News on Colour Evolution with CVolver

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Based on work in progress with Jeff Forshaw and Simon Plätzer

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Outline

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   - Virtual corrections
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Colour flow basis I

[Ángeles Martinez, De Angelis, Forshaw, Plätzer, Seymour, JHEP 1805 (2018) 044, arXiv: 1802.08531]

- Some key results concerning the colour flow basis

Some block

The basis tensors are labelled by permuations, σ, of the colour indices

$$|\sigma\rangle = \left| \begin{array}{ccc} 1 & \cdots & n \\ \sigma(1) & \cdots & \sigma(n) \end{array} \right\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}}$$

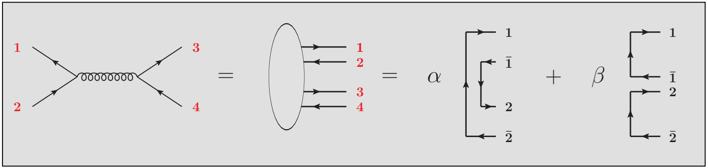
where α ($\bar{\alpha}$) are the fundamental (anti-fundamental) indices assigned to the colour (anti-colour) legs. There are $n!$ colour flows, corresponding to $n!$ basis tensors. The inner products of these basis tensors are given by

$$\langle\sigma|\tau\rangle = \delta^{\alpha_1}_{\bar{\alpha}_{\sigma(1)}} \cdots \delta^{\alpha_n}_{\bar{\alpha}_{\sigma(n)}} \delta^{\bar{\alpha}_{\tau(1)}}_{\alpha_1} \cdots \delta^{\bar{\alpha}_{\tau(n)}}_{\alpha_n} = N_c^{n-\#\text{transpositions}(\sigma,\tau)}$$

where $\#\text{transpositions}(\sigma,\tau)$ is the number of transpositions by which σ and τ differ.

Colour flow basis II

Example



$$|\mathcal{M}\rangle = \alpha \left| \begin{matrix} 1 & 2 \\ \bar{2} & \bar{1} \end{matrix} \right\rangle + \beta \left| \begin{matrix} 1 & 2 \\ \bar{1} & \bar{2} \end{matrix} \right\rangle$$
$$= \alpha \left| 2 \quad 1 \right\rangle + \beta \left| 1 \quad 2 \right\rangle$$

i	c_i	$\bar{c}_i$	λ_i	$\bar{\lambda}_i$
1	1	0	$\sqrt{T_R}$	0
2	0	$\bar{1}$	0	$\sqrt{T_R}$
3	2	0	$\sqrt{T_R}$	0
4	0	$\bar{2}$	0	$\sqrt{T_R}$

- A colour (anti-colour) index, c_i ($\bar{c}_i$) is assigned to each external leg i of a scattering amplitude.
- Colour index labels are counted from 1 and c_i ($\bar{c}_i$) $= 0$ indicates that i only carries anti-colour (colour).
- All momenta of the amplitude are taken to be outgoing.

The binary variables λ_i and $\bar{\lambda}_i$ can be summarised as $\lambda_i = \sqrt{T_R}$, $\bar{\lambda}_i = 0$ for a quark, $\lambda_i = 0$, $\bar{\lambda}_i = \sqrt{T_R}$ for a antiquark and $\lambda_i = \bar{\lambda}_i = \sqrt{T_R}$ for a gluon, where in QCD $T_R = 1/2$.

We want to compute

$$\mathrm{Tr}(\mathbf{O}) = \mathrm{Tr}([\tau|\,\mathbf{O}\,|\sigma]\,\langle\sigma|\tau\rangle) \quad (1)$$

where $\mathbf{O} = \sum_{\sigma,\tau} [\tau|\,\mathbf{O}\,|\sigma]\,|\tau\rangle\langle\sigma| = \sum_{\sigma,\tau} \mathcal{O}_{\tau\sigma}\,|\tau\rangle\langle\sigma|$.
Introduce dual basis vectors since basis is non-orthonormal

$$\sum_{\sigma} |\sigma\rangle\,[\sigma| = \mathbb{I}; \qquad [\sigma|\tau\rangle = \delta_{\sigma\tau}$$

As our operator, $\mathbf{O}$, can be written as a chain of operators, $\mathbf{R}$, which represent evolution operators (be them real or virtual), one can write

$$\mathbf{R}\,|\alpha\rangle = C_R^{\alpha}\,|\beta\rangle\,; \qquad C_R^{\alpha} = [\beta|\mathbf{R}|\alpha\rangle\,; \qquad [\tau|\,\mathbf{O}'\mathbf{R}\,|\sigma_3] = [\tau|\,\mathbf{O}'\,|\sigma_2]\,C_R^{\sigma_2}$$

In this way, we can recursively strip off evolution operators leaving behind reduced matrix elements and c-number factors.

