Double Parton Distributions for the Nucleon on the Lattice

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Work done with Gunnar S. Bali, Markus Diehl, Jonathan R. Gaunt, Benjamin Gläßle and Andreas Schäfer

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Double Parton Distributions:

- Crucial piece of information for description of Double Parton Scattering:

$$\sigma \sim \sigma_1 \sigma_2 \int \mathrm{d}^2 \vec{y}_\perp F(\vec{y}_\perp) \bar{F}(\vec{y}_\perp)$$

- Non-perturbative objects: largely unknown
- Several quark model studies:
  Chang et al, 2013; Rinaldi et al, 2013-2018; Broniowski et al 2014-2019 ; Kasemets, Mukherjee 2016; Courtoy et al 2019
- Lattice study for the case of the pion arXiv:1807.03073 , *talk last year*
- In this talk: information about Nucleon-DPDs from the lattice

DPDs: Definitions and Relations (see arXiv:1111.0910)

Definition (transverse momenta integrated out):

$$F_{ij}^{qq'}(x_1, x_2, \vec{y}_\perp) = 2p^+ \int \frac{\mathrm{d}z_1^- \; \mathrm{d}z_2^-}{4\pi^2} e^{ip^+(x_1 z_1^- + x_2 z_2^-)}$$
$$\times \int \mathrm{d}y^- \; \langle p| \mathcal{O}_i^q(0, z_1) \; \mathcal{O}_j^{q'}(y, z_2) \, |p\rangle \Big|_{\vec{z}_{1,2,\perp} = \vec{0}}$$

with light cone operators $\mathcal{O}_i^q(y, z) = \bar{q}(y - z/2) \; \Gamma_i \; q(y + z/2)$ with $y^+ = z^+ = 0$

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Leading twist:

	$\Gamma_i = \gamma^+$	$\Leftrightarrow$	$q \;\; (V)$	unpolarized
	$\Gamma_i = \gamma^+ \gamma_5$	$\Leftrightarrow$	$\Delta q \;\; (A)$	longitudinal polarization
$(j = 1, 2)$	$\Gamma_i = i\sigma^{j+} \gamma_5$	$\Leftrightarrow$	$\delta q \;\; (T)$	transverse polarization

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with light cone operators $\mathcal{O}_i^q(y, z) = \bar{q}(y - z/2) \, \Gamma_i \, q(y + z/2)$ with $y^+ = z^+ = 0$

First Mellin Moment ($y^2 := -\vec{y}_\perp^2$, since $y^+ = 0$, $\eta_C^{V/T} = -1$, $\eta_C^A = 1$):

$$M_{ij}^{qq'}(\vec{y}_\perp) = \int_0^1 \mathrm{d}x_1 \mathrm{d}x_2 \, \Big[F_{ij}^{qq'}(x_{1,2}, \vec{y}_\perp) - \eta_C^i F^{\bar{q}q'}{}_{ij}(x_{1,2}, \vec{y}_\perp)$$
$$- \eta_C^j F_{ij}^{q\bar{q}'}(x_{1,2}, \vec{y}_\perp) + \eta_C^i \eta_C^j F_{ij}^{\bar{q}\bar{q}'}(x_{1,2}, \vec{y}_\perp) \Big]$$
$$= (2p^+)^{-1} \int \mathrm{d}y^- \, \langle p | \, \mathcal{O}_i^q(0) \, \mathcal{O}_j^{q'}(y) \, | p \rangle$$

with $\mathcal{O}_i^q(y) = \bar{q}(y) \Gamma_i q'(y)$

$\Rightarrow$ **Operators become local**

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First Mellin Moment :

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$$= (2p^+)^{-1} \int \mathrm{d}y^- \mathcal{M}_{ij}^{qq'}(p, y)$$

Decompositions:

$$F_{qq}(x_i, \vec{y}_\perp) = f_{qq}(x_i, \vec{y}_\perp^2)$$
$$F_{\Delta q \ \Delta q}(x_i, \vec{y}_\perp) = f_{\Delta q \ \Delta q}(x_i, \vec{y}_\perp^2)$$
$$F_{q \ \Delta q}(x_i, \vec{y}_\perp) = 0$$
$$F_{q \ \delta q}^j(x_i, \vec{y}_\perp) = m \ y_\perp^l \epsilon^{lj} f_{q \ \delta q}(x_i, \vec{y}_\perp^2)$$
$$F_{\Delta q \ \delta q}^j(x_i, \vec{y}_\perp) = m \ y_\perp^j f_{\Delta q \ \delta q}(x_i, \vec{y}_\perp^2)$$
$$F_{\delta q \ \delta q}^{jl}(x_i, \vec{y}_\perp) = \delta^{jl} f_{\delta q \ \delta q}(x_i, \vec{y}_\perp^2) \quad + \quad 2m^2(y_\perp^j y_\perp^l - \delta^{jl}\vec{y}_\perp^2) f_{\delta q \ \delta q}^t(x_i, \vec{y}_\perp^2)$$

DPDs: Definitions and Relations (see arXiv:1111.0910)

First Mellin Moment :

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$$= (2p^+)^{-1} \int \mathrm{d}y^- \mathcal{M}_{ij}^{qq'}(p, y)$$

Decompositions (Mellin Moments, analogous):

$$M_{qq}(x_i, \vec{y}_\perp) = I_{qq}(x_i, \vec{y}_\perp^2)$$
$$M_{\Delta q \ \Delta q}(x_i, \vec{y}_\perp) = I_{\Delta q \ \Delta q}(x_i, \vec{y}_\perp^2)$$
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$$= (2p^+)^{-1} \int \mathrm{d}y^- \mathcal{M}_{ij}^{qq'}(p, y)$$

Decompose Local matrix elements , e.g. two vector currents ($V^\mu(y) = \bar{q}(y)\gamma^\mu q(y)$):

$$\mathcal{M}_{VV}^{\mu\nu} = \langle p| \, V^\mu(0) V^\nu(y) \, |p\rangle$$
$$= \left(2p^\mu p^\nu - \frac{p^2}{2} g^{\mu\nu}\right) \; A(py, y^2)$$
$$+ \left(p^\mu y^\nu + p^\nu y^\mu - \frac{py}{2} g^{\mu\nu}\right) m^2 \; B(py, y^2)$$
$$+ \left(2y^\mu y^\nu - \frac{y^2}{2} g^{\mu\nu}\right) m^4 \; C(py, y^2) + g^{\mu\nu} \, \mathrm{tr}\,\{\mathcal{M}_{VV}\}$$

Similar for other channels

DPDs: Definitions and Relations (see arXiv:1111.0910)

First Mellin Moment :

$$M_{ij}^{qq'}(\vec{y}_\perp) = (2p^+)^{-1} \int \mathrm{d}y^- \, \langle p| \, \mathcal{O}_i^q(0) \, \mathcal{O}_j^{q'}(y) \, |p\rangle$$
$$= (2p^+)^{-1} \int \mathrm{d}y^- \mathcal{M}_{ij}^{qq'}(p, y)$$

