Threshold factorization of the Drell-Yan process at NLP

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Threshold factorization of the Drell-Yan process at next-to-leading power

Martin Beneke, Alessandro Broggio, Sebastian Jaskiewicz and Leonardo Vernazza

To appear soon

Leading-logarithmic threshold resummation of the Drell-Yan process at next-to-leading power

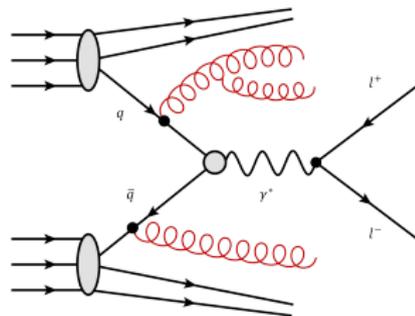Martin Beneke, Alessandro Broggio, Mathias Garny, Sebastian Jaskiewicz, Robert Szafron, Leonardo Vernazza and Jian Wang

Outline

- Motivations - a short overview $\to$ see also talk by Thomas Becher
- The Drell-Yan process - review of factorization at leading power within the position space SCET framework.
- The Drell-Yan process - new features at next-to-leading power
  - Accounting for power corrections
  - Appearance of collinear functions
  - Generalized soft functions
- Factorization formula at next-to-leading power

Power expansion for observables

Schematic form for production cross-sections near threshold, $z \to 1$:

$$\hat{\sigma}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[c_n \delta(1-z) + \sum_{m=0}^{2n-1} \left(c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+ + d_{nm} \ln^m(1-z) \right) + \dots \right]$$

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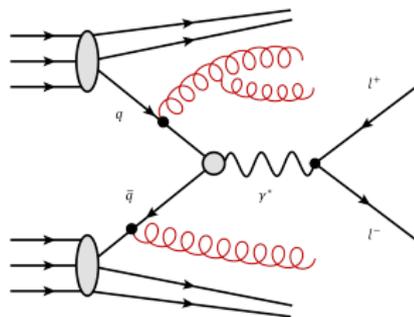Schematic form for production cross-sections near threshold, $z \to 1$:

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▶ Leading power (LP) logarithms

Threshold Drell-Yan (N^3LL/ NNLR) [T. Becher, M. Neubert, G. Xu, 0710.0680]

Higgs production with jet veto (NNLL/ NLR) [T. Becher, M. Neubert, 1205.3806] [C. Berger, C. Marcantonini, I. Stewart, F.Tackmann, W. Waalewijn, 1012.4480]

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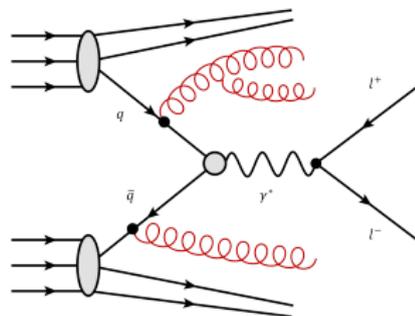
▶ *Next-to*-leading power (NLP) logarithms

Subleading power resummed thrust spectrum for $H \to gg$ [I. Moult, I. Stewart, G. Vita, H. Zhu, 1804.04665]
Drell-Yan production at threshold [M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]

Diagrammatic resummation of threshold effects [N. Bahjat-Abbas, D. Bonocore, J. Sinninghe Damste, E. Laenen, L. Magnea, L. Vernazza, C. White, 1905.13710]

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More on NLP:

▶ Power corrections for N-jettiness subtractions at $\mathcal{O}(\alpha_s)$ [M. Ebert, I. Moult, I. Stewart, F. Tackmann, G. Vita, H. Zhu, 1807.10764]

▶ Subleading power rapidity divergences and power corrections for q_T [M. Ebert, I. Moult, I. Stewart, F. Tackmann, G. Vita, H. Zhu, 1812.08189]

▶ Anomalous dimension of subleading-power N-jet operators II [M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

SCET formalism

SCET formalism: Lagrangian

In this talk we employ position-space SCET formalism

[M. Beneke, A. Chapovsky, M. Diehl, Th. Feldmann, 0206152]

$$\mathcal{L}_{\text{SCET}} = \sum_{i=1}^{N} \mathcal{L}_{c_i} + \mathcal{L}_{\text{soft}}$$

where each of the Lagrangians belonging to a collinear direction is expanded in small parameter λ:

$$\mathcal{L}_{c_i} = \underbrace{\mathcal{L}_{c_i}^{(0)}}_{\text{LP}} + \underbrace{\mathcal{L}_{c_i}^{(1)}}_{\mathcal{O}(\lambda^1)} + \underbrace{\mathcal{L}_{c_i}^{(2)}}_{\mathcal{O}(\lambda^2)} + ...$$

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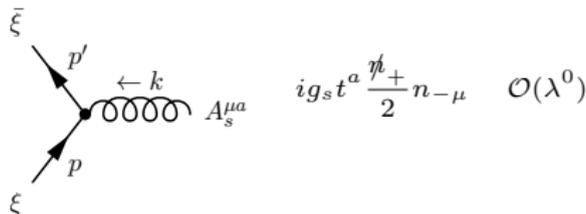where each of the Lagrangians belonging to a collinear direction is expanded in small parameter λ:

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Separate collinear sectors interact only through soft gluon interactions. Focusing in LP term:

$$\mathcal{L}_c^{(0)} = \bar{\xi} \left(in_- D_c + g\, n_- A_s(x_-) + i\slashed{D}_{\perp c} \frac{1}{in_+ D_c} i\slashed{D}_{\perp c} \right) \frac{\slashed{n}_+}{2} \xi + \mathcal{L}_{c,\mathrm{YM}}^{(0)}$$

with $in_- D_c = in_- \partial + g\, n_- A_c(x)$, $x_-^\mu = (n_+ x)\dfrac{n_-^\mu}{2}$.

The soft interaction with each collinear field at LP is given by the standard eikonal vertex

$$ig_s t^a \frac{\slashed{n}_+}{2} n_{-\mu} \qquad \mathcal{O}(\lambda^0)$$

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The decoupling transformation, $\chi_c^{(0)} = Y_+^{\dagger}(0)\chi_c$, separates soft and collinear sectors at LP

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)} \frac{\slashed{n}_+}{2} (n_- \mathcal{A}_c + n_- \partial) \chi^{(0)}(x)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

SCET formalism: Lagrangian

The structure of the SCET Lagrangian beyond LP is more intricate

[M. Beneke, Th. Feldmann, 0211358]

$$\mathcal{L}_c^{(1)} \quad = \quad \bar{\xi}\left(x_\perp^\mu\, n_-^\nu\, W_c\, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger\right) \frac{\slashed{n}_+}{2}\, \xi + \mathcal{L}_{\xi q}^{(1)} + \mathcal{L}_{\mathrm{YM}}^{(1)}$$

$$\begin{aligned}
\mathcal{L}_\xi^{(2)} \quad = \quad & \frac{1}{2}\,\bar{\xi}\left((n_-x)\, n_+^\mu n_-^\nu\, W_c\, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger + x_\perp^\mu x_{\perp\rho} n_-^\nu\, W_c\left[D_{\mathrm{us}}^\rho, g F_{\mu\nu}^{\mathrm{us}}\right] W_c^\dagger\right) \frac{\slashed{n}_+}{2}\, \xi \\
& + \frac{1}{2}\,\bar{\xi}\left(i\slashed{D}_{\perp c}\, \frac{1}{in_+ D_c}\, x_\perp^\mu \gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu\, W_c\, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger\, \frac{1}{in_+ D_c}\, i\slashed{D}_{\perp c}\right) \frac{\slashed{n}_+}{2}\, \xi
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& + \frac{1}{2} \bar{\xi}\left(i\slashed{D}_{\perp c} \frac{1}{in_+D_c} x_\perp^\mu \gamma_\perp^\nu \, W_c \, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger + x_\perp^\mu \gamma_\perp^\nu \, W_c \, g F_{\mu\nu}^{\mathrm{us}} W_c^\dagger \frac{1}{in_+D_c} i\slashed{D}_{\perp c}\right) \frac{\slashed{n}_+}{2} \, \xi
\end{aligned}$$

▶ Importantly, there are no purely collinear interactions at subleading powers. In each vertex there is at least one soft field.

SCET formalism: N-jet operator basis

Generic N-jet operator has the form:

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416]

$$J = \int \prod_{i=1}^{N} \prod_{k_i=1}^{n_i} dt_{ik_i} \ C(\{t_{ik_i}\}) \prod_{i=1}^{N} J_i(t_{i_1}, t_{i_2}, ..., t_{i_{n_i}})$$

where the Js are constructed by multiplying collinear gauge invariant building blocks in the same direction (up to $\mathcal{O}(\lambda^2)$)

$$\chi_i(t_i n_{i+}) \equiv W_i^\dagger \xi_i \qquad\qquad \mathcal{A}_{i\perp}^\mu \equiv W_i^\dagger [iD_{\perp i}^\mu, W_i]$$

and by acting on these with derivatives $i\partial_{\perp i}^\mu$.

