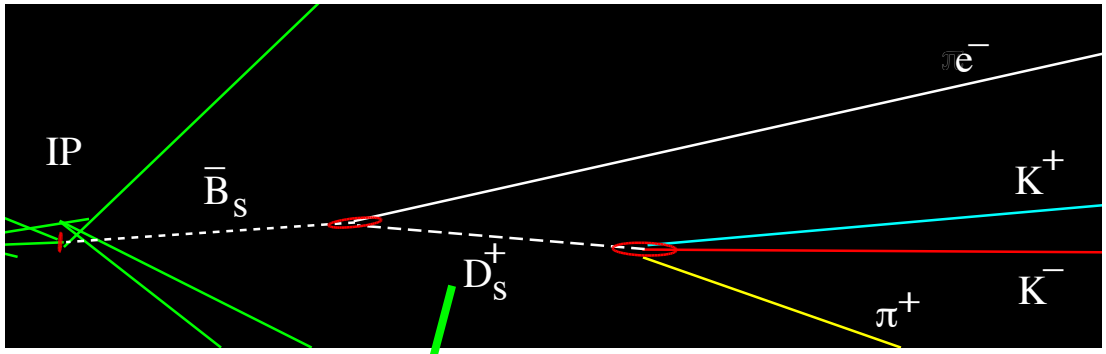
Semileptonic B decays from Lattice QCD

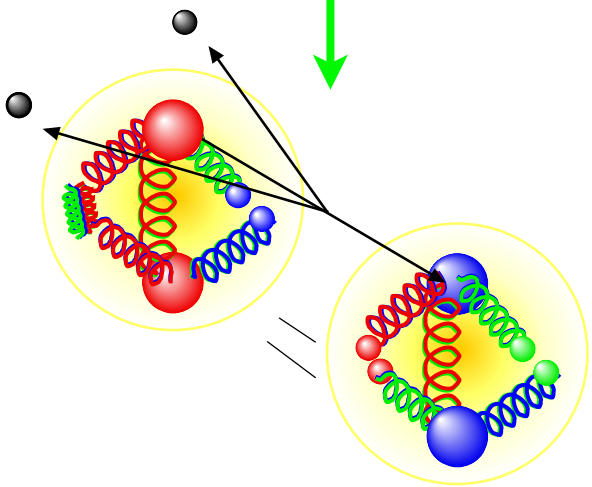
Christine Davies
University of Glasgow
HPQCD collaboration

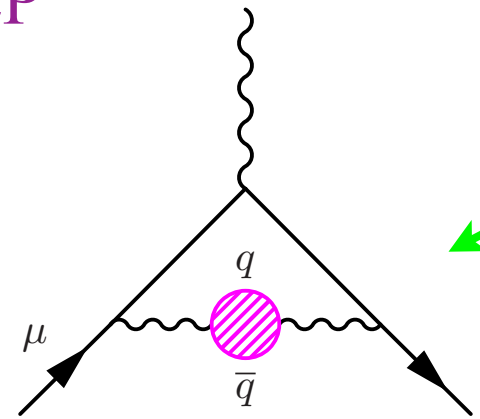
Orsay
Sept 2019

Quark low-energy strong interactions are a major complication in testing the Standard Model

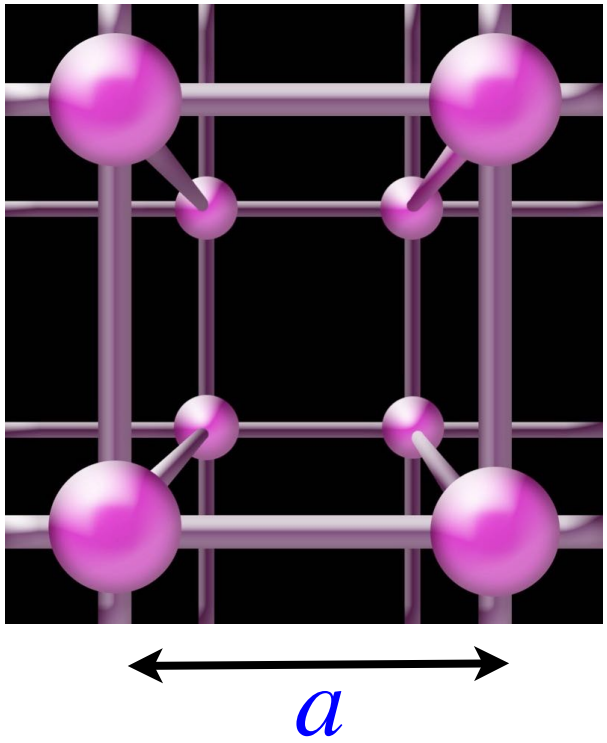


ALEPH@LEP





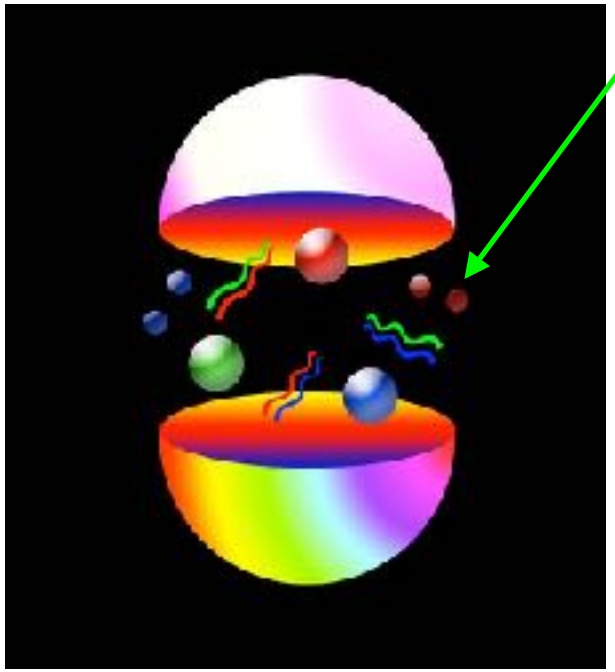
An accurate nonperturbative treatment of QCD is necessary to compare SM and low-energy experimental tests for new physics



Lattice QCD provides such an approach.
Must be able to determine results in physical continuum limit.

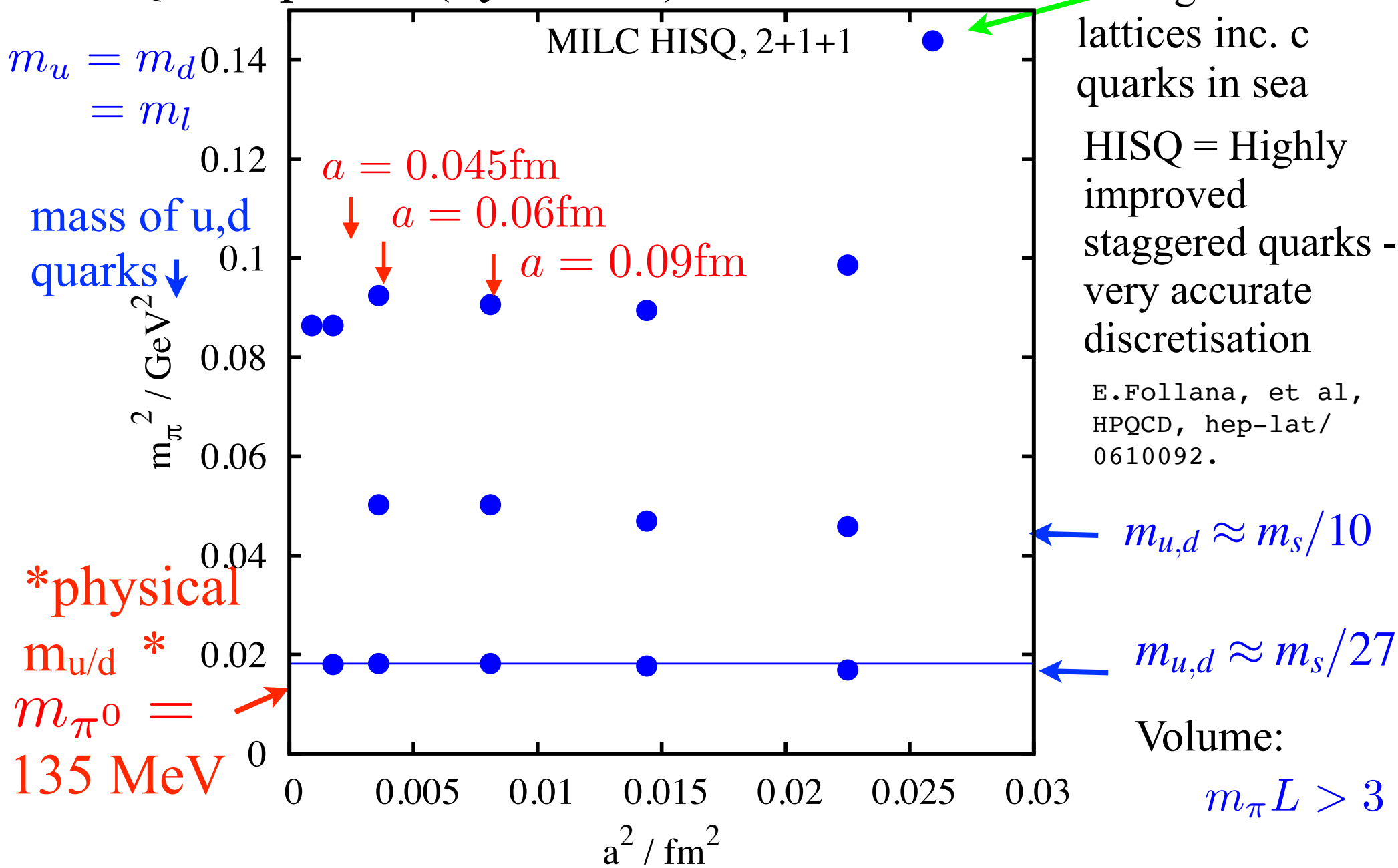
Final accuracy depends on :

- control of lattice spacing dependence
- tuning of quark masses
- normalisation of operators



Use HPQCD's Highly Improved Staggered Quark (HISQ) action since it has small discretisation errors, is numerically efficient, fully nonperturbative operator norm. is usually possible.

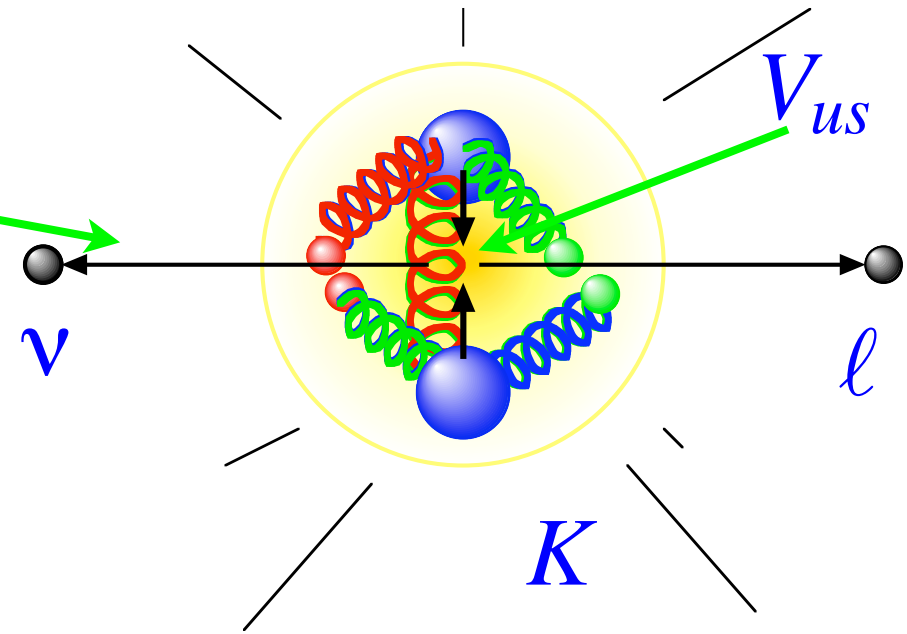
Parameters for gluon field configurations generated with HISQ sea quarks (by MILC)



n_f =1+1+1+1 also now being done

Weak decays probe hadron structure and quark couplings.
(Semi)-leptonic decays and mixing calculable in lattice QCD

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to l\nu & B \to \pi l\nu \\ & K \to \pi l\nu & \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to D l\nu \\ D \to \pi l\nu & D \to K l\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \overline{B}_d \rangle & \langle B_s | \overline{B}_s \rangle & \end{pmatrix}$$



$$Br(M \to \mu\nu) \propto V_{ab}^2 f_M^2$$

CKM matrix

Expt = CKM x theory(QCD)

Need precision lattice QCD to get accurate CKM elements to test Standard Model.

