# Hadronization in Terms of First-Order Phase Transition

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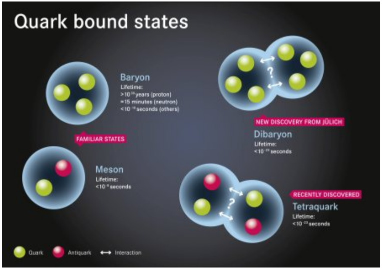
# Contents

# Introduction




Hadronization models:

1) Local parton-hadron duality approach

(i) String model(1+1) tube, Lund model

(ii) Claster model (fragmentation from the colorless clusters)

(iii) Glassma (fragmentation from the color condensate)

2) Dynamic approach

(i) Running gluon mass in the pure gluodynamic picture

ii) Renorm. group approach

# Quark-hadron duality and hadronization

$$\mathcal{M} =< out|in >=< \bar{q}_{HAD}|q_{QGP} >=< \bar{q}_{QGP}|U|q_{QGP} > \quad (1)$$

$$< |\mathcal{M}|^2 >= Tr|(\gamma^0 U) G^{-+}(x_1, x_2)|^2 \quad (2)$$

$$< |\mathcal{M}|^2 >= - Tr\left\{ G_{21}^{-+}\ \varrho(1,2)\ G_{12}^{-+}) \right\};$$

$$\varrho(1,2) = \gamma^0 U_1 U_2^{\dagger} \gamma^0 \quad (3)$$

# Quark-hadron duality and hadronization

When $G_{21}^{-+}; \varrho(1,2); G_{12}^{-+} \to G^{-+}, \varrho(x_1 - x_2)$

In the momentum representation

$$< |\mathcal{M}|^2 >= -Tr \left\{ \int \frac{dp_1 dp_2}{(2\pi)^8} \bar{G}^{-+}(p_1)\ \varrho(p_1,p_2)\ G^{-+}(p_2) \right\}, \tag{4}$$

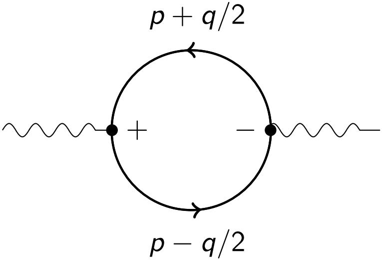
$$\frac{d < |\mathcal{M}|^2 >}{dp} = -Tr \left\{ \int \frac{dq}{(2\pi)^8} \bar{G}^{-+}(p+q/2)\varrho(p+q/2, p-q/2) G^{-+}(p-q/2) \right\} \tag{5}$$

.

$$p = \frac{p_1 + p_2}{2}; \quad q = p_1 - p_2, \tag{6}$$

# Quark-hadron duality and hadronization

## Diagramically

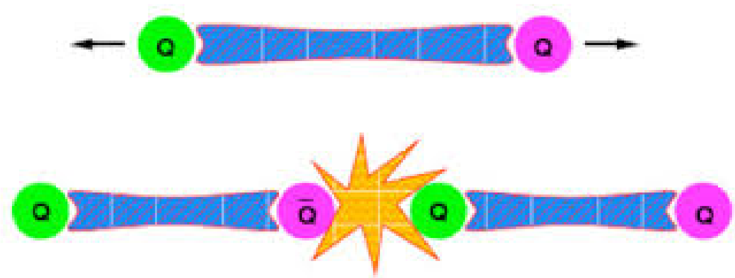



, (7)