For $\vec{p}_\perp = \vec{0}$ ($py = p^+ y^-$), one obtains:

$$I_{ij,qq'}(\vec{y}_\perp^2) = \int \mathrm{d}(py) \; A_{ij}^{qq'}(py, \vec{y}_\perp^2)$$
$$I^t_{\delta q \delta q'}(\vec{y}_\perp^2) = \int \mathrm{d}(py) \; B_{TT}^{qq'}(py, \vec{y}_\perp^2)$$

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On the lattice (Euclidean time):

- calculate $\mathcal{M}_{ij}(p, y)$ for $y^0 = 0$ (pion: 1807.03073 , nucleon: 1911.05051)
- extract $A_{ij}(py, y^2)$ for $y^2 \geq 0$ and $(py)^2 \leq p^2 y^2$;
  py, y^2 restricted by lattice volume
- information about Mellin Moments $I_{ij}(\vec{y}_\perp^2)$

Four Point Functions from the Lattice

On the lattice (Euclidean time) we can calculate:

$$C_{4\mathrm{pt}}^{ij,\vec{p}} = \langle 0| \mathcal{P}^{\vec{p}}(t)\ \mathcal{O}_i(\vec{y},\tau)\ \mathcal{O}_j(\vec{0},\tau)\ \overline{\mathcal{P}}^{\vec{p}}(0)\,|0\rangle$$

with proton sink operator (projection onto momentum $\vec{p}$)
$\mathcal{P}^{\vec{p}}(t) = \frac{a^3}{2}\sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{p}}\ \epsilon_{abc}\ (\mathbb{1}+\gamma_4) u_a(x)\left[u_b^T(x) i\gamma_2\gamma_4\gamma_5 d_c(x)\right]\ |_{t=x_4}$

$$\rightarrow \sum_{\pi\in S_4} \mathrm{sign}(\pi) \int \mathrm{D}[U]\ G_\pi[U]\ \det\{D[U]\} e^{-S_G[U]}$$

with $G_\pi[U]$ being products of (in our case four) propagators (Wick contractions, graphs) connecting the (anti-)quark fields according to the permutation π

Integral over gauge fields U solved numerically by Monte Carlo simulations

$\det\{D[U]\}e^{-S_G[U]}$ is real and positive for the case of Euclidean time $\Rightarrow$ Suitable weight factor.

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Extract ground state by calculating ratio:

$$R_{ij}^{\vec{p}}(\vec{y}) = 2V\sqrt{\vec{p}^2+m^2}\ \frac{C_{4\mathrm{pt}}^{ij,\vec{p}}(\vec{y},\tau,t)}{C_{2\mathrm{pt}}^{\vec{p}}(t)}\Bigg|_{0\ll\tau\ll t}$$
$$= \langle p(\vec{p})|\,\mathcal{O}_i(y)\ \mathcal{O}_j(0)\,|p(\vec{p})\rangle|_{\,y^0=0}$$

with

$$C_{2\mathrm{pt}}^{\vec{p}}(t) = \langle 0|\,\mathcal{P}^{\vec{P}}(t)\overline{\mathcal{P}}^{\vec{P}}(0)\,|0\rangle$$

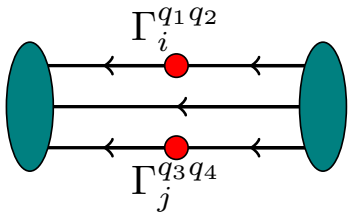
$\Rightarrow$ obtain desired matrix element for $y^0 = 0$.

More technical details on the simulation can be found in 1911.05051 / future publication

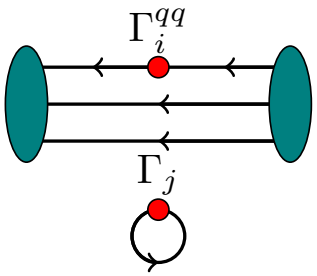
Contractions

Have 5 types of Wick contractions:

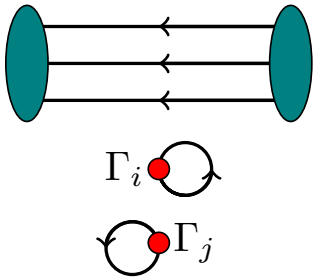
$C_1^{ij,q_1q_2q_3q_4} =$



$S_1^{ij,q} =$



$D^{ij} =$



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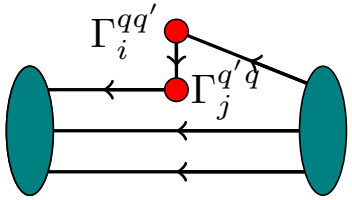
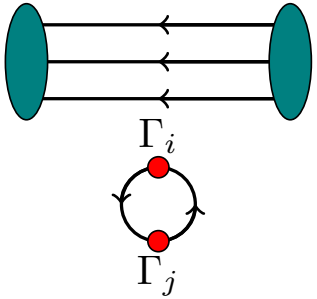


$S_2^{ij} =$



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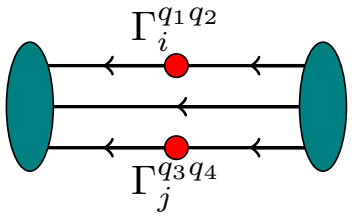

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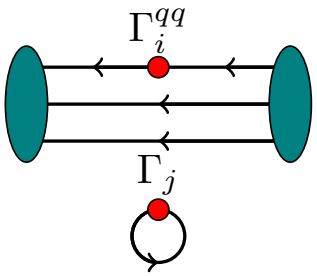

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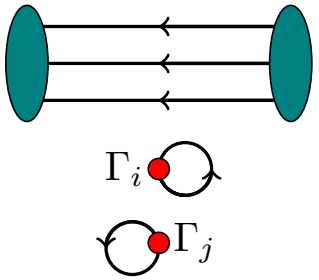
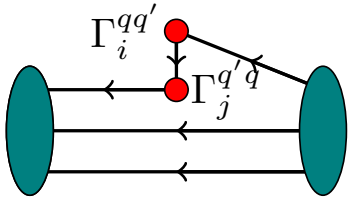


