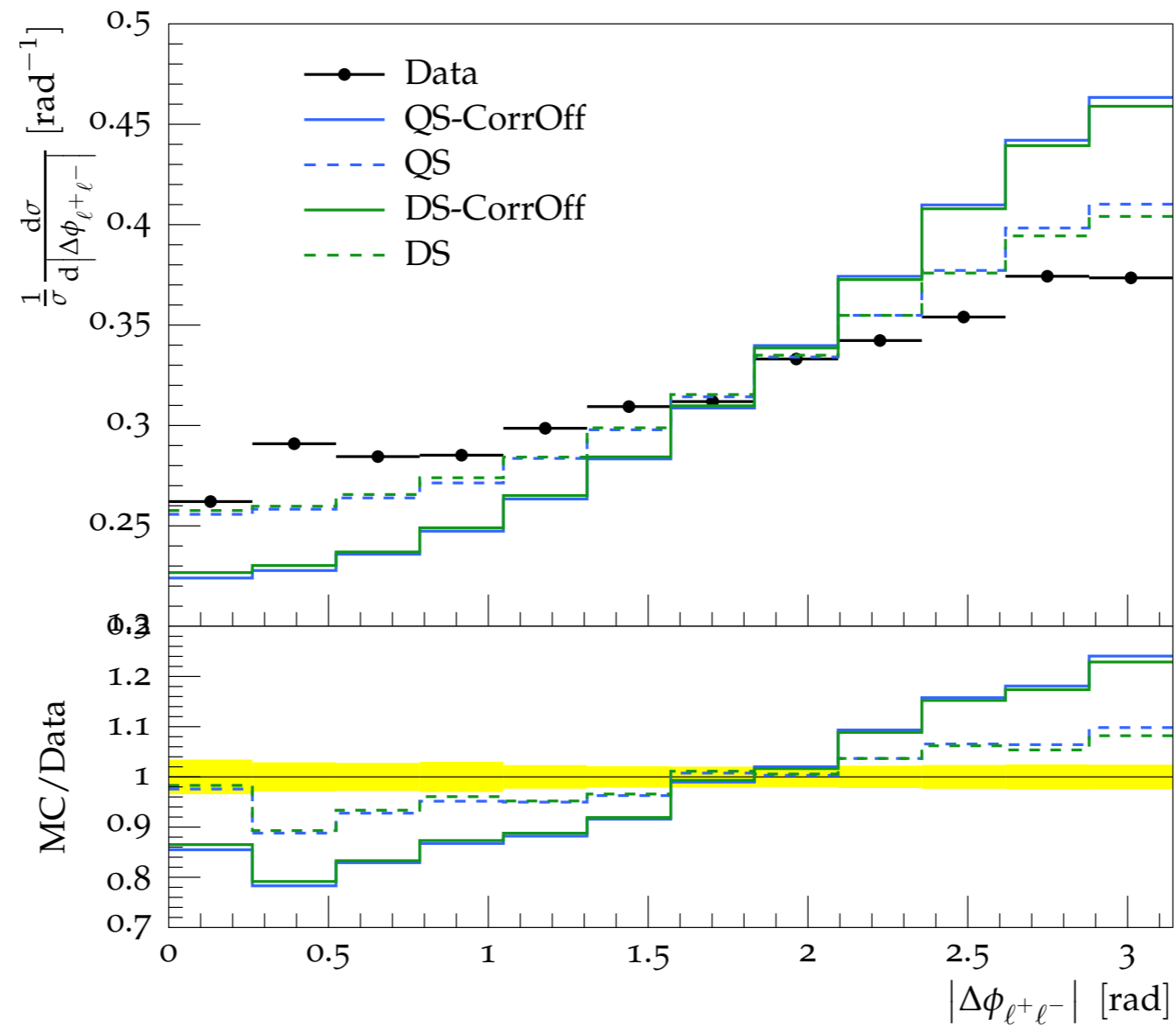
- Different dipole options illustrate recoil ambiguity



Richardson & Webster, 1807.01955

- Dilepton correlation in top decays

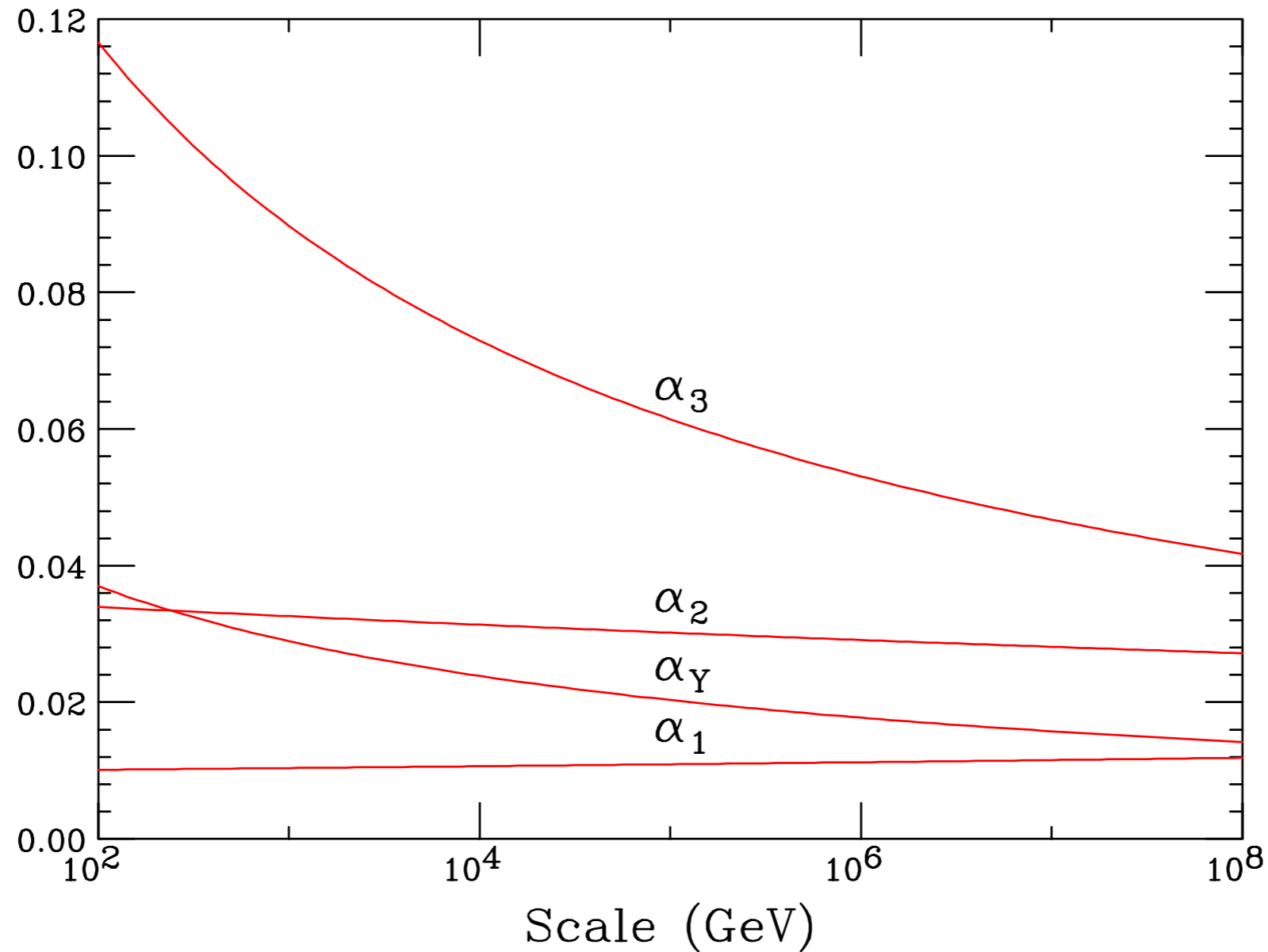
$$pp \rightarrow t\bar{t} \rightarrow b\bar{b} \ell^+ \ell^- \nu_\ell \bar{\nu}_\ell$$



