Two Loop Electroweak Corrections to Polarized Moller Scattering

in collaboration with
Ayres Freitas
Yong Du, Michael Ramsey-Musolf

Sept 16, 2019

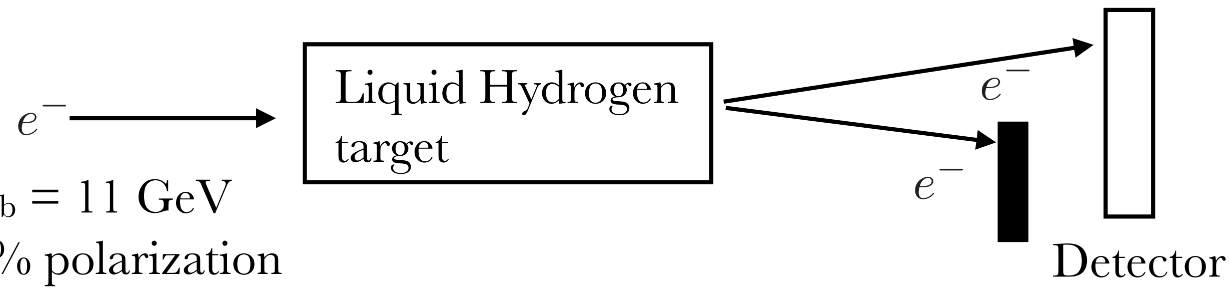
Hiren Patel
hpatel6@ucsc.edu

UC SANTA CRUZ

SCIPP
Santa Cruz Institute for Particle Physics

MOLLER Experiment

- Fixed target polarized Moller scattering experiment
- To measure the electron A_{LR} **(fractional accuracy 2.4%)**
- Goal to precisely determine $Q_W(e)$ and $\sin^2(\theta_W)$ far below the Z pole



E_{lab} = 11 GeV
80% polarization

5 to 17 mrad scattering angle

$Q^2 \sim (0.05\ \text{GeV})^2$

Definition:

$$A_{LR} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

Difference in measured rates for incident electrons of different polarization normalized by total rate

Motivation

1. Deviation from Standard Model prediction signals new physics --

$$\Delta\mathcal{L}_{\rm BSM} = \frac{c^2_{\pm,\pm}}{2\Lambda^2}(\bar{e}\gamma_\mu P_\pm e)(\bar{e}\gamma^\mu P_\pm e)$$

