Theory benchmarks for $\sin^2\theta_W$ measurement: genuine EW and lineshape corrections

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- **Quick reminder**
- **Benchmarks status (November/December'18)**
- **Updates on Powheg + wt^{EW} (March'19)**
- **Outlook**

Participants so far .....

PowhegZj: QCD NLO, Z+j
wt^{EW} : TauSpinner + Dizet 6.21

wt^{EW}

EW LO → $\alpha(0)$ v0: LO, NLO+HO
EW LO → $\alpha(0)$ v1: LO
EW LO → G_μ: LO

Arbitrary
EW setup

Powheg_ew: QCD LO, Z

- $\alpha(0)$ v0: LO
- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO

MCSANC: QCD LO, Z

- $\alpha(0)$ v1: LO, NLO, NLO+HO
- G_μ: LO, NLO, NLO+HO

DYTURBO: QCD LO, NLO, Z

- $\alpha(0)$ 0: LO
- $\alpha(0)$ v1: LO
- G_μ : LO

ZGRAD2: QCD LO, EW NLO + HO
expressed interest to participate too

EW schemes: input parameters

SM fundamental relation used to calculate EW parameters at LO in different EW schemes, on-mass-shell definition.

LEP legacy — LHC standard

EW scheme:	G_μ, α, M_Z	α, M_W, M_Z	G_μ, M_W, M_Z
	$\alpha(0)$ v0	$\alpha(0)$ v1	$G\mu$
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} M_W^2 s_W^2}$$

$$s_W^2 = 1 - m_W^2 / m_Z^2$$

Be aware: $\alpha(0)$ v1 comes with unphysical value of $G\mu$

DIZET library
exact $O(\alpha)$+ higher order terms

Powheg_ew, MCSANC
EW NLO, NLO+HO

EW schemes: input parameters

EW schemes: come with „on-shell" or „pole" definitions!

Table 44: The EW parameters used at tree-level EW, with on-mass-shell definition (LEP convention).

Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.23323
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.93886 GeV	80.385 GeV	80.385 GeV
s_W^2	0.2121517	0.2228972	0.2228972
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Runing Γ_Z in Z-propagator

Shift:

- -30 MeV for M_Z
- change on Γ_Z
- -0.00006 for $s^2 w$

Scaling

- 0.99906 for α

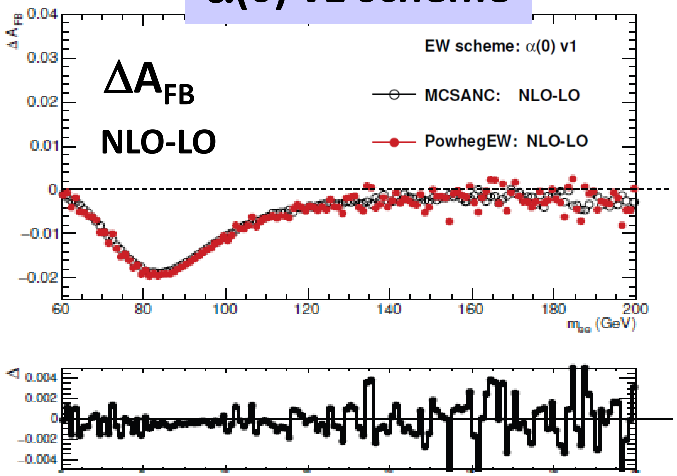
Table 45: The EW parameters used at tree-level EW, with pole definition of the Z, W masses.

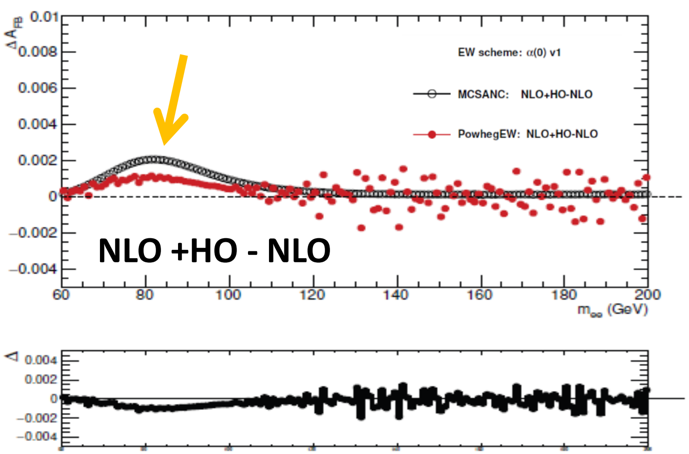
Parameter	$\alpha(0)$ v0	$\alpha(0)$ v1	G_μ
M_Z	91.15348 GeV	91.15348 GeV	91.15348 GeV
Γ_Z	2.494266 GeV	2.494266	2.494266 GeV
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV
α	1/137.03599	1/137.03599	1/132.3572336357709
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.126555497 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}
M_W	80.91191 GeV	80.35797 GeV	80.35797 GeV
s_W^2	0.21208680	0.22283820939	0.22283820939
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0

Fixed Γ_Z in Z-propagator

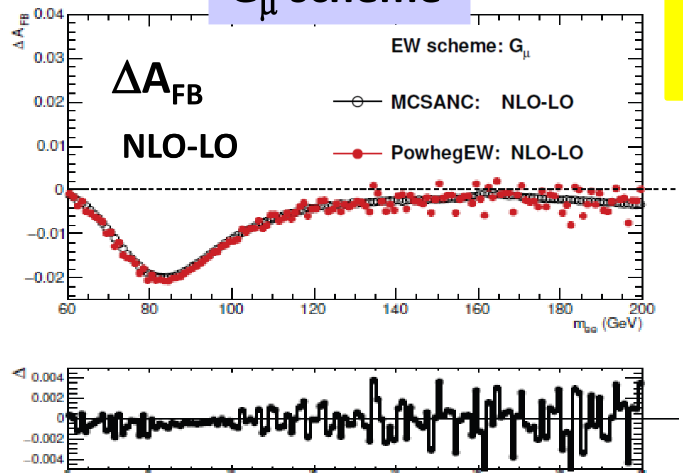
A_{FB}: EW LO, NLO, NLO+HO

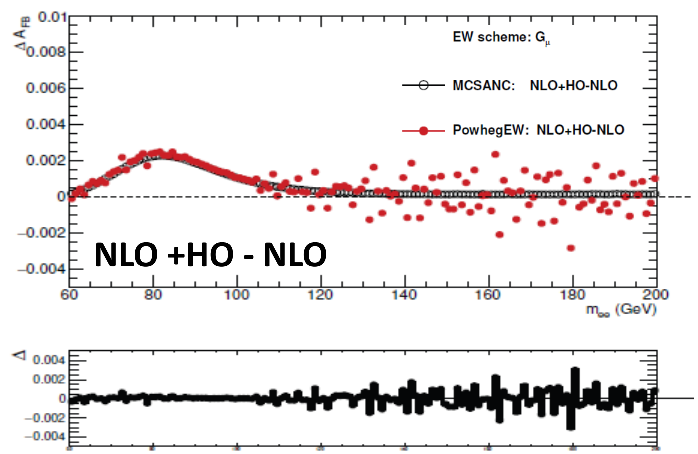
$\alpha(0)$ v1 scheme





G_μ scheme





m_{ll} (GeV)

Status:
December 2018

Powheg_ew
F. Piccinini et al.

