Double parton scattering: theory developments

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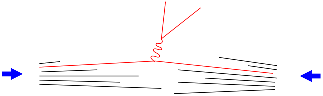
HELMHOLTZ RESEARCH FOR GRAND CHALLENGES

Hadron-hadron collisions

- standard description based on factorisation formulae

cross sect = parton distributions × parton-level cross sect



- net transverse momentum p_T of hard-scattering products:
  - p_T integrated cross sect $\rightsquigarrow$ collinear factorisation
  - $p_T \ll$ hard scale of interaction $\rightsquigarrow$ TMD factorisation

$\rightsquigarrow$ resummation of p_T logarithms

- particles resulting from interactions between spectator partons unobserved

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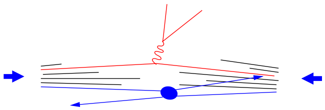


- net transverse momentum p_T of hard-scattering products:
  - p_T integrated cross sect $\rightsquigarrow$ collinear factorisation
  - $p_T \ll$ hard scale of interaction $\rightsquigarrow$ TMD factorisation

    $\rightsquigarrow$ resummation of p_T logarithms
- particles resulting from interactions between spectator partons unobserved
- spectator interactions can be soft $\rightsquigarrow$ underlying event
  or hard $\rightsquigarrow$ multiparton interactions
- here: double parton scattering with factorisation formula

  cross sect = double parton distributions × parton-level cross sections

Scope of this talk

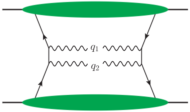
- theory of double parton scattering (DPS)
  with scales Q_1, Q_2 of both scatters $\gg$ soft scale Λ
  referred to as "perturbative phase" in Steffen Schumann's overview talk
- includes region $Q_1 \gg Q_2 \gg \Lambda$ relevant for "underlying event" in perturbative regime
- connections to this workshop:
  - resummation of DGLAP, rapidity, and p_T logarithms
  - higher-order corrections
  - parton shower algorithms for DPS
  - handling of colour structure

Not covered

- DPS phenomenology and experimental results
  much activity in both areas

Single vs. double parton scattering (SPS vs. DPS)

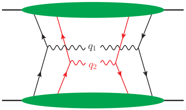
- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$



double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

- for transv. momenta $\sim \Lambda \ll Q$:

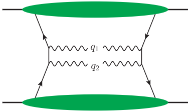
$$\frac{d\sigma_{\text{SPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{DPS}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{SPS}} \sim \frac{1}{Q^2} \gg \sigma_{\text{DPS}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double parton scattering (SPS vs. DPS)

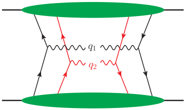
- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$



single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \sim$ hard scale Q

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q$



double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_2| \ll Q$

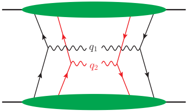
- for small parton mom. fractions x
  double scattering enhanced by parton luminosity
- depending on process: enhancement or suppression
  from parton type (quarks vs. gluons), coupling constants, etc.

  example: $\sigma(qq \to qq + W^- W^-) \propto \alpha_s^2$

  vs. $\sigma(d\bar{u} \to W^-) \times \sigma(d\bar{u} \to W^-) \propto \alpha_s^0$

  Kulsza, Stirling 2000; Gaunt, Kom, Kulesza, Stirling 2003

DPS cross section: collinear factorisation



$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{C}\; \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

C = combinatorial factor

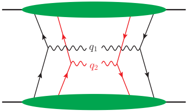
$\hat{\sigma}_i$ = parton-level cross sections

$F(x_1, x_2, \boldsymbol{y})$ = double parton distribution (DPD)

$\boldsymbol{y}$ = transv. distance between partons

- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F
- tree-level formula from Feynman graphs and kinematic approximations
  Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2011
- full factorisation proof for double Drell-Yan
  Vladimirov 2016, 2017; MD, Buffing, Gaunt, Kasemets, Nagar, Ostermeier, Plößl, Schäfer, Schönwald 2011–2018

DPS cross section: TMD factorisation



- for measured transv. momenta

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\, dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2$$
$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\, \frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, e^{-i(\boldsymbol{z}_1\boldsymbol{q}_1 + \boldsymbol{z}_2\boldsymbol{q}_2)} \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{z}_i, \boldsymbol{y})\, F(\bar{x}_i, \boldsymbol{z}_i, \boldsymbol{y})$$

