MC developments in small-x and diffraction

Patrick Kirchgaeßer(KIT)

Outline

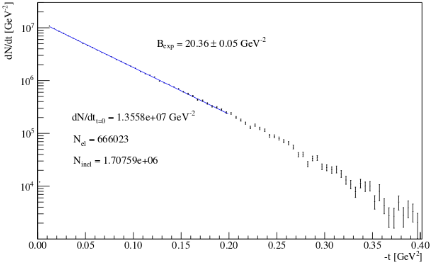
Introduction

Cross sections

- Optical theorem links the total cross section to the elastic scattering amplitude

$$\sigma_{\text{tot}} \propto \text{Im}[\mathcal{A}_{\text{el}}(t \to 0)]$$

$$\sigma_{\text{tot}} = \frac{16\pi(\hbar c)^2}{1+\rho^2} \frac{1}{N_{\text{el}} + N_{\text{inel}}} \frac{\mathrm{d}\sigma_{\text{el}}}{\mathrm{d}t}\bigg|_{t\to 0}$$



[TOTEM, Eur.Phys.J.C79(2019)no.2,103]

- Allows for a determination of the elastic slope parameter $\sigma_{\text{el}} \propto e^{-B|t|}$

Cross section parametrization (Input for MC)

- Commonly used Donnachie-Landshoff parametrization [Donnachie, Landshoff (1992)]

$$\sigma_{\mathrm{tot}}^{pp}(s) = \sigma_{\mathbb{P}} \left(\frac{s}{\mathrm{GeV}^2} \right)^{\epsilon}$$

  with $\sigma_{\mathbb{P}} = 21.7\mathrm{mb}$ and $\epsilon = 0.0808$ (soft pomeron)

- Extended by adding additional poles and branch-cuts in the complex plane

$$\sigma_{\mathrm{tot}}^{pp,\bar{p}p} = \sigma_{\mathbb{P}} \left(\frac{s}{\mathrm{GeV}^2} \right)^{\epsilon} + \sigma_{\mathbb{R}} \left(\frac{s}{\mathrm{GeV}^2} \right)^{-\eta}$$

  with $\eta = 0.4525$ and $\sigma_{\mathbb{R}} = 56\mathrm{mb}(pp), 98\mathrm{mb}(p\bar{p})$

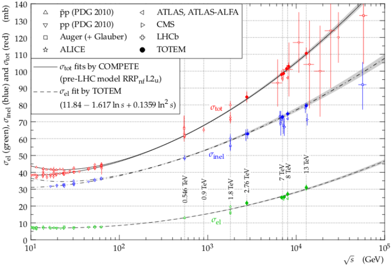
- Other parametrizations e.g implemented in Pythia [1]:
  - COMPAS group [Chin. Phys. C40 (2016) no.10]
  - ABMST [Eur. Phys. J C76 (2016) no.10,520]

[1] [Rasmussen, Sjöstrand, Eur.Phys.J. C78 (2018) no.6, 461]

Cross section parametrization (Input for MC)

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{inel}}$$

$$\sigma_{\text{tot}} > \sigma_{\text{inel}} > \sigma_{\text{el}} > \sigma_{\text{SD}} > \sigma_{\text{DD}} > \sigma_{\text{CD}}$$



[TOTEM, Eur.Phys.J.C79(2019)no.2,103]

- Satisfying description with 2-channel eikonal model [Khoze, Martin, Ryskin, Phys. Lett. B784(2018)192–198]

Theory (needed for later)

Froissart bound

Unitarity requirement of the total cross section leads to $\sigma_{\mathrm{tot}} \leq \ln^2(s)$.
Proton behaves asymptotically like a black disc (black-disc limit)

Pomeranchuk Theorem

Cross sections for all scattering processes involving any charge exchange vanish asymptotically

$\rightarrow$At high energies processes with the exchange of vacuum quantum numbers dominate the cross section

Multi Parton Interactions

- No first principle approach to MPI
- Simulation and study of MPI with Monte Carlo Event Generators (Pythia,Sherpa,Herwig,...)
- Perturbative aspects important (hard MPI) but additional input needed:
  - Colour connection between different scattering centers?
  - Form of soft MPIs (low $p_\perp$ interactions)?