Aim : sub-1% errors

If V_{ab} known, compare lattice to experiment to test QCD

ALSO - test corresponding electromagnetic processes

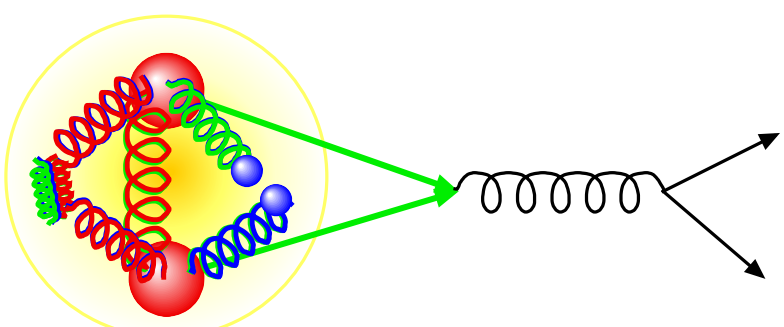
Most straightforward calculations are '2-point' current-current correlation functions - yield meson masses and decay constants, f

$$\langle 0|J^\dagger(T)J(0)|0\rangle = \sum_n a_n^2 e^{-m_n T}$$

masses of all hadrons with quantum numbers of J

$$a_n = \frac{\langle 0|J|n\rangle}{\sqrt{2m_n}} = \frac{f_n\sqrt{m_n}}{2}$$

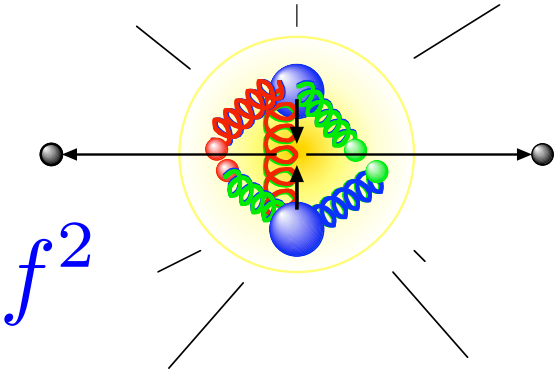
ground-state (n=0) most accurate



Amplitude to annihilate (W for charged pseudoscalar, photon for neutral vector) is proportional to f

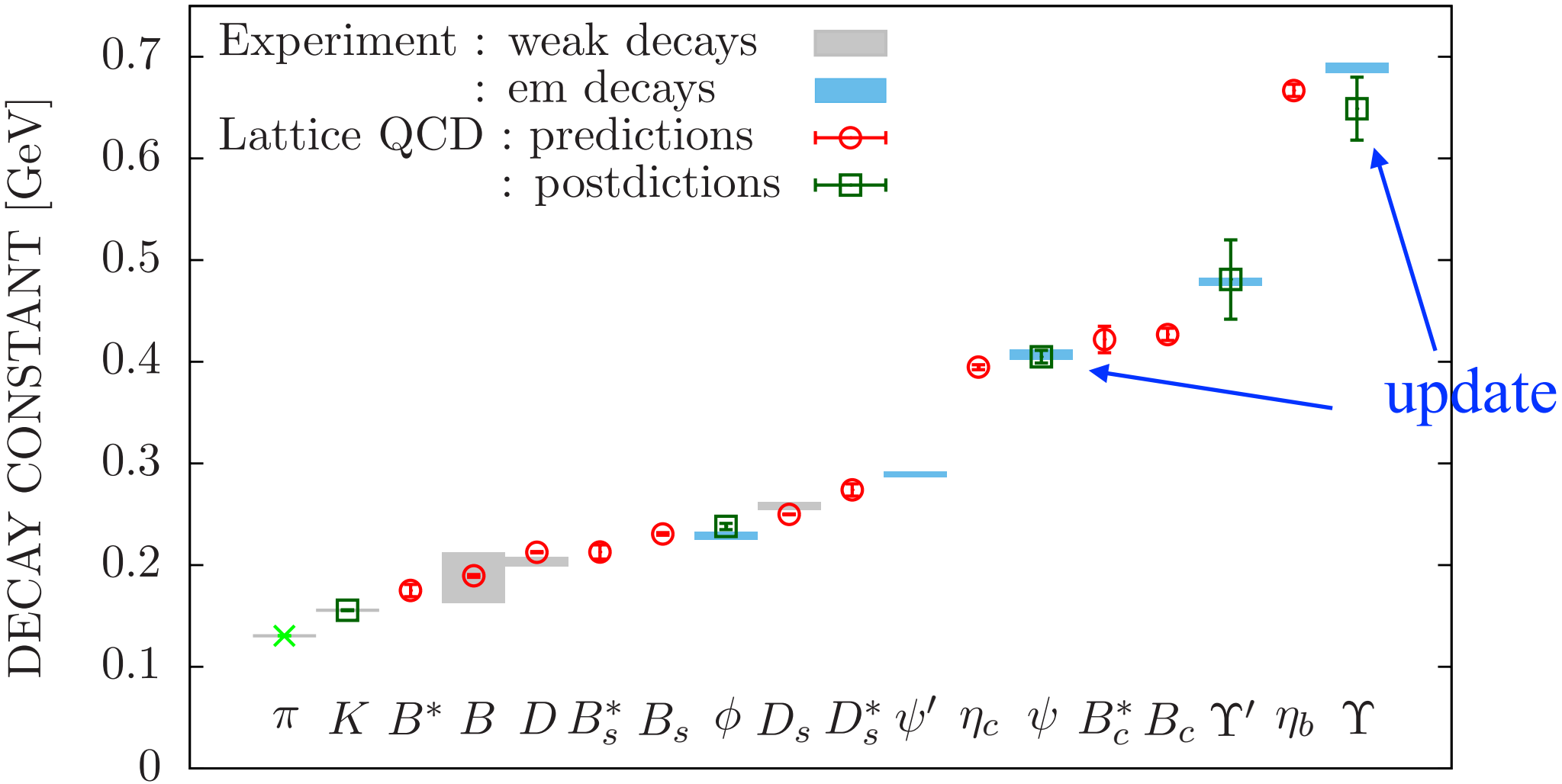
Meson decay constant summary

Parameterises hadronic information needed for annihilation rate to W or photon:



$$\Gamma \propto f^2$$

ordered by value:



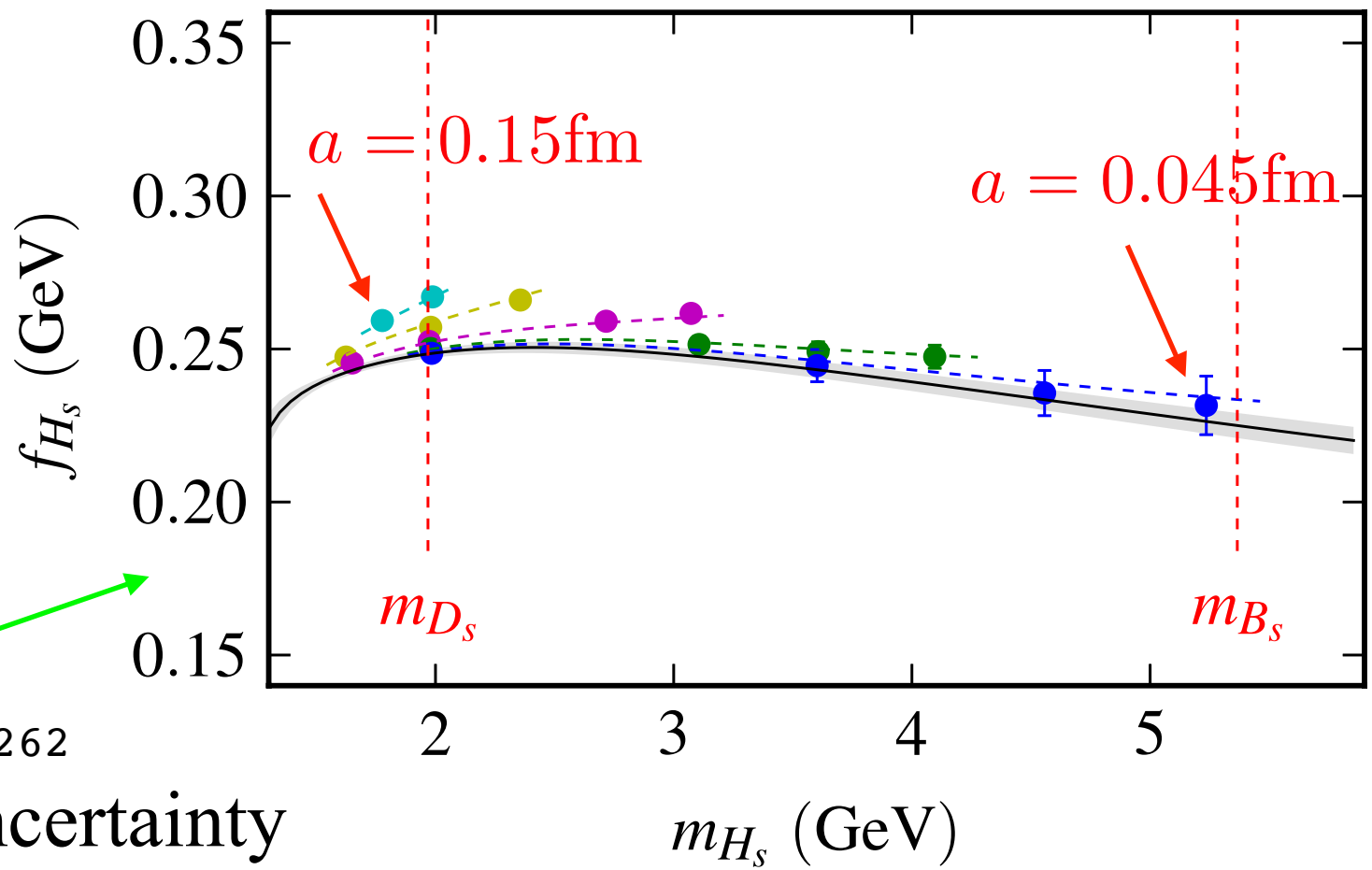
HPQCD, 1208.2855, 1312,5264, 1408.5768, 1503.05762, 1703.05552, FNAL/MILC 1712.09262

A key ingredient for accurate decay constants is having a fully nonperturbative normalisation for J.
Possible for relativistic formalisms such as HISQ.
To handle b quarks then requires very fine lattices ($am_b < 1$) and mapping out discretisation effects - 'heavy-HISQ' approach

Works well for heavy-light decay constants. (J abs. norm. by PCAC)