MCSANC
S. Bondarenko, L. Kalinovskaya

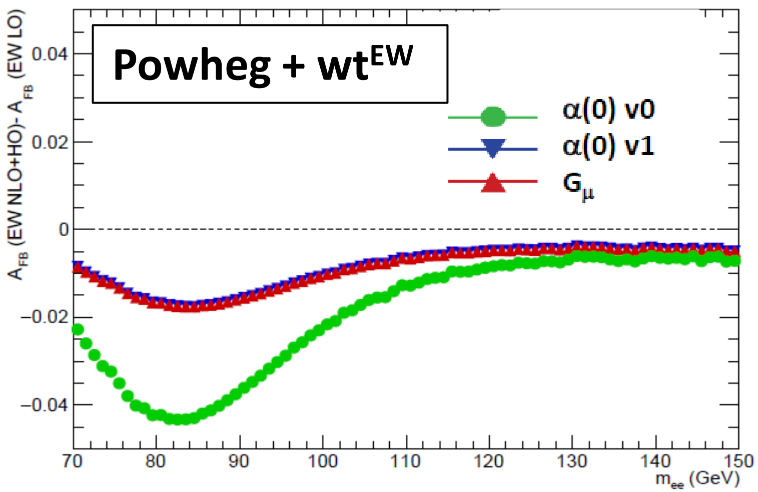
Pending investigating this discrepancy:
0.001 shift on ΔA_{FB} at Z-pole corresponds to shift ~ 30 10-5 on $\sin^2\theta_{eff}$

A_{FB}: EW LO, NLO+HO

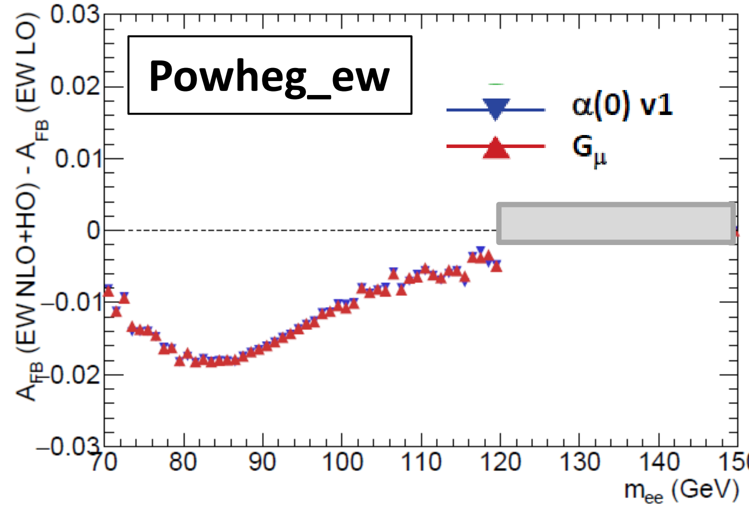
- **Comparing Powheg_ew and Powheg+wt^{EW}**

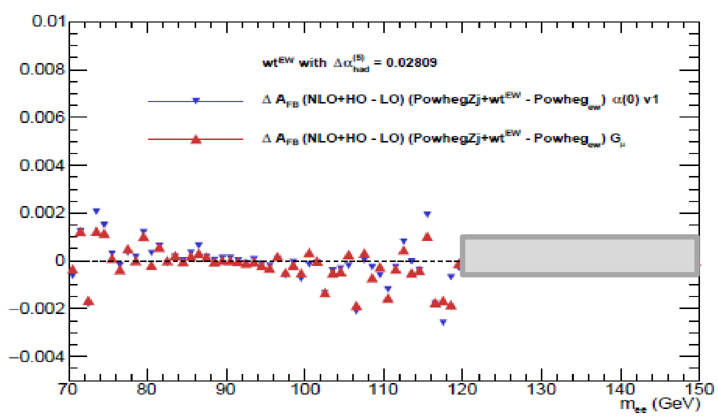
Status:
September 2018

ΔA_{FB} **NLO+HO - LO**



ΔA_{FB} **NLO+HO - LO**





wt^{EW} with $\Delta\alpha^{(5)}_{had}$ = 0.02809

ΔA_{FB} (NLO+HO - LO) (Powheg$_{ew}$ - PowhegZj+wt^{EW}) $\alpha(0)$ v1

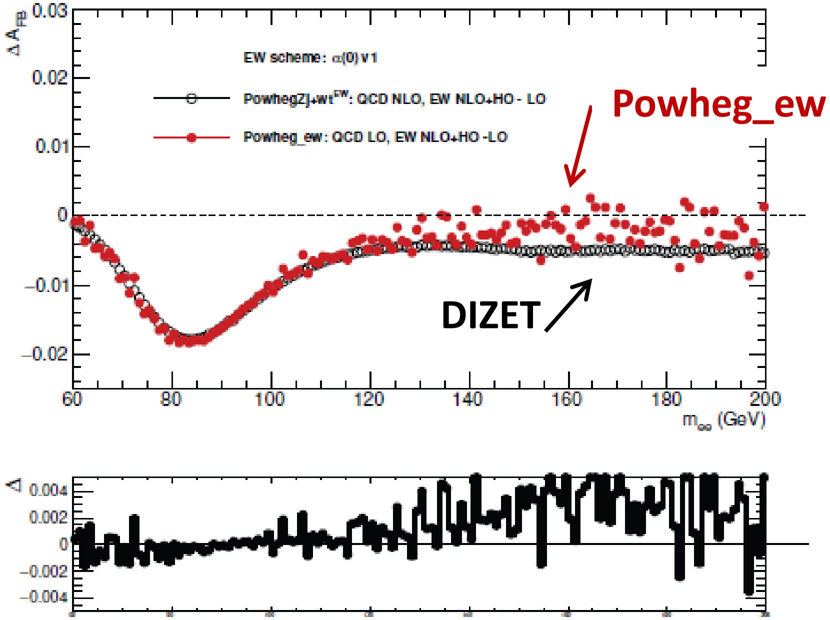
ΔA_{FB} (NLO+HO - LO) (Powheg$_{ew}$ - PowhegZj+wt^{EW}) G_{μ}

Excelent agreement on ΔA_{FB}! Is it accidental?