⇩

$$\frac{\Delta A_{LR}}{A_{\rm LR}} \sim \frac{c^2_{\pm\pm}}{G_F\Lambda^2}$$

MOLLER **2.4%**

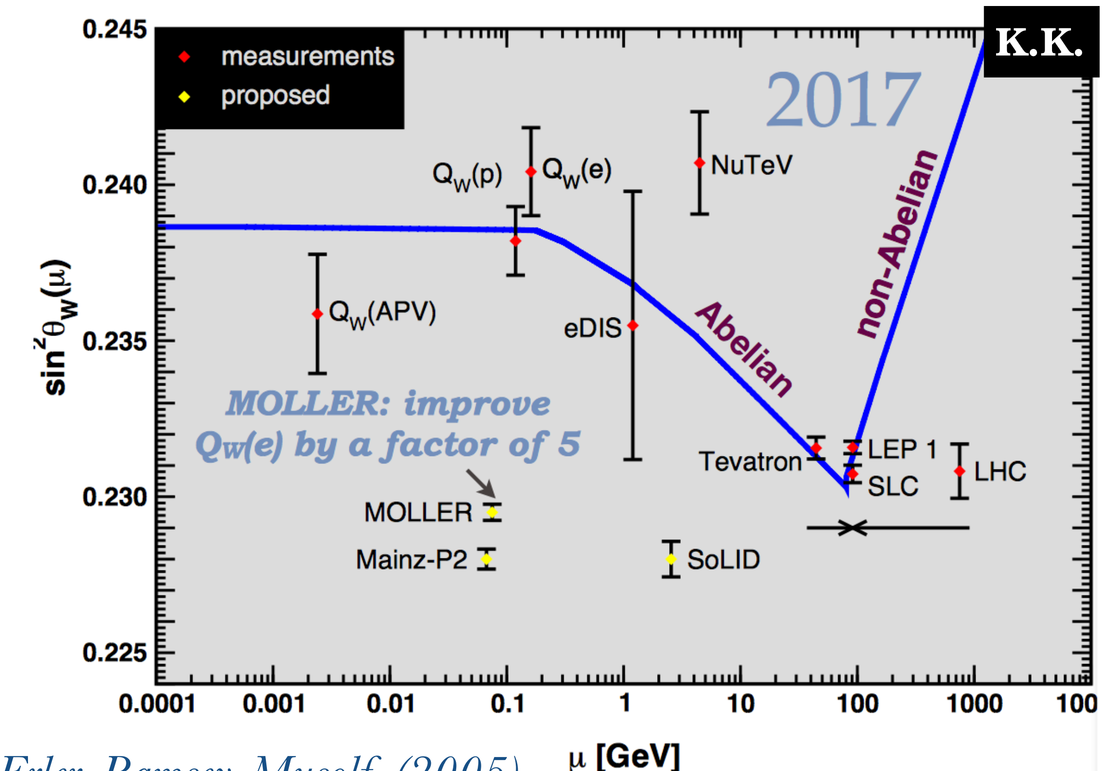
⇩

Roughly $\Lambda \sim \mathcal{O}(10\text{ TeV})$

(*naive* mass reach)

2. To date, Most precise measurement of $\sin^2(\theta_W)$ is at Z pole (LEP/SLC)

MOLLER will provide determination with similar accuracy **$\delta(\sin^2(\theta_W)) = 0.00036$**



Erler, Ramsey-Musolf (2005)

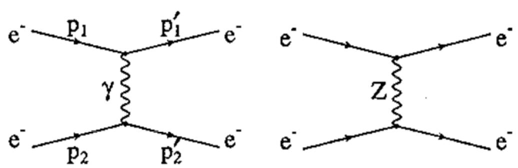
Erler, Ferro-Hernandez (2017)

Theoretical Input

$$A_{LR} = f(\sin(\theta_W), \ldots)$$

In order to extract $\sin^2(\theta_W)$ from A_{LR} with high accuracy, need similarly accurate SM prediction. Long history...

Tree-level: *Derman, Marciano (1979)*



$$A_{LR} = \frac{G_\mu Q^2 f(E_{\rm beam})}{\sqrt{2}\pi\alpha} \underbrace{(1 - 4\sin^2\theta_W)}_{Q_W^e}$$

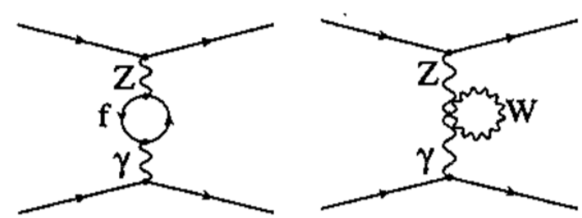
Q_W^e tree-level electron weak charge

Because $\sin^2(\theta_W) \sim 0.23$, tree-level relation 'accidentally' small

One-loop: virtual corrections

Czarnecki, Marciano (1996)
Denner, Pozzorini (1998)
Aleksejevs et. al. (2010), ...

Sizable shift due to corrections to Zγ propagator



$$Q_W^e \longrightarrow 1 - 4\kappa(0)\sin^2\theta_W$$

$\kappa(0) \sim 1.03$

=> **40% reduction in A_{LR}**

c.f. MOLLER **2.4%**

Interpreted as 'running' of $\sin^2(\theta_W)$

Are radiative corrections stable?

Main question essentially keeping the MOLLER experiment from getting the green light.

