

Jet TMDs and Non-global logs

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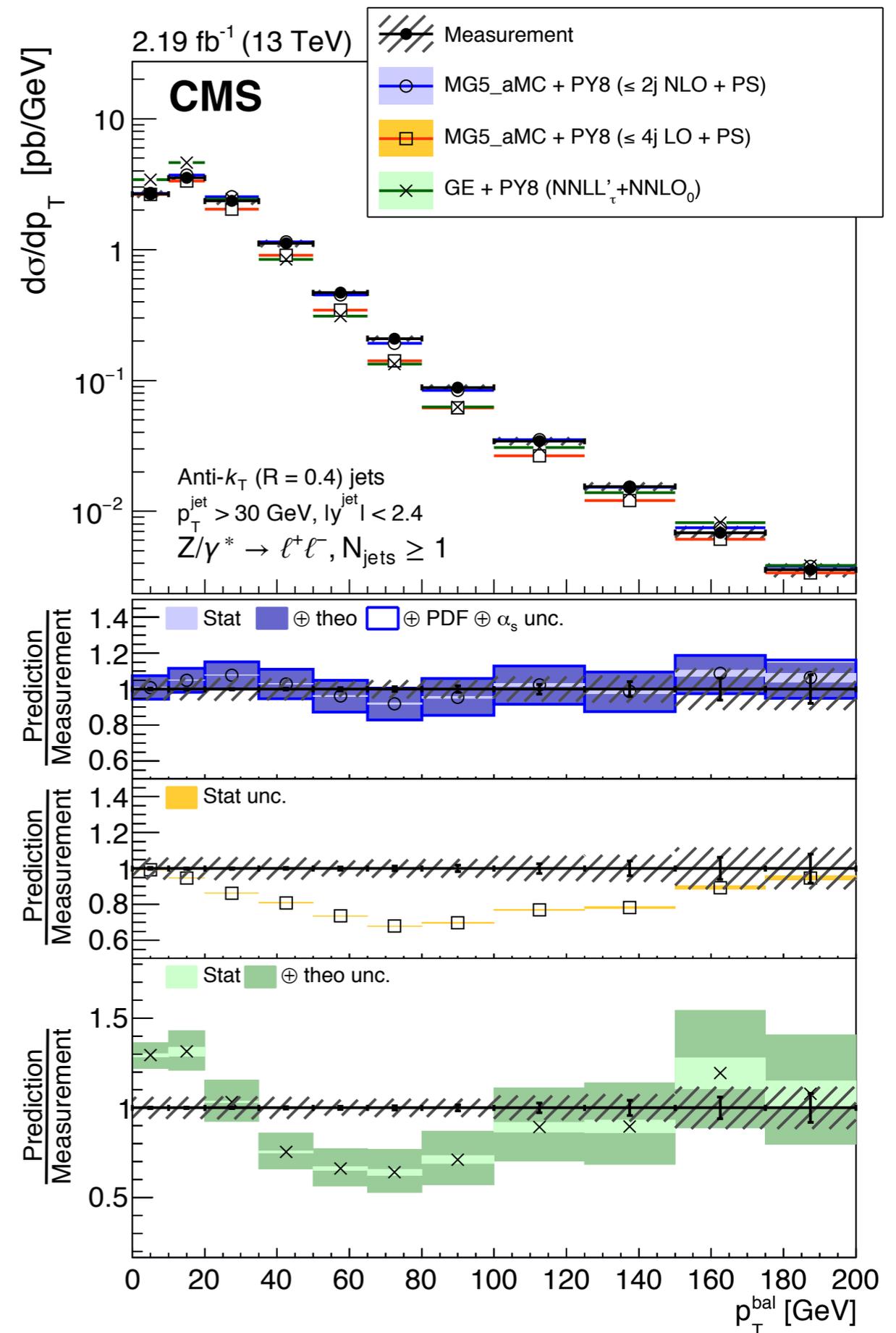
TMD factorization and resummation

- Drell-Yan processes: (Z, W, Higgs, VV, ...)
 - the all order structure is first understood by CSS at 1985
 - computer codes: CUTE, DYRES/DYTurbo, MATRIX, NangaParbat, RADISH, RESBOS, reSolve, SCETlib
 - N³LL for single-boson processes (CUTE, RADISH, SCETlib,...)
 - event-based automated framework (see Becher's talk); threshold and TMD joint resummation (see Hautmann's talk)
- Massive colored particles production: (ttbar, ...)
 - Soft radiations from final states (Li, Li, DYS, Yang, Zhu '12 & Catani, Grazzini & Sargsyan '18)
- Jet processes: (jet substructures, jet distributions, ...)
 - Jet pull (see Wu's talk); Jet shape (see Waalewijn's talk)
 - transverse momentum distribution of jets with the beam axis (this talk)

Transverse momentum balance measurement

$$p_T^{\text{bal}} = |\vec{p}_T(Z) + \sum_{\text{jets}} \vec{p}_T(j_i)|$$

This observable can be used for jet calibration at pp & energy loss study at PbPb

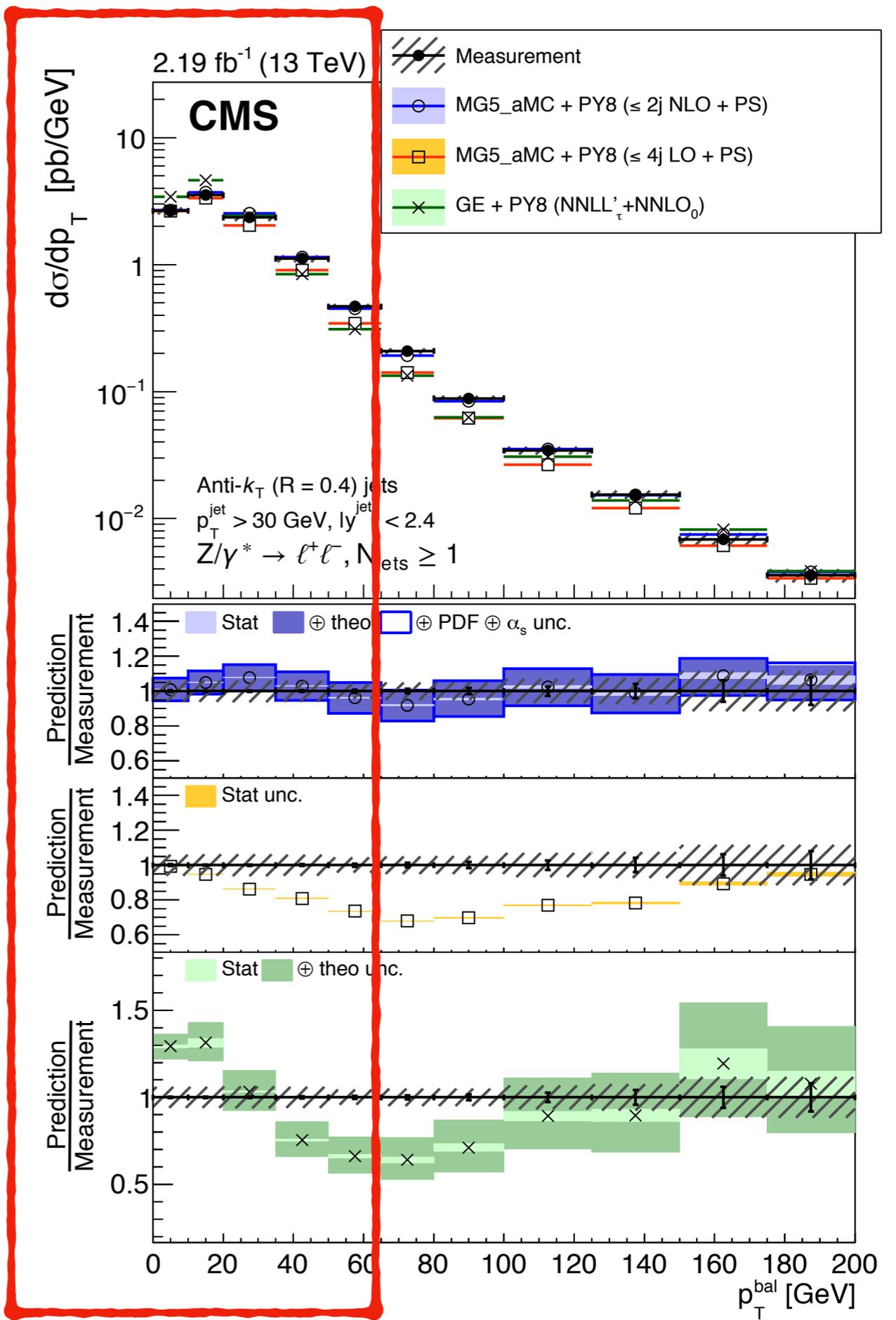


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In the small p_T^{bal} region, one needs all-order results ($\log(p_T^{\text{bal}}/Q)$ resummation)
 The predictions from different event generators are not consistent !!!