Eikonal model

- Scattering amplitude in impact parameter space in the high energy limit reads

$$a(s,\mathbf{b}) = \frac{1}{2\mathrm{i}}\left[\exp\left(-\frac{-\Omega(s,\mathbf{b})}{2} - 1\right)\right]$$

- Identify the parametrization with the eikonal function, then directly with the amplitude

$$\Omega(s,\mathbf{b}) \propto \sigma_{\mathbb{P}}\left(\frac{s}{\mathrm{GeV}^2}\right)^{\epsilon} + \sigma_{\mathbb{R}}\left(\frac{s}{\mathrm{GeV}^2}\right)^{\eta} + \ldots$$

- Cross section remains finite no unitarity violation

$$\sigma_{\mathrm{tot}}(s) = 2\int d^2\mathbf{b}[1 - \exp(-\frac{\Omega}{2})]$$

$$\sigma_{\mathrm{el}}(s) = \int d^2\mathbf{b}\left|\exp(-\frac{\Omega}{2})\right|^2$$

$$\sigma_{\mathrm{inel}}(s) = \sigma_{\mathrm{tot}} - \sigma_{\mathrm{el}} = \int d^2\mathbf{b}[1 - \exp(-\Omega)]$$

MPIs in Herwig

- Separate eikonal function of inelastic cross section in hard and soft part

$$\Omega_{\text{inel}} = \frac{1}{2}\left(A_{\text{hard}}(b,\mu)\sigma^{\text{inc}}_{\text{hard}}(s, p_\perp^{\text{min}}) + A_{\text{soft}}(b,\mu_{\text{soft}})\sigma^{\text{inc}}_{\text{soft}}(s)\right)$$

- σ_{soft} is determined such that $\sigma_{\text{tot}}(\text{DL})$ is reproduced
- Number of additional interactions

$$\langle N_{\text{hard}}\rangle = A_{\text{hard}}\sigma^{\text{inc}}_{\text{hard}}(s, p_\perp^{\text{min}})$$
$$\langle N_{\text{soft}}\rangle = A_{\text{soft}}\sigma^{\text{inc}}_{\text{soft}}(s)$$

- No relation to diffraction $\rightarrow$ Need additional framework (e.g. Good-Walker States)

Good-Walker States

- Physical states $|\Psi_j\rangle$ can be written as sums of the diffractive eigenstates $|\Phi_i\rangle$

$$|\Psi_j\rangle = \sum_{i=1}^{N} \alpha_{ij} |\Phi_i\rangle$$

- Assume states are orthogonal and normalized $\rightarrow$ unitary α_{ij} matrix
- For the elastic scattering amplitude follows

$$\langle\Psi|T|\Psi\rangle = \sum_i |\alpha_{1i}|^2 T_i = \langle T\rangle$$

$$\sigma_{\mathrm{el}} \propto \langle T\rangle^2$$

and diffractive producton of any other state $|\Psi_{k\neq1}\rangle$

$$\langle\Psi_k|T|\Psi\rangle = \sum_i \alpha_{1i}\alpha^*_{ik} T_i$$

$$\sum_k \langle\Psi|T|\Psi_k\rangle\langle\Psi_k|T|\Psi\rangle = ... = \langle T^2\rangle$$

$$\sigma_{\mathrm{diff}} \propto \langle T^2\rangle - \langle T\rangle^2$$

Good-Walker States

- Eikonal must be expressed in terms of the eigenstates that scatter
- Diffractive cross sections with eigenstate dependent eikonals Ω_{ik}

$$\frac{\mathrm{d}\sigma_{\mathrm{el+SD}}(Y)}{\mathrm{d}t} = \frac{1}{4\pi}\sum_{i,j,k} |\alpha_i|^2|\alpha_j|^2|\alpha_k|^2$$
$$\times \int d^2\mathbf{b}\, \exp(i\mathbf{q}_\perp \mathbf{b})[1-\exp(-\Omega_{ik}(Y,\mathbf{b}))]$$
$$\times \int d^2\mathbf{b'}\, \exp(i\mathbf{q}_\perp \mathbf{b'})[1-\exp(-\Omega_{jk}(Y,\mathbf{b'}))]$$

- More in Christine Rasmussen's talk

Small-x physics

Small-x overview[2]

- What is the **high-energy asymptotic description of QCD**?
- BFKL equation: describes high energy behaviour of pQCD for small x
- Solution to BFKL equation

