# Inclusive radiative and leptonic $B$ decays in the SM

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3. Status of the perturbative $b \to X_s^p\gamma$ calculations
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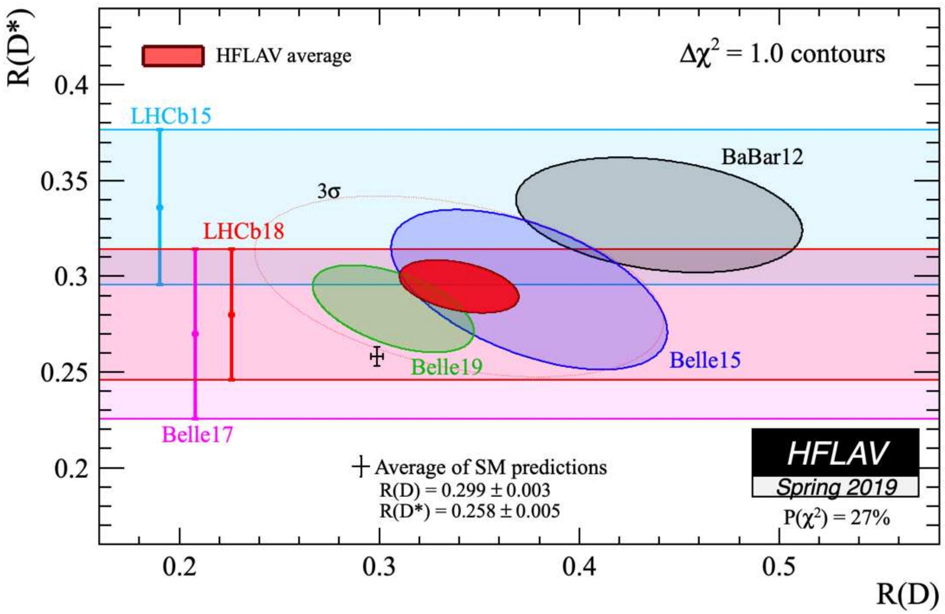
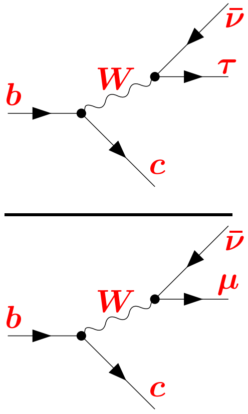
NARODOWE CENTRUM NAUKI

“HARMONIA” project UMO-2015/18/M/ST2/00518

# $R(D)$ and $R(D^*)$ "anomalies" [https://hflav.web.cern.ch] $(3.1\sigma)$



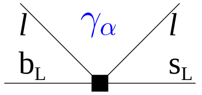


$$R(D^{(*)}) = \mathcal{B}(B \to D^{(*)}\tau\bar{\nu})/\mathcal{B}(B \to D^{(*)}\mu\bar{\nu})$$

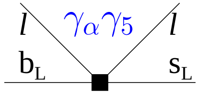
## $b \to s\ell^+\ell^-$ "anomalies" $(> 5\sigma)$

[see, e.g., J. Aebischer *et al.*, arXiv:1903.10434]

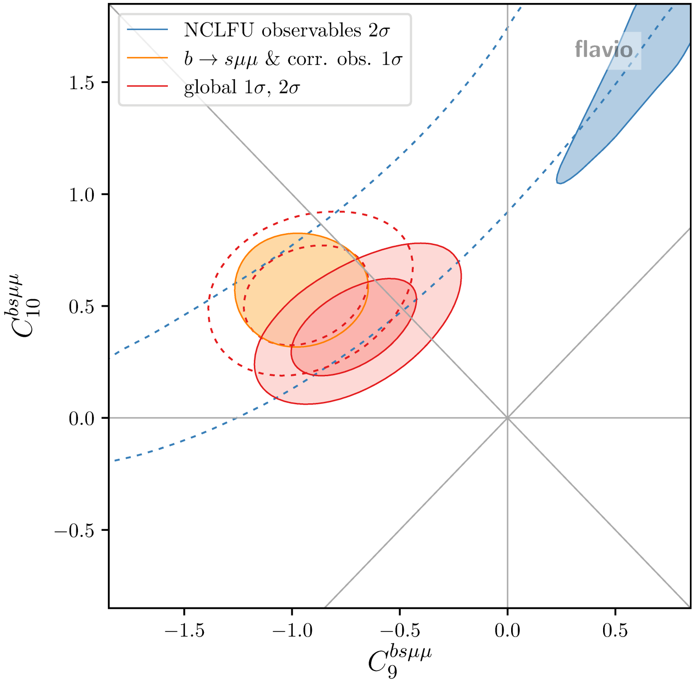
$Q_9^\ell =$



$Q_{10}^\ell =$



$\ell = e$ or $\mu$



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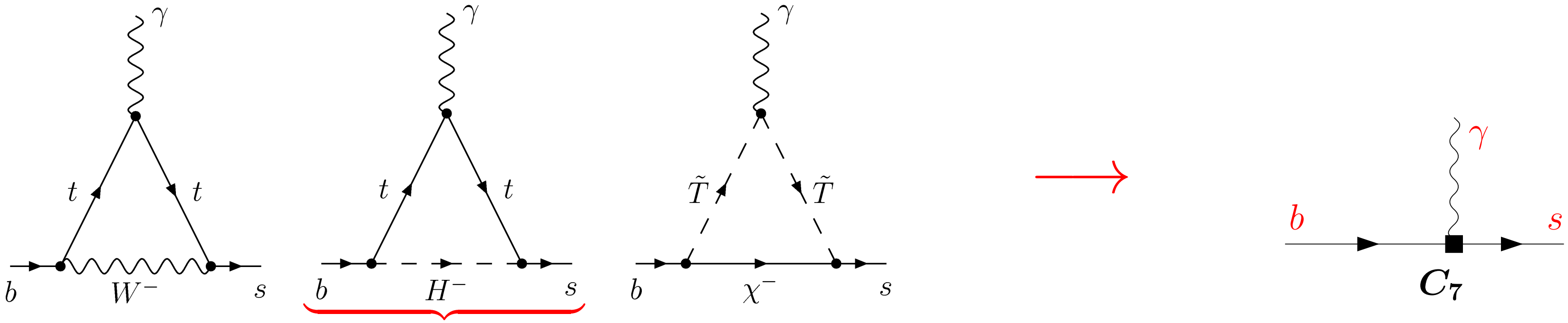

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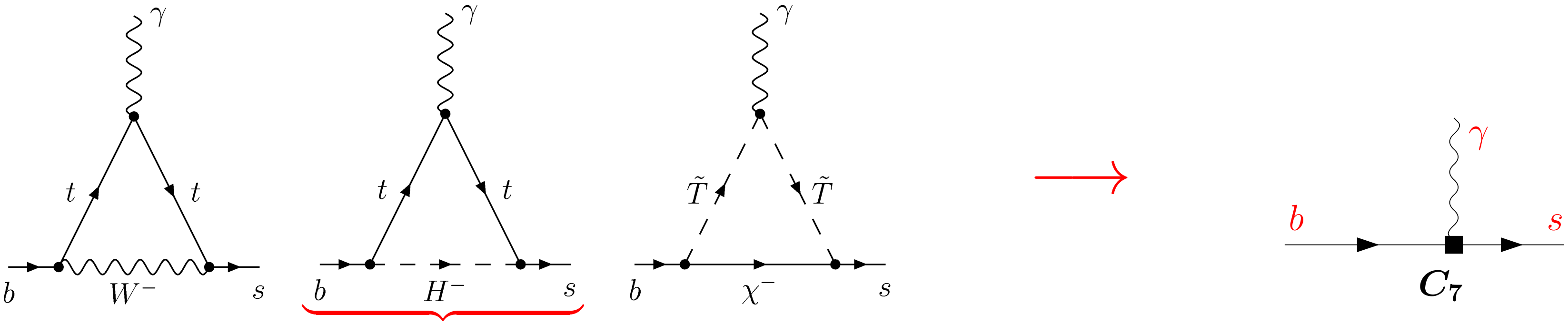

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The inclusive $\bar{B} \to X_s\gamma$ decay rate for $E_\gamma > E_0$ is well approximated by the corresponding perturbative decay rate of the $b$-quark:

$$\Gamma(\bar{B} \to X_s\gamma) \;=\; \Gamma(b \to X_s^p\gamma) \;+ \left( \begin{array}{c} \text{non-perturbative effects} \\ (5 \pm 3)\% \end{array} \right)$$

[G. Buchalla, G. Isidori and S.-J. Rey, Nucl. Phys. B511 (1998) 594]
[M. Benzke, S.J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099]
[A. Gunawardana and G. Paz, arXiv:1908.02812]

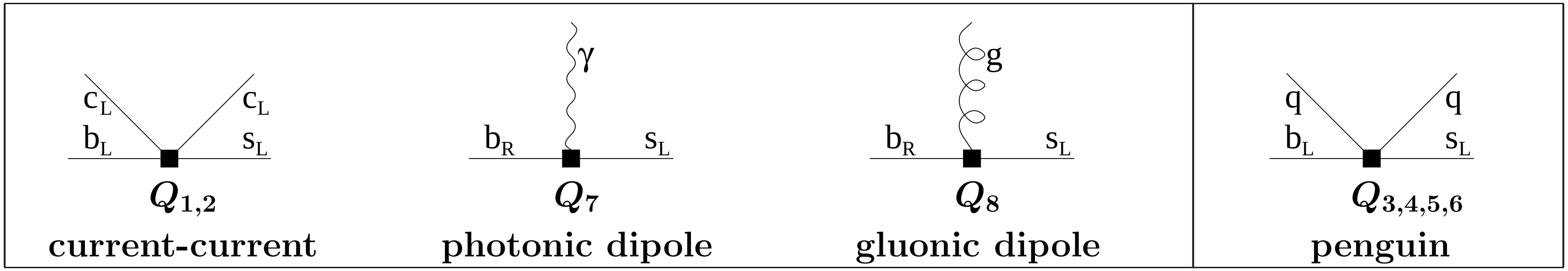
provided $E_0$ is large ($E_0 \sim m_b/2$)
but not too close to the endpoint $(m_b - 2E_0 \gg \Lambda_{\text{QCD}})$.