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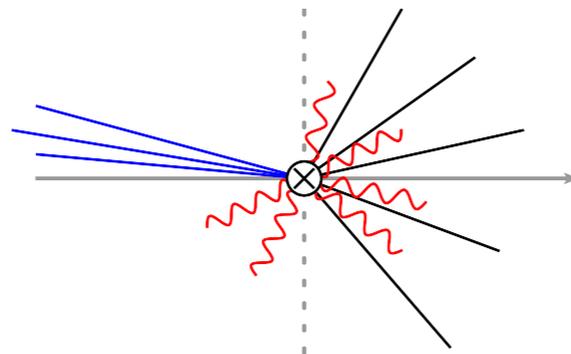
Jet TMDs and non-global logs

- **Non-global logs in jet TMD resummation** (Banfi, Dasgupta & Delenda '08)

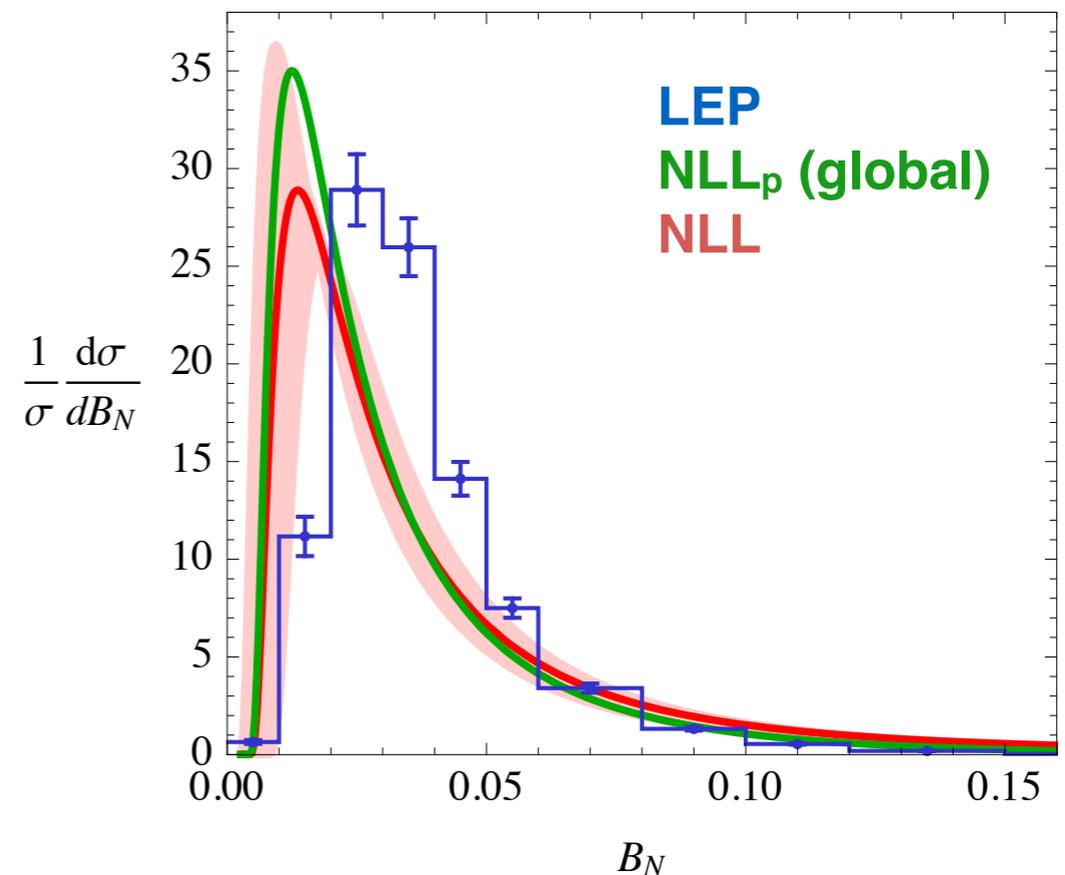
$$q_T = \left| \sum_{i \notin \text{jets}} \vec{k}_{T,i} \right| + \mathcal{O}(k_T^2)$$

- sum over all soft partons not combined with hard jets
- deviation from $q_T=0$ are only caused by particle flow outside the jet regions
- non-global observables (Dasgupta & Salam '01)
- E.g. narrow jet broadening resummation (Becher, Rahn & DYS '17)

$$b_N = \sum_{i \in \text{jets}} |\vec{p}_i^\perp|$$



$$\frac{d\sigma}{db_L} = \sum_{f=q,\bar{q},g} \int db_L^s \int d^{d-2} p_L^\perp \mathcal{J}_f(b_L - b_L^s, p_L^\perp) \times \sum_{m=1}^{\infty} \langle \mathcal{H}_m^f(\{\underline{n}\}, Q) \otimes \mathcal{S}_m(\{\underline{n}\}, b_L^s, -p_L^\perp) \rangle$$



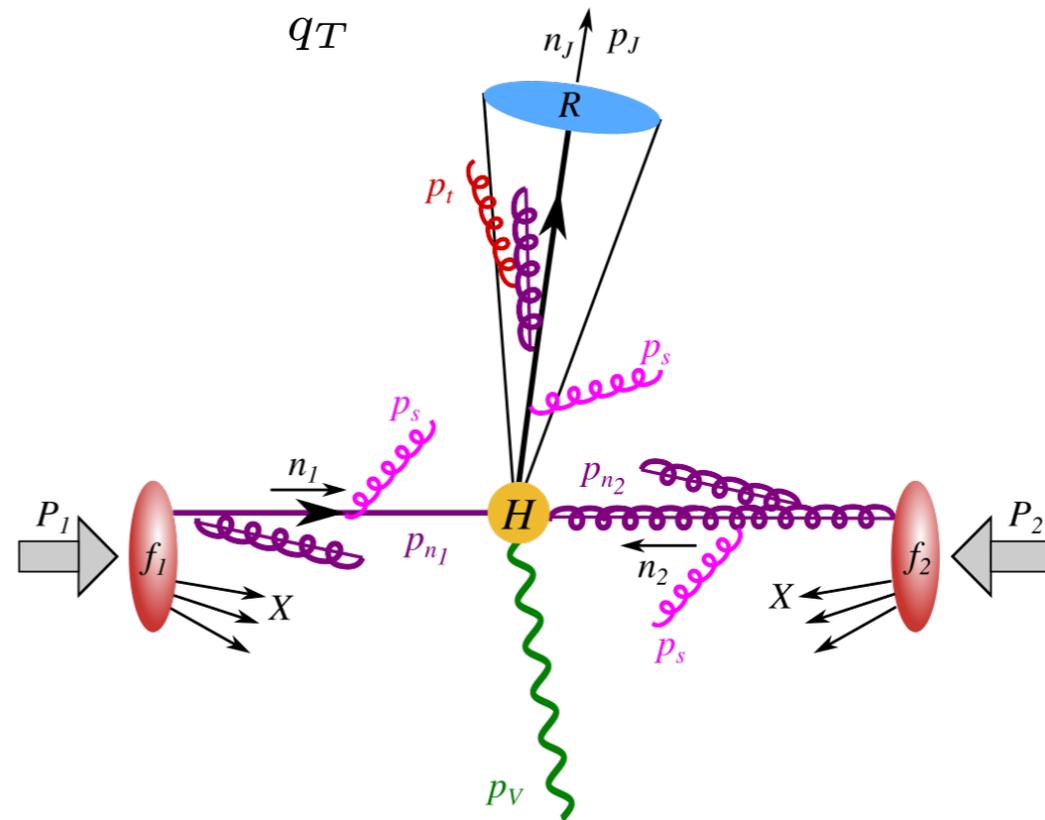
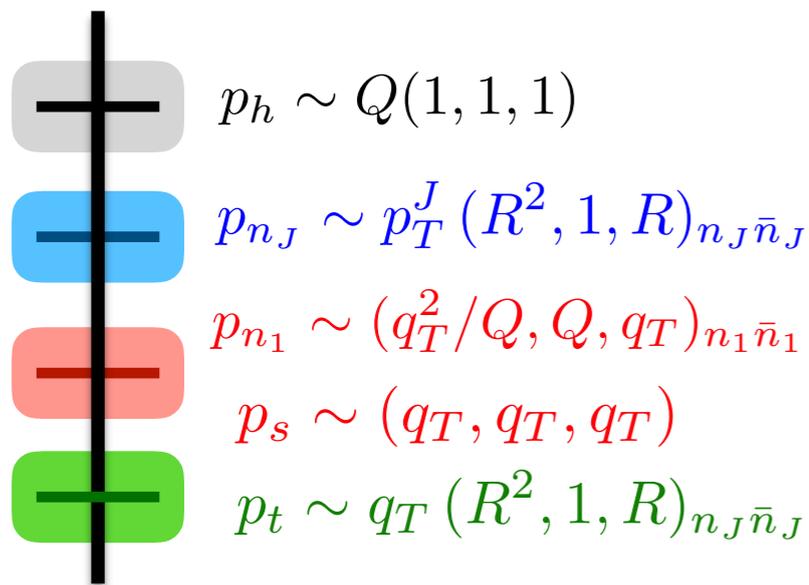
Previous works on the transverse momentum resummation for jet processes

