Semi-leptonic $B_{(s)}$ decays on the lattice

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RBC-UKQCD Collaborations

based on arXiv:1903.02100 (publication in preparation)

New physics at the low-energy precision frontier - LPT, Orsay

18 September 2019

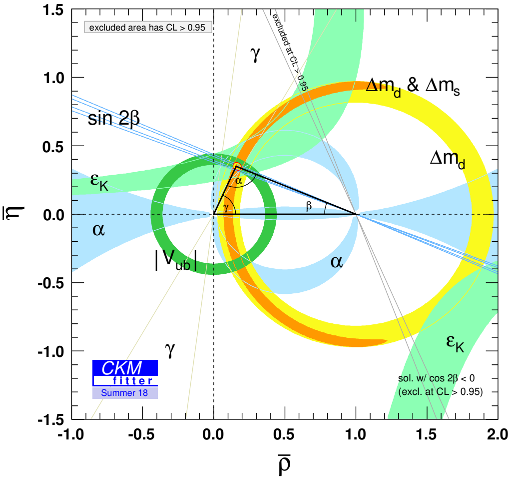
THE UNIVERSITY *of* EDINBURGH

Outline

How to find New Physics?

1. Direct searches:
   ⇒ *Bump in the spectrum*
2. Indirect searches:
   Precision Frontier:
   → Quantum corrections due to new particles modify SM predictions
   → NP shows as discrepancy between experiment and theory

⇒ **Over-constrain SM**



Why heavy quark sector?

- Huge experimental efforts:

Belle II at SuperKEKB, Tsukuba

First collision on 26/04/2018

LHC at CERN, Geneva

and CLEO-c, BaBar, BESIII, ...

Why heavy quark sector?

- Huge experimental efforts: $\Rightarrow$ LHC, Belle II, BESIII, ...
- Less explored than light quark sector

Absolute values (PDG 2018)

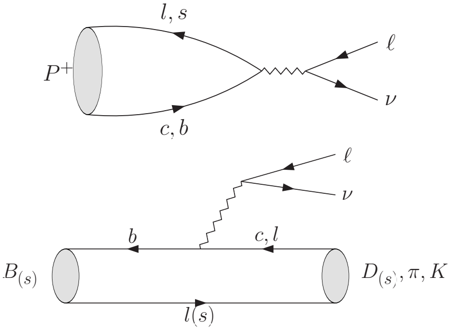
$$\begin{pmatrix} 0.97420(21) & 0.2243(5) & 0.00394(36) \\ 0.218(4) & 0.997(17) & 0.0422(8) \\ 0.0081(5) & 0.0394(23) & 1.019(25) \end{pmatrix}$$

Current uncertainties (PDG 2018)

$$\frac{|\delta V_{CKM}|}{|V_{CKM}|} = \begin{pmatrix} 0.02 & 0.22 & 9.1 \\ 1.8 & 1.7 & 1.9 \\ 6.2 & 5.8 & 2.5 \end{pmatrix} \%$$

Why heavy quark sector?

- Huge experimental efforts:
  $\Rightarrow$ LHC, Belle II, BESIII, ...
- Less explored than light quark sector
- Over-constrain the same CKM matrix elements from independent processes via (semi-)leptonic decays



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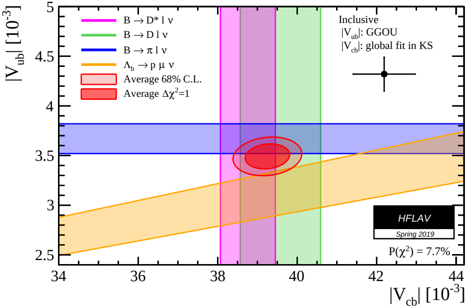
  ⇒ address inclusive vs exclusive



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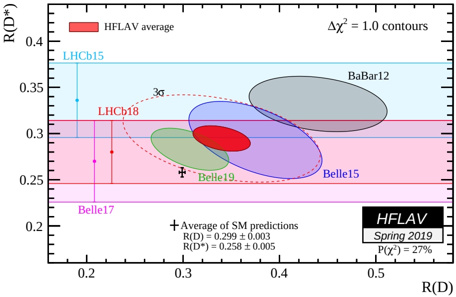
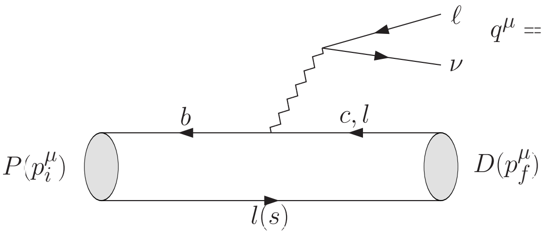
- Lepton Flavour Universality Violations?



$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu_\ell)} \quad (\ell = e, \mu)$$

How to extract CKM matrix elements?

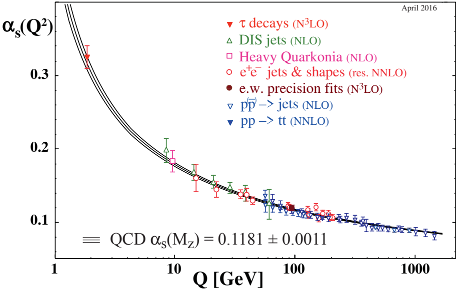
$$\text{experiment} \approx \text{CKM} \times \text{non}-\text{pert.} \times \text{known factors}$$



$$q^\mu = p_i^\mu - p_f^\mu$$

$$\frac{d\Gamma(P \to D\ell\nu_\ell)}{dq^2} \approx |V_{q_2 q_1}|^2 \times \left[\left| f_+(q^2) \right|^2 \mathcal{K}_1 + \left| f_0(q^2) \right|^2 \mathcal{K}_2 \right]$$

Non-Perturbative Physics and Lattice QCD



Source: PDG

BG/Q in Edinburgh

$\Rightarrow$ Large scale computing facilities

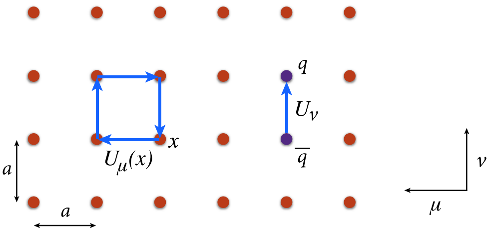
- At *low energy scales* perturbative methods fail
- Lattice QCD simulations provides **first principle precision predictions** for phenomenology
- Calculations need to be improved for observables where the error is dominated by **non-perturbative physics**...

Lattice QCD methodology

Wick rotate ($t \to i\tau$) Path Integral to Euclidean space:

$$\langle \mathcal{O} \rangle_E = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}]\, \mathcal{D}[U]\, \mathcal{O}[\psi, \overline{\psi}, U]\, e^{-S_E[\psi,\overline{\psi},U]}$$

Introducing lattice renders PI large **but finite** dimensional.



PDG

- Finite lattice spacing a
  $\Rightarrow$ UV regulator
- Finite Box of length L
  $\Rightarrow$ IR regulator

$\Rightarrow$ Calculate PI **explicitly** via Monte Carlo sampling:

b-physics on the lattice - disparity of scales

Control IR (Finite Size Effects) and UV (discretisation) effects

$$m_\pi L \gtrsim 4$$
$$a^{-1} \gg \text{Mass scale of interest}$$

For $m_\pi = m_\pi^{\mathrm{phys}} \sim 140\,\mathrm{MeV}$ and $m_b \approx 4.2\,\mathrm{GeV}$:

$$L \gtrsim 5.6\,\mathrm{fm}$$
$$a^{-1} \sim 4.2\,\mathrm{GeV} \approx (0.05\,\mathrm{fm})^{-1}$$

Requires $N \equiv L/a \gtrsim 120 \Rightarrow N^3 \times (2N) \gtrsim 4 \times 10^8$ lattice sites.

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EXPENSIVE to satisfy both constraints simultaneously...

(but we are getting there!)

Alternative: use effective action for the b-quark.