Conventionally, $E_0 = 1.6\,\text{GeV} \simeq m_b/3$ is chosen.

# The effective weak interaction Lagrangian for $\bar{B} \to X_s\gamma$

$$L_{\text{weak}} \sim \sum_i C_i Q_i$$

Eight operators $Q_i$ matter for $\mathcal{B}_{s\gamma}^{\text{SM}}$ when the NLO EW and/or CKM-suppressed effects are neglected:



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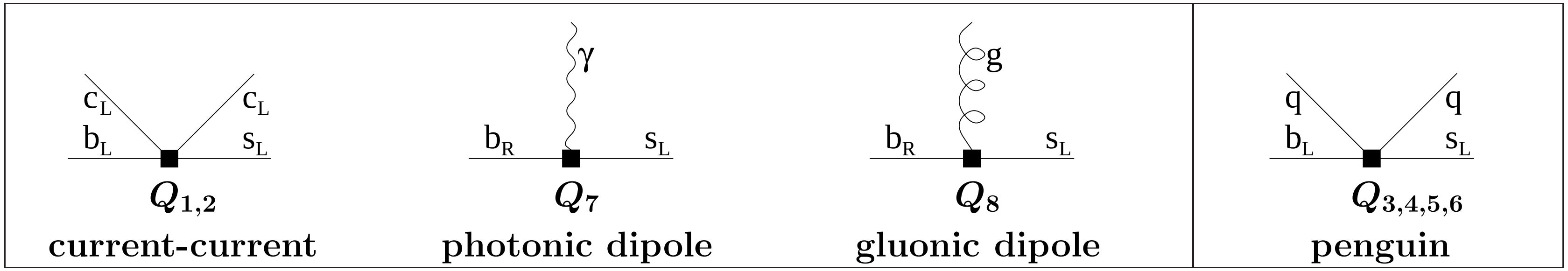

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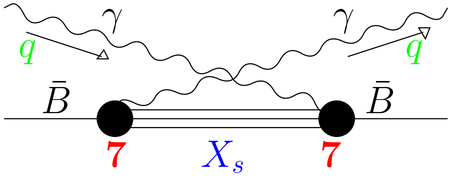
$$\Gamma(\bar{B} \to X_s\gamma)_{E_\gamma > E_0} = |C_7(\mu_b)|^2\, \Gamma_{77}(E_0) + (\text{other}) \qquad (\mu_b \sim m_b/2)$$

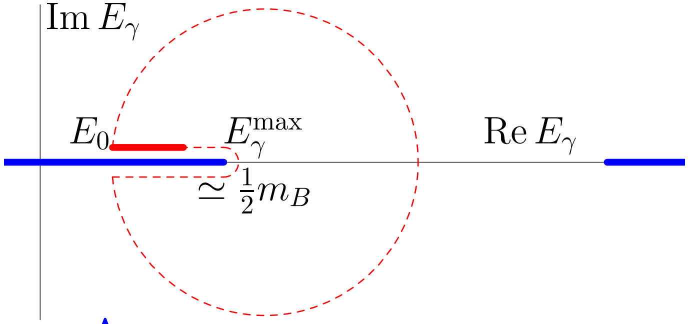
Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{\ \ \} \equiv \text{Im}\, A$$



J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

Integrating the amplitude $A$ over $E_\gamma$:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\rm QCD}}{m_b}$ and $\alpha_s$ that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3},\ldots;\ \frac{\alpha_s\mu_\pi^2}{(m_b-2E_0)^2},\ \frac{\alpha_s\mu_G^2}{m_b(m_b-2E_0)};\ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\rm QCD})$ are extracted from the semileptonic $\bar{B} \to X_c e\bar{\nu}$ spectra and the $B$–$B^\star$ mass difference.

**For operators other than $Q_7$, we encounter $\mathcal{O}\left(\frac{\Lambda}{m_b}\right)$ contributions from resolved photons (created away from the $b$-quark annihilation vertex):**

S.J. Lee, M. Neubert, G. Paz, PRD 75 (2007) 114005, hep-ph/0609224,
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Relative contributions to the branching ratio $\mathcal{B}_{s\gamma}^{\rm SM}$ for $E_\gamma > E_0 = 1.6\,\text{GeV}$:

| interference | ranges | | "TH $1\sigma$" | |
|---|---|---|---|---|
| | 2010 | 2019 | 2010 | 2019 |
| $Q_7$-$Q_8$ | $[-2.8, -0.3]\%$ | $[-0.6, 0.9]\%$ | $(-1.55 \pm 1.25)\%$ | $(0.16 \pm 0.74)\%$ |
| $Q_8$-$Q_8$ | $[-0.3, 1.9]\%$ | no change | $(0.80 \pm 1.10)\%$ | no change |
| $[Q_7$-$Q_{1,2}]^\star$ | $[-1.7, 4.0]\%$ | $[-0.3, 1.6]\%$ | $(1.15 \pm 2.85)\%$ | $(0.65 \pm 0.95)\%$ |
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⇐ Belle $\Delta_{0-}$
arXiv:1807.04236v4

⇐ arXiv:1908.02812

$^\star$ excluding the leading $\mathcal{O}\left(\frac{\mu_G^2}{m_c^2}\right)$ contribution $(\sim +3.2\%)$ [M.B. Voloshin, hep-ph/9612483], (...),
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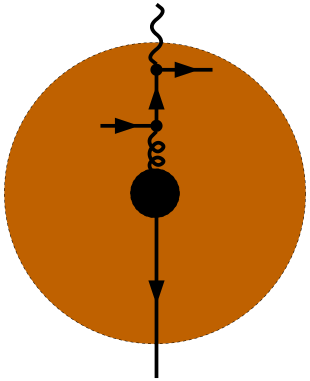
In the 2015 phenomenological update [arXiv:1503.01789, arXiv:1503.01791], $(0 \pm 5\%)$ of $\mathcal{B}_{s\gamma}^{\rm SM}$ was used, and combined in quadrature with other uncertainties: parametric (±2%), higher-order (±3%), and $m_c$-interpolation (±3%). The current experimental accuracy is ±4.5% [HFLAV].

# The resolved photon contribution to the $Q_7$-$Q_8$ interference.



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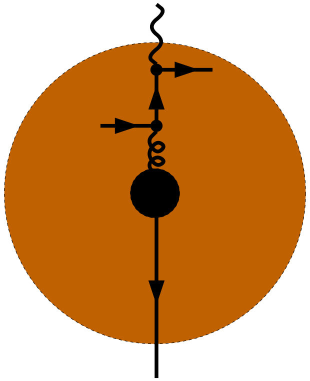

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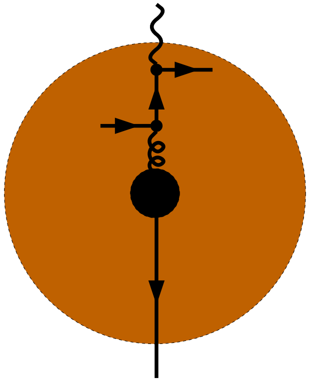

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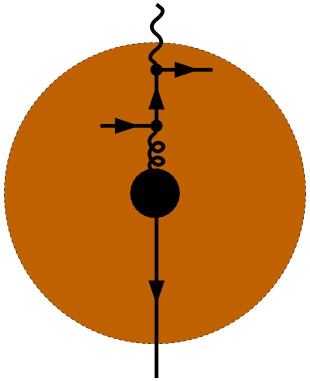

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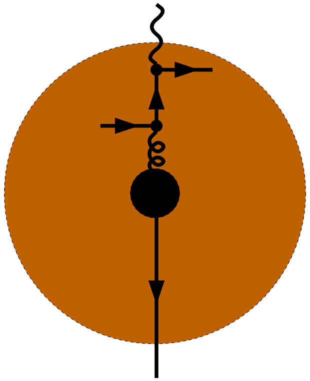