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We adopt the notation: J^{An}, J^{Bn}, J^{Cn} where:

- $A, B, C...$ refers to number of fields in a given collinear direction
- n is the power of λ suppression (relative to $A0$) in a given sector.

at $\mathcal{O}(\lambda^2)$ for example we can construct J^{A2}, J^{B2}, J^{C2} respectively:

$$i\partial_{\perp i}^\mu i\partial_{\perp i}^\nu \chi_i, \qquad\qquad \chi_i(t_{i_1}) i\partial_{\perp i}^\nu \mathcal{A}_{i\perp}^\mu(t_{i_2}), \qquad\qquad \chi_i(t_{i_1}) \chi_i(t_{i_2}) \mathcal{A}_{i\perp}^\mu(t_{i_3})$$

And a $\mathcal{O}(\lambda^2)$ 3-jet operator could be

$$J^{A2} J^{A0} J^{A0}, \qquad\qquad\qquad J^{A1} J^{A0} J^{B1}, ...$$

Drell-Yan process

The Drell-Yan process at threshold

$$A(p_A)B(p_B) \to \mathrm{DY}(Q) + X$$

$$z = \frac{Q^2}{\hat{s}} \qquad \lambda = \sqrt{(1-z)}$$

$$
\begin{aligned}
p_c &= (n_+ p_c, n_- p_c, p_{c\perp}) \sim Q(1, \lambda^2, \lambda) \\
p_{\bar{c}} &= (n_+ p_{\bar{c}}, n_- p_{\bar{c}}, p_{\bar{c}\perp}) \sim Q(\lambda^2, 1, \lambda) \\
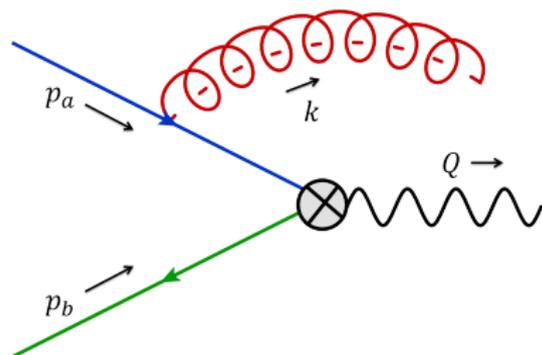p_s &= (n_+ p_s, n_- p_s, p_{s\perp}) \sim Q(\lambda^2, \lambda^2, \lambda^2)
\end{aligned}
$$

$$p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$$

$$\bar{\psi}\gamma_\mu\psi = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t})\, J_\mu^{A0}(t,\bar{t})$$

$$J_\rho^{A0}(t,\bar{t}) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\rho}\chi_c(tn_+)$$

Gauge invariant building block:

$$\chi(x) = W^\dagger(x)\,\psi(x)$$

The Drell-Yan proccess - the leading power result

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a\, dx_b\, f_{a/A}(x_a) f_{b/B}(x_b)\, \hat{\sigma}_{ab}^{\mathrm{LP}}(z)$$
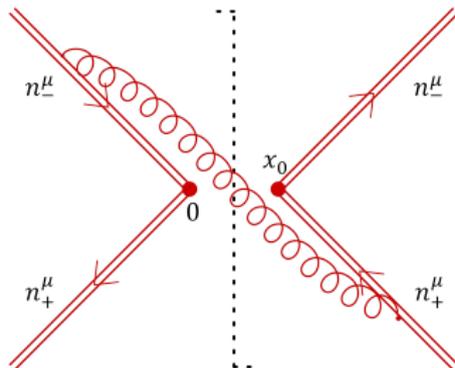
where

[G. P. Korchemsky *et al.*, 1993]
[T. Becher *et al.*, 0710.0680, S. Moch *et al.*, 0508265]

$$\hat{\sigma}^{\,\mathrm{LP}}(z) = |C(Q^2)|^2\, Q\, S_{\mathrm{DY}}(Q(1-z))$$

$$S_{\mathrm{DY}}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

Factorization formula at NLP

First a schematic formula:

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}^{\mathrm{NLP}}(z)$$

The $\hat{\sigma}_{ab}^{\mathrm{NLP}}(z)$ is given by

$$\hat{\sigma}^{\,\mathrm{NLP}} = \sum_{\mathrm{terms}} \left[C \otimes J \otimes \bar{J} \right]^2 \otimes S$$

- C is the hard Wilson matching coefficient
- S is the *generalized* soft function
- J is the collinear function

Let us now motivate the emergence of this structure at next-to-leading power.

Collinear functions at LP and NLP

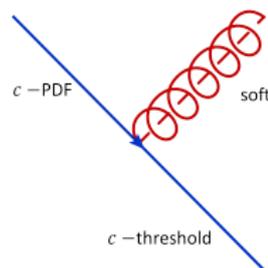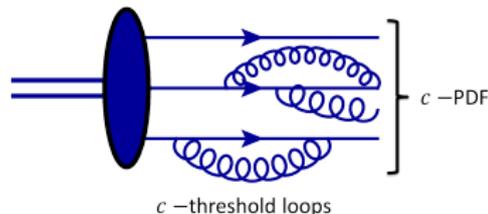- There is no collinear function present at LP because of decoupling transformation [C. Bauer, D. Pirjol, and I. Stewart, 0109045]

$$\left(J_{A0,2\xi}^{T2}(s,t)\right)^{\mu} = i \int d^4z \, \mathbf{T} \left[J_{A0}^{\mu}(s,t) \, \mathcal{L}_{2\xi}^{(2)}(z)\right]$$

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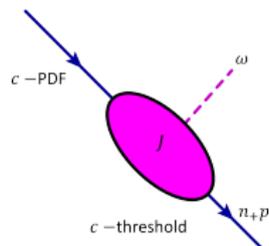
- There is no collinear function present at LP because of decoupling transformation [C. Bauer, D. Pirjol, and I. Stewart, 0109045]

- This is no longer true at NLP. Consider an example of subleading SCET Lagrangian: $\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c^{(0)} z_\perp^\mu z_\perp^\rho \left[i\partial_\rho in_-\partial \mathcal{B}_\mu^+(z_-) \right] \frac{\slashed{n}_+}{2} \chi_c^{(0)}, \ \ \mathcal{B}_\pm^\mu = Y_\pm^\dagger \left[iD_s^\mu Y_\pm \right]$
  [M. Beneke and Th. Feldmann, 0211358]

- Crucially, an insertion of a piece of a subleading lagrangian comes with an integral over its position, $\int d^4z$, which induces a collinear scale. Note, no J^{A2}, J^{B2}, J^{C2} currents appear at NLP.

c −threshold loops

c −PDF

soft

c −threshold

$$\left(J_{A0,2\xi}^{T2}(s,t) \right)^\mu = i \int d^4z \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_{2\xi}^{(2)}(z) \right]$$

Collinear functions at NLP

- PDF collinear modes *can* be radiated into the final state
  Modes: $p_c \sim Q(1, \lambda^2, \lambda)$ and $p_{c-\mathrm{PDF}} \sim (Q, \Lambda^2/Q, \Lambda)$

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▶ Hence we define the matching equation which gives a SCET definition of what is known as the "radiative jet function"

[D. Bonocore, E. Laenen, L. Magnea, S. Melville, L. Vernazza, C.D. White, 1503.05156]
see also [D. Bonocore, E. Laenen, L. Magnea, L. Vernazza, C.D. White, 1610.06842]
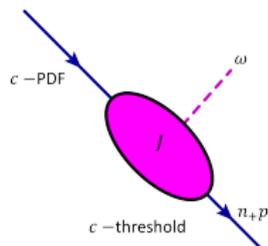
▶

$$i \int d^4 z \, \mathbf{T}\Big[\chi_{c,\gamma f}\,(tn_+)\,\mathcal{L}^{(2)}(z)\Big]$$

$$= 2\pi \sum_i \int du \int \frac{d(n_+ z)}{2} \, \tilde{J}_{i;\gamma\beta,\mu,fbd}\left(t, u; \frac{n_+ z}{2}\right) \chi_{c,\beta b}^{\text{PDF}}(un_+) \, \mathbf{\mathfrak{S}}_{i;\mu,d}(z_-)$$

$$\mathbf{\mathfrak{S}}_i(z_-) \in \left\{ \frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}_{\mu_\perp}^+(z_-), \, \frac{i\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}_{\nu_\perp]}^+(z_-), \, \frac{1}{(in_-\partial)}\big[\mathcal{B}_{\mu_\perp}^+(z_-), \mathcal{B}_{\nu_\perp}^+(z_-)\big], \dots \right\}$$

[M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]
[M. Beneke, A. Broggio, SJ, L. Vernazza, to appear soon]

Equation of motion:

$$n_+\mathcal{B}^+(z_-) = -2\frac{i\partial_\perp^\mu}{in_-\partial}\mathcal{B}_{\mu_\perp}^+(z_-)$$

$$-2\frac{\big[\mathcal{B}_\perp^+, \big[in_-\partial\mathcal{B}_{\mu_\perp}\big]\big]}{(in_-\partial)^2} + \dots$$

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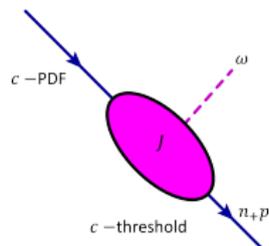
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$$\mathbf{S}_i(z_-) \in \left\{ \frac{i\partial_\perp^\mu}{in_-\partial} \mathcal{B}_{\mu_\perp}^+(z_-), \, \frac{i\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}_{\nu_\perp]}^+(z_-), \, \frac{1}{(in_-\partial)} \left[\mathcal{B}_{\mu_\perp}^+(z_-), \mathcal{B}_{\nu_\perp}^+(z_-) \right], \ldots \right\}$$

[M. Beneke, A. Broggio, M. Garny, SJ, R. Szafron, L. Vernazza, J. Wang, 1809.10631]
[M. Beneke, A. Broggio, SJ, L. Vernazza, to appear soon]

A similar definition appeared recently in
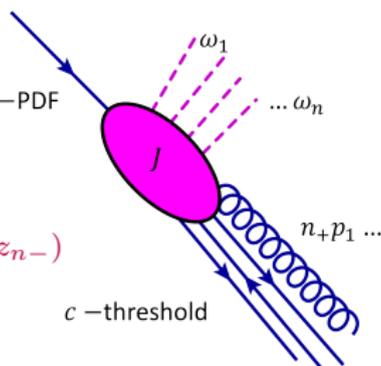[I. Moult, I.W. Stewart, G. Vita, 1905.07411]

