The Jet Shape at NLL′

Wouter Waalewijn

University of Amsterdam

Delta Institute for Theoretical Physics

NWO

erc

PSR 2019

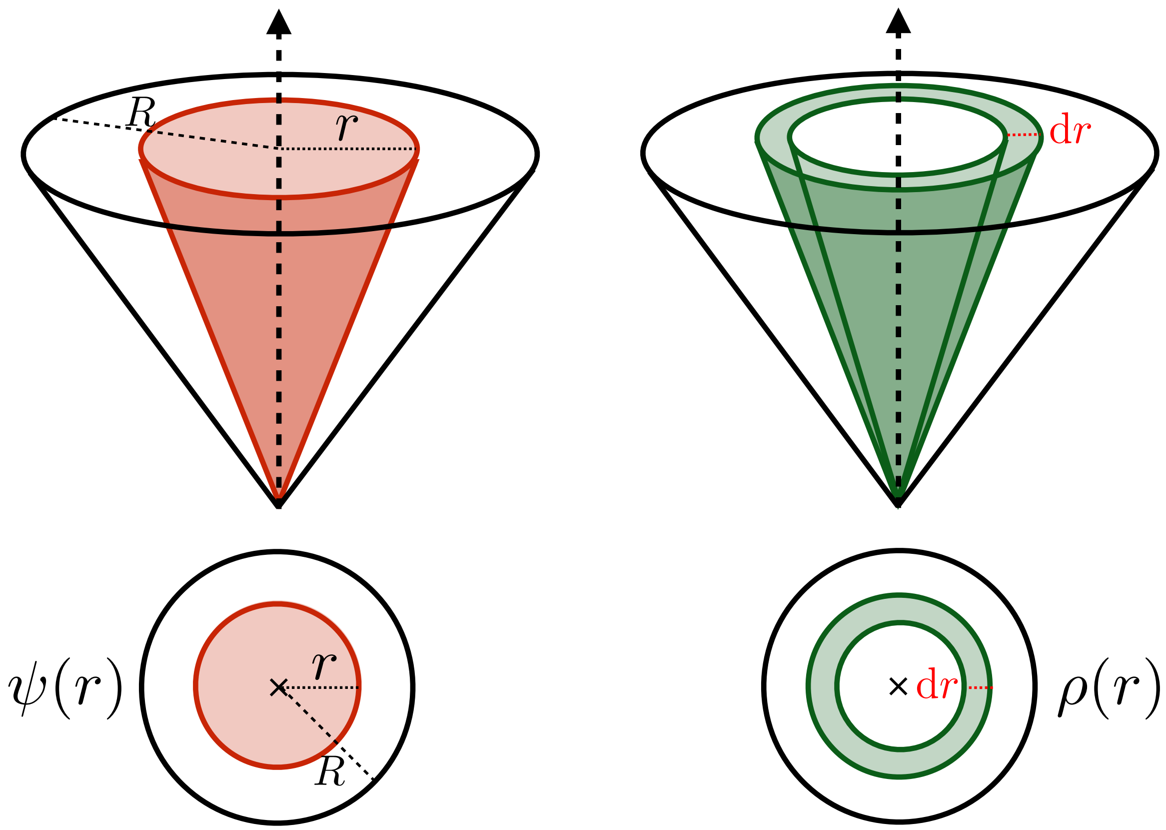
Outline

- Introduction
- Factorization
- Ingredients
- Implementation
- Results
- Conclusions

Based on arXiv:1901.06389 with P. Cal and F. Ringer

1. Introduction

Jet shape definition



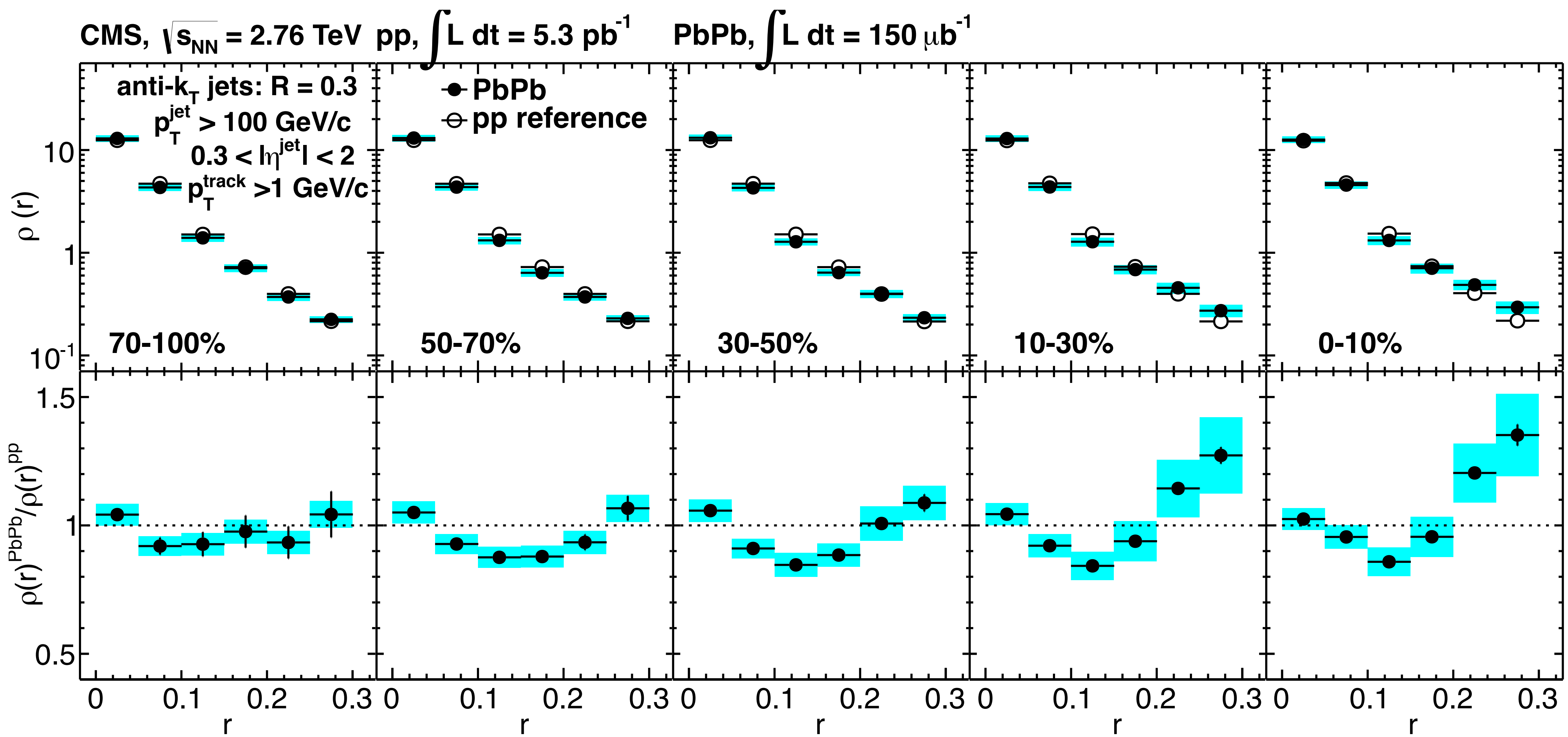
- Jet shape is average $z_r = p_T^{\text{subjet}} / p_T$

$$\psi(r) = \int_0^1 \mathrm{d}z_r \, z_r \, \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \, \mathrm{d}\eta \, \mathrm{d}z_r} \Bigg/ \frac{\mathrm{d}\sigma}{\mathrm{d}p_T \, \mathrm{d}\eta} \qquad \rho(r) = \frac{\mathrm{d}\psi}{\mathrm{d}r}$$

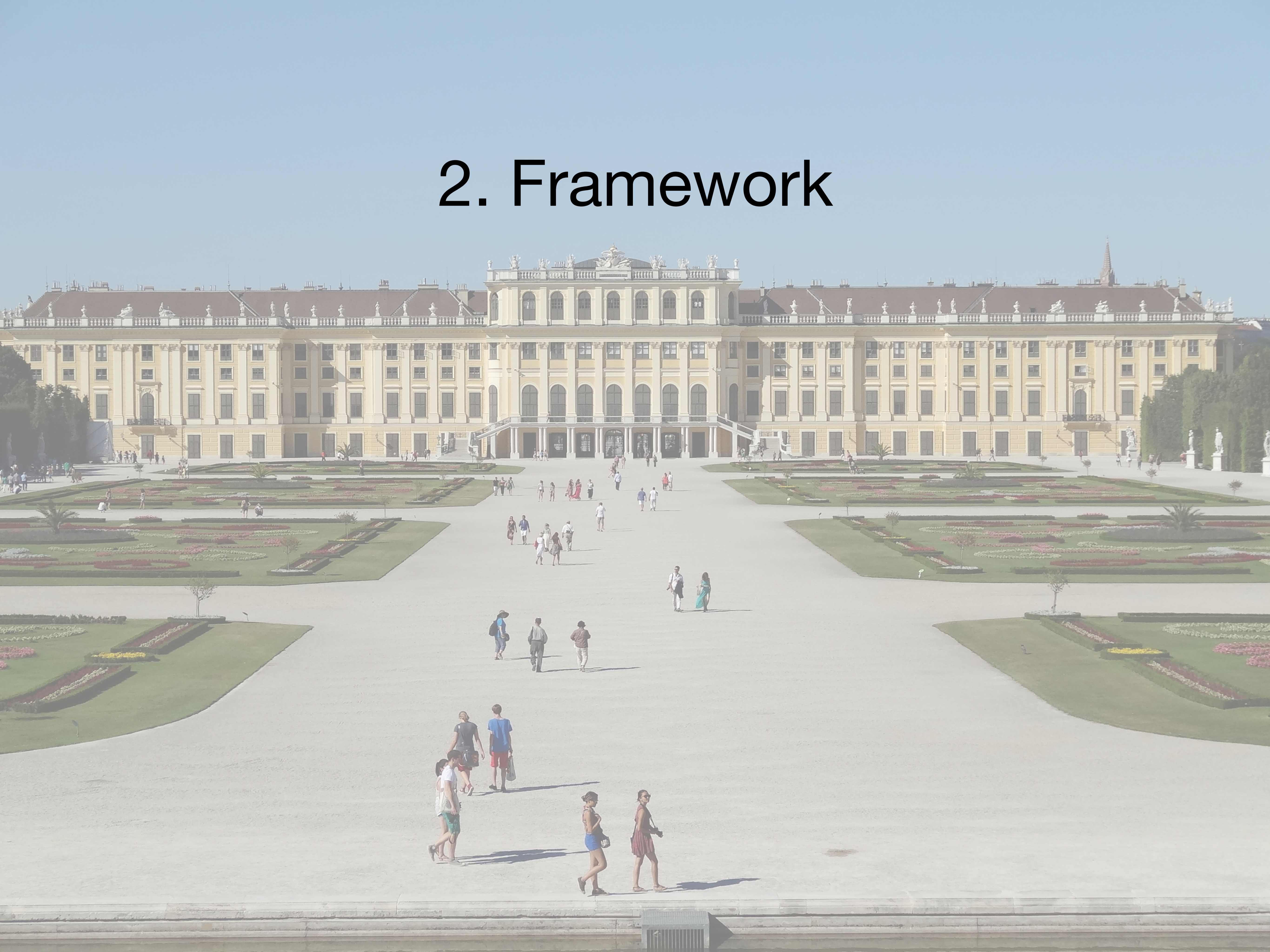
- Numerator & denominator integrated over jet kinematics p_T, η