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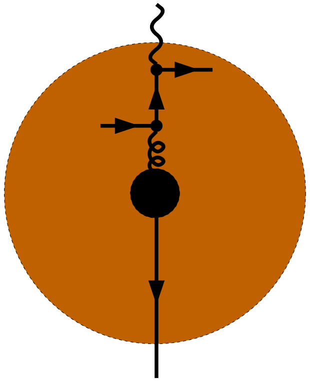

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$$\Rightarrow \frac{\delta\Gamma_{78\text{res}}/\Gamma}{\Delta_{0-}} \simeq \frac{(B+C)(Q_u+Q_d)+2DQ_s}{(C-B)(Q_u-Q_d)} \overset{Q_u+Q_d+Q_s=0}{=} \frac{Q_u+Q_d}{Q_d-Q_u}\left[1 + 2\,\overbrace{\frac{D-C}{C-B}}^{SU(3)_F \text{ violation}}\right]$$

MM, arXiv:0911.1651

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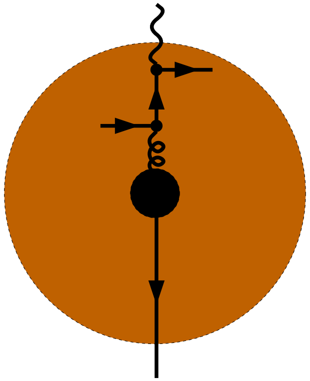

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$$\frac{\delta\Gamma_{78\text{res}}}{\Gamma} \simeq -\frac{1}{3}\Delta_{0-}\left[1 + 2\,\frac{D-C}{C-B}\right] = -\frac{1}{3}(\underbrace{-0.48 \pm 1.49 \pm 0.97 \pm 1.15}_{\text{Belle, arXiv:1807.04236, } E_0 = 1.9\,\text{GeV}})\% \times (1 \pm 0.3) = (0.16 \pm 0.74)\%$$

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**M. Benzke, S.J. Lee, M. Neubert, G. Paz, JHEP 1008 (2010) 099, arXiv:1003.5012,**
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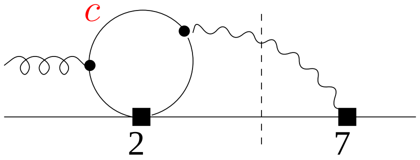

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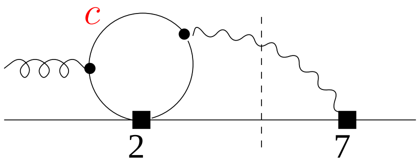

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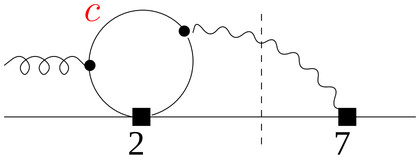

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Hermite polynomials

Constraints on moments (e.g.): $\qquad \int d\omega_1 h_{17} = \frac{2}{3}\mu_G^2, \qquad \int d\omega_1\omega_1^2 h_{17} = \frac{2}{15}(5m_5 + 3m_6 - 2m_9).$

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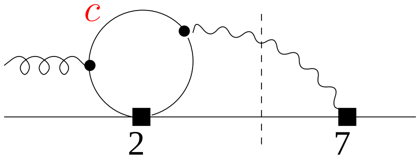

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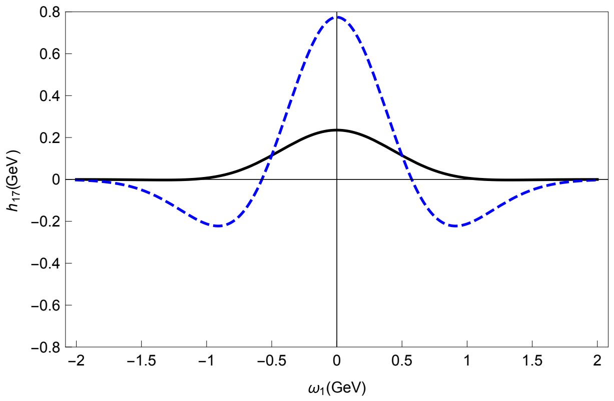

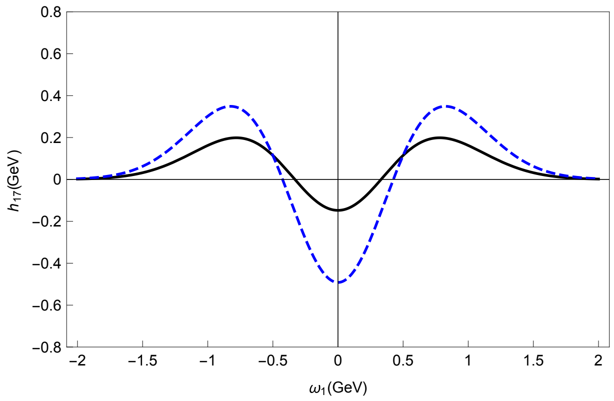

# NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity $P(E_0)$:

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Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ and $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\delta = 1 - \frac{2E_0}{m_b}$ and $z = \frac{m_c^2}{m_b^2}$.

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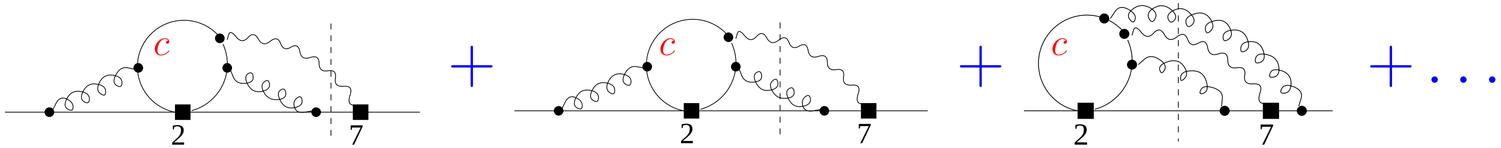
[MM, A. Rehman, M. Steinhauser, . . . ]
in progress



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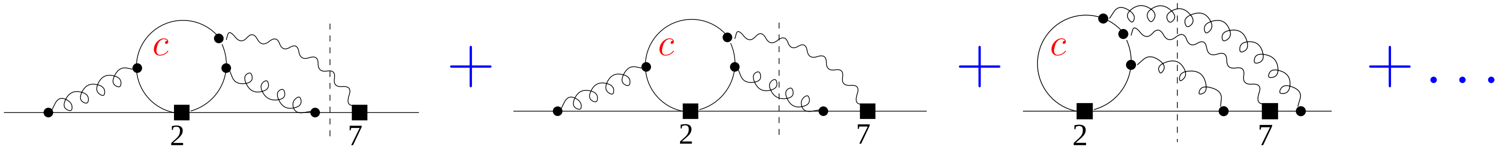

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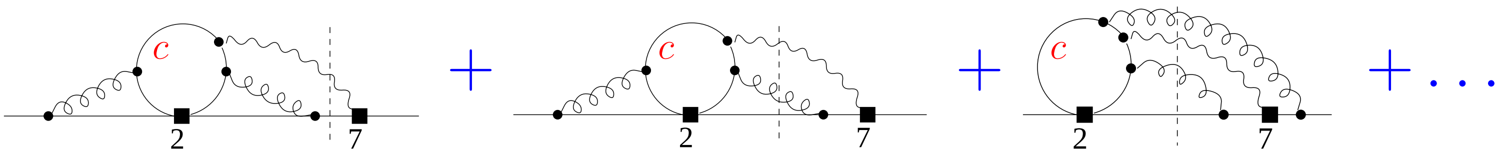

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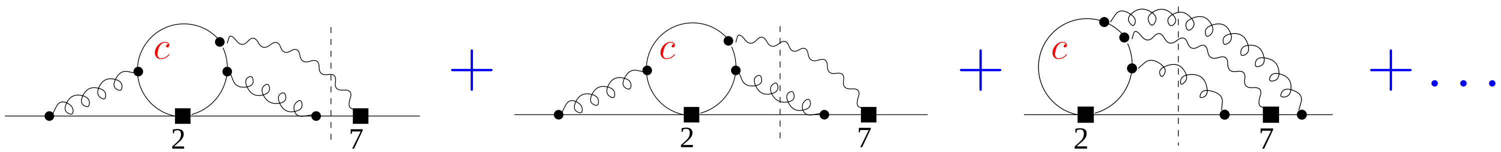

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**where $w_{nk}$ are rational functions of their arguments.**

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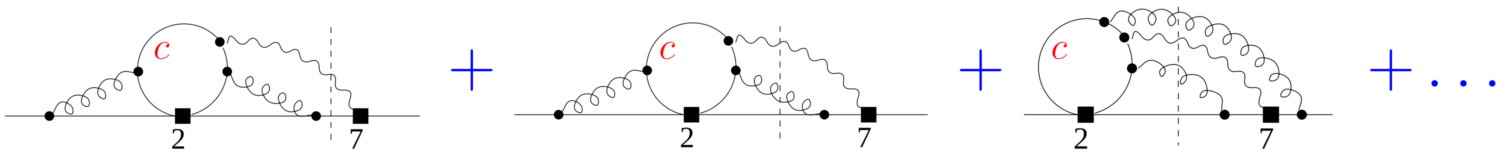