A Lattice Computation

Lattice vs Continuum

We simulate:

- at finite lattice spacing a
- in finite volume L^3

⇒ quantised momenta $2\pi\vec{n}/L$

- lattice regularised
- Some bare input quark masses am_l, am_s, am_h
  In general: $m_\pi \neq m_\pi^{\rm phys}$

We want:

- $a \to 0$
- $L \to \infty$
- continuous momenta $\vec{p}$
- some continuum scheme
- $m_l = m_l^{\rm phys}$
- $m_s = m_s^{\rm phys}$
- $m_h = m_c^{\rm phys}, m_b^{\rm phys}$

⇒ Need to control all limits!

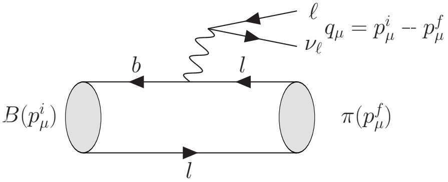
→ particularly simultaneously control FV and discretisation

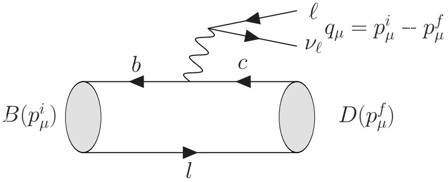
⇒ Decide on a fermion action:

Wilson, Staggered, Twisted Mass, **Domain Wall fermions**, $\cdots$

Why B_s decays?

To determine $|V_{ub}|$ and $|V_{cb}|$: interested in $b \to u$ and $b \to c$ transitions.





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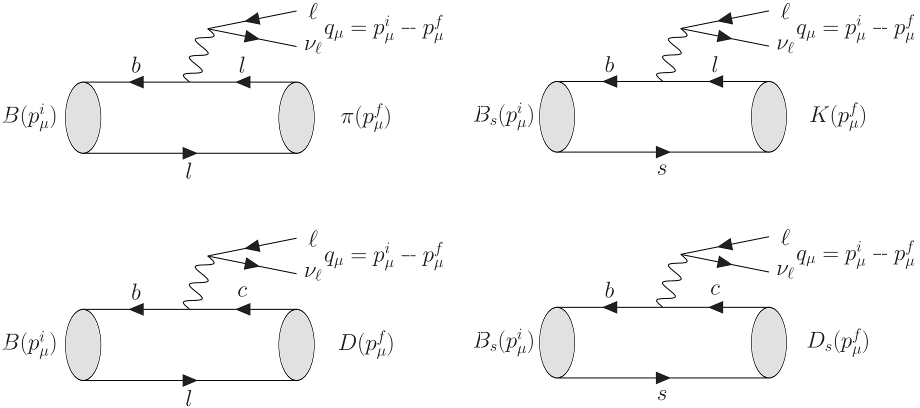

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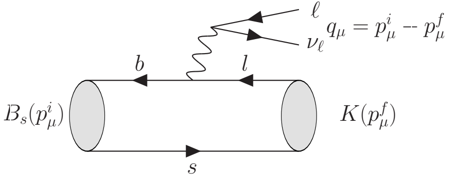
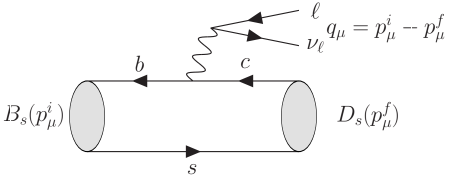
To determine $|V_{ub}|$ and $|V_{cb}|$: interested in $b \to u$ and $b \to c$ transitions.

- Only spectator quark differs
  $\Rightarrow$ complimentary to B decays
  $\Rightarrow$ $R(D_s^{(*)})$ good proxy?
- strange quarks are easier to deal with on the lattice:
  $\Rightarrow$ statistically cleaner
  $\Rightarrow$ computationally cheaper
- for $B_s \to D_s$
  $\Rightarrow$ chiral extrapolation only sea-quark effects:
- Gathering expertise for $B \to D$, $B \to \pi$.





RBC/UKQCD's $N_f = 2 + 1$ ensembles

name	L [fm]	a^{-1} [GeV]	m_π [MeV]
C0	5.476	**1.73**	**139**
C1	2.653	1.78	340
C2	2.653	1.78	430
M0	5.354	**2.36**	**139**
M1	2.649	2.38	300
M2	2.649	2.38	360
M3	2.649	2.38	410
F1	3.414	**2.77**	235

- Iwasaki gauge action
- Domain Wall Fermion action
- 2 ensembles with physical pion masses [PRD **93** (2016) 074505]
- $a \in 0.07 - 0.11\,\mathrm{fm}$
  third lattice spacing
  [JHEP **12** (2017) 008]

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- Iwasaki gauge action
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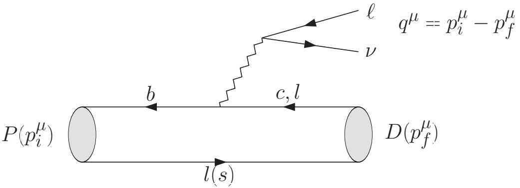
- 2 ensembles with physical pion masses [PRD **93** (2016) 074505]
- $a \in 0.07 - 0.11\,\mathrm{fm}$ third lattice spacing [JHEP **12** (2017) 008]
- $B_s \to K$: Update of [PRD **91** 074510] (third a, updated values of a + RHQ params)

✓ Measurements completed on C1, C2, M1, M2, M3, F1

(✓) Planned to include measurements on C0 in near future

Lattice set-up



$$\frac{d\Gamma(P \to D l \nu_l)}{dq^2} \approx |V_{q_2 q_1}|^2 \times \left[\left| f_+(q^2) \right|^2 \mathcal{K}_1 + \left| f_0(q^2) \right|^2 \mathcal{K}_2 \right]$$

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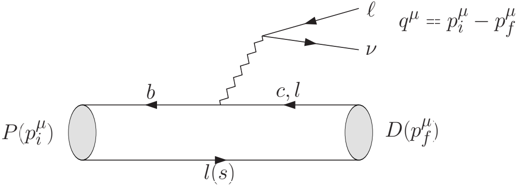

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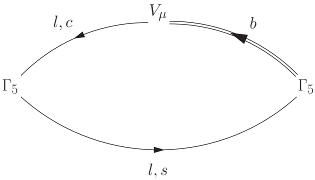

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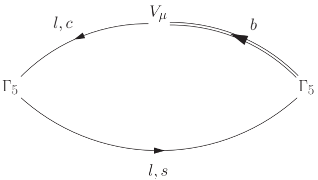

- l, s: DWF

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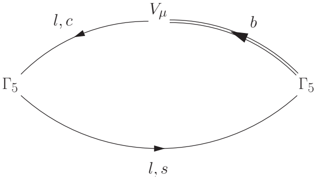

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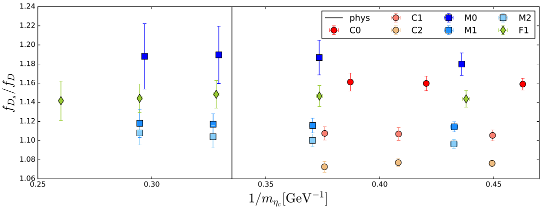
- c: *Heavy*-DWF optimised for charm [JHEP **05** (2015) 072, JHEP **04** (2016) 037]

Similar to [JHEP **12** (2017) 008]
Three m_c on **C**oarse ensemble $\Rightarrow$ extrapolate
Two m_c on **M**edium and **F**ine ensembles $\Rightarrow$ interpolate



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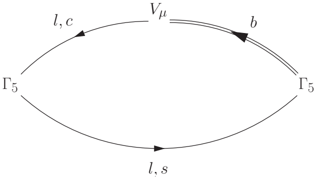

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  simulate range of $m_c \Rightarrow$ inter/extrapolate to m_c^{phys} [JHEP **12** (2017) 008]
- b: Relativistic Heavy Quark action [PRD **76** 074505, PRD **76** 074506]
  - based on Fermilab approach [PRD **64 014502**]
  - non-perturbatively tune 3 parameters [PRD **86 116003**]
  - smooth continuum limit, heavy quark treated to all orders in $(am_b)^n$.