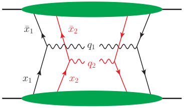
- $F(x_i, \boldsymbol{z}_i, \boldsymbol{y})$ = double-parton TMDs
  $\boldsymbol{z}_i$ = Fourier conjugate to parton transverse mom. $\boldsymbol{k}_i$
- operator definition as for TMDs: schematically have

$$F(x_i, \boldsymbol{z}_i, \boldsymbol{y}) = \underset{z_i^- \to x_i p^+}{\mathcal{FT}} \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\; \bar{q}\big(y - \tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y + \tfrac{1}{2}z_1\big)|p\rangle$$

  - to be completed by renormalisation, Wilson lines, soft factors in close analogy to single scattering
  - analogous definition for collinear distributions $F(x_i, \boldsymbol{y})$
  - essential for studying factorisation, scale and rapidity dependence

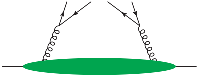
Double parton scattering: ultraviolet problems

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1 \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

$$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct.} \otimes \text{usual PDF}$$



first results at NLO ($\mathcal{O}(\alpha_s^2)$): MD, Gaunt, Plößl, Schäfer 2019

Double parton scattering: ultraviolet problems

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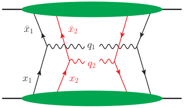

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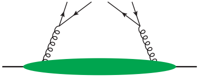
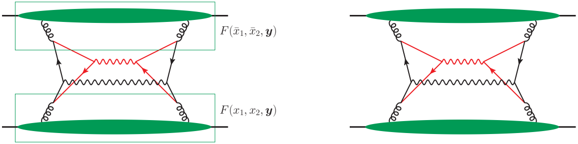


  gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$
  in fact, formula only valid for $|\boldsymbol{y}| \gg 1/Q$

- problem also for two-parton TMDs
  UV divergences logarithmic instead of quadratic

. . . and an identity crisis

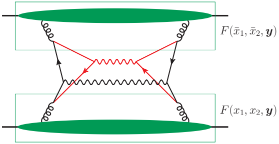


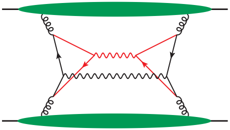
- double counting problem between double scattering with splitting (1v1) and single scattering at high-loop level (twisted box graphs)

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012; Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012; Manohar, Waalewijn 2012; noted earlier by Cacciari, Salam, Sapeta 2009

- how to separate DPS from SPS is a matter of definition/scheme choice intuitively: small $y \sim 1/Q$ is SPS, large $y \gg 1/Q$ is DPS

...and an identity crisis





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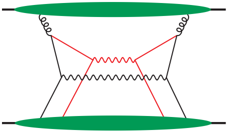
- also have graphs with splitting in one proton only: "2v1"

  $\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \times F_{\text{int}}(x_1, x_2, \boldsymbol{y})$

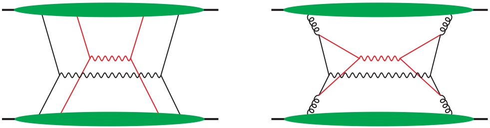
  Blok et al 2011-13, Blok, Gunnellini 2015
  Gaunt 2012

  skip here for reasons of time



A consistent scheme

MD, Gaunt, Schönwald 2017

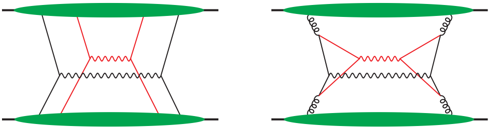


- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u - 1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions

  analogous regulator for transverse-momentum dependent DPDs
- keep definition of DPDs as operator matrix elements
  cutoff in $\boldsymbol{y}$ does not break symmetries that haven't already been broken

A consistent scheme

MD, Gaunt, Schönwald 2017



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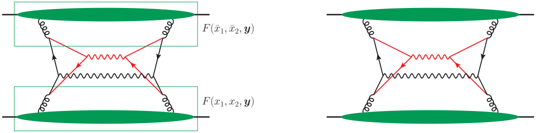
  analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order
  - σ_{SPS} defined as usual
    no new calculation needed

Subtraction formalism at work



$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \sim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \;\; \sigma \approx \sigma_{\text{SPS}}$ dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub}
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \;\; \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- subtraction formalism works order by order in perturb. theory
  Collins, Foundations of Perturbative QCD, Chapt. 10

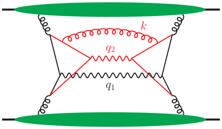
DGLAP evolution

- define DPDs as matrix elements of renormalised twist-two operators:

  $F(x_1, x_2, \boldsymbol{y}; \mu_1, \mu_2) \sim \langle p|\mathcal{O}_1(\boldsymbol{0}; \mu_1)\, \mathcal{O}_2(\boldsymbol{y}; \mu_2)|p\rangle \qquad f(x; \mu) \sim \langle p|\mathcal{O}(\boldsymbol{0}; \mu)|p\rangle$

