ARES for three-jet event-shapes

In collaboration with: A. Banfi, & B.K. El-Menoufi

Luke Arpino

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University of Sussex

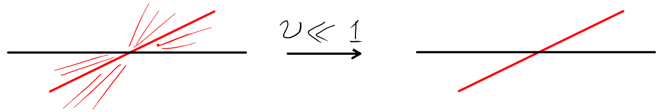
Outline

- Resummation of final state observables
- The ARES method
- The D-parameter in near-to-planar three-jet events
    1. Validation
    2. Phenomenology

Final-state observables

- Generic final-state observable, a function $V(p_1, \dots, p_n)$ of all possible final state momenta $p_1, \dots, p_n$
- Examples: Thrust, D-parameter

$$\Sigma(v) = \mathrm{Prob}\left[V(p_1, \dots, p_n) < v\right]$$



- Solved for a large class of observables at NNLL in ARES

The ARES master formula

$$\Sigma_{\rm NNLL}(v) = e^{-R_{\rm s}(v)-R_{\rm hc}(v)}\times$$
$$\times\left[\mathcal{F}_{\rm NLL}(v)\left(1+\frac{\alpha_s(Q)}{2\pi}H^{(1)}+\sum_{\ell=1}^{n_{\rm legs}}\frac{\alpha_s\left(Qv^{\frac{1}{a+b_\ell}}\right)}{2\pi}C^{(1)}_{{\rm hc},\ell}\right)\right.$$
$$\left.+\frac{\alpha_s(Q)}{\pi}\delta\mathcal{F}_{\rm NNLL}(v)\right]$$

$$\delta\mathcal{F}_{\rm NNLL} = \delta\mathcal{F}_{\rm sc}+\delta\mathcal{F}_{\rm rec}+\delta\mathcal{F}_{\rm hc}+\delta\mathcal{F}_{\rm wa}+\delta\mathcal{F}_{\rm correl}+\delta\mathcal{F}_{\rm clust}$$

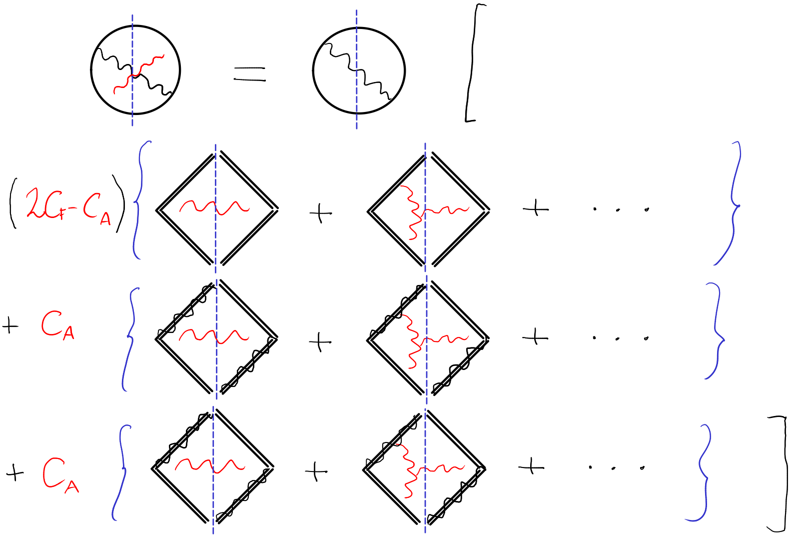
Banfi, Salam, Zanderighi '04
Banfi, McAslan, Monni, Zanderighi '14
Banfi, Monni, El-Menoufi '18

ARES for three-jet events

$$\Sigma(v) \propto \int \mathrm{d}\Phi_B \, \mathcal{V}(\Phi_B) \sum_{n=0}^{\infty} \int \prod_{i=1}^{n} [\mathrm{d}k_i] \, |\mathcal{M}(\Phi_B; k_1, \ldots, k_n)|^2 \times \Theta(v - V(\Phi_B; k_1, \ldots, k_n))$$

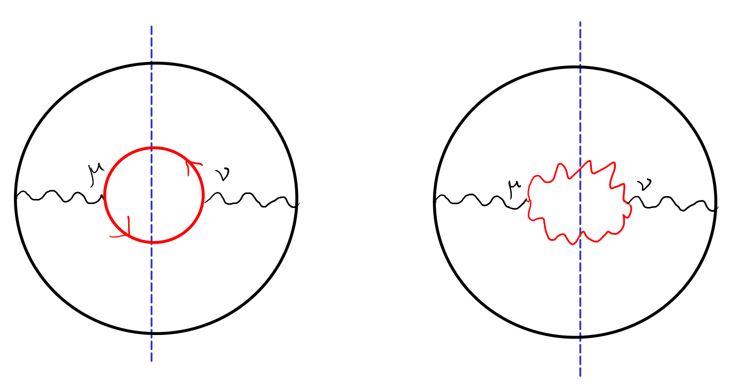
- Consider matrix element structure in different limits