- **CSS framework ($NLL_p \log(q_T/Q)$ w.o. non-global logs; partial $\log(R)$ resummed)**
 - **dijet** (Sun, Yuan & Yuan '14 & '15)
 - **jet + V** (Sun, Yuan & Yuan '18; Chen, Qin, Wang, Wei, Xiao, Zhang '18)
 - **lepton + jet (two-loop NGLs)** (Liu, Yuan & Felix '19)
 - **jet + top** (Cao, Sun, Yan, Yuan & Yuan '18 & '19)
- **SCET framework**
 - **photon + jet ($NLL_p \log(q_T/Q)$ & $\log(R)$; coft modes; w.o. non-global logs)** (Buffing, Kang, Lee & Liu '18)

Jet radius and q_T joint resummation for boson-jet correlation

(Chien, DYS & Wu 1905.01335)

$$N_1(P_1) + N_2(P_2) \rightarrow \underbrace{\text{boson}(p_V) + \text{jet}(p_J)}_{q_T} + X$$



- **Coft modes:** $p_t^\mu \sim q_T (R^2, 1, R)_{n_J \bar{n}_J}$ **for the jet radius R resummation** (Becher, Neubert, Rothen & DYS '15; Chien, Hornig & Lee '15; Kolodrubetz, Pietrulewicz, Stewart, Tackmann & Waalewijn '16;
- **Multi-Wilson-Line operators describe radiations along the jet direction for NGLs resummation** (Caron-Hout '15; Becher, Neubert, Rothen & DYS '15;

Jet radius and q_T joint resummation for boson-jet correlation

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$$N_1(P_1) + N_2(P_2) \rightarrow \underbrace{\text{boson}(p_V) + \text{jet}(p_J)}_{q_T} + X$$

Factorization formula (neglecting glauber modes):

$$\frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} = \sum_{ijk} \int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} \mathcal{S}_{ij \rightarrow V k}(\vec{x}_T, \epsilon) \mathcal{B}_{i/N_1}(\xi_1, x_T, \epsilon) \mathcal{B}_{j/N_2}(\xi_2, x_T, \epsilon) \times \mathcal{H}_{ij \rightarrow V k}(\hat{s}, \hat{t}, m_V, \epsilon) \sum_{m=1}^{\infty} \langle \mathcal{J}_m^k(\{n_J\}, R p_J, \epsilon) \otimes \mathcal{U}_m^k(\{n_J\}, R \vec{x}_T, \epsilon) \rangle$$

$p_h \sim Q(1, 1, 1)$
 $p_{n_J} \sim p_T^J (R^2, 1, R)_{n_J \bar{n}_J}$
 $p_{n_1} \sim (q_T^2/Q, Q, q_T)_{n_1 \bar{n}_1}$
 $p_s \sim (q_T, q_T, q_T)$
 $p_t \sim q_T (R^2, 1, R)_{n_J \bar{n}_J}$

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RG evolution and resummation

- **Resummation formula:**

$$\begin{aligned} \frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} &= \sum_{ijk} \int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} e^{\int_{\mu_h}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{H_{ij \rightarrow V_k}}(\bar{\mu})} \mathcal{H}_{ij \rightarrow V_k}(\hat{s}, \hat{t}, m_V, \mu_h) \\ &\times \left(\frac{x_T^2 \hat{s}}{b_0^2} \right)^{-(C_i + C_j) F_{\perp}(\mu)} e^{\int_{\mu_b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{W_{ij \rightarrow V_k}}(\bar{\mu})} S_{ij \rightarrow V_k}(\vec{x}_T, \mu_b) B_{i/N_1}(\xi_1, x_T, \mu_b) B_{j/N_2}(\xi_2, x_T, \mu_b) \\ &\times e^{\int_{\mu_t}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{U_k}(\bar{\mu}) + \int_{\mu_j}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{J_k}(\bar{\mu})} U_{\text{NG}}^k(\mu_t, \mu_j), \end{aligned}$$

- **Typical scales:** $\mu_h \sim Q$, $\mu_b \sim b_0/x_T$, $\mu_j \sim R p_T$, $\mu_t \sim R b_0/x_T$,

- **NGL resummation:** (Becher, Neubert, Rothen & DYS '15)

$$U_{\text{NG}}(\mu_t, \mu_j) \equiv \sum_{l=1}^{\infty} \langle \mathcal{J}_l(\{\underline{n}'\}, R p_T, \mu_j) \otimes \sum_{m \geq l}^{\infty} U_{lm}(\{\underline{n}\}, \mu_t, \mu_j) \hat{\otimes} \mathcal{U}_m(\{\underline{n}\}, R \vec{x}_T, \mu_t) \rangle$$

- **At the NLL(LR) level and large Nc limit the evolution factor reduces to Dasgupta-Salam shower and BMS equation**

$$U_{\text{NG}}(\mu_t, \mu_j) \xrightarrow{\text{NLL}} \sum_{m \geq 1}^{\infty} \langle U_{1m}(\{\underline{n}\}, \mu_t, \mu_j) \hat{\otimes} \mathbf{1} \rangle$$

RG evolution and resummation

- Resummation formula:

Logs from different scales

$$\begin{aligned} \frac{d\sigma}{d^2q_T d^2p_T d\eta_J dy_V} &= \sum_{ijk} \int \frac{d^2x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} e^{\int_{\mu_h}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{H_{ij \rightarrow V k}}(\bar{\mu})} \mathcal{H}_{ij \rightarrow V k}(\hat{s}, \hat{t}, m_V, \mu_h) \\ &\times \left(\frac{x_T^2 \hat{s}}{b_0^2} \right)^{-(C_i + C_j) F_{\perp}(\mu)} e^{\int_{\mu_b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{W_{ij \rightarrow V k}}(\bar{\mu})} \mathcal{S}_{ij \rightarrow V k}(\vec{x}_T, \mu_b) B_{i/N_1}(\xi_1, x_T, \mu_b) B_{j/N_2}(\xi_2, x_T, \mu_b) \\ &\times e^{\int_{\mu_t}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{U_k}(\bar{\mu}) + \int_{\mu_j}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{J_k}(\bar{\mu})} U_{\text{NG}}^k(\mu_t, \mu_j), \end{aligned}$$

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Rapidity logs resummation

$$\times \left(\frac{x_T^2 \hat{s}}{b_0^2} \right)^{-(C_i + C_j) F_{\perp}(\mu)} e^{\int_{\mu_b}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{W_{ij \rightarrow V k}}(\bar{\mu})} S_{ij \rightarrow V k}(\vec{x}_T, \mu_b) B_{i/N_1}(\xi_1, x_T, \mu_b) B_{j/N_2}(\xi_2, x_T, \mu_b)$$

$$\times e^{\int_{\mu_t}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{U_k}(\bar{\mu}) + \int_{\mu_j}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma^{J_k}(\bar{\mu})} U_{\text{NG}}^k(\mu_t, \mu_j),$$