Richardson & Webster, 1807.01955

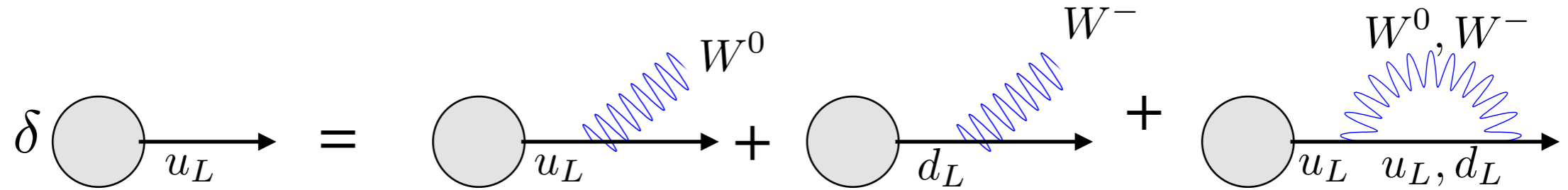
# Electroweak Showering

## Standard Model couplings



- Far above EW scale, at  $q \gg m_W$ , we have approximately unbroken  $SU(3) \times SU(2) \times U(1)$
- Corrections  $\sim m_W/q$

- Real-virtual emission mismatch leads to **double logarithms** of  $q/m_W$



$$q \frac{\partial}{\partial q} u_L(x, q) = \frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} \frac{dz}{z} P_{ff}(z) \left[ \frac{1}{3} u_L(x/z, q) + \frac{2}{3} d_L(x/z, q) - z u_L(x, q) \right]$$

$$q \frac{\partial}{\partial q} d_L(x, q) = \frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} \frac{dz}{z} P_{ff}(z) \left[ \frac{1}{3} d_L(x/z, q) + \frac{2}{3} u_L(x/z, q) - z d_L(x, q) \right]$$

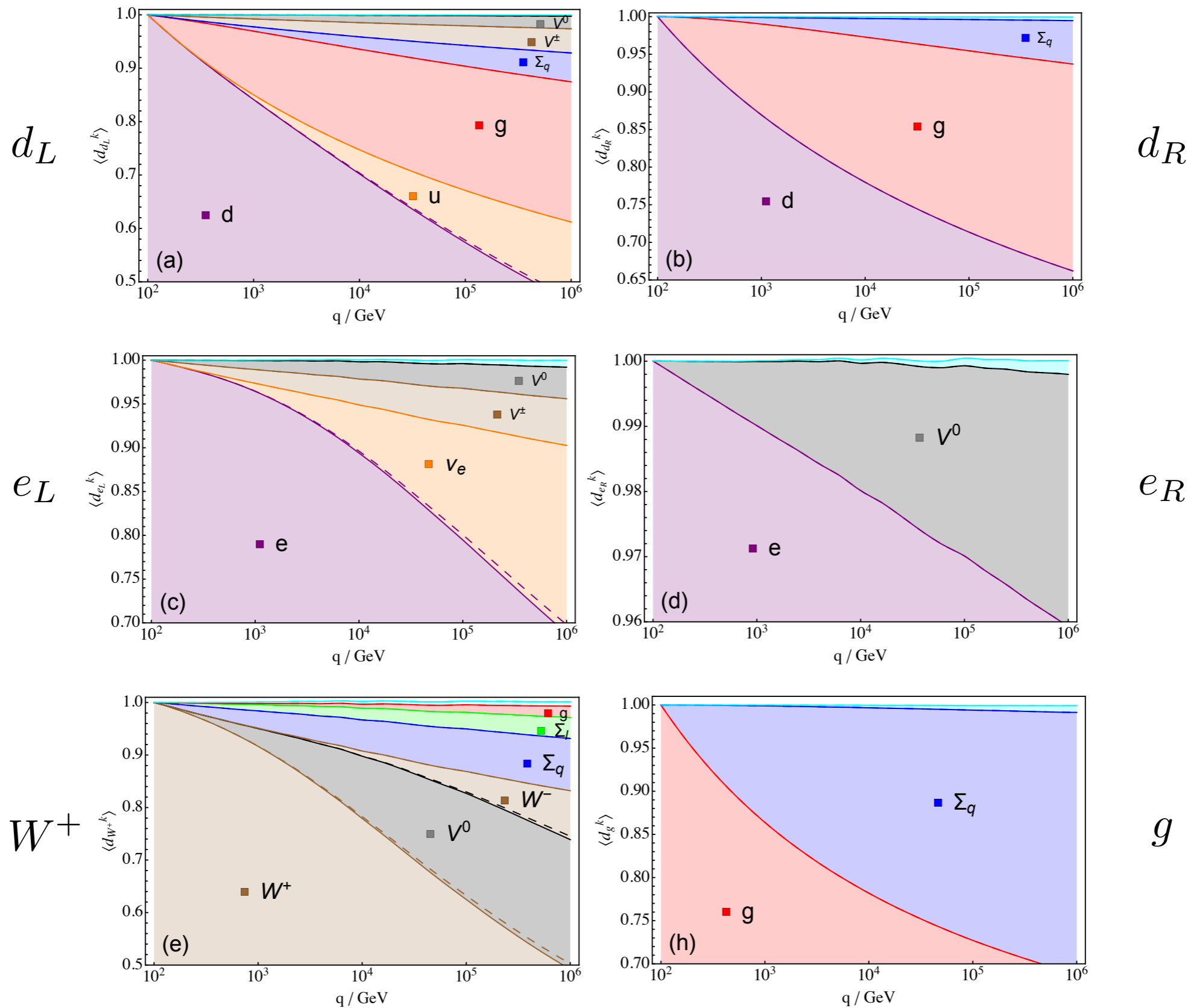
- Define  $Q^\pm = \frac{1}{2} (u_L \pm d_L)$

$$q \frac{\partial}{\partial q} Q^+(x, q) = \frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} \frac{dz}{z} P_{ff}(z) [Q^+(x/z, q) - z Q^+(x, q)]$$

$$q \frac{\partial}{\partial q} Q^-(x, q) = -\frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} \frac{dz}{z} P_{ff}(z) \left[ \frac{1}{3} Q^-(x/z, q) + z Q^-(x, q) \right]$$

