Searching for the critical point of strongly interacting matter in nucleus-nucleus collisions at CERN SPS

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1. Critical point search strategies
2. The NA61/SHINE experiment
3. Strongly intensive quantities
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Exploring the phase diagram with heavy-ion collisions

Objective: detection of the Critical End Point (CP) of strongly interactive matter in the phase diagram

Hill of fluctuations expected around the CP;

2nd order phase transition → scale invariance → power-law form of correlation function for large distances ⇔ small momentum transfer $\Delta k$

Look for observables tailored for the CP; scan phase diagram by varying energy and size of collision system ⇒

Change freeze-out conditions $(T, \mu_B)$

Quark-Gluon Plasma

hadronic matter

CP (2nd order)
cross-over

$T_c$

$\mu_c$

$\mu_B$

$T$

Collision energy

System size

$\mu_B$

Critical Point (2nd order)

Hill of fluctuations expected around the CP;

2nd order phase transition → scale invariance → power-law form of correlation function for large distances ⇔ small momentum transfer $\Delta k$

[Asakawa, Yazaki NPA 504 (1989) 668; Barducci et al., PLB 231 (1989) 463]

[M. Gazdzicki et al., APPB 47 (2016) 1201]
The NA61/SHINE experiment

- Fixed-target, high-energy collision experiment at CERN SPS;
- Large variety of beams / hydrogen & nuclear targets;
- Large acceptance; good hadron ID; almost complete coverage in the projectile hemisphere; good momentum resolution;
- Particle ID through $dE/dx$ & TOF in the TPCs;
- Centrality determination through energy deposit of projectile spectators;
- **Goals:** neutrino – cosmic ray – strong interactions programme
  - Study of strong + EM effects in a variety of nucleus-nucleus, proton-proton & proton-nucleus collisions;
  - Search for **critical point** signatures;

![Diagram of colliding nuclei with beam momenta and years of data collection]

- $p+p$
- $p+Pb$
- $Be+Be$
- $Ar+Sc$
- $Xe+La$
- $Pb+Pb$

- 2009/10/11
- 2011/12/13
- 2012/14/16/17
- 2015
- 2017
- 2018
- 2021-2024

Particle ID through $dE/dx$ & TOF in the TPCs; Centrality determination through energy deposit of projectile spectators;
Intensive vs strongly intensive quantities

1. Extensive quantities ⇒ proportional to $W$ (WNM) or $V$
   - $\langle N \rangle$, $\sigma^2$, $S\sigma^3$, $\kappa\sigma^4$ ...

2. Intensive quantities ⇒ ratios of extensive, e.g.
   - scaled variance $\omega[N] = \frac{\sigma^2[N]}{\langle N \rangle}$
     - still depend on $P(W)$

3. Strongly intensive quantities ⇒ Independent of $P(W)$, e.g.
   - $\langle K \rangle/\langle \pi \rangle$
   - $\Sigma[P_T, N] = \frac{1}{\omega[P_T]\langle N \rangle} [\langle N \rangle\omega[P_T] + \langle P_T \rangle\omega[N] - 2(\langle P_T N \rangle - \langle P_T \rangle\langle N \rangle)]$
   - $P_T \equiv \sum_{i=1}^{N} p_{Ti}$

[Gazdzicki, Gorenstein, PRC 84 (2011) 014904];
[Gazdzicki, Gorenstein, Mackowiak-Pawlowska, PRC 88 (2013) 024907]
(Strongly) intensive measures in NA61/SHINE

No prominent structures that could be related to the critical point are observed so far...
Self-similar density fluctuations near the CP

Critical Point \rightarrow \text{Universality Class & space dimensionality}

\text{divergent correlation length } \xi \rightarrow \infty

determines

Critical exponents (power-law) \rightarrow \text{Correlations in configuration space}

dictate

Correlations in momentum space

\sigma\text{-field:}

\langle n_\sigma(k)n_\sigma(k') \rangle \sim |k-k'|^{-4/3},

n_\sigma(k) = \sigma^2(k)

3D-Ising, infinite size system

Baryons:

\langle n_B(k)n_B(k') \rangle \sim |k-k'|^{-5/3},

n_B = \text{net baryon density at midrapidity}
Observing power-law fluctuations: Factorial moments

Experimental observation of local, power-law distributed fluctuations ⇒ Intermittency\(^1\)\(^-\)\(^3\) in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions\(^3\))

- Net protons used as proxy for net baryons (same critical fluctuations\(^4\)); finally, protons can be used (dominant contribution) & anti-protons dropped.
- Transverse momentum space is partitioned into \(M^2\) cells
- Calculate second factorial moments \(F_2(M)\) as a function of cell size ⇔ number of cells \(M\):

\[
F_2(M) \equiv \left( \frac{1}{M^2} \sum_{i=1}^{M^2} n_i (n_i - 1) \right) \left( \frac{1}{M^2} \sum_{i=1}^{M^2} n_i \right)^2,
\]

where \(\langle \ldots \rangle\) denotes averaging over events.