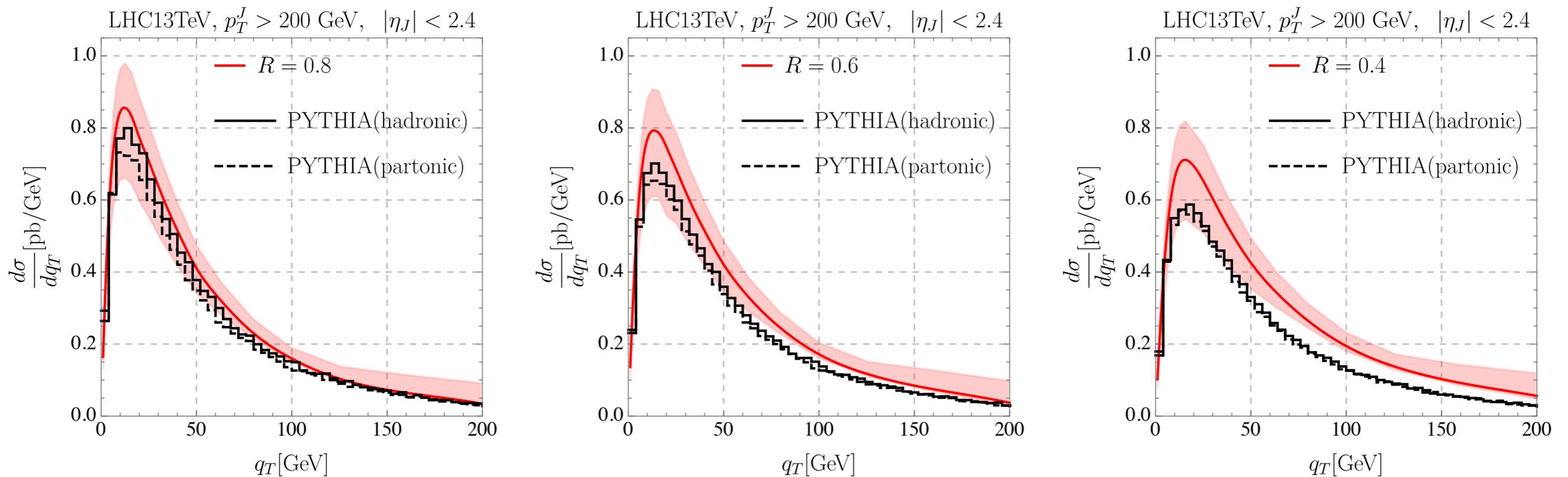
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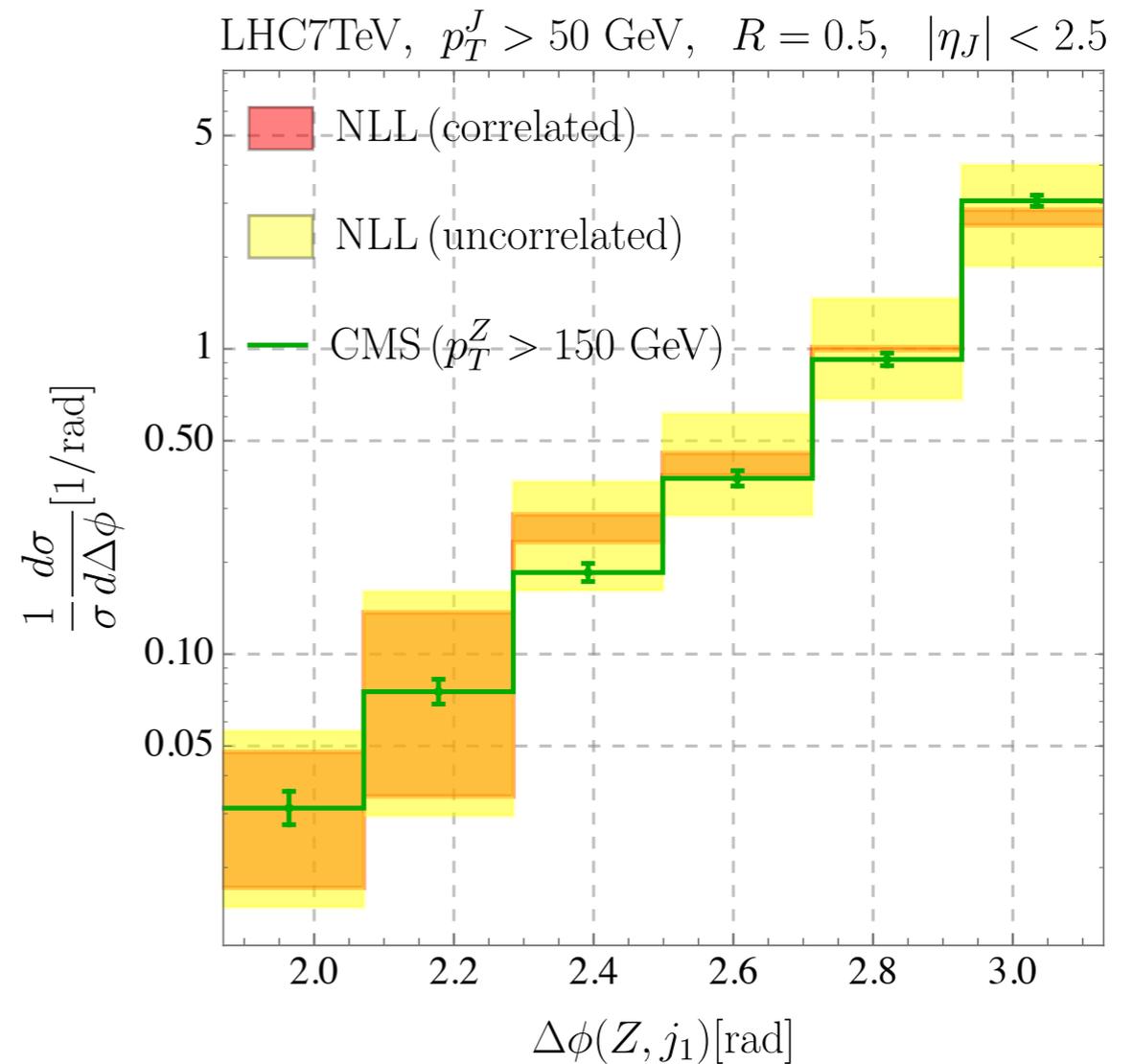
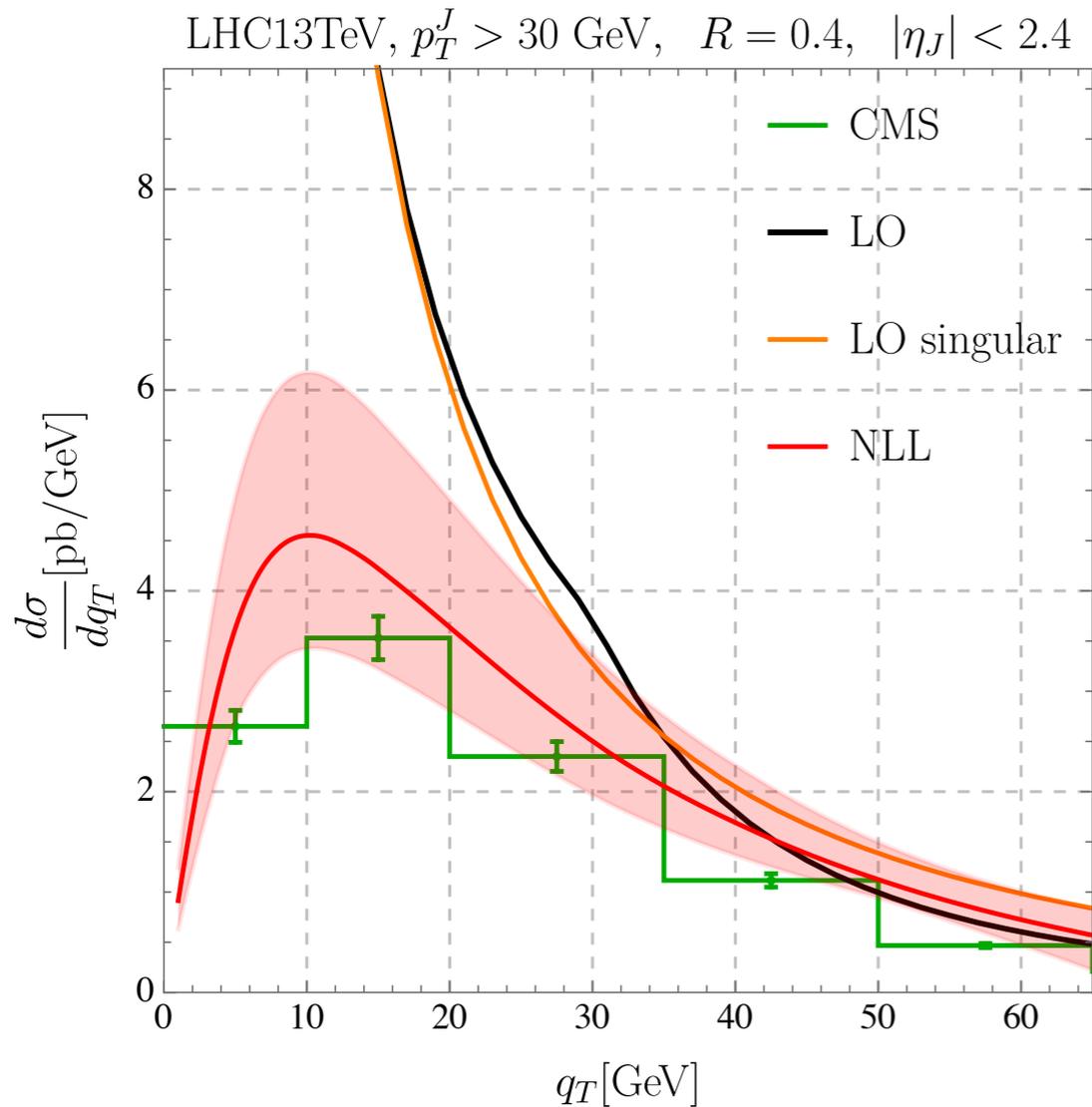
$$U_{\text{NG}}(\mu_t, \mu_j) \xrightarrow{\text{NLL}} \sum_{m \geq 1}^{\infty} \langle U_{1m}(\{\underline{n}\}, \mu_t, \mu_j) \hat{\otimes} \mathbf{1} \rangle$$

