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AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

RESUMMATION, NUMERICS, AND SOFTSERVE

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Based on work with
Guido Bell, Bahman Dehnadi, Tobias Mohrmann (Siegen), and Jim Talbert (DESY)

OUTLINE

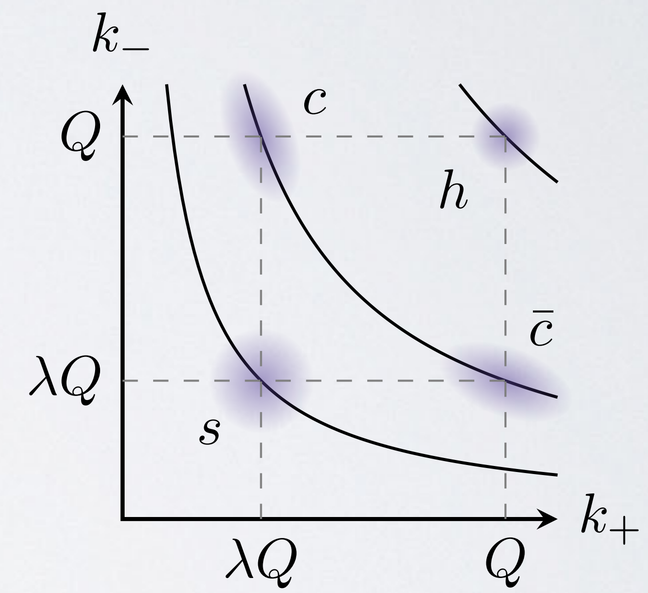
- Motivation
- SoftSERVE
- A few results
- Where to?

MOTIVATION

- Starting point:

  Global Sudakov Logarithms

$$\alpha_s^n \log^{2n} \lambda$$



- Analytic resummation via SCET

- Typical picture:

  Hard, collinear, soft modes

- Decouple, and factorise

$$\Sigma(\tau) = H(\tau, \mu, \mu_h) J(\tau, \mu, \mu_J) \bar{J}(\tau, \mu, \mu_J) S(\tau, \mu, \mu_s)$$

RUNNING GEAR

- For NNLL, we need
  - 3-loop Cusp anomalous dimension
  - 2-loop ingredient anomalous dimensions
  - 1-loop matching corrections / renormalised ingredient functions

RUNNING GEAR

- For NNLL NLR, we need
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RUNNING GEAR

- For NNLL NLR', we need
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RUNNING GEAR

- For ~~NNLL~~ NLR’, we need
  - 3-loop Cusp anomalous dimension
  - 2-loop ingredient anomalous dimensions
  - 2-loop matching corrections / renormalised ingredient functions
- Look at soft functions: Can we streamline this?

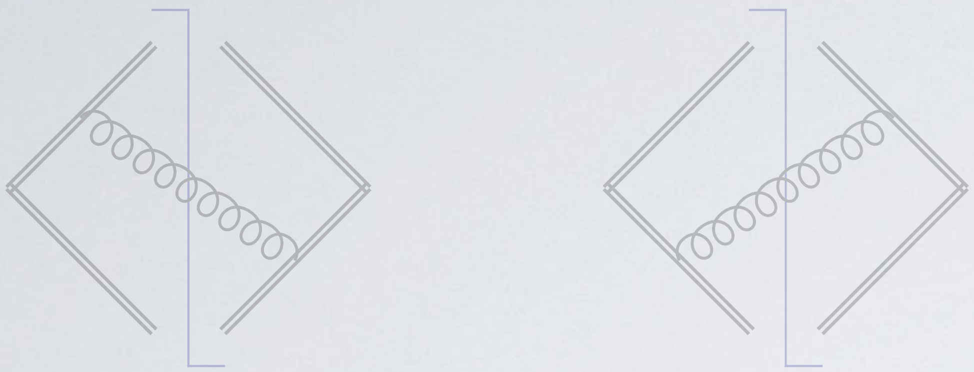
YES

- Universal dijet Laplace space soft functions:

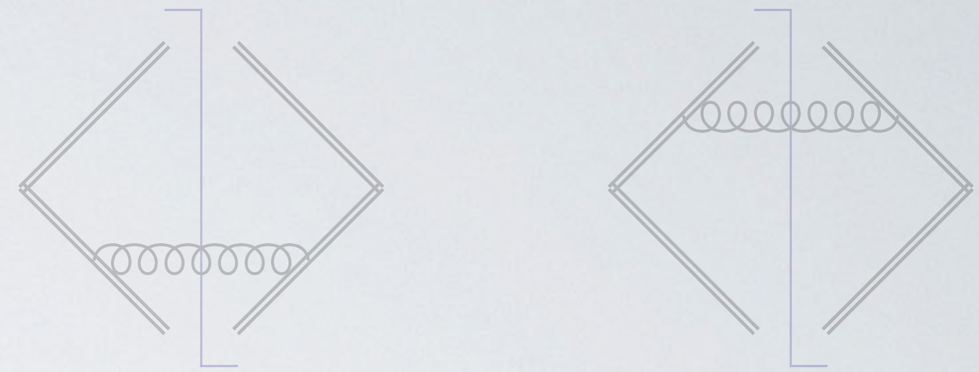
$$S(\tau,\mu) = \frac{1}{N_c}\sum_X \mathcal{M}(\tau,\{k_i\})\ \mathrm{Tr}\,\langle 0|S_{\bar{n}}^\dagger(0)S_n(0)|X\rangle\langle X|S_n^\dagger(0)S_{\bar{n}}(0)|0\rangle$$

  - Matrix element is not nice, but fixed
  - Measurement is harmless, but observable-dependent
- Extract divergences, expand to Laurent series:

$$S_2 \sim \frac{\int \mathrm{d}\{x\} f_4(\{x\})}{\epsilon^4} + \frac{\int \mathrm{d}\{x\} f_3(\{x\})}{\epsilon^3} + \frac{\int \mathrm{d}\{x\} f_2(\{x\})}{\epsilon^2} + \ldots$$



NLO



- Matrix element:

$$|\mathcal{A}(k)|^2 \sim \frac{\alpha_s C_F}{k_+ k_-}$$

- Parametrise:

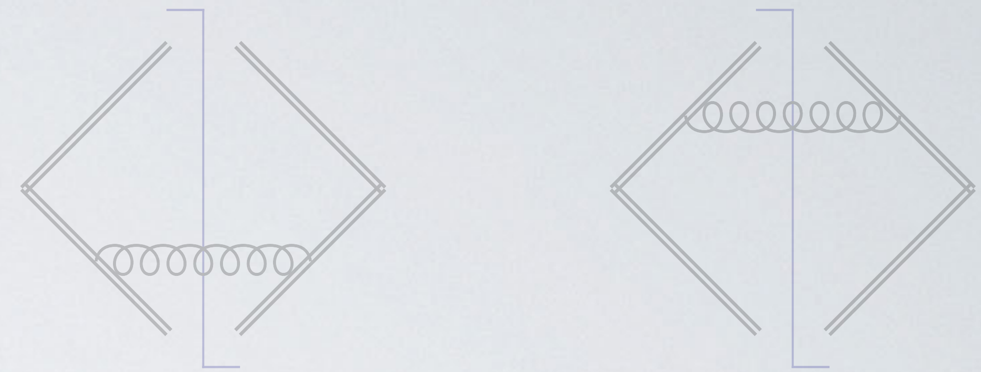
$$y_k = \frac{k_+}{k_-}, \quad k_T = \sqrt{k_+ k_-}$$

- (Informed) Assumption: $\mathcal{M}^{(1)}(\tau, k) = e^{-\tau\, k_T\, y_k^{\frac{n}{2}}\, f(y_k, \vartheta_k)}$

- Master formula after integrating:

$$S_R(\tau,\mu) \sim \Gamma(-2\epsilon-\alpha) \int_0^1 \mathrm{d}t_k \sqrt{4t_k\bar{t_k}}^{\,-1-2\epsilon} \int_0^1 \mathrm{d}y_k\, y_k^{-1+n\epsilon+\alpha/2}\, \frac{f(y_k,t_k)^{2\epsilon+\alpha}}{(1+y_k)^{-\alpha}}$$

NLO



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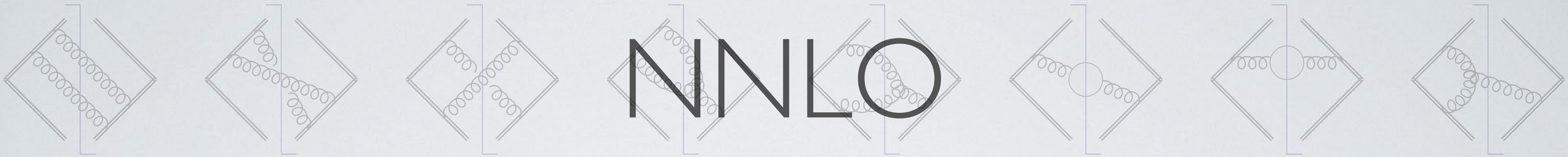