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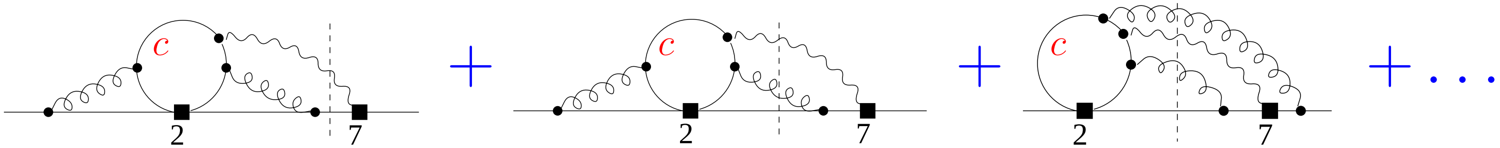

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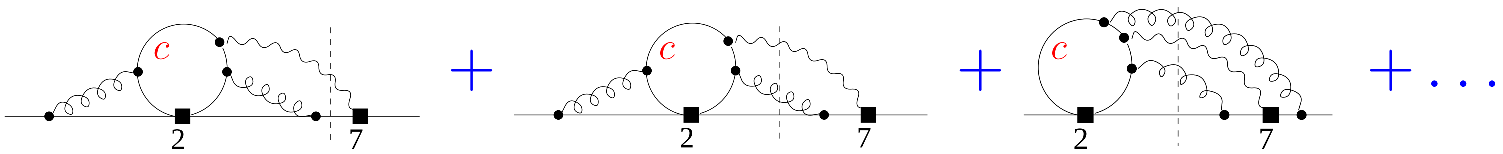

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6. Solving the system $(*)$ numerically [A.C. Hindmarsch, `http://www.netlib.org/odepack`] along an ellipse in the complex $z$ plane. Doing so along several different ellipses allows us to estimate the numerical error.

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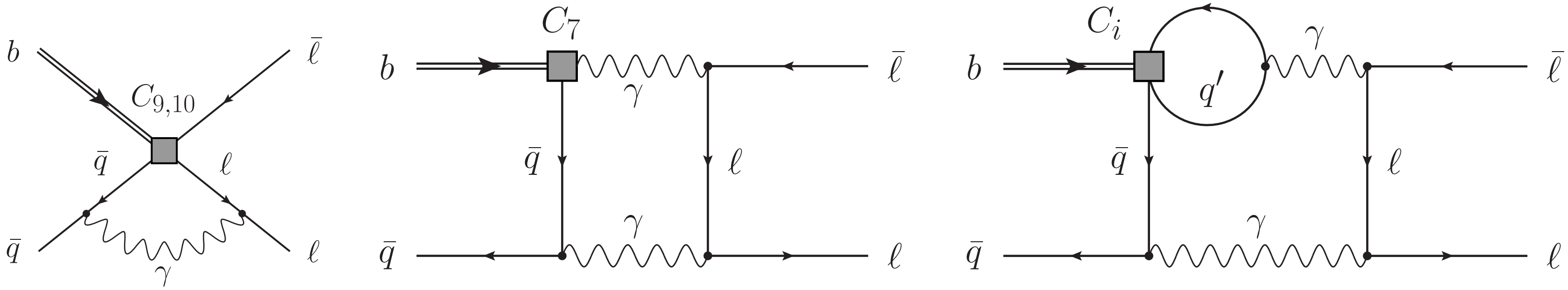

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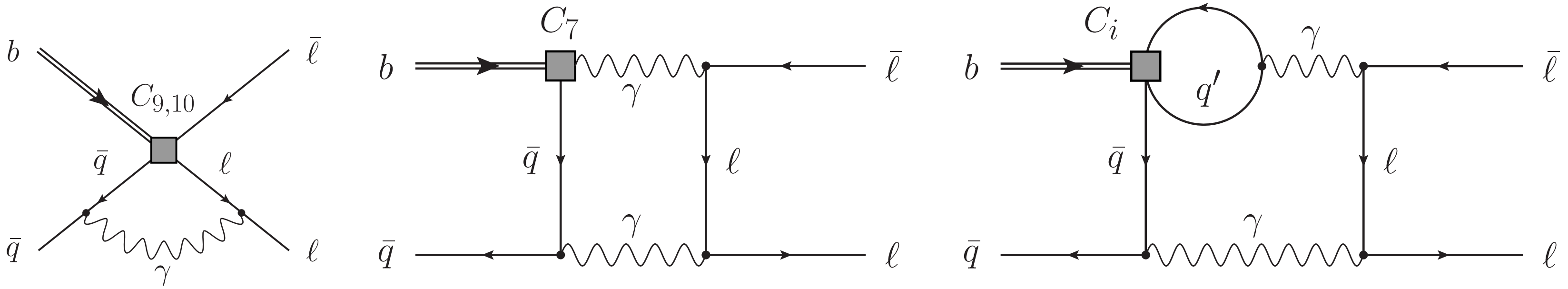

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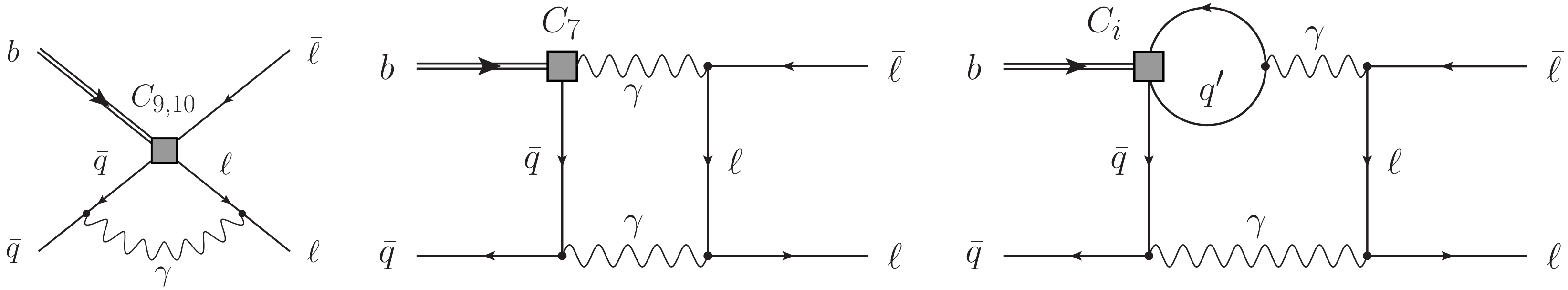

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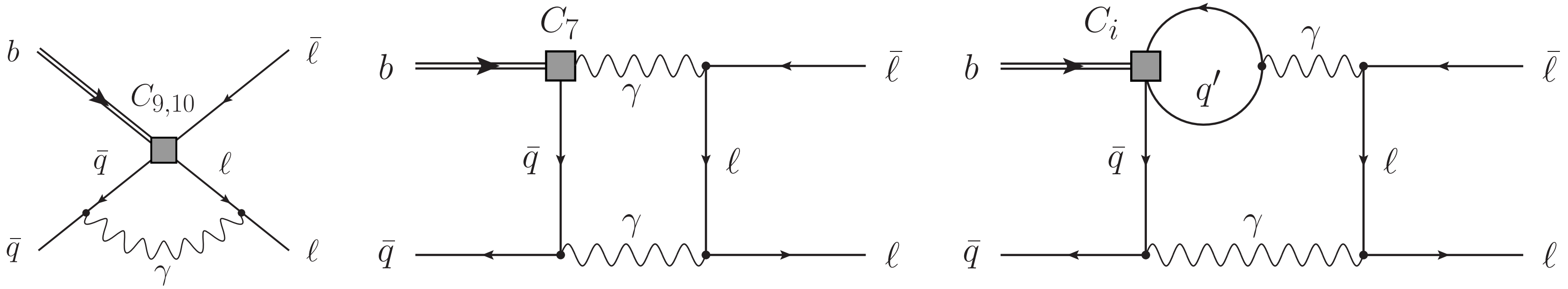

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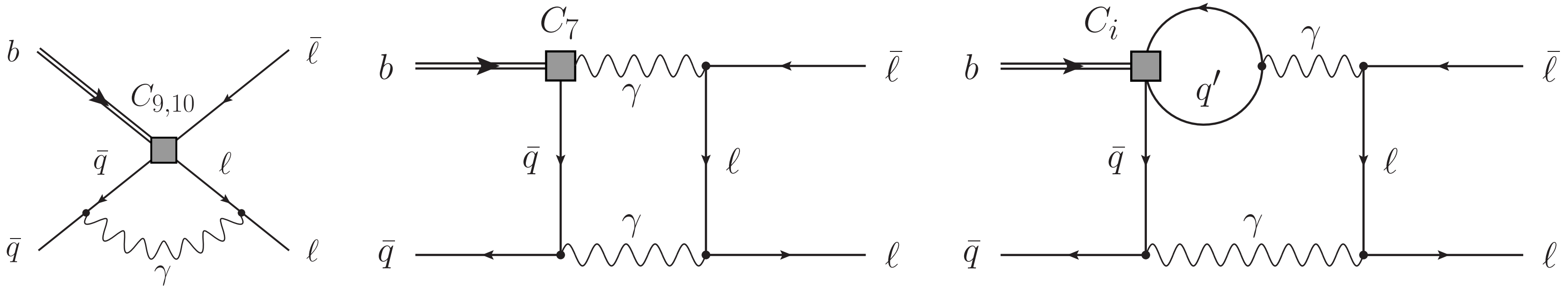

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However, it is larger than $\pm 0.3\%$ due to scale-variation of the Wilson coefficient $C_A(\mu_b)$.