Numerical results



- Resummation is “consistent” with pythia (discrepancy in normalization)
- At NLL theoretical uncertainties mainly come from scale variations
- Non-perturbative corrections mainly comes from MPI contributions

Numerical results



- **NLL resummation is consistent with the LHC data (q_T & $\Delta\Phi$)**
- q_T distribution can be a clean probe of *factorization violation* (Collins & Qiu '07, Rogers & Mulders '10,
- **NLL result has 20-30% scale uncertainties. Higher-order resummation is necessary**

NLL'(LR') resummation of the jet mass

Balsiger, Becher & DYS (JHEP04(2019)020)

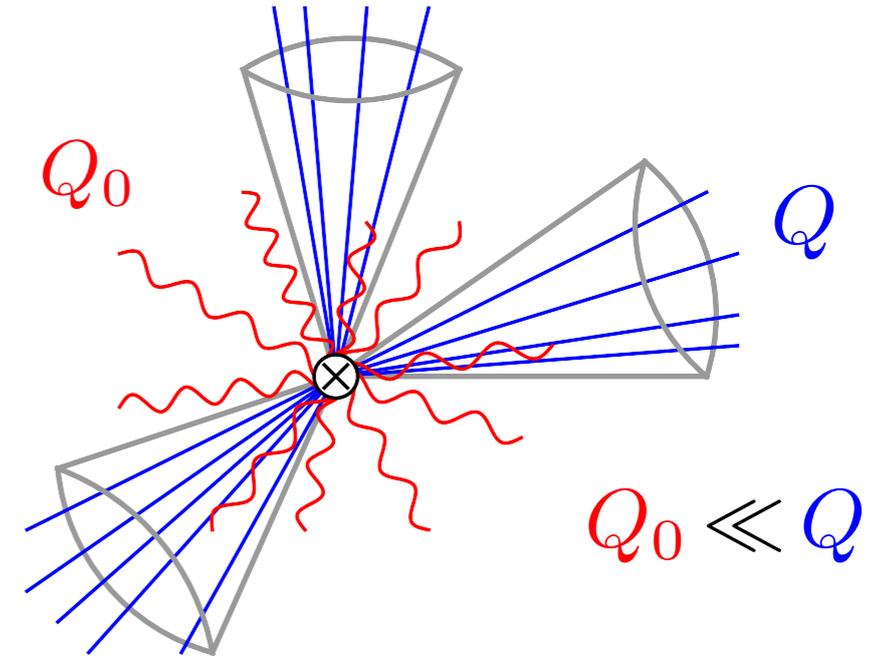
Factorization in SCET

(Becher, Neubert, Rothen & DYS '15)

- Energy flow outside jet (single log observable)

$$d\sigma(Q, Q_0) = \sum_{m=k}^{\infty} \langle \mathcal{H}_m(\{\underline{n}\}, Q, \mu) \otimes \mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) \rangle$$

$$\{\underline{n}\} = \{n_1, n_2, \dots, n_m\}$$



- Soft function:

$$\mathcal{S}_m(\{\underline{n}\}, Q_0, \mu) = \int_{X_s} \langle 0 | \mathcal{S}_1^\dagger(n_1) \dots \mathcal{S}_m^\dagger(n_m) | X_s \rangle \langle X_s | \mathcal{S}_1(n_1) \dots \mathcal{S}_m(n_m) | 0 \rangle \theta(Q_0 - E_{\text{out}})$$

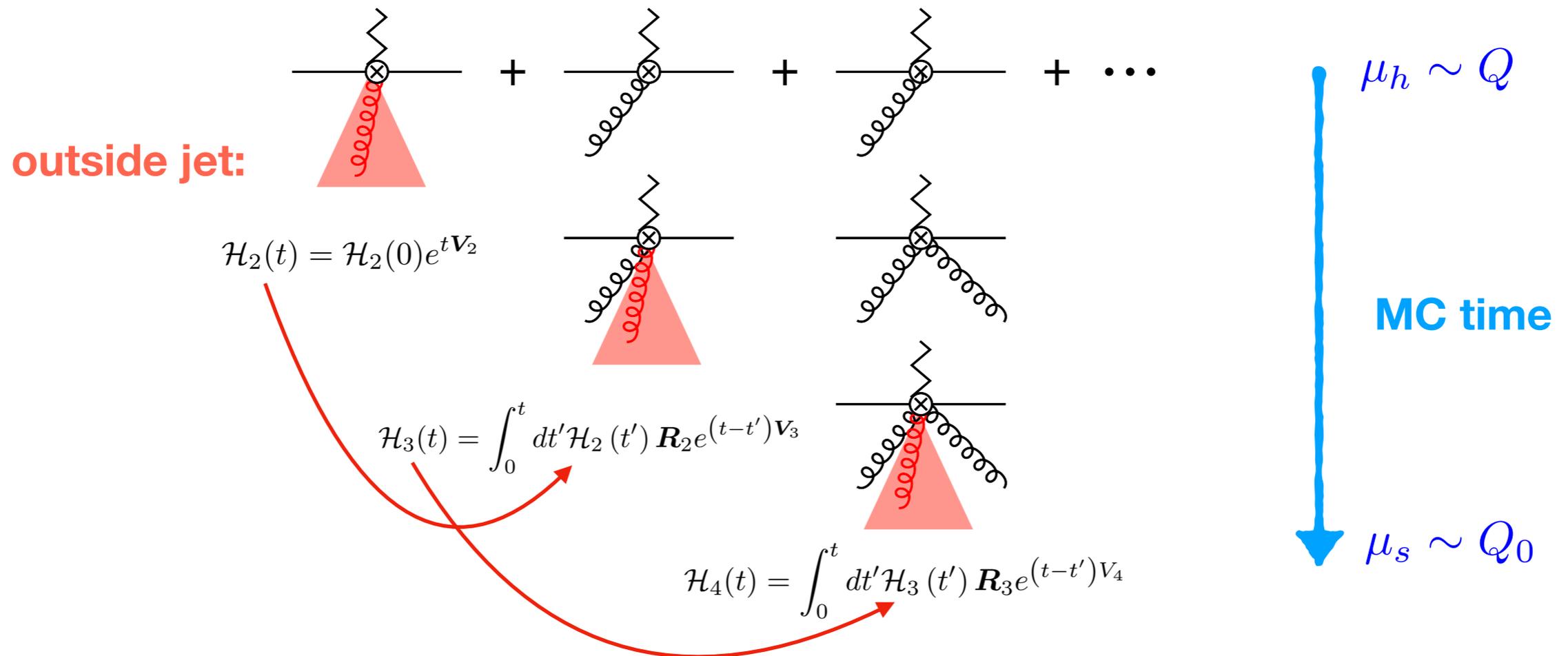
- Hard function: integrating over the energies of the hard particles, while keeping their direction fixed

$$\mathcal{H}_m(\{\underline{n}\}, Q, \mu) = \frac{1}{2Q^2} \sum_{\text{spins}} \prod_{i=1}^m \int \frac{dE_i E_i^{d-3}}{(2\pi)^{d-2}} |\mathcal{M}_m(\{\underline{p}\})\rangle \langle \mathcal{M}_m(\{\underline{p}\})| (2\pi)^d \delta\left(Q - \sum_{i=1}^m E_i\right) \delta^{(d-1)}(\vec{p}_{\text{tot}}) \Theta_{\text{in}}(\{\underline{p}\})$$