Jet shape measurements

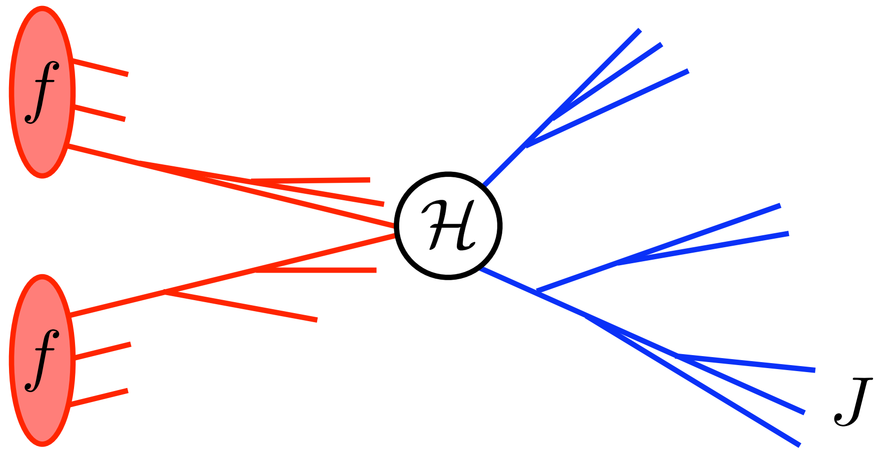
- Jet shape is classic jet substructure observable, measured in $pp, p\bar{p}, ep, e^+e^-$ and heavy ion collisions
- Constrain parton shower event generators [e.g. ATL-PHYS-PUB-2011-008]
- Study medium modification in heavy ion collisions



2. Framework



Factorization for inclusive sample of jets with $R \ll 1$



- $pp \to \text{jet} + X$ for $R \ll 1$ [Kaufmann et al, Kang et al, Dai et al]

$$\frac{d\sigma}{d\eta\, dp_T} = \sum_{a,b,c} f_a(x_a) \otimes f_b(x_b) \otimes \mathcal{H}_{ab\to c}(x_a, x_b, \eta, p_T/z)$$

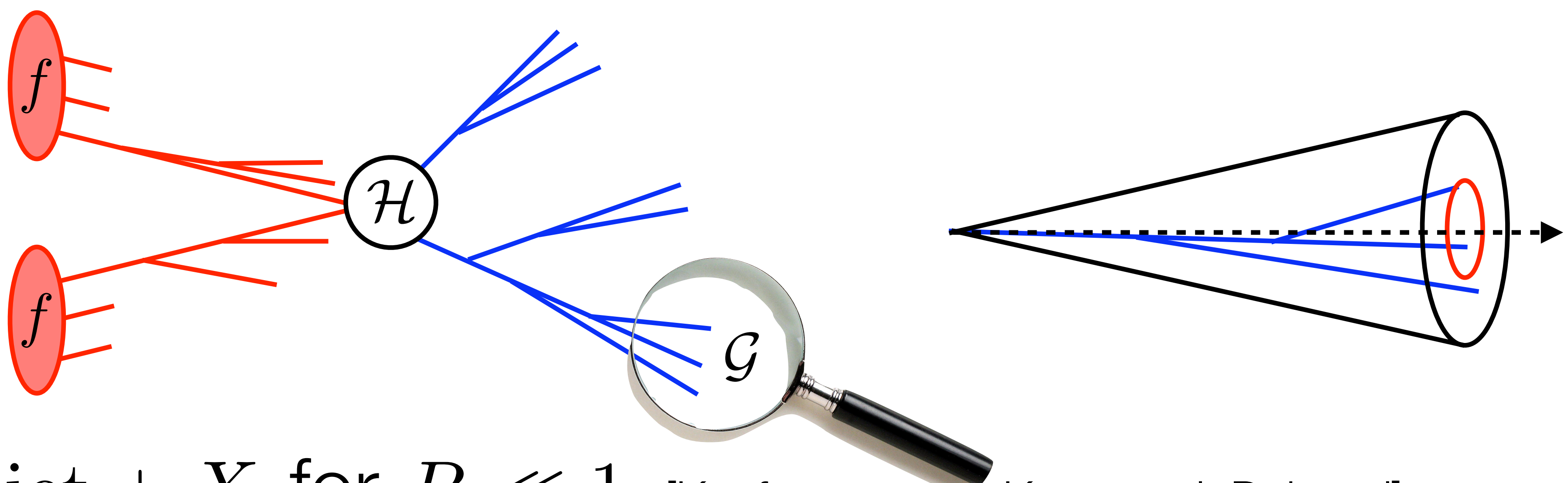
$$\otimes J_c(z, p_T R)$$

- Resum logarithms of $\mu_J/\mu_{\mathcal{H}} \sim (p_T R)/p_T \sim R$ with DGLAP

[see also Dasgupta et al]

$$\frac{d}{d\ln\mu} J_i(z, p_T R, \mu) = \sum_j \int_z^1 \frac{dz'}{z'} \frac{\alpha_s}{\pi} P_{ji}(z/z')\, J_j(z', p_T R, \mu)$$

Factorization for inclusive sample of jets with $R \ll 1$



- $pp \to \text{jet} + X$ for $R \ll 1$ [Kaufmann et al; Kang et al; Dai et al]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\,\mathrm{d}p_T\,\mathrm{d}z_r} = \sum_{a,b,c} f_a(x_a) \otimes f_b(x_b) \otimes \mathcal{H}_{ab\to c}(x_a, x_b, \eta, p_T/z)$$
$$\otimes\, \mathcal{G}_c(z, z_r, p_T R, r/R)$$

Jet shape measurement

- Resum logarithms of $\mu_J/\mu_{\mathcal{H}} \sim (p_T R)/p_T \sim R$ with DGLAP

[see also Dasgupta et al]

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} J_i(z, p_T R, \mu) = \sum_j \int_z^1 \frac{\mathrm{d}z'}{z'} \frac{\alpha_s}{\pi} P_{ji}(z/z')\, J_j(z', p_T R, \mu)$$

Separating jet production from jet shape

- At $\mathcal{O}(\alpha_s)$, real emission is either in or out of jet. Schematically,

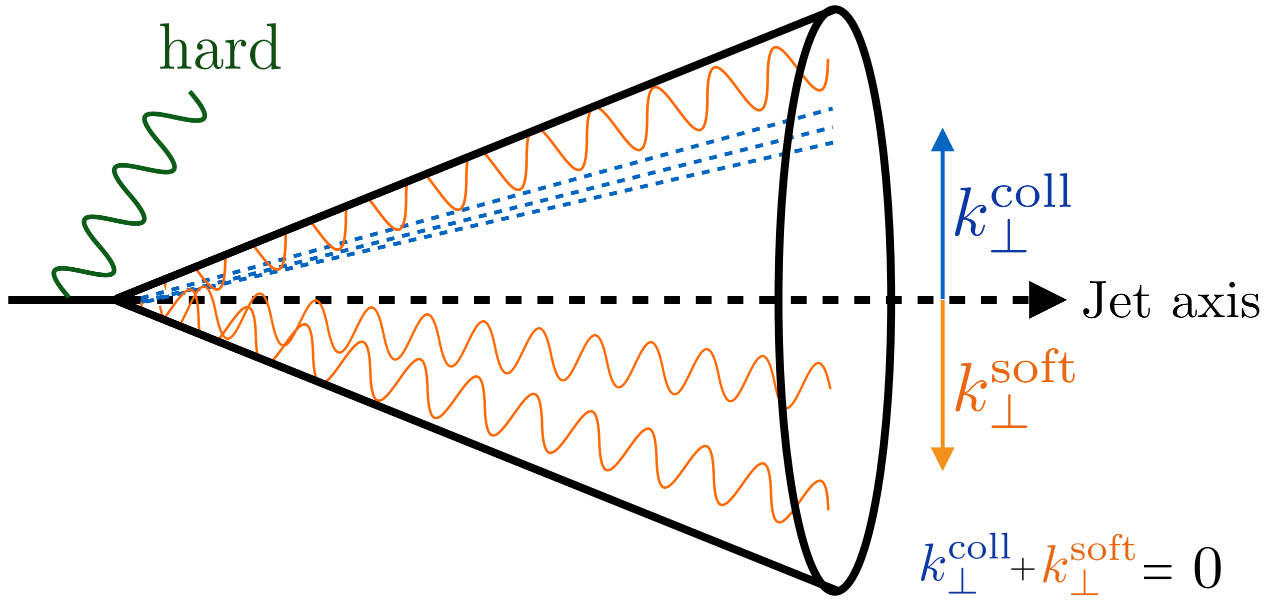
$$\begin{aligned}\mathcal{G} &= \delta(1-z)\delta(1-z_r) + J^{(1)}(z)\,\delta(1-z_r) + \delta(1-z)\Delta\mathcal{G}^{(1)}(z_r) \\ &= \underbrace{\left(\delta(1-z) + J^{(1)}(z)\right)}_{\text{jet production}}\underbrace{\left(\delta(1-z_r) + \Delta\mathcal{G}^{(1)}(z_r)\right)}_{\text{jet shape}} + \mathcal{O}(\alpha_s^2)\end{aligned}$$