- Same idea, two structures: $C_F C_A, C_F T_f n_f$ & C_F^2

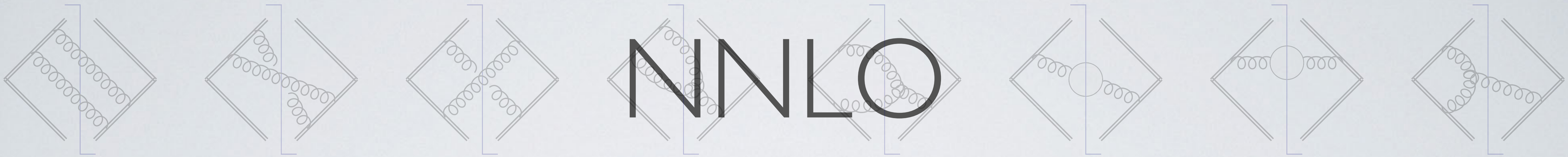
- Divergences Correlated
  - Collinear to { Wilson line / Each other
  - Soft { Globally / Relatively

[1812.08690]

- Divergences Uncorrelated
  - Collinear to { Wilson line / Wilson line
  - Soft { Globally / Relatively

[In preparation.]

NNLO



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  - Soft { Mass dim. / IRC safety

[1812.08690]

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  - Soft { Mass dim. / IRC safety

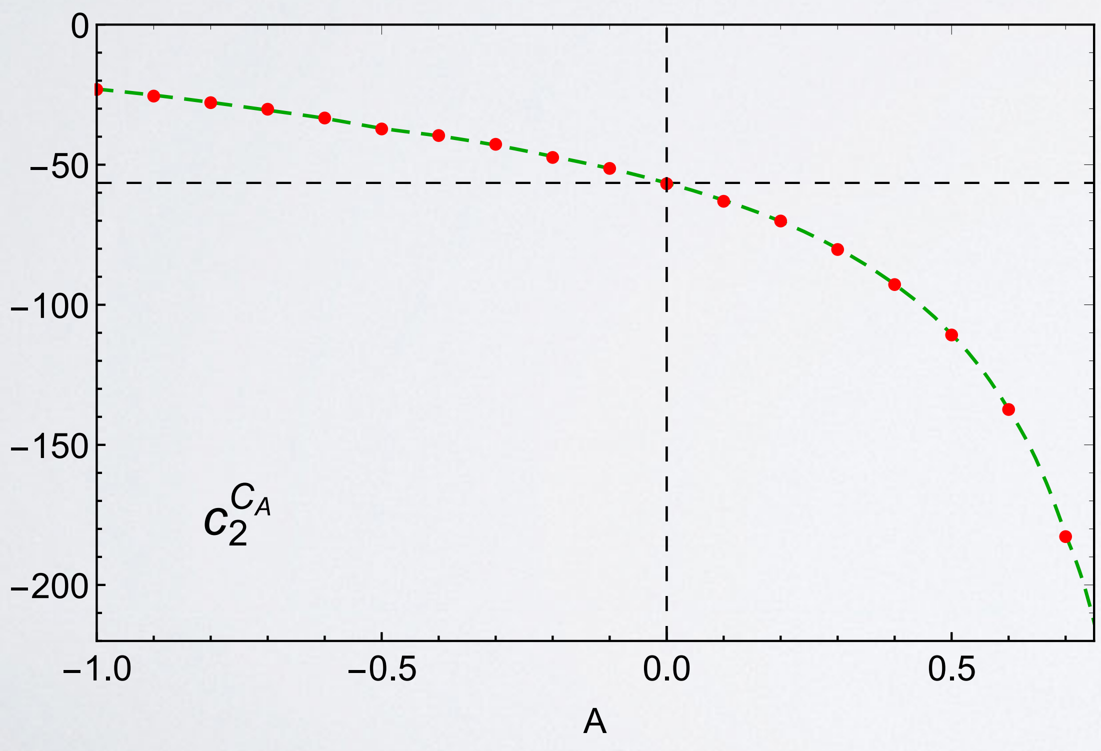
[In preparation.]