Rough answer: largest shift to A_{LR} comes from shift in $\sin^2(\theta_W)$.
So, size of higher order corrections to $\sin^2(\theta_W)$ should provide estimate of higher order corrections to A_{LR}.

But to avoid any surprises, a full two-loop evaluation of the asymmetry must be carried out.

Gong to two-loop:

Currently only two groups working on evaluation

Aleksejevs et. al. (2011, 2012, 2015)

*Yong Du, Ayres Freitas, **Hiren Patel**, Michael Ramsey-Musolf (2019 TBA)*
+ Jia Zhou (2020 TBA)

Method

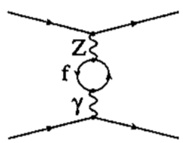
Three challenges in doing calculation:

1. Organization
2. Bookkeeping of $O(10^3)$ diagrams,
3. Evaluation of two-loop integrals.

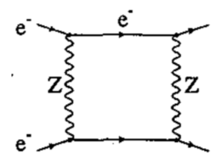
1. Organization

In addition to powers of G_F and e^2, we organize the evaluation by counting number of closed loops in the two loop diagrams.

<u>One loop</u>

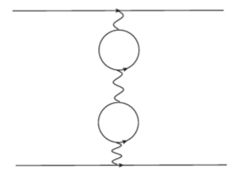


One fermion loop

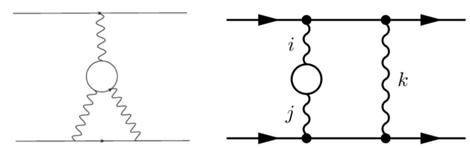


Zero fermion loop (bosonic)

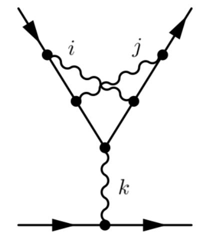
<u>Two loop</u>



Two fermion loops



One fermion loop



Zero fermion loop (bosonic)

Diagrams drawn by Aleksejevs

Reasons:

- Diagrams with closed fermion loops are expected to be 'bigger'
- Diagrams with fewer closed fermion loops are harder to calculate

Schematically,

one-loop | two-loop

$$A_{LR} = \frac{G_F}{\alpha}(\text{tree})\left[1 + \alpha(N_F a_1^{(1)} + a_0^{(1)}) + \alpha^2(N_F^2 a_2^{(2)} + N_F a_1^{(2)} + a_0^{(2)})\right]$$

1.00 -0.39 +0.04

we calculated these first

c.f.
MOLLER
2.4%

SCIPP

2. Bookkeeping

Everything done using computer (*Mathematica*)

Generate diagrams and amplitudes using FeynArts

T. Hahn (2000)

Two independent codes to proceed with calculation

- In house codes developed by A. Freitas
- Modified versions of *Package*-X

Hiren Patel (2016)

<u>Regularization</u>:

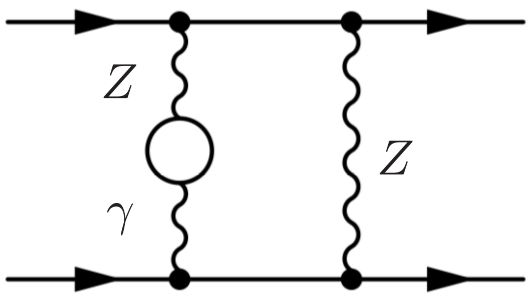
Naive Dim. Reg. for UV divergences

Photon/Electron mass for IR divergences

3. Evaluation of Feynman Diagrams

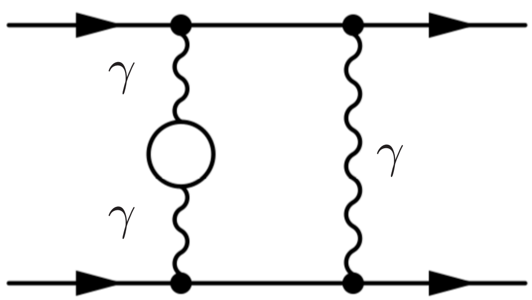
1. Take advantage of separation of scales in problem

$$m_e^2 \ll s, Q^2 \ll M_W^2, M_Z^2, M_H^2, M_t^2$$



Construct an analytic expansion in large heavy masses (expansion by regions)