- \otimes indicates integration over the direction of the energetic partons
- $\langle \dots \rangle$ taking the color trace

A brief review of the leading log resummation

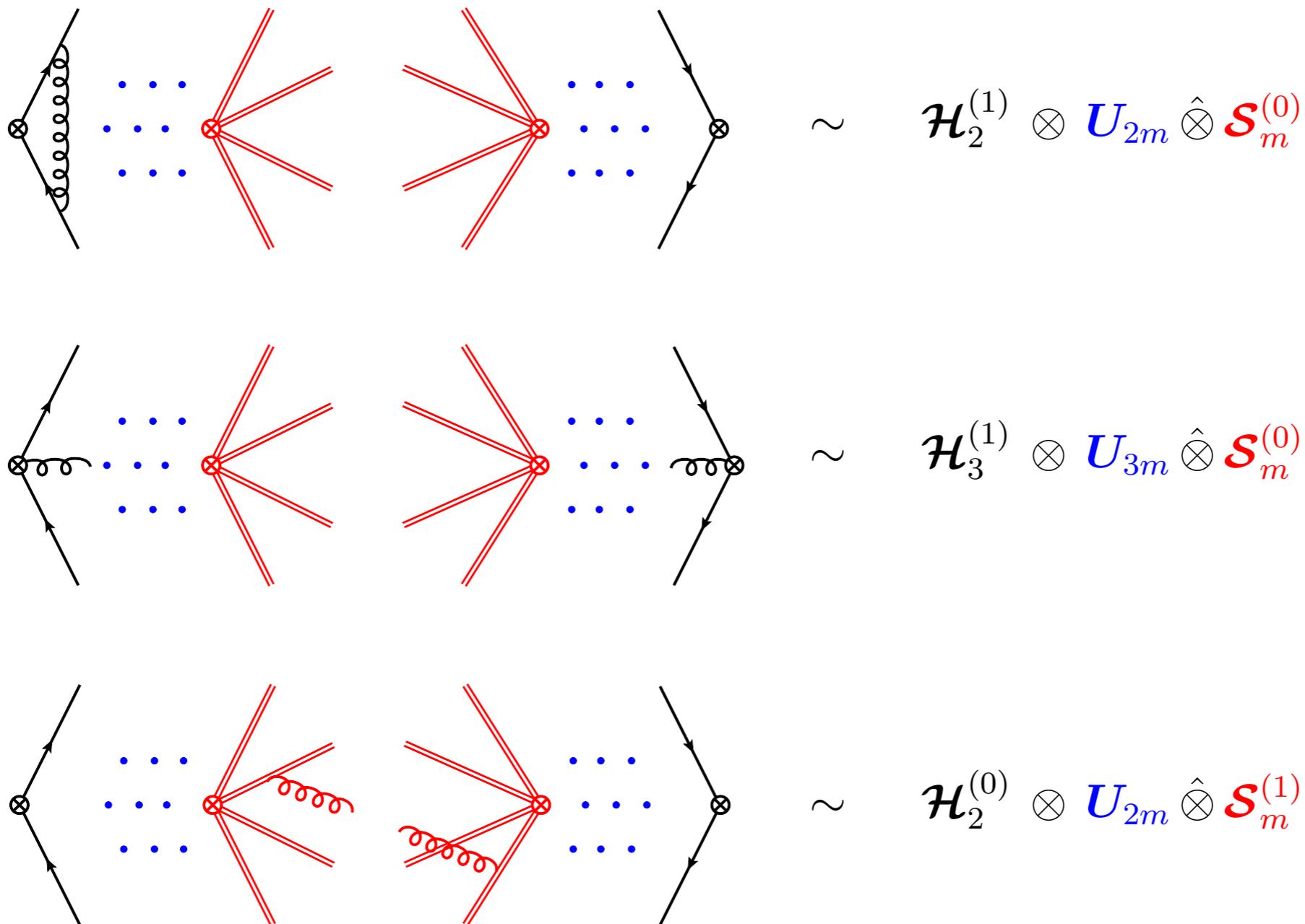
$$\sigma^{\text{LL}}(Q, Q_0) = \sum_{m=2}^{\infty} \langle \mathcal{H}_2(\{n_1, n_2\}, Q, \mu_h) \otimes \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle$$



1. start at $t = 0$ with initial event $E = \{n_1, n_2\}$ and weight $w = 1$
2. increase Δt according to $V_E \exp(-V_E t)$
3. choose a dipole $\{n_i, n_j\}$ with probability V_{ij}/V_E
4. generate a new vector, if it's inside the jet, add it to the event, and return step 2. Otherwise, stop and go to step 1

LL'(LR') resummation

$$\mu_h \sim Q \longrightarrow \mu_s \sim Q_0$$



Soft corrections

One-loop soft function corrections:

$$\sum_{m=2}^{\infty} \langle \mathcal{H}_m(t) \hat{\otimes} \mathcal{S}_m^{(1)} \rangle = \langle \mathcal{H}_2(t) \mathcal{S}_2^{(1)} + \int \frac{d\Omega_1}{4\pi} \mathcal{H}_3(t) \mathcal{S}_3^{(1)} + \int \frac{d\Omega_1}{4\pi} \int \frac{d\Omega_2}{4\pi} \mathcal{H}_4(t) \mathcal{S}_4^{(1)} + \dots \rangle$$

Definition:

$$\frac{\alpha_s}{4\pi} \mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \epsilon) = -g_s^2 \tilde{\mu}^{2\epsilon} \sum_{(ij)} \mathbf{T}_{i,L} \cdot \mathbf{T}_{j,R} \int \frac{d^d k}{(2\pi)^{d-1}} \delta(k^2) \theta(k^0) \frac{n_i \cdot n_j}{n_i \cdot k n_j \cdot k} \Theta_{\text{out}}(n_k) \theta(Q_0 - E_k)$$

$$\mathbf{T}_{i,L} \cdot \mathbf{T}_{j,R} \rightarrow -\frac{N_c}{2} \delta_{i,j\pm 1}$$



c.m. frame of parent dipole (n_i, n_j)

$$\mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \mu) = \frac{N_c}{2} \sum_{i,j=1}^m \delta_{i,j\pm 1} \int d\hat{y} \int_0^{2\pi} \frac{d\hat{\phi}}{2\pi} \left[-4 \ln \frac{\mu}{Q_0} + 4 \ln \frac{2 |\sin \hat{\phi}|}{f_{ij}(\hat{\phi}, \hat{y})} \right] \Theta_{\text{out}}^{\text{lab}}(\hat{y}, \hat{\phi})$$

weight factor

Soft corrections

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1. start at $t = 0$ with initial event $E = \{n_1, n_2\}$ and weight $w = 1$
2. increase Δt according to $V_E \exp(-V_E t)$
3. choose a dipole $\{n_i, n_j\}$ with probability V_{ij}/V_E
4. generate a new vector, if it's inside the jet, add it to the event, and return step 2. Otherwise, add the **weight factor** at time t , go to step 1

$$\mathcal{S}_m^{(1)}(\{\underline{n}\}, Q_0, \mu) = \frac{N_c}{2} \sum_{i,j=1}^m \delta_{i,j\pm 1} \int d\hat{y} \int_0^{2\pi} \frac{d\hat{\phi}}{2\pi} \left[-4 \ln \frac{\mu}{Q_0} + 4 \ln \frac{2 |\sin \hat{\phi}|}{f_{ij}(\hat{\phi}, \hat{y})} \right] \Theta_{\text{out}}^{\text{lab}}(\hat{y}, \hat{\phi})$$