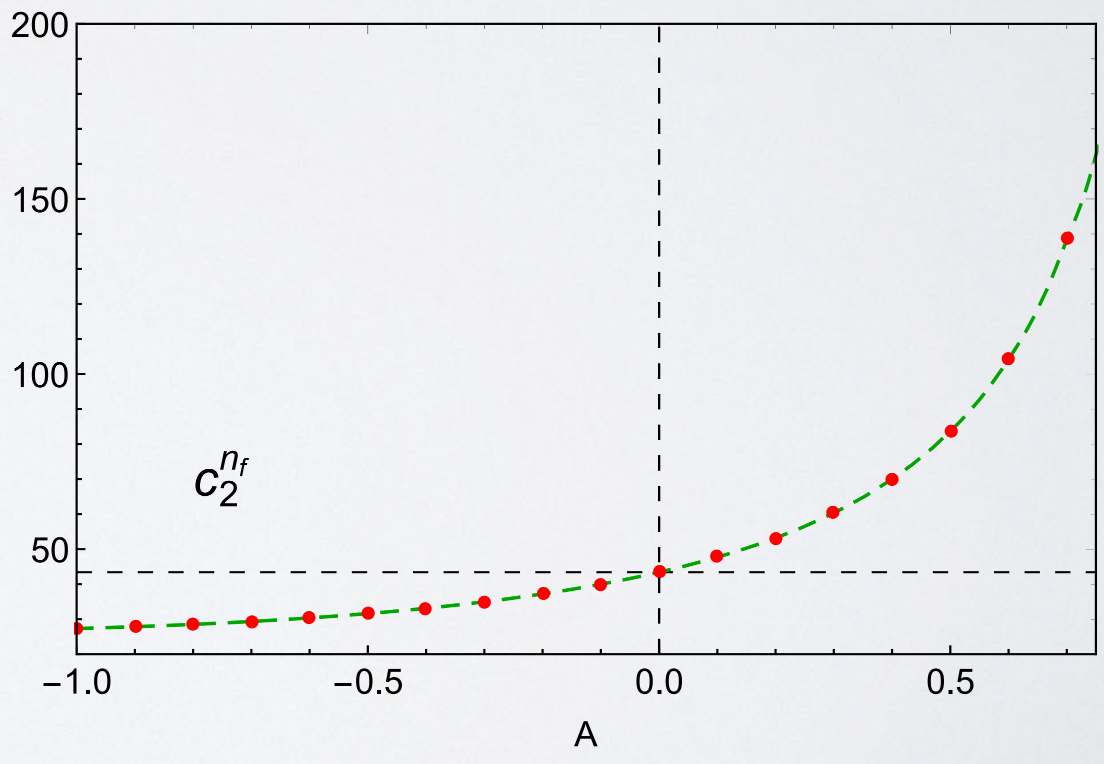
SOFTSERVE

- C++ (well, C) implementation, using Cuba library (Divonne)
- A recipe to generate numerical integration binaries
- Manual user input required (apply parametrisation to definition)
- Scripts for renormalisation, Fourier transforms
- Uses substitutions to flatten integrable divergences
- Can be found on softserve.hepforge.org (for now only correlated)

VANILLA RESULT: ANGULARITIES

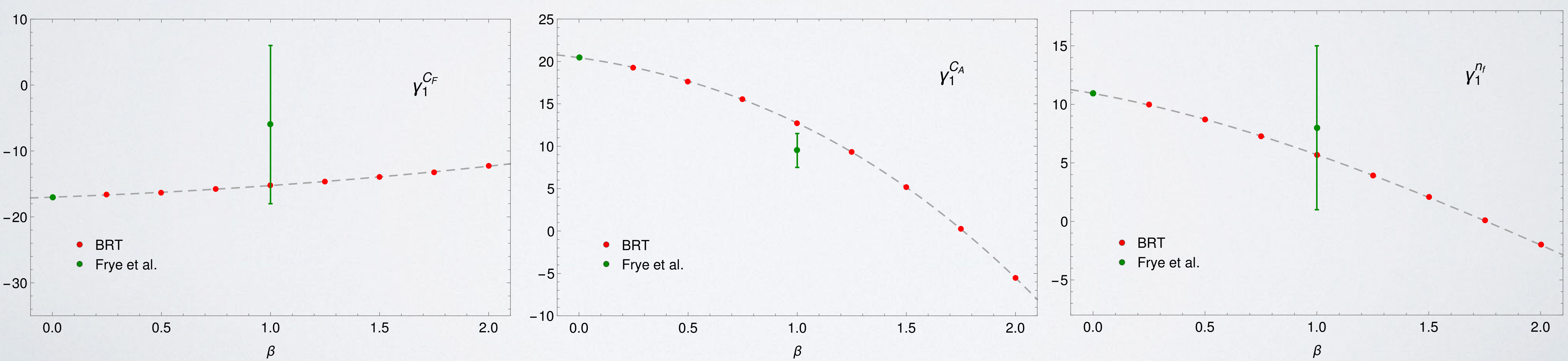
- Generalisation of Thrust
- Used for resummation in [Bell et al., 1808.07867]