$C_2^{ij,q} =$



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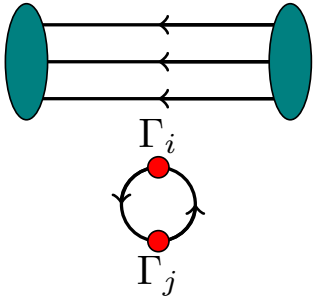
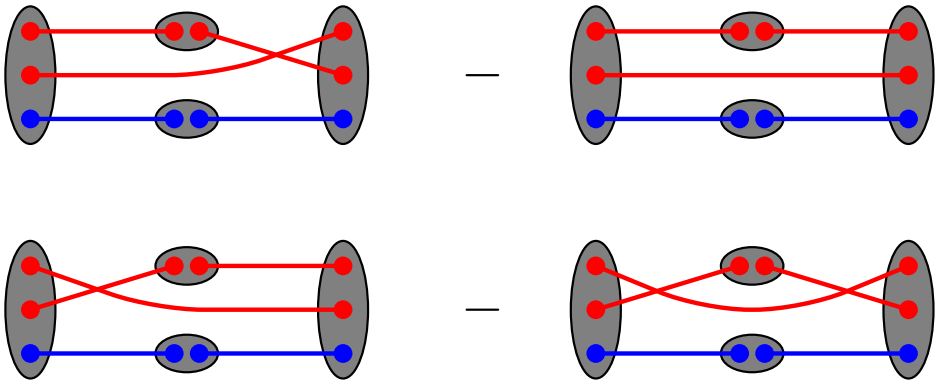
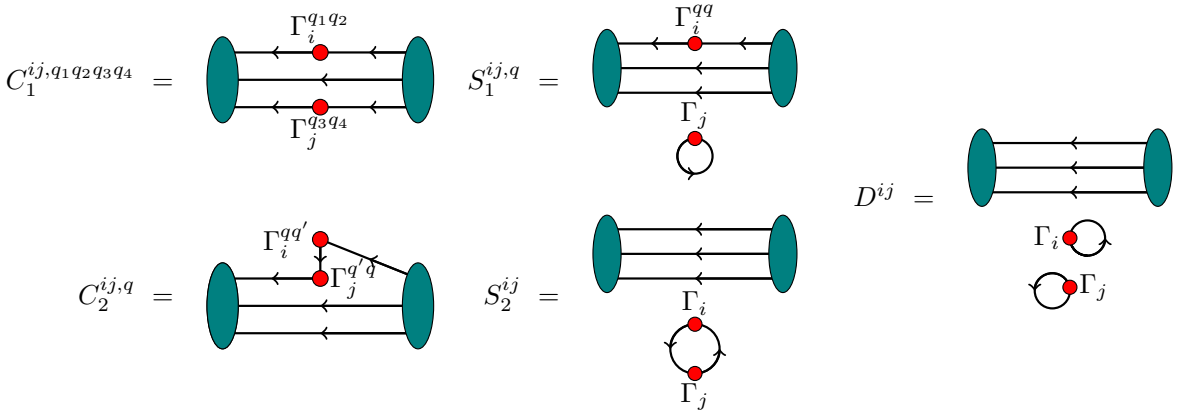


Explicit contraction depends on the operator flavor, e.g.:

$C_1^{ij,uudd} =$



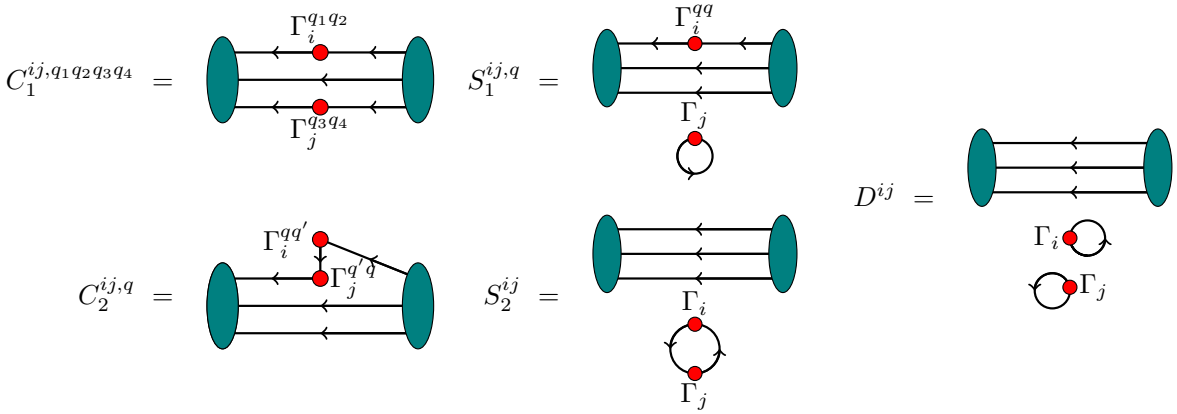
Contributions to Physical Matrix Elements



Depend on the given quark flavor. For light quarks we have:

$$
\begin{aligned}
\langle p|\, \mathcal{O}_j^{dd}(\vec{y})\ \mathcal{O}_i^{uu}(\vec{0})\, |p\rangle &= C_1^{ij,uudd}(\vec{y}) + S_1^{ij,u}(\vec{y}) + S_1^{ji,d}(-\vec{y}) + D^{ij}(\vec{y})\ , \\
\langle p|\, \mathcal{O}_j^{uu}(\vec{y})\ \mathcal{O}_i^{uu}(\vec{0})\, |p\rangle &= C_1^{ij,uuuu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,u}(-\vec{y}) \\
&\quad + S_1^{ij,u}(\vec{y}) + S_1^{ji,u}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})\ , \\
\langle p|\, \mathcal{O}_j^{dd}(\vec{y})\ \mathcal{O}_i^{dd}(\vec{0})\, |p\rangle &= C_2^{ij,d}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_1^{ij,d}(\vec{y}) + S_1^{ji,d}(-\vec{y}) \\
&\quad + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})\ , \\
\langle p|\, \mathcal{O}_j^{du}(\vec{y})\ \mathcal{O}_i^{ud}(\vec{0})\, |p\rangle &= C_1^{ij,uddu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y})\ .
\end{aligned}
$$

Contributions to Physical Matrix Elements



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$$\langle p|\, \mathcal{O}_j^{uu}(\vec{y})\ \mathcal{O}_i^{uu}(\vec{0})\, |p\rangle = C_1^{ij,uuuu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,u}(-\vec{y}) + S_1^{ij,u}(\vec{y}) + S_1^{ji,u}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p|\, \mathcal{O}_j^{dd}(\vec{y})\ \mathcal{O}_i^{dd}(\vec{0})\, |p\rangle = C_2^{ij,d}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_1^{ij,d}(\vec{y}) + S_1^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y}) + D^{ij}(\vec{y})$$

$$\langle p|\, \mathcal{O}_j^{du}(\vec{y})\ \mathcal{O}_i^{ud}(\vec{0})\, |p\rangle = C_1^{ij,uddu}(\vec{y}) + C_2^{ij,u}(\vec{y}) + C_2^{ji,d}(-\vec{y}) + S_2^{ij}(\vec{y})$$