# SM predictions for all the branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}(B_q^0 \to \ell^+\ell^-)$ including 2-loop electroweak and 3-loop QCD matching at $\mu_0 \sim m_t$

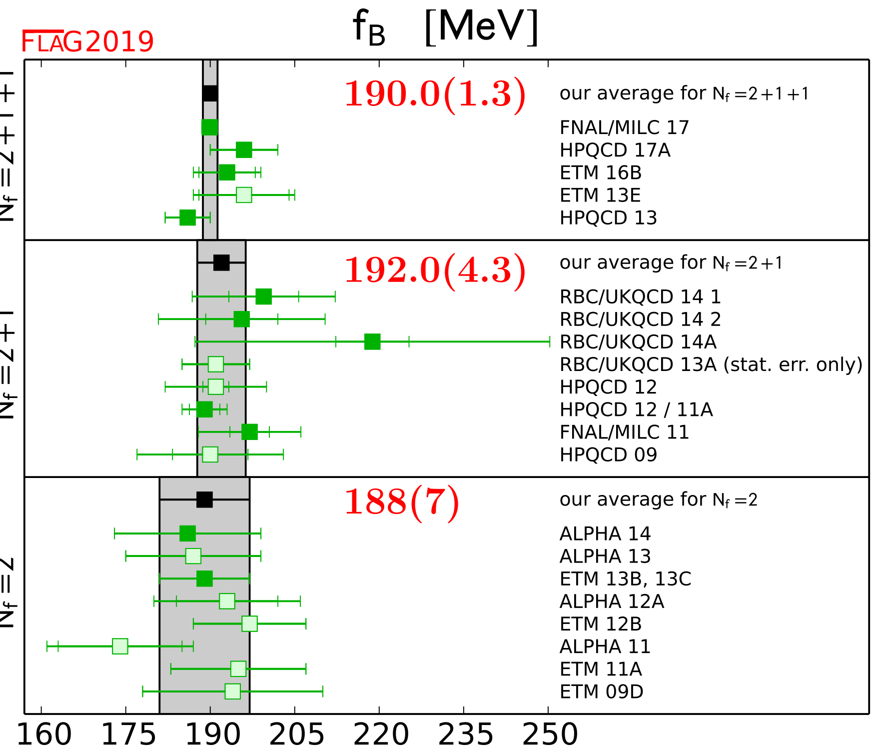
[ C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser, PRL 112 (2014) 101801]

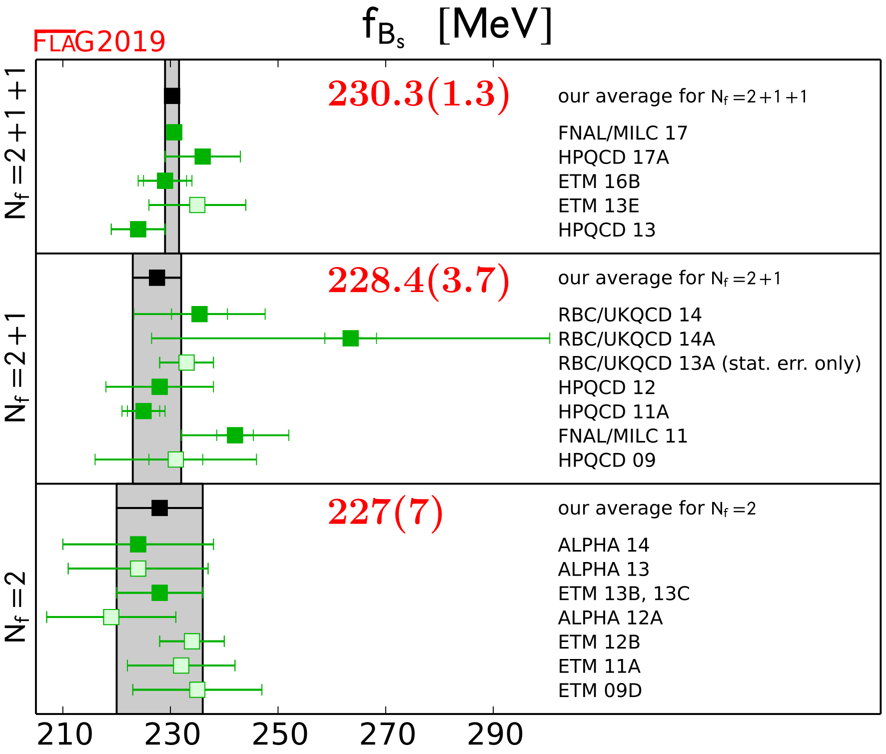
$$\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= \eta_{\rm QED}(8.54 \pm 0.13)\, R_{t\alpha}\, R_s,\\
\overline{\mathcal{B}}_{s\mu} \times 10^{9} &= \eta_{\rm QED}(3.65 \pm 0.06)\, R_{t\alpha}\, R_s,\\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= \eta_{\rm QED}(7.73 \pm 0.12)\, R_{t\alpha}\, R_s,\\
\overline{\mathcal{B}}_{de} \times 10^{15} &= \eta_{\rm QED}(2.48 \pm 0.04)\, R_{t\alpha}\, R_d,\\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= \eta_{\rm QED}(1.06 \pm 0.02)\, R_{t\alpha}\, R_d,\\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= \eta_{\rm QED}(2.22 \pm 0.04)\, R_{t\alpha}\, R_d,
\end{aligned}$$

where

$$\begin{aligned}
R_{t\alpha} &= \left(\frac{M_t}{173.1\ \text{GeV}}\right)^{3.06} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18},\\
R_s &= \left(\frac{f_{B_s}[\text{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\ [\text{ps}]}{1.615},\\
R_d &= \left(\frac{f_{B_d}[\text{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\rm av}\ [\text{ps}]}{1.519}.
\end{aligned}$$

# Inputs from FLAG, arXiv:1902.08191, Figs. 23 and 33





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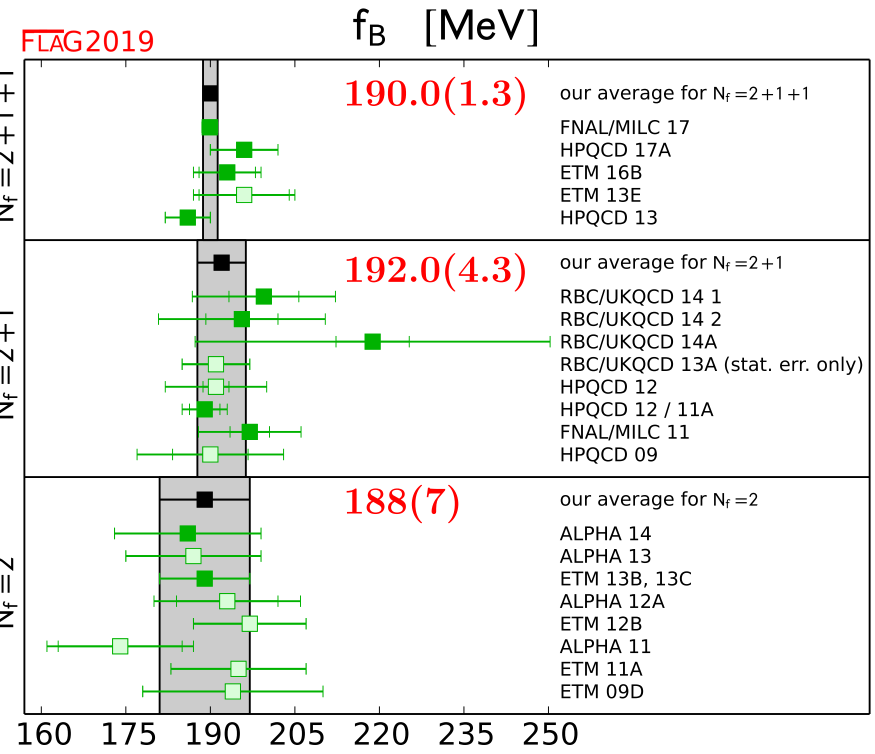