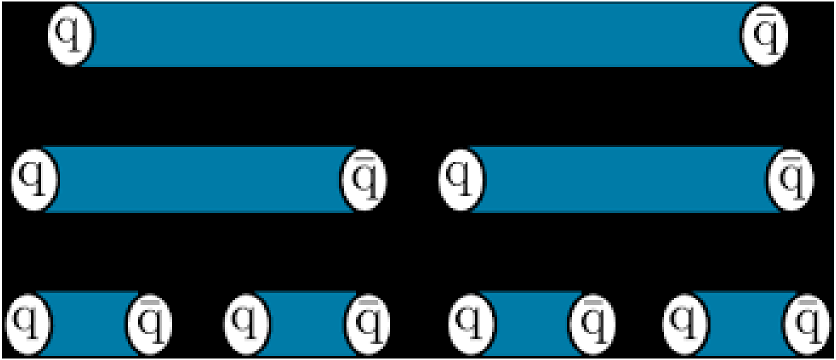
$$\frac{E(\boldsymbol{p})dN_h}{d^3p} = \int dp^0 (E(\boldsymbol{p})) \frac{d < |\mathcal{M}|^2 >}{dp}, \qquad (8)$$

# Flux tube pictutre

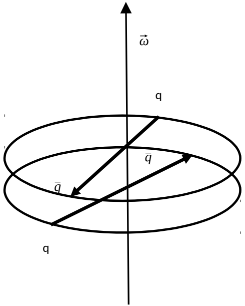
$$\sqrt{s} \simeq 8 - 20 GeV \tag{9}$$

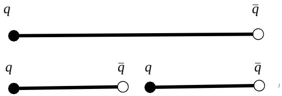


# Flux tube pictutre



# Flux tube pictutre



# Flux tube pictutre

1. Supported in experiments on hadronic, $e^+e^-$ and $pp$ collisions
2. Theoretical background - E.Whitten papeprs

$$j^\mu = -m\varepsilon^{\mu\nu}\partial_\nu\phi, \tag{10}$$

Conclusion

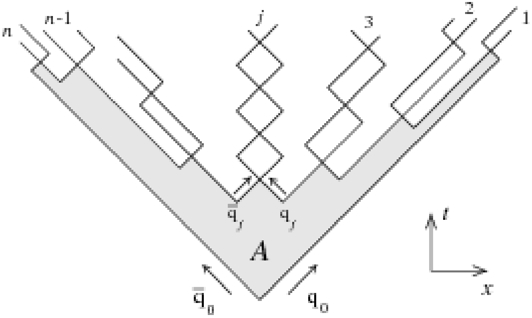
I. The longitudinal dominance

II. The transverse confinement

# Flux tube pictutre

I,II have taken into account in LUND model.

i) entirely phenomenological;
ii) the rapidity distribution only;
iii) no mechanism of "o-o" states.



## $QCD_4 \to QCD_{xy} + QCD_{zt}$ compactification

HOW TO GET IT

1. To start from the exact (3+1) QCD
2. To reduce it up to (1+1) QCD

# $QCD_4 \to QCD_{xy} + QCD_{zt}$ compactification

$QCD_4 \to QCD_{xy} + QCD_{zt}$ compactification
(A.V.Koshelkin, C.-Y.Wong)

$$\Psi(x) = \Psi(\boldsymbol{r}_\perp)\psi(z,t), \tag{11}$$

Transverse motion

$$\begin{aligned}(p_1 + ip_2)\Psi_+(\boldsymbol{r}_\perp) &= (m(\boldsymbol{r}_\perp) + E_\nu^2)\Psi_-(\boldsymbol{r}_\perp),\\ (p_1 - ip_2)\Psi_-(\boldsymbol{r}_\perp) &= (E_\nu^2 - m(\boldsymbol{r}_\perp))\Psi_+(\boldsymbol{r}_\perp),\end{aligned} \tag{12}$$