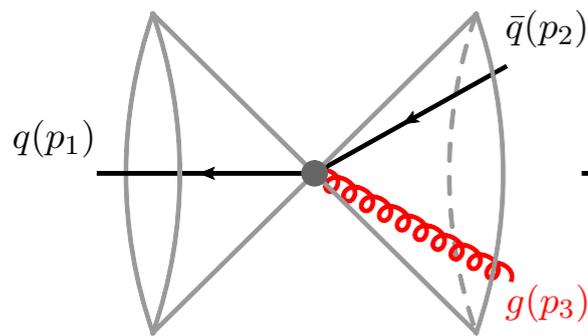
weight factor

Hard corrections

Virtual corrections to \mathcal{H}_2 give trivial refactor $H_2(Q^2, \mu) = 1 + \frac{\alpha_s}{4\pi} C_F \left[-8 \ln^2 \frac{\mu}{Q} - 12 \ln \frac{\mu}{Q} - 16 + \frac{7}{3} \pi^2 \right]$

$$\sum_{m=2}^{\infty} \langle \mathcal{H}_2(\{n_1, n_2\}, Q, \mu_h) \otimes \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle = \sigma_0 H_2(Q, \mu_h) \langle \mathbf{U}_{2m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle$$

\mathcal{H}_3 is a distribution of two angles: $\mathcal{H}_3 \left[\delta(u), \delta(v), \left(\frac{1}{v} \right)_+, \left(\frac{\log v}{v} \right)_+, \dots \right]$

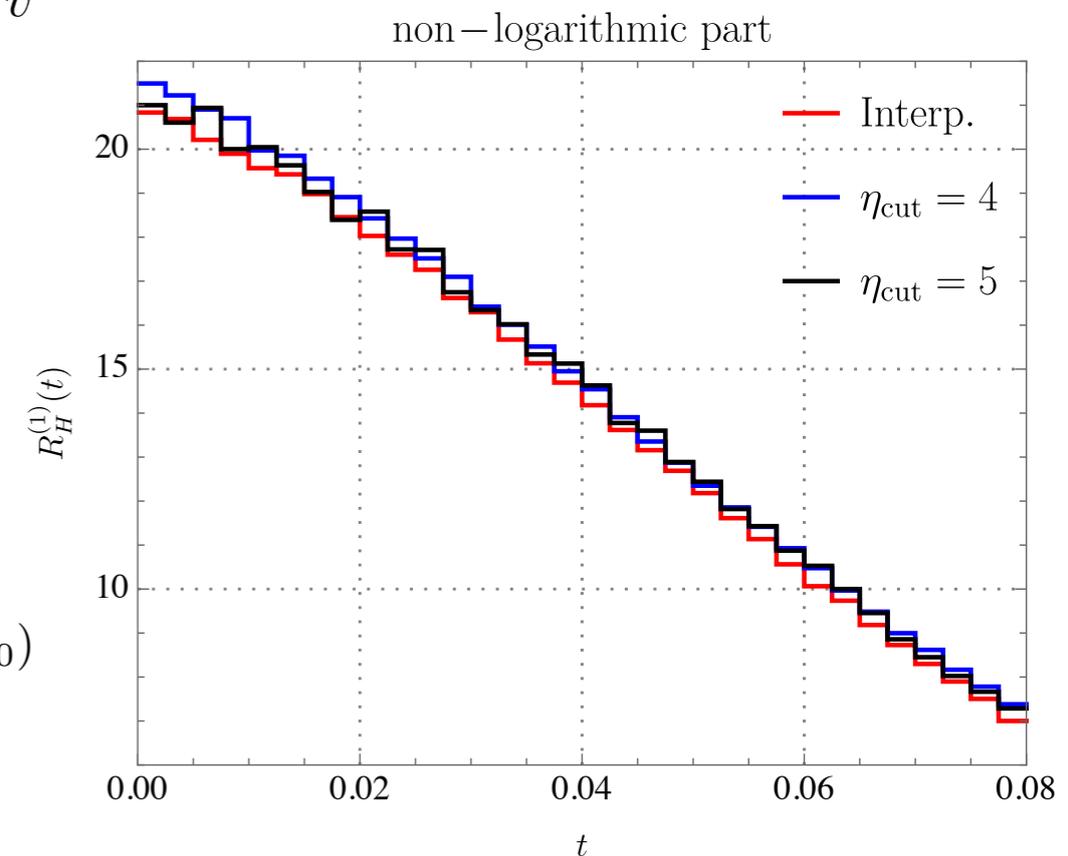


$$\hat{\theta}_2 \equiv \tan \frac{\theta_2}{2} = uv, \quad \hat{\theta}_3 \equiv \tan \frac{\theta_3}{2} = v$$

$$\int_0^1 du \int_0^1 dv \langle \mathcal{H}_3^{(1)}(u, v, Q, \mu_h) \sum_{m=3}^{\infty} \mathbf{U}_{3m}(\{\underline{n}\}, \mu_s, \mu_h) \hat{\otimes} \mathbf{1} \rangle$$

Slicing method:

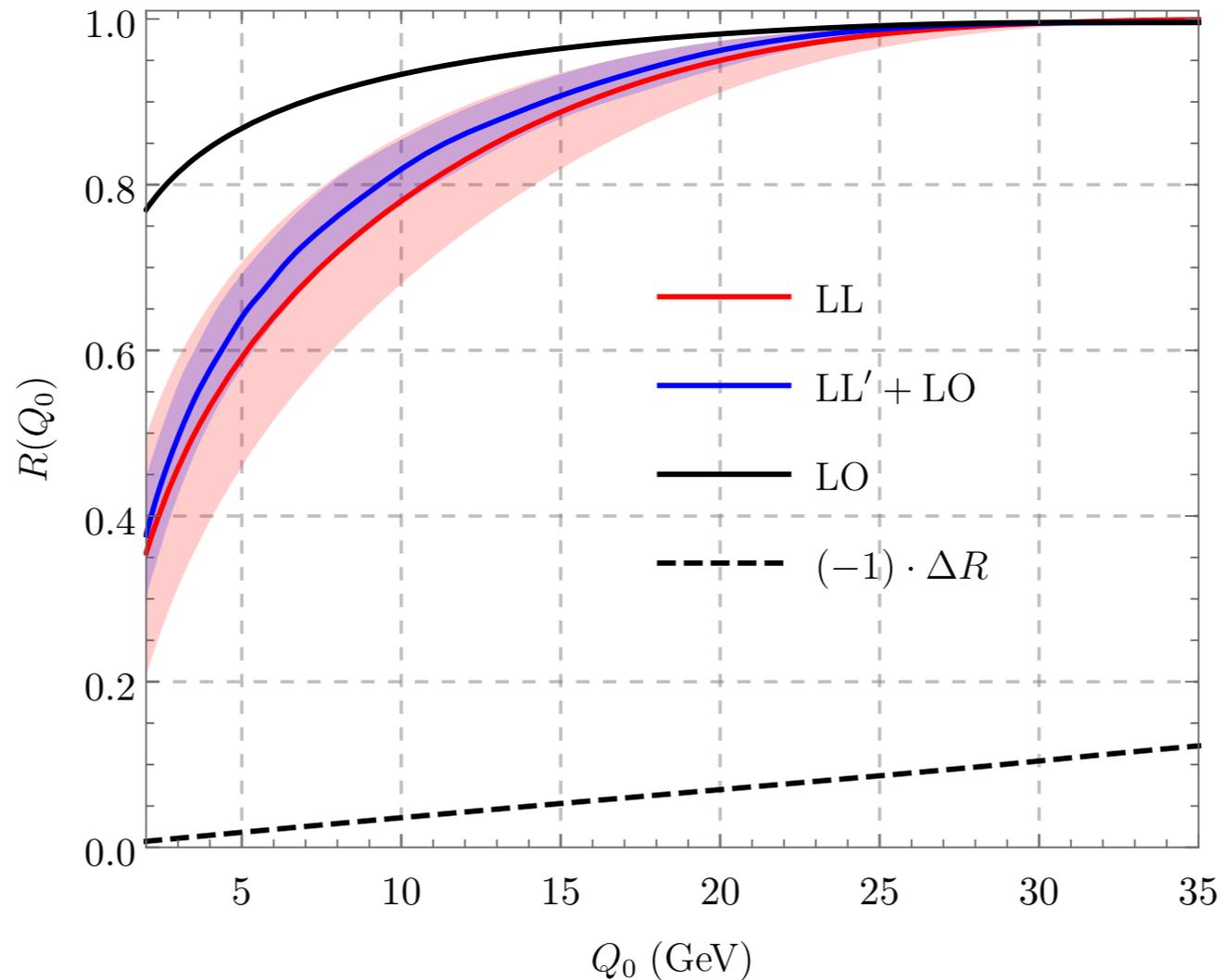
$$\int_0^1 dv \left[\frac{1}{v} \right]_+ \hat{\mathcal{S}}_3(v) = \int_0^1 \frac{dv}{v} \left[\hat{\mathcal{S}}_3(v) - \hat{\mathcal{S}}_2 \right] = \int_{v_0}^1 \frac{dv}{v} \hat{\mathcal{S}}_3(v) + \ln v_0 \hat{\mathcal{S}}_2 + \mathcal{O}(v_0)$$