HPQCD 1110.4510

FNAL/MILC 1712.09262



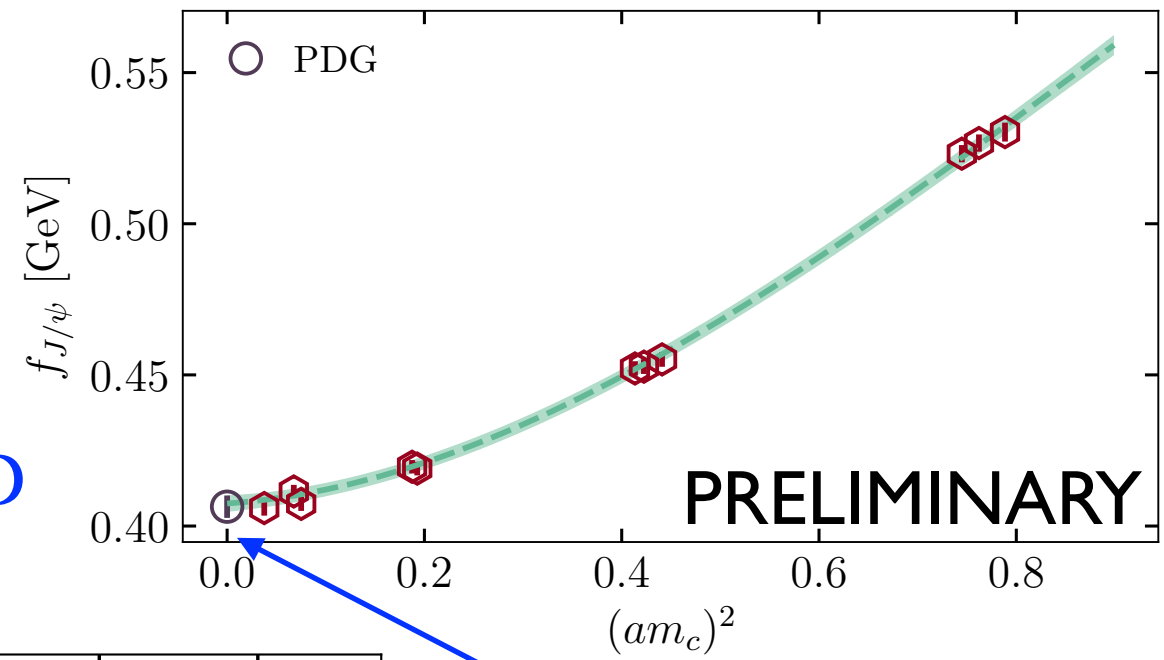
<1% uncertainty

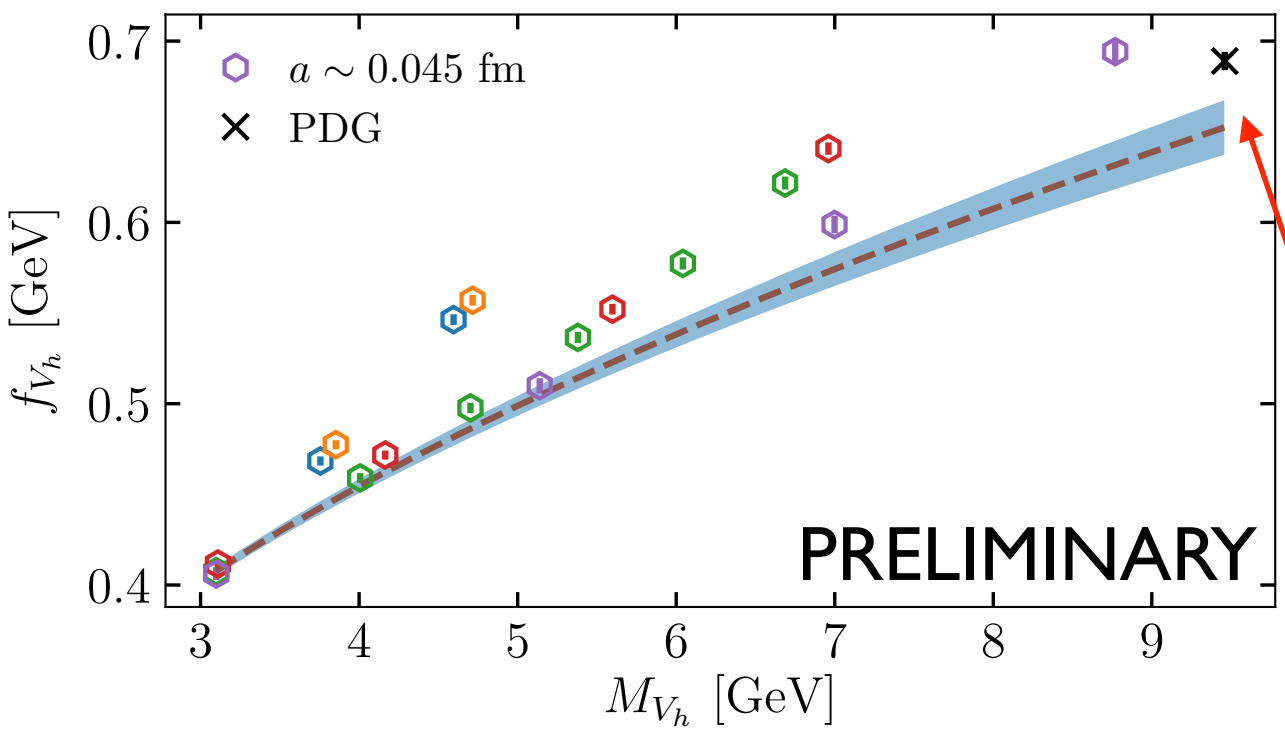
Improved lattice QCD results for vector heavyonium

D. Hatton et al, HPQCD, 1908.10116, 1909.00756

Use RI-SMOM Z_v (with exact lattice Ward-Takahashi identity)

Now adding quenched QED - impact < 0.5%.





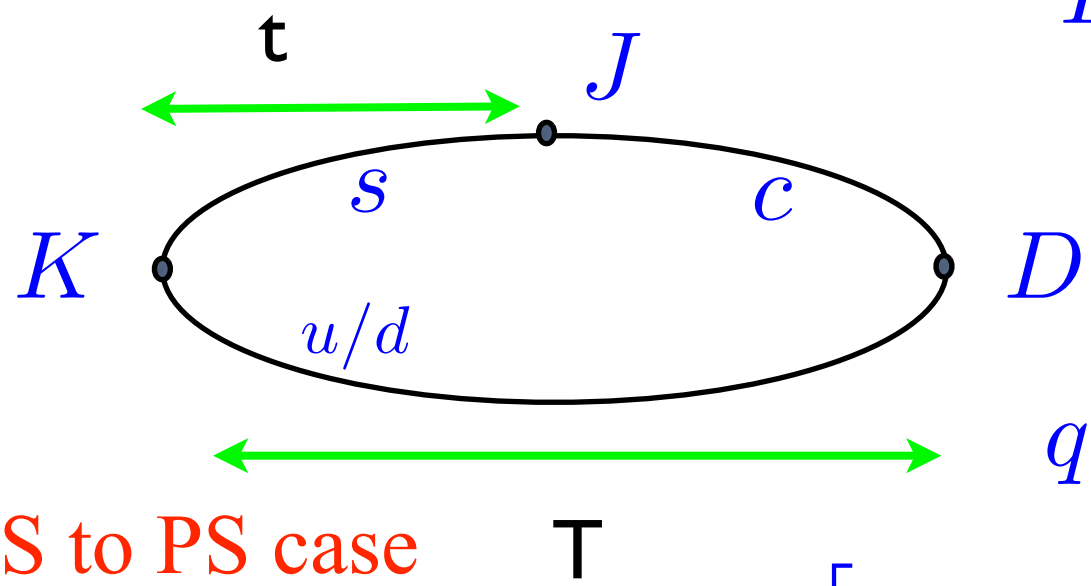
J/ψ result agrees at < 1% with PDG

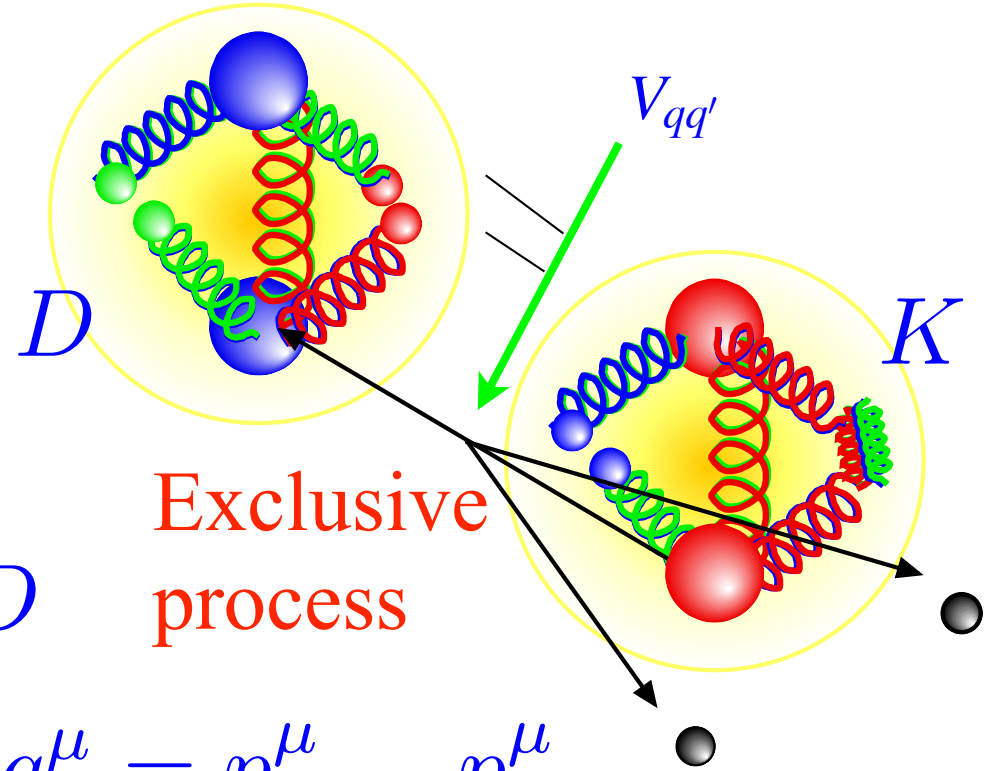
$$\Gamma(V \to e^+e^-) = \frac{4\pi}{3}\alpha^2 Q^2 \frac{f_V^2}{M_V}$$

In Υ case discrepancy is emerging - working on even finer lattices

Semileptonic form factors

QCD info. encoded in form factors, functions of q^2





Exclusive process

$$q^\mu = p_D^\mu - p_K^\mu$$

PS to PS case

$$< K|V^\mu|D >= f_+(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right]$$

$$+ f_0(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu$$

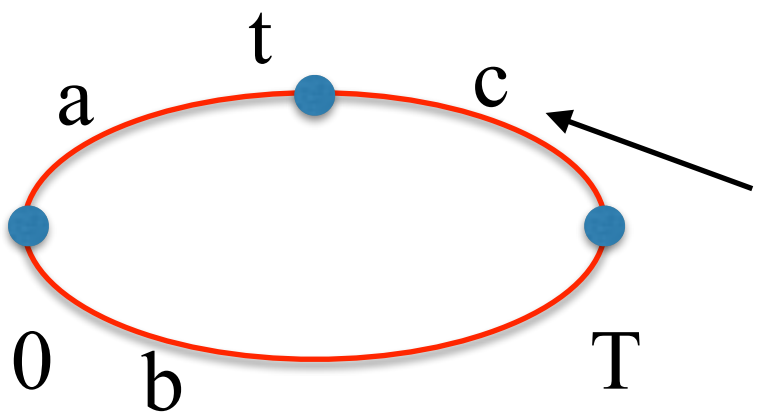
measured by expt through rate for e.g $D \to Kl\nu$

$$f_0(0) = f_+(0)$$

$$< K|S|D >= \frac{M_D^2 - M_K^2}{m_{0c} - m_{0s}} f_0(q^2)$$

abs. norm. for same c/s action HPQCD:1008.4562. Gives V normln at q^2max

Lattice QCD calculation - needs 3-point function.



- Calculate a, b propagators from 0
- Calculate c prop from T using b as source and Γ as req'd to make meson bc
- Tie a and c props together at t with Γ as req'd for current (V-A)
- For heavy-HISQ approach use multiple masses for a, b or c

- Average results over ensemble

Fit 3-point to: (simultaneously with 2-point)

$$C_{3\mathrm{pt}} = \sum_{j,k} a_j^{ab} J_{jk} a_k^{bc} f(E_j^{ab}, t) f(E_k^{bc}, T-t)$$

$$f(E,t) = e^{-Et} + e^{-E(L_t - t)}$$

$$\langle ab|J|bc\rangle = 2\sqrt{E_0^{ab} E_0^{bc} J_{00}}$$

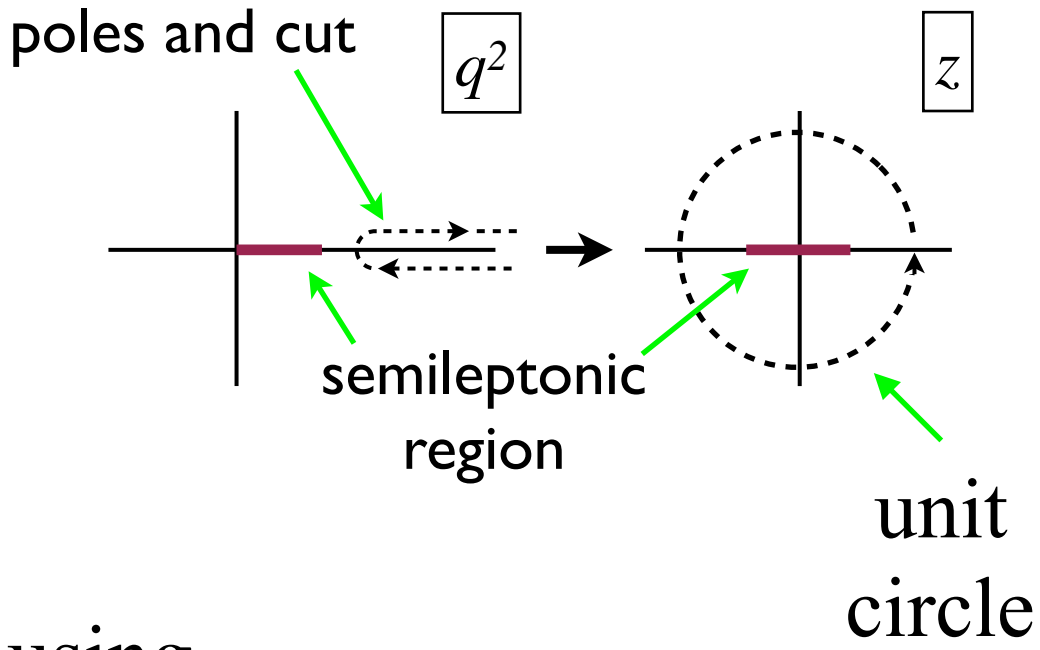
Give momentum to quark a to cover q^2 range

Fitting as a function of q^2

$$H \to L$$

decay amplitude has cut starting at

$$q^2 = (M_H + M_L)^2$$



map q^2=t plane to z-plane using

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (M_H \pm M_L)^2$$

we take $t_0 = 0$

SL region then well within unit circle. Fit form factors to:

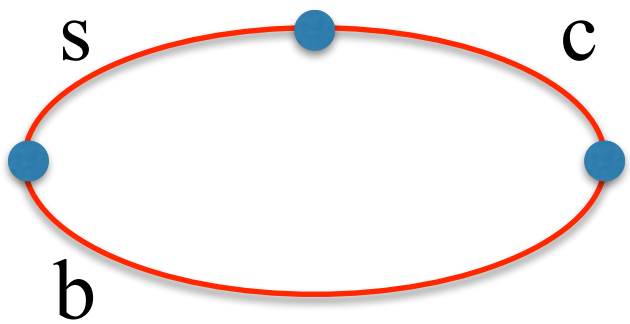
$$F(t) = \frac{1}{P(t)} \sum b_k z^k$$

Allow for disc. effects and m_h dependence in b_k

$B_c \to B_s \ell \nu$

on LHCb list for near future

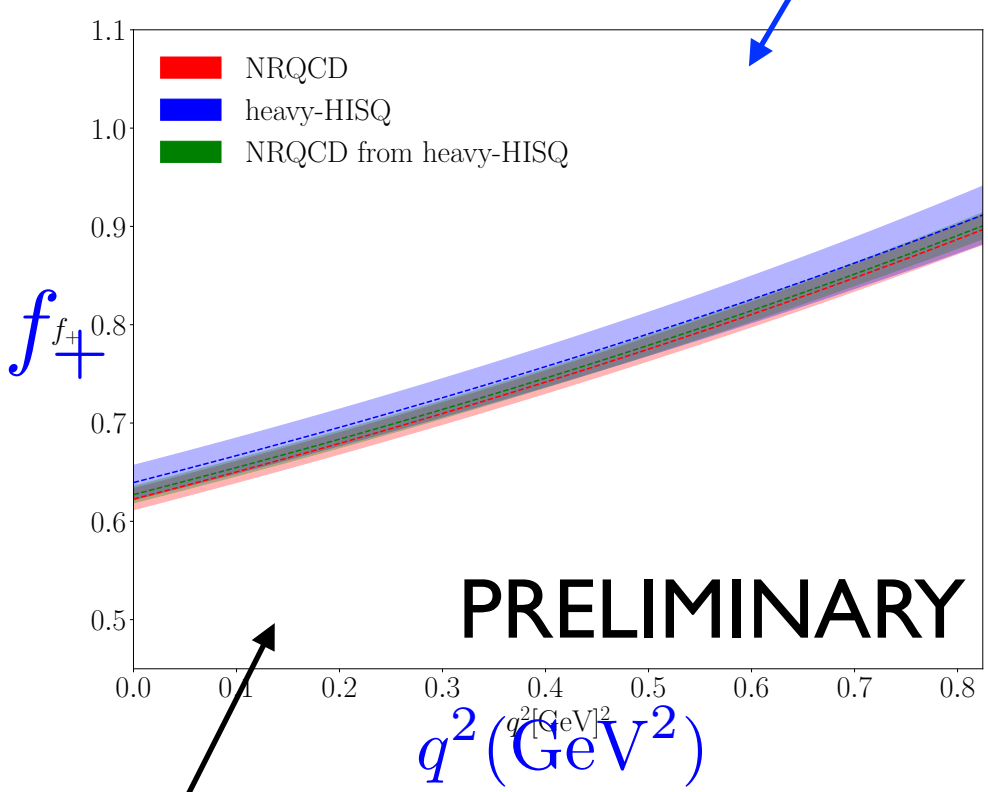
L. Cooper et al, HPQCD, Lattice2019

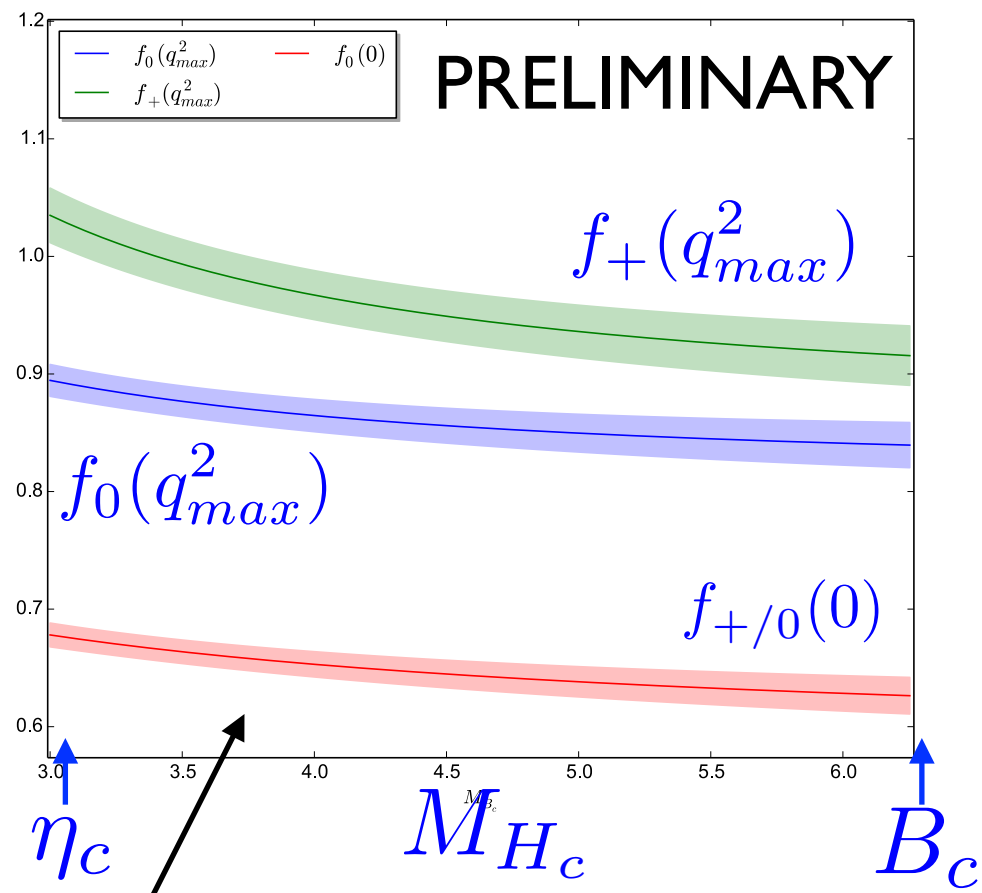


small q^2 range easily covered on lattice

can compare to $D \to K \ell \nu$

B. Chakraborty et al, HPQCD, Lattice2019





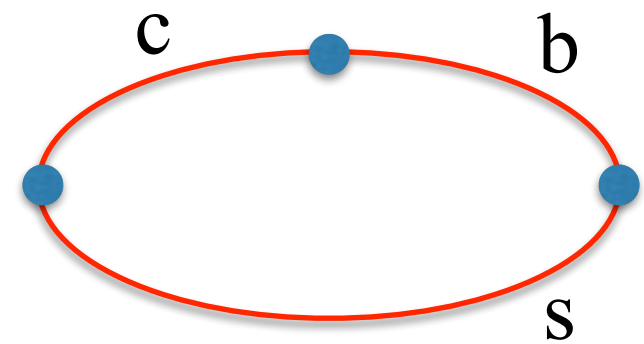
heavy HISQ and NRQCD b agree

spectator mass dependence is mild

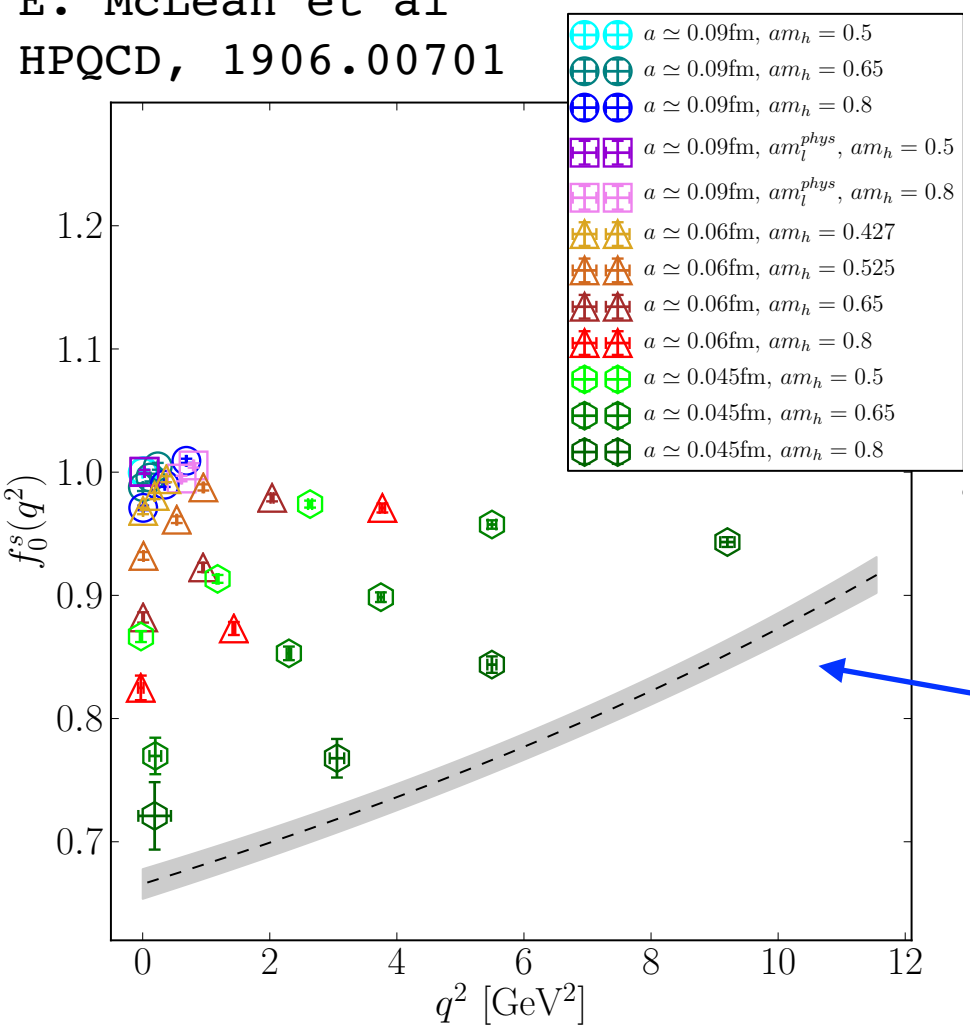
B meson form factors from b quark decay - much bigger q^2 range. How to cover this range in lattice QCD?