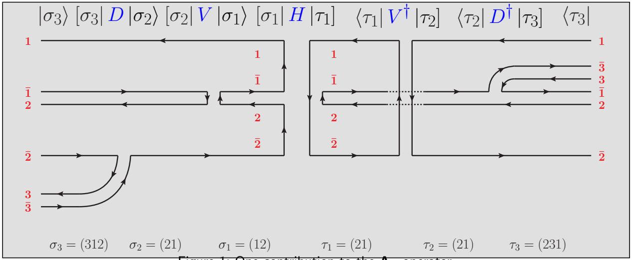


Figure 1: One contribution to the $\mathbf{A}_1$ operator

The matrix element for colour reconnectors is given by:

$$[\tau|\mathbf{T}_i \cdot \mathbf{T}_j|\sigma\rangle = -N_c\delta_{\tau\sigma}\left(\lambda_i\bar{\lambda}_j\ \delta_{c_i,\sigma^{-1}(\bar{c}_j)} + \lambda_j\bar{\lambda}_i\ \delta_{c_j,\sigma^{-1}(\bar{c}_i)} + \frac{1}{N_c^2}(\lambda_i - \bar{\lambda}_i)(\lambda_j - \bar{\lambda}_j)\right)$$
$$+ \sum_{(ab)} \delta_{\tau_{(ab)},\sigma}\left(\lambda_i\lambda_j\delta_{(ab),(c_ic_j)} + \bar{\lambda}_i\bar{\lambda}_j\delta_{(ab),(\sigma^{-1}(\bar{c}_i)\sigma^{-1}(\bar{c}_j))}\right.$$
$$\left. - \lambda_i\bar{\lambda}_j\delta_{(ab),(c_i,\sigma^{-1}(\bar{c}_j))} - \lambda_j\bar{\lambda}_i\delta_{(ab),(c_j,\sigma^{-1}(\bar{c}_i))}\right)$$

This allows us to write the general form of the anomalous dimension matrix as:

$$[\tau|\,\mathbf{\Gamma}\,|\sigma\rangle = N_c\delta_{\tau\sigma}\Gamma_\sigma + \Sigma_{\sigma\tau} + \frac{1}{N_c}\delta_{\tau\sigma}\rho$$

where $\mathbf{\Gamma}$ is the anomalous dimension matrix which contains all $\mathbf{T}_i \cdot \mathbf{T}_j$ [Plätzer, Eur. Phys. JC (2014) 74, arXiv: 1312.2448] . The off-diagonal elements in the matrix representation of $\mathbf{T}_i \cdot \mathbf{T}_j$ are non-zero only if σ and τ differ by at most one transposition.

Subleading contributions I

The main challenge is to compute the Sudakov matrix elements as this involves the exponentiation of a possibly large colour matrix [Plätzer, Eur. Phys. JC (2014) 74] :

$$[\tau|\exp(\mathbf{\Gamma})|\sigma\rangle = \sum_{l=0}^{\infty} \frac{(-1)^l}{N_c^l} \sum_{\sigma_0,\ldots,\sigma_l} \delta_{\tau\sigma_0}\delta_{\sigma_l\sigma} \times \left(\prod_{\alpha=0}^{l-1} \Sigma_{\sigma_\alpha,\sigma_{\alpha+1}} \right) \times R(\{\sigma_0,\ldots,\sigma_l\}) \qquad (2)$$

- Define succesive summations at (next-to)^d-leading colour (N^d LC) by truncating the sum over l at $l = d$.

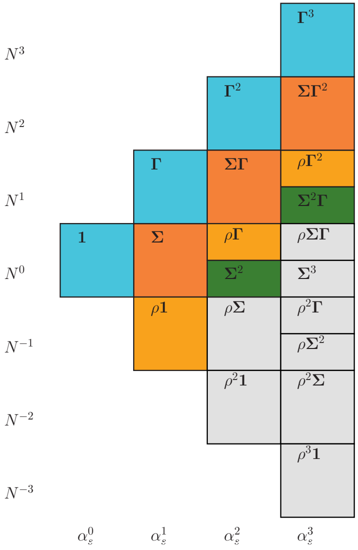
Example of $d = 1$

$$[\tau|\exp(\mathbf{\Gamma})|\sigma\rangle|_{\text{NLC}} = \delta_{\tau\sigma} e^{(-N_c\Gamma'_\sigma)} - \frac{1}{N_c}\Sigma_{\tau\sigma} \frac{e^{(-N_c\Gamma'_\tau)} - e^{(-N_c\Gamma'_\sigma)}}{\Gamma'_\tau - \Gamma'_\sigma}; \qquad \Gamma'_\sigma = \Gamma_\sigma(1-\rho/N_c^2)$$