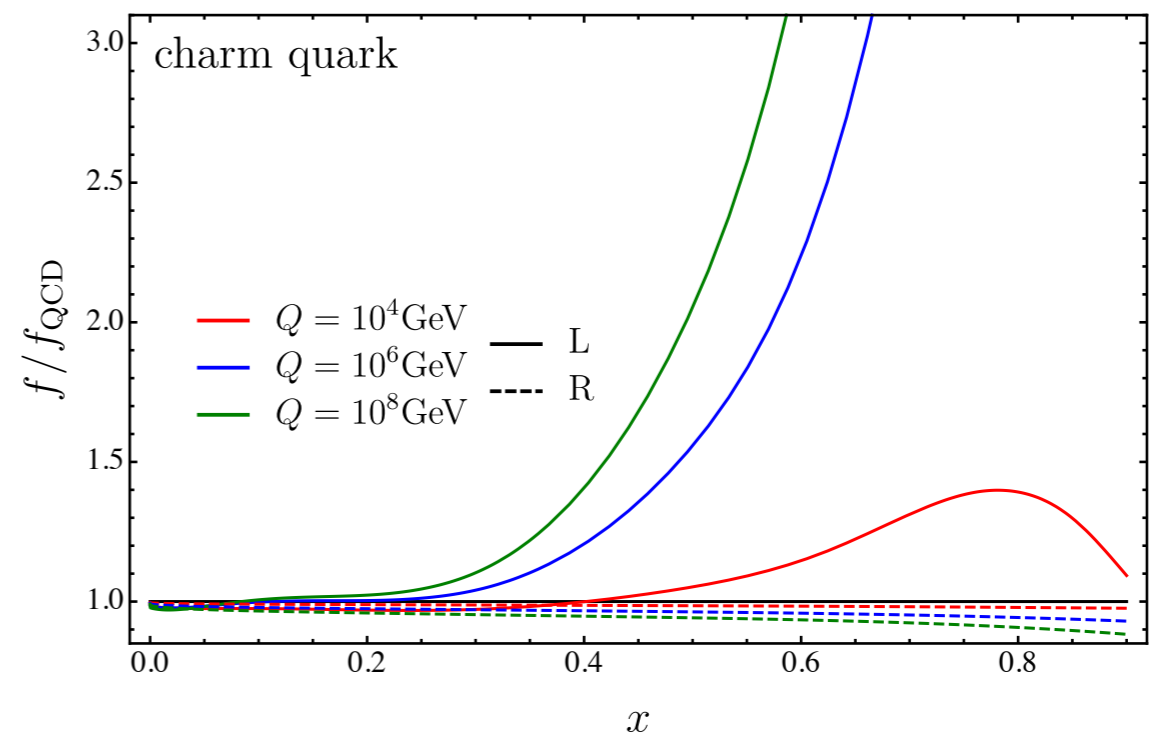
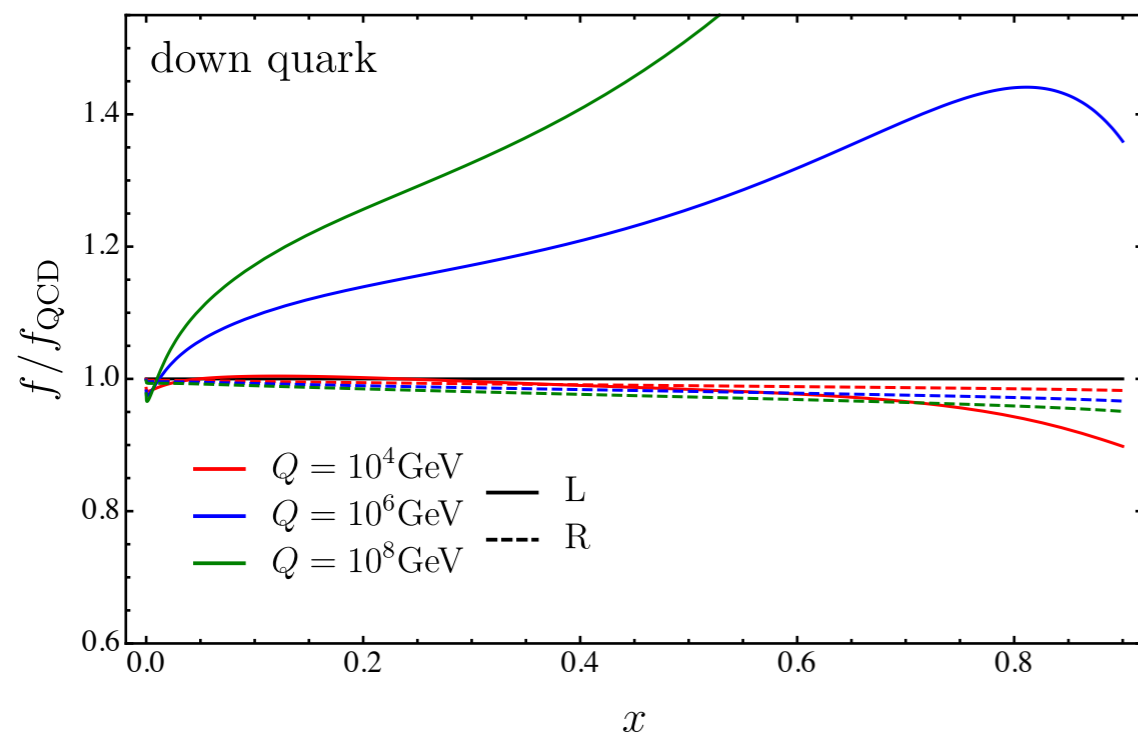
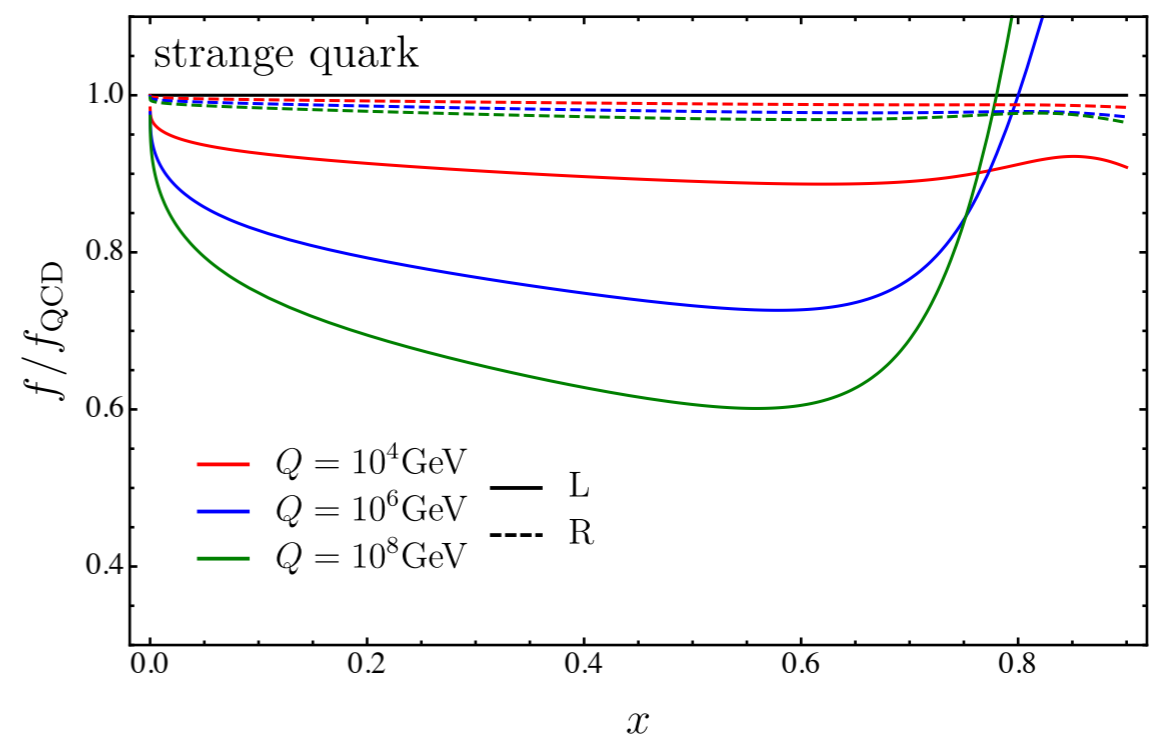
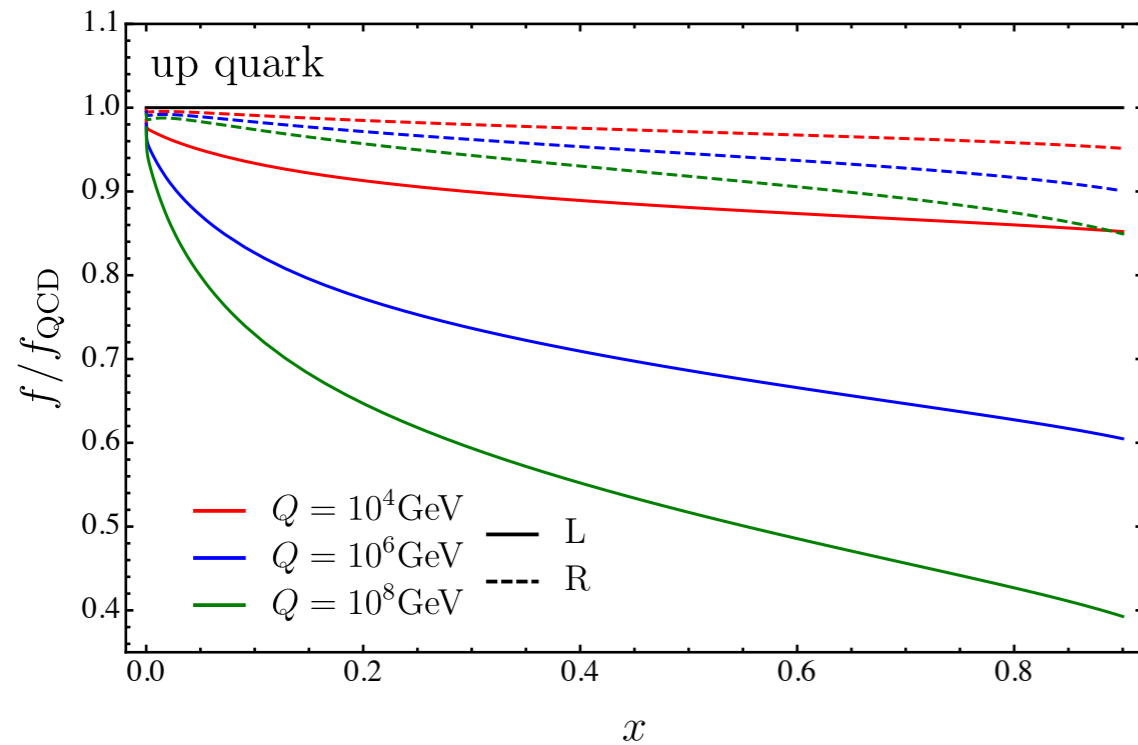
- $Q^+$  has DGLAP (single-log) evolution
- $Q^-$  has double-log damping (asymptotic symmetry)