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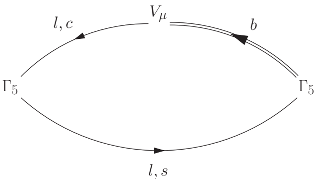

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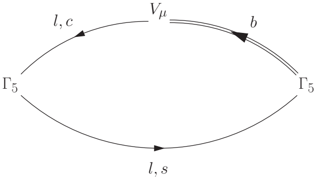
  - non-perturbatively tune 3 parameters [PRD **86 116003**]
  - smooth continuum limit, heavy quark treated to all orders in $(am_b)^n$.
- renormalisation: "mostly non-perturbative"
- Parent at rest ($\mathbf{p}_i = \mathbf{0}$), Daughter carries momentum:

$$q^2 = m_P^2 + m_D^2 - 2m_P E_D$$

$B_s \to K$ form factors $f_\parallel$ and $f_\perp$: data



$$f_\parallel(E_K^2) = \frac{\langle K | \, V_0 \, | B_s \rangle}{\sqrt{2 m_{B_s}}}$$

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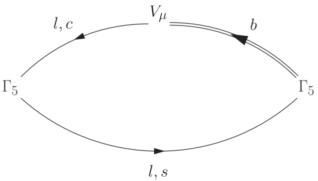

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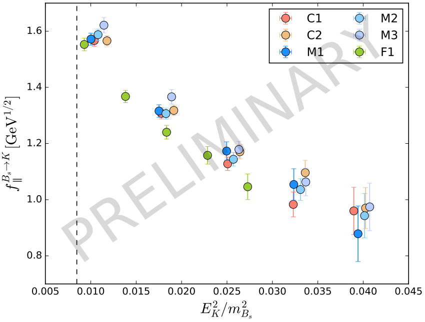
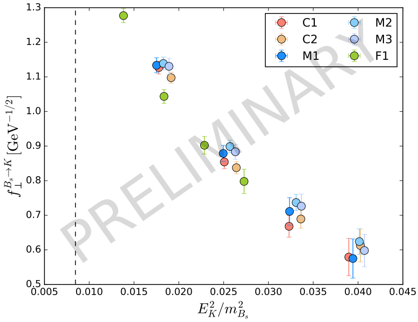
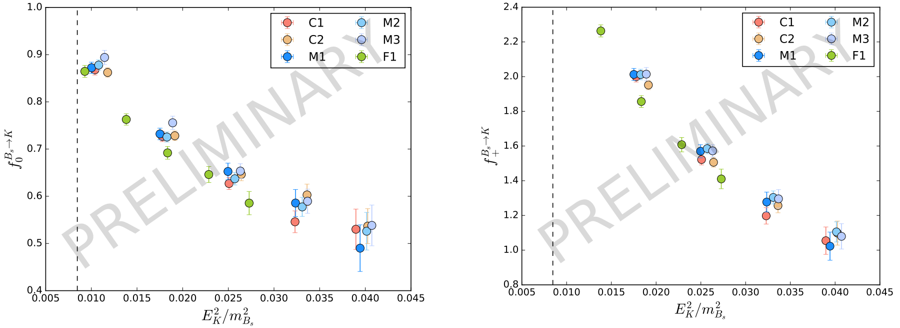




$B_s \to K$ form factors f_0 and f_+

$$f_+(E_K^2) = \frac{1}{\sqrt{2m_{B_s}}}\left[f_\parallel(E_K) + (m_{B_s} - E_K)\, f_\perp(E_K)\right]$$

$$f_0(E_K^2) = \frac{\sqrt{2m_{B_s}}}{m_{B_s}^2 - m_K^2}\left[(m_{B_s} - E_{Ds})f_\parallel(E_K) + \left(E_{D_s}^2 - m_K^2\right) f_\perp(E_K)\right]$$



$B_s \to K$ form factors f_+ and f_0: Strategy

1. Chiral-continuum limit fit to remove lattice artifacts
2. Introduce reference energies $E_K^{\mathrm{ref}} \leftrightarrow$ reference q^2 values

$$q^2_{\mathrm{ref}} = m^2_{B_s} + m^2_K - 2 m_{B_s} E_K^{\mathrm{ref}}$$

   At these, read off

   - central value, statistical errors and **statistical correlation**
   - fit systematic at each reference value and
   - estimate remaining systematic errors.

3. Use statistical **correlation matrix** to combine the above

   Obtain $f_+(q^2_{\mathrm{ref}})$ and $f_0(q^2_{\mathrm{ref}})$ in the range $q^2/\mathrm{GeV}^2 \in [17.2, 23.7]$

4. Model low-q^2 region to obtain f_0, f_+ for full range of q^2.

$B_s \to K$ form factors: chiral-continuum limit ansatz

We need to

- remove lattice artifacts ($O(a^2)$).
- extrapolate to physical quark masses.
- describe the behaviour with Kaon energy in available range.

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$\Rightarrow$ Functional ansatz from NLO $SU(2)$ χPT:

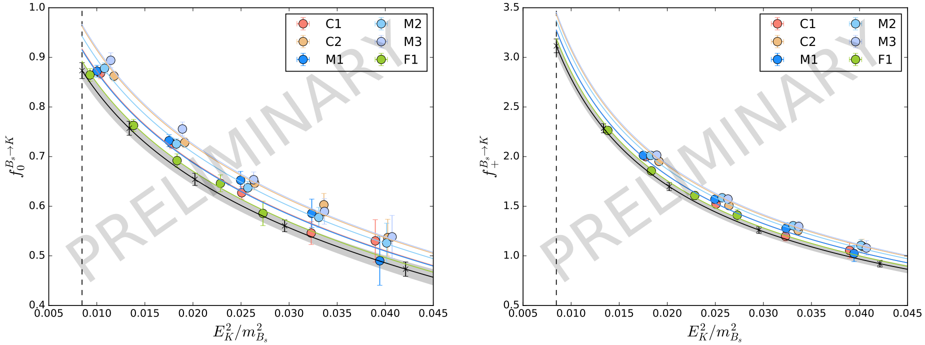
$$f^{B_s\to K}(m_\pi, E_K, a^2) = \frac{\alpha_0}{E_K + \Delta} \times \left(1 + \frac{\delta f}{(4\pi f_\pi)^2} + \alpha_1 \frac{m_\pi^2}{\Lambda^2} + \alpha_2 \frac{E_K}{\Lambda} + \alpha_3 \frac{E_K^2}{\Lambda^2} + \alpha_4 (\Lambda a)^2\right)$$

with

$$\delta f = \frac{3}{4} m_\pi^2 \log\left(\frac{m_\pi^2}{\Lambda^2}\right)$$

$\Delta_+ = m_{B_s} - m_{B^*} \approx 0.263\,\mathrm{GeV}$ $\quad$ $\Delta_0 = m_{B_s} - m_{B^*(0^+)} \approx -0.0416\,\mathrm{GeV}$