$$|\vec{p}_X|_{q^2=0} = \frac{M_B}{2} - \frac{M_X^2}{2M_B}$$

Large momenta need fine lattices



E. McLean et al
HPQCD, 1906.00701



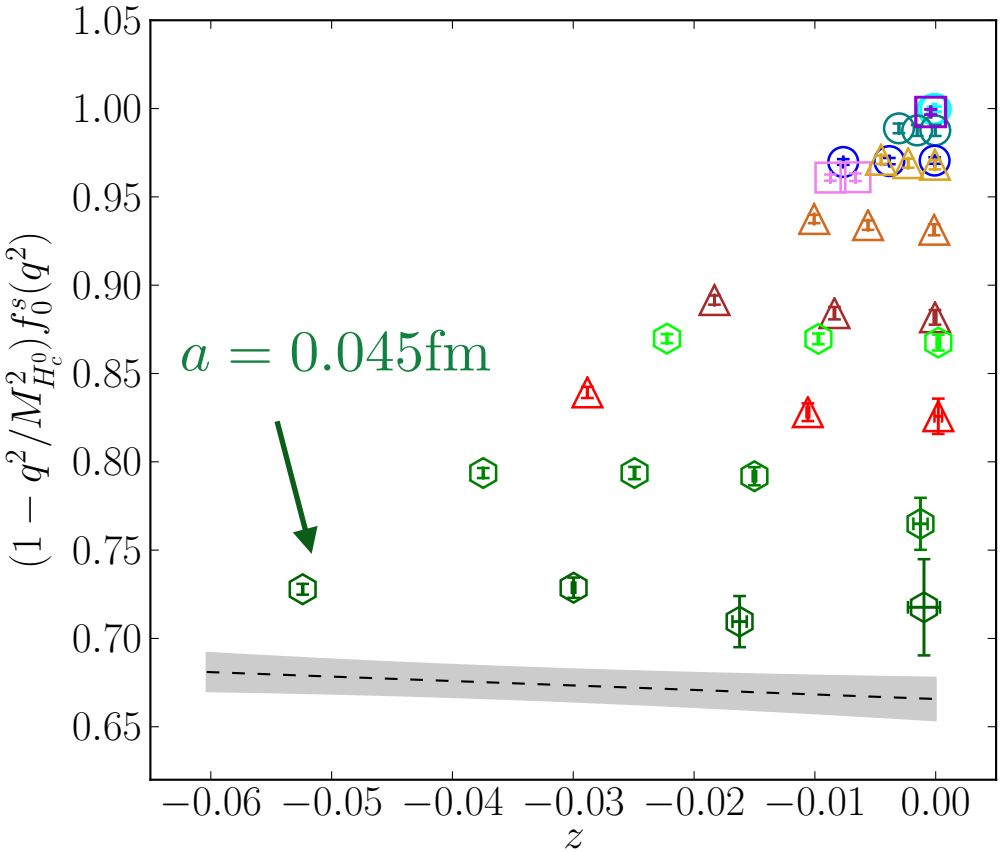
$B_s \to D_s \ell \nu$ for Vcb

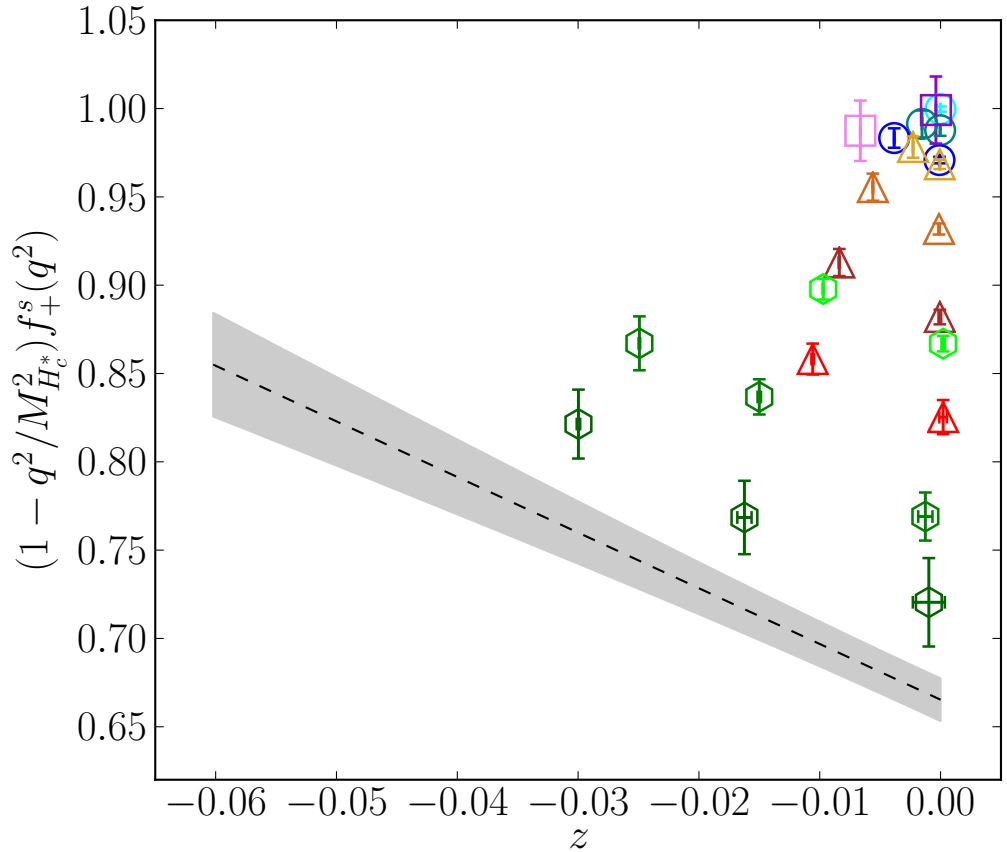
calculate for a range of heavy quark masses ($am_h < 1$) for a range of fine lattice spacings.

Fit $f(m_h, a, z)$ and extract physical curve at $m_h = m_b$.

q^2 range grows with m_h as lattices get finer

Interpolate in z-space and fit dependence on heavy quark mass and lattice spacing





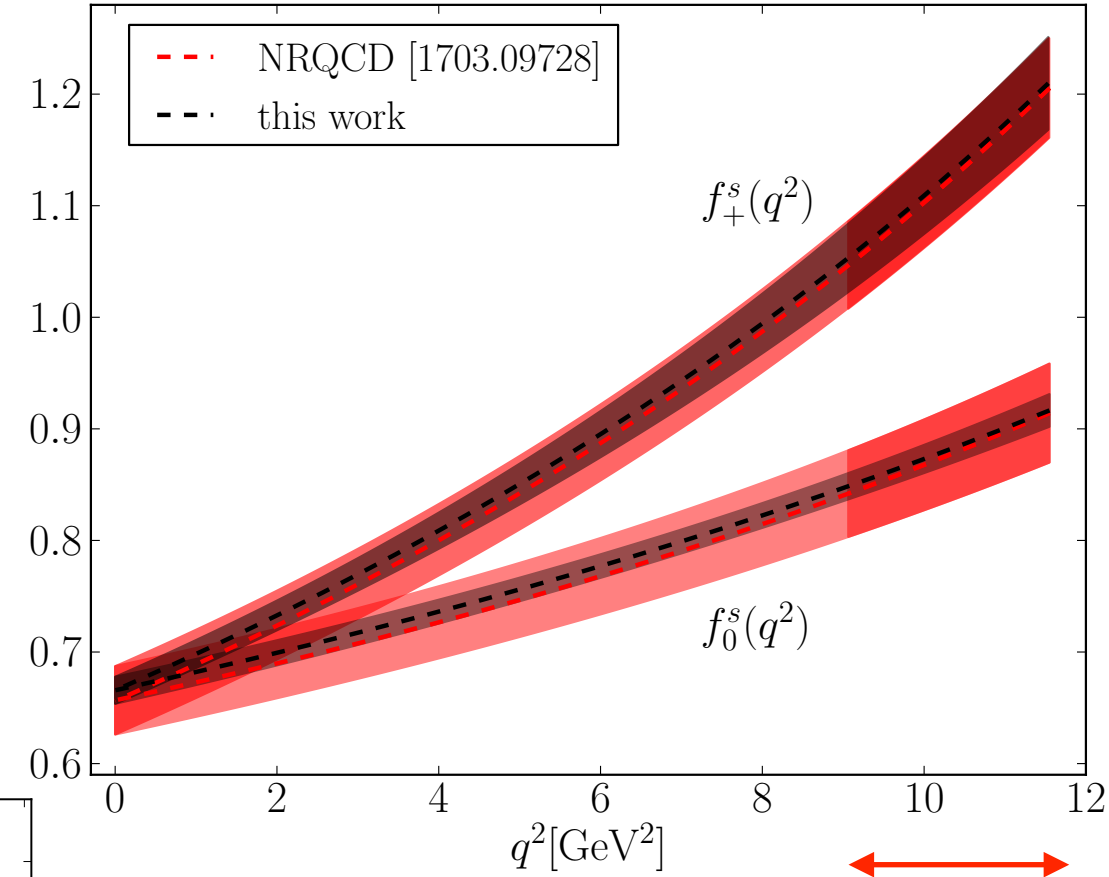
Form of coefficients of z-expansion:

$$a_n^{0,+} = \left(1 + \rho_n^{0,+} \log\left(\frac{M_{\eta_h}}{M_{\eta_c}}\right)\right) \times \sum_{i,j,k=0}^{2,2,2} d_{ijkn}^{0,+} \left(\frac{2\Lambda_{\text{QCD}}}{M_{\eta_h}}\right)^i \left(\frac{am_{h0}^{\text{val}}}{\pi}\right)^{2j} \left(\frac{am_{c0}^{\text{val}}}{\pi}\right)^{2k} \times \left(1 + \mathcal{N}_{\text{mistuning},n}^{0,+}\right) . \quad (29)$$

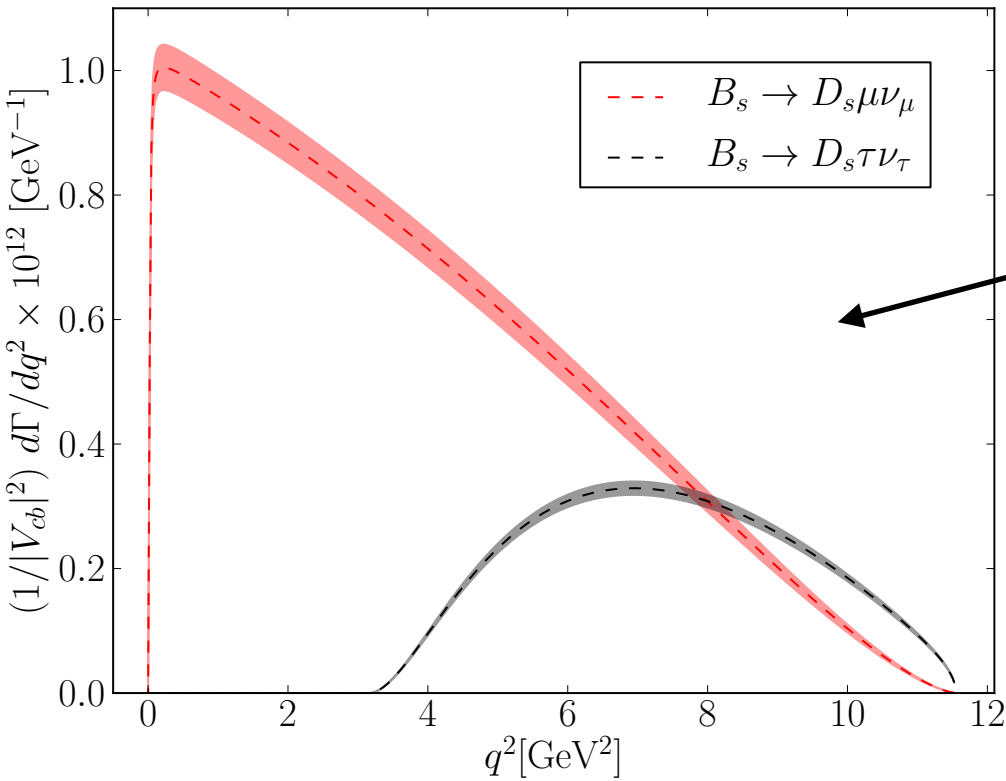
E. McLean et al
HPQCD, 1906.00701

Final result agrees with (but more accurate than) non relativistic methods that have to extrapolate in q^2 from close to zero-recoil.

E. McLean et al
HPQCD, 1906.00701



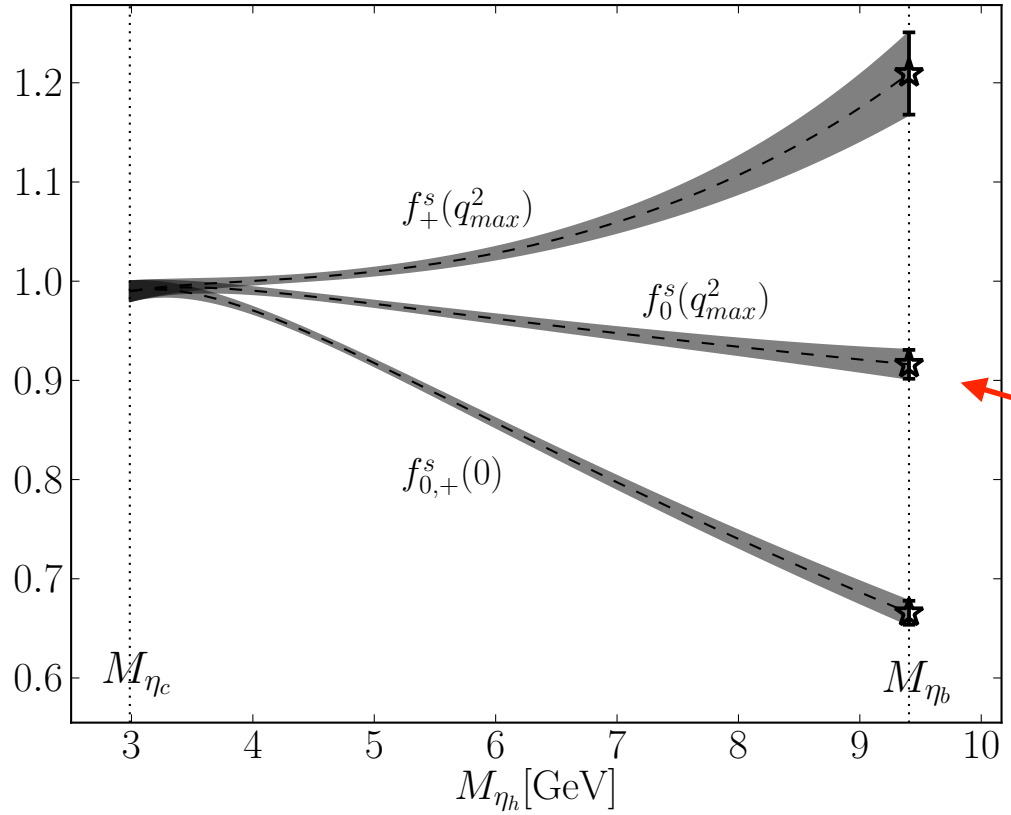
NRQCD
q^2 range



Ratio of branching fractions to τ and μ

$$R(D_s) = 0.2987(46)$$

With expt, can give Vcb.
Extend to B to D case.