# Momentum fractions in jets

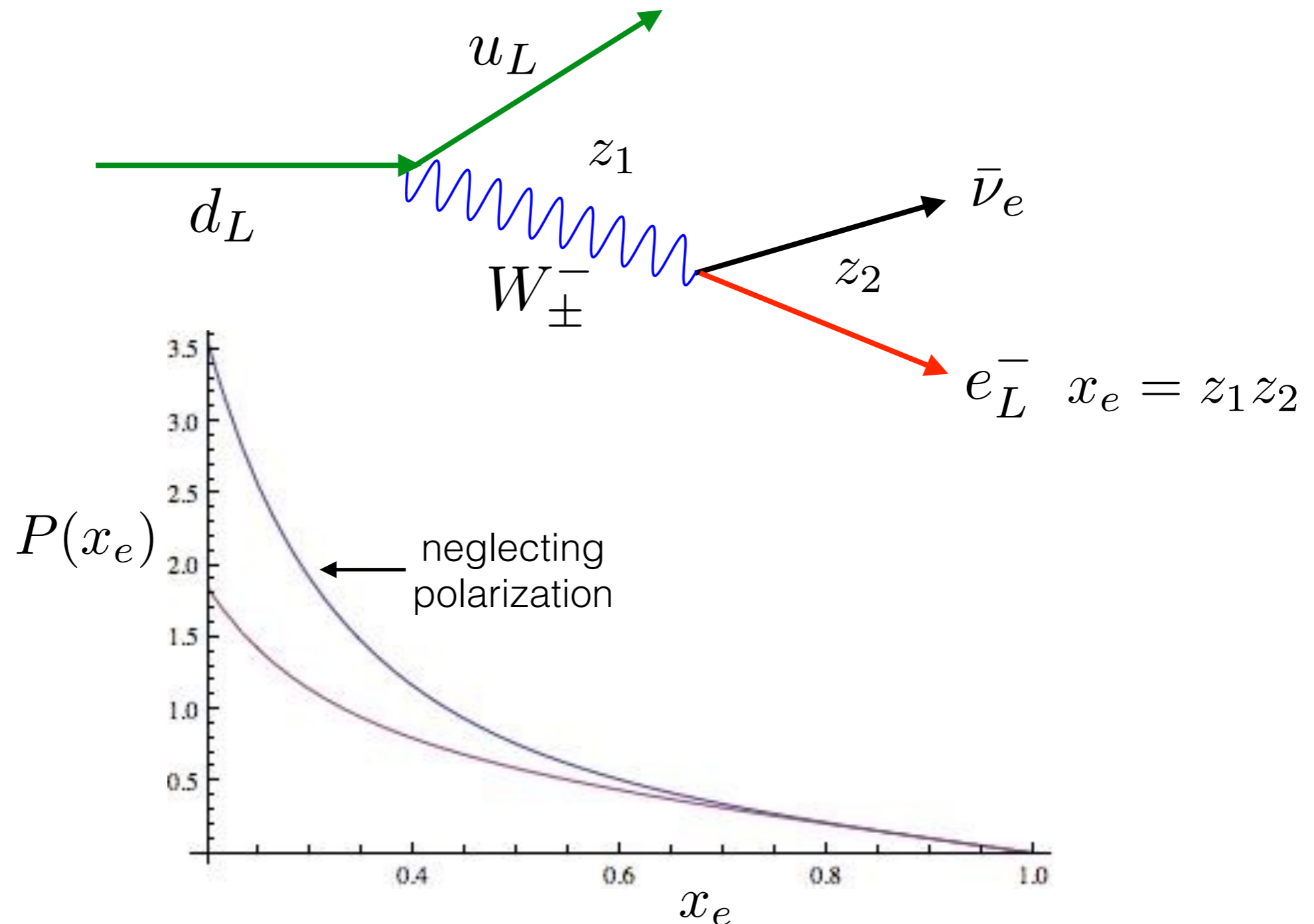




- Similarly in initial-state showering (PDF evolution)
  - $u_L$ - $d_L$  (&  $s_L$ - $c_L$ ) has double-log damping