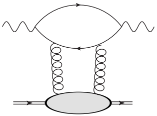
$$\sigma \sim s^{\mathrm{const}\,\alpha_s}$$

- Violates unitarity bound (Froissart bound $\sigma \leq \ln^2(s)$)
- Corrections to BFKL equation at high energy to restore unitarity (BK/JIMWLK)
- PDFS at small-x grow with $\frac{1}{x} \rightarrow$ enormous density of quarks and gluons leads to saturation
- High density proton/nucleus $\rightarrow$ implications for MPI (more MPIs)

[2] Follows book [Quantum Chromodynamics at High Energies (Kovchegov, Levin)]

Small-x basics

- From DIS($\gamma^* A/\gamma^* p$) at small-x $\rightarrow$ gluon exchange in t-channel dominates via quark loop virtual photon fluctuates into $q\bar{q}$ long before the interactions $\rightarrow$ **dipole interaction**



- DIS cross section at small-x:

$$\sigma_{T,L}^{\gamma^* A}(x, Q^2) = \int \frac{d^2 x_\perp}{4\pi} \int_0^1 \frac{dz}{z(1-z)} |\Psi_{T,L}^{\gamma^* \rightarrow q\bar{q}}(\mathbf{x}_\perp, z)|^2 \sigma_{\rm tot}^{q\bar{q}A}(\mathbf{x}_\perp, Y)$$

- DIS structure functions:

$$F_2(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\rm em}} \sigma_{\rm tot}^{\gamma^* A}$$

$$2xF_1(x, Q^2) = \frac{Q^2}{4\pi^2 \alpha_{\rm em}} \sigma_{T}^{\gamma^* A}$$

$\rightarrow$ **Need to calculate** $\sigma_{\rm tot}^{q\bar{q}A}$ and $\sigma_{T,L}^{\gamma^* A}$!

Small-x basics

- Rewrite the cross section as

$$\sigma_{\rm tot}^{q\bar{q}A}(\mathbf{x}_\perp, Y) = 2\int \mathrm{d}^2\mathbf{b}_\perp [1 - \mathrm{Re}[S(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)]]$$

  where

$$S(\mathbf{x}_\perp, \mathbf{b}_\perp, Y) = 1 + \mathrm{i}A(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)$$

- Define imaginary part of A by $N = \mathrm{Im}[A]$ s.t.:

$$N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y) = 1 - S(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)$$

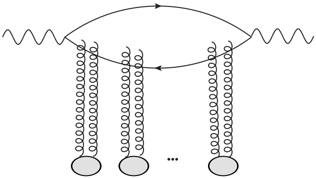
$$\to \sigma_{\rm tot}^{q\bar{q}A}(\mathbf{x}_\perp, Y) = 2\int \mathrm{d}^2\mathbf{b}_\perp N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)$$

Small-x basics

- DIS 2 gluon exchange at LO:

$$N_{\mathrm{LO}}(\mathbf{x}_\perp, \mathbf{b}, Y) = \frac{\pi \alpha_s^2 C_F}{N_c} T(\mathbf{b}_\perp)\mathbf{x}_\perp^2 \ln\left(\frac{1}{\mathbf{x}_\perp \Lambda}\right)$$

- Valid for $\mathbf{x}_\perp < 1/\Lambda \rightarrow$ perturbative dipole!
- $T(\mathbf{b}_\perp)$: Nuclear profile function
- **Problem:** increase transverse distance between quarks $\mathbf{x}_\perp$ and N_{LO} violates black-disc limit!
- **Solution:** Multiple interactions of the $q\bar{q}$ dipole with the target (p, A)



Small-x basics

- Diagrams need to be resummed [Mueller 1990]
- $\rightarrow$ exponentiaion of N_{LO}

$$N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y=0) = 1 - \exp\{-\frac{\alpha_s^2 C_F}{N_c} T(\mathbf{b}_\perp)\mathbf{x}_\perp^2 \ln\frac{1}{x_\perp \Lambda}\} \quad (1)$$

- Define saturation scale: $Q_s^2(\mathbf{b}_\perp) = \frac{4\pi\alpha_s^2 C_F}{N_c} T(\mathbf{b}_\perp)$