- If $\sigma = \tau$, the R functions revert to their degenerate form:

$$R(\{\sigma,\sigma\}) = -N_c e^{-N_c\Gamma'_\sigma}$$

- Note that as we treat the real emissions, scalar product matrix and the diagonal part of the anomalous dimension matrix without any approximation, and N^d LC approximation involves at most d swaps for each Sudakov operator, this is much more than N^d LC for observables.



virtuals	reals
(0 flips) $\times 1 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{t}\|_{2\text{ flips}} \times 1$ $(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{s}\|_{1\text{ flip}} \times N^{-1}$ $(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\ \mathbf{s}[...]\mathbf{s}\|_{0\text{ flips}} \times N^{-2}$
(1 flip) $\times \alpha_s \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{s}\|_{1\text{ flip}} \times N^{-1}$
(0 flips) $\times \alpha_s N^{-1} \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r}$
(0 flips) $\times \alpha_s^2 \times (\alpha_s N)^n$ (2 flips) $\times \alpha_s^2 \times (\alpha_s N)^n$	$(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r}$ $(\mathbf{t}[...]\mathbf{t}\|_{0\text{ flips}})^{r-1}\ \mathbf{t}[...]\mathbf{t}\|_{2\text{ flips}}$

Subleading contributions III

Subleading colour contributions arise from the hard scattering matrix, from the $1/N_c$ and $1/N_c^2$ terms in both the real emission and virtual evolution operators and from scalar product matrix.

- Pure $1/N_c$ corrections can only originate from interference contributions in the hard process matrix; we will ignore subleading colour contributions from this source here.
- The leading colour contributions from the virtual evolution operator come from Γ, so are all enhanced by powers of $\alpha_s N_c$, and owing to the fact that the leading contribution is diagonal, can easily be accounted to all orders in an exponential. This evolution does not change the colour structure in the amplitude or its conjugate.
- Subleading colour contributions (suppressed by $1/N_c^2$) due to real emissions come from three sources: two flips - $\mathbf{t}[\ldots]\mathbf{t}$, one flip and a factor of $1/N_c$ - e.g $\mathbf{t}[\ldots]\mathbf{s}$, zero flips and a factor of $1/N_c^2$ - $\mathbf{s}[\ldots]\mathbf{s}$.
- An insertion of a perturbation, Σ, comes with a factor of $(\alpha_s N_c)/N_c$ and induces a flip. This can also undo flips induced by real emissions of the type $\mathbf{s}[\ldots]\mathbf{t}$. Whilst we rid ourselves of a factor of $1/N_c$, the $\mathbf{s}$ introduces another.
- A similar reasoning applies to a single ρ perturbation, which contributes at same order.
- With two Σ insertions, we can have a net zero or two flips. Zero flips contributes a $(\alpha_s N_c)^2/N_c^2$ correction, whereas two flips contributes if it compensates a $\mathbf{t}[\ldots]\mathbf{t}$ two-flip real emission.

Grey contributions lead to factors of $(\alpha_s N_c)^2/N_c^4$ and are beyond NLC.

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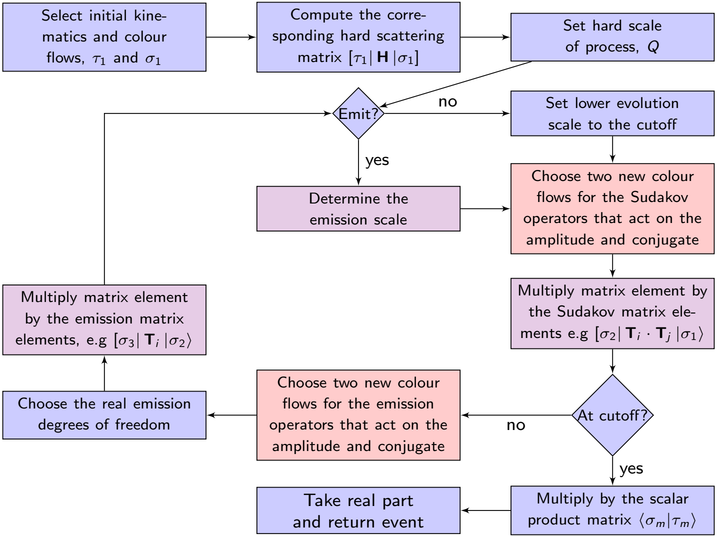
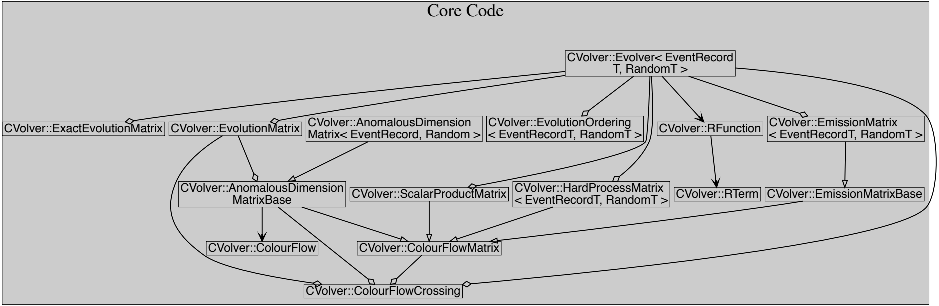
We can Monte Carlo over the intermediate colour states (based heavily on CVolver code [Plätzer, Eur. Phys. JC (2014) 74]) [Work in progress: De Angelis, Forshaw and Plätzer] :