Longitudinal motion

$$\partial_z^2\psi(t,z) - \partial_t^2\psi(t,z) = m_{qT}^2\psi(t,z), \tag{13}$$

# $QCD_4 \to QCD_{xy} + QCD_{zt}$ compactification

$$\tau = \sqrt{t^2 - z^2}, \quad \eta = \frac{1}{2} \ln \left( \frac{t+z}{t-z} \right) \tag{14}$$

$$\tau^2 \frac{\partial^2 \psi(\tau,\eta)}{\partial \tau^2} + \tau \frac{\partial \psi(\tau,\eta)}{\partial \tau} - \frac{\partial^2 \psi(\tau,\eta)}{\partial \eta^2} = 0. \tag{15}$$

$$\psi_+(\tau,\eta) = f(\eta - \ln(\tau/\tau_0)); \quad \psi_-(\tau,\eta) = g(\eta + \ln(\tau/\tau_0))$$

$$\psi(\tau = \tau_0, \eta) = \frac{1}{\sqrt{\sigma \pi^{1/2}}} \exp\left( -\frac{\eta^2}{2\sigma^2} \right), \tag{16}$$

$$\psi_\pm(\tau,\eta) = \frac{1}{\sqrt{\sigma \pi^{1/2}}} \exp\left( -\frac{(\eta \mp \ln(\tau/\tau_0))^2}{2\sigma^2} \right). \tag{17}$$

# Hadron production

$$\frac{E(\boldsymbol{p})dN_h}{d^3p} = -Tr\int dp^0(E(\boldsymbol{p}))\int\frac{dq}{(2\pi)^8}\times$$
$$\left\{\bar{G}^{-+}(p+q/2)\varrho(p+q/2,p-q/2)G^{-+}(p-q/2)\right\}.(18)$$

$$G^{-+}(p) = 2\pi i|\Psi(\boldsymbol{p})|^2\delta(p^0-\mu+\varepsilon(\boldsymbol{p})). \qquad (19)$$

# Hadron production

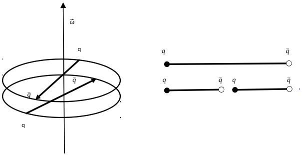
I. The longitudinal dominance

II. The transverse confinement

$$\frac{E(\boldsymbol{p})dN_h}{d^3p} = \sum_{a=1}^{N} \int \frac{d^2q_\perp}{(2\pi)^6}(E(\boldsymbol{p}))\varrho_a(\boldsymbol{p};\boldsymbol{q}) \times$$
$$|\Psi_{\bar{q}_a}(\boldsymbol{p}_\perp + \boldsymbol{q}_\perp/2)|^2 \; |\Psi_{q_a}(\boldsymbol{p}_\perp - \boldsymbol{q}_\perp/2)|^2 \times$$
$$\int dq_z |\psi_{\bar{q}_a}(p_z + q_z/2)|^2 \; |\psi_{q_a}(p_z - q_z/2)|^2, \qquad (20)$$

# Hadron production

$$\rho_a(\boldsymbol{p}, q_z) = \int \frac{d^2 q_\perp}{(2\pi)^6} E(\boldsymbol{p}) \varrho_a(\boldsymbol{p}; \boldsymbol{q}) |\Psi_{\bar{q}_a}(\boldsymbol{p}_\perp + \frac{\boldsymbol{q}_\perp}{2})|^2 |\Psi_{q_a}(\boldsymbol{p}_\perp - \frac{\boldsymbol{q}_\perp}{2})|^2 \tag{21}$$




$$\rho_a(\boldsymbol{p}, q_z) = \mathcal{P}_a(\boldsymbol{p}) \delta(q_z). \tag{22}$$

# Hadron production

$$\frac{dN_h}{d^2p dy} = \sum_{a=1}^{N} \mathcal{P}_a(\boldsymbol{p}_\perp, y)|\Psi_{\bar{q}_a}(y)|^2 \ |\Psi_{q_a}(y)|^2. \tag{23}$$

$$y = \frac{1}{2} \ln \left( \frac{E(\boldsymbol{p}) + p_z}{E(\boldsymbol{p}) - p_z} \right). \tag{24}$$

# Probability

$$\mathcal{P}_a(\boldsymbol{p}_T) = A\exp\left(-\frac{\Delta G_a}{T}\right) = A\exp\left(-\frac{E_a + \alpha\ln\left(\frac{C_{q_h}^{(a)}(P,T)}{C_{qq}^{(a)}(P,T)}\right)}{T}\right) \quad (25)$$

i) no clusters
ii) no interaction in the final state

$$\Delta G^{(a)}(E_T(p_T = 0)) = m_h\sqrt{g_{00}} = m_h\sqrt{1 - v^2}, \quad (26)$$

where $m_h$ is the mass of a hadron, $g_{00}$ is the zeroth component of the metric tensor $g_{\mu\nu}$ for the rotating frame of reference.