This talk: Focus on C_1 and C_2, only flavor conserving currents

Lattice Setup

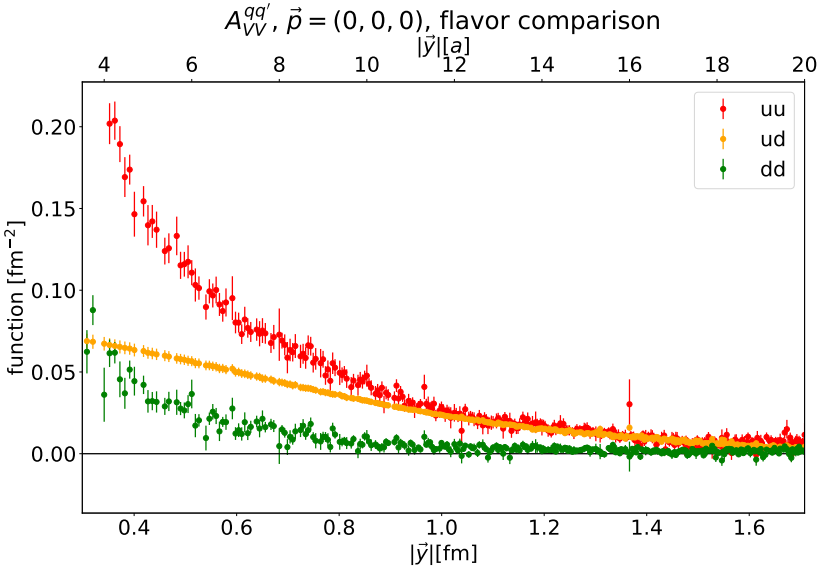
Two Ensembles (Wilson-clover action, $n_f = 2 + 1$, CLS [arXiv:1411.3982]):

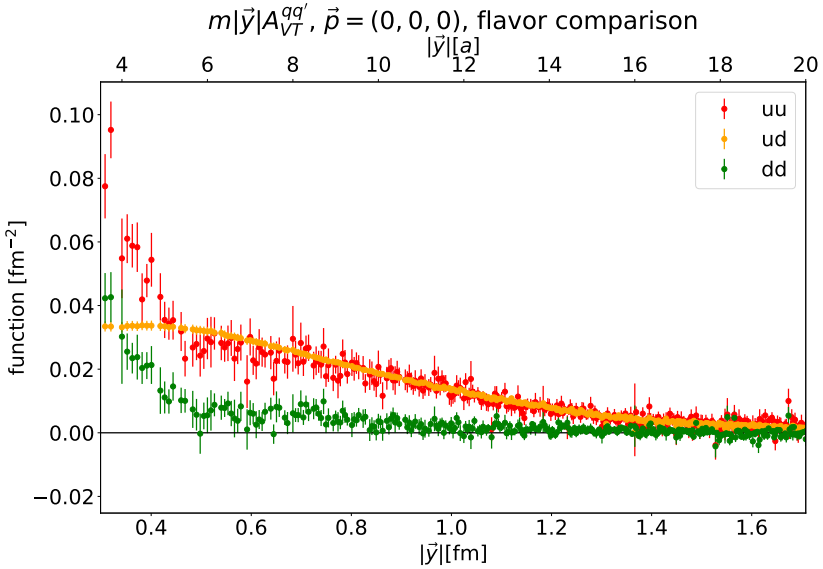
ensemble	β	a [fm]	$\kappa_{l/s}$	$L^3 \times T$	$m_{\pi/K}$ [MeV]	Lm_π	N_{used}
H102	3.4	0.0854	0.136865	$32^3 \times 96$	356	4.9	960
			0.136549339		442		

- source - sink separation: $10a$ (non-zero momentum) or $12a$ (zero momentum)
- Momenta: $\vec{p} = \vec{k} \cdot 2\pi/L$ with $\vec{k} \in \{(0,0,0),(1,1,1),(2,2,2),(2,-2,-2),(-2,2,-2),(-2,-2,2)\}$
- Considered insertions: $\mathcal{O}^q_{i,j} = \bar{q}\Gamma_{i,j}q$ $\rightarrow$ combinations $(\Gamma_i, \Gamma_j) = (\gamma^\mu, \gamma^\nu), (\gamma^\mu\gamma_5, \gamma^\nu\gamma_5), (\gamma^\mu, \sigma^{\nu\rho}), (\sigma^{\mu\nu}, \sigma^{\rho\lambda})$
- Renormalization scale: $2\mathrm{GeV}$ (for renormalization on the lattice see, e.g. arXiv:1903.12590 and references therein)

Results for $A_{ij}(py, y^2)$ (preliminary)

All results without disconnected contributions:





- Comparable correlations for uu/ud for large distances, small correlations for dd,
- Small distances: steep increase of uu / dd

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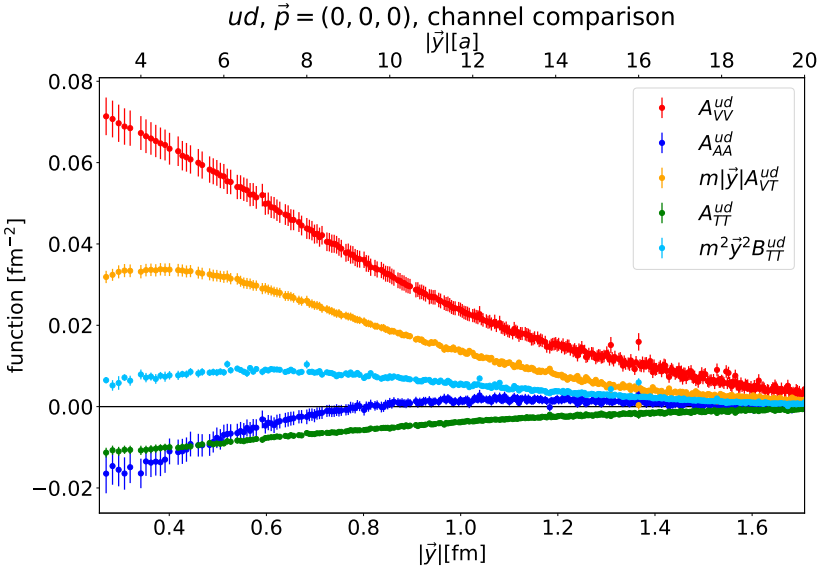

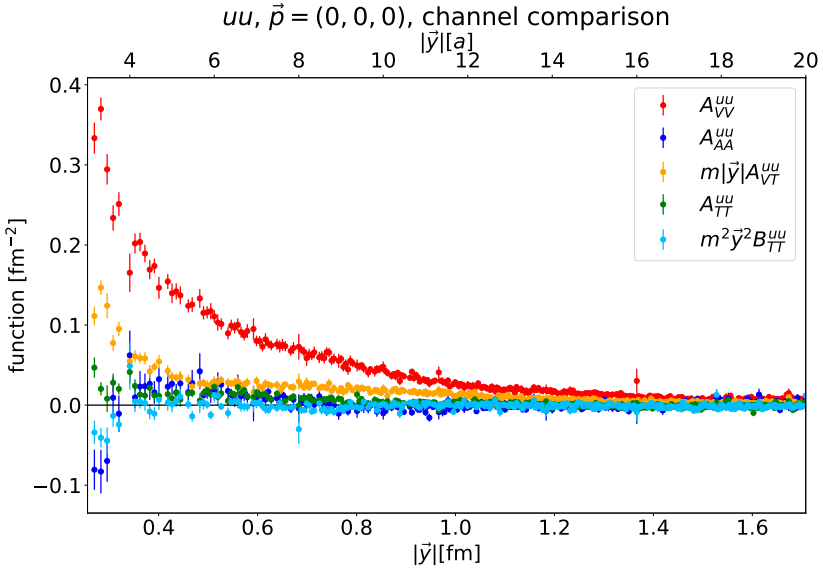

- Comparable correlations for uu/ud for large distances, small correlations for dd,
- Small distances: steep increase of uu / dd
- Signal for unpolarized quarks dominant
- Visible polarization effects for ud, while they are small for uu