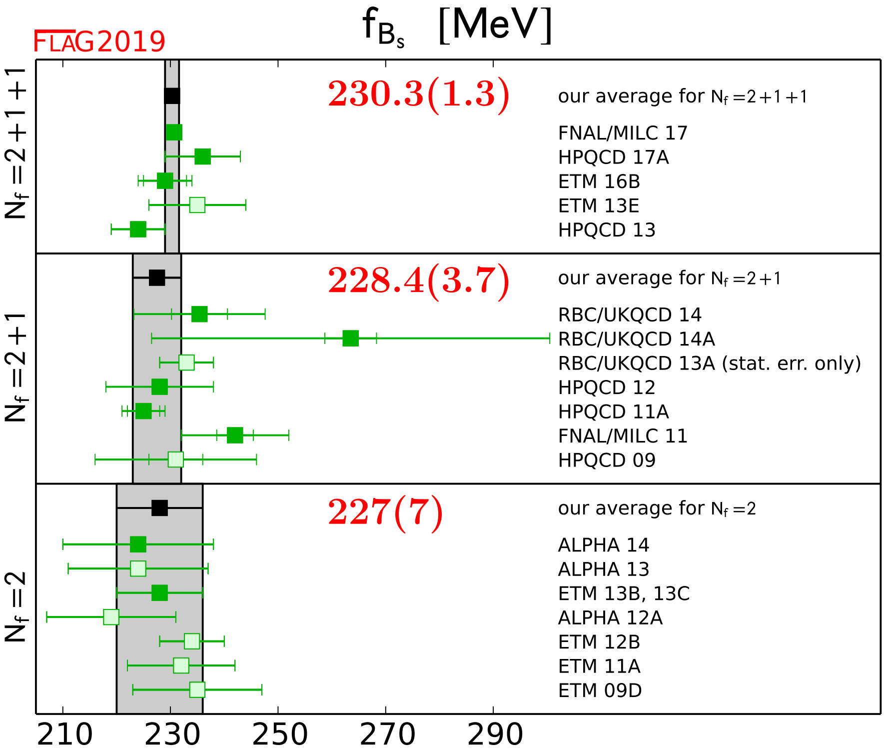

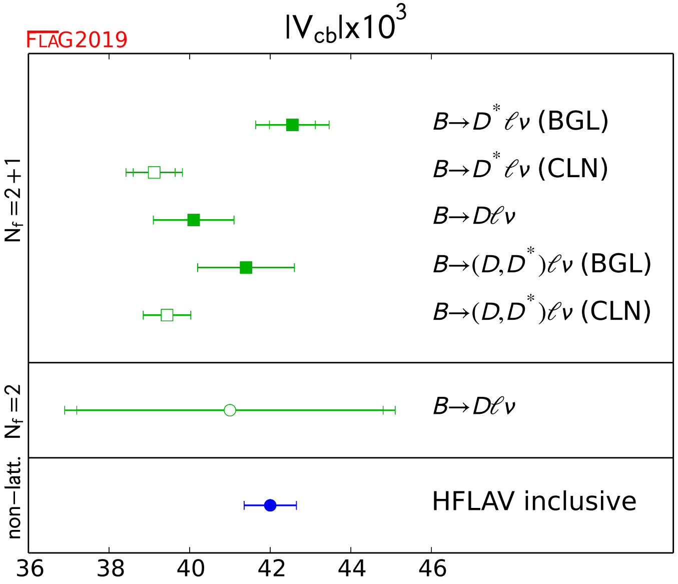

⟶ 0.04200(64) **from P. Gambino, K. J. Healey and S. Turczyk, arXiv:1606.06174.**

# Update of the input parameters

| | 2014 paper | this talk | source |
|---|---|---|---|
| $M_t$ [GeV] | 173.1(9) | 172.9(4) | PDG 2019, `http://pdglive.lbl.gov` |
| $\alpha_s(M_Z)$ | 0.1184(7) | 0.1181(11) | arXiv:1907.01435 |
| $f_{B_s}$ [GeV] | 0.2277(45) | 0.2303(13) | FLAG, arXiv:1902.08191 |
| $f_{B_d}$ [GeV] | 0.1905(42) | 0.1900(13) | FLAG, arXiv:1902.08191 |
| $\|V_{cb}\| \times 10^3$ | 42.40(90) | 42.00(64) | inclusive, arXiv:1606.06174 |
| $\|V_{tb}^* V_{ts}\|/\|V_{cb}\|$ | 0.9800(10) | 0.9819(5) | derived from CKMfitter 2019, `http://ckmfitter.in2p3.fr` |
| $\|V_{tb}^* V_{td}\| \times 10^4$ | 88(3) | $87.1^{+0.86}_{-2.46}$ | CKMfitter 2019, `http://ckmfitter.in2p3.fr` |
| $\tau_H^s$ [ps] | 1.615(21) | 1.615(9) | HFLAV 2019, `https://www.slac.stanford.edu/xorg/hflav` |
| $\tau_H^d$ [ps] | 1.519(7) | 1.520(4) | HFLAV 2019, `https://www.slac.stanford.edu/xorg/hflav` |
| $\overline{\mathcal{B}}_{s\mu} \times 10^9$ | 3.65(23) | 3.64(14) | |
| $\overline{\mathcal{B}}_{d\mu} \times 10^{10}$ | 1.06(9) | $1.02^{+0.03}_{-0.06}$ | |

| Sources of uncertainties | $f_{B_q}$ | CKM | $\tau_H^q$ | $M_t$ | $\alpha_s$ | other parametric | non-parametric | $\sum$ |
|---|---|---|---|---|---|---|---|---|
| $\overline{\mathcal{B}}_{s\ell}$ | 1.1% | 3.1% | 0.6% | 0.7% | 0.2% | $< 0.1\%$ | 1.5% | 3.8% |
| $\overline{\mathcal{B}}_{d\ell}$ | 1.4% | $\left({}^{+2.0}_{-5.6}\right)\%$ | 0.3% | 0.7% | 0.2% | $< 0.1\%$ | 1.5% | $\left({}^{+3.0}_{-5.9}\right)\%$ |

# LHC measurements of $\overline{\mathcal{B}}_{q\mu}$:

| | $\overline{\mathcal{B}}_{s\mu} \times 10^9$ | $\overline{\mathcal{B}}_{d\mu} \times 10^{10}$ |
|---|---|---|
| LHCb, PRL 118 (2017) 191801 | $3.0 \pm 0.6^{+0.3}_{-0.2}$ | $1.5^{+1.2+0.2}_{-1.0-0.1}$ |
| ATLAS, JHEP 1904 (2019) 098 | $2.8^{+0.8}_{-0.7}$ | $-1.9 \pm 1.6$ |
| CMS, PRL 111 (2013) 101804 | $3.0^{+1.0}_{-0.9}$ | $3.5^{+2.1}_{-1.8}$ |
| CMS-PAS-BPH-16-004, Aug'19 | $2.9^{+0.7}_{-0.6} \pm 0.2$ | $0.8^{+1.4}_{-1.3}$ |

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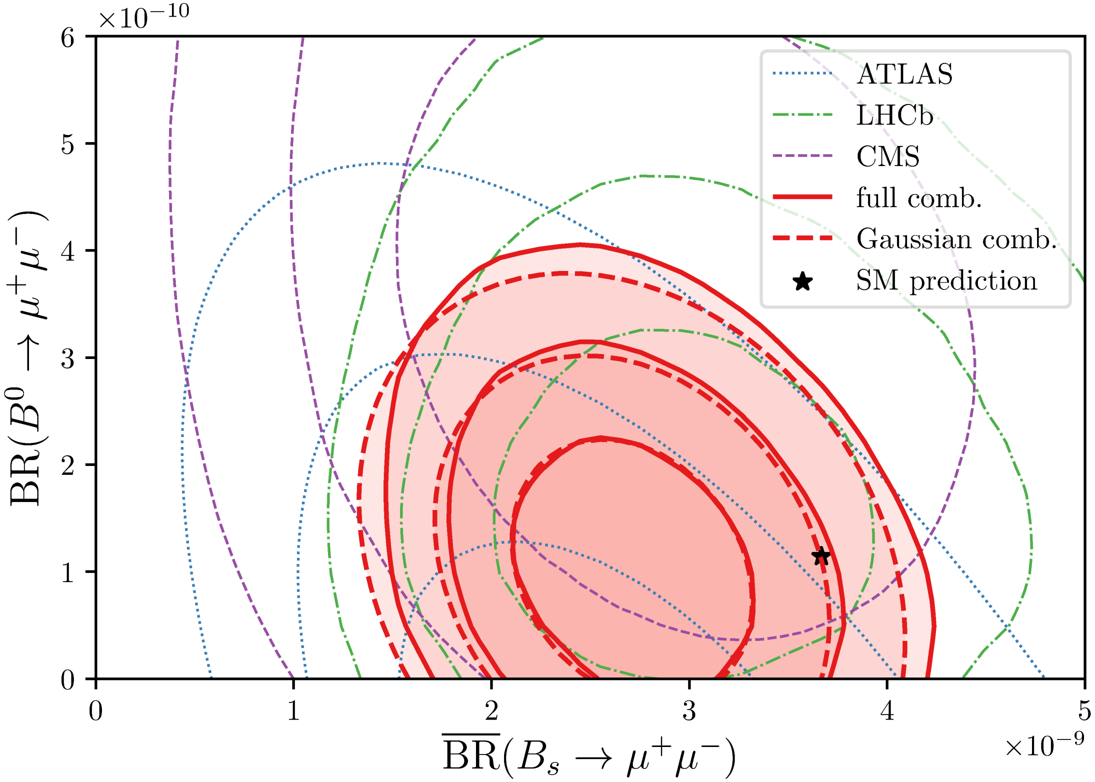
**Combination (with CMS from 2013) in Appendix A of arXiv:1903.10434:**



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- Perturbative NNLO calculations of $\bar{B} \to X_s\gamma$ for arbitrary $m_c$ are close to the point of completing the IBP reduction.