\(^3\)F. K. Diakonos, N. G. Antoniou and G. Mavromanolakis, *PoS (CPOD2006) 010, Florence*
Factorial moments – removal of noncritical background

- Non-critical pairs & experimental noise must be subtracted from $F_2(M)$.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.
- Crucial parameter: Ratio $\lambda$ of background to total proton multiplicity.
- For $\lambda \lesssim 1$ (background domination), non-critical background is approximated by (uncorrelated) mixed event moments; Critical Monte Carlo (CMC) then shows we can write:

$$\Delta F_2(M) \simeq \Delta F_2^{(e)}(M) \equiv F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

- For a critical system, $\Delta F_2$ scales with cell size (number of cells, $M$) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}; \quad \varphi_2 : \text{intermittency index}$$

Theoretical prediction for $\varphi_2$

For net baryons (protons):

$$\varphi_{2,cr}^{(p)} = \frac{5}{6} \approx 0.833 \ldots$$

Bootstrap method used to calculate statistical uncertainties

Bootstrap samples of events created by sampling of events with replacement

$\Delta F_2(M)$ calculated for each bootstrap sample; variance of sample values provides statistical error of $\Delta F_2(M)$


Systematic uncertainties arise from:
- Misidentification of protons & detector effects (e.g. acceptance)
- The fact that $F_2(M)$ are correlated for different bin sizes $M$
- Selection of $M$-range to fit for power-law

Bin correlations are partially handled by the bootstrap $\varphi_2$ distribution$^1$, but that is insufficient! The effect of bin correlation has to be investigated through Critical and background Monte Carlo simulation.

Other systematic uncertainties are estimated by varying proton and $M$-range selection

Indication of intermittency effect in $\Delta F_2(M)$ – however, systematic uncertainties are hard to estimate due to strong correlation of points for different $M$
\( \Delta F_2(M) \) – Be+Be, “C”+C, Pb+Pb, Ar+Sc (central)

- Mid-rapidity protons @ 17 GeV

No signal visible in central Be+Be, “C”+C, Ar+Sc and Pb+Pb

$\Delta F_2(M)$ – “Si”+Si, Ar+Sc (mid-central)

- Mid-rapidity protons @ 17 GeV

A deviation of $\Delta F_2(M)$ from zero seems apparent in central “Si”+Si and mid-central Ar+Sc

Ar+Sc 150 $\Delta F_2(M)$ – statistical significance of signal

1. $\Delta F_2(M)$, NA61 Ar+Sc @ 150 GeV/c bootstrap distributions
2. $\Delta F_2(M)$, random background sub-sample distributions

- Contour map of $\Delta F_2(M)$ distributions

1. $\gtrsim 95\%$ of $\Delta F_2(M)$ values above zero in Ar+Sc 150
2. $\gtrsim 95 - 99\%$ of random background $\Delta F_2(M)$ values below Ar+Sc 150 average
Conclusions

- NA61/SHINE pursues a program for the critical point search using a variety of observables;
- Studies of intensive and strongly intensive quantities show no evidence of the “critical hill” predicted for the CP;
- A possible non-zero signal in proton intermittency could be present for “Si”+Si and Ar+Sc collisions, in a well-defined region of the phase diagram;
- **No intermittency signal** is observed for other systems (Be+Be, “C”+C, Pb+Pb);
- Work on intermittency analysis is ongoing:
  - Plans to expand the analysis to other energies for Ar+Sc, as well as different system sizes (e.g. Xe+La);
  - A consistent method of estimating systematic uncertainties of factorial moments is being sought – must take into account bin correlation.
Thank You!
Acknowledgements

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Backup Slides
Subtracting the background from factorial moments

- Experimental data is noisy ⇒ a background of non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.

**Partitioning of pairs into critical/background**

\[
\langle n(n-1) \rangle = \langle n_c(n_c-1) \rangle + \langle n_b(n_b-1) \rangle + 2\langle n_b n_c \rangle
\]

- Critical
- Background
- Cross term

\[
\Delta F_2(M) = F_2^{(d)}(M) - \lambda(M)^2 \cdot F_2^{(b)}(M) - 2 \cdot \lambda(M) \cdot (1 - \lambda(M)) f_{bc}
\]

- Correlator
- Data
- Background
- Ratio

- The cross term can be neglected under certain conditions (non-trivial! Justified by Critical Monte Carlo* simulations)

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC\(^*\) code:
  - Only protons produced
  - One cluster per event, produced by random Lévy walk:
    \[ \bar{d}^{(B,2)}_F = 1/3 \Rightarrow \phi_2 = 5/6 \]
  - Lower / upper bounds of Lévy walks \( p_{\text{min, max}} \) plugged in.
  - Cluster center exponential in \( p_T \), slope adjusted by \( T_c \) parameter.
  - Poissonian proton multiplicity distribution.

