Semileptonic B decays from Lattice QCD

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Orsay
Sept 2019
Quark low-energy strong interactions are a major complication in testing the Standard Model.

An accurate nonperturbative treatment of QCD is necessary to compare SM and low-energy experimental tests for new physics.
Lattice QCD provides such an approach. Must be able to determine results in physical continuum limit.

Final accuracy depends on:
- control of lattice spacing dependence
- tuning of quark masses
- normalisation of operators

Use HPQCD’s Highly Improved Staggered Quark (HISQ) action since it has small discretisation errors, is numerically efficient, fully nonperturbative operator norm. is usually possible.
Parameters for gluon field configurations generated with HISQ sea quarks (by MILC)

\[ m_u = m_d = m_l \]

mass of u,d quarks

\[ m_{\pi}^{0} = 135 \text{ MeV} \]

*physical

\[ m_{u/d} \]

\[ m_{u,d} \approx m_s/10 \]

\[ m_{u,d} \approx m_s/27 \]

Volume:

\[ m_{\pi} L > 3 \]

\[ n_f = 1+1+1+1 \text{ also now being done} \]

HISQ = Highly improved staggered quarks - very accurate discretisation

E. Follana, et al, HPQCD, hep-lat/0610092.

“2nd generation” lattices inc. c quarks in sea
Weak decays probe hadron structure and quark couplings. (Semi)-leptonic decays and mixing calculable in lattice QCD

\[
\begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
\pi \to l\nu & K \to l\nu & B \to \pi l\nu \\
K \to \pi l\nu & V_{cd} & V_{cs} & V_{cb} \\
D \to l\nu & D_s \to l\nu & B \to D l\nu \\
D \to \pi l\nu D \to Kl\nu & V_{td} & V_{ts} & V_{tb} \\
\langle B_d|\overline{B}_d \rangle & \langle B_s|\overline{B}_s \rangle
\end{pmatrix}
\]

CKM matrix

Need precision lattice QCD to get accurate CKM elements to test Standard Model.

If \( V_{ab} \) known, compare lattice to experiment to test QCD

\[ Br(M \to \mu\nu) \propto V_{ab}^2 f_M^2 \]

Expt = CKM x theory(QCD)

\[ \text{Aim : sub-1\% errors} \]

ALSO - test corresponding electromagnetic processes
Most straightforward calculations are ‘2-point’ current-current correlation functions - yield meson masses and decay constants, \( f \)

\[
\langle 0 | J^\dagger(T) J(0) | 0 \rangle = \sum_n a_n^2 e^{-m_n T}
\]

\[
a_n = \frac{\langle 0 | J | n \rangle}{\sqrt{2m_n}} = \frac{f_n \sqrt{m_n}}{2}
\]

masses of all hadrons with quantum numbers of \( J \)
ground-state (n=0) most accurate

Amplitude to annihilate (W for charged pseudoscalar, photon for neutral vector) is proportional to \( f \)
Meson decay constant summary
Parameterises hadronic information needed for annihilation rate to $W$ or photon:

$$\Gamma \propto f^2$$

ordered by value:

<table>
<thead>
<tr>
<th>Decay Constant [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment: weak decays</td>
</tr>
<tr>
<td>: em decays</td>
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<tr>
<td>Lattice QCD: predictions</td>
</tr>
<tr>
<td>: postdictions</td>
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</tbody>
</table>

HPQCD, 1208.2855, 1312.5264, 1408.5768, 1503.05762, 1703.05552, FNAL/MILC 1712.09262
A key ingredient for accurate decay constants is having a fully nonperturbative normalisation for J. Possible for relativistic formalisms such as HISQ. To handle b quarks then requires very fine lattices (am_b < 1) and mapping out discretisation effects - ‘heavy-HISQ’ approach

Works well for heavy-light decay constants. (J abs. norm. by PCAC)

\[ \begin{align*}
\text{HPQCD} & \quad 1110.4510 \\
\text{FNAL/MILC} & \quad 1712.09262
\end{align*} \]

<1% uncertainty

\[ f_{H_s} \text{ (GeV)} \]

\[ m_{D_s} \]

\[ m_{B_s} \]

\[ a = 0.15\text{fm} \]

\[ a = 0.045\text{fm} \]
Improved lattice QCD results for vector heavyonium

D. Hatton et al, HPQCD, 1908.10116, 1909.00756

Use RI-SMOM $Z_v$ (with exact lattice Ward-Takahashi identity)

Now adding quenched QED - impact < 0.5%.