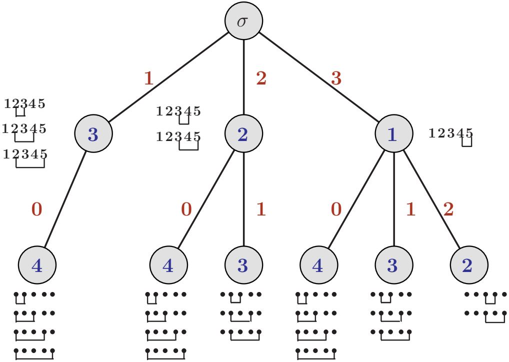


- The CVolver framework is highly robust and modular with core-code dedicated to colour book-keeping, algorithms to sample colour structures and event processing.
- Observable-dependent code inherits from this core.
- This plugin structure is currently used to accomodate NGLs in a soft-gluon-only shower, but a fully-fledged shower is currently in the works.
- The framework can compute the real emission contributions at both amplitude and cross-section level.

- We choose the new colour flows from the set of all possible colour flows that can be accessed after the action of a Sudakov or emission operator. The main challenge is to account for the independent colour evolution in the amplitude and the conjugate amplitude.
- For emissions, the next pair of chosen colour states, σ_n and τ_n, differ by n, $n+1$ or $n+2$ transpositions, where $n = \#(\sigma_{n-1}, \tau_{n-1})$.
- For virtuals, choose a number of flips to make, p, from a $(1/N_c)^p$ distribution up to d in an attempt to prevent $\#(\sigma_n, \tau_n)$ from becoming too large.

- How to select σ_{n+1} from σ_n such that the number of transpositions between the two, $\#(\sigma_{n+1}, \sigma_n)$, equals L?

For example, $n = 5$, $L = 2$ and $\sigma = |12345\rangle$:



- Particles in a singlet state are ignored when considering the colour flow to evolve to with a Sudakov or emission operator.

Use of subtraction scheme to expose and simplify the colour structure of collinear singularities to better handle them. This amounts to, for the Sudakov exponent:

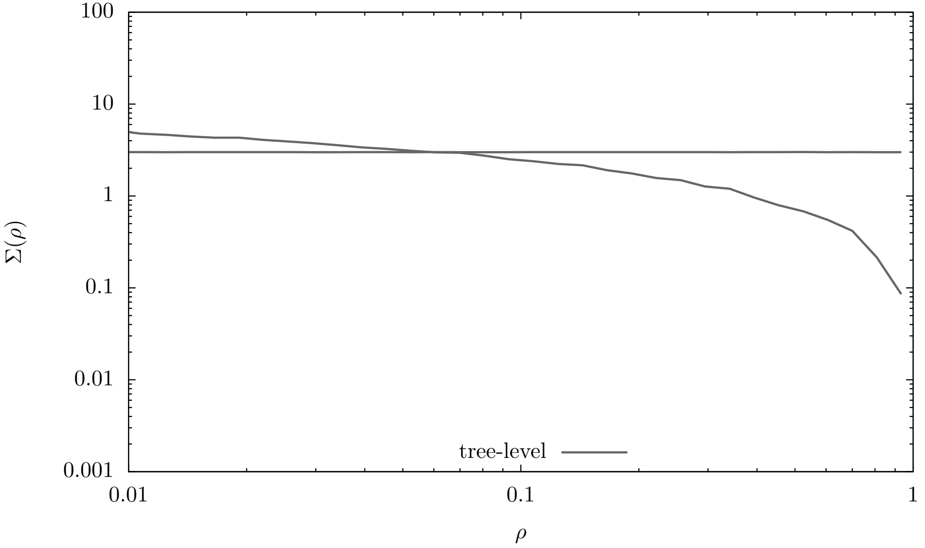
$$\sum_{i<j} \frac{n_i \cdot n_j}{(n_i \cdot n)(n_j \cdot n)} \theta_{\text{cut}} T_i \cdot T_j \to \sum_{i<j} \left(\frac{n_i \cdot n_j}{(n_i \cdot n)(n_j \cdot n)} - \frac{1}{n_i \cdot n} - \frac{1}{n_j \cdot n} \right) T_i \cdot T_j - \sum_i \frac{1}{n_i \cdot n} T_i^2 \theta(n_i \cdot n - \lambda), \tag{3}$$