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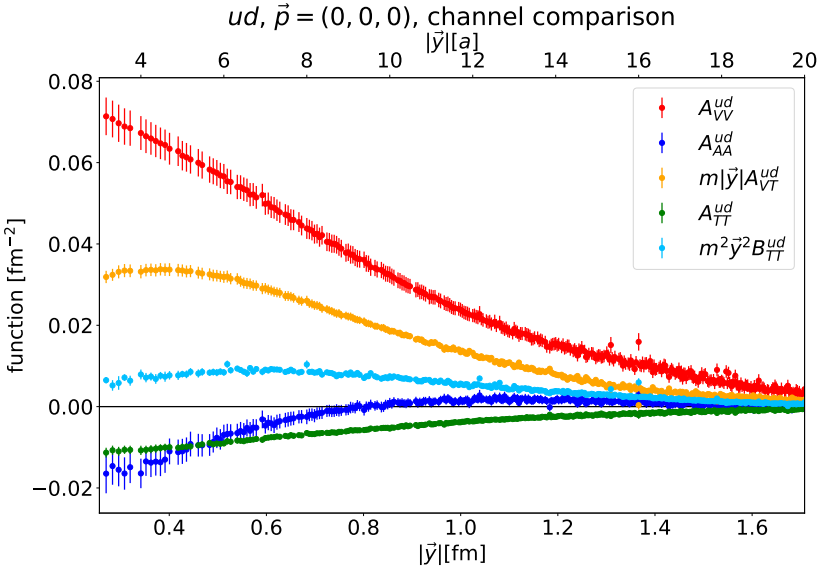
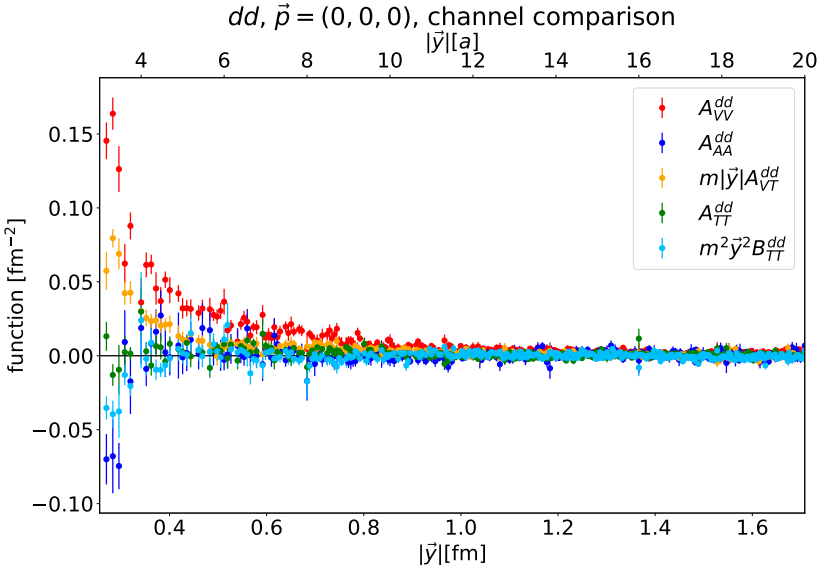
All results without disconnected contributions:





- Comparable correlations for uu/ud for large distances, small correlations for dd,
- Small distances: steep increase of uu / dd
- Signal for unpolarized quarks dominant
- Visible polarization effects for ud, while they are small for uu

Factorization Test (preliminary)

Simplest ansatz: Full factorization of DPDs in terms of PDFs:

$$F^{ij}(x_1, x_2, \vec{y}_\perp) = f^i(x_1) f^j(x_2) f_\perp(\vec{y}_\perp)$$

Our implementation: Insert complete set of states between the operators of a **light cone** matrix element, and **neglect all non-nucleon-states**:

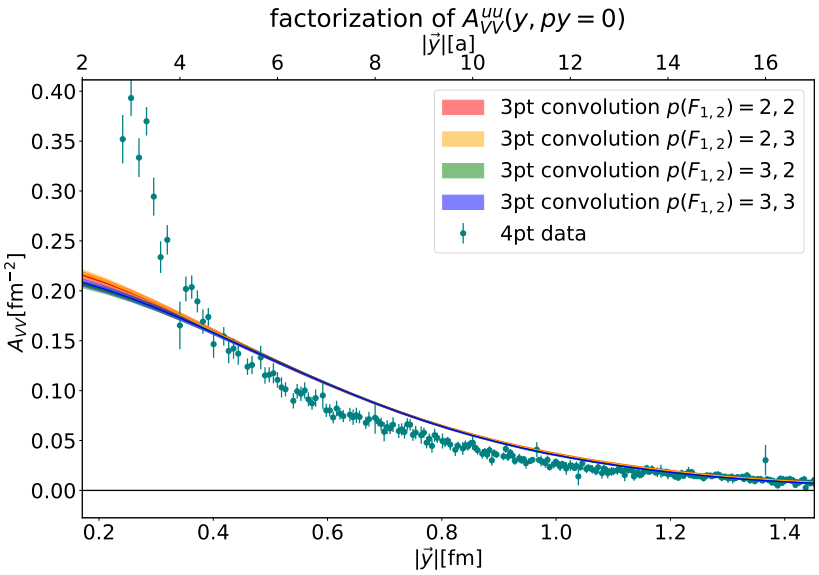
$$\mathcal{M}_{ij}(z_{1,2}, y) \stackrel{?}{=} \frac{1}{2} \sum_{\lambda\lambda'} \int \frac{\mathrm{d}^4 p'}{(2\pi)^4} \langle p, \lambda | \mathcal{O}_i(0, z_1) | p', \lambda' \rangle \langle p', \lambda' | \mathcal{O}_j(0, z_2) | p, \lambda \rangle \, e^{-iy(p'-p)} \delta(p'^2 - m^2)$$