VANILLA RESULT: SOFT DROP JET MASS

- Violates Non-abelian exponentiation
- Green: Extraction using EVENT2 in [Frye et al, 1603.09338]



ASIDE: COMPLEXITY

- Performance of SoftSERVE is essentially uncorrelated with complexity of input
- Angularities measurement (momentum space):

$$\delta\big(\omega_A - \min(k_+, k_-)^{1-A/2} \max(k_+, k_-)^{A/2} - \min(l_+, l_-)^{1-A/2} \max(l_+, l_-)^{A/2}\big)$$

SOFT DROP MEASUREMENT

- A bit more complex

$$
\begin{aligned}
&\theta(k_- - k_+)\theta(l_+ - l_-) \\
&\times \Big[\theta(k_0^{1+\beta/2}k_+^{-\beta/2} - l_0^{1+\beta/2}l_-^{-\beta/2})\delta(\omega_{SD} - k_0^{1+\beta/2}k_+^{-\beta/2}) + \\
&\quad + \theta(l_0^{1+\beta/2}l_-^{-\beta/2} - k_0^{1+\beta/2}k_+^{-\beta/2})\delta(\omega_{SD} - l_0^{1+\beta/2}l_-^{-\beta/2})\Big] \\
&+ \theta(k_- - k_+)\theta(l_- - l_+) \\
&\times \Big[\theta(k_+ - \frac{k\cdot l}{l_0})\theta(l_+ - \frac{k\cdot l}{k_0})\delta(\omega_{SD} - (k_0 + l_0)^{1+\beta/2}(k_+ + l_+)^{-\beta/2}) \\
&\quad + \big[1 - \theta(k_+ - \frac{k\cdot l}{l_0})\theta(l_+ - \frac{k\cdot l}{k_0})\big]\times \\
&\qquad \Big[\theta(k_0^{1+\beta/2}k_+^{-\beta/2} - l_0^{1+\beta/2}l_+^{-\beta/2})\delta(\omega_{SD} - k_0^{1+\beta/2}k_+^{-\beta/2}) \\
&\quad + \theta(l_0^{1+\beta/2}l_+^{-\beta/2} - k_0^{1+\beta/2}k_+^{-\beta/2})\delta(\omega_{SD} - l_0^{1+\beta/2}l_+^{-\beta/2})\Big]\Big] \\
&+ (n \leftrightarrow \bar{n})
\end{aligned}
$$

SCET-2

- SCET-2 observables exhibit rapidity divergences
- Resum using *collinear anomaly* or *Rapidity renormalisation group*
  [Becher, Neubert, '11] [Chiu et al., '12]
- Second regulator required
- We use a phase space regulator

$$\mathrm{dPS}^{(n)} = \prod_{i=1}^{n} \frac{\mathrm{d}^d k_i}{(2\pi)^d} \left(\frac{\nu}{2E_{k_i}} \right)^{\alpha} \delta^+(k_i^2)$$

CALAMITY!

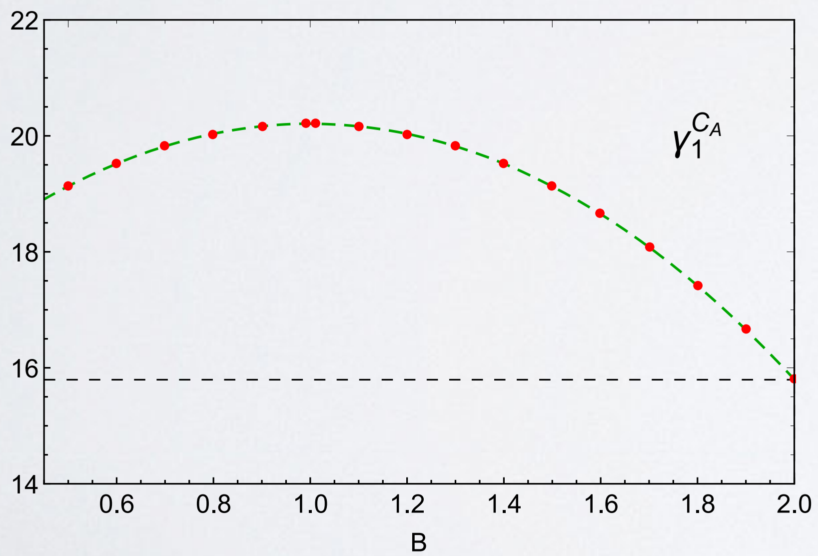
- Transverse momentum resummation using phase space regulator and RRG: poles don't cancel!
- RRG requires regulator on connected webs
- Using that we reproduce the correct result:

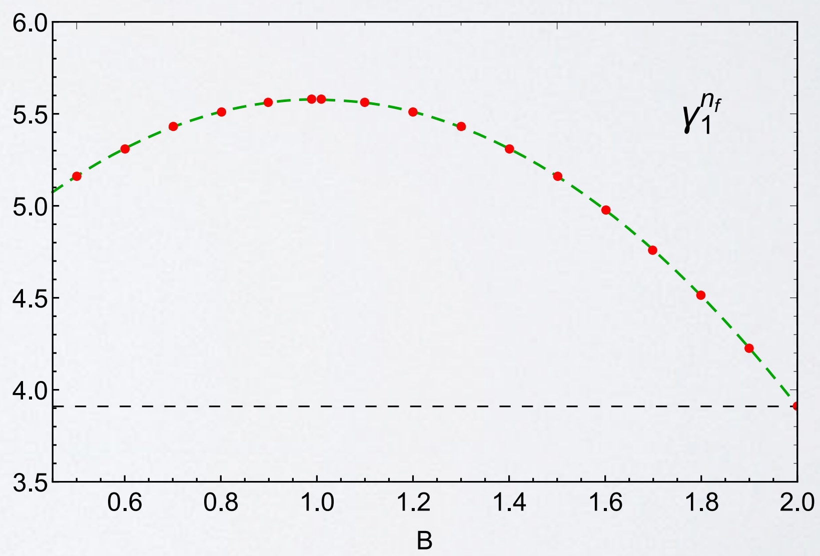
$$S_2 = 10.352(8)[10.347]C_F T_f n_F - 16.517(59)[16.507]C_f C_A + \frac{\pi^4}{18}C_F^2$$

[Lübbert et al, 1602.01829]

SCET-2, PART DEUX

- Angularities are parametric across a spectrum
- One point is SCET-2
- We can interpolate! [1805.12414]: $d_2 = -\gamma^S_{1,\mathrm{IP}} + \beta_0 d'_1$