# Probability

$$\Delta G^{(a)}(E_T(p_T=0)) = g_{\mu\nu} U^\mu p^\nu, \tag{27}$$

where $p^\nu = (m_h, 0)$ , $U^\mu$ is the hydrodynamic velocity

$$U^\mu = \left( \frac{1}{\sqrt{1-v^2}}, \frac{v^1}{\sqrt{1-v^2}}, \frac{v^2}{\sqrt{1-v^2}}, 0 \right), v^2 = (v^1)^2 + (v^2)^2. \tag{28}$$

Since the created hadrons are assumed to leave instantly the hadronization area, there is no motion of the hadronic matter as a whole in the laboratory frame of reference. Therefore, we have for the hydrodynamic velocity in this frame of reference

$$U^\mu = (1, 0, 0, 0). \tag{29}$$

# Probability

Taking into account that $g_{\mu\nu} = diag(1, -1, -1, -1)$ in the laboratory frame of reference we find

$$\Delta G^{(a)} = E_T, \tag{30}$$

where $E_T = \sqrt{\boldsymbol{p}_T^{\,2} + m_h^2}$ is the transverse energy of a hadron, $\boldsymbol{p}_T = m_h \boldsymbol{v} / \sqrt{1 - v^2}$.

# Probability

When matter moves along the collision axes with a velocity $v^3$, we have in the laboratory frame

$$p^{\nu} = (E_T \cosh y_{v^3}, 0, 0, E_T \sinh y_{v^3}). \qquad (31)$$

However, then

$$U^{\nu} = (\cosh y_{v^3}, 0, 0, \sinh y_{v^3}). \qquad (32)$$

Since $g_{\mu\nu}$ still is $g_{\mu\nu} = diag(1, -1, -1, -1)$

$$\Delta G^{(a)} = g_{\mu\nu} U^{\mu} p^{\nu} = E_T, \qquad (33)$$

where $E_T = \sqrt{\boldsymbol{p}_T^2 + m_h^2}$ is the transverse energy of a hadron, $\boldsymbol{p}_T = m_h \boldsymbol{v}/\sqrt{1 - v^2}$.

# Hadron rate

$$\frac{dN_h}{dyd^2p_\perp} = \frac{N_f N_c^2 \theta(T_c - T)}{512\pi^8 m_h T K_1(m_h/T)} \exp\left(-\frac{E_T}{T}\right)$$

$$\sum_{a=1}^{N} \frac{\Sigma_a^2}{\sigma_a^2} \left\{ \exp\left(-\frac{2(y - y_a)^2}{\sigma_a^2}\right) + \exp\left(-\frac{2(y + y_a)^2}{\sigma_a^2}\right) \right\} (34)$$

# Hadron rate

$$\frac{dN_h}{dy} = \frac{N_f N_c^2 \theta(T_c - T)(1 + (T/m_h))}{256\pi^7 K_1(m_h/T)\exp(m_h/T)}$$

$$\sum_{a=1}^{N} \frac{1}{\sigma_a^2} \left\{ \exp\left( -\frac{2(y - y_a)^2}{\sigma_a^2} \right) + \exp\left( -\frac{2(y + y_a)^2}{\sigma_a^2} \right) \right\} \quad (35)$$

# Relation to the experiment

N.Abgrall et al, Eur.Phys.J. **C74**, 2794 (2014).