$B_s \to K$ form factors f_+ and f_0: chiral-continuum fit



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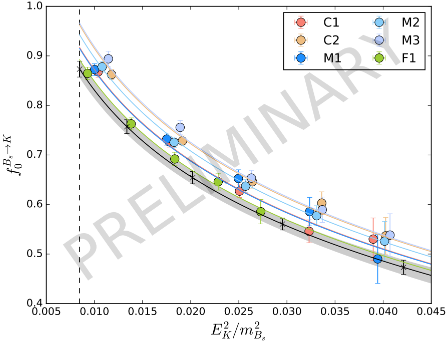
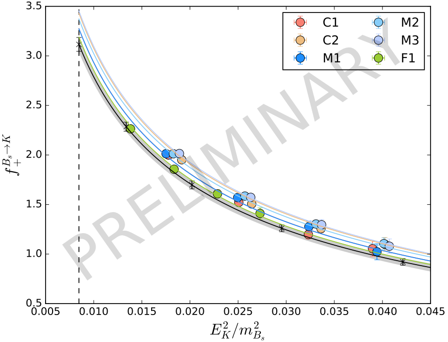
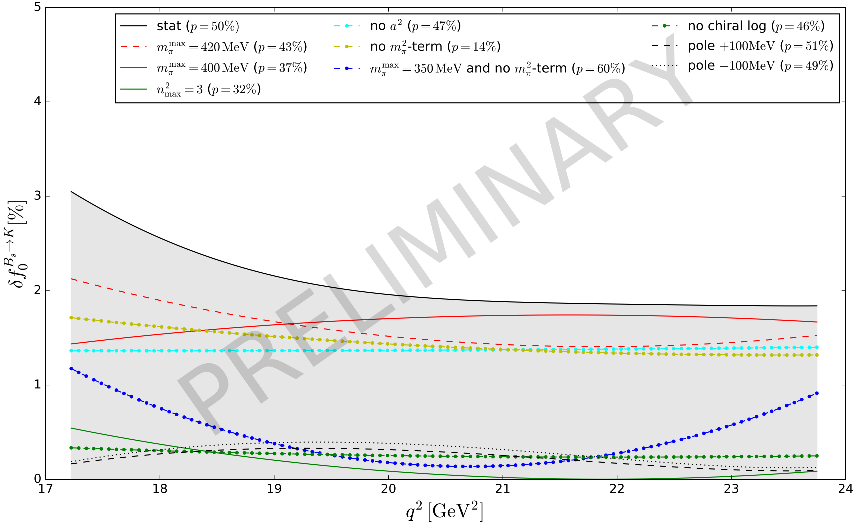




- vary pole masses by $\pm 100\,\mathrm{MeV}$
- include/exclude terms in the fit function
- include/exclude data points (heaviest pion mass, largest momenta, coarsest ensemble)

$B_s \to K$: Systematic errors for f_0



$$\delta f = \left| f^{\mathrm{variation}} - f^{\mathrm{central}} \right| / f^{\mathrm{central}}$$

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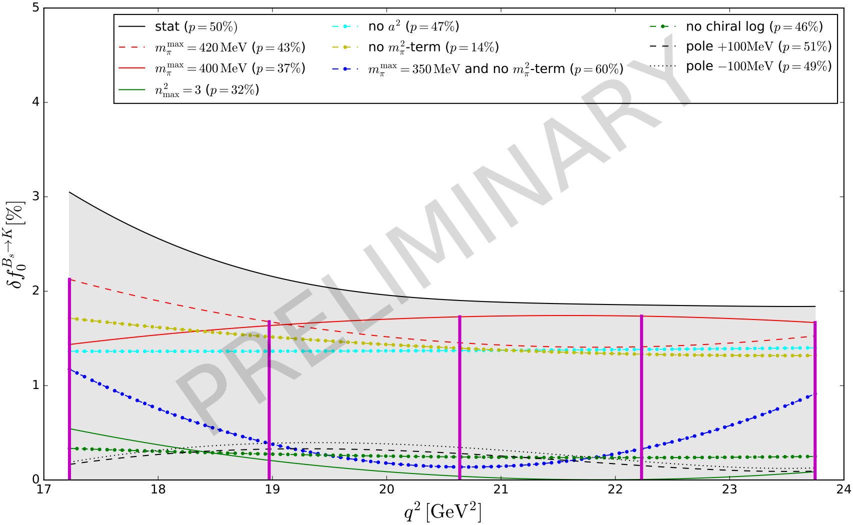
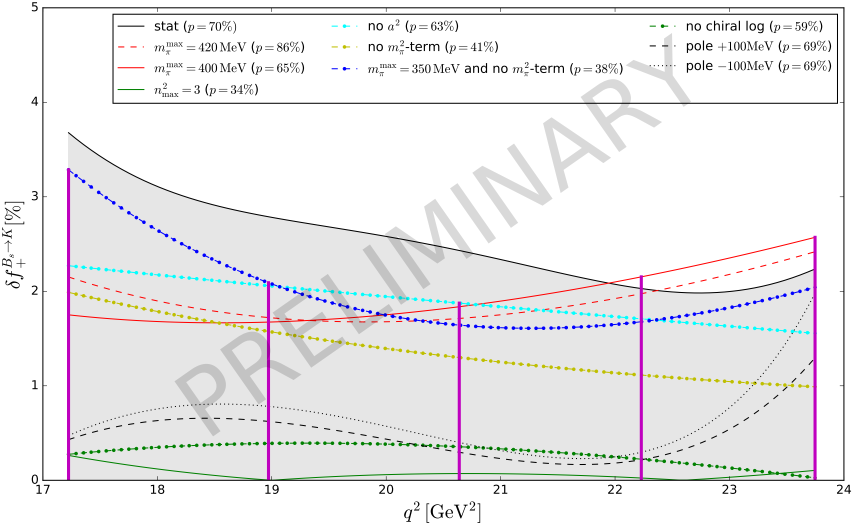


$\Rightarrow$ Read off maximal deviation at reference q^2 values.

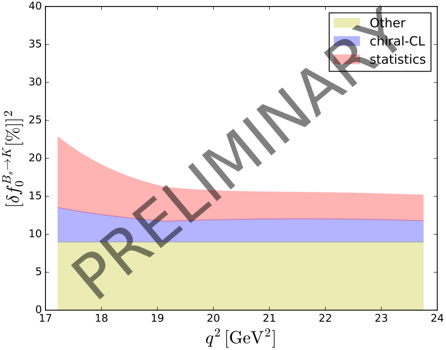
$B_s \to K$: Systematic errors for f_+

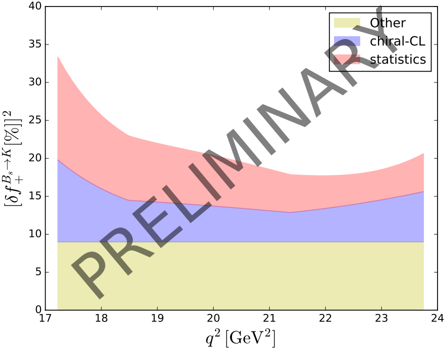


$\Rightarrow$ Read off maximal deviation at reference q^2 values.

$B_s \to K$: Systematic error budget

VERY PRELIMINARY: Error budget still under investigation.

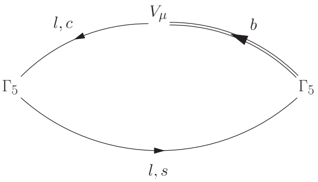




"Other": Higher order corrections, lattice spacing uncertainties, FV, ...
Currently just taken 3% as **placeholder**

$\Rightarrow$ Read off total uncertainty at reference values q^2_{ref}.

$B_s \to D_s$ form factors $f_\parallel$ and $f_\perp$: data



$$f_\parallel(q^2) = \frac{\langle D_s | V_0 | B_s \rangle}{\sqrt{2m_{B_s}}}$$

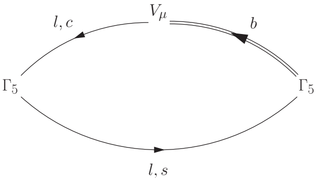
$$f_\perp(q^2) = \frac{\langle D_s | V_i | B_s \rangle}{\sqrt{2m_{B_s}}} \frac{1}{k_{D_s,i}}$$

Simulate multiple charm masses $\Rightarrow$ **range of** m_{D_S}

$$q^2 = q^2(m_{B_s}, m_{D_s}, \mathbf{p}_{D_s}) = m_P^2 + m_D^2 - 2m_P E_D$$

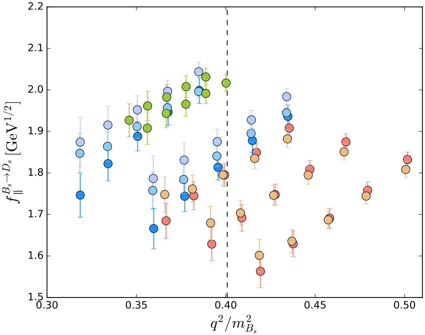
$\Rightarrow$ Different masses give different values of q^2_{max}