- The accuracy of SM predictions for $B_s \to \ell^+\ell^-$ has significantly improved, mainly due to more precise lattice determinations of the decay constants. Power-enhanced QED corrections have been identified and included.

# BACKUP SLIDES

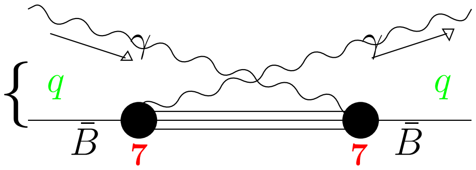
# The “hard” contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

**Goal: calculate the inclusive sum** $\Sigma_{X_s} \left| C_7(\mu_b) \langle X_s\gamma|O_7|\bar{B}\rangle + C_2(\mu_b)\langle X_s\gamma|O_2|\bar{B}\rangle + \ldots \right|^2$

**The “77” term in this sum is “hard”. It is related via the optical theorem to the imaginary part of the elastic forward scattering amplitude $\bar{B}(\vec{p}=0)\gamma(\vec{q}) \to \bar{B}(\vec{p}=0)\gamma(\vec{q})$:**

Im{



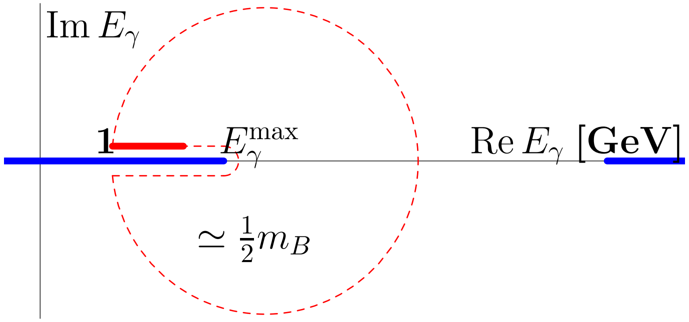
} $\equiv$ Im $A$

**When the photons are soft enough, $m^2_{X_s} = |m_B(m_B - 2E_\gamma)| \gg \Lambda^2 \Rightarrow$ Short-distance dominance $\Rightarrow$ OPE.**

**However, the $\bar{B} \to X_s\gamma$ photon spectrum is dominated by hard photons $E_\gamma \sim m_b/2$.**

**Once $A(E_\gamma)$ is considered as a function of arbitrary complex $E_\gamma$, Im$A$ turns out to be proportional to the discontinuity of $A$ at the physical cut. Consequently,**

$$\int_{1\,\mathrm{GeV}}^{E_\gamma^{\max}} dE_\gamma \; \mathrm{Im}\, A(E_\gamma) \sim \oint_{\mathrm{circle}} dE_\gamma \; A(E_\gamma).$$



**Since the condition $|m_B(m_B - 2E_\gamma)| \gg \Lambda^2$ is fulfilled along the circle, the OPE coefficients can be calculated perturbatively, which gives**

$$A(E_\gamma)\big|_{\mathrm{circle}} \simeq \sum_j \left[ \frac{F^{(j)}_{\mathrm{polynomial}}(2E_\gamma/m_b)}{m_b^{n_j}(1 - 2E_\gamma/m_b)^{k_j}} + \mathcal{O}\left(\alpha_s(\mu_{\mathrm{hard}})\right) \right] \langle \bar{B}(\vec{p}=0)|Q^{(j)}_{\mathrm{local\ operator}}|\bar{B}(\vec{p}=0)\rangle.$$

**Thus, contributions from higher-dimensional operators are suppressed by powers of $\Lambda/m_b$.**

**At $(\Lambda/m_b)^0$:** $\langle \bar{B}(\vec{p})|\bar{b}\gamma^\mu b|\bar{B}(\vec{p})\rangle = 2p^\mu \quad \Rightarrow \quad \Gamma(\bar{B} \to X_s\gamma) = \Gamma(b \to X_s^{\mathrm{parton}}\gamma) + \mathcal{O}(\Lambda/m_b).$

**At $(\Lambda/m_b)^1$:** **Nothing! All the possible operators vanish by the equations of motion.**

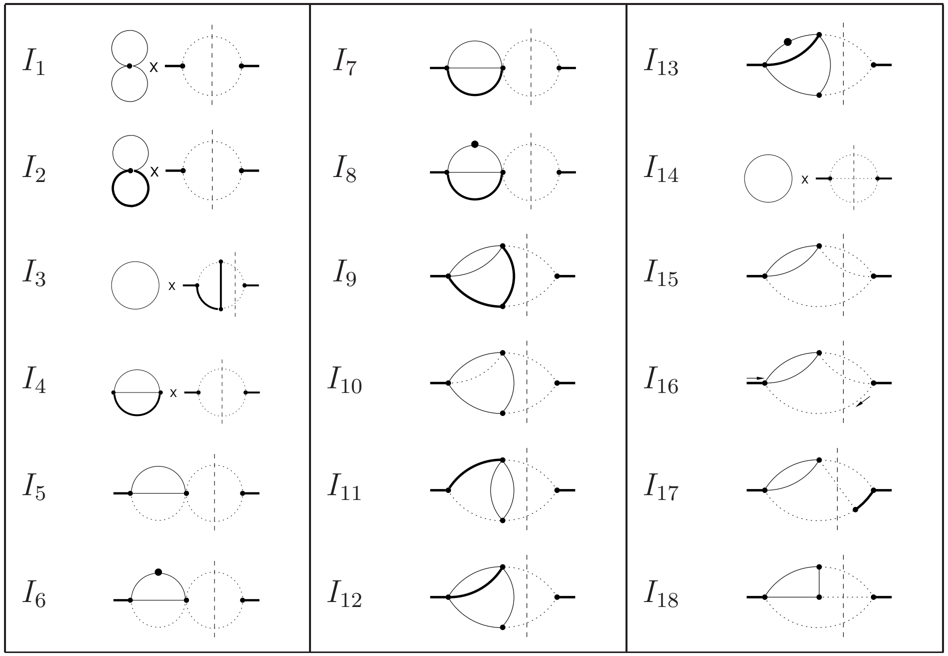
**At $(\Lambda/m_b)^2$:** $\langle \bar{B}(\vec{p})|\bar{b}_v D^\mu D_\mu b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_\pi^2,$
$\langle \bar{B}(\vec{p})|\bar{b}_v g_s G_{\mu\nu}\sigma^{\mu\nu} b_v|\bar{B}(\vec{p})\rangle \sim m_B\,\mu_G^2,$

**The HQET heavy-quark field:** $b_v(x) = \frac{1}{2}(1 + \not{v})b(x)\exp(i m_b\, v\cdot x)$ **with** $v = p/m_B$.

# The same method has been applied to the 3-loop counterterm diagrams

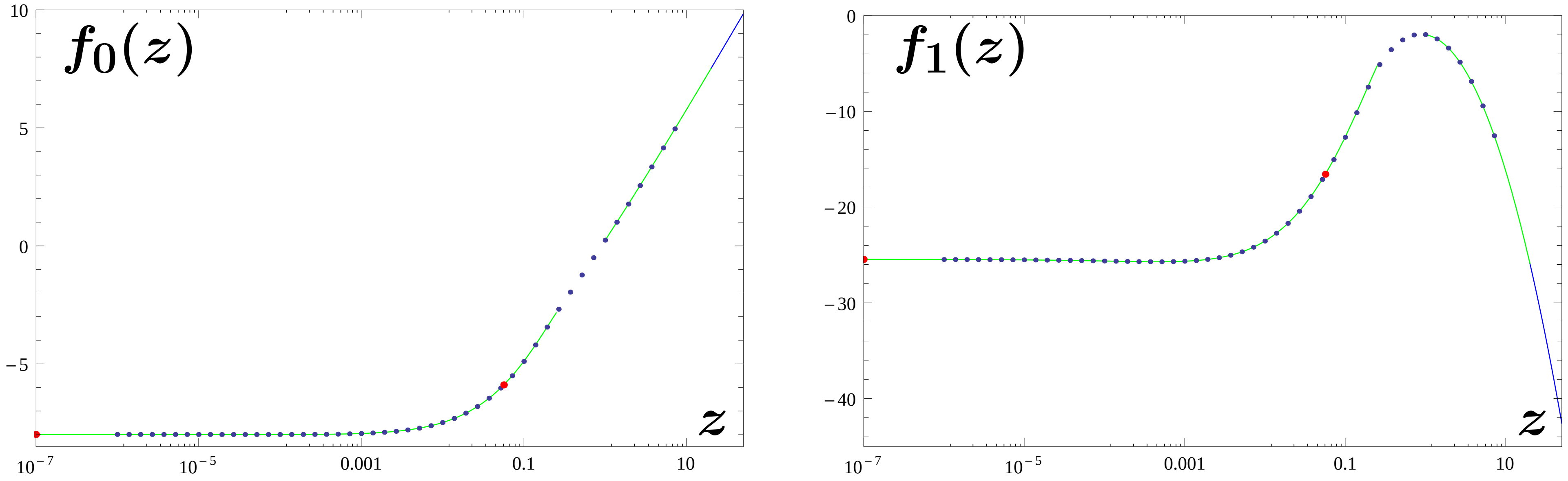
**[MM, A. Rehman, M. Steinhauser, PLB 770 (2017) 431]**

# Master integrals:



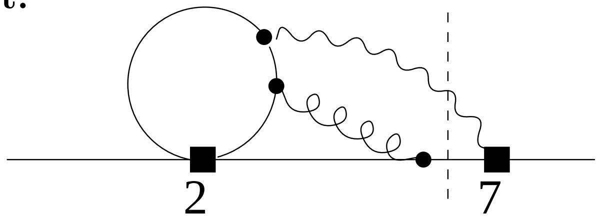
# Results for the bare NLO contributions up to $\mathcal{O}(\epsilon)$:

$$\hat{G}_{27}^{(1)2P} = -\frac{92}{81\epsilon} + f_0(z) + \epsilon f_1(z) \xrightarrow{z\to 0} -\frac{92}{81\epsilon} - \frac{1942}{243} + \epsilon\left(-\frac{26231}{729} + \frac{259}{243}\pi^2\right)$$



Dots: solutions to the differential equations and/or the exact $z \to 0$ limit.