  $\Rightarrow$ separate DGLAP evolution for partons 1 and 2:

$$\frac{\partial}{\partial \log \mu_i^2} F(x_i, \boldsymbol{y}; \mu_i) = P \underset{x_i}{\otimes} F \qquad \text{for } i = 1, 2$$



- DGLAP logarithm from strongly ordered region $|\boldsymbol{q}_1| \ll |\boldsymbol{k}| \sim |\boldsymbol{q}_2| \ll Q_2$ repeats itself at higher orders (ladder graphs)
- resummed by DPD evolution in σ_{DPS} if take $\nu \sim \mu_1 \sim Q_1$, $\mu_2 \sim Q_2$ and appropriate initial conditions ($\rightarrow$ next slide)
- can enhance DPS region over SPS region $|\boldsymbol{q}_1| \sim |\boldsymbol{q}_2| \sim Q_{1,2}$ which dominates by power counting

A model study

- take DPD model with $F = F_{\text{spl}} + F_{\text{int}}$

$$F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; 1/y^*, 1/y^*) = F_{\text{perturb.}}(y^*)\, e^{-y^2 \Lambda^2} \quad \text{with} \quad y^* = \frac{y}{\sqrt{1 + y^2/y_{\text{max}}^2}}$$

inspired by b^* of Collins, Soper, Sterman

$$F_{\text{int}}(x_1, x_2, \boldsymbol{y}; \mu_0, \mu_0) = f(x_1; \mu_0) f(x_2; \mu_0)\, \Lambda^2 e^{-y^2 \Lambda^2}/\pi$$

description simplified, actual model slightly refined

- $F_{\text{perturb.}}(y)$ ensures correct perturbative behaviour at small y

  DGLAP logarithms built up between splitting scale $\sim 1/y^*$ and $\sim Q$
- in SPS subtraction term take instead

  $F_{\text{spl}}(x_1, x_2, \boldsymbol{y}; Q, Q) = F_{\text{perturb.}}(y)$

  hard scattering at fixed order, no resummation here
- following plots show double parton luminosity

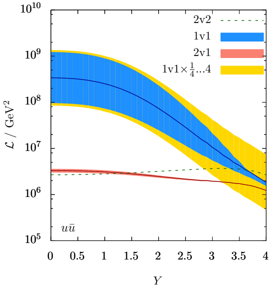
  $\mathcal{L} = \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$

  with separate contributions from 1v1, 2v1, 2v2

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity Y of q_1; with q_2 central
  take $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14\,\mathrm{TeV}$
- blue band: vary ν from $0.5\,M_W \ldots 2\,M_W$
  yellow band: naive scale variation for $\sigma_{\text{1v1}} \propto \nu^2$ from $\int_{b_0^2/\nu^2} dy^2 \, (1/y^2)^2$

$u\bar{u}$



- large ν variation $\rightsquigarrow$ need $-\sigma_{\text{sub (1v1)}} + \sigma_{\text{SPS}}$
  $\rightsquigarrow$ use 1v1 to estimate importance of SPS at high orders
- large rapidity separation $\rightsquigarrow$ very small x_1 or x_2
  $\rightsquigarrow$ region $y \gg 1/\nu$ in 1v1 is enhanced by DPD evolution

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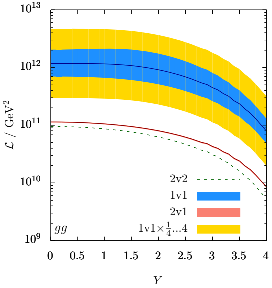
gg



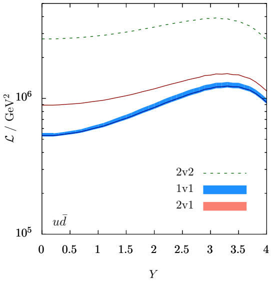
- gluons: prominent evolution effects at all Y