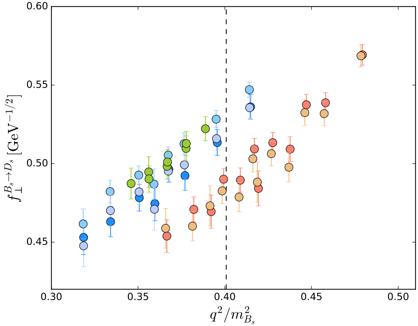
$B_s \to D_s$ form factors $f_\parallel$ and $f_\perp$: data



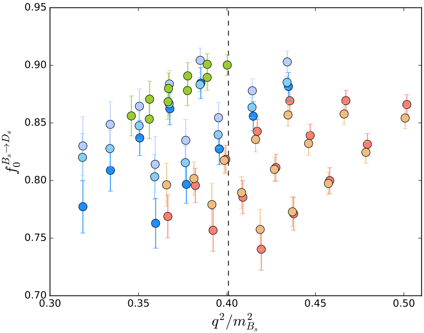
$$f_\parallel(q^2) = \frac{\langle D_s | \, V_0 \, | B_s \rangle}{\sqrt{2 m_{B_s}}}$$

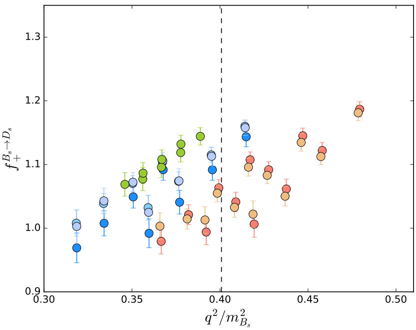
$$f_\perp(q^2) = \frac{\langle D_s | \, V_i \, | B_s \rangle}{\sqrt{2 m_{B_s}}} \frac{1}{k_{D_s, i}}$$





$B_s \to D_s$ form factors f_0 and f_+: data

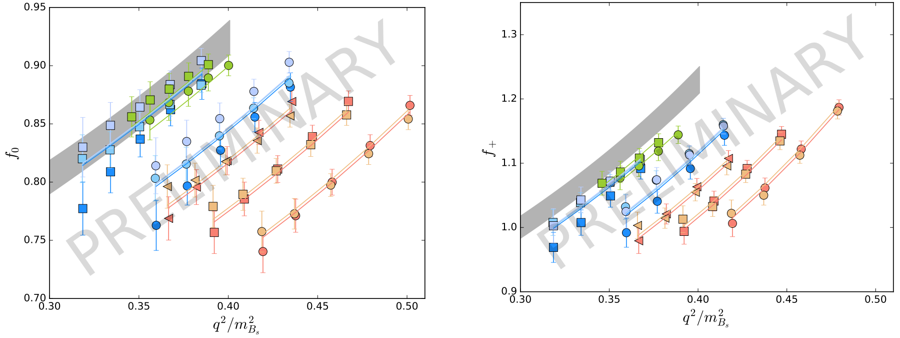




$$f_+(q^2) = \frac{1}{\sqrt{2m_{B_s}}} \left[f_\parallel(E_{D_s}) + (m_{B_S} - E_{D_s})\, f_\perp(E_{D_s}) \right]$$

$$f_0(q^2) = \frac{\sqrt{2m_{B_s}}}{m^2_{B_s} - m^2_{D_s}} \left[(m_{B_s} - E_{Ds}) f_\parallel(E_{D_s}) + \left(E^2_{D_S} - m^2_{D_s}\right) f_\perp(E_{D_s}) \right]$$

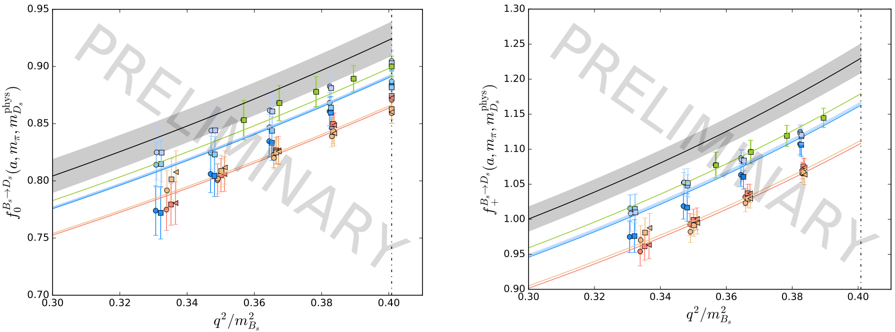
$B_s \to D_s$ form factors f_0 and f_+: fit



$$f(a, m_\pi, m_{D_s}, q^2) = \left[\alpha_1 + \alpha_2 m_\pi^2 + \alpha_{3,j} \sum_{j=1}^{n_{D_s}} [\Delta m_{D_s}^{-1}]^j + \alpha_4 a^2\right] P_{a,b}\left(\frac{q^2}{m_{B_s}^2}\right)$$

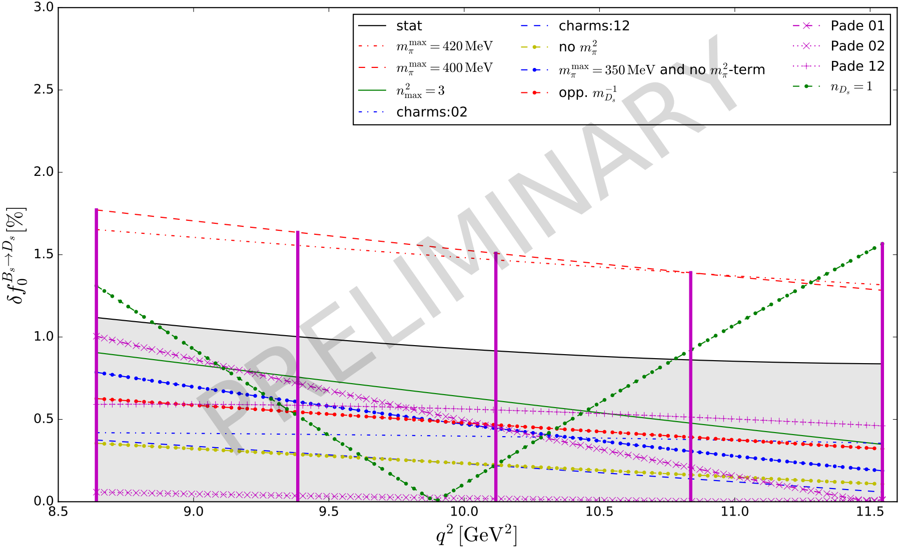
$$\Delta m_{D_s}^{-1} \equiv \left(\frac{1}{m_{D_s}} - \frac{1}{m_{D_s}^{\mathrm{phys}}}\right), \qquad P_{a,b}(x) = \frac{1 + \sum_{i=1}^{a} a_i x^i}{1 + \sum_{i=1}^{b} b_i x^i}$$