General collinear functions

- The discussed construction is actually general at subleading powers, not only next-to-leading power
- There can be many Lagrangian insertions at various positions each with its own ω_i conjugate to the large component of threshold collinear momentum

We can separate the Lagrangian insertions

$$\mathcal{L}_V^{(n)}(z) = \mathcal{L}_c^{(n)}(z) \otimes \mathcal{L}_s^{(n)}(z_-)$$

$$i^n \left(\prod_{j=1}^n \int d^4 z_j \right)$$

$$\times \mathbf{T} \left[\chi_c(t_1 n_+) \chi_c(t_2 n_+) ... \times \mathcal{L}^{(n)}(z_1) \times ... \times \mathcal{L}^{(m)}(z_n) \right]$$

$$= 2\pi \sum_i \int du \left(\prod_{j=1}^n \int dz_{j-} \right) \tilde{J}_i(t_1, t_2..., u; z_{1-}, ..., z_{n-})$$

$$\times \chi_c^{\mathrm{PDF}}(u n_+) \, \mathbf{S}_i(z_{1-}, ..., z_{n-})$$
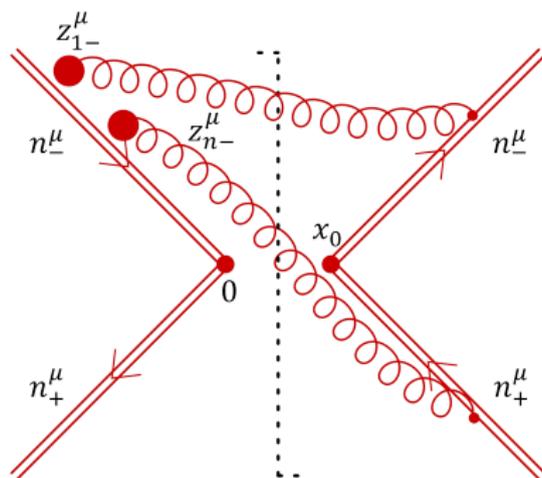

Sebastian Jaskiewicz

Generalized soft functions

The necessary presence of the time-ordered products at NLP gives rise to the concept of generalized soft functions with explicit gauge fields. Schematically we have

$$S(\Omega, \omega) = \int \frac{dx^0}{4\pi} e^{ix^0 \Omega/2} \left(\prod_{j=1}^{n} \int \frac{d(n_+ z_j)}{4\pi} e^{-i\omega_j (n_+ z_j)/2} \right)$$

$$\times \operatorname{Tr} \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \times \mathcal{L}_s^{(n)}(z_{1-}) \times ... \times \mathcal{L}_s^{(n)}(z_{n-}) \right] | 0 \rangle$$

$\mathcal{L}_s^{(n)}(z_{j-})$ contains $\mathcal{B}_{\perp\nu}^+(z_{j-})$ fields, not only Wilson lines

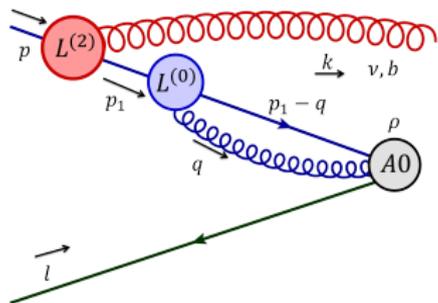[M. Beneke , F. Campanario, T. Mannel, B.D. Pecjak, 0411395]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.
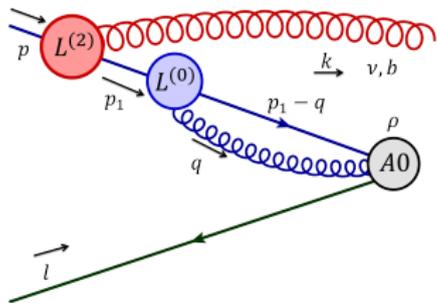
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$$ig_s t^a \begin{cases} \dfrac{\not{n}_+}{2} n_{-\mu} & \mathcal{O}(\lambda^0) \\[2mm] \dfrac{\not{n}_+}{2} X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\[2mm] S^{\rho\nu}(k, p, p') \dfrac{\not{n}_+}{2} (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$

$$S^{\rho\nu}(k, p, p') \equiv \frac{1}{2} \left[(n_- X) n_+^\rho n_-^\nu + (k X_\perp) X_\perp^\rho n_-^\nu + X_\perp^\rho \left(\frac{\not{p}'_\perp}{n_+ p'} \gamma_\perp^\nu + \gamma_\perp^\nu \frac{\not{p}_\perp}{n_+ p} \right) \right]$$

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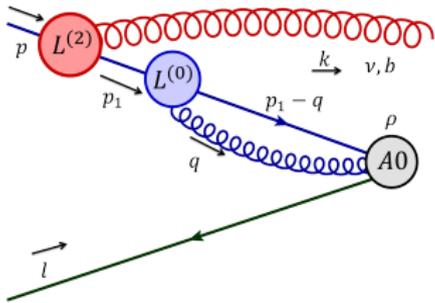

$$\mathcal{L}_\xi^{(2)} = \frac{1}{2}\bar\chi_c i\ (n_-x)\ n_+^\mu \left[in_-\partial\mathcal{B}_\mu^+(x_-) \right] \frac{\not n_+}{2}\chi_c + \dots$$
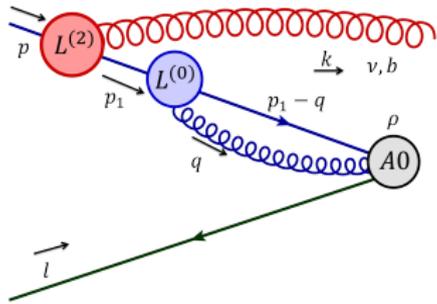
[M. Beneke and Th. Feldmann, 0211358]

$$X^\alpha = -\frac{\partial}{\partial p_{1\alpha}}\left\{ (2\pi)^4\delta^4(p-k_+-p_1) \right\}$$

$$ig_s t^a \begin{cases} \frac{\not n_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0) \\[2mm] \frac{\not n_+}{2}X_\perp^\rho n_-^\nu (k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda) \\[2mm] S^{\rho\nu}(k,p,p')\frac{\not n_+}{2}(k_\rho g_{\nu\mu} - k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2) \end{cases}$$
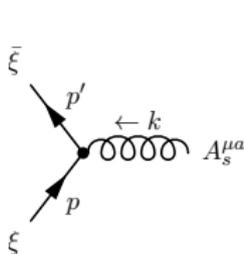
$$S^{\rho\nu}(k,p,p') \equiv \frac{1}{2}\left[(n_-X)n_+^\rho n_-^\nu\ +\ (kX_\perp)X_\perp^\rho n_-^\nu\ +\ X_\perp^\rho\left(\frac{\not p'_\perp}{n_+p'}\gamma_\perp^\nu + \gamma_\perp^\nu\frac{\not p_\perp}{n_+p}\right) \right]$$

Sebastian Jaskiewicz

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

Power suppressed amplitude calculation

Example: Take one of the diagrams with a $\mathcal{L}^{(2)}$ insertion - power suppression in the form of a time ordered product.

$$\bar{v}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

$$\times\Bigg\{\; \big((n_+k)n_{-\nu}-(n_-k)n_{+\nu}\big)\;\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$

$$+\left(\frac{k_\perp^2}{(n_-k)}\,n_{-\nu}-k_{\perp\nu}\right)\;\left(\frac{2}{\epsilon^2}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^0)\right)$$

$$+\left[\gamma_{\perp\nu}\,,\,\slashed{k}_\perp\right]\;\left(\frac{1}{\epsilon^2}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^0)\right)\Bigg\}u_c(p)$$

$$ig_s t^a\begin{cases}\dfrac{\slashed{n}_+}{2}n_{-\mu} & \mathcal{O}(\lambda^0)\\[2mm]\dfrac{\slashed{n}_+}{2}X_\perp^\rho n_-^\nu(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda)\\[2mm]S^{\rho\nu}(k,p,p')\dfrac{\slashed{n}_+}{2}(k_\rho g_{\nu\mu}-k_\nu g_{\rho\mu}) & \mathcal{O}(\lambda^2)\end{cases}$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\;(n_-X)n_+^\rho n_-^\nu\;+\;(kX_\perp)X_\perp^\rho n_-^\nu\;+\;X_\perp^\rho\left(\frac{\slashed{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\slashed{p}_\perp}{n_+p}\right)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

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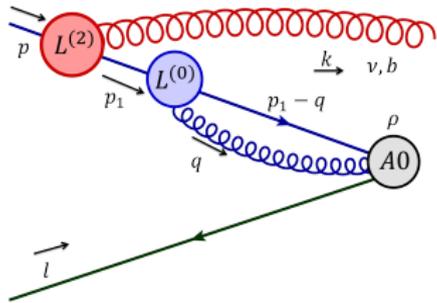

$$\bar{v}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

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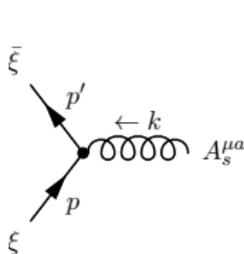$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[\ (n_-X)n_+^\rho n_-^\nu\ +\ (kX_\perp)X_\perp^\rho n_-^\nu\ +\ X_\perp^\rho\left(\frac{\slashed{p}'_\perp}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\slashed{p}_\perp}{n_+p}\right)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