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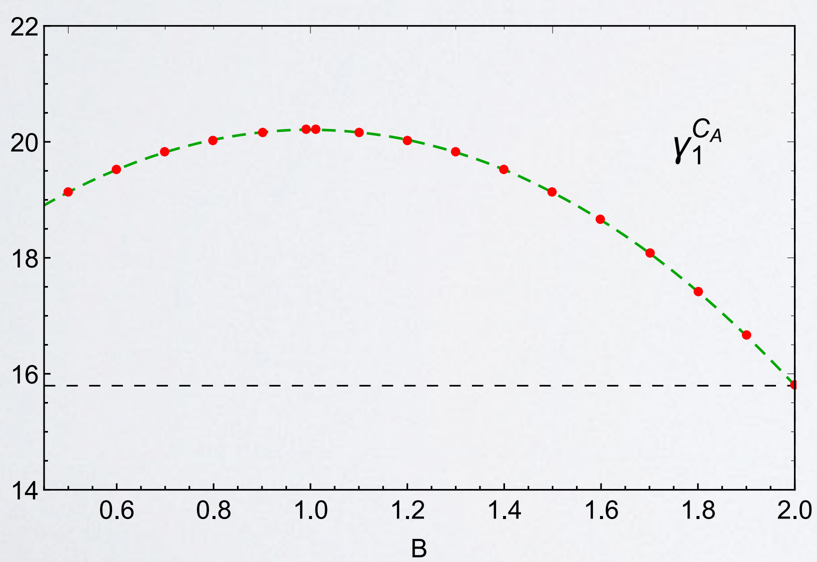
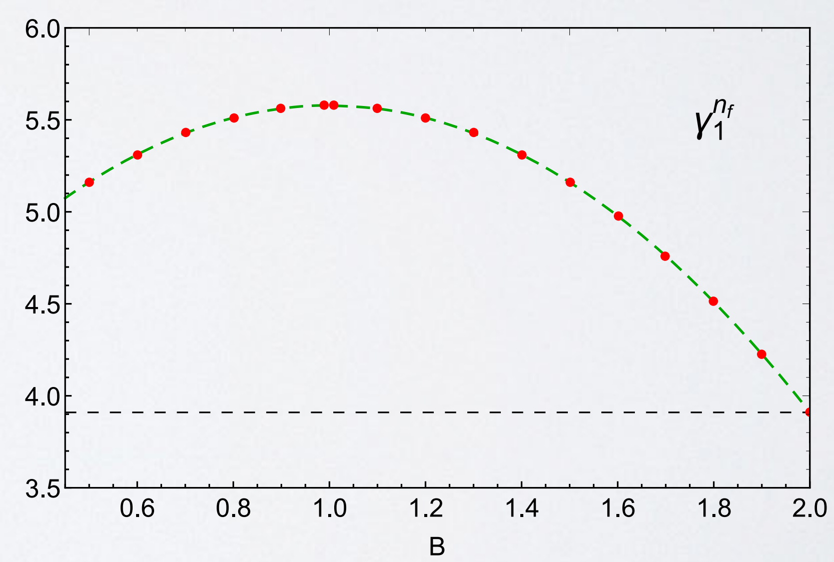
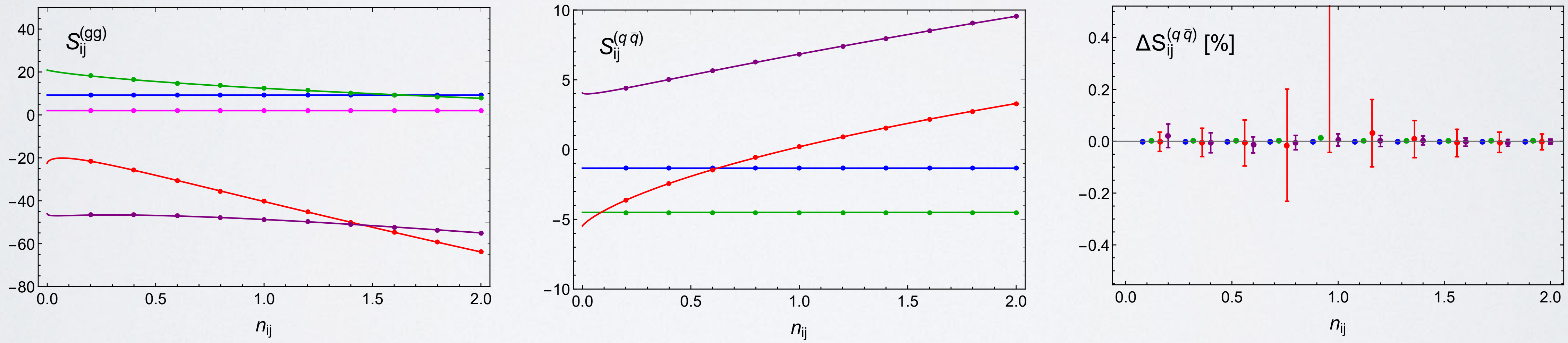




- But only for anomaly exponent/anomalous dimension

DIJET NON-BACK-2-BACK

- Boost to B2B frame
- Transverse space more complicated, more angular dependence (unless we're lucky): 5 dynamic angles, not 3
- If we're lucky: [Caola et al., 1807.05835], $\epsilon^{-4,3,2,1,0}$



- Not even an SCET calculation:
  "The double-soft integral for an arbitrary angle between hard radiators"