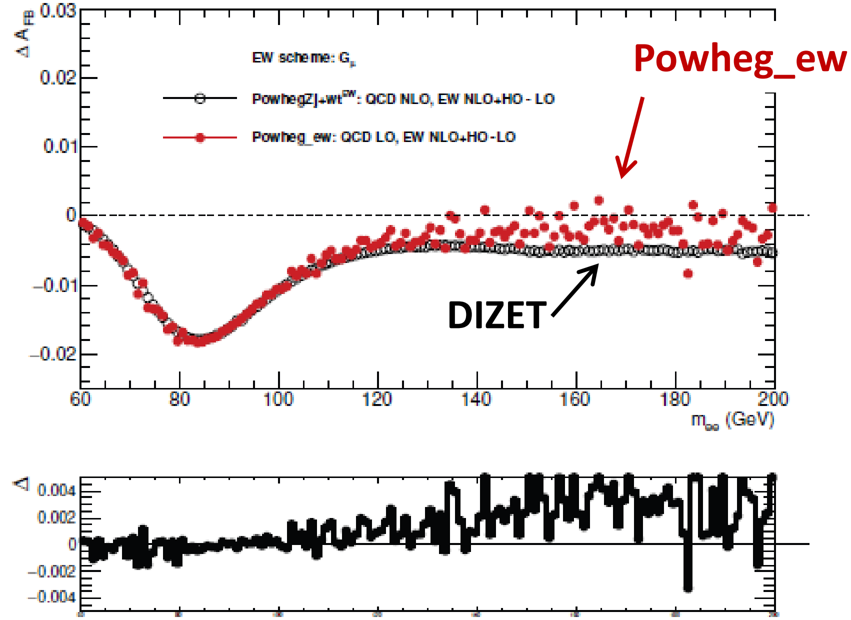
A_{FB} : EW LO, NLO+HO

Status:
December 2018

α(0) v1 scheme



G_μ scheme



Around Z-pole:

-> good agreement between Powheg_ew and DIZET

-> MCSANC shifted by 0.001

Pending invetigating this discrepancy!

At higher masses:

-> DIZET predicts stable (NLO+HO – LO) corr. of 0.005

-> PowhegEW and MCSANC predicts (NLO+HO – LO) being close to zero.

Powheg + wt^{EW}: updates since December

- EW weights calculated now with running Γ_Z in Z-boson propagator

- Updated parametrisation of $\Delta\alpha_{had}^{(5)}(s)$
  - Jegerlehner 2017 (arXiv:1711.06089)
  - Burkhard&Pietrzyk 2001 (Phys.LettB 513 (2001) 46)

- Updated Dizet 6.21 -> Dizet 6.42
  - AMT4=4: Subleading two-loop corrections and re-summation recipe
  - AMT4=6: Complete two-loop corrections to M_W and fermionic two-loop corrections to $\sin^2\theta_{eff}^{lep}$

Impact of $\Delta\alpha_{had}^{(5)}(M_Z^2)$

Parameter	Dizet 6.21, 6.42 $\Delta\alpha_h^{(5)}(M_Z^2) = 0.0280398$ (param. Jegerlehner 1995)	NEW $\Delta\alpha_h^{(5)}(M_Z^2) = 0.0275762$ (param. Jegerlehner 2017)	Δ
$\alpha(M_Z^2)$	0.00775884	0.00775492	
$1/\alpha(M_Z^2)$	128.885224	128.9503292	
s_W^2	0.22351946	0.22332758	- 0.00019
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23175990	0.23158294	- 0.00018
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23164930	0.23147645	- 0.00017
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23152214	0.23134945	- 0.00017
M_W	80.35281 GeV	80.36274 GeV	+10 MeV
Δr	0.03694272	0.036609	
Δr_{rem}	0.01169749	0.01170287	

$$M_W = \frac{M_Z}{\sqrt{2}}\sqrt{1+\sqrt{1-\frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

Impact of AMT= 4 –> AMT=6

Updated to Dizet 6.42, which comes with more complete two-loop corrections

NEW

Parameter	AMT4= 4	AMT4 = 6	Δ
$\alpha(M_Z^2)$	0.00775492	0.00775492	
$1/\alpha(M_Z^2)$	128.9503239	128.9503239	
s_W^2	0.22332758	0.22343647	+ 0.00011
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23158294	0.23153917	-0.000044
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23147645	0.23143261	-0.000044
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23134945	0.23130551	-0.000044
M_W	80.36274 GeV	80.35710 GeV	- 5.6 MeV
Δr	0.036609	0.03636609	
Δr_{rem}	0.01170287	0.01170287	

$$M_W = \frac{M_Z}{\sqrt{2}}\sqrt{1+\sqrt{1-\frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$\Delta r = \Delta\alpha(M_Z^2) + \Delta r_{EW}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

- **AMT4=4: Subleading two-loop corrections and re-summation recipe (best option Dizet 6.21)**
- **AMT4=6: Complete two-loop corrections to M_W and fermionic two-loop corrections to $\sin^2\theta_{eff}^{lep}$ (best option in Dizet 6.42)**