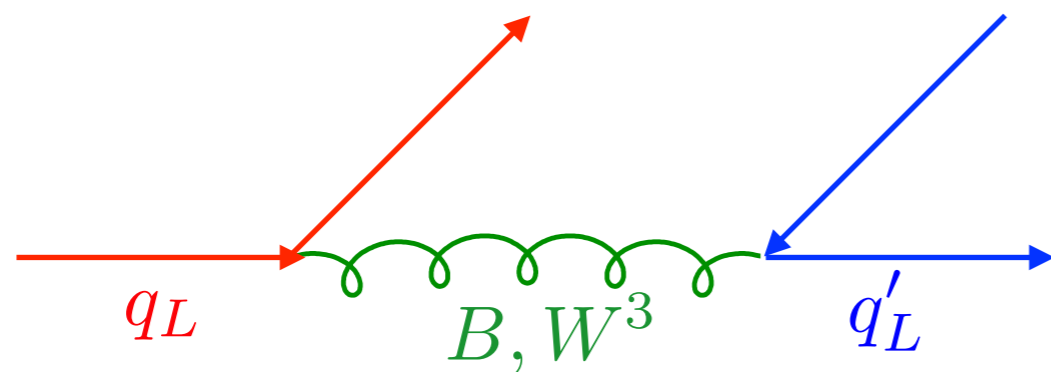


- Parity violation implies large **polarisation effects**
- Azimuthal integration cancels helicity interference (could be handled by CKR method)

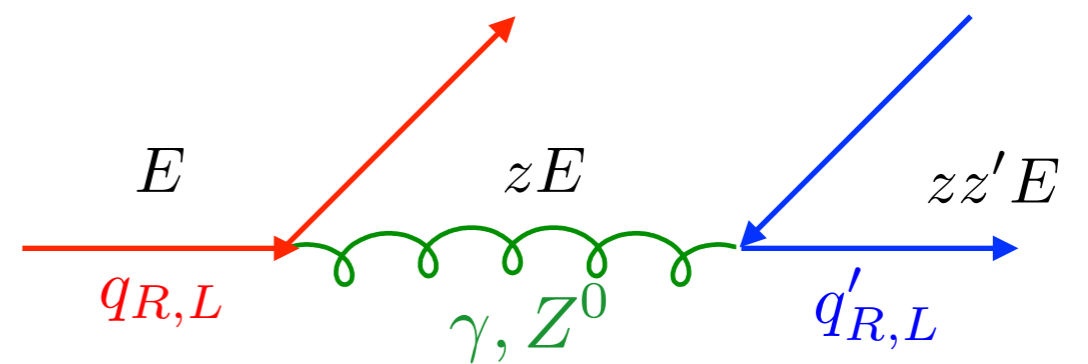


# Mixed State Showering

- Mixed states have different couplings



Unbroken phase

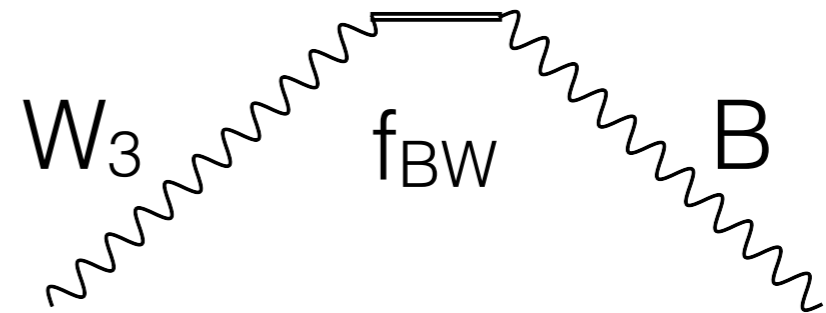
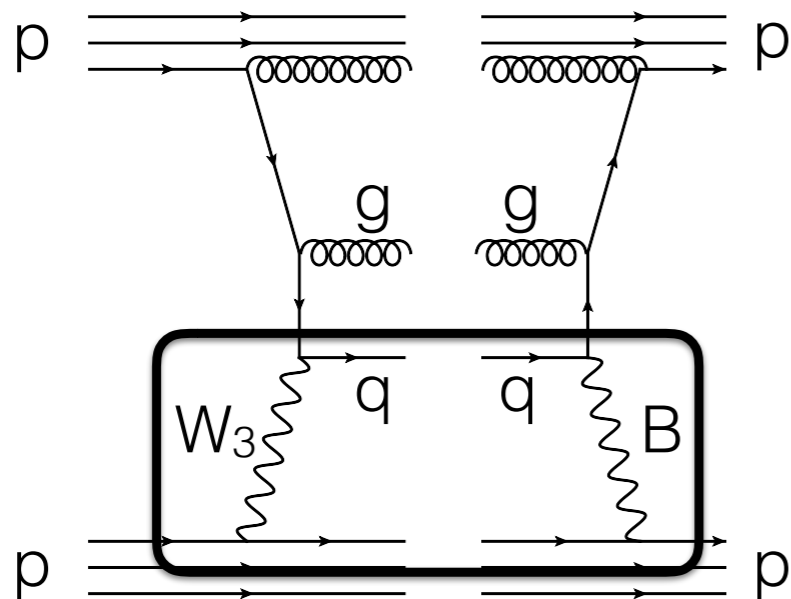


Broken phase

- Is there an analog of CKM?

# Mixed U(1)xSU(2) PDF

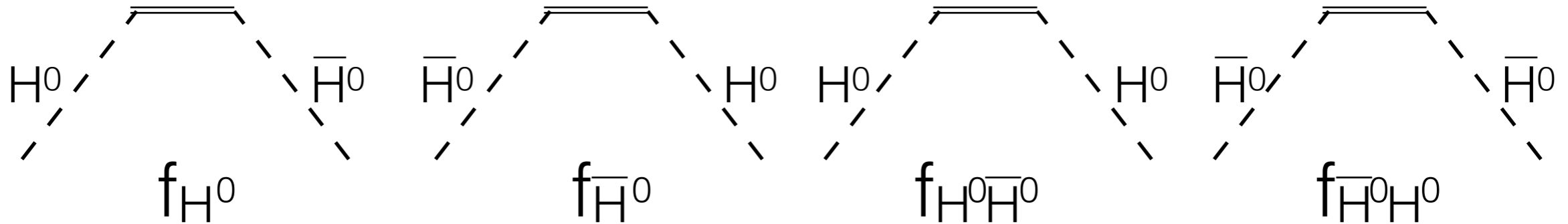
$$f_{BW}(x) = \frac{2}{\bar{n} \cdot p} \int \frac{dy}{2\pi} e^{-i 2x \bar{n} \cdot p y} \bar{n}^\mu \bar{n}_\nu \langle p | B_{\mu\lambda}(y) W_3^{\lambda\nu}(-y) | p \rangle + \text{h.c.}$$



$$\alpha_M = \sqrt{\alpha_1 \alpha_2}$$

- Left-handed quarks have isospin and hypercharge, so they can generate  $f_{BW}$  ( $I^{CP} = 1^+$ )
- This means in broken basis we have  $f_y$ ,  $f_z$  and  $f_{yz}$