Determine dependence of form factors on heavy quark mass

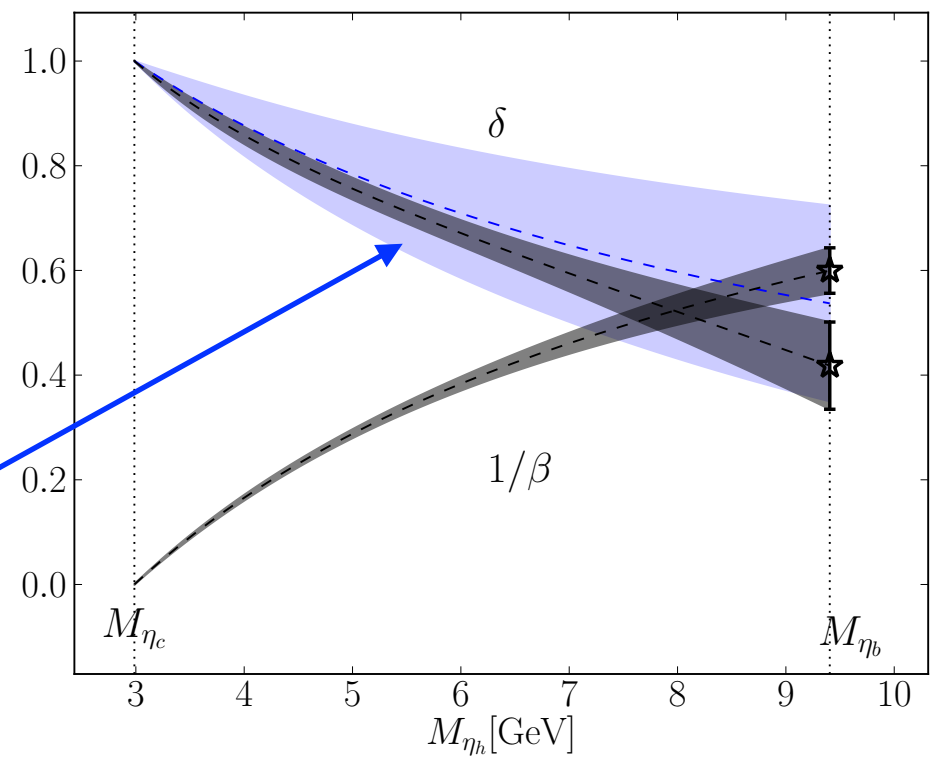
E. McLean et al
HPQCD, 1906.00701

$$\frac{1}{\beta(m_h)} \equiv \frac{M_{H_s}^2 - M_{D_s}^2}{f_+^s(0)} \left.\frac{df_+^s}{dq^2}\right|_{q^2=0}, \tag{43}$$

$$\delta(m_h) \equiv 1 - \frac{M_{H_s}^2 - M_{D_s}^2}{f_+^s(0)} \left(\left.\frac{df_+^s}{dq^2}\right|_{q^2=0} - \left.\frac{df_0^s}{dq^2}\right|_{q^2=0} \right). \tag{44}$$

LO HQET

R.Hill hep-ph/0606023



Pseudoscalar to vector meson semileptonic decay

e.g. $B_{(s)} \to D^*_{(s)} \ell\nu \qquad B_c \to J/\psi \ell\nu$

To map full q^2 range, need both V and A matrix elements and calculation of 4 form factors:

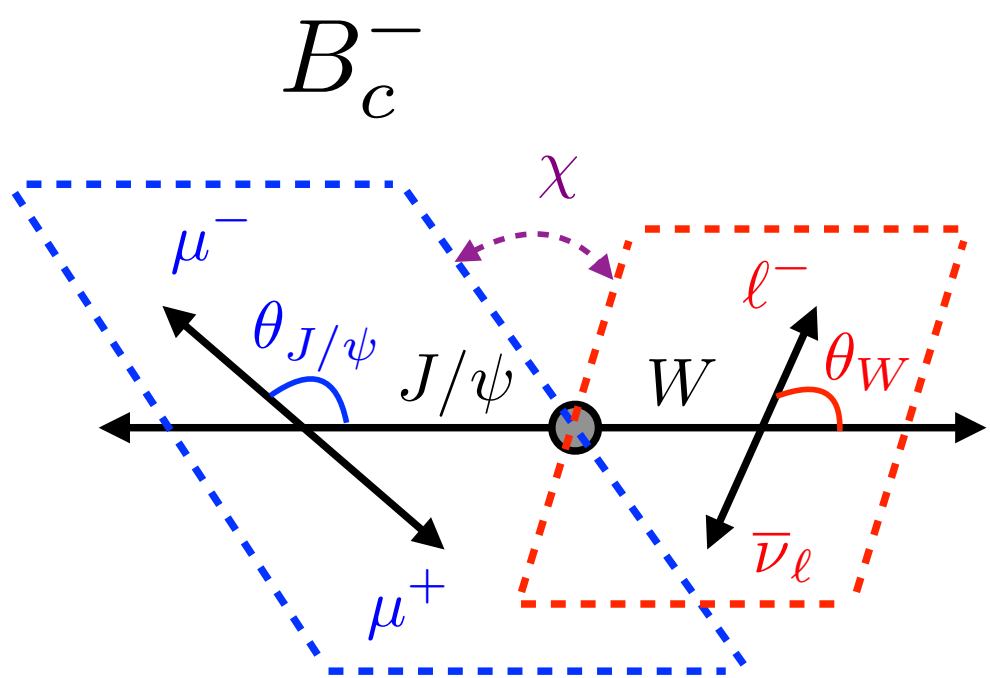
A_0, A_1, A_2, V .

For heavy-HISQ follow method developed for $D_s \to \phi\ell\nu$ with nonpert. mormon of V, A