Powheg + wt^{EW}: updates since December

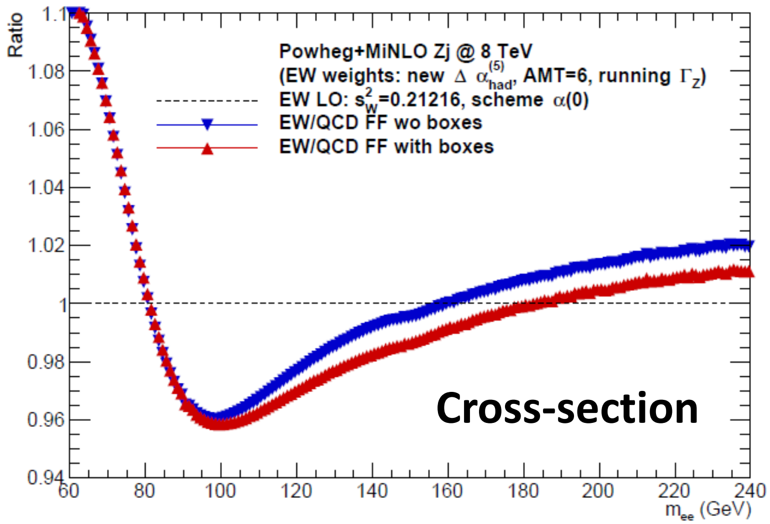
All updates together:

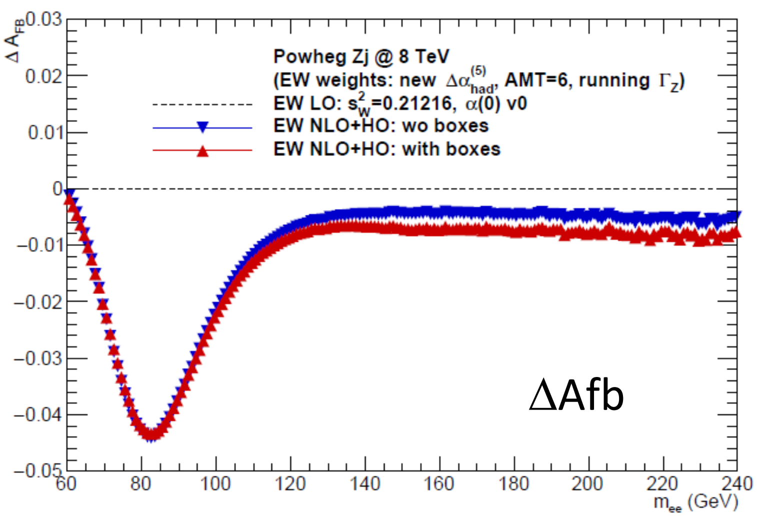
Parameter	September'18 m_{ll} = 89-93 GeV	May'19 m_{ll} = 89-93 GeV	
$\alpha(0)$ scheme			
σ NLO+HO/LO (no boxes)	0.96536	0.96503	0.03%
σ NLO+HO/LO (with boxes)	0.96536	0.96508	
A_{fb} NLO+HO - LO (no boxes)	-0.03529	-0.03496	+0.0003 →
A_{fb} NLO+HO - LO (with boxes)	-0.03526	-0.03495	
G_μ scheme			
σ NLO+HO/LO (no boxes)	0.99243	0.99204	0.04%
σ NLO+HO/LO (with boxes)	0.99244	0.99209	
A_{fb} NLO+HO - LO (no boxes)	-0.01553	-0.01514	+0.0004 →
A_{fb} NLO+HO - LO (with boxes)	-0.01550	-0.01512	

About + 0.00010 on $\sin^2_{eff}$

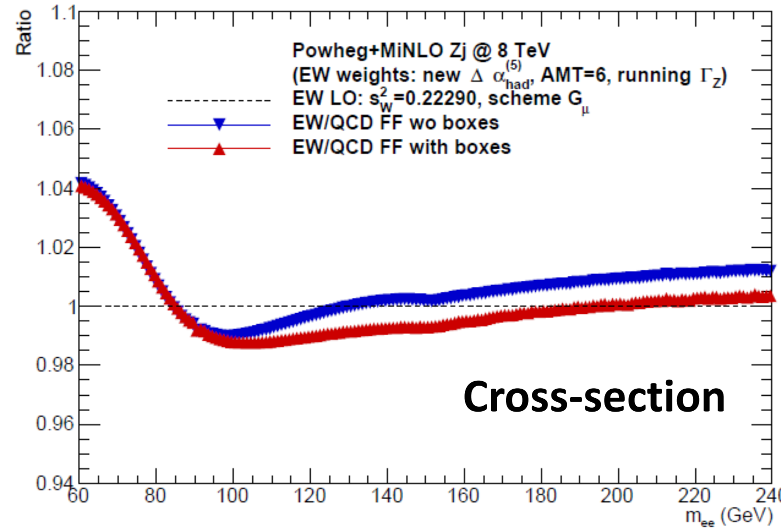
Powheg + wt^{EW}: EW NLO+HO corr.

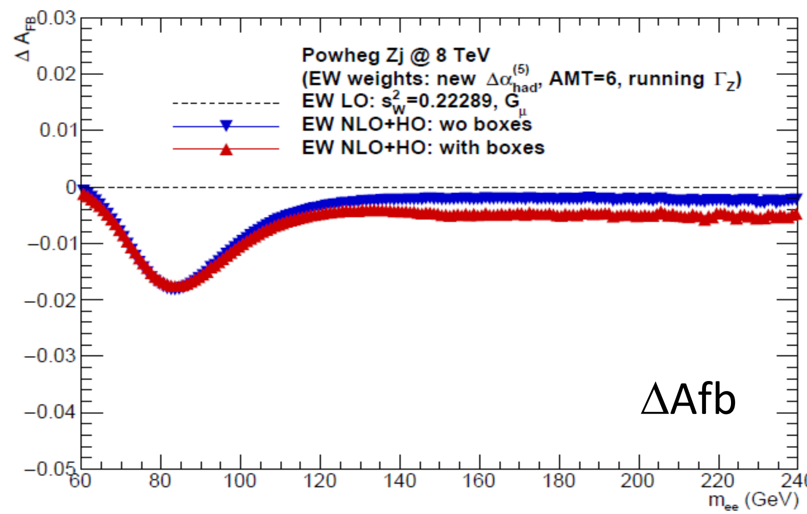
$\alpha(0)$ v0 scheme





G_μ scheme





Benchmarking: outlook

- **Shall we continue with the present setup or there is different/better proposal?**

- **In Durham WS, Fulvio presented proposal for new EW scheme implemented in Powheg_ew. We can include it as well, need to define table for EW LO parameters below, and follow with standard plots/tables for EW NLO+HO - LO**