Lines: large- and small-$z$ asymptotic expansions



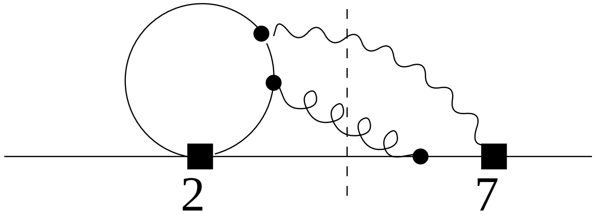
Small-$z$ expansions of $\hat{G}_{27}^{(1)2P}$:

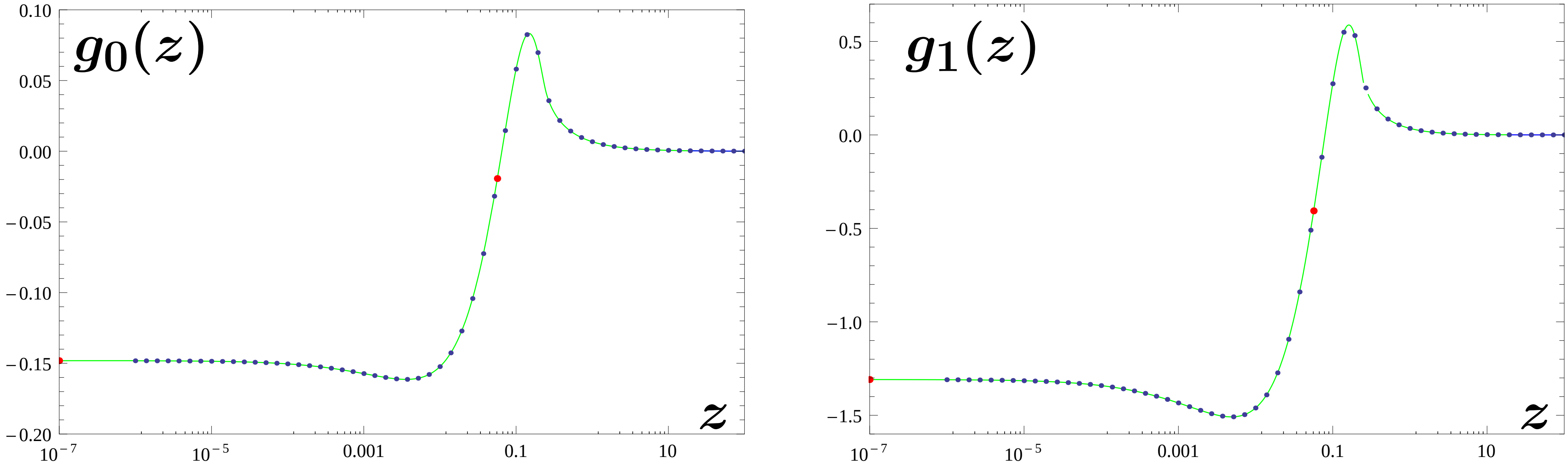
$f_0$ from C. Greub, T. Hurth, D. Wyler, hep-ph/9602281, hep-ph/9603404,
A. J. Buras, A. Czarnecki, MM, J. Urban, hep-ph/0105160,

$f_1$ from H.M. Asatrian, C. Greub, A. Hovhannisyan, T. Hurth and V. Poghosyan, hep-ph/0505068.

# Analogous results for the 3-body final state contributions ($\delta = 1$):

$$\hat{G}_{27}^{(1)3P} = g_0(z) + \epsilon g_1(z) \xrightarrow{z\to 0} -\frac{4}{27} - \frac{106}{81}\epsilon$$



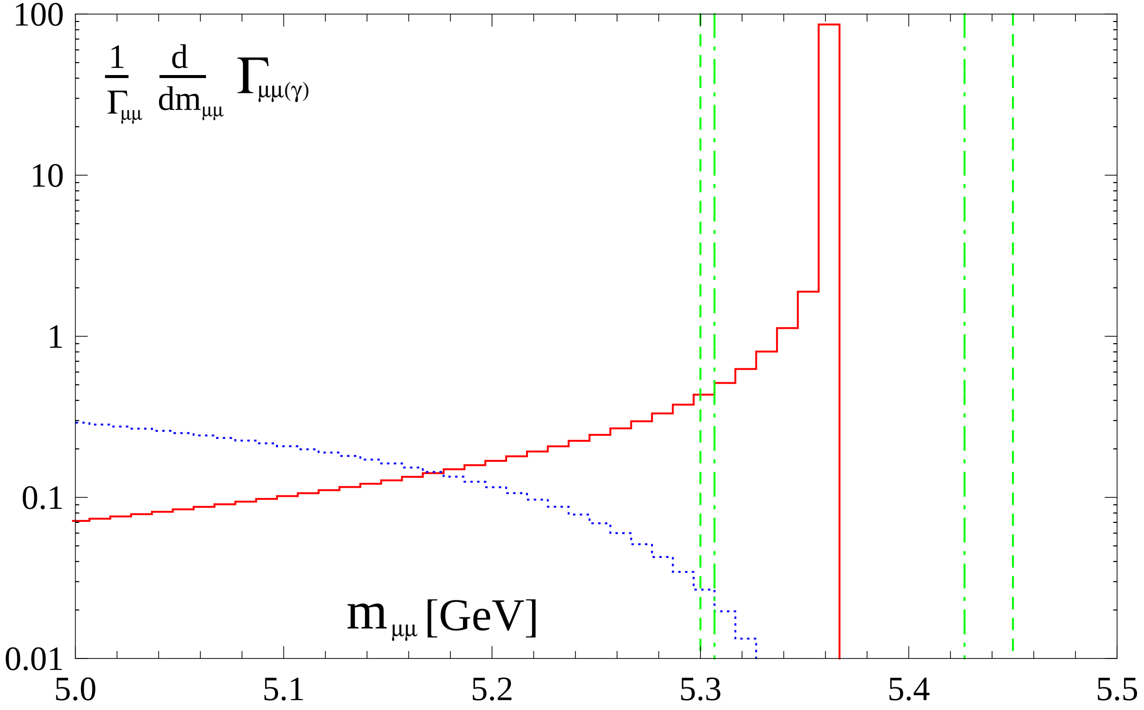


**Dots:** solutions to the differential equations and/or the exact $z \to 0$ limit.
**Lines:** exact result for $g_0$, as well as large- and small-$z$ asymptotic expansions for $g_1$.

$$g_0(z) = \begin{cases} -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, s\, L + \frac{16}{9}z(6z^2-4z+1)\left(\frac{\pi^2}{4} - L^2\right), & \text{for } z \leq \frac{1}{4}, \\ -\frac{4}{27} - \frac{14}{9}z + \frac{8}{3}z^2 + \frac{8}{3}z(1-2z)\, t\, A + \frac{16}{9}z(6z^2-4z+1)\, A^2, & \text{for } z > \frac{1}{4}, \end{cases}$$

where $s = \sqrt{1-4z}$, $L = \ln(1+s) - \frac{1}{2}\ln 4z$, $t = \sqrt{4z-1}$, and $A = \arctan(1/t)$.

# Radiative tail in the dimuon invariant mass spectrum



Green vertical lines – experimental “blinded” windows [CMS and LHCb, Nature 522 (2015) 68]

Red line – no real photon and/or radiation only from the muons. It vanishes when $m_\mu \to 0$.

[A.J. Buras, J. Girrbach, D. Guadagnoli, G. Isidori, Eur.Phys.J. C72 (2012) 2172]

[S. Jadach, B.F.L. Ward, Z. Was, Phys.Rev. D63 (2001) 113009], Eq. (204) as in PHOTOS

Blue line – remainder due to radiation from the quarks. IR-safe because $B_s$ is neutral.

Phase-space suppressed but survives in the $m_\mu \to 0$ limit.

[Y.G. Aditya, K.J. Healey, A.A. Petrov, Phys.Rev. D87 (2013) 074028]

[D. Melikhov, N. Nikitin, Phys.Rev. D70 (2004) 114028]

Interference between the two contributions is negligible – suppressed both by phase-space and $m_\mu^2/M_{B_s}^2$.