where we ignore terms of order of the collinear cutoff, λ, after angular integration. Similarly, we have for the real emission kinematic part:

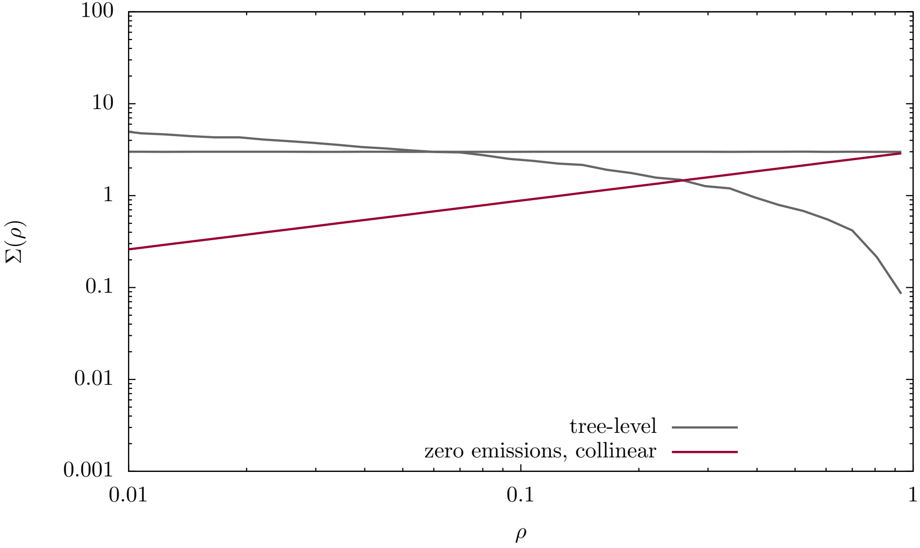
$$\sum_{i<j} \left(\frac{n_i \cdot n_j}{(n_i \cdot n)(n_j \cdot n)} - \theta(\lambda - n_i \cdot n) \frac{1}{n_i \cdot n} - \theta(\lambda - n_j \cdot n) \frac{1}{n_j \cdot n} \right)$$

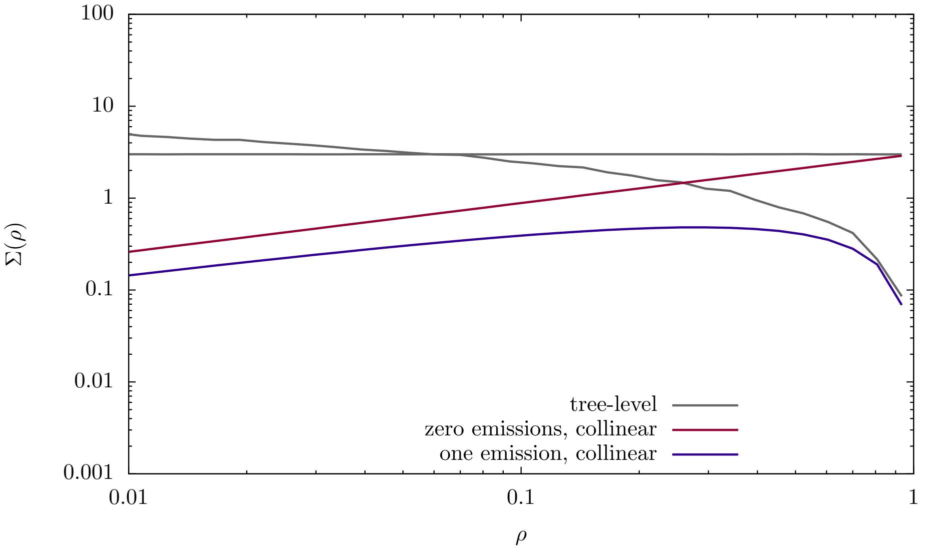
- The first term of Eq. 3 is reminiscent of the fifth form factor [Dokshitzer, Marchesini, Phys. Lett. B631 (2005) 118-125] .
- Second term is a probabilistic factor included in the MC scale selection weight and used to sample energy scales.
- Including the hard-collinear emissions for the spin-average result amounts to changing the integral over this second term to the full splitting functions [Forshaw, Holguin, Plätzer, arXiv:1905.08686] .