	LEP legacy		LHC standard	
EW scheme:	G_μ, α, M_Z	α, M_W, M_Z	G_μ, M_W, M_Z	
	α(0) v0	α(0) v1	Gμ	sin2w
M_Z	91.1876 GeV	91.1876 GeV	91.1876 GeV	
Γ_Z	2.4952 GeV	2.4952 GeV	2.4952 GeV	
Γ_W	2.085 GeV	2.085 GeV	2.085 GeV	
α	1/137.03599	1/137.03599	1/132.23323	
G_μ	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	$1.1254734 \cdot 10^{-5}$ GeV^{-2}	$1.1663787 \cdot 10^{-5}$ GeV^{-2}	
M_W	80.93886 GeV	80.385 GeV	80.385 GeV	
s_W^2	0.2121517	0.2228972	0.2228972	
$\frac{G_\mu \cdot M_Z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha}$	1.0	1.0	1.0	

Benchmarking: outlook

- The **wt^{EW} calculations are updated**.
- Comparison done so far still not detailed enough, agreement achieved (slide 6) seemed accidental.
- To complete, we need to break it into a few checkpoints:
  - Predicted α_{QED} at Z-pole
  - Predicted $\sin^2\theta_{eff}$ at Z-pole
  - Corrections due to Z self-energy (running width propagator)
  - What else?
- We need to complete and agree upon evaluation of parametric and theory uncertainties of each EW scheme.
  - For the $\alpha(0)$ v0 scheme (DIZET)
    - Parametric uncertainties: $4\ 10^{-5}$ for $\sin^2\theta_{eff}$
    - Theory uncertainities: $5\ 10^{-5}$ for $\sin^2\theta_{eff}$ [LEP legacy]
  - For the G_μ scheme
    - Parametric uncertainties: ±15 MeV on M_W -> $30\ 10^{-5}$ on $\sin^2\theta_W$

Theoretical and parametric uncertainties

ALDO+ SLD + Tevatron, arXiv:1012.2367

- **The remaining theoretical uncertainties were estimated to be 4 MeV on m_W and 0.000049 on $\sin^2\theta_{eff}^{lep}$. We can use this estimate @ Z-pole.**
- **The parametric uncertainties were dominated by $\Delta\alpha_{had}(M_Z^2)$. The uncertainty of ±0.00035 caused an error of ±0.00013 on $\sin^2\theta_{eff}^{lep}$.**

EW weights from TauSpinner/DIZET, $\alpha(0)$ scheme:

- **The same code as ALDO for calculating EW genuine corrections. We can use this estimate.**

  Theoretical uncertainties (EW, @Z-pole): 5 10^{-5} on $\sin^2\theta_{eff}^{lep}$
- **The parametric uncertanties should be updated for better precision on $\Delta\alpha_{had}(M_Z^2)$ and m_t.**

Parametric uncertainties: $\Delta\alpha_{had}$, m_t

Parameter	$\Delta\alpha_h^{(5)}(M_Z^2)$ - 0.0001	$\Delta\alpha_h^{(5)}(M_Z^2) = 0.0275762$	$\Delta\alpha_h^{(5)}(M_Z^2)$ + 0.0001	$\Delta/2$
$\alpha(M_Z^2)$	0.0077540999	0.0077549240	0.0077557482	
$1/\alpha(M_Z^2)$	128.9640328306	128.9503292550	128.9366256793	
s_W^2	0.22340146	0.22343647	0.22347148	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23150412	0.23153917	0.23157421	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23139759	0.23143261	0.23146763	0.000035
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23127052	0.23130551	0.23134049	0.000035
M_W	80.35892 GeV	80.35710 GeV	80.35529 GeV	1.8 MeV
Δr	0.03625683	0.036609	0.03647535	
Δr_{rem}	0.01170310	0.01170287	0.01170264	

Parameter	m_t - 0.5 GeV	m_t = 173.2 GeV	m_t + 0.5 GeV	$\Delta/2$
$\alpha(M_Z^2)$	0.0077549205	0.0077549240	0.0077549274	
$1/\alpha(M_Z^2)$	128.9503873792	128.9503292550	128.9502716590	
s_W^2	0.22349450	0.22343647	0.22337836	0.000058
$sin^2\theta_W^{eff}(M_Z^2)$ (electron, muon)	0.23155486	0.23153917	0.23152344	0.000016
$sin^2\theta_W^{eff}(M_Z^2)$ (up-quark)	0.23144830	0.23143261	0.23141688	0.000016
$sin^2\theta_W^{eff}(M_Z^2)$ (down-quark)	0.23132119	0.23130551	0.23128979	0.000016
M_W	80.354102 GeV	80.35710 GeV	80.360111 GeV	3 MeV
Δr	0.03654697	0.036609	0.03618477	
Δr_{rem}	0.01169343	0.01170287	0.01171229	

@ Z-pole:

Parametric uncertainties: 4 10-5 [35 10-6 (from $\Delta\alpha_{had}$) and 16 10-6 (from m_t)]

SPARE slides

EW schemes: LEP and LHC paradigms

LEP

- EW scheme: „on-mass-schell" regularisation; input: ($\alpha(0)$, G_F, M_Z), the most precisely known quantities:

  $\Delta\alpha/\alpha$ ~ 3.6 10-9; $\Delta G_F/G_F$ ~8.6 10-6; $\Delta M_Z/M_Z$ ~ 2.4 10-5
- TH precision @Z-pole: 4 MeV on m_W; 5 10-5 on $\sin^2\theta_{eff}$
- Parametric precision on $\sin^2\theta_{eff}$: 4 10-5 (dominated by $\Delta\alpha_{had}$)
- Used and develloped still today: GFitter, FCCee preparation.

LHC

- EW scheme: „pole-mass" regularisation; input: (G_F, M_Z, M_W)

  $\Delta G_F/G_F$ ~8.6 10-6; $\Delta M_Z/M_Z$ ~ 2.4 10-5; $\Delta M_W/M_W$ ~ 1.9 10-4
- Relations to EWPO's: not established.
- EW corrections complete at EW NLO, only some at EW NLO+HO
- TH precision @Z-pole: not established.
- Parametric precision: ±15 MeV on M_W -> 30 10-5 on $\sin^2\theta_W$