LL'(LR') + LO resummation

(Balsiger, Becher, DYS,'19)

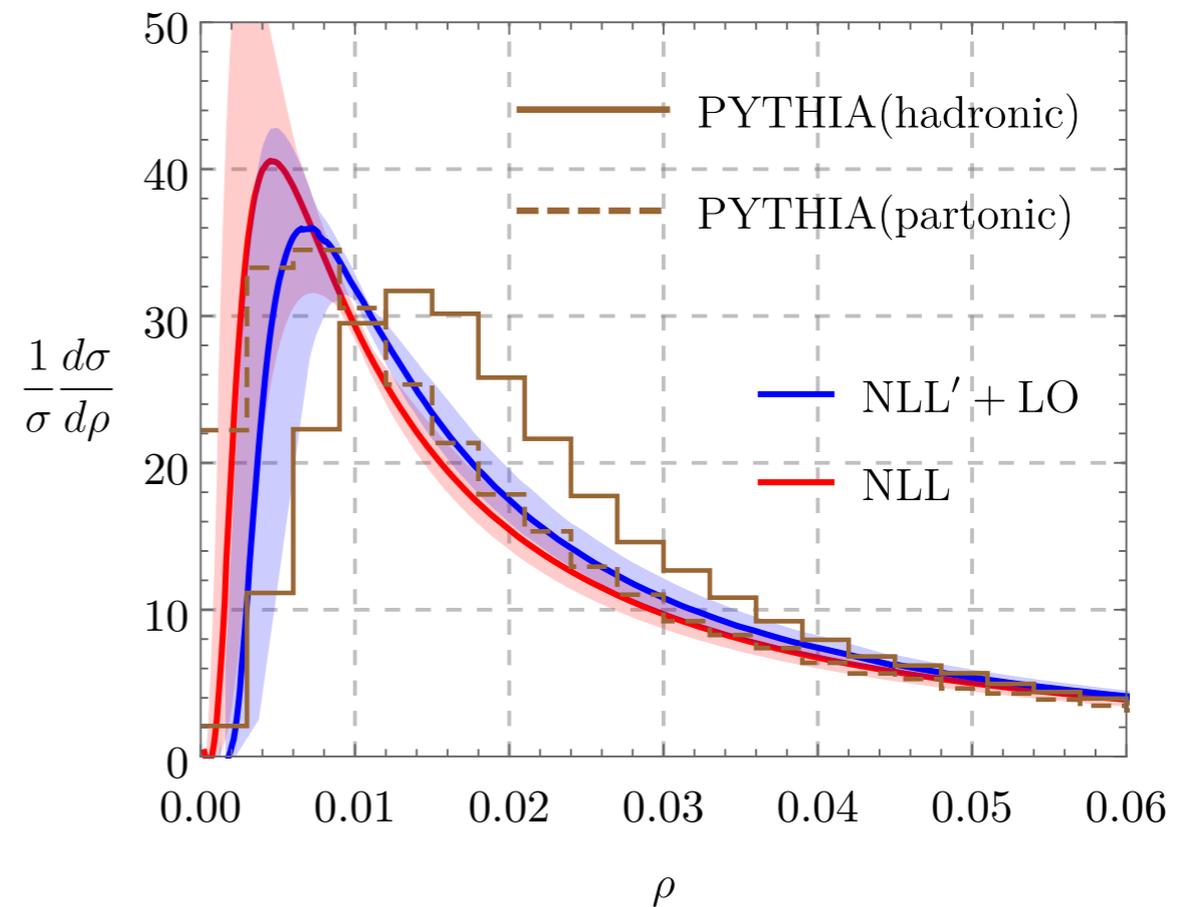
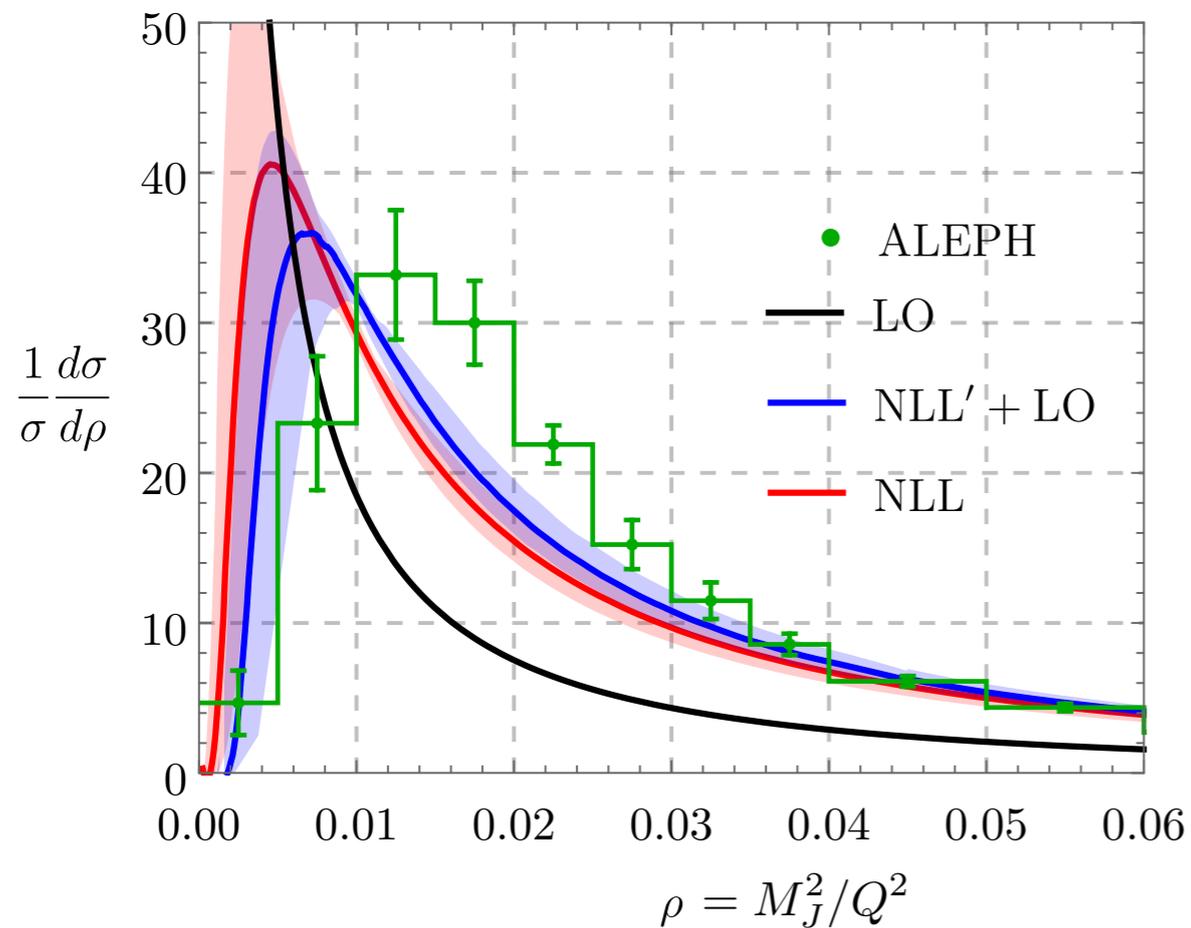
$\sigma(Q, Q_0)/\sigma$



- We match LL' to LO and use a profile function to switch off resummation
- No Exp. data

NLL'(LR') resummation for the jet mass

(Balsiger, Becher, DYS,'19)



- **Peak at $\rho \sim 0.006$ corresponds to $\mu_s \sim 0.5\text{GeV}$. Non-perturbative effects are important and shift the peak**
- **Partonic PYTHIA is close to NLL'+LO**

Conclusions

- **Jet radius and q_T joint resummation for boson-jet correlation**
 - including NGLs resummation
 - small N.P. corrections, perturbative predictions are reliable
 - the winner-take-all axis? glauber modes? factorization violation effects?
- **First results for non-global observables which go beyond the (N)LL (LR) accuracy**
 - full one-loop corrections to matching coefficients
 - implemented in Monte-Carlo framework
 - higher order corrections improve results significantly

Thank you!