[Kaufmann et al; Cal, Ringer, WW]

- This is NOT a factorization of scales
- Jet shape has large logarithms for $r \ll R$. E.g. for quark jet

$$\psi_q(r) = 1 + \frac{\alpha_s C_F}{2\pi}\left(-2\ln^2\frac{r}{R} - 3\ln\frac{r}{R} - \frac{9}{2} + \frac{6r}{R} - \frac{3r^2}{2R^2}\right)$$

Factorization for jet shape with $r \ll R$



	$(k^-, k^+, k_\perp^\mu)$
hard(-collinear)	$p_T(1, R^2, R)$
collinear	$p_T(1, r^2, r)$
(collinear-)soft	$p_T(r/R, rR, r)$

[Kang, Ringer, WW]

- Hard emissions must be out of the jet. Only collinear radiation contributes to jet shape, but soft radiation displaces jet axis

$$\mathcal{G}_c(z, z_r, p_T R, r/R, \mu) = \sum_d H_{cd}(z, p_T R, \mu) \int \mathrm{d}^2 k_\perp \, C_d(z_r, p_T r, k_\perp, \mu, \nu) \times S_d(-k_\perp, \mu, \nu R) \left[1 + \mathcal{O}\left(\frac{r}{R}\right)\right]$$

- Fine print: nonglobal logarithms of r/R

Resummation for $r \ll R$

- Resum logarithms of $\mu_C/\mu_H \sim \mu_S/\mu_H \sim \nu_S/\nu_C \sim r/R$ with

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} H_{cd}(z, p_T R, \mu) = \sum_e \int_z^1 \frac{\mathrm{d}z'}{z'} \, \gamma_{ce}^H\Big(\frac{z}{z'}, p_T R, \mu\Big) \, H_{ed}(z', p_T R, \mu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_d(z_r, p_T r, k_\perp, \mu, \nu) = \gamma_d^C(\mu, \nu/p_T) \, C_d(z_r, p_T r, k_\perp, \mu, \nu)$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} S_d(k_\perp, \mu, \nu R) = \gamma_d^S(\mu, \nu R) \, S_d(k_\perp, \mu, \nu R)$$

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} C_d(z_r, p_T r, k_\perp, \mu, \nu) = -\int \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} \, \gamma_d^\nu(k_\perp - k'_\perp, \mu) \, C_d(z_r, p_T r, k'_\perp, \mu, \nu)$$

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} S_d(k_\perp, \mu, \nu R) = \int \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} \, \gamma_d^\nu(k_\perp - k'_\perp, \mu) \, S_d(k'_\perp, \mu, \nu R)$$

- Anomalous dimension add up to that of $\mathcal{G}$, not zero

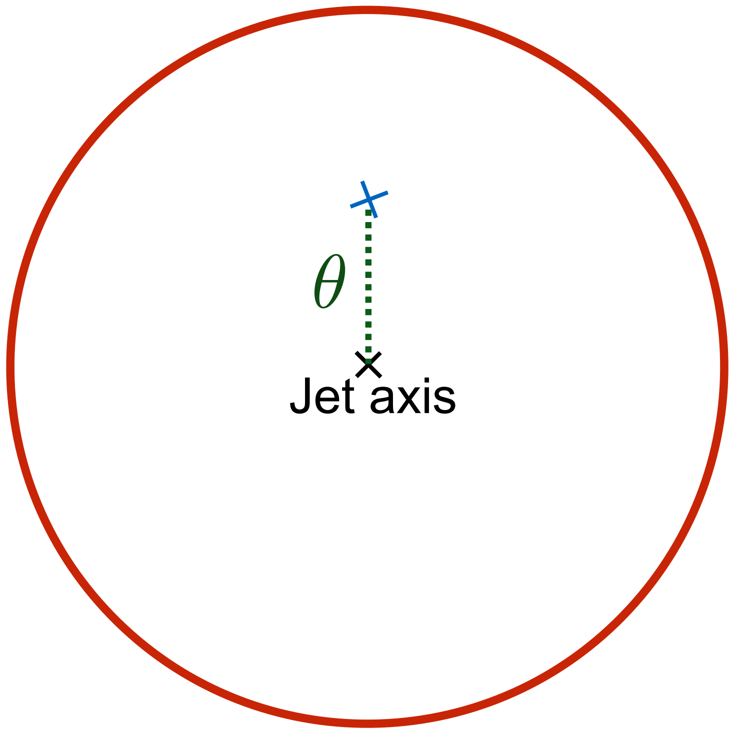
Resummation orders

		Fixed-order	β	γ_μ	γ_ν	NGLs
$\ln R$	LL	tree	1-loop	1-loop	-	-
	NLL	1-loop	2-loop	2-loop	-	-
	NNLL	2-loop	3-loop	3-loop	-	-
$\ln(r/R)$	LL	tree	1-loop	1-loop	-	-
	NLL	tree	2-loop	2-loop	1-loop	LL
	NLL$'$	1-loop	2-loop	2-loop	1-loop	LL
	NNLL	1-loop	3-loop	3-loop	2-loop	NLL

- Single logarithms of R, double logarithms of r/R
- Non-cusp part of γ_μ is only needed at one less loop

3. Ingredients

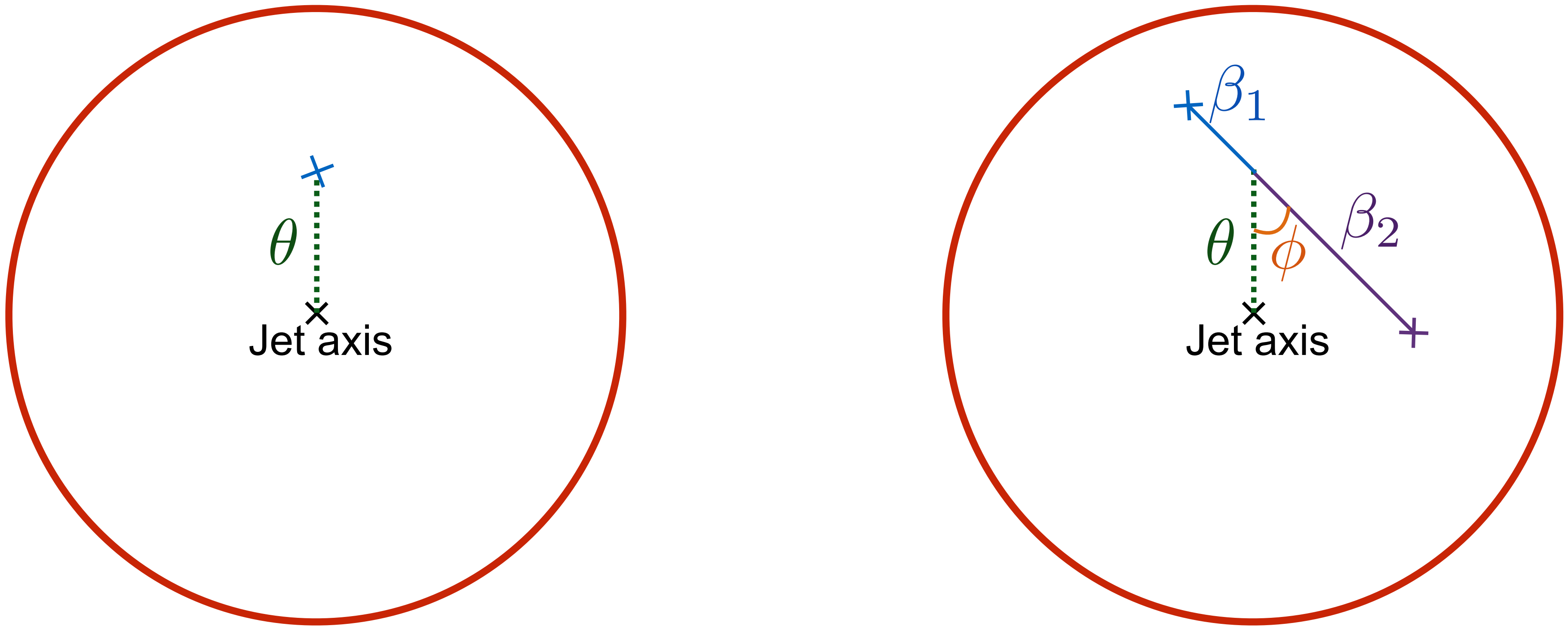
Collinear function



- At tree level, parton is in/out depending on recoil $\theta = k_\perp / p_T$

$$C_d^{(0)} = \delta(1 - z_r)\,\Theta(\theta < r)$$

Collinear function



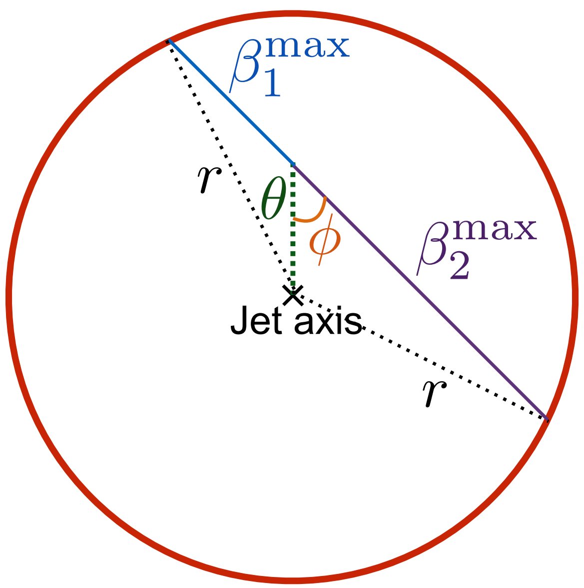
- At tree level, parton is in/out depending on recoil $\theta = k_\perp / p_T$

$$C_d^{(0)} = \delta(1 - z_r)\,\Theta(\theta < r)$$

- At $\mathcal{O}(\alpha_s)$, determining which partons are in/out involves nontrivial ϕ dependence, due to recoil