$\Gamma(V \rightarrow e^+ e^-) = \frac{4\pi}{3} \alpha^2 Q^2 \frac{f_V^2}{M_V}$

In $Y$ case discrepancy is emerging - working on even finer lattices
Semileptonic form factors

QCD info. encoded in form factors, functions of $q^2$

$$t \quad J$$

$$<K|V^\mu|D> = f_+(q^2) \left[ p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_D^2 - M_K^2}{q^2} q^\mu$$

measured by expt through rate for e.g $D \rightarrow K l \nu$

$$<K|S|D> = \frac{M_D^2 - M_K^2}{m_0 c - m_0 s} f_0(q^2)$$

Exclusive process

PS to PS case

$$q^\mu = p_D^\mu - p_K^\mu$$

$D \rightarrow K l \nu$

abs. norm. for same c/s

action HPQCD:1008.4562.

Gives V normln at $q^2_{max}$
Lattice QCD calculation - needs 3-point function.

- Calculate $a$, $b$ propagators from 0
- Calculate $c$ prop from $T$ using $b$ as source and $\Gamma$ as req’d to make meson bc
- Tie $a$ and $c$ props together at $t$ with $\Gamma$ as req’d for current (V-A)
- For heavy-HISQ approach use multiple masses for $a$, $b$ or $c$

Fit 3-point to: (simultaneously with 2-point)

$$C_{3pt} = \sum_{j,k} a^a_j J_{jk} a^b_k f(E^a_j, t) f(E^b_k, T - t)$$

$$f(E, t) = e^{-Et} + e^{-E(L_t - t)}$$

$$\langle ab| J| bc \rangle = 2\sqrt{E_0^{ab} E_0^{bc}} J_{00}$$

Give momentum to quark $a$ to cover $q^2$ range
Fitting as a function of $q^2$

$H \rightarrow L$

decay amplitude has cut starting at

$$q^2 = (M_H + M_L)^2$$

map $q^2 = t$ plane to $z$-plane using

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

SL region then well within unit circle. Fit form factors to:

$$F(t) = \frac{1}{P(t)} \sum b_k z^k$$

Allow for disc. effects and $m_h$ dependence in $b_k$
$B_c \rightarrow B_s \ell \nu$
on LHCb list for near future

L. Cooper et al, HPQCD, Lattice2019

small $q^2$ range easily covered on lattice

B. Chakraborty et al, HPQCD, Lattice2019

can compare to

$D \rightarrow K \ell \nu$

PRELIMINARY

heavy HISQ and NRQCD $b$ agree

spectator mass dependence is mild

PRELIMINARY
B meson form factors from b quark decay - much bigger $q^2$ range. How to cover this range in lattice QCD?

$|\vec{p}_X|_{q^2=0} = \frac{M_B}{2} - \frac{M_X^2}{2M_B}$

Large momenta need fine lattices

$B_s \rightarrow D_s s \nu$ for $V_{cb}$

calculate for a range of heavy quark masses ($a m_h < 1$) for a range of fine lattice spacings.

Fit $f(m_h,a,z)$ and extract physical curve at $m_h = m_b$.

$q^2$ range grows with $m_h$ as lattices get finer
Interpolate in z-space and fit dependence on heavy quark mass and lattice spacing

\[ a = 0.045 \text{fm} \]

Form of coefficients of z-expansion:

\[
a_{n}^{0,+} = \left(1 + \rho_{n}^{0,+}\log\left(\frac{M_{\eta_{n}}}{M_{\eta_{c}}}\right)\right) \times \sum_{i,j,k=0}^{2,2,2} d_{i,j,k}^{0,+} \left(\frac{2\Lambda_{\text{QCD}}}{M_{\eta_{n}}}\right)^{i} \left(\frac{am_{h_{0}}^{\text{val}}}{\pi}\right)^{2j} \left(\frac{am_{c_{0}}^{\text{val}}}{\pi}\right)^{2k} \times \left(1 + N_{\text{mistuning},n}^{0,+}\right).
\]
Final result agrees with (but more accurate than) non relativistic methods that have to extrapolate in $q^2$ from close to zero-recoil.