$B_s \to D_s$ form factors f_0 and f_+: fit

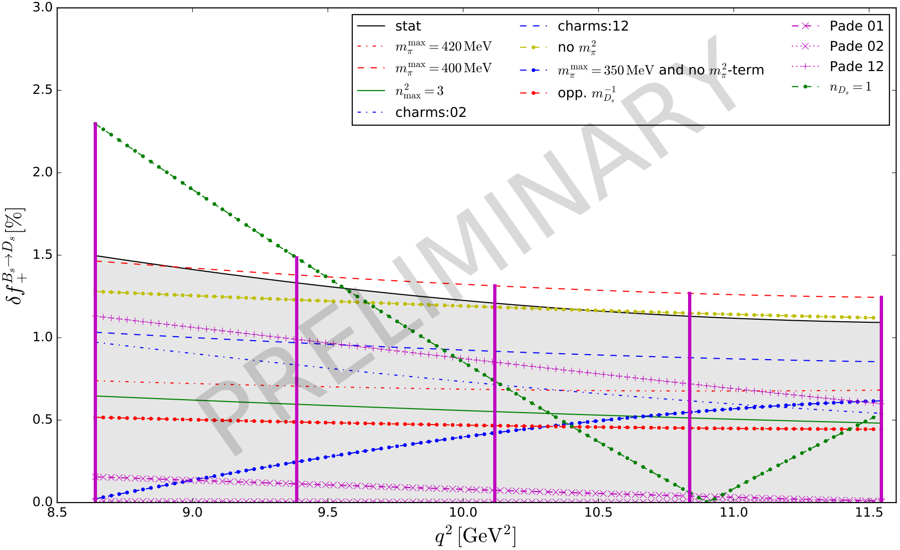


- Uncorrelated for illustration (in practice only use two m_c on coarse ensembles)
- Projected data points to the physical m_{D_s} mass with the fit result overlayed.
- Vary ansatz to assess systematics errors from chiral-CL fit

$B_s \to D_s$: Systematic errors for f_0

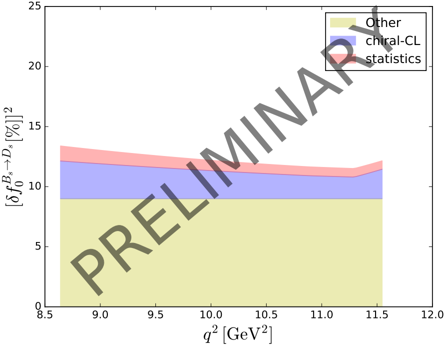


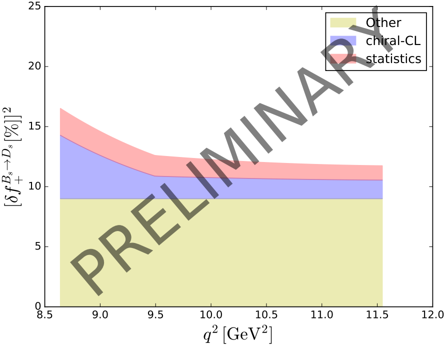
$B_s \to D_s$: Systematic errors for f_+



$B_s \to D_s$: Systematic error budget

VERY PRELIMINARY: Error budget still under investigation.





"Other": Higher order corrections, lattice spacing uncertainties, FV, ...
Currently just taken 3% as **placeholder**

$\Rightarrow$ Read off total uncertainty at reference values q^2_{ref}.

z-expansion: BGL vs BCL

- Completed "lattice analysis" to get form factors for $[q^2_{\mathrm{min,sim}}, q^2_{\mathrm{max}}]$.
- BUT interested in form factors over **full** range $[0, q^2_{\mathrm{max}}]$

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- Map $q^2 \in [0, q^2_{\mathrm{max}}]$ to $z \in [z_{\mathrm{min}}, z_{\mathrm{max}}]$ with $|z| < 1$

$$z(q^2; t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

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BGL: Boyd, Grinstein, Lebed [PRL **74** 4603]:

$$f_X(q^2) = \frac{1}{B_X(q^2)\phi_X(q^2, t_0)} \sum_{n\geq 0} a_n(t_0) z^n$$

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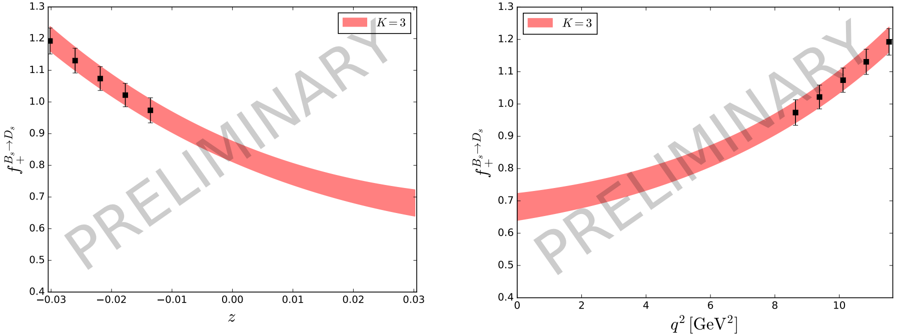
$$f_X(q^2) = \frac{1}{B_X(q^2)\phi_X(q^2, t_0)} \sum_{n\geq 0} a_n(t_0) z^n$$

BCL: Bourrely, Lellouch, Caprini [PRD **82** 099902]:

$$f_+^{BCL}(q^2) = \frac{1}{1 - q^2/m^2_{\mathrm{pole}}} \left[\sum_{k=0}^{K-1} b_k(t_0) \left(z^k - \frac{k}{K}(-1)^{K+k} z^K \right) \right]$$

$B_s \to D_s$: z-expansions in practice

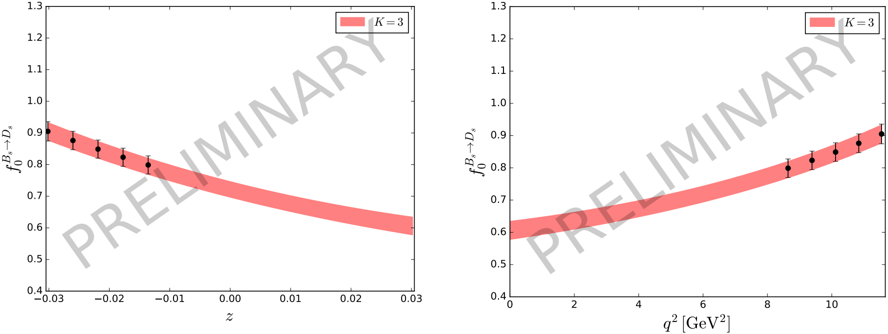
Individual fit of $f_+^{B_s \to D_s}(q^2)$ for BCL with $K = 3$



PRELIMINARY: Error budget will still change

$B_s \to D_s$: z-expansions in practice

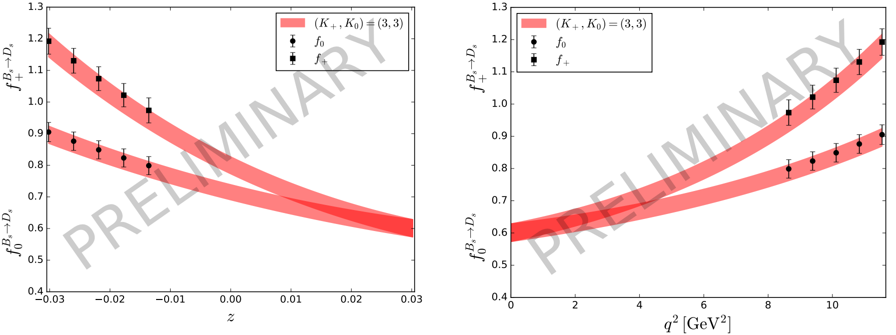
Individual fit of $f_0^{B_s \to D_s}(q^2)$ for BCL with $K = 3$



PRELIMINARY: Error budget will still change

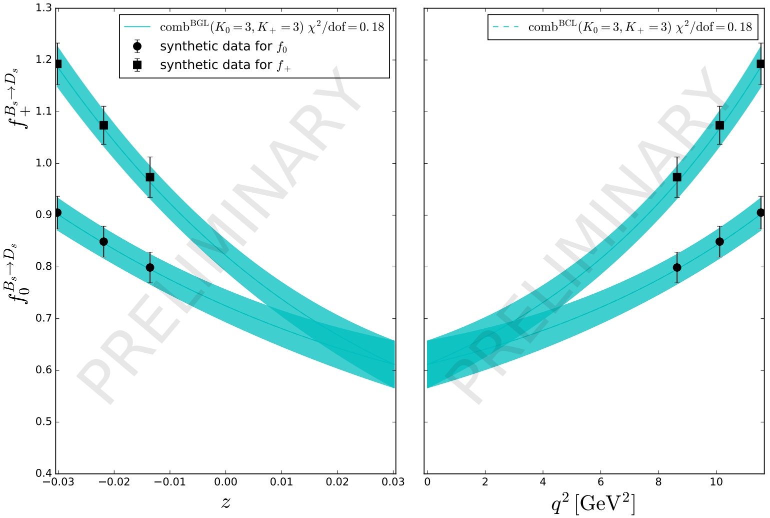
$B_s \to D_s$: z-expansions in practice

Combined fit using $f_0^{B_s \to D_s}(0) = f_+^{B_s \to D_s}(0)$ with BGL $K_+ = 3$, $K_0 = 3$