HPQCD, 1311.6669

Combine ffs into helicity amplitudes, H_0, H_+, H_- , H_t

Use these to determine differential rate in q^2 and angular space

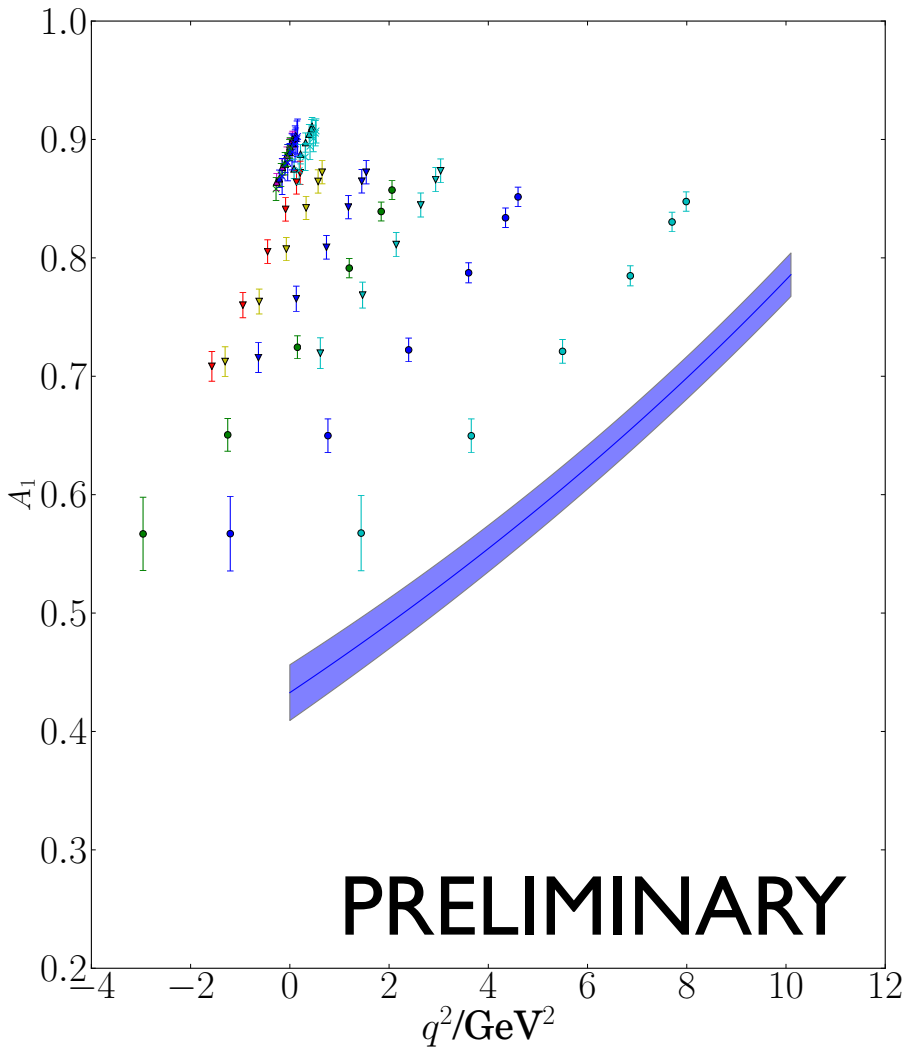


$B_c \to J/\psi \ell \nu$

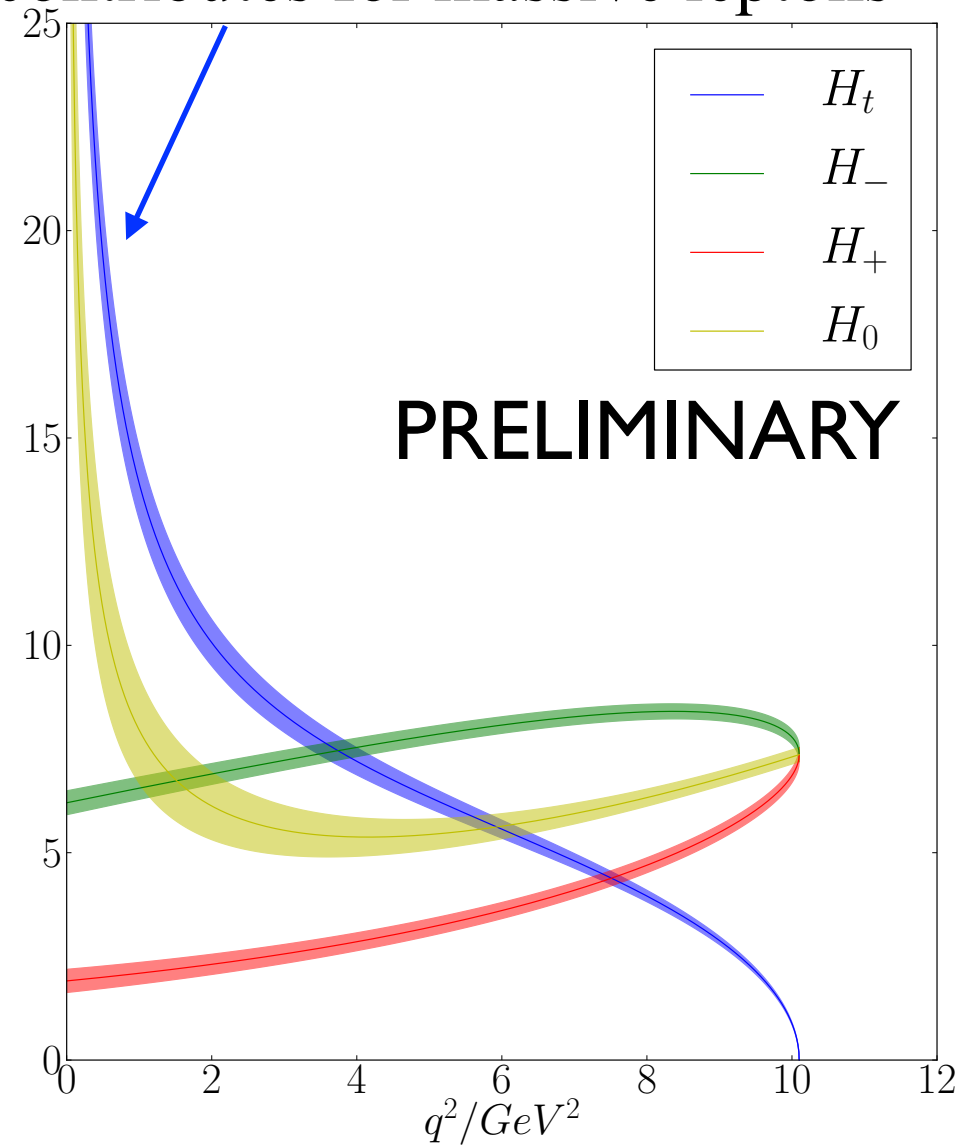
J. Harrison
et al, HPQCD,

resulting
helicity amplitudes

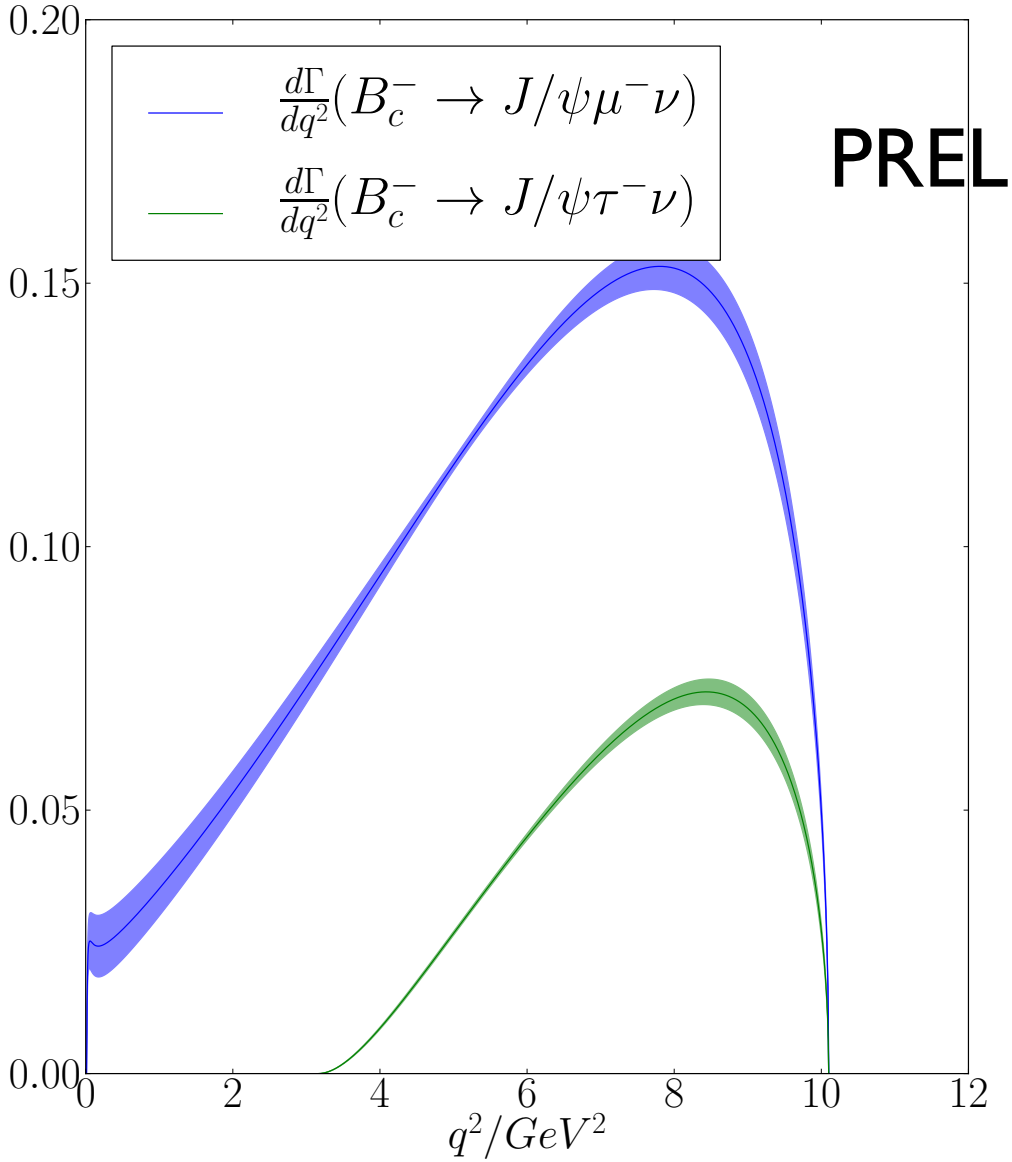
A_1 raw results



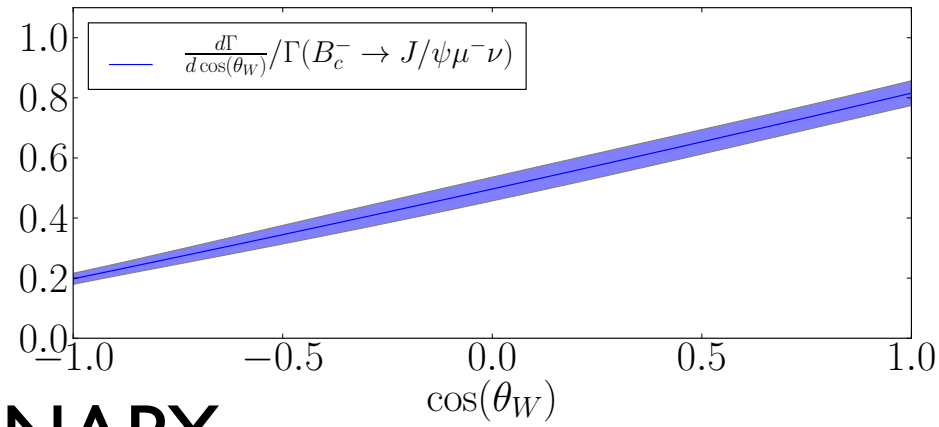
H_t contributes for massive leptons

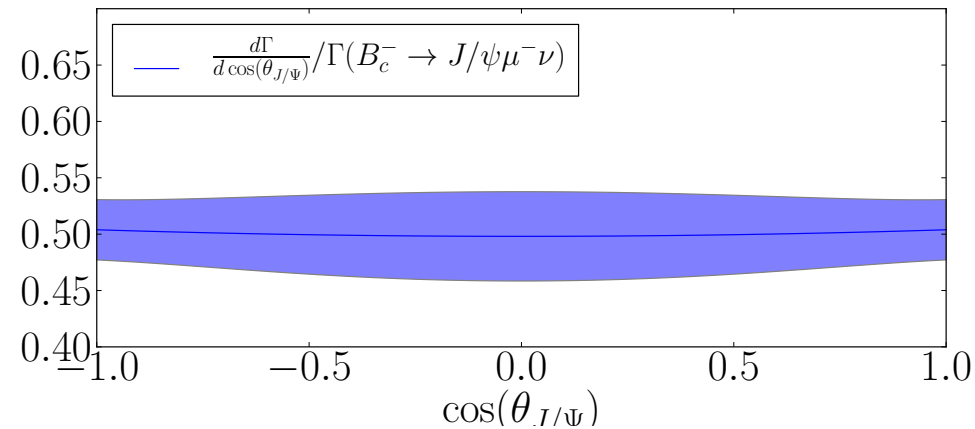


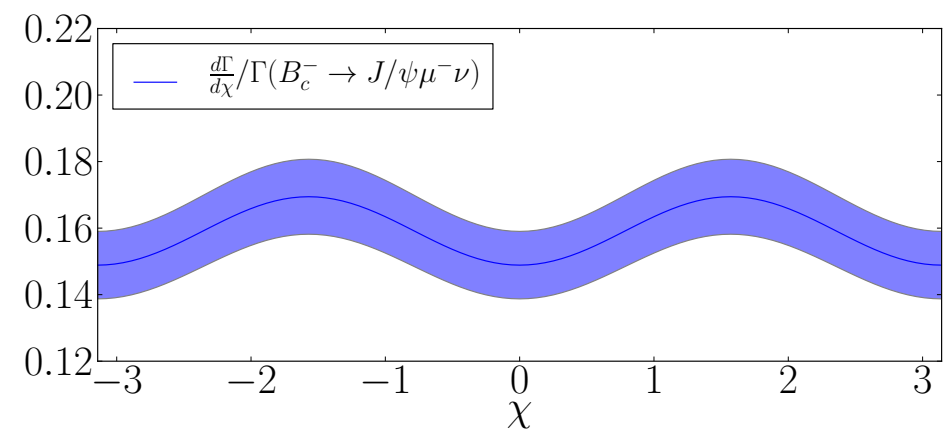
differential distributions for W decay to (massless) μ or (massive) τ



PRELIMINARY







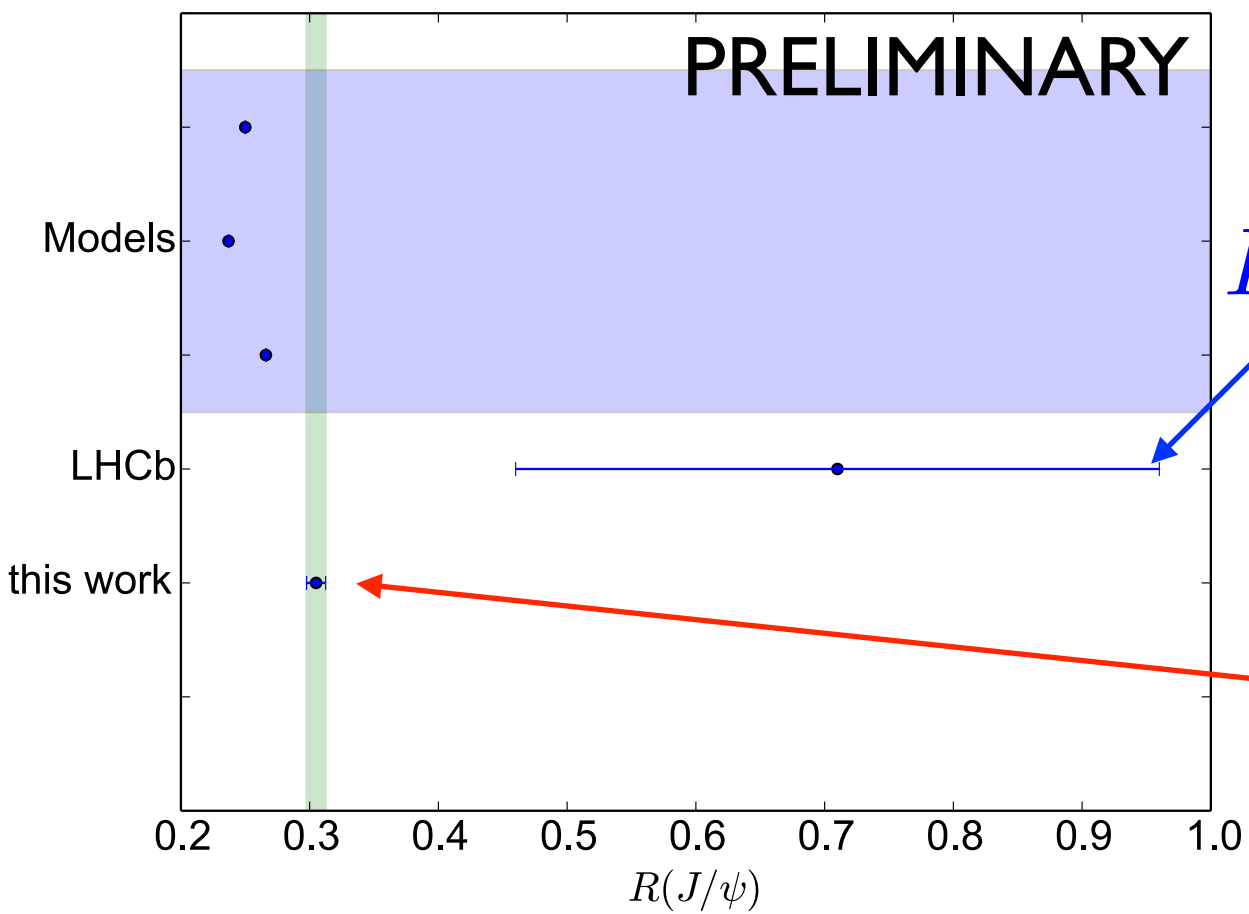
J. Harrison et al, HPQCD, in prep.

Bc to J/ψ form factors

Can contribute to tests of lepton universality.