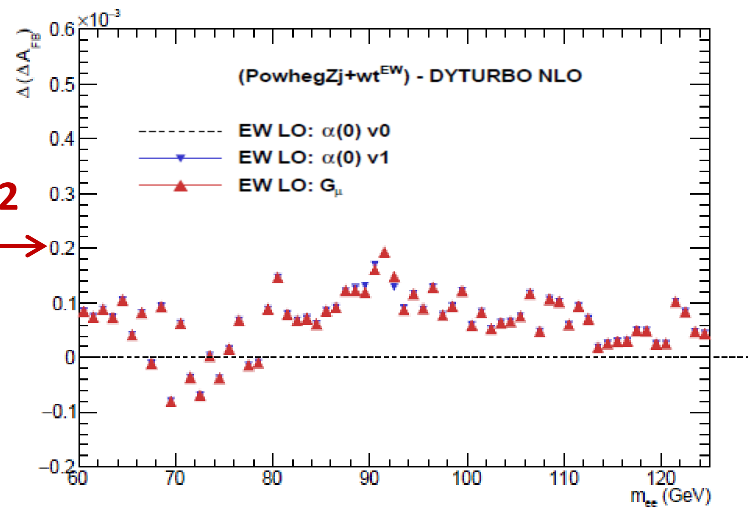
Strategy for comparisons

- **Scope:**
  - Genuine EW and lineshape corrections to Drell-Yan production at NLO QCD.
  - Three EW LO schemes chosen to allow for straightforward interpretation of results. We tuned EW LO parameters, otherwise out-of-the-box.
  - The highest available corrections in a given approach used.
  - QED FRS/ISR not included, so called double-deconvoluted observables.
- **Observables:**
  - Lineshape (cross-section) and forward-backward asymmetry A_{FB} in the full phase-space.
  - Compared ratios or absolute differences between different EW LO schemes and/or between NLO, NLO+HO predictions within each EW scheme and same MC generator. Allows to minimize sensitivity to QCD details.
- **Goals:**
  - Check if reweighting with wt^{EW} (TauSpinner) works for NLO QCD MC's. Compared distributions at EW LO (DYTURBO, Powheg_ew).
  - Establish how consistent are predictions between different EW schemes with EW NLO corrections (Powheg_ew, MCSANC).
  - Establish how consistent are EW NLO+HO corrections of Dizet 6.21 form-factors implemented in wt^{EW} and those of Powheg_ew.

Validating reweighting with wt^{EW}: EW LO

ΔA_{FB}: driven by s^2_W value (same for $\alpha(0)$ v1 and G_μ schemes)

0.0002



Benchmark for wt^{EW} reweighting

Double difference:

ΔA_{FB} (DYTURBO) - ΔA_{FB} (PowhegZj+wt^{EW})

$\alpha(0)$ v1 - $\alpha(0)$ v0

G_μ - $\alpha(0)$ v0

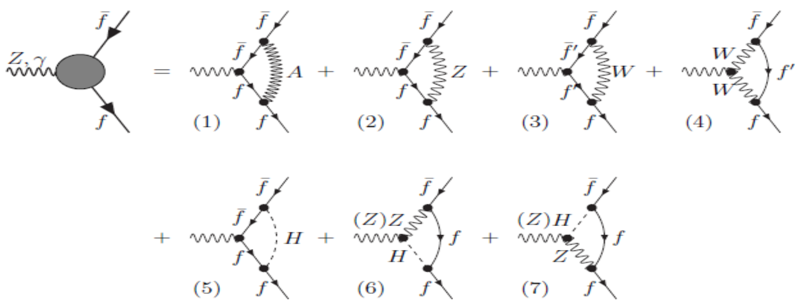
Agreement on $\Delta(\Delta A_{FB})$ within ± 0.0002

Should redo it with much finer binning around Z-pole to better estimate precision.

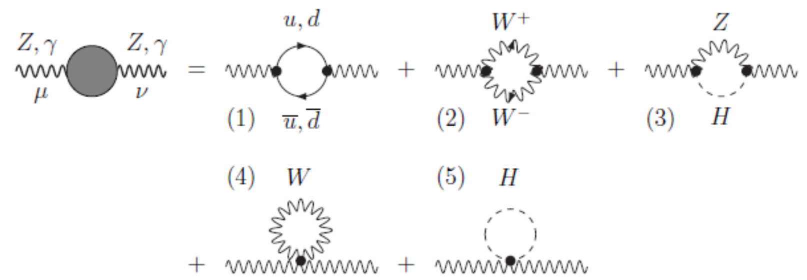
Genuine EW and lineshape corrections

Gauge-invariant set of diagrams. Calculated as form-factor corrections to couplings, propagators and masses.
Eg. running $\alpha_{QED}(s)$, $\alpha_{QED}(M_Z) = 1./128.86674175$

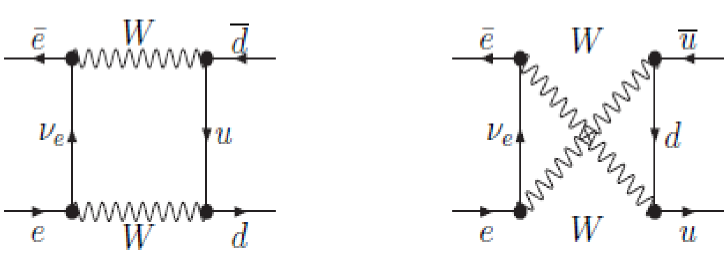
Zff and γ ff vertices



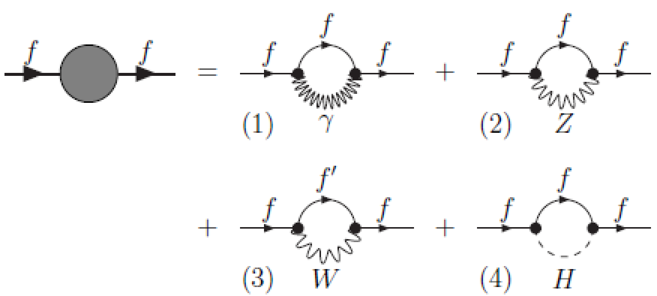
Bosonic self-energies



WW, ZZ boxes (shown only WW diagrams)



Fermionic self-energies



From Zfitter/Dizet documentation

D. Bardin et al. arXiv:9908433

Zfitter is a semi-analytical program for calculating total cross-sections and pseudo-observables (eg. A_{fb}, $\sin^2\theta_W^{eff}$), used by LEP1, and to a lesser degree by LEP2.
DIZET is a library for calculating form-factors and some other corrections. Provides complete EW O(α) weak-loop corrections supplemented with selected higher order terms (eg. vacum polarisation, $\alpha_{QED}(Q^2)$).
For analyses at LEP1, LEP2 used aways in parallel with MC generators (KoralZ, KoralW) eg. to evaluate systematics of simplified cuts used in analysis integration.