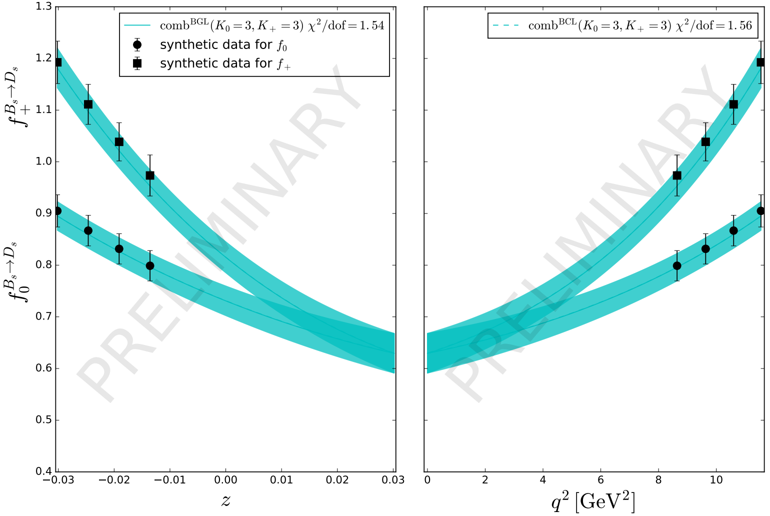
PRELIMINARY: Error budget will still change

$B_s \to D_s$: z-expansions for different number of $N_{\rm ref}$



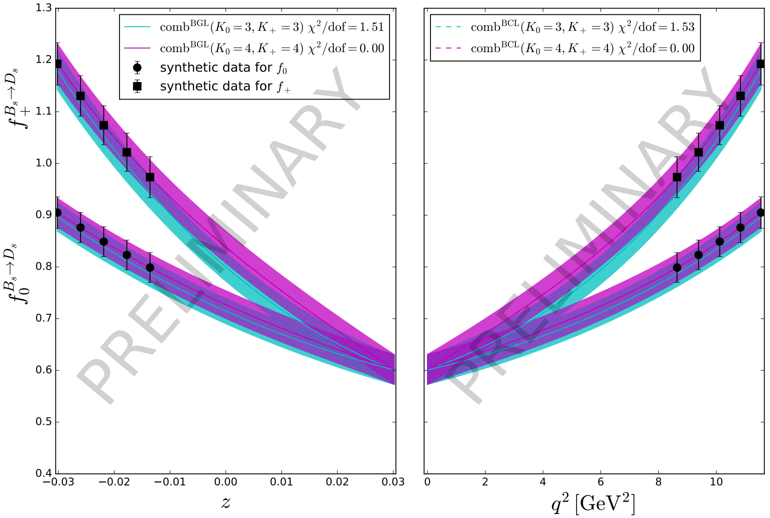
Stable with respect to K, $N_{\rm ref}$ and BGL vs BCL.

$B_s \to D_s$: z-expansions for different number of N_{ref}



Stable with respect to K, N_{ref} and BGL vs BCL.

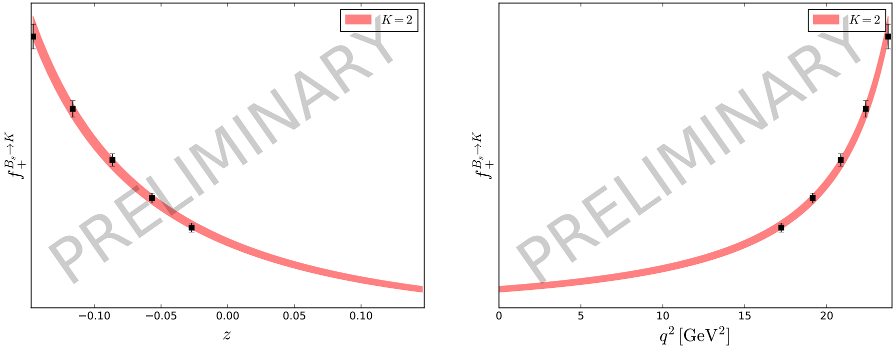
$B_s \to D_s$: z-expansions for different number of N_{ref}



Stable with respect to K, N_{ref} and BGL vs BCL.

$B_s \to K$: z-expansions in practice

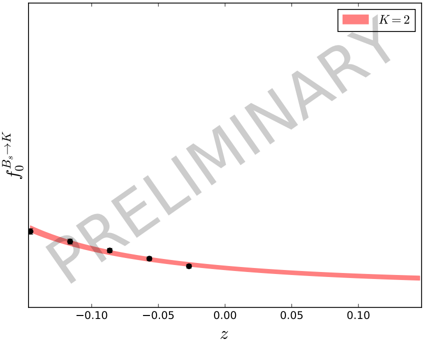
Individual fit of $f_+^{B_s\to K}(q^2)$ for BGL with $K = 2$

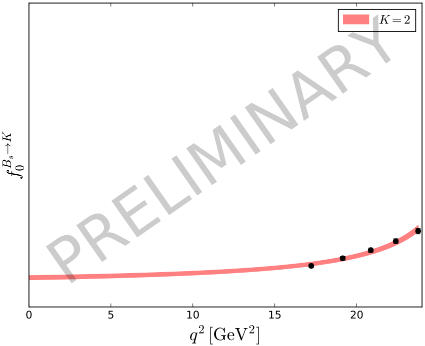


PRELIMINARY: Error budget will still change

$B_s \to K$: z-expansions in practice

Individual fit of $f_0^{B_s \to K}(q^2)$ for BGL with $K = 2$

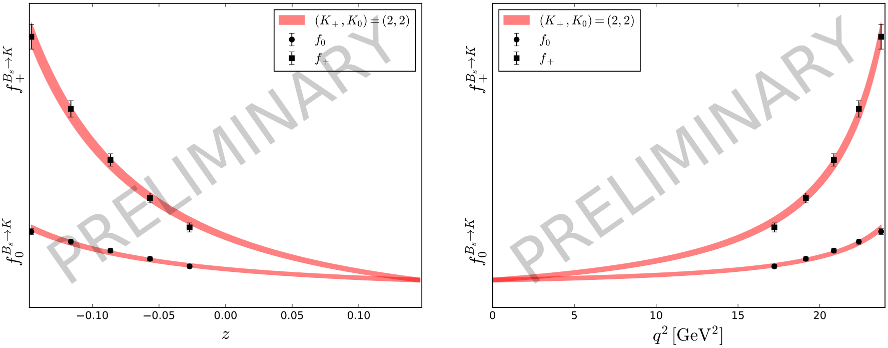




PRELIMINARY: Error budget will still change

$B_s \to K$: z-expansions in practice

Combined fit using $f_0^{B_s\to K}(0) = f_+^{B_s\to K}(0)$ with BGL $K_+ = 2$, $K_0 = 2$



PRELIMINARY: Error budget will still change

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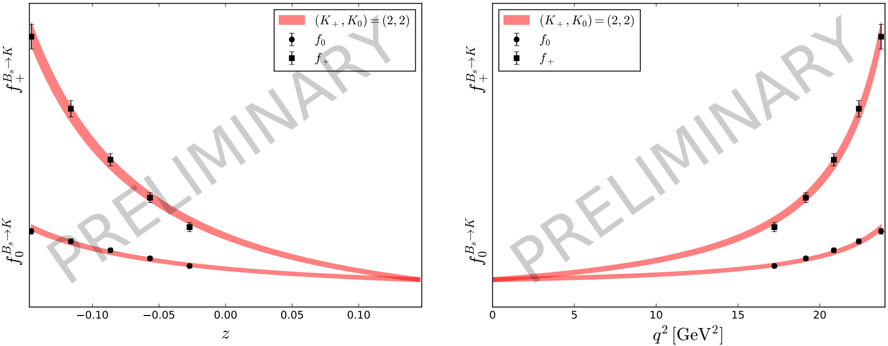