R_X shows tensions with SM for X=D,D*

$$R_X = \frac{\mathrm{Br}(B \to X\tau\overline{\nu}_\tau)}{\mathrm{Br}(\to X\ell\overline{\nu}_\ell)}$$



First LHCb result :

$$R_{J/\psi} = 0.71(17)(18)$$

LHCb, 1711.05623

In SM we find

$$R_{J/\psi} = 0.3050(74)$$

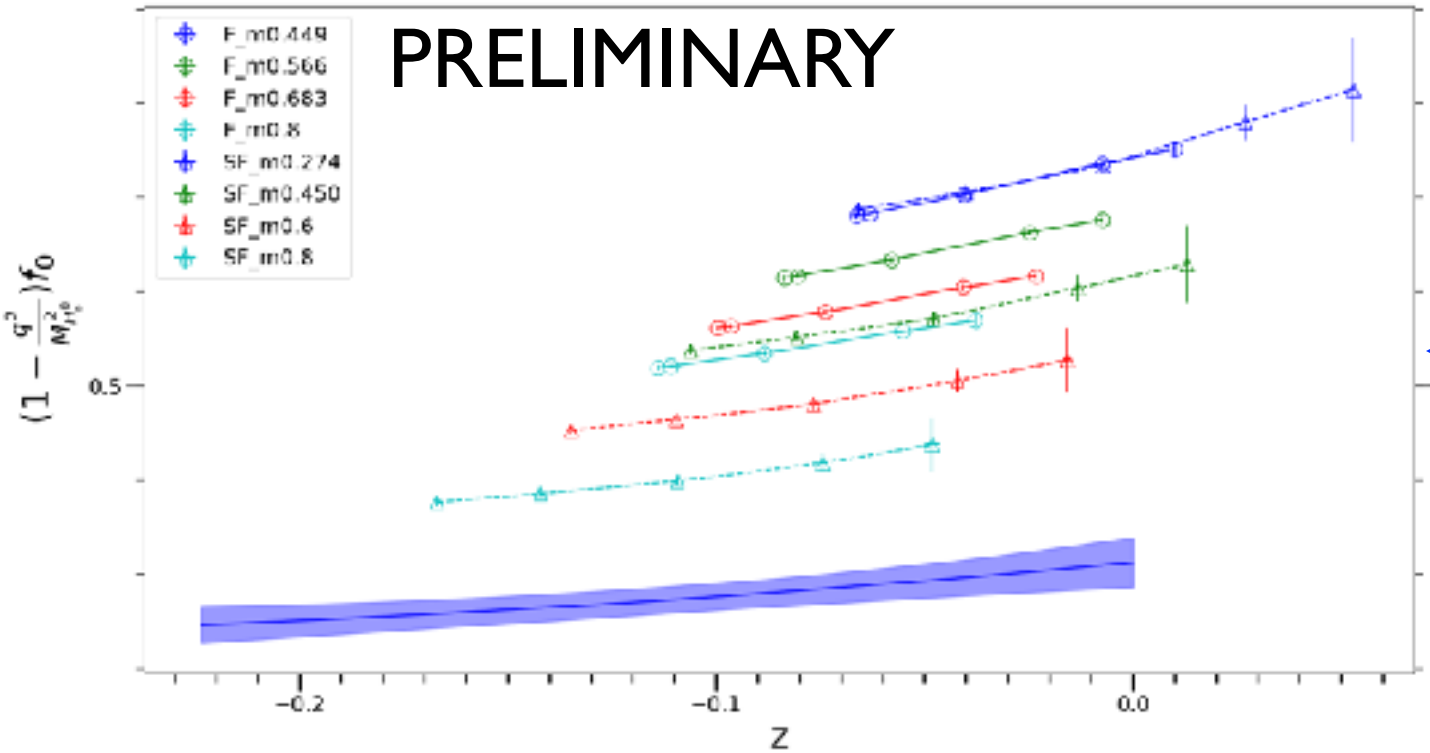
LHCb aim for 2% in R with 300 fb^{-1}

$B_{(s)} \to D^*_{(s)}\ell\nu$ calculation underway

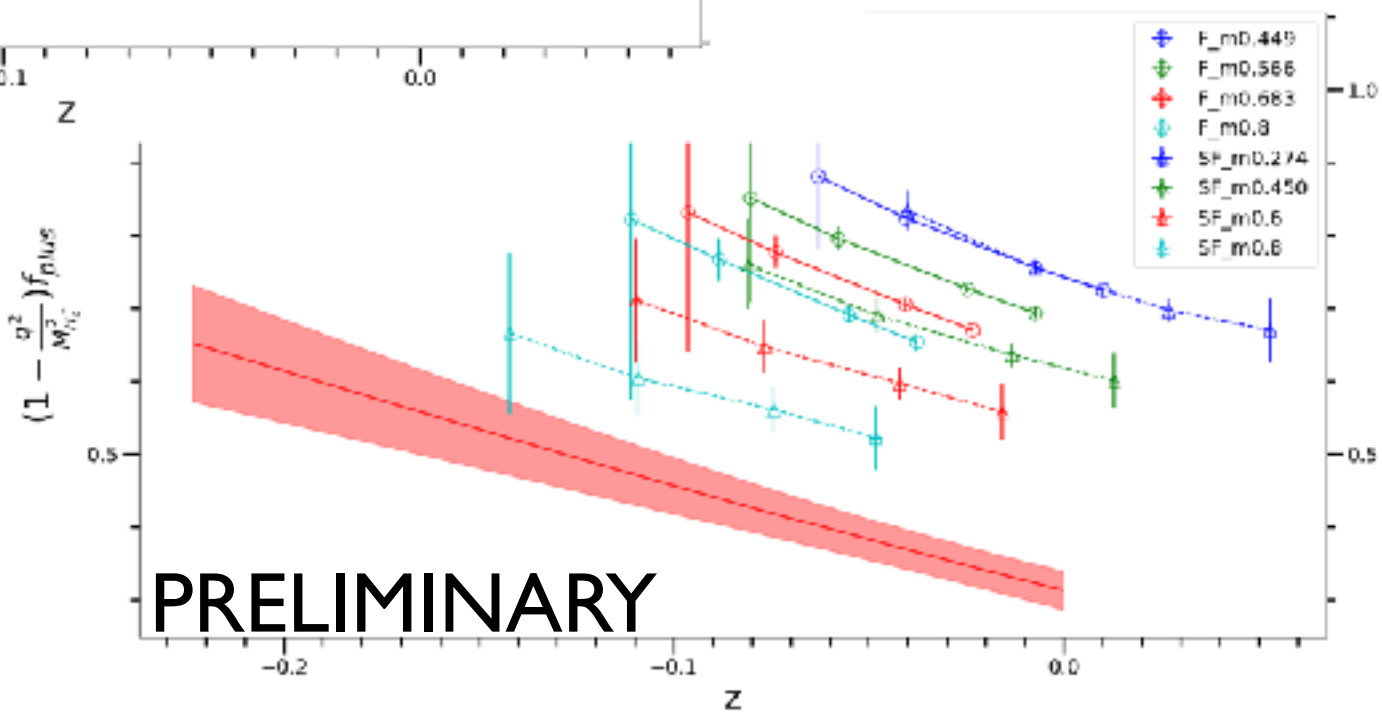
LHCb, 1808.08865

$B_s \to \eta_s \ell \nu$ is an unphysical process, but use to test heavy-HISQ method for b to light decays.

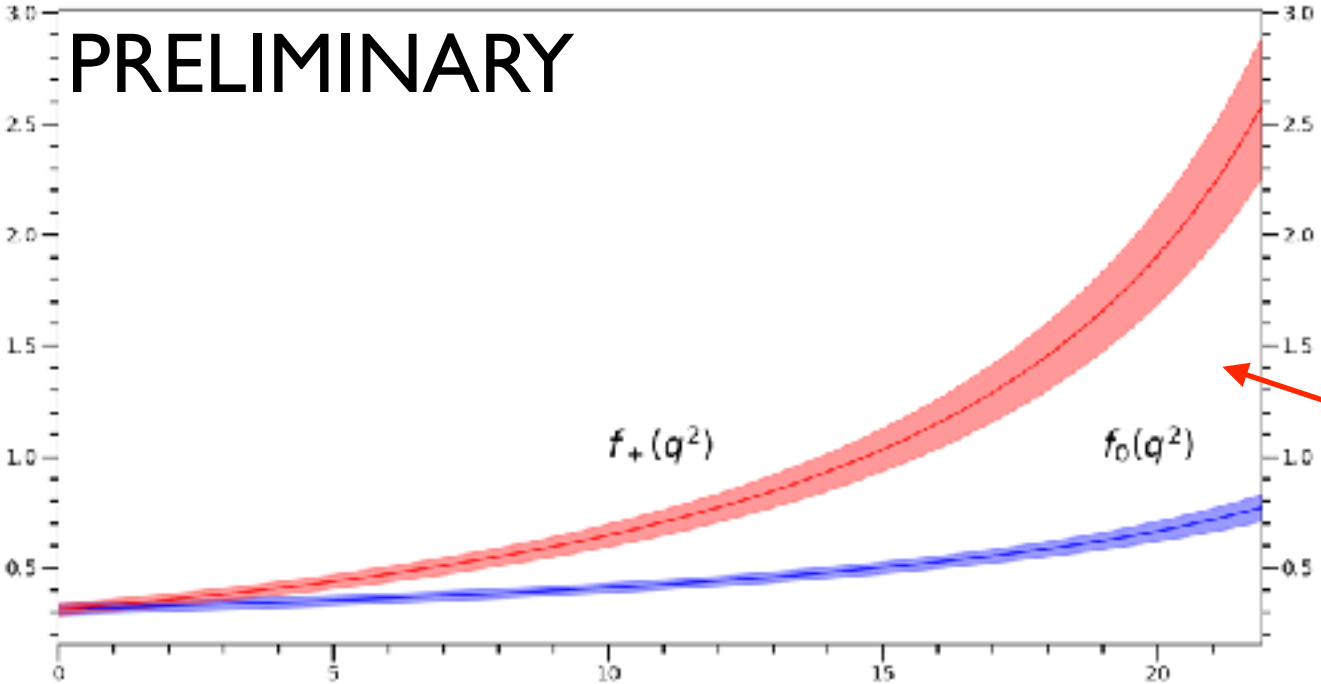
W. Parrott et al, HPQCD.



z-space fits



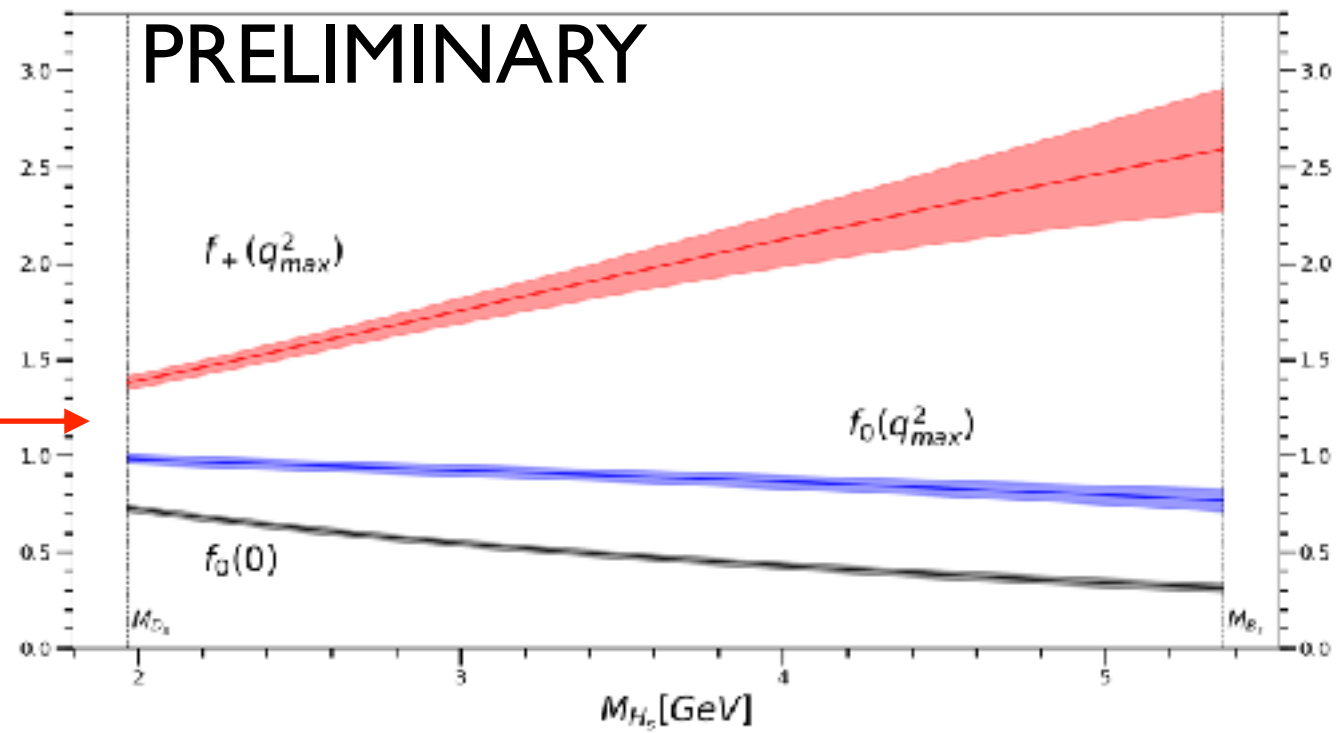
So far, results at a=0.09fm and 0.06fm for a variety of am_h values



$B_s \to \eta_s \ell \nu$

f_+ and f_0 form factors will improve with results at a=0.045fm

Dependence of form factors at q^2=0 and q^2_{max} on heavy quark mass



Conclusion

- Success of heavy-HISQ for heavy-light decay constants being extended to heavyonium = tension with experiment for Y? - and to semileptonic form factors.
- allows full q^2 range to be covered for $B_{(c/s)}$ decays. Currents normalised fully nonperturbatively (except tensor). Aim for 1% accuracy

Future

- Complete f_Y with QCD+QED.
- Complete form factors for $B_{(s)} \to D^*_{(s)} \ell \nu$
- Push forward on b to light processes e.g. $B \to K \mu^+ \mu^-$