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$$\times\Bigg\{\left((n_{+}k)n_{-\nu}-(n_{-}k)n_{+\nu}\right)\;\left(\frac{2}{\epsilon}+\mathcal{O}(\epsilon^{0})\right)$$

$$+\left(\frac{k_{\perp}^{2}}{(n_{-}k)}n_{-\nu}-k_{\perp\nu}\right)\;\left(\frac{2}{\epsilon^{2}}+\frac{4}{\epsilon}+\mathcal{O}(\epsilon^{0})\right)$$

$$+\boxed{\left[\gamma_{\perp\nu}\,,\,\slashed{k}_{\perp}\right]}\left(\frac{1}{\epsilon^{2}}+\frac{1}{\epsilon}+\mathcal{O}(\epsilon^{0})\right)\Bigg\}u_{c}(p)$$

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Sebastian Jaskiewicz
[M. Beneke, M. Garny, R. Szafron, J. Wang, 1808.04742]

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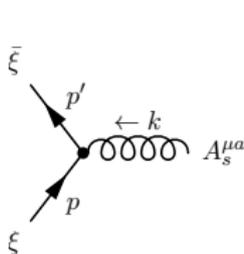

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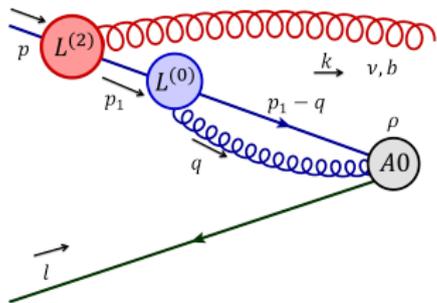
$$(n_+k)(n_-\epsilon^*)=2\left(-\frac{(n_-k)(n_+\epsilon^*)}{2}-k_\perp\cdot\epsilon_\perp^*\right)$$

$$S^{\rho\nu}(k,p,p')\equiv\frac{1}{2}\left[(n_-X)n_+^\rho n_-^\nu+(kX_\perp)X_\perp^\rho n_-^\nu+X_\perp^\rho\left(\frac{\slashed{p}_\perp'}{n_+p'}\gamma_\perp^\nu+\gamma_\perp^\nu\frac{\slashed{p}_\perp}{n_+p}\right)\right]$$

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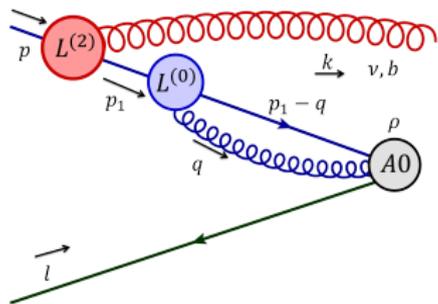

$$\bar{\nu}_{\bar{c}}(l)\gamma_\perp^\rho \frac{i\,g\,\alpha}{4\pi}\left[\frac{(n_+p)(n_-k)}{\mu^2}\right]^{-\epsilon}\frac{C_F t^b}{(n_+p)(n_-k)}$$

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Amplitude calculation: 1-real emission

$$\mathcal{A} = \ C \ \otimes \ J_{2\xi} \ \otimes \langle X| \ \frac{\partial_\perp^\mu}{in_-\partial} \mathcal{B}_{\mu_\perp}^+ \ |0\rangle + \ C \ \otimes \ J_{4\xi} \ \otimes \langle X| \ \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}_{\nu_\perp]}^+ \ |0\rangle + \ C \ \otimes \ J_\xi \ \otimes \langle X| \ \mathcal{B}_{\mu_\perp}^+ \ |0\rangle$$

1-loop collinear $\otimes$ 1-real soft emission

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes \boxed{J_{2\xi}} \otimes \langle X| \ \frac{\partial^{\mu}_{\perp}}{in_-\partial} \mathcal{B}^+_{\mu_\perp} \ |0\rangle + C \otimes \boxed{J_{4\xi}} \otimes \langle X| \ \frac{\partial_{[\mu_\perp}}{in_-\partial} \mathcal{B}^+_{\nu_\perp]} \ |0\rangle + C \otimes \boxed{J_\xi} \otimes \langle X| \ \mathcal{B}^+_{\mu_\perp} \ |0\rangle$$

1-loop collinear ⊗ 1-real soft emission

Extract 1-loop collinear functions

Amplitude calculation: 1-real emission

$$\mathcal{A} = C \otimes J_{2\xi} \otimes \langle X | \boxed{\frac{\partial_\perp^\mu}{in_-\partial}\mathcal{B}_{\mu_\perp}^+} |0\rangle + C \otimes J_{4\xi} \otimes \langle X | \boxed{\frac{\partial_{[\mu_\perp}}{in_-\partial}\mathcal{B}_{\nu_\perp]}^+} |0\rangle + C \otimes J_\xi \otimes \langle X | \boxed{\mathcal{B}_{\mu_\perp}^+} |0\rangle$$

1-loop collinear ⊗ 1-real soft emission
Extract 1-loop collinear functions

1-loop soft ⊗ 1-real
soft emission

Amplitude calculation: 1-real emission

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1-loop collinear ⊗ 1-real soft emission
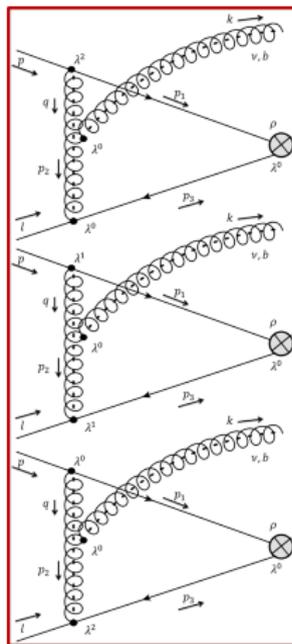Extract 1-loop collinear functions

1-loop soft ⊗ 1-real
soft emission

1-loop hard ⊗ 1-real
soft emission

Amplitude calculation: 1-real emission

We find agreement with the method of regions expansion for the 1-real 1-virtual amplitude. For explicit results see back up slides.

1-loop collinear ⊗ 1-real soft emission
Extract 1-loop collinear functions

1-loop soft ⊗ 1-real soft emission

1-loop hard ⊗ 1-real soft emission

Cross section

Hadronic tensor is given by

$$W_{\mu\rho} = \int d^4x\, e^{-iq\cdot x} \langle A(p_A)B(p_B)|J_\mu^\dagger(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

Cross section

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which is combined with the part from the lepton tensor

$$d\sigma = \frac{d^4q}{(2\pi)^4}\frac{4\pi\alpha^2}{3sq^2}\big(-g^{\mu\rho}W_{\mu\rho}\big)$$

Hadronic tensor is given by

$$W_{\mu\rho} = \int d^4x e^{-iq\cdot x} \langle A(p_A)B(p_B)|J_\mu^\dagger(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

Contributions from power suppressed currents can start contributing at NNLP only!

Cross section

Hadronic tensor is given by

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Cross section

Hadronic tensor is given by

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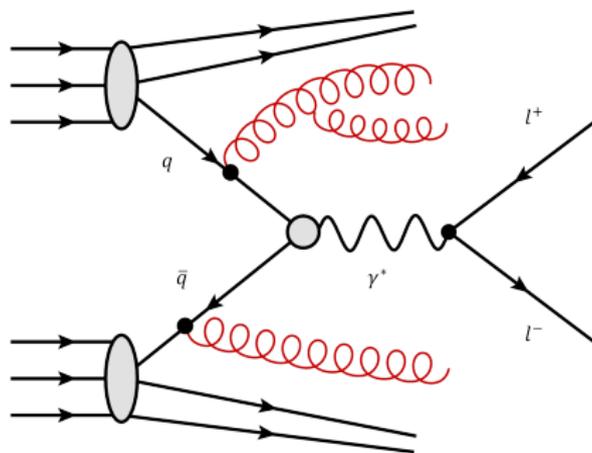# Cross section

Hadronic tensor is given by

$$W_{\mu\rho} = \int d^4x e^{-iq\cdot x} \langle A(p_A)B(p_B)|J_\mu^\dagger(x)J_\rho(0)|A(p_A)B(p_B)\rangle$$

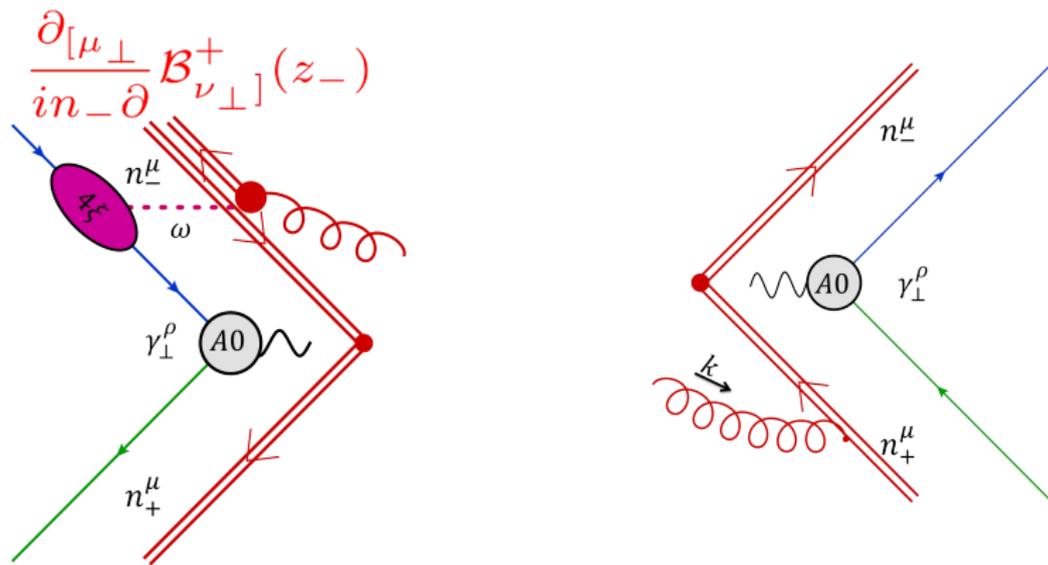# Final result

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_cQ^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