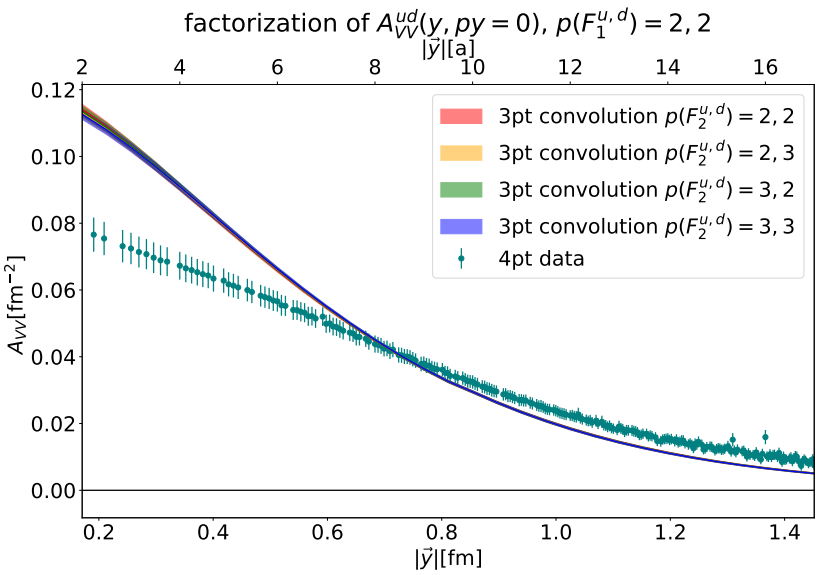
For A_{VV} we obtain:

$$A_{VV}(py=0, y^2) \stackrel{?}{=} \frac{1}{2\pi^2} \int_0^1 \mathrm{d}\zeta \frac{(1-\frac{\zeta}{2})^2}{1-\zeta} \int \mathrm{d}r_\perp \; r_\perp \; J_0(y_\perp r_\perp) \times$$
$$\times \left[(1-Z) F_1^i(t) F_1^j(t) - 2Z F_1^{\{i}(t) F_2^{j\}}(t) + \left(\frac{Z^2}{1-Z} - \frac{r_\perp^2}{4m^2} \right) F_2^i(t) F_2^j(t) \right]$$

where $Z = \zeta^2/(2-\zeta)^2$ and $t = (\zeta^2 m^2 + r_\perp^2)/(\zeta - 1)$
Expressions are similar for A_{AA}, where g_A and g_P are convoluted
Form factors F_1, F_2, g_A, g_P are determined by p-pole fits on lattice 3-point data *T. Wurm (RQCD), private communication*

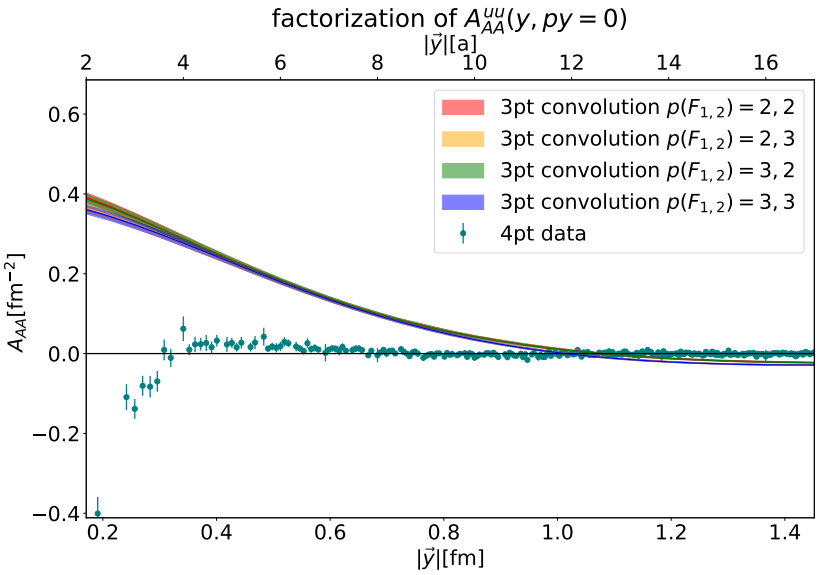
Factorization Test (preliminary)

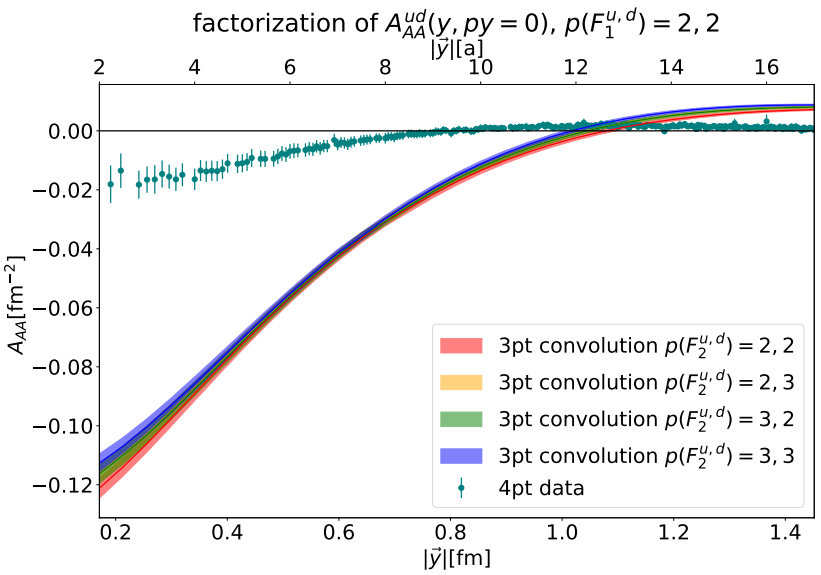




- No significant dependence on the fit ansatz
- Good agreement for large distances in the VV-channel
- Strong deviations for smaller distances

Factorization Test (preliminary)





- No significant dependence on the fit ansatz
- Good agreement for large distances in the VV-channel
- Strong deviations for smaller distances
- Completely fails for AA

Summary and Outlook

Achieved/Observed:

- Calculated two-current matrix elements for the nucleon on the lattice (4 of 5 contractions)
- Obtained Lorentz invariant combinations related to DPD Mellin moments for specific quark polarizations and flavor (C_1 and C_2 contributions)
- Found dominance of the signal of unpolarized quarks, for the case of *ud* polarization effects visible
- Factorization into one-current matrix elements works for large distances in the VV channel (unpolarized quarks), but fails for AA