Gribov-Glabuber Mueller (GGM) formula

$$N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y=0) = 1 - \exp\{-\frac{1}{4}\mathbf{x}_\perp^2 Q_s^2(\mathbf{b}_\perp) \ln\frac{1}{x_\perp \Lambda}\} \quad (2)$$

- As $\mathbf{x}_\perp \rightarrow 0$, $N \rightarrow 0$ (dipoles very close and colours cancel which means no interaction with target)(“Colour Transparency”)
- As $\mathbf{x}_\perp \rightarrow \infty$, $N \rightarrow 1$
- Saturation region: $1/Q_s \leq x_\perp << 1/\Lambda$

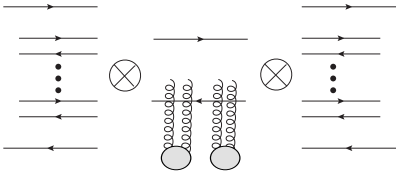
Small-x basics

- **Problem:** Amplitude N_{LO} (GGM formula) does not depend on CME or Y
- Also need to calculate radiative corrections (come in with factor of $\alpha_s \ln 1/x$)
- $\alpha_s << 1$ (pQCD regime) compensated by $\ln(1/x) >> 1$ $\rightarrow \alpha_s \ln(1/x) \sim 1$
- **Muellers Dipole Model**[3] :Original dipole splits and turns into dipole cascade (interaction between different dipoles N_C suppressed)
- Resum infinite cascade of gluons (large N_C limit, only planar diagrams, gluons=dipoles)
- Each dipole of the cascade interacts via the GGM multiple scatterings

[3] MC implementation of Muellers dipole model→ Christine Rasmussen's Talk

Small-x basics

- Original dipole turns into a dipole cascade
- Need to sum up a cascade of dipoles each interacting independently with the GGM multiple scatterings



- Explicit calculation leads to evolution equation of the dipole scattering amplitude

BK equation [Balitsky(1996,1999)Kovchegov(1999,2000)]

$$\frac{\partial N(\mathbf{x}_{1\perp},\mathbf{x}_{0\perp},Y)}{\partial Y} = \frac{\alpha_s N_c}{2\pi}\int d^2x_2 \frac{x_{10}^2}{x_{20}^2 x_{21}^2}[N(\mathbf{x}_{1\perp},\mathbf{x}_{2\perp},Y)$$

$$+ \mathrm{N}(\mathbf{x}_{2\perp},\mathbf{x}_{0\perp},Y) - N(\mathbf{x}_{1\perp},\mathbf{x}_{0\perp},Y) - N(\mathbf{x}_{1\perp},\mathbf{x}_{2\perp},Y)N(\mathbf{x}_{2\perp},\mathbf{x}_{0\perp},Y)]$$

Small-x basics

- Initial conditions for BK evolution at $Y = 0$ from GGM formula
- Now can calculate DIS structure functions of protons and nuclei at small-x:
- Solve BK equation with initial conditions at $Y = 0 \to N$
- Use resulting amplitude N for $\sigma_{\mathrm{tot}}^{q\bar{q}A}(\mathbf{x}_\perp, Y) = 2 \int d^2\mathbf{b} N(\mathbf{x}_\perp, \mathbf{b}_\perp, Y)$ and $\sigma_{TL}^{\gamma^* A}$
- Calculate structure functions F_1 and F_2

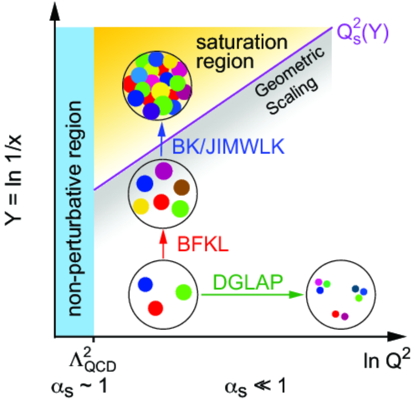
Small-x: Unitarity and Saturation

- Neglect quadratic term in BK equation $\rightarrow$ BFKL equation in dipole form
- Quadratic term becomes important for very small-x
- Qudadratic term accounts for saturation effects at large N_c
- BK equation does not violate unitarity
- Saturation scale ($Q_s(Y)$) as transition point between region where N grows with $\mathbf{x}_\perp$ and $N = \mathrm{const}$

$$Q_s(Y) \approx Q_s(0) e^{2.44 \frac{\alpha_s N_C Y}{\pi}}$$

- Saturation scale grows with Y
- Saturated gluonic matter: Colour Glass Condensate (CGC)