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. rapidity Y of q_1; with q_2 central
  take $\mu_{1,2} = Q_{1,2} = M_W$ at $\sqrt{s} = 14\,\text{TeV}$
- blue band: vary ν from $0.5\,M_W \ldots 2\,M_W$
  yellow band: naive scale variation for $\sigma_{1\text{v}1} \propto \nu^2$

from $\int_{b_0^2/\nu^2} dy^2 \, (1/y^2)^2$

$u\bar{d}$

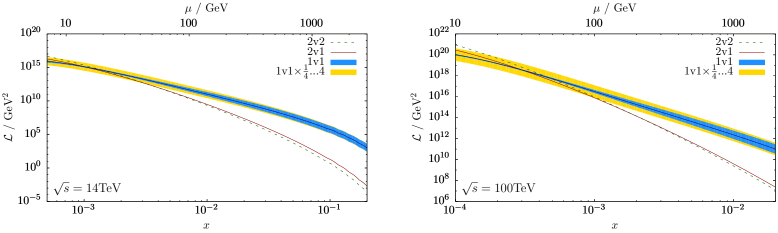


- $u\bar{d}$ induced by splitting at $\mathcal{O}(\alpha_s^2)$
  e.g. by $u \to ug \to ud\bar{d}$

DPS parton luminosities for illustration, model parameters not tuned

- plot $\mathcal{L}$ vs. $x = x_1 = x_2 = \bar{x}_1 = \bar{x}_2$ at fixed $\sqrt{s}$
  $\mu_{1,2} = Q_{1,2} = x\sqrt{s}$

gg



- DPS region enhanced for small x by evolution slide provided by J. R. Gaunt

A Monte Carlo implementation of the DGS framework

Advantages of MC parton shower implementation: exclusive final states, can implement arbitrary cuts

DGS framework implemented as dShower: Cabouat, Gaunt, Ostrolenk, 2019

- Select kinematics of hard processes and parton separation y according to DGS DPS formula.
- Backward evolution from hard process using homogeneous double DGLAP equations:

$$\mathrm{d}\mathcal{P}_{ij}^{\mathrm{ISR}} = \mathrm{d}\mathcal{P}_{ij} \exp\left(- \int_{Q^2}^{Q_h^2} \mathrm{d}\mathcal{P}_{ij} \right)$$

$$\mathrm{d}\mathcal{P}_{ij} = \frac{\mathrm{d}Q^2}{Q^2} \left(\sum_{i'} \int_{x_1}^{1-x_2} \frac{\mathrm{d}x_1'}{x_1'} \frac{\alpha_s(p_\perp^2)}{2\pi} P_{i'\to i}\left(\frac{x_1}{x_1'}\right) \frac{F_{i'j}(x_1', x_2, \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)} + \sum_{j'} \int_{x_2}^{1-x_1} \frac{\mathrm{d}x_2'}{x_2'} \frac{\alpha_s(p_\perp^2)}{2\pi} P_{j'\to j}\left(\frac{x_2}{x_2'}\right) \frac{F_{ij'}(x_1, x_2', \boldsymbol{y}, Q^2)}{F_{ij}(x_1, x_2, \boldsymbol{y}, Q^2)} \right)$$

'Guided' by some DPD set

- $2 \to 1$ 'mergings' in backward evolution at scale $\mu_y \sim 1/y$, with probability given by splitting DPD/total DPD.

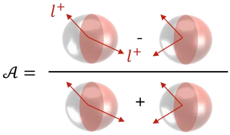
Some discussion of mergings ('joined interactions') given already in Sjöstrand, Skands, 2004, but here y-dependence of mergings taken into account.

slide provided by J. R. Gaunt

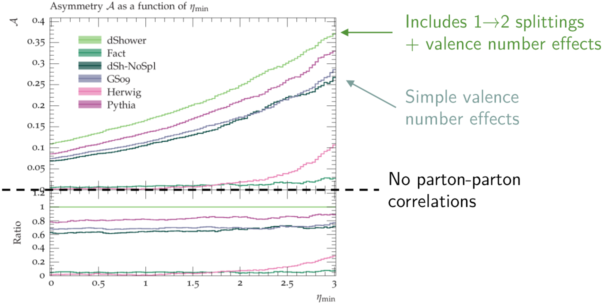
A Monte Carlo implementation of the DGS framework

First numerical investigation:

- same-sign WW $\quad pp \to W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$
- 3 quark flavours
- DPDs from DGS paper, with modifications to very approximately take account of number & momentum sum rule constraints Gaunt, Stirling, 2010, Blok, Dokshitzer, Frankfurt, Strikman, 2013, Ceccopieri, 2014, MD, Plößl, Schäfer, 2018

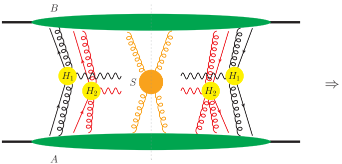


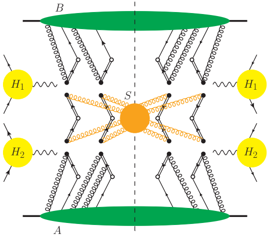
Gaunt, Kom, Kulesza, Stirling, 2010



DPS: factorisation and colour

- can generalise treatment of Collins, Soper, Sterman from single to double Drell-Yan and other DPS processes