STEP FURTHER: N-JET

- In addition to before: tripoles, quadrupoles

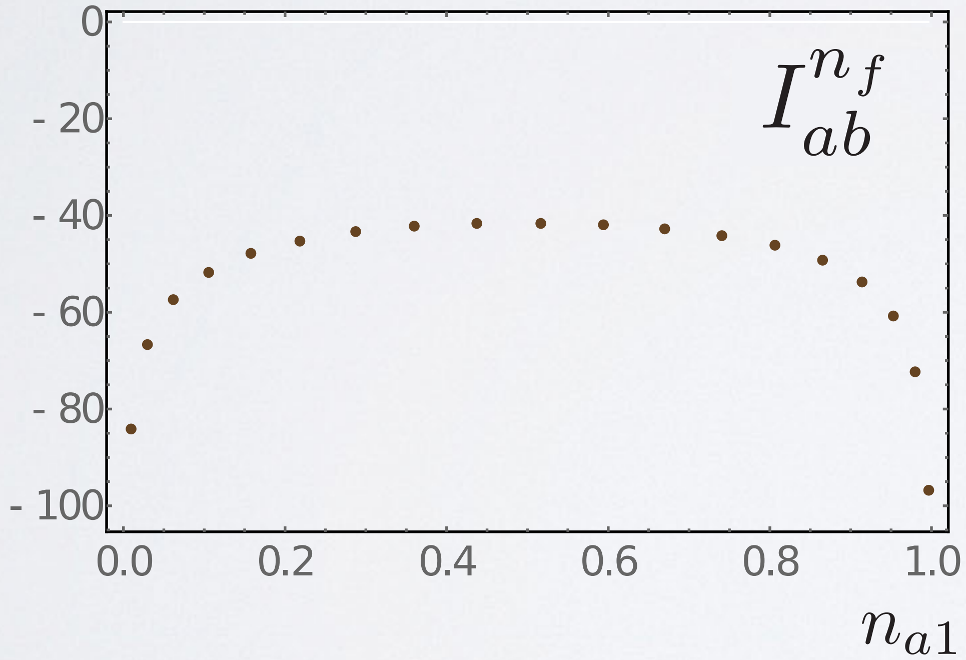
STEP FURTHER: N-JET

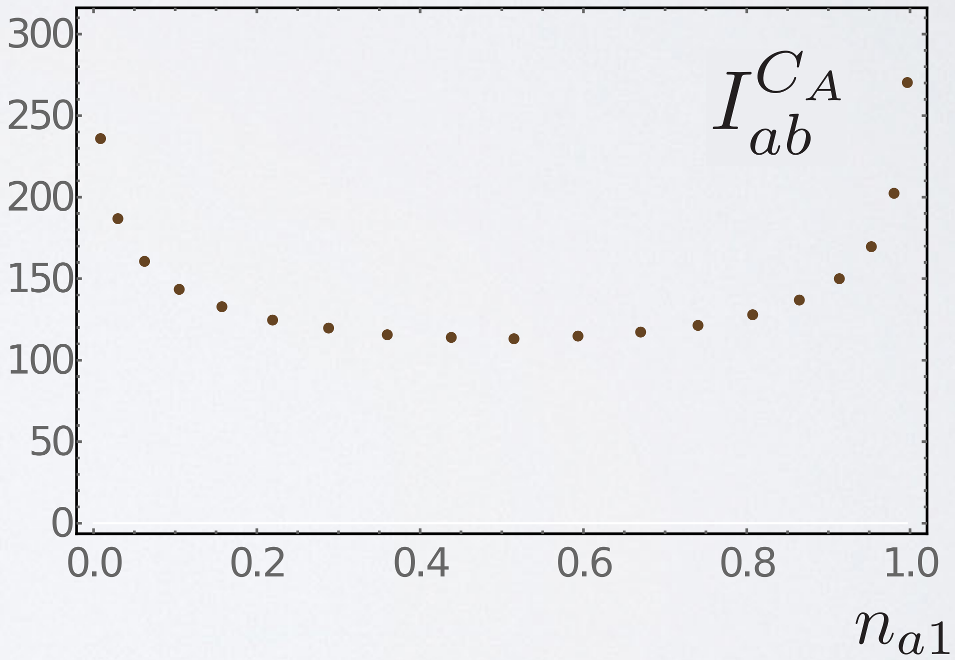
- In addition to before: tripoles, quadrupoles Assume NAE!
    - Only contributes to RV: NLO like
- Angular integrations still more complex
    - a numerical nightmare: $x^{-3/2-\epsilon} f(x, \epsilon)$
- My substitutions don't work…

PRELIMINARY: N-JETTINESS

- Preliminary results for 1- and 2-jettiness available

- 2-jettiness (no error bars yet):





- Higher N: #grid points, #dipoles

CONCLUSION

- We have developed an approach to streamline the calculation of dijet soft functions in (mostly) SCET
- SoftSERVE is publicly available for correlated emissions, and will soon be for the uncorrelated ones
- SCET-2 continues to surprise us
- N-jet extension is in the works