Collinear function at $\mathcal{O}(\alpha_s)$

- Quark jet with $\theta < r$



$$C_q^{(\theta<r)} = \frac{\alpha_s C_F}{2\pi^2} \int_0^{2\pi} \mathrm{d}\phi \left\{ \delta(1-z_r) \left[\left(\frac{1}{\eta} + \ln\frac{\nu}{2p_T} + \frac{3}{4} \right) \left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{p_T^2 (\beta_1^{\max})^2} \right) \right.\right.$$
$$\left. - \ln^2(1-\widetilde{\beta}) + 2\ln\widetilde{\beta}\ln(1-\widetilde{\beta}) - \frac{3}{2}\ln\widetilde{\beta} + 2\mathrm{Li}_2(1-\widetilde{\beta}) - \frac{\widetilde{\beta}}{2} - \frac{\pi^2}{3} + 2 \right]$$
$$+ \Theta(z_r > \widetilde{\beta}) \left[-(1+z_r^2) \left(\frac{\ln(1-z_r)}{1-z_r} \right)_+ + \ln\left(\frac{z_r(1-\widetilde{\beta})}{\widetilde{\beta}} \right) \frac{1+z_r^2}{(1-z_r)}_+ \right]$$
$$\left. + \Theta(z_r > 1-\widetilde{\beta}) \left[\frac{1+(1-z_r)^2}{z_r} \ln\left(\frac{z_r\widetilde{\beta}}{(1-z_r)(1-\widetilde{\beta})} \right) \right] \right\}$$

$$\widetilde{\beta} = \frac{\beta_2^{\max}}{\beta_1^{\max} + \beta_2^{\max}}$$

- Residual ϕ integral, but $1/\epsilon, 1/\eta$ can be calculated analytically

- Simplifies when averaging over z_r

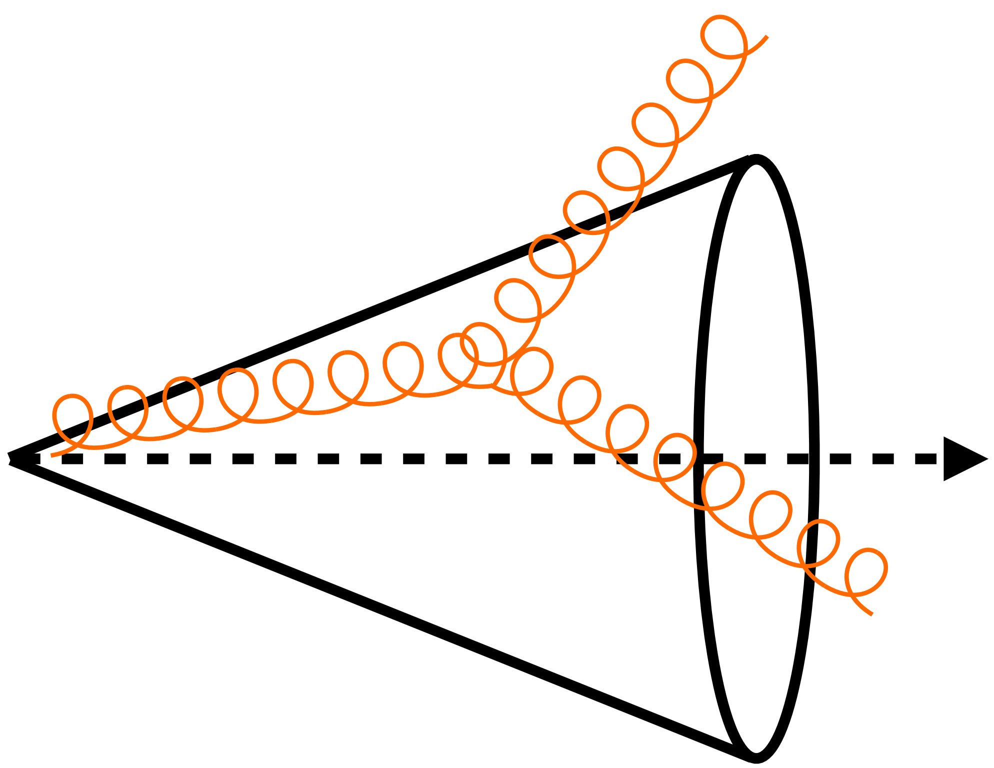
Soft function

- Only soft radiation inside jet recoils jet axis. Up to $\mathcal{O}(\alpha_s)$,

$$S_q(k_\perp, \mu, \nu R) = \delta^2(k_\perp) + \frac{\alpha_s C_F}{2\pi^2}\left[-\frac{1}{\mu^2}\left(\frac{\ln(k_\perp^2/\mu^2)}{k_\perp^2/\mu^2}\right)_+ + \frac{1}{\mu^2}\,\frac{1}{(k_\perp^2/\mu^2)}_+ \ln\frac{\nu^2 R^2}{4\mu^2} - \frac{\pi^2}{12}\delta(\vec{k}_\perp^{\,2})\right]$$

- Nonglobal logarithms [Dasgupta, Salam]

$$-\frac{\alpha_s^2 C_F C_i}{24\pi}\,\frac{1}{(p_T R)^2}\left(\frac{\ln(k_\perp^2/(p_T R)^2)}{k_\perp^2/(p_T R)^2}\right)_+$$



Soft function and nonglobal logarithms

- Only soft radiation inside jet recoils jet axis. Up to $\mathcal{O}(\alpha_s)$,

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- Nonglobal logarithms [Dasgupta, Salam]

$$\int \mathrm{d}^2 k_\perp \Theta(k_\perp < p_T r) \times -\frac{\alpha_s^2 C_F C_i}{24\pi}\,\frac{1}{(p_T R)^2}\left(\frac{\ln(k_\perp^2/(p_T R)^2)}{k_\perp^2/(p_T R)^2}\right)_+ = -\frac{\alpha_s^2 C_F C_i}{12}\ln^2\frac{R}{r}$$

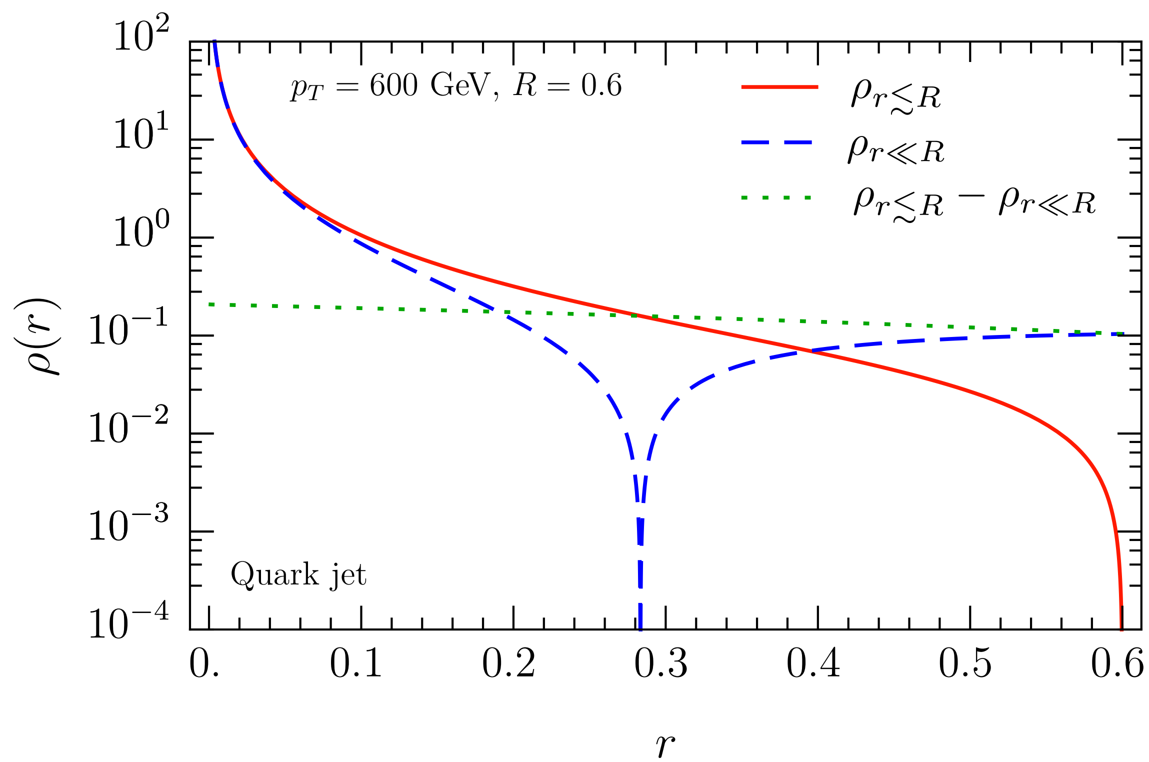
- Upon integrating with the collinear function, this is the same as the hemisphere case [Banfi, Dasgupta, Khelifa-Kerfa, Marzani]