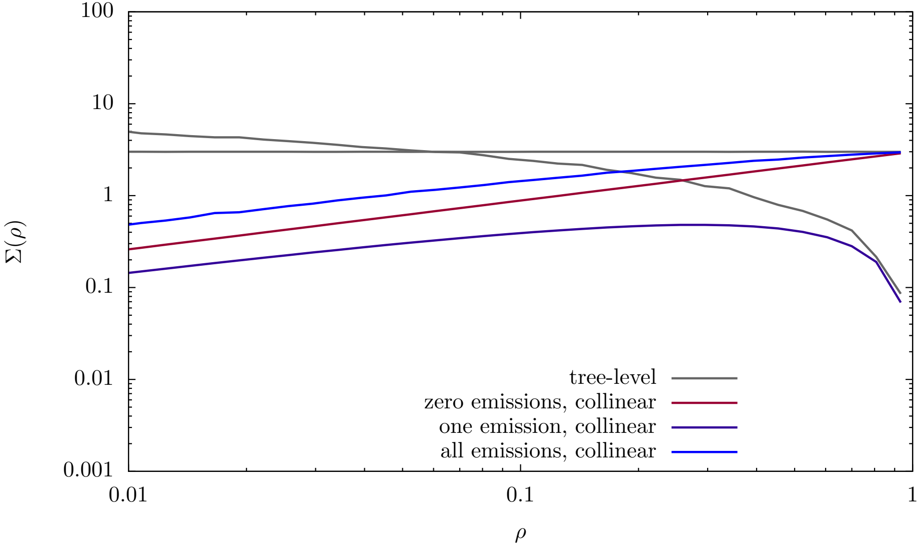
Results I



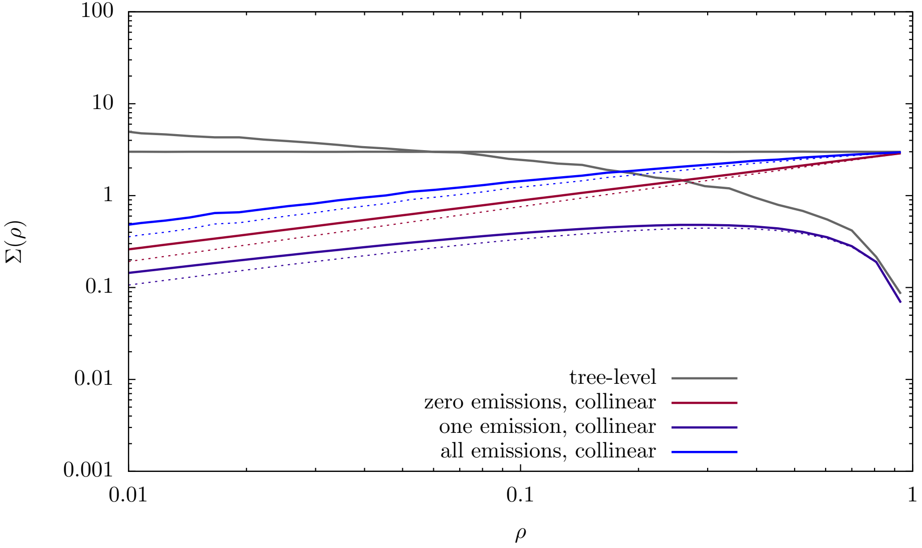
Results II


100
10
1
0.1
0.01
0.001
0.01
0.1
1
Σ(ρ)
ρ
tree-level
zero emissions, collinear


100
10
1
0.1
0.01
0.001
0.01
0.1
1
$\Sigma(\rho)$
ρ
tree-level
zero emissions, collinear
one emission, collinear