# Mixed Higgs PDF



$$H^0 = \frac{1}{\sqrt{2}} (h - iZ_L), \quad \bar{H}^0 = \frac{1}{\sqrt{2}} (h + iZ_L)$$

$$h = \frac{1}{\sqrt{2}} (H^0 + \bar{H}^0), \quad Z_L = \frac{i}{\sqrt{2}} (H^0 - \bar{H}^0)$$

$$f_{HH}^{1\pm} = \frac{1}{2} (f_{H^0 \bar{H}^0} \pm f_{\bar{H}^0 H^0})$$

- $f_{HH}^{1+}$  distinguishes between Higgs and  $Z_L$

$$f_{Z_L} = f_H^{0+} - f_H^{1+} - f_{HH}^{1+},$$

$$f_h = f_H^{0+} - f_H^{1+} + f_{HH}^{1+}.$$

# Amplitude-level PS

$$\begin{aligned}
\mathbf{P}_{ij} = & \delta_{s_j, \frac{1}{2}} \delta_j^{\text{final}} \left( \sqrt{\frac{\mathcal{P}_{qq}}{2\mathcal{C}_F(1+z_i^2)}} \frac{1}{\langle q_i \tilde{p}_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2 \mathcal{P}_{qq}}{2\mathcal{C}_F(1+z_i^2)}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \right. \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_F(2-2z_i+z_i^2)}} \frac{1}{\langle \tilde{p}_j q_i \rangle} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{2\mathcal{C}_F(2-2z_i+z_i^2)}} \frac{1}{[q_i \tilde{p}_j]} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \left. \right) \\
& + \delta_{s_j, -\frac{1}{2}} \delta_j^{\text{final}} \left( \sqrt{\frac{\mathcal{P}_{qq}}{2\mathcal{C}_F(1+z_i^2)}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{z_i^2 \mathcal{P}_{qq}}{2\mathcal{C}_F(1+z_i^2)}} \frac{1}{\langle q_i \tilde{p}_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \right. \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_F(2-2z_i+z_i^2)}} \frac{1}{[q_i \tilde{p}_j]} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{2\mathcal{C}_F(2-2z_i+z_i^2)}} \frac{1}{\langle \tilde{p}_j q_i \rangle} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \left. \right) \\
& + \delta_{s_j, 1} \delta_j^{\text{final}} \left( \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{2T_R(1-2z_i(1-z_i))}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{W}^{ij} - \mathbb{1}) (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{P}_j^2 \mathbb{S}^{+\frac{1}{2}i}) \right. \\
& + \sqrt{\frac{z_i^2 \mathcal{P}_{gg}}{2T_R(1-2z_i(1-z_i))}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{W}^{ij} - \mathbb{1}) (\mathbb{T}_j^g \otimes \mathbb{P}_j^2 \mathbb{S}^{-\frac{1}{2}i}) \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{\langle q_i \tilde{p}_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^4 \mathcal{P}_{gg}}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[q_i \tilde{p}_j]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{S}^{+1_i}) \left. \right) \\
& + \delta_{s_j, -1} \delta_j^{\text{final}} \left( \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{2T_R(1-2z_i(1-z_i))}} \frac{1}{\langle q_i \tilde{p}_j \rangle} (\mathbb{W}^{ij} - \mathbb{1}) (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{P}_j^2 \mathbb{S}^{-\frac{1}{2}i}) \right. \\
& + \sqrt{\frac{z_i^2 \mathcal{P}_{gg}}{2T_R(1-2z_i(1-z_i))}} \frac{1}{\langle q_i \tilde{p}_j \rangle} (\mathbb{W}^{ij} - \mathbb{1}) (\mathbb{T}_j^g \otimes \mathbb{P}_j^2 \mathbb{S}^{+\frac{1}{2}i}) \\
& + \sqrt{\frac{\mathcal{P}_{gg}}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[\tilde{p}_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{z_i^4 \mathcal{P}_{gg}}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{\langle \tilde{p}_j q_i \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{2\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[q_i \tilde{p}_j]} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{S}^{-1_i}) \left. \right) \\
& + \delta_{s_j, \frac{1}{2}} \delta_j^{\text{initial}} \sqrt{\frac{1}{z_i}} \left( \sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_F(1+z_i^2)}} \frac{1}{\langle q_i p_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{z_i^2 \mathcal{P}_{qq}}{\mathcal{C}_F(1+z_i^2)}} \frac{1}{[p_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \right. \\
& + \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{n_f \mathcal{C}_F(1-2z_i(1-z_i))}} \frac{1}{[p_j q_i]} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^2 \mathcal{P}_{gg}}{n_f \mathcal{C}_F(1-2z_i(1-z_i))}} \frac{1}{\langle q_i p_j \rangle} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \left. \right) \\
& + \delta_{s_j, -\frac{1}{2}} \delta_j^{\text{initial}} \sqrt{\frac{1}{z_i}} \left( \sqrt{\frac{\mathcal{P}_{qq}}{\mathcal{C}_F(1+z_i^2)}} \frac{1}{[p_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{z_i^2 \mathcal{P}_{qq}}{\mathcal{C}_F(1+z_i^2)}} \frac{1}{\langle q_i p_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \right. \\
& + \sqrt{\frac{(1-z_i)^2 \mathcal{P}_{gg}}{n_f \mathcal{C}_F(1-2z_i(1-z_i))}} \frac{1}{\langle q_i p_j \rangle} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{z_i^2 \mathcal{P}_{gg}}{n_f \mathcal{C}_F(1-2z_i(1-z_i))}} \frac{1}{[p_j q_i]} \mathbb{W}^{ij} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \left. \right) \\
& + \delta_{s_j, 1} \delta_j^{\text{initial}} \sqrt{\frac{1}{z_i}} \left( \sqrt{\frac{2n_f \mathcal{P}_{gg}}{T_R(2-2z_i+z_i^2)}} \frac{1}{\langle p_j q_i \rangle} (\mathbb{T}_j^g \otimes \mathbb{P}_j^2 \mathbb{S}^{+\frac{1}{2}i}) \right. \\
& + \sqrt{\frac{2n_f(1-z_i)^2 \mathcal{P}_{gg}}{T_R(2-2z_i+z_i^2)}} \frac{1}{[q_i p_j]} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{P}_j^2 \mathbb{S}^{-\frac{1}{2}i}) + \sqrt{\frac{\mathcal{P}_{gg}}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{\langle q_i p_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) \\
& + \sqrt{\frac{z_i^4 \mathcal{P}_{gg}}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[q_i p_j]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{\langle q_i p_j \rangle} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{S}^{-1_i}) \left. \right) \\
& + \delta_{s_j, -1} \delta_j^{\text{initial}} \sqrt{\frac{1}{z_i}} \left( \sqrt{\frac{2n_f \mathcal{P}_{gg}}{T_R(2-2z_i+z_i^2)}} \frac{1}{[q_i p_j]} (\mathbb{T}_j^g \otimes \mathbb{P}_j^2 \mathbb{S}^{-\frac{1}{2}i}) \right. \\
& + \sqrt{\frac{2n_f(1-z_i)^2 \mathcal{P}_{gg}}{T_R(2-2z_i+z_i^2)}} \frac{1}{\langle p_j q_i \rangle} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{P}_j^2 \mathbb{S}^{+\frac{1}{2}i}) + \sqrt{\frac{\mathcal{P}_{gg}}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[p_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{S}^{-1_i}) \\
& + \sqrt{\frac{z_i^4 \mathcal{P}_{gg}}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{\langle p_j q_i \rangle} (\mathbb{T}_j^g \otimes \mathbb{S}^{+1_i}) + \sqrt{\frac{\mathcal{P}_{gg}(1-z_i)^4}{\mathcal{C}_A(1-z_i+z_i^2)^2}} \frac{1}{[p_j q_i]} (\mathbb{T}_j^g \otimes \mathbb{P}_j^1 \mathbb{S}^{+1_i}) \left. \right).
\end{aligned} \tag{A.1}$$