- Extends to leading nonglobal logs: [using Schwartz, Zhu; see also Dingyu's talk]

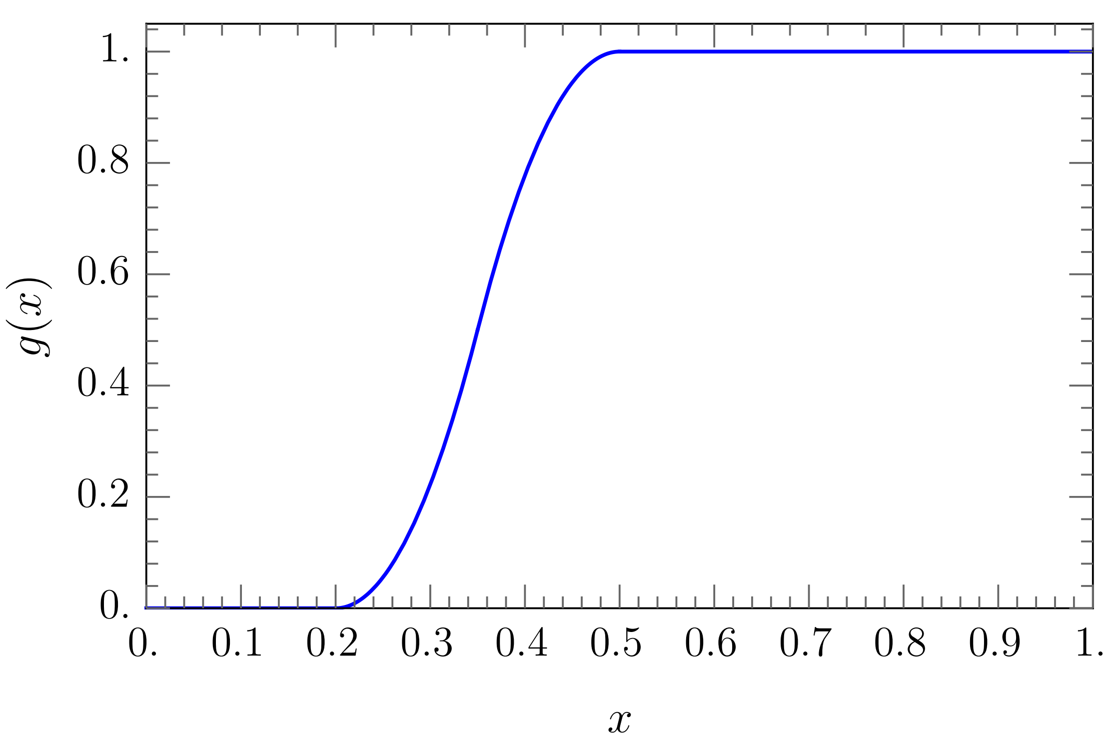
$$S_q^{\mathrm{NG}}(\widehat{L}) = 1 - \frac{\pi^2}{24}\widehat{L}^2 + \frac{\zeta_3}{12}\widehat{L}^3 + \frac{\pi^4}{34560}\widehat{L}^4 + \dots \qquad \widehat{L} = \frac{\alpha_s N_c}{\pi}\ln\frac{R}{r}$$

4. Implementation

Matching $r \ll R$ and $r \lesssim R$



$$\psi(r) = \left[1 - g\left(\frac{r}{R}\right)\right]\psi_{r \ll R}(r) + g\left(\frac{r}{R}\right)\psi_{r \lesssim R}(r)$$



- Choose transition function based on jet shape at $\mathcal{O}(\alpha_s)$
- Avoids complications from profile scales

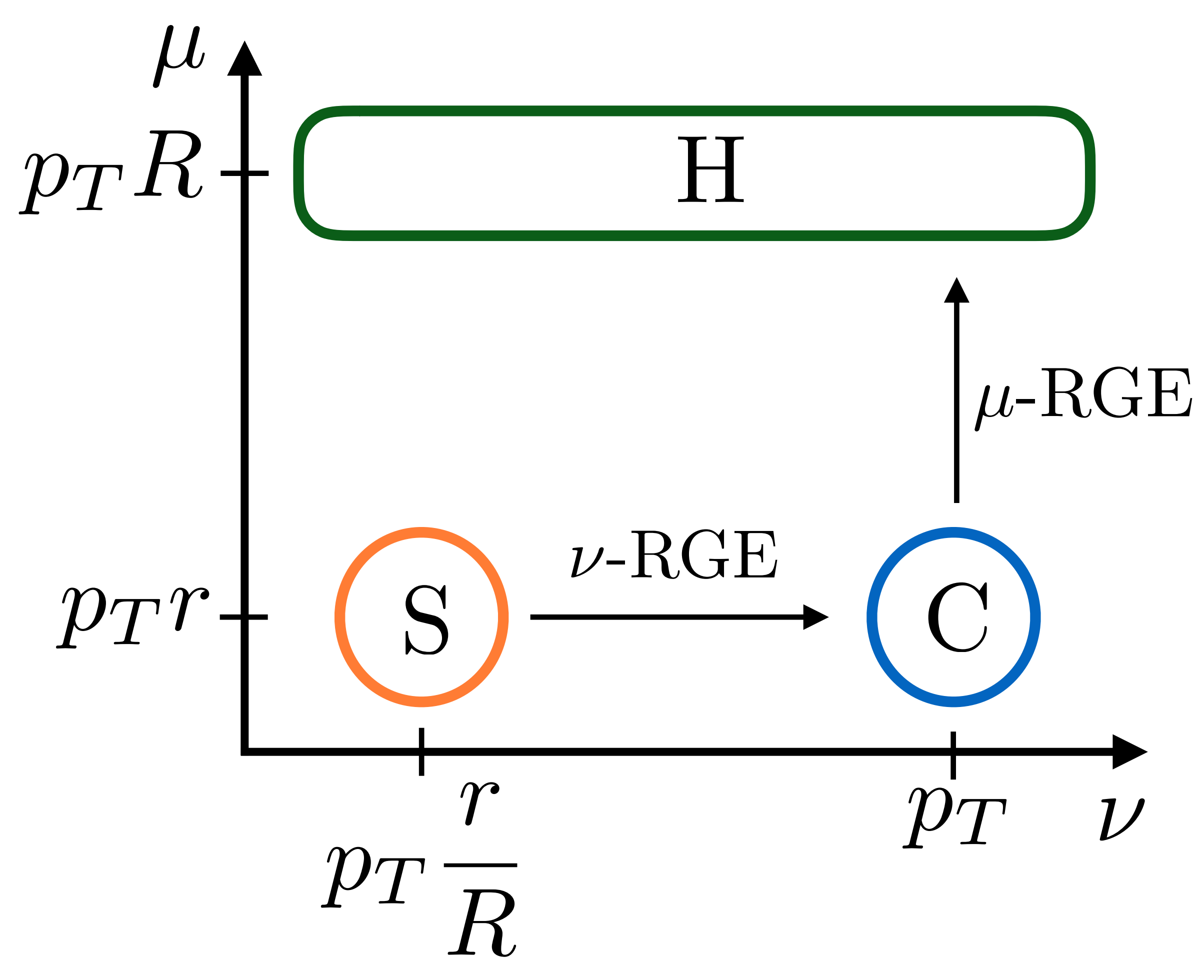
Scale choices and perturbative uncertainties

- Central scale choice:

$$\mu_{\mathcal{H}} = p_T \qquad \mu_H = p_T R \qquad \mu_C = p_T r \qquad \mu_S = p_T r$$

$$\nu_C = p_T \qquad \nu_S = \frac{1}{b_\perp R}$$

Fourier conjugate of $k_\perp$



Scale choices and perturbative uncertainties

- Central scale choice:

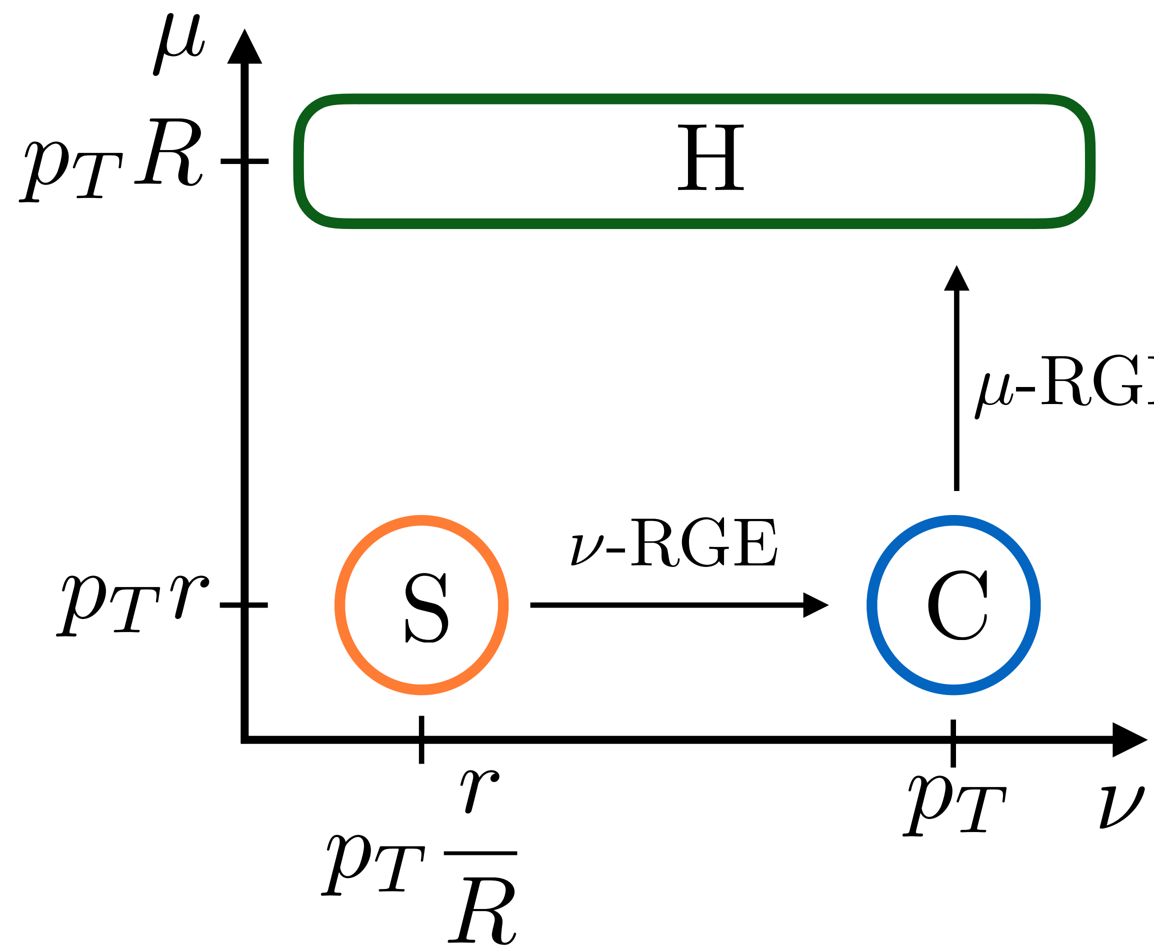
$$\mu_{\mathcal{H}} = p_T \qquad \mu_H = p_T R \qquad \mu_C = p_T r \qquad \mu_S = p_T r$$