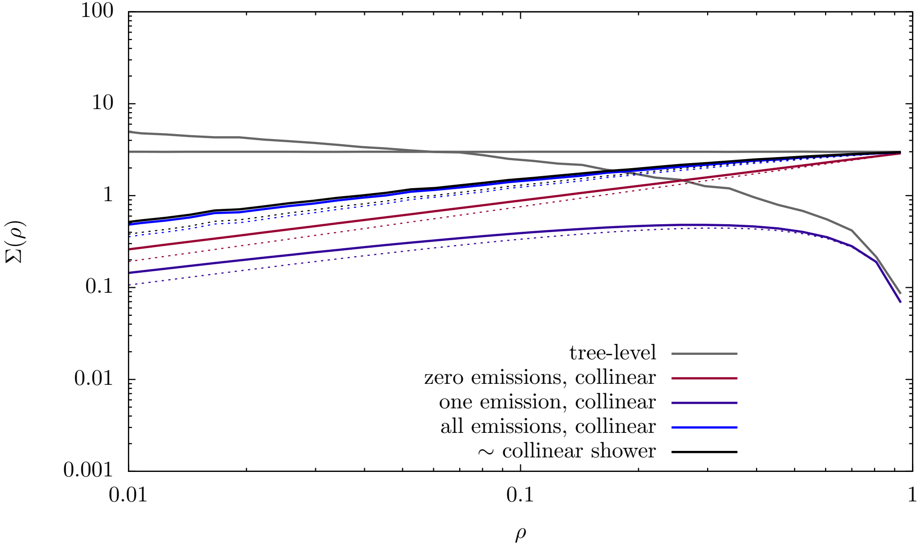

100
10
1
0.1
0.01
0.001
$\Sigma(\rho)$
0.01
0.1
1
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear

Results V



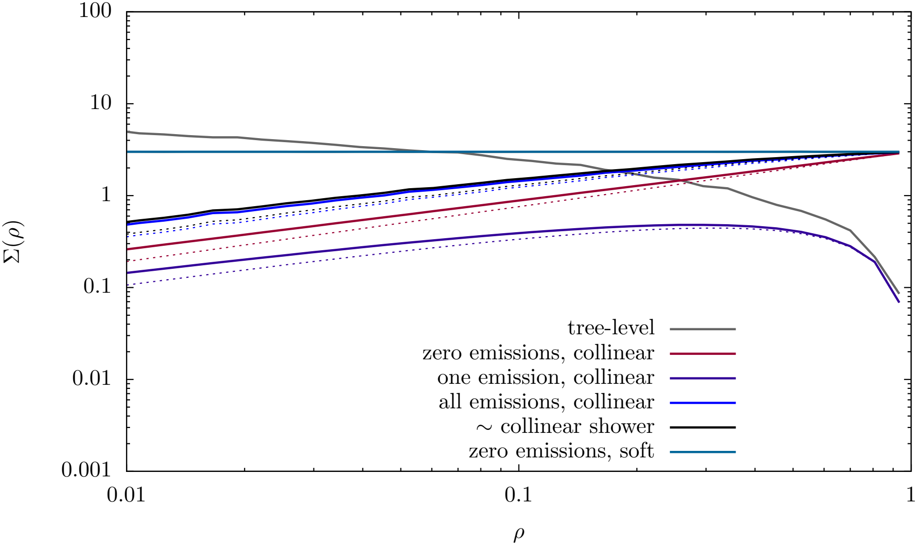
100
10
1
0.1
0.01
0.001
$\Sigma(\rho)$
0.01
0.1
1
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear

Results VI

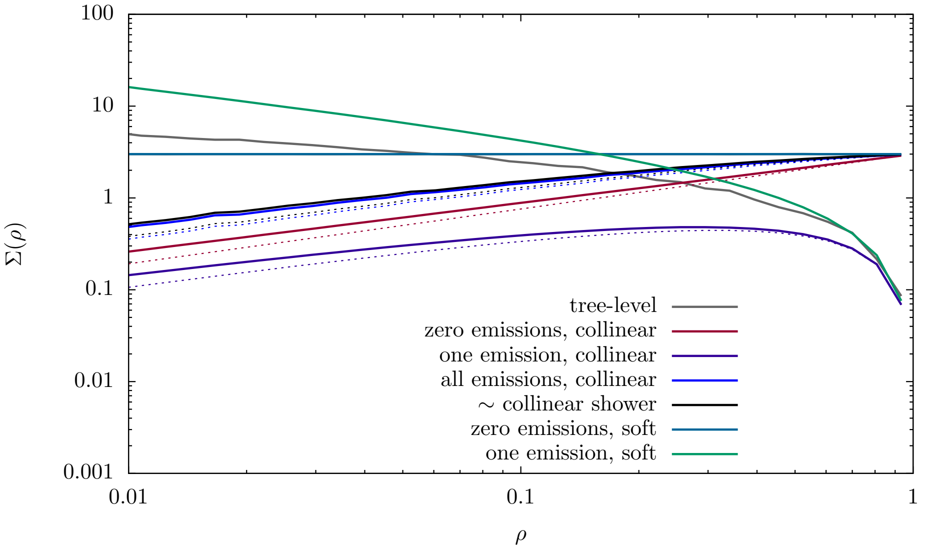


100
10
1
0.1
0.01
0.001
0.01
0.1
1
Σ(ρ)
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear
∼ collinear shower