Soft matrix element



Soft emissions from three-jet events factorise into sums over two-jet events: modifies soft radiator R_s

Hard-collinear matrix element



$$|\mathcal{M}(p_q, p_{\bar{q}}, p_g; k)|^2 \simeq 8\pi\alpha_s \mu^{2\epsilon} \frac{1}{k_t^2} \times$$

$$\times \Big[|\mathcal{M}(p_q, p_{\bar{q}}, p_g)|^2 P_{gq}(z;\epsilon) + |\mathcal{M}(p_q, p_{\bar{q}}, p_g)|^2 P_{g\bar{q}}(z;\epsilon)$$

$$+ \mathcal{T}_{\mu\nu}(p_q, p_{\bar{q}}, p_g) \left(\hat{P}_{gg}^{\mu\nu}(z, k_t;\epsilon) + \hat{P}_{qg}^{\mu\nu}(z, k_t;\epsilon) \right) \Big]$$

Observables sensitive to orientation have additional contributions due to spin dependence: modifies $C^{(1)}_{\mathrm{hc},\ell}$

An example three-jet event-shape: D-parameter

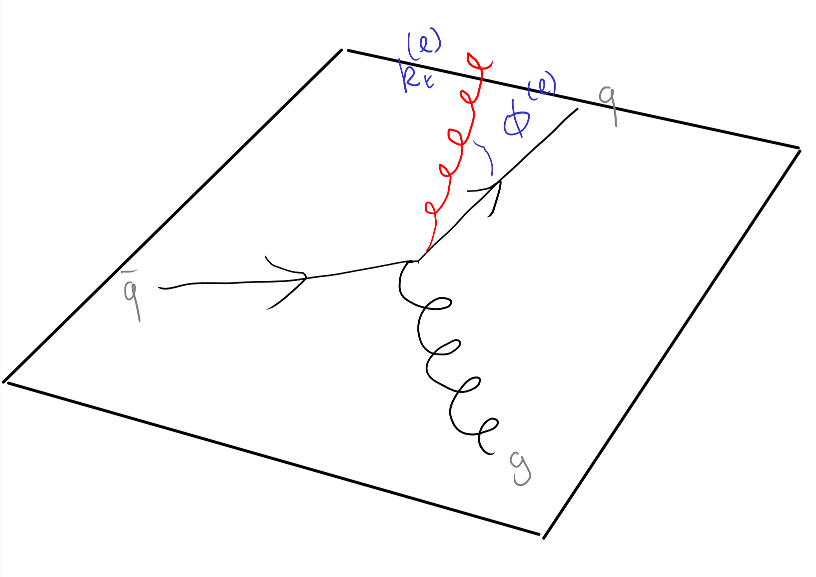
$$D = 27 \det \Theta = 27\lambda_1\lambda_2\lambda_3\,, \qquad \Theta^{ij} = \frac{1}{\sum_h |\vec{p}_h|} \sum_h \frac{p_h^i p_h^j}{|\vec{p}_h|}$$

$$D_s(\Phi_B; \{k_i\}) = \frac{27\lambda_1\lambda_2}{Q} \sum_i \frac{k_{xi}^2}{\omega_i}$$

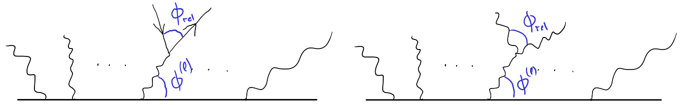
- Sensitive to out of plane radiation
- Additive
- Resum in near-to-planar three-jet region

Banfi, Dokshitzer, Marchesini, Zanderighi '01
Larkoski, Procita '18

Event geometry



$\delta\mathcal{F}_{\mathrm{correl}}$



$$\delta\mathcal{F}_{\mathrm{correl}} = \left\langle \Theta\left(1 - \lim_{v\to 0}\frac{V_{\mathrm{sc}}\left(\{\tilde{p}\}, k_a, k_b, \{k_i\}\right)}{v}\right) - \Theta\left(1 - \lim_{m^2\to 0}\lim_{v\to 0}\frac{V_{\mathrm{sc}}\left(\{\tilde{p}\}, k_a + k_b, \{k_i\}\right)}{v}\right)\right\rangle$$

- Value of $\delta\mathcal{F}_{\mathrm{correl}}$ sensitive to ϕ and $\phi^{(\ell)}$