# Quantum MC

- Bauer, Nachman, Provasoli & de Jong, 1904.03196

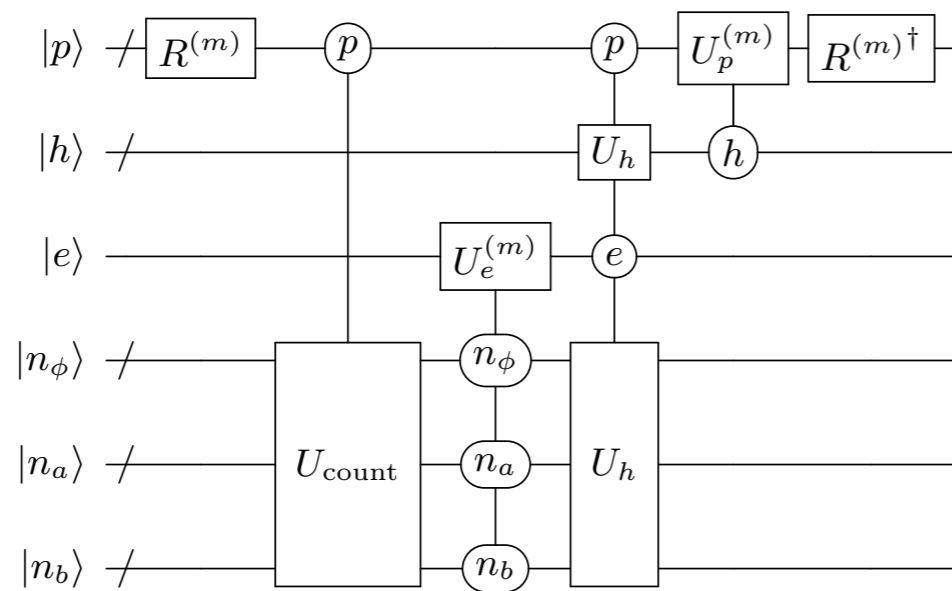


FIG. 1: Quantum circuit block for one step, to be repeated  $N$  times for the full circuit.

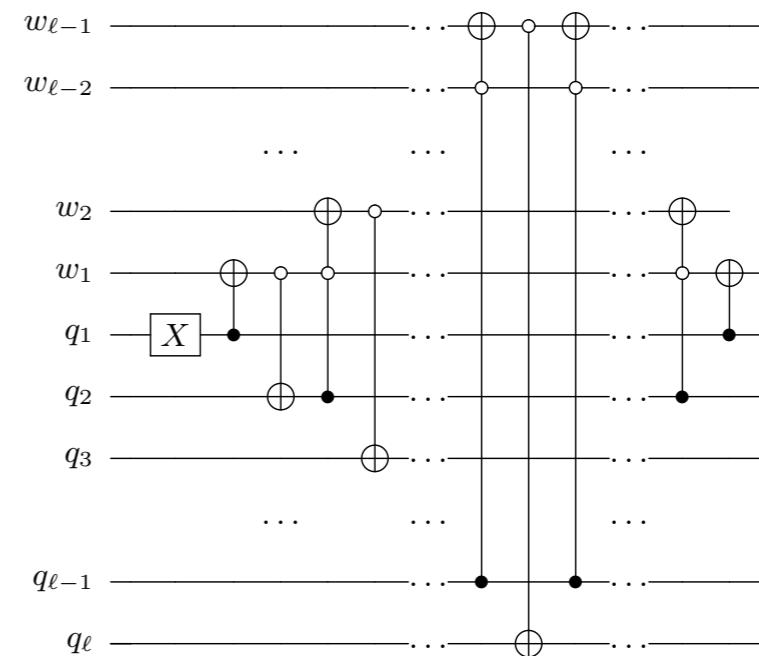
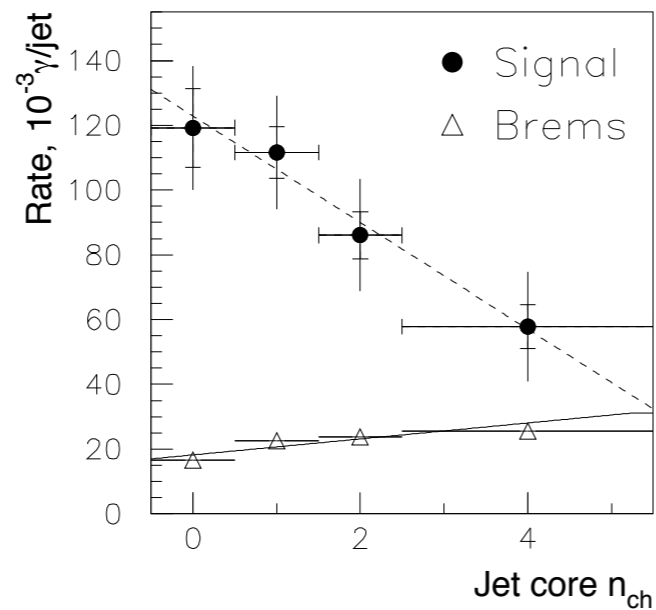