$$\nu_C = p_T \qquad \nu_S = \frac{1}{b_\perp R}$$

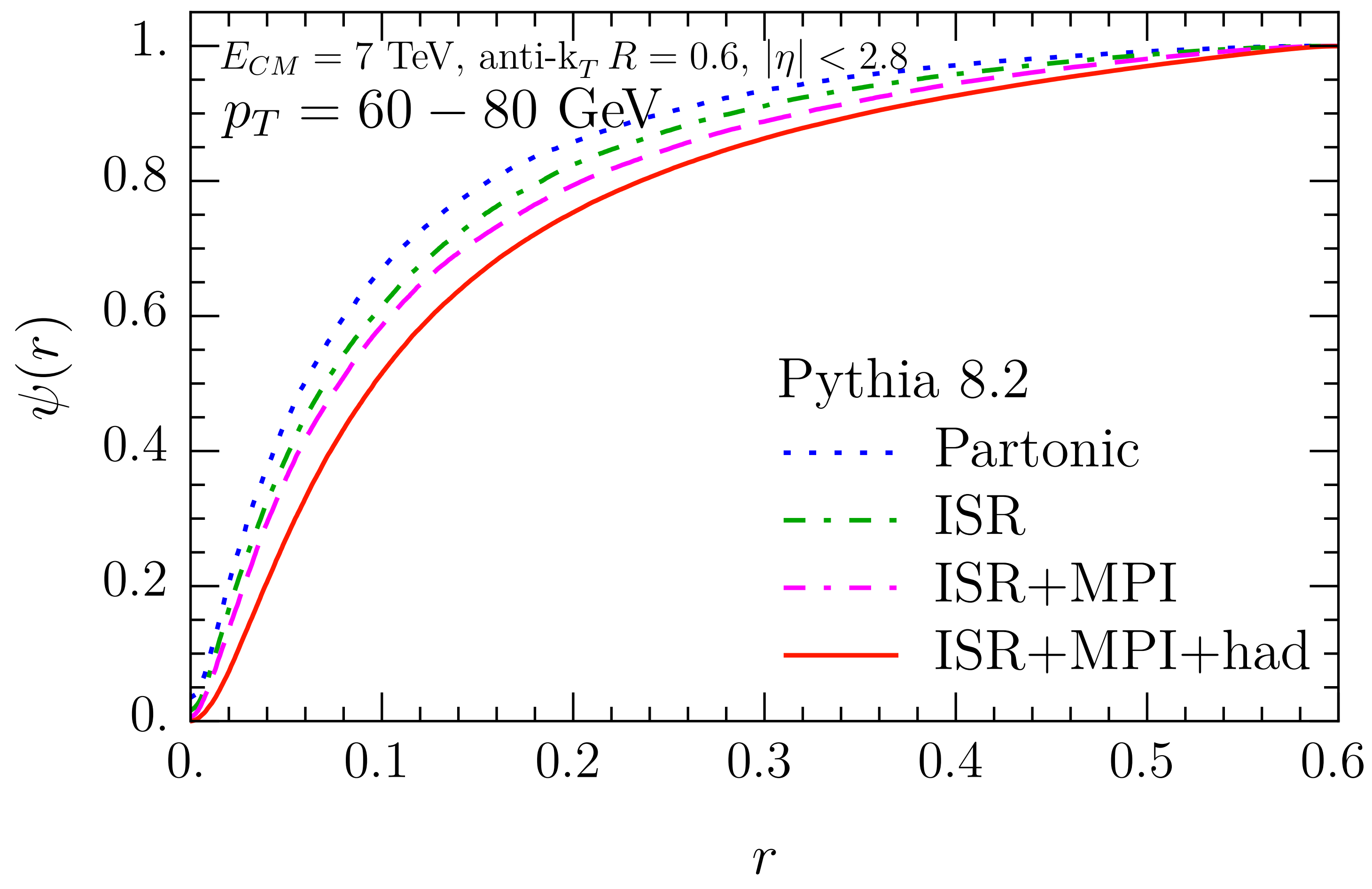
Fourier conjugate of $k_\perp$

- Scale variations:

1. All scales $\times 2, \frac{1}{2}$
2. $\mu_{\mathcal{H}}, \mu_H$ both $\times 2, \frac{1}{2}$
3. μ_C, μ_S both $\times 2, \frac{1}{2}$
4. ν_C $\times 2, \frac{1}{2}$
5. ν_S $\times 2, \frac{1}{2}$
6. Vary transition in matching