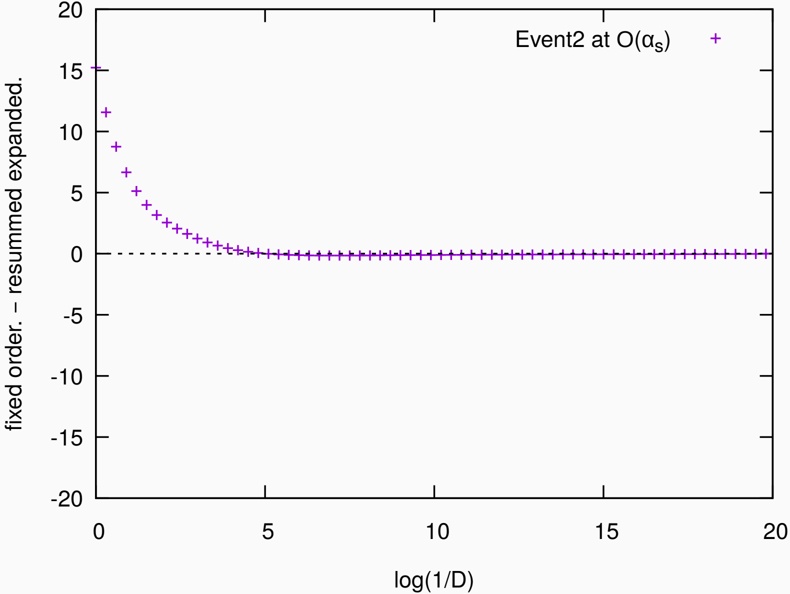
Validation

- First three-jet resummation at NNLL accuracy in ARES & of the D-parameter
- Validate our resummation as best we can by computing:

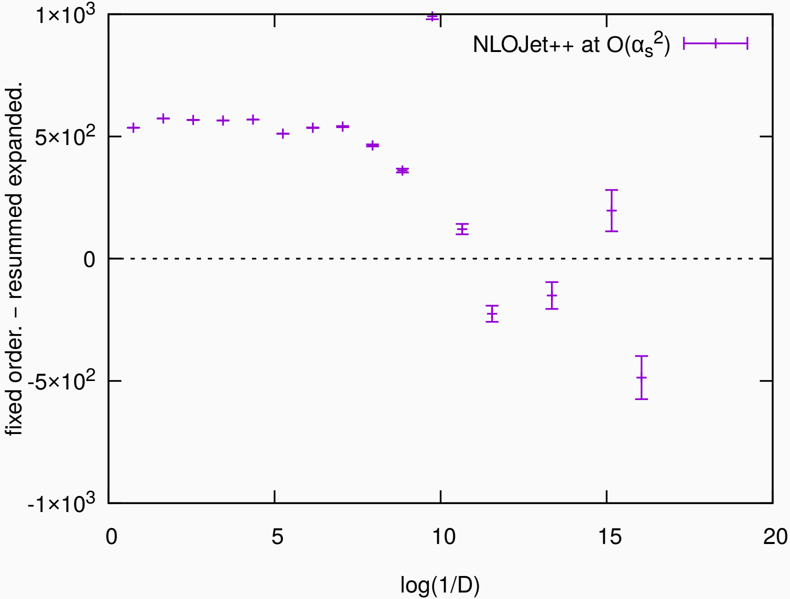
$$\Sigma_{\text{fixed-order}}(v) - \Sigma_{\text{resummed}}(v)\Big|_{\text{expanded in } \alpha_s}$$

- Need exact cancellation order-by-order in powers of α_s
  1. Check $\mathcal{O}(\alpha_s)$ terms with EVENT2
  2. Check $\mathcal{O}(\alpha_s^2)$ terms with NLOJET++
  3. No check of $\mathcal{O}(\alpha_s^3)$ terms possible
- This is sufficient to check all new terms in ARES master formula