$$\mathcal{A}_Z^{OLA}(s,t) = i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z(s)\rho_{ef}(s,t)\Big\{\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu(1+\gamma_5) -4|Q_e|s_W^2\kappa_e(s,t)\gamma_\mu\otimes\gamma_\mu(1+\gamma_5) - 4|Q_f|s_W^2\kappa_f(s,t)\gamma_\mu(1+\gamma_5)\otimes\gamma_\mu +16|Q_eQ_f|s_W^4\kappa_{e,f}(s,t)\gamma_\mu\otimes\gamma_\mu\Big\}. \quad (A.4.75)$$

one loop amplitude

$$\mathcal{A}_\gamma^{OLA} = i\chi_\gamma(s)\alpha(s)\gamma_\mu\otimes\gamma_\mu\,. \quad (2.2.36)$$

Dyson summation leads to the change of α into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha^{\rm fer}(s)} = \frac{\alpha(0)}{1-\Delta\alpha^{(5)}(s)-\Delta\alpha^{t}(s)-\Delta\alpha^{\alpha\alpha_s}(s)}, \quad (2.2.37)$$

Vacuum polarisation corrections

LEP legacy: from Zfitter/Dizet documentation

After some trivial algebra one derives the final expressions:

$$\rho_{ef} = 1 + \frac{g^2}{16\pi^2}\Big\{-\Delta\rho_z^F + \mathcal{D}_z^F(s) + \frac{5}{3}B_0^F(-s;M_W,M_W) - \frac{9}{4}\frac{c_W^2}{s_W^2}\ln c_W^2 - 6$$
$$+\frac{5}{8}c_W^2\left(1+c_W^2\right) + \frac{1}{4c_W^2}\left(3v_e^2+a_e^2+3v_f^2+a_f^2\right)\mathcal{F}_z(s) + \hat{\mathcal{F}}_W^0(s) + \hat{\mathcal{F}}_W(s)$$
$$-\frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right] - c_W^2(R_z-1)\,s\hat{\mathcal{B}}_{WW}^d(s,t)\Big\}, \qquad \text{(A.4.80)}$$

$$\kappa_e = 1 + \frac{g^2}{16\pi^2}\Big\{-\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s;M_W,M_W) - \frac{1}{9} - \frac{v_e\sigma_e}{2c_W^2}\mathcal{F}_z(s)$$
$$-\hat{\mathcal{F}}_W^0(s) + (R_z-1)\Big[\frac{|Q_f|}{2}\left(1-4|Q_f|s_W^2\right)\mathcal{F}_z(s) + c_W^2\Big[\hat{\mathcal{F}}_{W_n}(s)$$
$$-|Q_{f'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}_{WW}^d(s,t)\Big]\Big]\Big\}, \qquad \text{(A.4.81)}$$

$$\kappa_f = 1 + \frac{g^2}{16\pi^2}\Big\{-\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi_{Z\gamma}^F(s) - \frac{1}{6}B_0^F(-s;M_W,M_W) - \frac{1}{9} - \frac{v_f\sigma_f}{2c_W^2}\mathcal{F}_z(s)$$
$$-\hat{\mathcal{F}}_W(s) + (R_z-1)\Big[\frac{|Q_e|}{2}\left(1-4|Q_e|s_W^2\right)\mathcal{F}_z(s) + c_W^2\Big[\hat{\mathcal{F}}_{W_n}^0(s)$$
$$-|Q_{e'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}_{WW}^d(s,t)\Big]\Big] - \frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right]\Big\}, \qquad \text{(A.4.82)}$$

$$\kappa_{ef} = 1 + \frac{g^2}{16\pi^2}\Big\{-2\frac{c_W^2}{s_W^2}\Delta\rho^F - 2\Pi_{Z\gamma}^F(s) - \frac{1}{3}B_0^F(-s;M_W,M_W) - \frac{2}{9}$$
$$-\frac{1}{4c_W^2}\left[\frac{\delta_e^2+\delta_f^2}{s_W^2}(R_W-1) + 3v_e^2+a_e^2+3v_f^2+a_f^2\right]\mathcal{F}_z(s)$$
$$-\hat{\mathcal{F}}_W^0(s) - \hat{\mathcal{F}}_W(s) - \frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right]$$
$$+c_W^2(R_z-1)\left[\frac{2}{3} - \hat{\Pi}_{\gamma\gamma}^{\text{bos},F}(s) + s\hat{\mathcal{B}}_{WW}^d(s,t)\right]\Big\}. \qquad \text{(A.4.83)}$$

interference

BOX

Fermionic loops in γ propagator

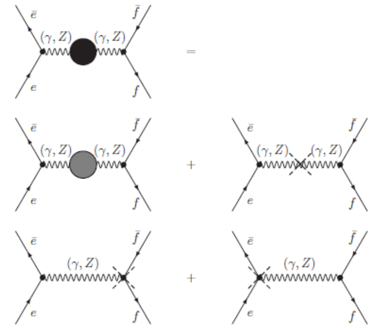


Figure A.11. *Bosonic self-energies and bosonic counter-terms for $e\bar{e} \to (Z,\gamma) \to f\bar{f}$*

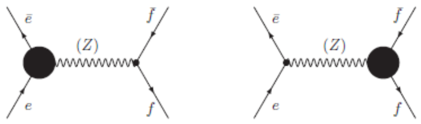


Figure A.10. *Electron (a) and final fermion (b) vertices in $e\bar{e} \to (Z) \to f\bar{f}$*

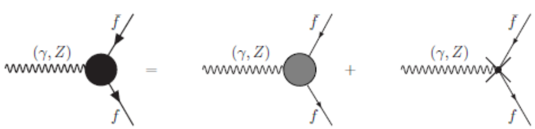


Figure A.6. *Off-shell $Zf\bar{f}$ and $\gamma f\bar{f}$ vertices*

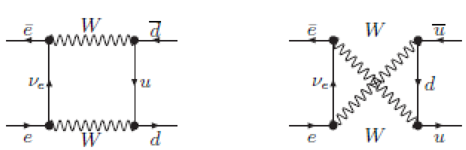


Figure A.7. *The WW boxes*

etc. etc.