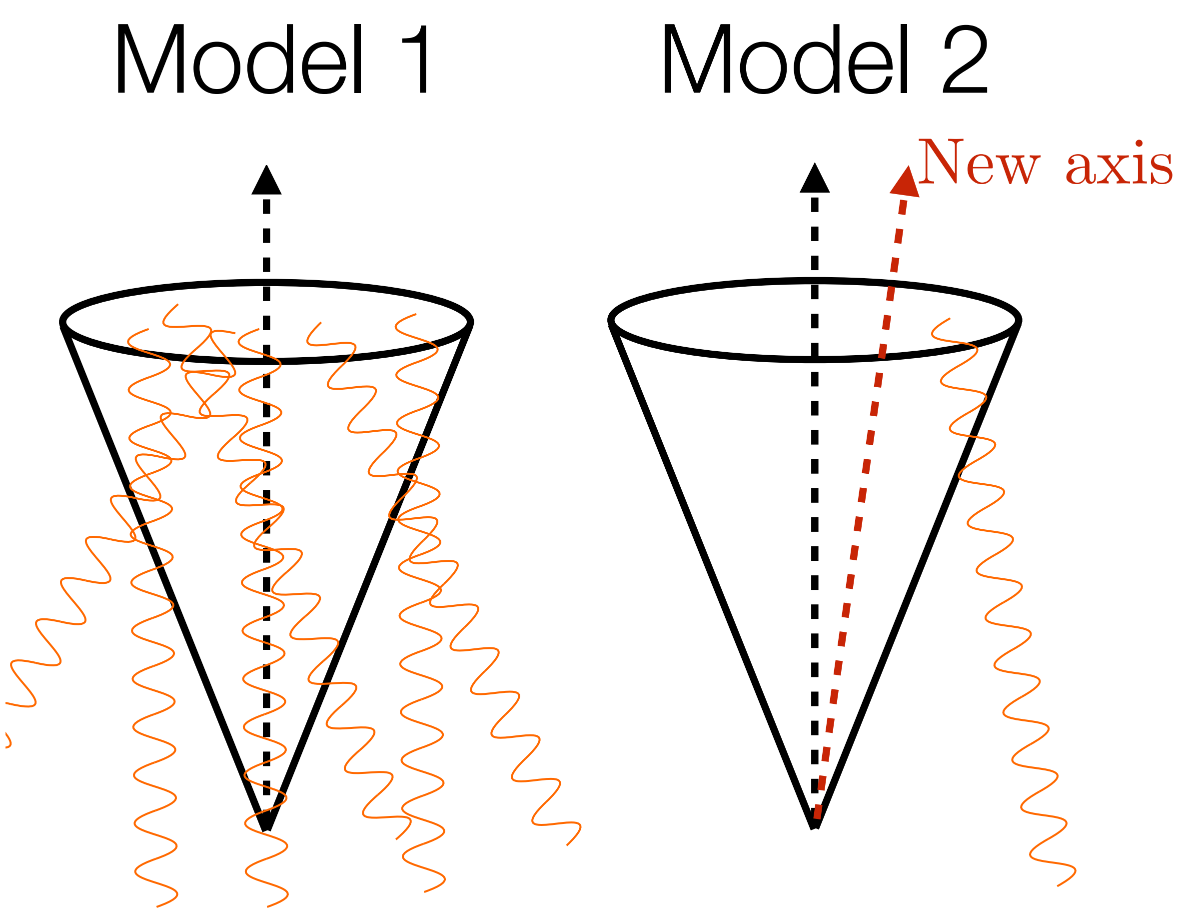


Nonperturbative effects



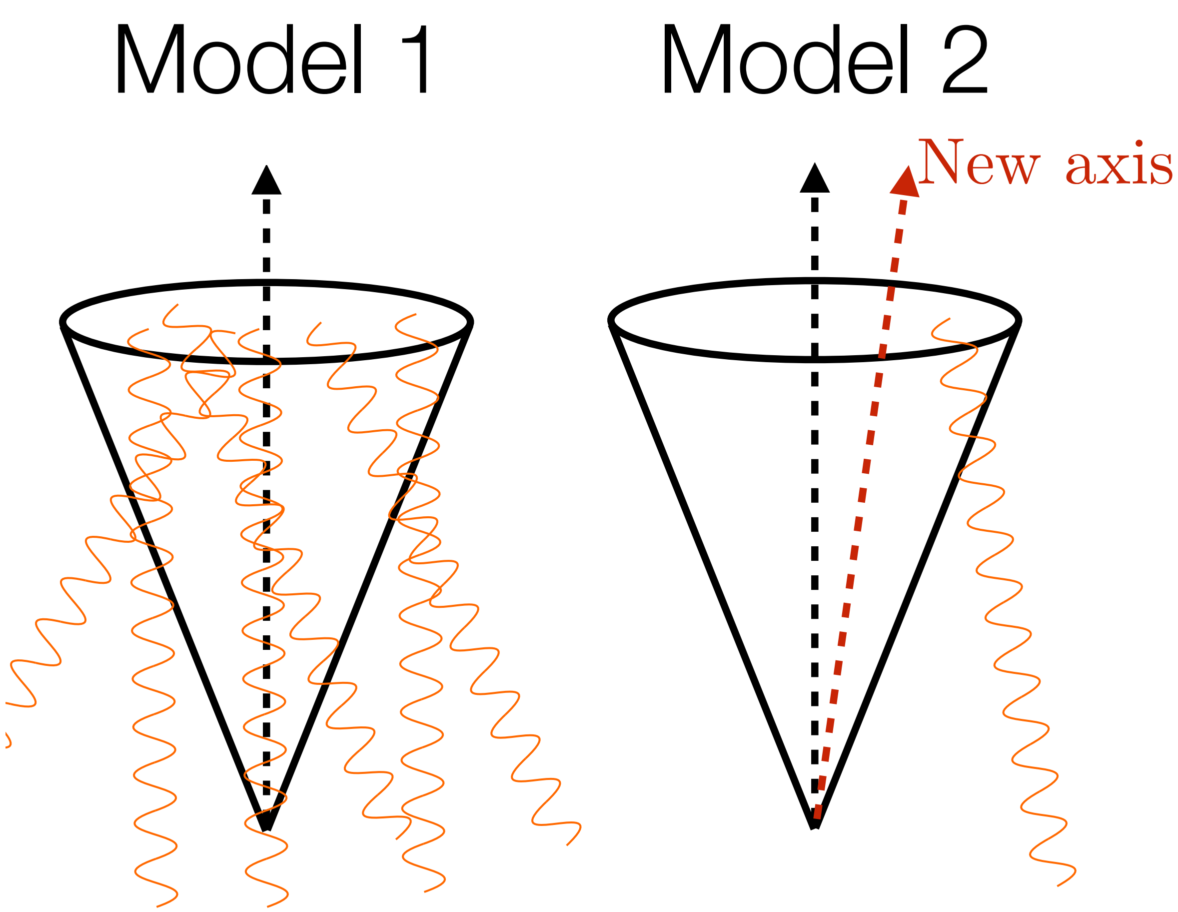
- Significant effects from soft radiation: initial-state radiation, multi-parton interactions and hadronization effects

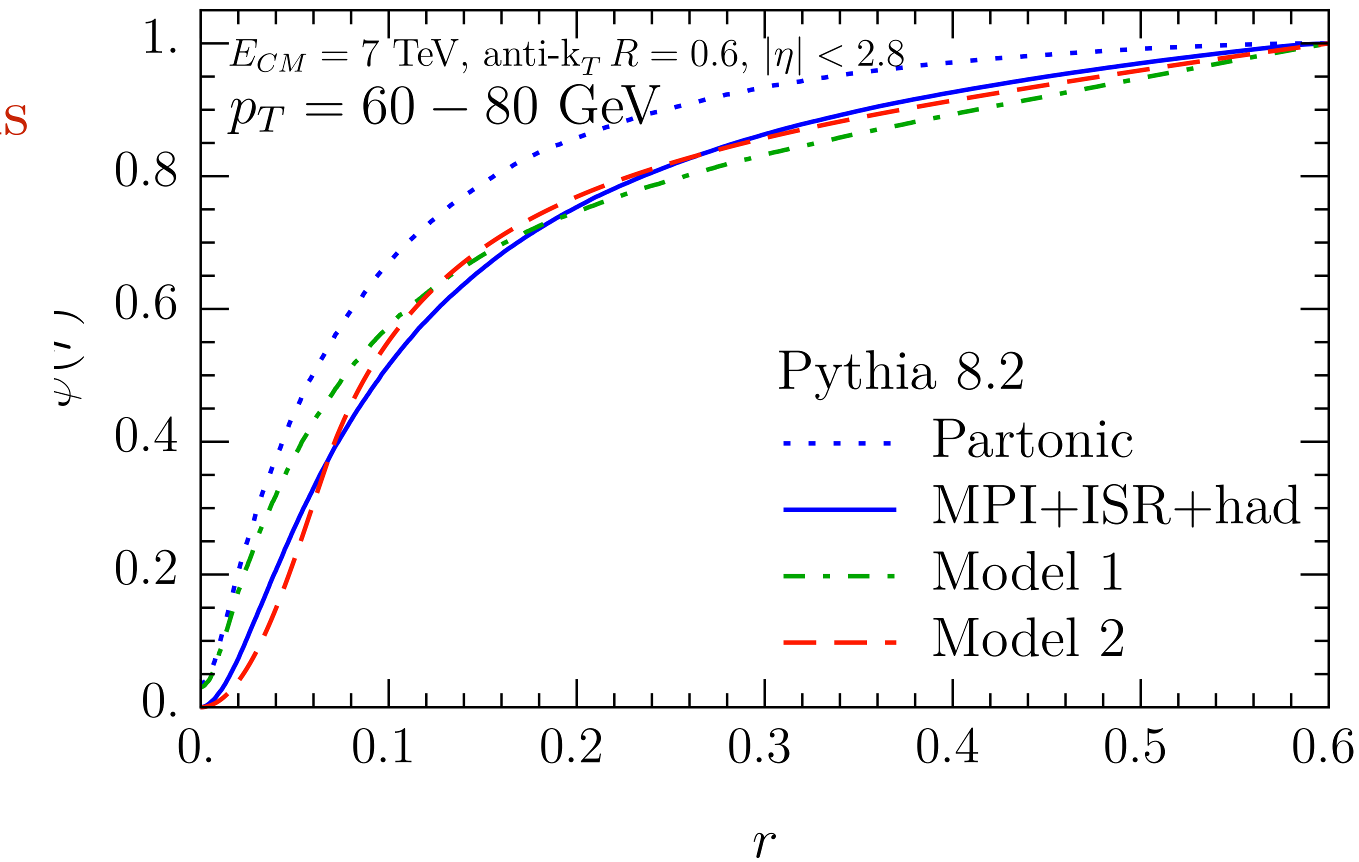
Nonperturbative model



- 1: uniform contamination, $\psi(r) \to \frac{1}{1+f}\,\psi(r) + \frac{f}{1+f}\left(\frac{r}{R}\right)^2$
- 2: localized contamination, also displaces jet axis