Check of $\mathcal{O}(\alpha_s)$ terms with Event2



Check of $\mathcal{O}(\alpha_s^2)$ terms with NLOJet++

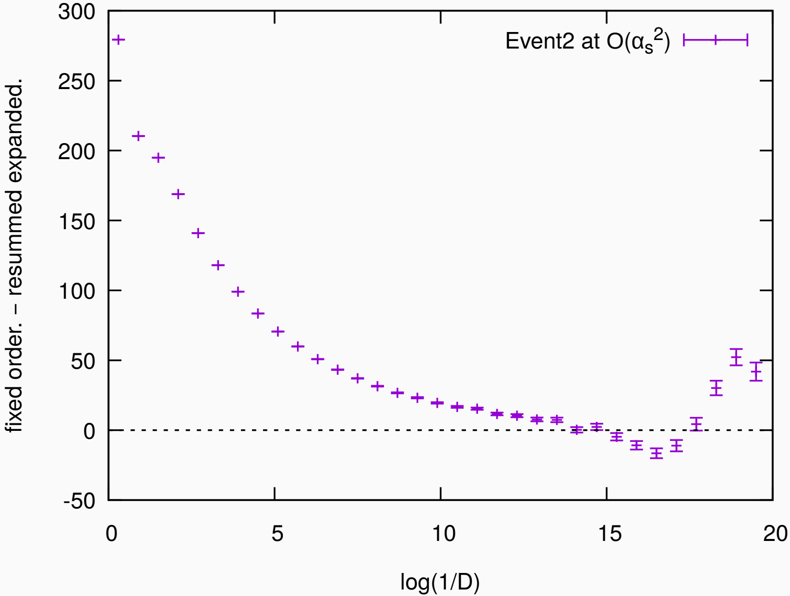


"Fake" D-parameter

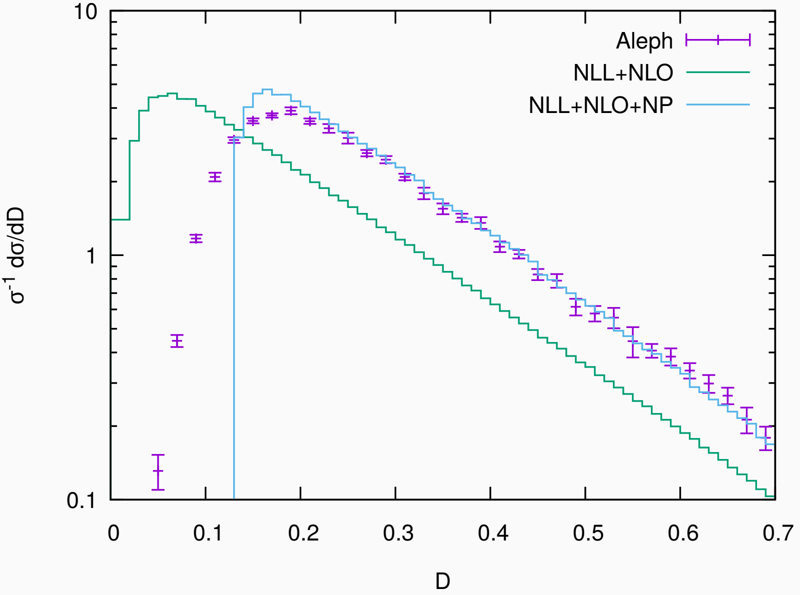
$$D = \sum_{i<j} \frac{\left| \hat{\vec{n}}_{\mathrm{beam}} \times (\vec{p}_i \times \vec{p}_j) \right|^2}{E_i E_j Q^2}$$

- Same scaling behaviour as three-jet D-parameter

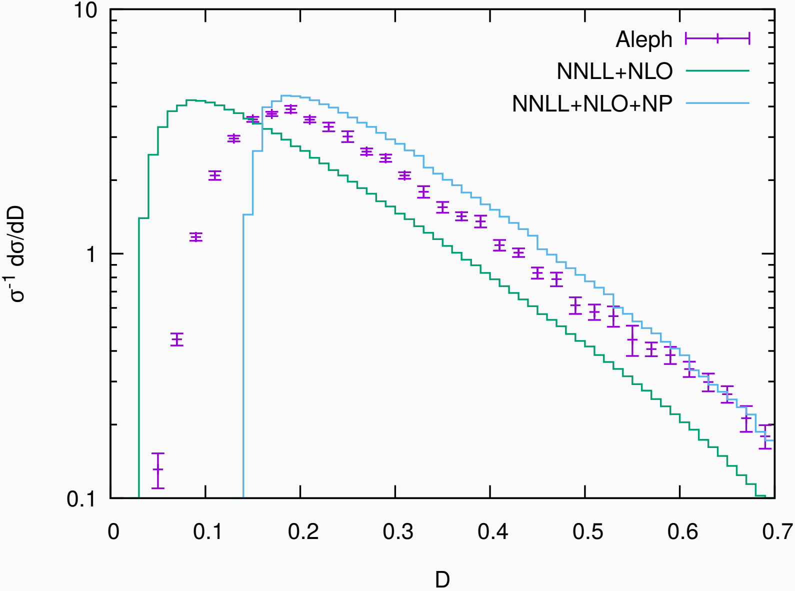
Check of $\mathcal{O}(\alpha_s^2)$ terms with Event2



Phenomenology of the D-parameter



Phenomenology of the D-parameter



Some open questions to the audience

Very large sub-leading effects

- What scale for α_s is most appropriate?
  - $\alpha_s\,(27\lambda_1\lambda_2)$
  - $\alpha_s(k_{t,\text{hard-gluon}}) \sim \alpha_s\left(\sqrt{27\lambda_1\lambda_2}\right)$
- What logarithms do we resum?
  - D
  - λ_3

Any questions?