After combination with leptonic part and stripping off the PDFs

$$\hat{\sigma}(z) = H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2 + \vec{q}^2}} \, \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x}$$

$$\times \left\{ \widetilde{S}_0(x) + \, 2\frac{1}{2} \int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x,\omega) \, + \bar{c}\text{-term} \right\}$$

$$\mathbb{S} = \frac{\partial_\perp^\mu}{in_-\partial} \mathcal{B}_{\mu\perp}^+(z_-)$$

Final result

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

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$$
\begin{aligned}
\hat{\sigma}(z) &= H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2 + \vec{q}^{\,2}}} \, \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q) \cdot x} \\
&\quad \times \left\{ \widetilde{S}_0(x) + \; 2\frac{1}{2} \int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x, \omega) \; + \; \bar{c}\text{-term} \right\}
\end{aligned}
$$

$$
\begin{aligned}
J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) &= \; -\frac{2}{x_a(n_+ p_A)} - 2\frac{\partial}{x_a \partial(n_+ p_A)} \\
&\quad + \frac{\alpha}{4\pi} \frac{2}{x_a(n_+ p_A)} \left[\frac{\omega(x_a n_+ p_A)}{\mu^2} \right]^{-\epsilon} \left(C_A \left(5 + \mathcal{O}(\epsilon^1)\right) - C_F \left(\frac{4}{\epsilon} + 5 + \mathcal{O}(\epsilon^1) \right) \right)
\end{aligned}
$$

where the scalar collinear function in the factorization theorem is defined as:

$$J_{2\xi, \gamma\beta, fb}^{A}(n_+ p, n_+ p_a; \omega) = J_{2\xi}^{(O)}(n_+ p_a; \omega) \, \mathbf{T}_{fb}^{A} \, \delta_{\gamma\beta} \delta(n_+ p - n_+ p_a)$$
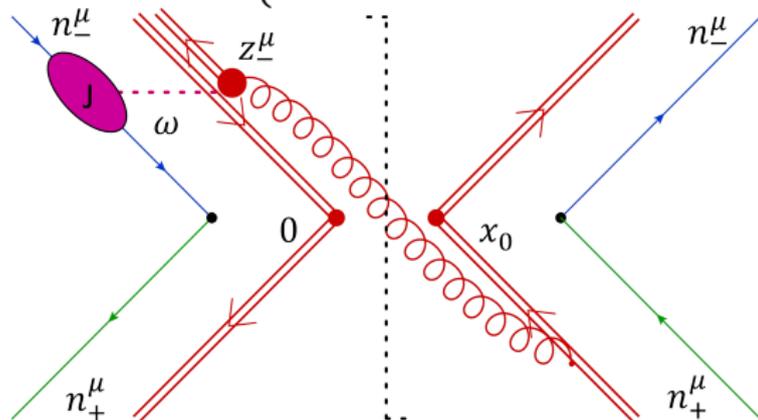
Final result

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_c Q^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

After combination with leptonic part and stripping off the PDFs

$$
\begin{aligned}
\hat{\sigma}(z) \;=\;& H(\hat{s}) \times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2 + \vec{q}^2}} \, \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x} \\
&\times \left\{ \widetilde{S}_0(x) + \boxed{2\frac{1}{2}\int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x,\omega)} + \bar{c}\text{-term} \right\}
\end{aligned}
$$

$$
\begin{aligned}
J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \;=\;& -\frac{2}{x_a(n_+ p_A)} - 2\frac{\partial}{x_a \partial(n_+ p_A)} \\
&+ \frac{\alpha}{4\pi} \frac{2}{x_a(n_+ p_A)} \left[\frac{\omega(x_a n_+ p_A)}{\mu^2} \right]^{-\epsilon} \left(C_A \left(5 + \mathcal{O}(\epsilon^1)\right) - C_F \left(\frac{4}{\epsilon} + 5 + \mathcal{O}(\epsilon^1) \right) \right)
\end{aligned}
$$

where the scalar collinear function in the factorization theorem is defined as:

$$J_{2\xi,\gamma\beta,fb}^A \left(n_+ p, n_+ p_a; \omega\right) = J_{2\xi}^{(O)} \left(n_+ p_a; \omega\right) \mathbf{T}_{fb}^A \, \delta_{\gamma\beta}\delta\left(n_+ p - n_+ p_a\right)$$

Comments on the final result

$$\int d\omega\, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega)\, \widetilde{S}_{2\xi}(x, \omega)$$

$$
\begin{aligned}
J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) &= -\frac{2}{x_a(n_+ p_A)} - 2\frac{\partial}{x_a \partial(n_+ p_A)} \\
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\end{aligned}
$$

$$S_{2\xi}(\Omega, \omega) = \frac{\alpha\, C_F}{2\pi} \frac{\mu^{2\epsilon}\, e^{\epsilon\gamma_E}}{\Gamma[1-\epsilon]} \frac{1}{\omega^{1+\epsilon}} \frac{1}{(\Omega-\omega)^{\epsilon}} \theta(\omega)\theta(\Omega-\omega) + \mathcal{O}(\alpha^2)$$

Comments on the final result

$$\int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x, \omega)$$

The factorization formula is valid for unrenormalized objects. Performing the convolution in d - dimensions reproduces fixed NNLO result:

$$\Delta_{\text{NLP-coll}}^{(2)} = \frac{\alpha_s^2}{(4\pi)^2} \bigg(C_A C_F \left(\frac{20}{\epsilon} - 60 \log(1-z) + 8 + \mathcal{O}(\epsilon^1) \right)$$

$$+ C_F^2 \left(\frac{-16}{\epsilon^2} - \frac{20}{\epsilon} + \frac{48}{\epsilon} \log(1-z) + 60 \log(1-z) - 72 \log^2(1-z) + \mathcal{O}(\epsilon^1) \right) \bigg)$$

after we set the scale to hard. In agreement with equation (4.22) of 1503.05156

New soft structures can appear at N^3LO: see C.White talk from NLP Corrections in Particle Physics workshop, Amsterdam 2018.

Comments on the final result

$$\int d\omega \, J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \, \widetilde{S}_{2\xi}(x, \omega)$$

For resummation, we treat the two objects independently, and expand in ϵ prior to performing the final convolution. However, there is a problem! At two loops:

$$J_{2\xi}^{(O)}(x_a n_+ p_A; \omega) \sim \alpha \log(\omega)$$

and

$$S_{2\xi}(\Omega, \omega) \sim \alpha \, \delta(\omega) + \mathcal{O}(\alpha^2)$$

For LL resummation, only tree level collinear function is needed, since the soft function begins at one loop due to the explicit field insertions. Beyond this not yet understood.

Summary

- Introduction of the collinear functions at *amplitude* level and explicit computation within SCET framework.
- Presented general factorization formula for DY threshold production at next-to-leading power, checked its validity up to fixed NNLO and identified a problem for resummation at NLL
- For leading logarithmic resummation - see Leonardo's talk next

Thank you

Back up slides

The Drell-Yan process - Decoupling transformation

We define the decoupled field

$$\chi_c^{(0)}(tn_+) = Y_+^\dagger(0)\chi_c(tn_+)$$

[C. Bauer, D. Pirjol, and I. Stewart, 0109045]

where

$$Y_\pm(x) = \mathbf{P}\exp\left[ig_s \int_{-\infty}^0 ds\, n_\mp A_s\left(x + sn_\mp\right)\right]$$

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The LP quark Lagrangian is

$$\mathcal{L}_{\mathrm{LP}} = \bar{\chi}\left(in_- D + i\slashed{D}_{\perp c}\frac{1}{in_+ D_c}\, i\slashed{D}_{\perp c}\right)\frac{\slashed{n}_+}{2}\,\chi$$

[M. Beneke and Th. Feldmann, 0211358]

where

$$in_- D = in_-\partial + g\, n_- A_c(x) + g\, n_- A_s(x_-)$$

and *after* the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\slashed{n}_+}{2}(\,n_-\mathcal{A}_c + n_-\partial\,)\,\chi^{(0)}(x)$$

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$$in_-D = in_-\partial + g\,n_-A_c(x) + g\,n_-A_s(x_-)$$

and *after* the decoupling transformation we have

$$\mathcal{L}_{c+s} \to \bar{\chi}^{(0)}\frac{\slashed{n}_+}{2}\left(n_-\mathcal{A}_c + n_-\partial\right)\chi^{(0)}(x)$$

From now on we use decoupled fields. Leading power current becomes

$$J_\rho^{A0}(t,\bar{t}) = \bar{\chi}_{\bar{c}}^{(0)}(\bar{t}n_-)Y_-^\dagger(0)\gamma_{\perp\rho}Y_+(0)\chi_c^{(0)}(tn_+)$$

Matching to quark current at NLP

N-jet operators are built out of following relevant building blocks.

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742.]

(A1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i\slashed{\partial}_\perp]\chi_c(tn_+), \ \bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho(-i)\overleftarrow{\slashed{\partial}}_\perp]\chi_c(tn_+)$$

(B1-type)

$$\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \slashed{\mathcal{A}}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+), \ \bar{\chi}_{\bar{c}}(\bar{t}_1 n_-)[n_\pm^\rho \slashed{\mathcal{A}}_{\bar{c}\perp}(\bar{t}_2 n_-)]\,\chi_c(tn_+)$$

With the the scaling

$$[n_\pm^\rho i\slashed{\partial}_\perp]\,\chi_c(tn_+) \sim \lambda$$
$$[n_\pm^\rho \slashed{\mathcal{A}}_{c\perp}(t_2 n_+)]\,\chi_c(tn_+) \sim \lambda$$

relative to LP.