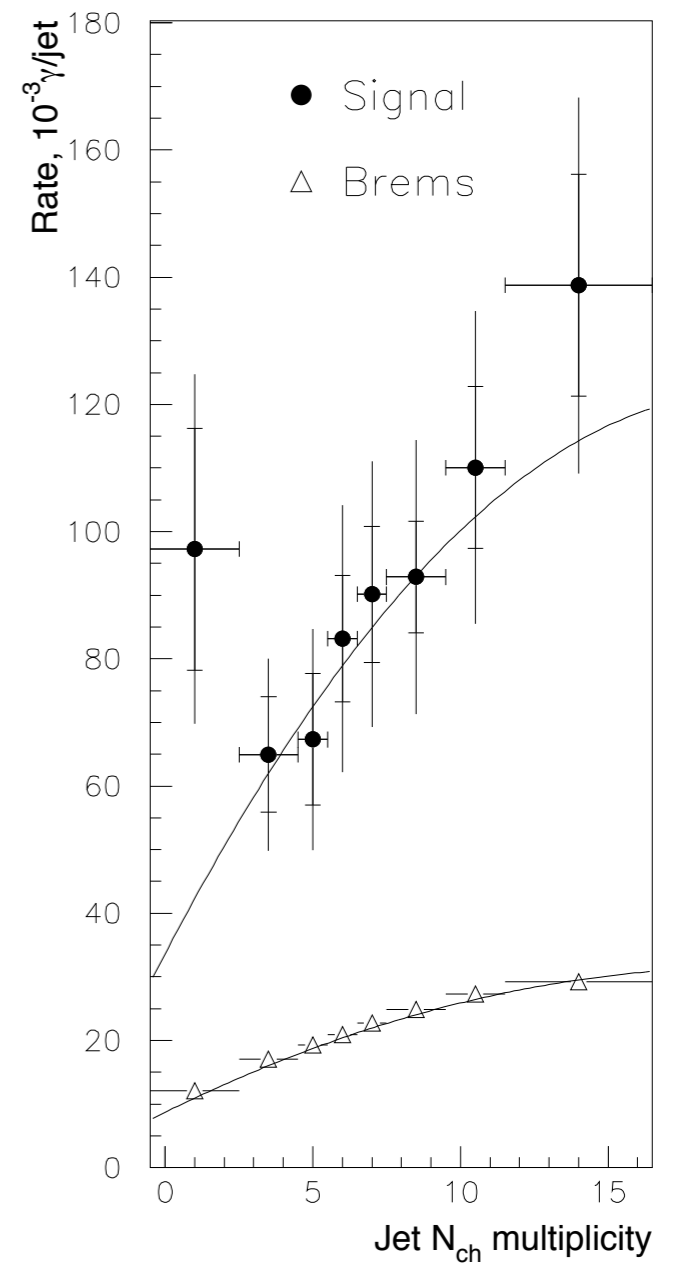
FIG. 8: Decomposition of the  $U_+$  gate for integers as large as  $a$ , where  $\ell = \lceil \log_2(a) \rceil$ .

# Hadronisation

- Unexplained features:  
e.g. soft photon excess



Core:  $\Delta\theta < 0.1$



DELPHI, 1004.1587



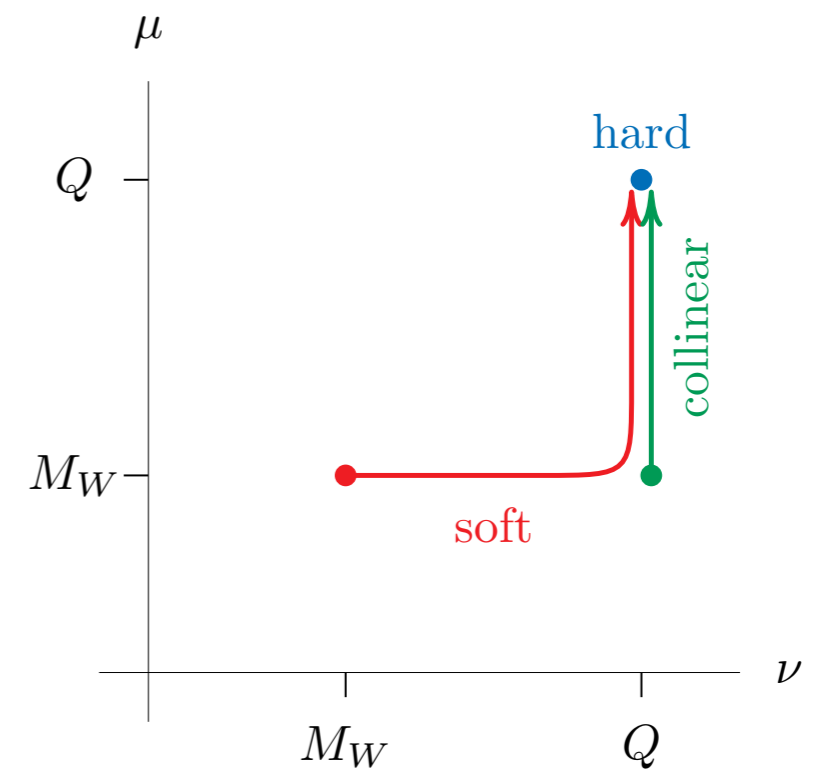
# Resummation Issues

- SCET vs “traditional”: EW Sudakov
- Coloured final states
- Factorisation

# Electroweak Sudakov

- SCET approach (Manohar & Waalewijn, 1802.08687)
  - $\mu$ -anomalous dimension has  $\ln v$  terms and v.v.

- Resum logs by double evolution:



- Equivalent to angular evolution with  $\alpha_w(p_t)$  (Bauer & BW, 1808.08831) - why?

# Coloured Final States

- Resummation:
  - Soft function incorporates soft wide-angle ISR and FSR and interference
- Parton Showers:
  - How well do different dipole initial scales ( $p_i \cdot p_j$ ) approximate this?

# Factorisation

- MPI/UE leads to violation for certain observables, e.g.  $ET = \sum |p_T|$ , beam thrust, ...
- Does it make sense to use these as factorisation scales?

# Conclusions?

- Both PS and R are (still) very active fields
- Aiming for precision to make full use of LHC data
- PS issues: recoils, correlations, NLO, ...
- R issues: SCET vs trad, coloured FS, multivariable, ...
- Plenty to be done (by you!)

Thanks

Backup

$$q \frac{\partial}{\partial q} Q^-(x, q) = -\frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} \frac{dz}{z} P_{ff}(z) \left[ \frac{1}{3} Q^-(x/z, q) + z Q^-(x, q) \right]$$

- Define  $F(q) = \int_0^1 dx x Q^-(x, q) = \int_0^1 dx x [u_L(x, q) - d_L(x, q)]$

- Then  $q \frac{dF}{dq} = -\frac{\alpha_2}{\pi} C_F \int_0^{1-m_W/q} dz P_{ff}(z) \frac{4}{3} F(q)$

where  $C_F \int_0^{1-m_W/q} dz P_{ff}(z) \sim \frac{3}{2} \ln \left( \frac{q}{m_W} \right)$  [ $C_F = 3/4$  for SU(2)]

- Hence

$$F(q) \sim F(m_W) \exp \left[ -\frac{\alpha_2}{\pi} \ln^2 \left( \frac{q}{m_W} \right) \right]$$

- For LLA resummation:  $\alpha_2 \rightarrow \alpha_2(q(1-z))$



# Polarised Splitting Functions

- For any gauge interaction  $G=SU(3), SU(2), U(1)$   
(neglecting azimuthal correlations)

$$P_{f_L f_L, G}^R(z) = P_{f_R f_R, G}^R(z) = \frac{2}{1-z} - (1+z),$$

$$P_{V_+ f_L, G}^R(z) = P_{V_- f_R, G}^R(z) = \frac{(1-z)^2}{z},$$

$$P_{V_- f_L, G}^R(z) = P_{V_+ f_R, G}^R(z) = \frac{1}{z},$$

$$P_{f_L V_+, G}^R(z) = P_{f_R V_-, G}^R(z) = \frac{1}{2}(1-z)^2,$$

$$P_{f_L V_-, G}^R(z) = P_{f_R V_+, G}^R(z) = \frac{1}{2}z^2,$$

$$P_{V_+ V_+, G}^R(z) = P_{V_- V_-, G}^R(z) = \frac{2}{1-z} + \frac{1}{z} - 1 - z(1+z),$$

$$P_{V_+ V_-, G}^R(z) = P_{V_- V_+, G}^R(z) = \frac{(1-z)^3}{z},$$

$$P_{HH, G}^R(z) = \frac{2}{1-z} - 2,$$