The yield of the negatively charged pions, arising in the result of inelastic *pp* interaction of the target protons and the incident protons, having the momenta $20, 31, 40, 80, 158\ GeV/c$ (which are $\sqrt{s} = 6.3, 7.7, 8.8, 12.3, 17.3\ GeV$), has been measured at the beam rapidities $y_b = 1.877, 2.094, 2.223, 2.569, 2.909$.

# Relation to the experiment

$$\frac{dN_h(m_T, y=0)}{dy m_T dm_T} = \frac{2 < N_{ch}(s) > \exp\left(m_h/T - \frac{4y_0^2}{y_b^2}\right)}{(2\pi)^{1/2}\sigma m_h T(1 + (T/m_h))} \exp\left(-\frac{m_T}{T}\right) \tag{36}$$

$$\frac{dN_h}{dy} = \frac{< N_{ch}(s) >}{\pi^{1/2} y_b} \left\{ \exp\left(-\frac{4(y-y_0)^2}{y_b^2}\right) + \exp\left(-\frac{4(y+y_0)^2}{y_b^2}\right) \right\} \tag{37}$$

# Relation to the experiment

$$\zeta\sqrt{s} = \int E_T \cosh y \frac{dN_h}{dy d^2 p_\perp} =$$
$$< N_{ch}(s) > m_h \cosh(y_0) \exp\left(\frac{\sigma^2}{8}\right) \frac{1 + 2(T/m_h) + 2(T/m_h)^2}{(1 + (T/m_h))} \quad (38)$$

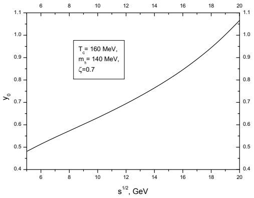



Figure: 1. Dependence of $y_0$ on the energy of a proton beam.

# Relation to the experiment

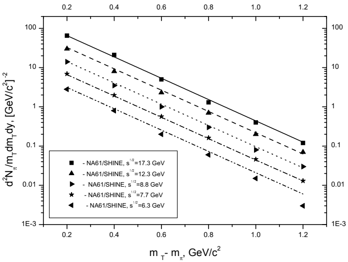



Figure: 2. The lines of various types are the $p_T$ distributions of pions which are given by Eq.(11) at $T_c = 160MeV$ and $\kappa = 1Gev/F$, which are normalized by the experimental value of the pion rate at $(m_T - m_\pi) = 0.2GeV/c$ v.s. the pion rate in $p - p$ collisions(the scattered symbols) at the same projectile energies.

# Relation to the experiment

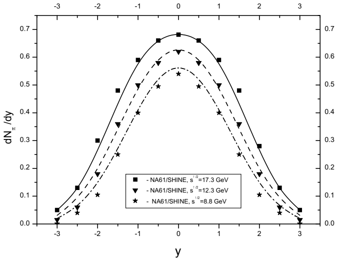



Figure: 3. The rapidity distributions given by Eq.(12) at $\sqrt{s} = 17,3 GeV$ (solid lines), $\sqrt{s} = 12,3 GeV$ (dashed line), $\sqrt{s} = 8,8 GeV$ (dot-dashed line), and at $T_c = 160 MeV$ v.s. the rapidity distributions in $p - p$ collisions

# Conclusion

1. Based on the quark-hadron duality concept the new approach to the problem hadronization of the deconfinement is developed.

2. Under the longitudinal dominance and transverse confinement both the rapidity and $p_T$ distributions of the hadron created in collisions of high energy particles are explicitly derived when the hadronization is governed by the first order phase transition.

3. The pion production in high energy proton collisions is studied in detail.

4. The derived hadron rate has been compared with the experimental results on pion production in *pp* collisions. A good relation to the experimental data is found along the whole range of the energies, $\sqrt{s} = 6.3 - 17.3 GeV$, of the proton projectile used in the experiments.

# Acknowledgment

THANK YOU FOR YOUR ATTENTION!!!