Nonperturbative model

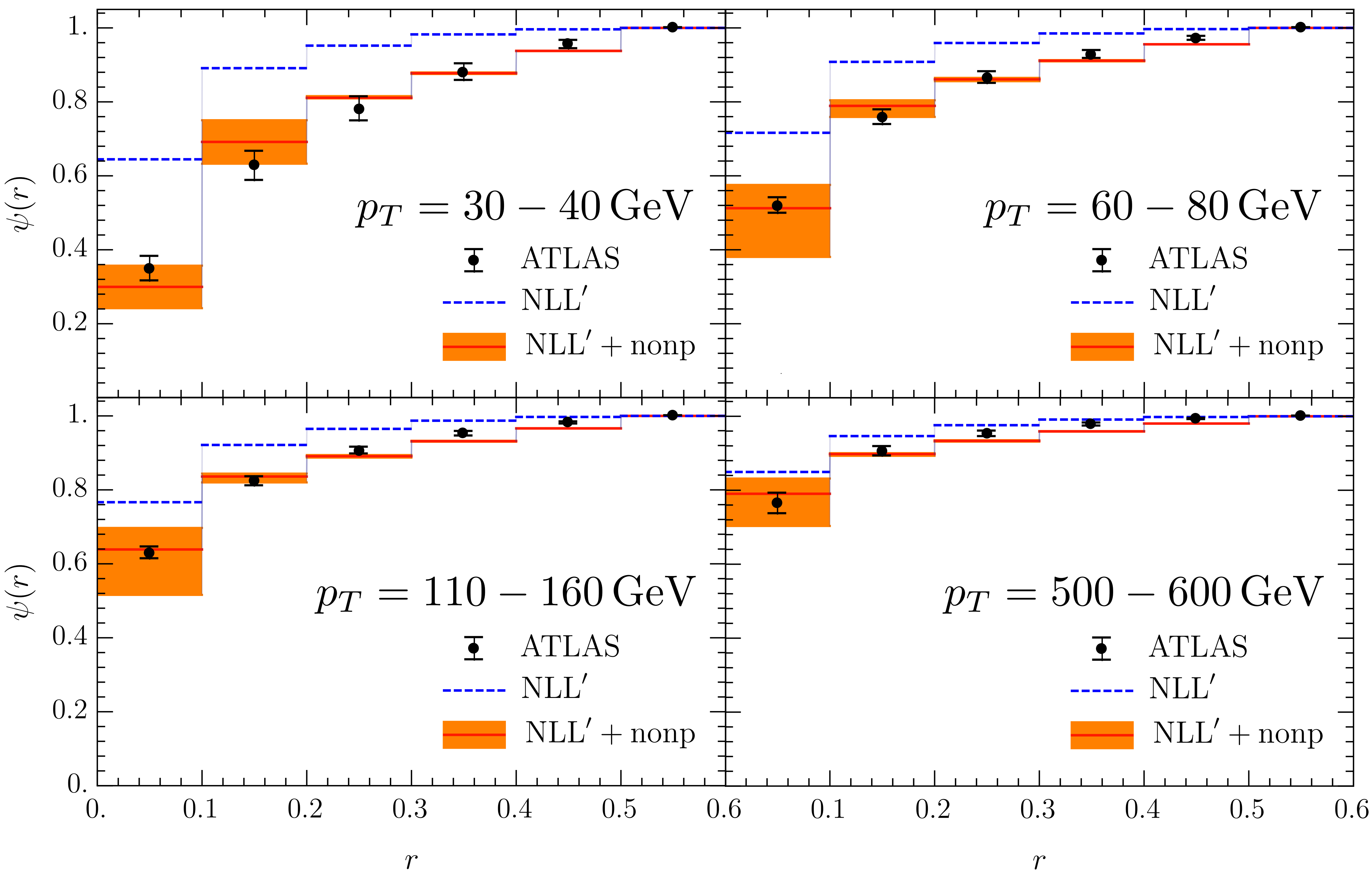




- 1: uniform contamination, $\psi(r) \to \frac{1}{1+f}\,\psi(r) + \frac{f}{1+f}\left(\frac{r}{R}\right)^2$
- 2: localized contamination, also displaces jet axis
- Model 2 agrees better, used when comparing to LHC data

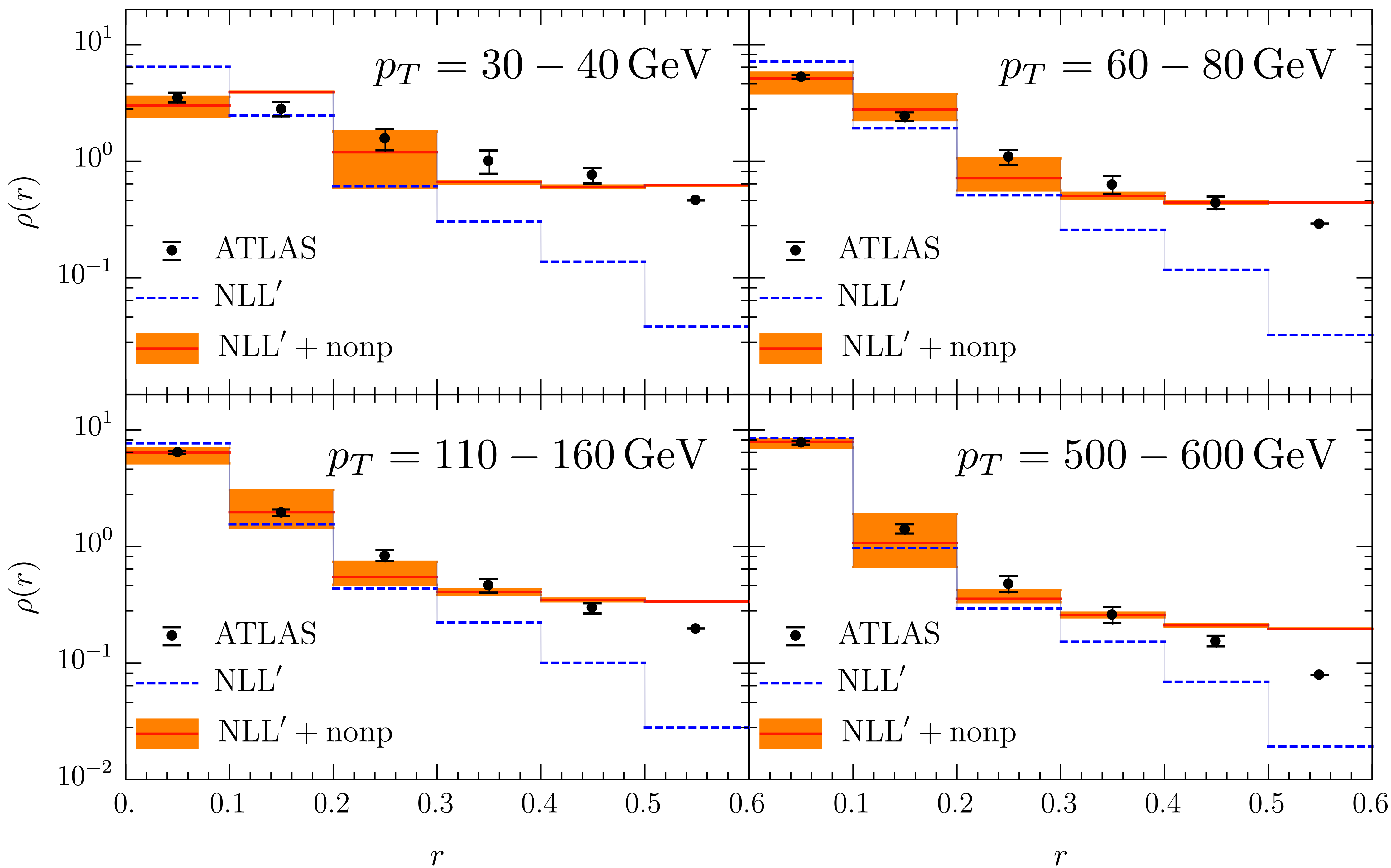
5. Results

ATLAS integrated jet shape



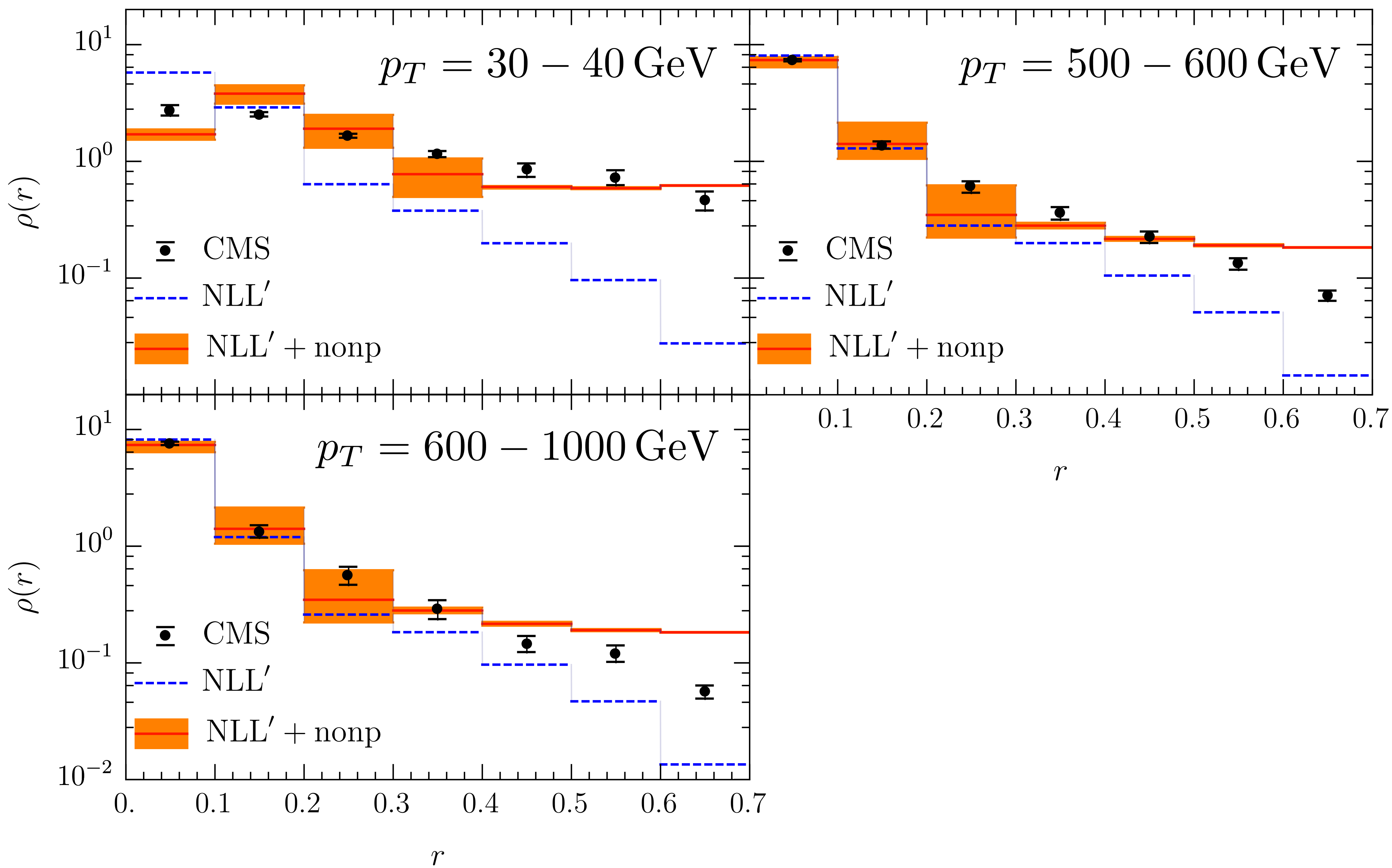
- Good agreement. Perturbative uncertainty largest for small r
  Nonperturbative effects $\propto 1/p_T$

ATLAS differential jet shape



- Nonperturbative effects particularly important in tail (not the region where r/R resummation is important)

CMS differential jet shape



- Similar level of agreement
  Slightly larger R and nonperturbative effects

Conclusions

- First jet shape calculation beyond LL: recoil of soft radiation
  - Collinear function with recoil is more complicated
  - Rapidity resummation
  - Nonglobal logarithms are fortunately same as hemisphere case, to the order we are working at
- Good agreement with data when using nonperturbative model
  - Extending to groomed jet shape