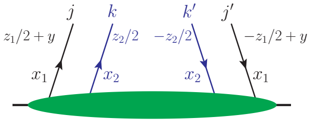


- basic steps:
  - collinear gluons ⇝ Wilson lines in DPDs
  - Glauber gluons cancel
  - soft gluons ⇝ soft factor = vevs of Wilson lines
    rapidity dependence ⇝ Collins-Soper eq'n (rapidity renormalisation)
    ⇝ Sudakov logarithms

MD, Ostermeier, Schäfer 2011; MD, Gaunt, Ostermeier, Plößl, Schäfer 2015
Vladimirov 2016, 2017; Buffing, Kasemets, MD 2017

DPS: colour complications

- DPDs have several colour combinations of partons



- colour projection operators
- singlet: $P_1^{jj',kk'} = \delta^{jj'}\delta^{kk'}/3$ as in usual PDFs
- octet: $P_8^{jj',kk'} = 2t_a^{jj'}t_a^{kk'}$
- for gluons: $8_A, 8_S, 10, \overline{10}, 27$
- we use the multiplet basis

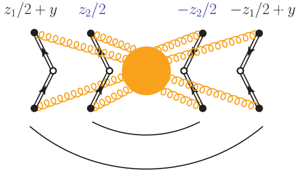
  $\rightarrow$ talk Stefan Keppeler

- corresponding combinations in soft factor
  - soft factor $\rightarrow$ matrix in colour space
  - in collinear factorisation $z_1 = z_2 = 0$

    for colour singlet: $W\,W^\dagger = 1$

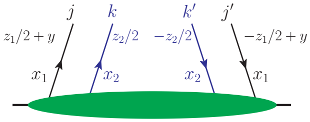
    $\Rightarrow \quad S = 1$

    $\rightsquigarrow$ soft gluon effects cancel
    no Sudakov logarithms



DPS: colour complications

- DPDs have several colour combinations of partons



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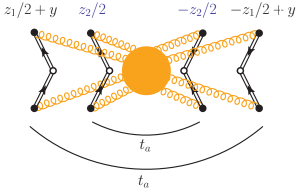
$\rightarrow$ talk Stefan Keppeler

- corresponding combinations in soft factor
  - soft factor $\rightarrow$ matrix in colour space
  - in collinear factorisation $z_1 = z_2 = 0$

    for colour octet: $W\,t^a W^\dagger \neq 1$

    $$\Rightarrow \quad S \neq 1$$

    $\rightsquigarrow$ Sudakov factors even in collinear factoris'n
    M Mekhfi 1988; A Manohar, W Waalewijn 2012



Summary

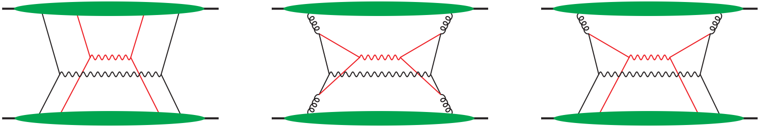
- double parton scattering important
  in specific kinematics/for specific processes
- recent years: progress towards systematic factorisation in QCD
- UV problem of DPS ↔ double counting with SPS
  ⇝ must define distinction between DPS and SPS

  our scheme (DGS):
  - simple UV regulator for DPS (cutoff in distance y between partons)
  - simple subtraction term to avoid double counting

  naturally includes DGLAP logarithms in DPS
  - at large scales Q find dominant 1v1 contributions in many cases
    ⇝ SPS required at high order in α_s before DPS becomes important
  - evolution ⇝ DPS can dominate for small x_1 and/or x_2
- ongoing work on a DPS parton shower (dShower)
- soft factor and rapidity evolution: matrix structure in colour space

Double counting: include 2v1

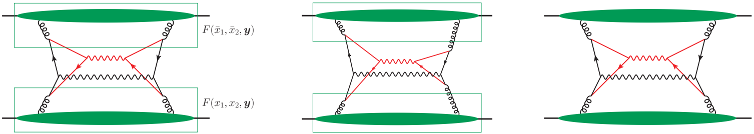
MD, Gaunt, Schönwald 2017



- on slide 13 had cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
- full version: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1v1 + 2v1)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  - includes twist 2 × twist 4 contribution and double counting subtraction for 2v1 term

Double counting: TMD factorisation

Buffing, MD, Kasemets 2017



- left and right box can independently be collinear or hard:
  $\rightsquigarrow$ DPS, DPS/SPS interference and SPS
- get nested double counting subtractions