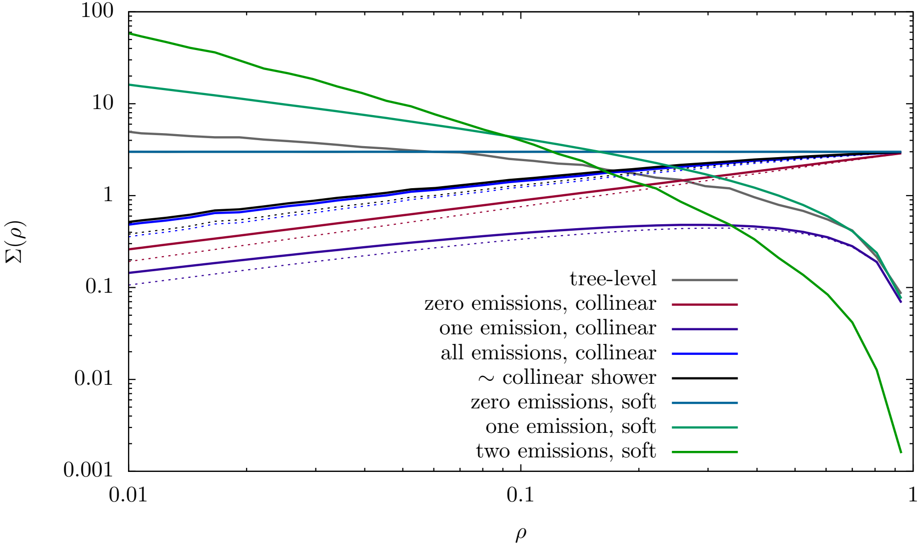
Results VII



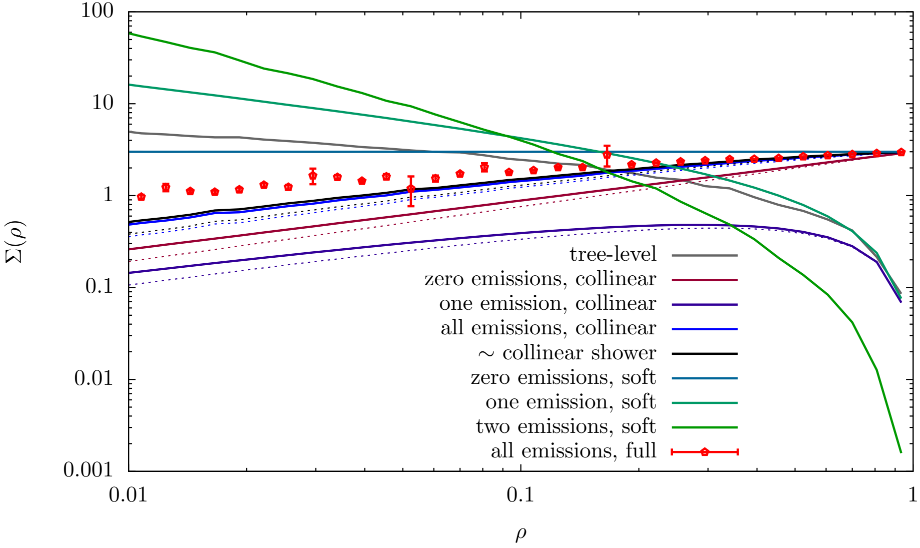
100
10
1
0.1
0.01
0.001
0.01
0.1
1
$\Sigma(\rho)$
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear
∼ collinear shower
zero emissions, soft


100
10
1
0.1
0.01
0.001
0.01
0.1
1
$\Sigma(\rho)$
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear
∼ collinear shower
zero emissions, soft
one emission, soft

Results IX



100
10
1
0.1
0.01
0.001
Σ(ρ)
0.01
0.1
1
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear
∼ collinear shower
zero emissions, soft
one emission, soft
two emissions, soft



100
10
1
0.1
0.01
0.001
0.01
0.1
1
$\Sigma(\rho)$
ρ
tree-level
zero emissions, collinear
one emission, collinear
all emissions, collinear
∼ collinear shower
zero emissions, soft
one emission, soft
two emissions, soft
all emissions, full

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Conclusions

- Iterative form of the algorithm is well suited to a Monte Carlo implementation.
- Colour flow basis facilitates numerical implementation to arbitrary order colour expansion.
- Currently handling soft gluons in e^+e^- but implementation of a fully-fledged shower is underway.
- Inclusion of spin-averaged hard-collinear emissions is imminent and we want to include NLL soft emissions, go beyond e^+e^- and include spin correlations.

Thanks for listening!