Time-ordered products

$$\left(J_{W,V}^{Tm}(s,t)\right)^{\mu} = i \int d^4x \, \mathbf{T}\left[J_W^{\mu}(s,t) \, \mathcal{L}_V^{(n)}(x)\right]$$

[M. Beneke, M. Garny, R. Szafron, J. Wang, 1712.04416, 1808.04742]

The NLP soft-collinear SCET quark-gluon interaction Lagrangian written in terms of building blocks $\mathcal{B}_{\pm}^{\mu} = Y_{\pm}^{\dagger}\left[iD_s^{\mu}Y_{\pm}\right]$ and $q^{\pm}(x) = Y_{\pm}^{\dagger}(x)\, q_s(x)$ is

[M. Beneke, F. Campanario, T. Mannel, B.D. Pecjak , 0411395]

$$\mathcal{L}_{\xi}^{(1)} = \bar{\chi}_c ix_{\perp}^{\mu}\left[in_-\partial\mathcal{B}_{\mu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c$$

$$\mathcal{L}_{1\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c in_-xn_+^{\mu}\left[in_-\partial\mathcal{B}_{\mu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c$$

$$\mathcal{L}_{2\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c x_{\perp}^{\mu}x_{\perp}^{\rho}\left[i\partial_{\rho}in_-\partial\mathcal{B}_{\mu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c$$

$$\mathcal{L}_{3\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c x_{\perp}^{\mu}x_{\perp}^{\rho}\left[\mathcal{B}_{\rho}^+(x_-),\, in_-\partial\mathcal{B}_{\mu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c$$

$$\mathcal{L}_{4\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial}ix_{\perp}^{\mu}\gamma_{\perp}^{\nu}\left[i\partial_{\nu}\mathcal{B}_{\mu}^+(x_-) - i\partial_{\mu}\mathcal{B}_{\nu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.}$$

$$\mathcal{L}_{5\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c\left(i\not{\partial}_{\perp} + \not{\mathcal{A}}_{c\perp}\right)\frac{1}{in_+\partial}ix_{\perp}^{\mu}\gamma_{\perp}^{\nu}\left[\mathcal{B}_{\nu}^+(x_-),\mathcal{B}_{\mu}^+(x_-)\right]\frac{\not{n}_+}{2}\chi_c + \text{h.c.}$$

$$\mathcal{L}_{\xi q}^{(1)} = \bar{q}_+(x_-)\mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$

$$\mathcal{A}_{\perp\mu} = Y_+^\dagger W_c^\dagger \left[i\, D_c\, W_c \right] Y_+$$
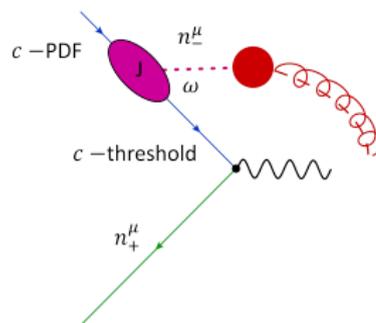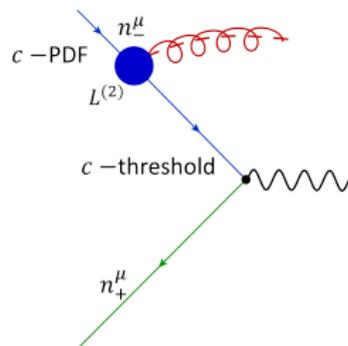
$$\langle A(p_A)|\bar{\chi}_{c,\alpha a}(x + u'\,n_+)\chi_{c,\beta b}(un_+)|A(p_A)\rangle = \frac{\delta_{ba}}{N_c} \left(\frac{\slashed{n}_-}{4} \right)_{\beta\alpha} n_+ p_A$$

$$\times \int_0^1 dx_a\, f_{a/A}(x_a)\, e^{i(x + u'\,n_+ - u\,n_+)\cdot x_a p_A}$$

Collinear functions

Threshold collinear fields are matched to collinear-PDF fields

$$\int dt \, e^{i(n_+p)t} \, i \int d^4z \, e^{i\omega(n_+z)/2} \, \mathbf{T} \left[\chi_c(tn_+) \times \mathcal{L}_c^{(n)}(z) \right]$$

$$= \int d(n_+p') \int dt \, e^{i(n_+p')t} \, J(n_+p, n_+p'; \omega) \, \chi_c^{\mathrm{PDF}}(tn_+)$$

Soft functions

We introduce the soft operator

$$\widetilde{\mathcal{S}}_{2\xi}\left(x, z_-\right) = \bar{\mathbf{T}}\left[Y_+^\dagger(x)Y_-(x)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}_{\perp\nu}^+(z_-)\right]$$

and the Fourier transform of its (colour-traced) vacuum matrix element
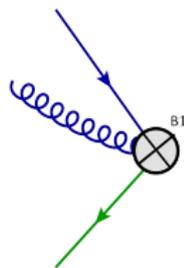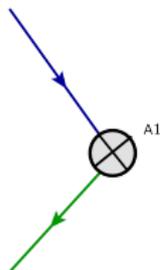
$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+z)}{4\pi} \, e^{ix^0\Omega/2 - i\omega(n_+z)/2} \, \frac{1}{N_c} \, \mathrm{Tr}\, \langle 0|\widetilde{\mathcal{S}}_{2\xi}(x^0, z_-)|0\rangle$$

Possible contributing structures

First we check whether subleading power contributions start at order λ.

- ▶ Consider A1 and B1 type currents:
  A1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho i \overleftrightarrow{\partial}_\perp]\chi_c(tn_+)$ B1-type: $\bar{\chi}_{\bar{c}}(\bar{t}n_-)[n_\pm^\rho \mathcal{A}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$
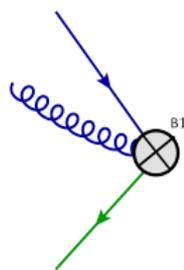
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- Another possibility is a single power suppressed time-ordered product of the form $\left(J_{A0,\xi}^{T1}(s,t)\right)^\mu = i \int d^4x\, \mathbf{T}\left[J_{A0}^\mu(s,t)\, \mathcal{L}_\xi^{(1)}(x)\right]$

Only one possibility

$$\mathcal{L}_\xi^{(1)} \quad = \quad \bar{\chi}_c i x_\perp^\mu \left[in_-\partial \mathcal{B}_\mu^+\right] \frac{\not{n}_+}{2} \chi_c$$

Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

▶ B1-type current, $\bar{\chi}_{\bar{c}}(\bar{t}n_-)\,[n_\pm^\rho \mathcal{A}_{c\perp}(t_2 n_+)]\chi_c(t_1 n_+)$, with Lagrangian insertion $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[in_-\partial \mathcal{B}_\mu^+\right] \dfrac{\slashed{n}_+}{2}\chi_c$

Possible contributing structures

First subleading contributions are found at λ^2 order. This we call next-to-leading power.

- A1-type current with $\mathcal{L}_\xi^{(1)} = \bar{\chi}_c i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\slashed{n}_+}{2} \chi_c$ insertion

Feynman rule for emission of a soft gluon from $\mathcal{B}_\mu^+$ is

$$g\, T^A \left[-\frac{k_\perp^\mu n_{-\nu}}{(n_- k)} + g_\perp^{\mu\nu} \right] \epsilon_\nu^* \, e^{+i k \cdot z_-}$$

Considering the Lagrangian insertions

Previous arguments allow us also to drop following possible contributions

$$\left(J_{A0,\xi}^{T1}(s,t)\right)^{\mu} = i \int d^4x_1 \, \mathbf{T}\left[J_{A0}^{\mu}(s,t)\,\mathcal{L}_{\xi}^{(1)}(x_1)\right]$$

$$\left(\bar{J}_{A0,\xi}^{T1}(\bar{s},\bar{t})\right)^{\mu} = (-i) \int d^4x_2 \, \mathbf{T}\left[\bar{J}_{A0}^{\mu}(\bar{s},\bar{t})\,\mathcal{L}_{\xi}^{(1)}(x_2)\right]$$

$$\mathcal{L}_{\xi}^{(1)} = \bar{\chi}_c i x_{\perp}^{\mu} \left[i n_-\partial \mathcal{B}_{\mu}^{+}\right] \frac{\not{n}_+}{2} \chi_c$$
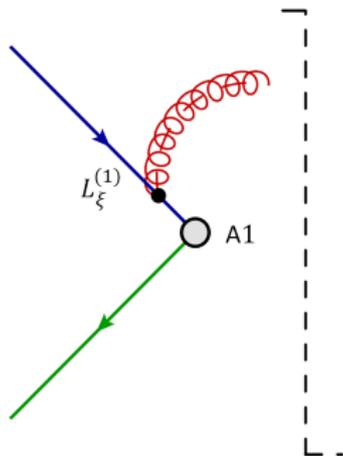