E. McLean et al

HPQCD, 1906.00701

NRQCD

$0 \leq q^2 \leq 12$ [GeV$^2$]

Ratio of branching fractions to $\tau$ and $\mu$

$$R(D_s) = 0.2987(46)$$

With expt, can give $V_{cb}$.

Extend to $B$ to $D$ case.
Determine dependence of form factors on heavy quark mass

E. McLean et al
HPQCD, 1906.00701

\[ \frac{1}{\beta(m_h)} = \frac{M_{H_s}^2 - M_{D_s}^2}{f_+^s(0)} \frac{df_+^s}{dq^2} \bigg|_{q^2=0}, \]

\[ \delta(m_h) = 1 - \frac{M_{H_s}^2 - M_{D_s}^2}{f_+^s(0)} \left( \frac{df_+^s}{dq^2} \bigg|_{q^2=0} - \frac{df_0^s}{dq^2} \bigg|_{q^2=0} \right). \]

LO HQET

R. Hill hep-ph/0606023
Pseudoscalar to vector meson semileptonic decay
e.g. $B(s) \rightarrow D_s^* \ell \nu$  $B_c \rightarrow J/\psi \ell \nu$

To map full $q^2$ range, need both V and A matrix elements
and calculation of 4 form factors:
$A_0, A_1, A_2, V$.

For heavy-HISQ follow method developed
for $D_s \rightarrow \phi \ell \nu$ with nonpert. morman of V, A

Combine ffs into helicity
amplitudes, $H_0, H_+, H_-, H_t$

Use these to determine
differential rate in $q^2$ and
angular space
$B_c \to J/\psi \ell \nu$  

J. Harrison et al, HPQCD,  

resulting helicity amplitudes  

$A_1$ raw results  

H$_t$ contributes for massive leptons

PRELIMINARY
differential distributions for $W$ decay to (massless) $\mu$ or (massive) $\tau$

PRELIMINARY

J. Harrison et al, HPQCD, in prep.
**Bc to J/ψ form factors**

Can contribute to tests of lepton universality.

$R_X$ shows tensions with SM for $X=D,D^*$

$$R_X = \frac{\text{Br}(B \to X\tau\bar{\nu}_\tau)}{\text{Br}(\to X\ell\bar{\nu}_\ell)}$$

First LHCb result:

$$R_{J/\psi} = 0.71(17)(18)$$

In SM we find

$$R_{J/\psi} = 0.3050(74)$$

LHCb aim for 2% in $R$ with 300 fb$^{-1}$

$B_{(s)} \to D_{(s)}^* \ell\nu$ calculation underway
$B_s \to \eta_s \ell \nu$ is an unphysical process, but use to test heavy-HISQ method for b to light decays.

So far, results at $a=0.09\text{fm}$ and $0.06\text{fm}$ for a variety of $am_h$ values

W. Parrott et al, HPQCD.
Dependence of form factors at $q^2=0$ and $q^2_{\text{max}}$ on heavy quark mass

$B_s \to \eta_s \ell\nu$

$\ f_+ \text{ and } f_0 \text{ form factors will improve with results at } a=0.045\text{fm}$
Conclusion

• Success of heavy-HISQ for heavy-light decay constants being extended to heavyonium = tension with experiment for $\Upsilon^-$ and to semileptonic form factors.
• allows full $q^2$ range to be covered for $B_{(c/s)}$ decays. Currents normalised fully nonperturbatively (except tensor). Aim for 1% accuracy

Future

• Complete $f_\Upsilon$ with QCD+QED.
• Complete form factors for $B_{(s)} \to D_{(s)}^* \ell \nu$
• Push forward on $b$ to light processes e.g. $B \to K \mu^+ \mu^-$