Summary and Outlook

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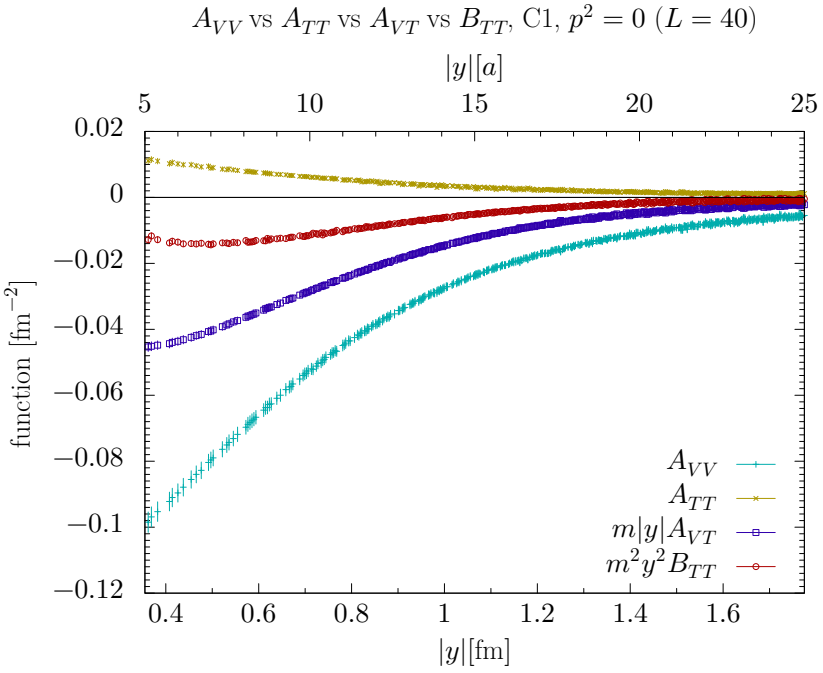
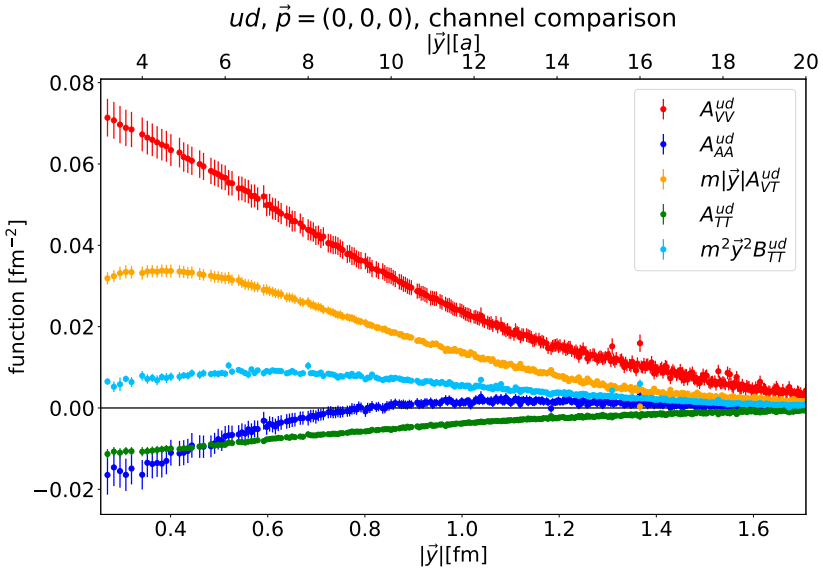
Further discussion beyond this talk / future work:

- Extraction of the first Mellin moments
- Derivative contributions ($\rightarrow$ higher moments)
- Analysis on larger lattices closer to the physical point

Thank you for your attention!