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Input parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( p_{\text{min}} ) (MeV)</th>
<th>( p_{\text{max}} ) (MeV)</th>
<th>( \lambda_{\text{Poisson}} )</th>
<th>( T_c ) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.1 → 1</td>
<td>800 → 1200</td>
<td>( \langle p \rangle_{\text{non-empty}} )</td>
<td>163</td>
</tr>
</tbody>
</table>


NA61/SHINE CP task force created to verify and extend these results. Task force is spearheaded by IFJ Krakow group, with important contributions from Athens (NKUA), Warsaw (WUT, NCNR) and Frankfurt (FIAS).

Intermittency analysis process:
- Proton selection via particle energy loss $dE/dx$
- Removal of split tracks – $q_{inv}$ distribution & cut of proton pairs
- Probe $\Delta p_T$ distribution of proton pairs for power-law like behaviour in the limit of small $p_T$ differences
- Calculate factorial moments $F_2(M), \Delta F_2(M)$ for selected protons
- Calculate intermittency index $\phi_2$ (when possible) & estimate its statistical uncertainty

Results were obtained for:
- 0-5%, 5-10% and 10-15% centrality bins
- 80%, 85% and 90% minimum proton purity selections
Proton selection

- Employ $p_{tot}$ region where Bethe-Bloch bands do not overlap
  $(3.98 \text{ GeV/c} \leq p_{tot} \leq 126 \text{ GeV/c})$

- Fit $dE/dx$ distribution with 4-gaussian sum for $\alpha = \pi, K, p, e$ — Bins: $p_{tot}, p_T$

- 30 Bins in $Log_{10}(p_{tot})$: $10^{0.6} \rightarrow 10^{2.1}$ GeV/c

- 20 Bins in $p_T$: $0.0 \rightarrow 2.0$ GeV/c

- Proton purity: probability for a track to be a proton, $P_p = p/(\pi + K + p + e)$

- Additional cut along Bethe-Blochs
  (avoid low-reliability region between $p$ and $K$ curves)
### EPOS – proton $p_T$ statistics

| Centrality | #events | $\langle p \rangle_{|p_T| \leq 1.5 \text{ GeV}, |y_{CM}| \leq 0.75}$ | $\Delta p_{x,y}$ |
|------------|---------|-------------------------------------------------|-----------------|
| Non-empty  |         |                                                 |                 |
| 0-5%       | 293,412 | 3.06 ± 1.60                                     | 0.35 - 0.43     |
| 5-10%      | 252,362 | 2.72 ± 1.45                                     | 0.35 - 0.43     |
| 10-15%     | 274,072 | 2.45 ± 1.33                                     | 0.35 - 0.43     |
| With empty |         |                                                 |                 |
| 0-5%       | 2.89 ± 1.70 |                                                |                 |
| 5-10%      | 2.49 ± 1.58 |                                                |                 |
| 10-15%     | 2.16 ± 1.48 |                                                |                 |

### $^{40}Ar + ^{45}Sc$ NA61 data – proton $p_T$ statistics

| Centrality | #events | $\langle p \rangle_{|p_T| \leq 1.5 \text{ GeV}, |y_{CM}| \leq 0.75}$ | $\Delta p_{x,y}$ |
|------------|---------|-------------------------------------------------|-----------------|
| Non-empty  |         |                                                 |                 |
| 0-5%       | 144,362 | 3.44 ± 1.79                                     | 0.46 - 0.58     |
| 5-10%      | 148,199 | 3.00 ± 1.61                                     | 0.46 - 0.58     |
| 10-15%     | 142,900 | 2.81 ± 1.53                                     | 0.45 - 0.57     |
| With empty |         |                                                 |                 |
| 0-5%       | 3.30 ± 1.89 |                                                |                 |
| 5-10%      | 2.79 ± 1.73 |                                                |                 |
| 10-15%     | 2.58 ± 1.66 |                                                |                 |
$q_{inv}$ proton distributions – NA61/SHINE

Ar+Sc NA61, cent.0-5%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.80%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.0-5%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.85%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.0-5%, pur.90%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.5-10%, pur.90%, $q_{inv}$ Ratio

Ar+Sc NA61, cent.10-15%, pur.90%, $q_{inv}$ Ratio
$\Delta p_T$ proton distributions – NA61/SHINE