Sebastian Jaskiewicz

Considering the Lagrangian insertions

The following contributions start at $\mathcal{O}(\alpha^2)$

$$\left(J_{A0,\xi}^{T2}(s,t) \right)^\mu = i \int d^4 x_1 \, i \int d^4 x_2 \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_\xi^{(1)}(x_1) \, \mathcal{L}_\xi^{(1)}(x_2) \right]$$

$$\mathcal{L}_\xi^{(1)} = \bar{\chi}_c \, i x_\perp^\mu \left[i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\slashed{n}_+}{2} \chi_c$$

$$\left(J_{A0,V}^{T2}(s,t) \right)^\mu = i \int d^4 x \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_V^{(2)}(x) \right]$$

$$\mathcal{L}_{3\xi}^{(2)} = \frac{1}{2} \bar{\chi}_c x_\perp^\mu x_\perp^\rho \left[\mathcal{B}_\rho^+ , \, i n_- \partial \mathcal{B}_\mu^+ \right] \frac{\slashed{n}_+}{2} \chi_c$$

$$\mathcal{L}_{5\xi}^{(2)} = \frac{1}{2} \bar{\chi}_c \left(i \slashed{\partial}_\perp + \slashed{\mathcal{A}}_{c\perp} \right) \frac{1}{i n_+ \partial} i x_\perp^\mu \gamma_\perp^\nu$$

$$\times \left[\mathcal{B}_\nu^+ , \mathcal{B}_\mu^+ \right] \frac{\slashed{n}_+}{2} \chi_c + \text{h.c.}$$

Considering the Lagrangian insertions

It is also possible to construct diagrams containing soft quarks

$$\left(J_{A0,\xi q}^{T2}(s,t) \right)^\mu = i \int d^4 x_1 \, i \int d^4 x_2 \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_{\xi q}^{(1)}(x_1) \, \mathcal{L}_{\xi q}^{(1)}(x_2) \right]$$

$$\mathcal{L}_{\xi q}^{(1)} = \bar{q}_+ \mathcal{A}_{c\perp} \chi_c + \text{h.c.}$$

These contributions also start at $\mathcal{O}(\alpha^2)$

Considering the Lagrangian insertions

Two more possible contributions with following Lagrangian terms making up the time-ordered product

$$\mathcal{L}_{1\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c in_- x n_+^\mu \left[in_-\partial\mathcal{B}_\mu^+\right]\frac{\slashed{n}_+}{2}\chi_c$$

$$\mathcal{L}_{4\xi}^{(2)} = \frac{1}{2}\bar{\chi}_c \left(i\slashed{\partial}_\perp + \mathcal{A}_{c\perp}\right)\frac{1}{in_+\partial}ix^\mu_\perp\gamma_\perp^\nu$$
$$\times \left[i\partial_{\nu_\perp}\mathcal{B}_{\mu_\perp}^+ - i\partial_{\mu_\perp}\mathcal{B}_{\nu_\perp}^+\right]\frac{\slashed{n}_+}{2}\chi_c + \text{h.c.}$$

Conclusion

We therefore find that for LL resummation at NLP in the quark-antiquark channel only the single time-ordered product contribution:

$$\left(J_{A0,2\xi}^{T2}(s,t) \right)^\mu = i \int d^4x \, \mathbf{T} \left[J_{A0}^\mu(s,t) \, \mathcal{L}_{2\xi}^{(2)}(x) \right]$$

To NLP LL accuracy the matching equation is then extended to

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt \, d\bar{t} \, \widetilde{C}^{A0}(t,\bar{t}) \left[J_{A0}^\mu(t,\bar{t}) + \left(J_{A0,2\xi}^{T2}(t,\bar{t}) \right)^\mu + \bar{c}\text{-term} \right]$$

Again we consider

$$\langle X | \bar{\psi}\gamma^\mu\psi(0) | A(p_A)B(p_B) \rangle$$

A power suppressed amplitude

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\,d\bar{t}\,\widetilde{C}^{A0}(t,\bar{t})\left[J_{A0}^\mu(t,\bar{t}) + \boxed{i\int d^4x\,\mathbf{T}\left[J_{A0}^\mu(s,t)\,\mathcal{L}_{2\xi}^{(2)}(x)\right]} + \ \bar{c}\text{-term}\right]$$

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\,d\bar{t}\,e^{it\,n_+\,p}e^{i\bar{t}n_-\bar{p}}C^{A0}(n_+p,n_-\bar{p})$$

$$\times\langle X|\mathbf{T}\left[\underbrace{\bar{\chi}_{\bar{c}}(\bar{t}n_-)Y_-^\dagger(0)\gamma_\perp^\mu\,Y_+(0)\chi_c\,(tn_+)}_{J_{A0}^\mu(t,\bar{t})}\,i\int d^4z\,\bar{\chi}_{c,e}\,(z)\,\frac{1}{2}z_\perp^\nu z_\perp^\rho\right.$$

$$\times\left[\boxed{\left(\frac{in_-\partial_z}{in_-\partial_z}\right)}(in_-\partial_z)i\partial_\perp^\rho\mathcal{B}_{\perp\nu,ed}^+(z_-)\right]\frac{\slashed{n}_+}{2}\chi_{c,d}\,(z)\right]|A(p_A)B(p_B)\rangle$$

A power suppressed amplitude

The states factorize as at leading power: $\langle X| = \langle X_{\bar{c}}^{\mathrm{PDF}}| \, \langle X_c^{\mathrm{PDF}}| \, \langle X_s|$ as they are eigenstates of the LP Lagrangian

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{d(n_+p)}{2\pi}\frac{d(n_-\bar{p})}{2\pi}\int dt\,d\bar{t}\,e^{it\,n_+p}e^{i\bar{t}n_-\bar{p}}C^{A0}(n_+p, n_-\bar{p})$$

$$\times\langle X_{\bar{c}}^{\mathrm{PDF}}|\bar{\chi}_{\bar{c},\alpha a}(\bar{t}n_-)|B(p_B)\rangle\gamma^\mu_{\perp,\alpha\gamma}$$

$$\times i\int d^4z\langle X_c^{\mathrm{PDF}}|\frac{1}{2}z_\perp^\nu z_\perp^\rho\,(in_-\partial_z)^2\,\mathbf{T}\left[\chi_{c,\gamma f}\,(tn_+)\,\bar{\chi}_{c,e}\,(z)\,\frac{\not{n}_+}{2}\chi_{c,d}\,(z)\right]|A(p_A)\rangle$$

$$\times\langle X_s|\mathbf{T}\left(\left[Y_-^\dagger(0)Y_+(0)\right]_{af}\frac{i\partial_\perp^\rho}{in_-\partial_z}\mathcal{B}^+_{\perp\nu,ed}\,(z_-)\right)|0\rangle$$

Amplitude with collinear function

$$\langle X | \bar{\psi} \gamma^\mu \psi(0) | A(p_A) B(p_B) \rangle = \int \frac{d(n_+ p)}{2\pi} \frac{d(n_- \bar{p})}{2\pi} \int d(n_+ p_a) d(n_- p_b)$$

$$\delta \left(n_- \bar{p} + (n_- p_b) \right) C^{A0}(n_+ p, n_- \bar{p})$$

$$\times \int \frac{d\omega}{2\pi} J^{\rho\nu}_{2\xi, \gamma\beta, fbed} (n_+ p, n_+ p_a; \omega) \langle X^{\mathrm{PDF}}_{\bar{c}} | \hat{\bar{\chi}}^{\mathrm{PDF}}_{\bar{c}, \alpha a}(n_- p_b) | B(p_B) \rangle$$

$$\times \gamma^\mu_{\perp, \alpha\gamma} \langle X^{\mathrm{PDF}}_c | \hat{\chi}^{\mathrm{PDF}}_{c, \beta b}(n_+ p_a) | A(p_A) \rangle$$

$$\times \int \frac{dn_+ z}{2} e^{-i\omega \frac{n_+ z}{2}} \langle X_s | \mathbf{T} \left(\left[Y^\dagger_-(0) Y_+(0) \right]_{af} \frac{i\partial^\rho_\perp}{in_- \partial} \mathcal{B}^+_{\perp\nu, ed}(z_-) \right) | 0 \rangle$$

Computation of collinear function

The short-distance coefficient can be extracted by computing the partonic matrix element $\langle 0|\mathcal{J}^{\rho\nu}_{\gamma,fed}\left(n_+q_a,\omega\right)|q(q)_q\rangle$. Running to collinear scale: only tree level collinear function is necessary.

Collinear function:

$$J^{\rho\nu}_{2\xi,\gamma\beta,fbed}\left(n_+q_a,(n_+q);\omega\right) = -\delta_{bd}\delta_{fe}\delta_{\beta\gamma}\delta\left((n_+q_a)-(n_+q)\right)\frac{g^{\nu\rho}_\perp}{(n_+q)}$$

LP + NLP amplitude

We are considering the matching up to NLP

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}) \left[J^\mu_{A0}(t,\bar{t}) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}) \right)^\mu + \bar{c}\text{-term} \right]$$

For which we obtained

$$\langle X|\bar{\psi}\gamma^\mu\psi(0)|A(p_A)B(p_B)\rangle = \int \frac{dn_+p_a}{2\pi}\frac{dn_-p_b}{2\pi}\, C^{A0}(n_+p_a, -n_-p_b)$$

$$\times \langle X_{\bar{c},\text{PDF}}|\hat{\bar{\chi}}^{\text{PDF}}_{\bar{c},\alpha a}(n_-p_b)|B(p_B)\rangle\, \gamma^\mu_{\perp\alpha\beta}\, \langle X_{c,\text{PDF}}|\hat{\chi}^{\text{PDF}}_{c,\beta b}(n_+p_a)|A(p_A)\rangle$$

$$\times \left\{ \langle X_s|\mathbf{T}\left[Y^\dagger_-(0)Y_+(0) \right]_{ab}|0\rangle \right.$$

$$+ \frac{1}{2}\int \frac{d\omega}{4\pi}\, J^{(O)}_{2\xi}(n_+p_a;\omega)\int d(n_+z)\, e^{-i\omega(n_+z)/2}$$

$$\times \langle X_s|\mathbf{T}\left(\left[Y^\dagger_-(0)Y_+(0) \right]_{af} \frac{i\partial^\nu_\perp}{in_-\partial_z}\mathcal{B}^+_{\perp\nu;fb}(z_-) \right)|0\rangle \right\} + \bar{c}\text{-term}$$

Note that $J^{(O)}_{2\xi}(n_+p_a;\omega) = -\dfrac{2}{n_+p_a}$

Relevant soft function

The generalized soft function at cross section level here is

$$S_{2\xi}(\Omega, \omega) = \int \frac{dx^0}{4\pi} \int \frac{d(n_+ z)}{4\pi} e^{ix^0 \Omega/2 - i\omega(n_+ z)/2}$$

$$\times \frac{1}{N_c} \operatorname{Tr} \langle 0 | \bar{\mathbf{T}} \left[Y_+^\dagger(x^0) Y_-(x^0) \right] \mathbf{T} \left[Y_-^\dagger(0) Y_+(0) \frac{i\partial_\perp^\nu}{in_- \partial} \mathcal{B}_{\perp\nu}^+(z_-) \right] | 0 \rangle$$

For details on renormalization of soft functions and resummation see Robert's talk.