Small-x: Unitarity and Saturation



From [Cyrille Marquet,1212.3482]

- Study high energy asymptotics of QCD$\rightarrow$ small-x evolution equations (BFKL/BK)
- More in EIC talk from Elke-Caroline Aschenauer (next talk)

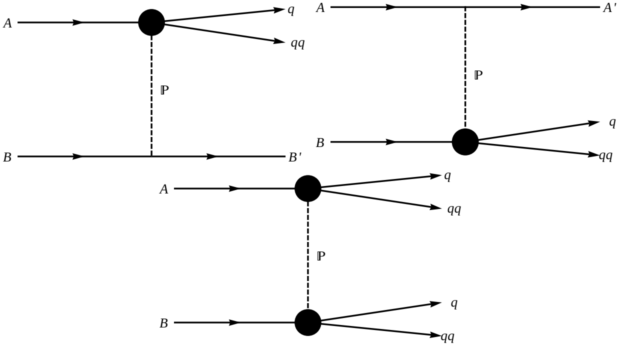
Diffraction

Diffraction

- Diffraction depends on experimental definition: rapidity gaps ($\Delta\eta$) mesurements
- MPIs can destroy experimental signatures of diffraction
- Rapidity gaps get contributions from MPI
- Colour reconnection effects?

Model for soft diffraction in Herwig

- Soft diffraction (small $p_\perp$) in Herwig by modelling it with the following matrix elements



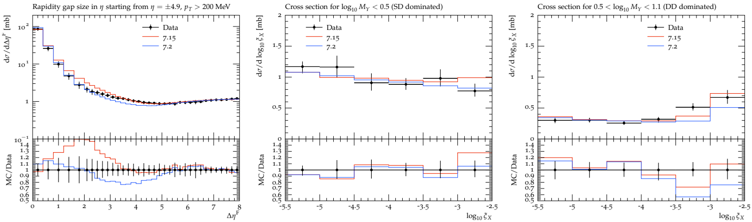
- Final state treated fully non-perturbatively - Quark (q) and diquark (qq) form a cluster with diffractive mass M and stretched along the direction of the dissociated proton

Diffraction

- Combine existing MPI model with diffraction
- DiffractionRatio: fraction of diffractive cross section w.r.t non-diffractive cross section
- Modifies cross sections of hard and soft MPIs in the eikonal model s.t σ_{tot} is reproduced after eikonalization
- Tunable parameter
- Every event either diffractive or non-diffractive

Tuning and results

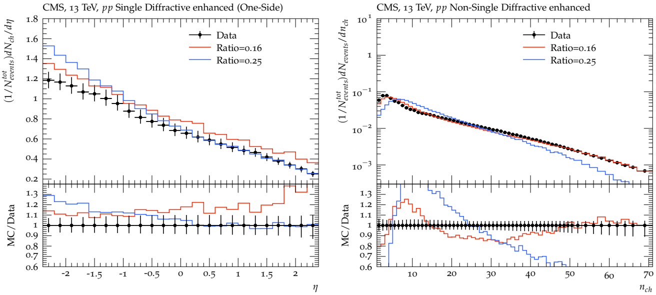
- Tune DiffractionRatio to sensible data ($\Delta\eta$, SD,DD dominated cross sections)
- Data at 7 TeV: DiffractionRatio$=$ 0.257 ($\sigma_{\text{Diff}} \approx 17\text{mb}$)
- Data at 13 TeV: DiffractionRatio$=$ 0.167 ($\sigma_{\text{Diff}} \approx 14\text{mb}$)
- No dedicated diffractive measurements at 13 TeV: large uncertainty on diffraction



[ATLAS, Eur.Phys.J. C72 (2012) 1926] [ALICE, Eur.Phys.J. C73 (2013) no.6, 2456]

Tuning and results

- Fix DiffractionRatio to 7 TeV measurment and tune remaining MPI parameters
- Diffraction at 13 TeV not well constrained



[CMS, Phys.Lett. B751 (2015) 143-163]

Summary

- Overview about MPI, small-x physics and diffraction
- Many interesting talks this week!
- Dedicated 13 TeV diffraction measurements needed