PRELIMINARY: Error budget will still change

- Everything looks sensible so far.
- Finalise error budget
- Repeat same analysis as for $B_s \to D_s$ and perform stability checks

$B_s \to D_s$ and $B_s \to K$: Current status - and to do list

- $B_s \to K$ chiral-continuum limit fit ✓
- $B_s \to D_s$ chiral-continuum-charm inter/extrapolation ✓
- Full systematic error (at synthetic data points ✓)
  - chiral-continuum fit: cut to data, different fit forms, $\cdots$ ✓
  - charm fit: use all m_c or subsets on coarse ✓
  - RHQ, FV, HO disc. errors, isospin, quark mass tunings, $\cdots$ (✓)
- z-expansion over full range
  - BGL vs BCL (✓) vs CLN (✗)
  - Vary number of synthetic data points and different truncations ✓
  - inc vs exc $f_+ \equiv f_0$ at $q^2 = 0$ ✓
- Pheno quantities, e.g. $R(D_s)$
- Comparison to existing literature
  $B_s \to K$: FNAL/MILC 19, HPQCD 14, RBC/UKQCD 15
  $B_s \to D_s$: HPQCD 19 $B_s \to D_s$, JLQCD
- Cross checks, **completely independent second analysis code** ✓

Related HQ projects with DWF by RBC/UKQCD

Edinburgh - Southampton

Peter Boyle, Luigi Del Debbio, Andreas Jüttner, Ava Khamseh, Francesco Sanfilippo, JTT

[JHEP **04** (2016) 037, JHEP **12** (2017) 008]

Edinburgh - Liverpool - Southampton - Boulder - BNL

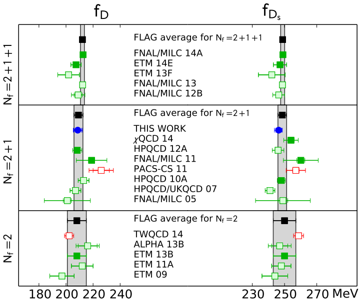
Peter Boyle, Luigi Del Debbio, Nicolas Garron, Andreas Jüttner, Amarjit Soni, JTT, Oliver Witzel

[1712.00862, 1812.08791]

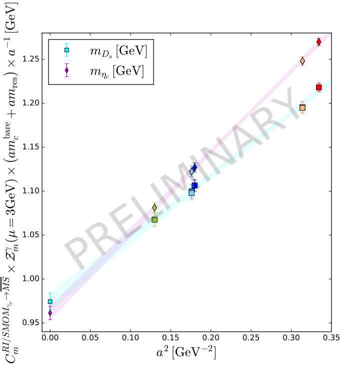
RBC-UKQCD's HQ-DWF program I

- Choice of DW parameters for charm [JHEP **05** (2015) 072,JHEP **04** (2016) 037].
- Leptonic decay constants $f_{D_{(s)}}$, f_{D_s}/f_D [JHEP **12** (2017) 008].
- $a_\mu^{LOHVP;c}$ [PRL **121** (2018) no.2 022003]
- charm mass m_c - ongoing

[JHEP **12** (2017) 008]



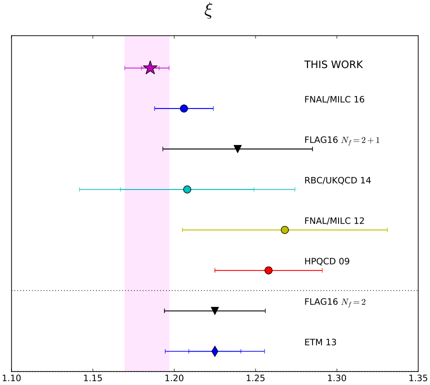
[arXiv:1712:00862]

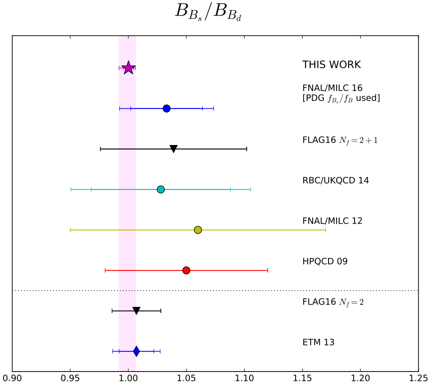


RBC-UKQCD's HQ-DWF program II

Extrapolation from heavier-than-charm to bottom

- $SU(3)$-breaking ratios for $D_{(s)}$ and $B_{(s)}$ mesons [arXiv:1812.08791]
  - Ratios of decay constants: f_{D_s}/f_D, f_{B_s}/f_B $(\Rightarrow |V_{cd}/V_{cs}|)$
  - Ratios of bag parameters, ξ $(\Rightarrow |V_{td}/V_{ts}|)$





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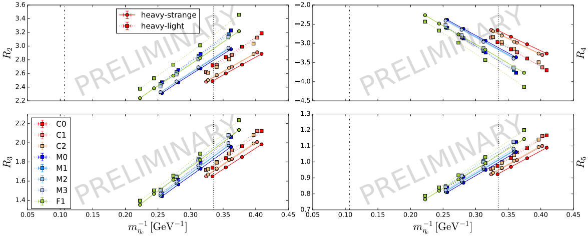
- Analogous study to $K - \bar{K}$ BSM mixing analysis [1812.04981]
  - $D^0 - \bar{D}^0$ (short-distance part)
  - $B^0_{(s)} - \bar{B}^0_{(s)}$ (supplement with very fine JLQCD ensembles)



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  - $D^0 - \bar{D}^0$ (short-distance part)
  - $B^0_{(s)} - \bar{B}^0_{(s)}$ (supplement with very fine JLQCD ensembles)
- Combined fit of two data-sets for m_c, $a_\mu^{LOHVP,c}$
- Individual decay constants, bag parameters (as opposed to ratios)
- $D_{(s)}$ semi-leptonic runs planned

Conclusions and Outlook

$B_s \to K$, $B_s \to D_s$ (now)

- Data on 6 ens ($N_f = 2+1$)
  $a \in 0.07 - 0.11\,\mathrm{fm}$
  $m_\pi \in 235 - 430\,\mathrm{MeV}$
  DWF for l,s,c, RHQ for b
- "Lattice analysis":
  Very nearly done ✓
- "Continuum analysis":
  z-expansion for $B_s \to D_s$ ✓
  z-expansion for $B_s \to K$ (✓)
- draft in preparation

Conclusions and Outlook

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$B_s \to K$, $B_s \to D_s$ (future)

- Add m_π^{phys} ensemble

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$B_s \to K$, $B_s \to D_s$ (future)

- Add m_π^{phys} ensemble

Other processes

ANALYSIS UNDERWAY

- $B \to \pi\ell\nu$
- $B \to D\ell\nu$

MORE DATA ON DISK

- $B \to D^*\ell\nu$
- $B_s \to D_s^*\ell\nu$

Conclusions and Outlook

$B_s \to K$, $B_s \to D_s$ (now)

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- $B \to D\ell\nu$

MORE DATA ON DISK

- $B \to D^*\ell\nu$
- $B_s \to D_s^*\ell\nu$

Complementary DWF program

- $f_{D_{(s)}}$, $f_{B_{(s)}}$, B_{B_s}/B_{B_d}, ξ
- m_c, $a_\mu^{LOHVP,c}$
- BSM mixing for $K - \bar{K}$, $D - \bar{D}$, $B_{(s)} - \bar{B}_{(s)}$