2. Take advantage of closed fermion loops



Represent fermion sub-loop integral as a dispersion relation

Infrared divergences

Amplitudes and cross section contain soft (photon mass) and collinear (electron mass) singularities

However, soft/collinear factorization ensures that these singularities cancel out in the asymmetry

$$A_{LR} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

Yennie, Frautschi, Suura (1961)
Kinoshita (1962)

We are able to confirm exact analytical cancellation of IR singularities, providing a crosscheck on our calculations

Renormalization

Hybrid renormalization scheme:

On shell renormalization of couplings and masses (subtraction point at Z pole),

MS-bar renormalization for $\sin^2(\theta_W)$.

Preliminary Results

one-loop | two-loop

$$A_{LR} = \frac{G_F}{\alpha}(\text{tree})\left[1 + \alpha(N_F a_1^{(1)} + a_0^{(1)}) + \alpha^2(N_F^2 a_2^{(2)} + N_F a_1^{(2)} + a_0^{(2)})\right]$$

1.00 -0.39 $+0.04$ -0.03 $+0.02$??

(for one representative kinematic point)

c.f.
A_{LR} MOLLER
2.4%

Individually, N_F^2 and N_F pieces at level of accuracy of MOLLER experiment.
But sum largely cancels, and is below sensitivity (good news).

We don't fully understand the cancellation yet (to be analyzed).

These results are preliminary, and we are still performing crosschecks.
Paper with results to come out soon (this year).

Next:
Complete evaluation of purely bosonic contribution

Backup

Expansion by Regions

Smirnov (1999)

$$I = \int (dk) \frac{k^\mu k^\nu}{[k^2 - M_Z^2][(k+p_1)^2 + m_e^2][(k+p_2)^2 - m_e^2][(k+p_3)^2 - m_\gamma^2]}$$

Big...

...small
=>drop

$$\sim \frac{1}{-M_Z^2} \int (dk) \frac{k^\mu k^\nu}{[(k+p_1)^2 + m_e^2][(k+p_2)^2 - m_e^2][(k+p_3)^2 - m_\gamma^2]}$$

Approximation leads to UV divergent integral

$$\sim \frac{-1}{M_Z^2} \Big(\frac{1}{\epsilon} + \ln\big(\frac{\mu^2}{-t}\big) + \ldots \Big)$$

However, loop momentum may also be large, and may also give a contribution at same order in $1/M_Z$.

Expansion by Regions

Smirnov (1999)

$$I = \int (dk) \frac{k^\mu k^\nu}{[k^2 - M_Z^2][(k+p_1)^2 + m_e^2][(k+p_2)^2 - m_e^2][(k+p_3)^2 - m_\gamma^2]}$$

Small Small Small

Big Big Big

$$\sim \int (dk) \frac{k^\mu k^\nu}{[k^2 - M_Z^2][k^2][k^2][k^2]}$$

Approximation leads to IR divergent integral

$$\sim \frac{1}{M_Z^2}\Big(\frac{1}{\epsilon} + \ln\big(\frac{\mu^2}{M_Z^2}\big) + \ldots\Big)$$

But, after adding both approximations together

(small k, previous slide) (large k, this slide)

$$I \sim \frac{-1}{M_Z^2}\Big(\frac{1}{\epsilon} + \ln\big(\frac{\mu^2}{-t}\big) + \ldots\Big) + \frac{1}{M_Z^2}\Big(\frac{1}{\epsilon} + \ln\big(\frac{\mu^2}{M_Z^2}\big) + \ldots\Big) = -\frac{1}{M_Z}\Big(\ln\big(\frac{m_Z^2}{-t}\big) + \ldots\Big)$$

cancel

obtain correct asymptotic approximation