Comparison pion vs proton: polarization effects

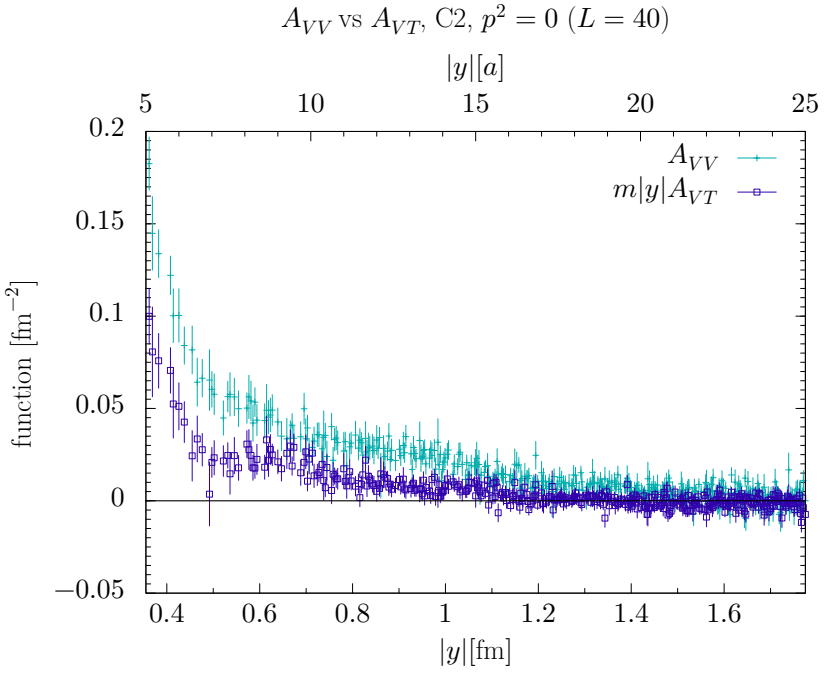


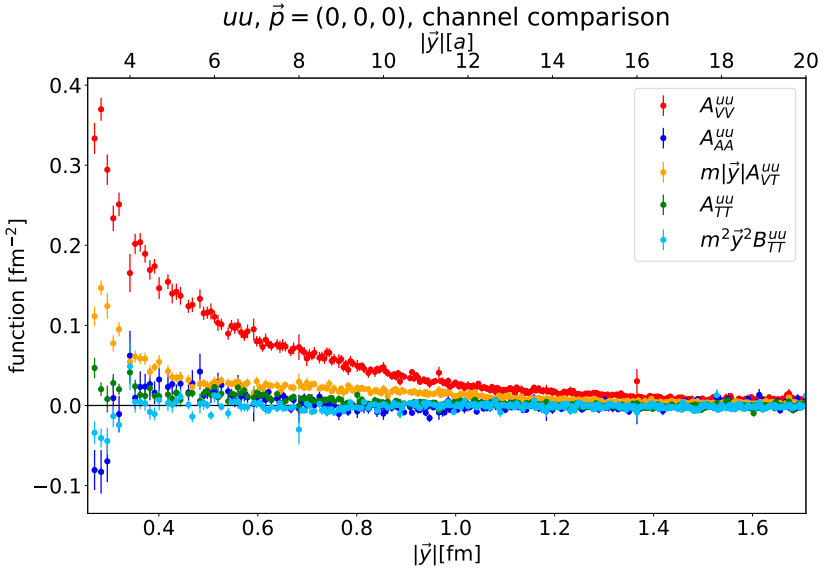


$ud(\pi^+)$ **vs** $ud(p)$ **:**

- Situation similar
- Extra sign because of anti-d in π^+

Comparison pion vs proton: polarization effects

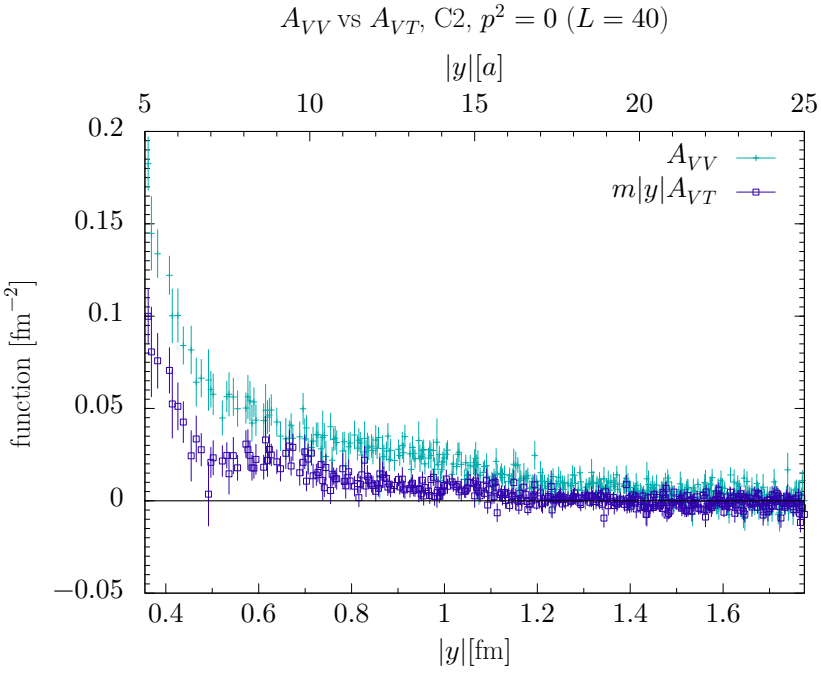


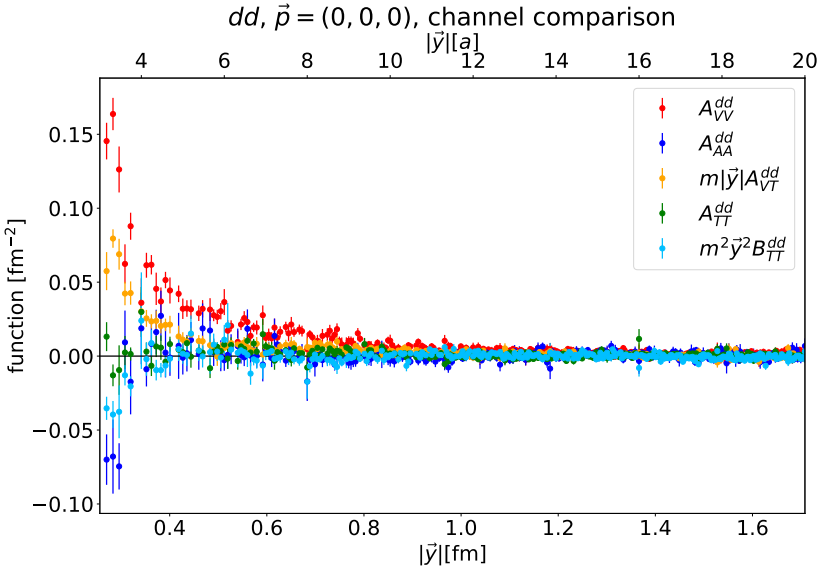


$uu/dd(\pi^+)$ **vs** $uu(p)$ **:**

- signal smaller for the pion
- polarization effects more present in the pion

Comparison pion vs proton: polarization effects

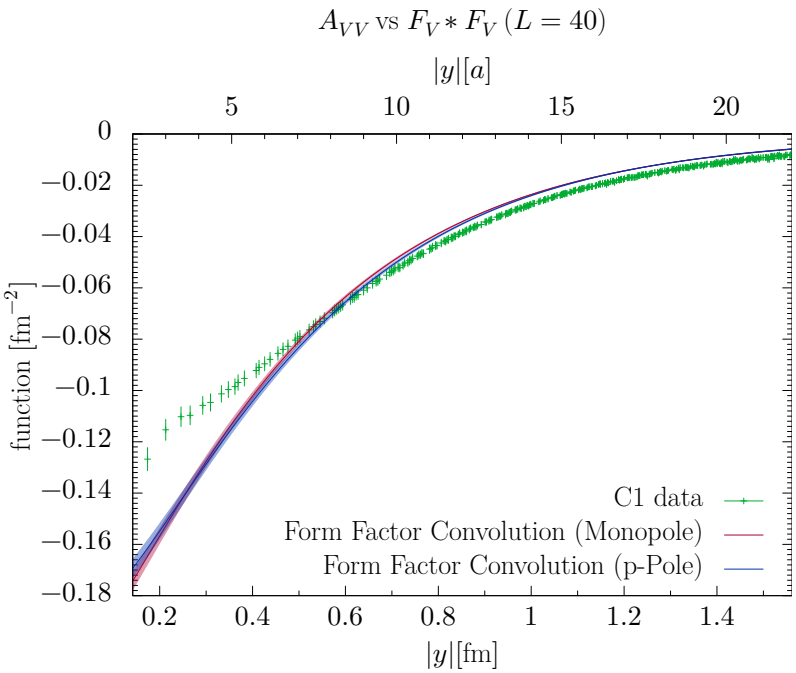


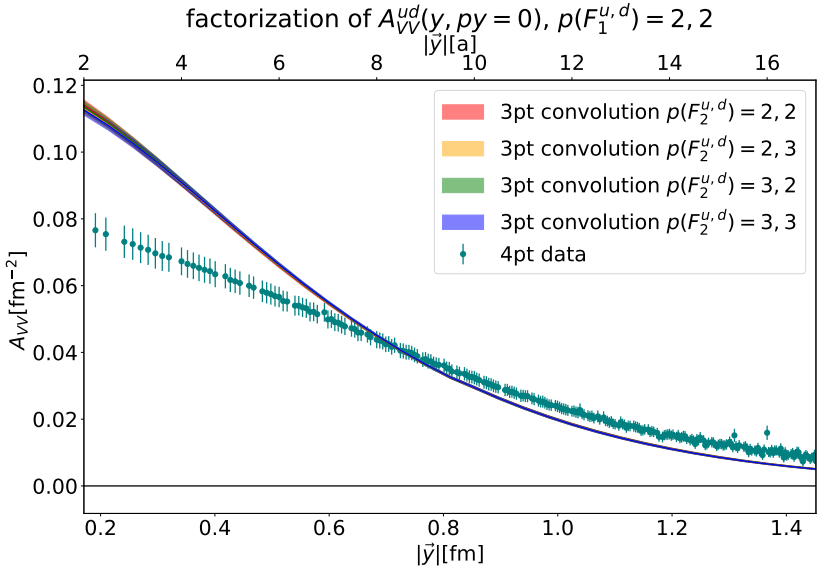


$uu/dd(\pi^+)$ **vs** $dd(p)$ **:**

- signals comparable

Comparison pion vs proton: Factorization for ud





Comparable results for factorization in the case of the pion or nucleon, respectively. Deviations slightly stronger for the nucleon.