Result for the power suppressed amplitude: C_F

$$i\, C_F\, \gamma_{\perp\rho}\, \frac{1}{(n_+p)(n_-k)} \bigg((n_+k)n_{-\nu} \left(\frac{3}{\epsilon} + 2 - \frac{3}{2}\zeta(2)\epsilon + \left(-\zeta(2) - \frac{21\zeta(3)}{3} - 4 \right)\epsilon^2 \right)$$

$$+ (n_-k)n_{+\nu} \left(-\frac{1}{\epsilon} - 3 + (-6 + \frac{1}{2}\zeta(2))\epsilon + \left(\frac{3}{2}\zeta(2) - 12 + \frac{7\zeta(3)}{3} \right)\epsilon^2 \right)$$

$$+ k_{\perp\nu} \left(+\frac{2}{\epsilon} - 1 + (-6 - \zeta(2))\epsilon + \left(+\frac{\zeta(2)}{2} - \frac{14\zeta(3)}{3} - 16 \right)\epsilon^2 \right)$$

$$+ [\slashed{k}_\perp, \gamma_{\perp\nu}] \left(+\frac{1}{2} + \epsilon + \left(-\frac{1}{4}\zeta(2) + 2 \right)\epsilon^2 \right) \bigg)$$

$$i\, C_F\, n_{-\rho}\, \frac{1}{n_-l} \left(\gamma_{\perp\nu} - \frac{\slashed{k}_\perp n_{-\nu}}{(n_-k)} \right) \left(+1 + 4\epsilon - \frac{1}{2}(\zeta(2) - 20)\epsilon^2 \right)$$

$$i\, C_F\, n_{+\rho}\, \frac{1}{n_+p} \left(\gamma_{\perp\nu} - \frac{\slashed{k}_\perp n_{-\nu}}{(n_-k)} \right) \left(-1 - 4\epsilon + \frac{1}{2}(\zeta(2) - 20)\epsilon^2 \right)$$

Result for the power suppressed amplitude: C_A

$$i\,C_A\,\gamma_{\perp\rho}\,\frac{1}{(n_+p)(n_-k)}\left((n_+k)n_{-\nu}\left(-\frac{1}{2\,\epsilon^2}-\frac{3}{2\,\epsilon}+\frac{1}{4}(\zeta(2)-18)\right.\right.$$

$$+\frac{1}{12}(9\zeta(2)+14\zeta(3)-48)\epsilon+\frac{1}{32}(72\zeta(2)+112\zeta(3)+47\zeta(4)-288)\epsilon^2\bigg)$$

$$+(n_-k)n_{+\nu}\left(-\frac{1}{2\epsilon^2}-\frac{3}{2\epsilon}+\frac{1}{4}(\zeta(2)+2)+\frac{1}{12}(9\zeta(2)+14\zeta(3)-24)\epsilon\right.$$

$$-\frac{1}{32}(8\zeta(2)-112\zeta(3)-47\zeta(4)+32)\epsilon^2\bigg)$$

$$+k_{\perp\nu}\left(-\frac{1}{\epsilon^2}-\frac{3}{\epsilon}+\frac{1}{2}(\zeta(2)-8)\right.$$

$$+\left(\frac{3\zeta(2)}{2}+\frac{7\zeta(3)}{3}-6\right)\epsilon+\left(2\zeta(2)+7\zeta(3)+\frac{47\zeta(4)}{16}-10\right)\epsilon^2\bigg)$$

$$+[\slashed{k}_{\perp},\gamma_{\perp\nu}]\left(\frac{1}{4}\left(-2-4\epsilon+(\zeta(2)-8)\epsilon^2\right)\right)\bigg)$$

$$i\,C_A\,n_{-\rho}\,\frac{1}{n_-l}\left(\gamma_{\perp\nu}-\frac{\slashed{k}_{\perp}n_{-\nu}}{(n_-k)}\right)\left(+\frac{1}{\epsilon}+2-\frac{1}{2}(\zeta(2)-6)\epsilon+\left(-\zeta(2)-\frac{7\zeta(3)}{3}+5\right)\epsilon^2\right)$$

$$i\,C_A\,n_{+\rho}\,\frac{1}{n_+p}\left(\gamma_{\perp\nu}-\frac{\slashed{k}_{\perp}n_{-\nu}}{(n_-k)}\right)\left(-\frac{1}{\epsilon}-2+\frac{1}{2}(\zeta(2)-6)\epsilon+\left(\zeta(2)+\frac{7\zeta(3)}{3}-5\right)\epsilon^2\right)$$

Results for power suppressed amplitude: Soft

$$
i\, C_A \gamma_{\perp\rho} \frac{1}{(n_+ p)(n_- k)} \left((n_+ k)\, n_{-\nu} \left(+\frac{1}{\epsilon^2} + \frac{\zeta(2)}{2} - \frac{7}{3}\zeta(3)\epsilon - \frac{1}{16}39\zeta(4)\epsilon^2 \right) \right.
$$

$$
+ (n_- k)\, n_{+\nu} \left(0 \right)
$$

$$
+ k_{\perp\nu} \left(\frac{1}{\epsilon^2} + \frac{1}{2}\zeta(2) - \frac{7}{3}\zeta(3)\epsilon - \frac{39}{16}\zeta(4)\epsilon^2 \right)
$$

$$
\left. + [\slashed{k}_\perp, \gamma_{\perp\nu}] \left(+\frac{1}{2\,\epsilon^2} + \frac{1}{4}\zeta(2) - \frac{7}{6}\zeta(3)\epsilon - \frac{1}{32}39\zeta(4)\epsilon^2 \right) \right)
$$

$$
g\, \frac{C_A}{2} t^b \frac{1}{(n_+ p)} n_+^\rho \frac{i\alpha\, e^{\epsilon\gamma_E}}{(4\pi)} \frac{1}{\epsilon^2} \frac{\Gamma[1-\epsilon]^3}{\Gamma[2-2\epsilon]} \Gamma[1+\epsilon]^2
$$

$$
\times \left[-(1-2\epsilon)\frac{\slashed{k}_\perp n_{-\nu}}{(n_- k)} + 2\gamma_{\perp\nu} + \frac{2\slashed{k}_\perp k_{\perp\nu}}{(n_- k)(n_+ k)} + \frac{\slashed{k}_\perp n_{+\nu}}{(n_+ k)}(1-2\epsilon) \right]
$$

Sebastian Jaskiewicz

Results for power suppressed amplitude: soft × hard

$$i C_F \, \gamma_\perp^\rho \, \frac{1}{(n_+p)(n_-k)} \left((n_+k)n_{-\nu}\left(\frac{2}{\epsilon^2} + \frac{1}{\epsilon} + 5 - \frac{1}{6}\pi^2 + \mathcal{O}(\epsilon) \right) \right.$$

$$+(n_-k)n_{+\nu}\left(+\frac{2}{\epsilon} + 3 + \mathcal{O}(\epsilon) \right)$$

$$+k_{\perp\nu}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{6} + 8 + \mathcal{O}(\epsilon) \right)$$

$$\left. +[\slashed{k}_\perp, \gamma_{\perp\nu}]\left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} - \frac{\pi^2}{12} + 4 + \mathcal{O}(\epsilon) \right) \right)$$

$$i \, g t^b \, n_+^\rho \, C_F \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6} + \mathcal{O}(\epsilon) \right)$$

$$\frac{1}{(n_+p)(n_-k)}\left(\slashed{k}_\perp n_{-\nu} - (n_-k)\gamma_{\perp\nu} \right)$$

NLP factorization formula

$$\frac{d\sigma_{\mathrm{DY}}}{dQ^2} = \frac{4\pi\alpha_{\mathrm{em}}^2}{3N_cQ^4} \sum_{a,b} \int_0^1 dx_a dx_b \, f_{a/A}(x_a) f_{b/B}(x_b) \, \hat{\sigma}_{ab}(z)$$

The $\hat{\sigma}_{ab}(z)$ is now

$$
\begin{aligned}
\hat{\sigma}(z) &= \sum_{\mathrm{terms}} \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i' \, D(-\hat{s}; \omega_i, \bar{\omega}_i) D^*(-\hat{s}; \omega_i', \bar{\omega}_i') \\
&\times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2+\vec{q}^2}} \frac{1}{2\pi} \int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x} \\
&\times \widetilde{S}(x; \omega_i, \bar{\omega}_i, \omega_i', \bar{\omega}_i')
\end{aligned}
$$

and

$$
\begin{aligned}
D(-\hat{s}; \omega_i, \bar{\omega}_i) &= \int d(n_+ p_i) d(n_- \bar{p}_i) \, C(n_+ p_i, n_- \bar{p}_i) \\
&\times J(n_+ p_i, x_a n_+ p_A; \omega_i) \, \bar{J}(n_- \bar{p}_i, -x_b n_- p_B; \bar{\omega}_i)
\end{aligned}
$$