Constructing wt^{EW}: per-event weight

ERW and Z.Was, arXiv: 1808.08616

Define per event electroweak weight

$wt^{EW} = \sigma^{new}_{Born} / \sigma^{old}_{Born}$

Approach developed in TauSpinner, arXiv:1802.05459

$$wt^{EW} = \frac{d\sigma_{Born+EW}(x_1, x_2, \hat{s}, \cos\theta\ , s_W^2)}{d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta\ , s_W^2)}$$

$$d\sigma_{Born}(x_1, x_2, \hat{s}, \cos\theta^*, s_W^2) = \sum_{q_f, \bar{q}_f} [f^{q_f}(x_1, ...) f^{\bar{q}_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, \cos\theta\ , s_W^2) + f^{q_f}(x_2, ...) f^{\bar{q}_f}(x_1, ...) d\sigma^{\bar{q}_f q_f}_{Born}(\hat{s}, -\cos\theta\ , s_w^2)]$$

x_1, x_2, $\cos\theta$ (symmetrised) calculated using 4-momenta of outgoing leptons; asymmetry in sign of $\cos\theta$ from weighted average over PDFs

Allows to reweight MC event generated between different EW LO scheme and to Improved Born Approximation in EW scheme used for form-factors calculation.

Constructing wt^{EW}: EW Improved Born (IBA)

ERW and Z.Was, arXiv: 1808.08616

$$\mathscr{A}^{Born+EW} = \frac{\alpha}{s}\{ \quad [\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u] \cdot (q_\ell \cdot q_f) \cdot \Gamma_{V_\Pi} \cdot \chi_\gamma(s)$$
$$+[\bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (v_\ell \cdot v_f \cdot vv_{\ell f}) + \bar{u}\gamma^\mu v g_{\mu\nu} \bar{v}\gamma^\nu\gamma^5 u \cdot (v_\ell \cdot a_f)$$
$$+\bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu u \cdot (a_\ell \cdot v_f) + \bar{u}\gamma^\mu\gamma^5 v g_{\mu\nu} \bar{v}\gamma^\nu\gamma^5 u \cdot (a_\ell \cdot a_f)] \cdot Z_{V_\Pi} \cdot \chi_Z(s) \quad \}$$

$$\chi_\gamma(s) = 1$$

$$\chi_Z(s) = \frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}$$

$$v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2 \cdot K_\ell(s,t))/\Delta$$
$$v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot K_f(s,t))/\Delta$$
$$a_\ell = (2 \cdot T_3^\ell)/\Delta$$
$$a_f = (2 \cdot T_3^f)/\Delta$$

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}$$

$$Z_{V_\Pi} = \rho_{e,f}(s,t)$$

EW form-factors , functions of $(s,t)=(m_{ll}, \cos\theta)$
Calculated with Dizet 6.21 library.

$$\Gamma_{V_\Pi} = \frac{1}{2 - (1 + \Pi_{\gamma\gamma}(s))}$$

$$vv_{\ell f} = \frac{1}{v_\ell \cdot v_f}[(2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_\ell \cdot s_W^2 \cdot K_f(s,t)(2 \cdot T_3^\ell) - 4 \cdot q_f \cdot s_W^2 \cdot K_\ell(s,t)(2 \cdot T_3^f)$$
$$+(4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2 K_{\ell f}(s,t)]\frac{1}{\Delta^2}$$

Vacuum polarisation corrections, used low-energy experiment input.
Warning: problem for analytic continuation.

EW schemes

D. Bardin et al.
arXiv:9908433

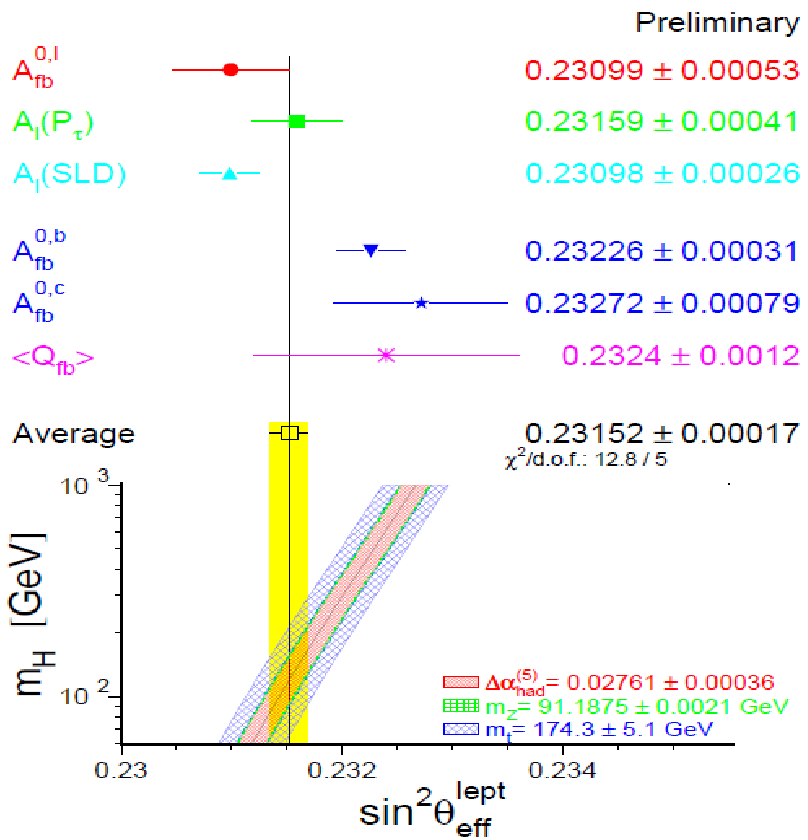
- **LEP legacy: ($\alpha(0)$, G_μ, M_Z)**
  - Inputs are very precisely measured physics quantities
  - M_Z, M_W are on-shell masses
  - Genuine EW and lineshape corrections in form of (multiplicative) form-factors to LO couplings

S. Dittmaier, M. Huber
arXiv:0911.2329

- **LHC paradigm: (G_μ, M_Z, M_W).**
  - M_Z, M_W are pole-masses or complex masses.
  - Absorbs most of universal corrections into lowest-order couplings
  - Higher-order corrections redefine couplings in non-multiplicative manner

Re-discovered impact of $\Delta\alpha_{had}^{(5)}(M_Z^2)$

arXiv:hep-ex/0112021



Recent measurements:

arXiv: 1706.09436

$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02753 ± 0.00009

arXiv: 1711.06089

$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02774 ± 0.00016

= 0.02752 ± 0.00012

arXiv: 1802.02995

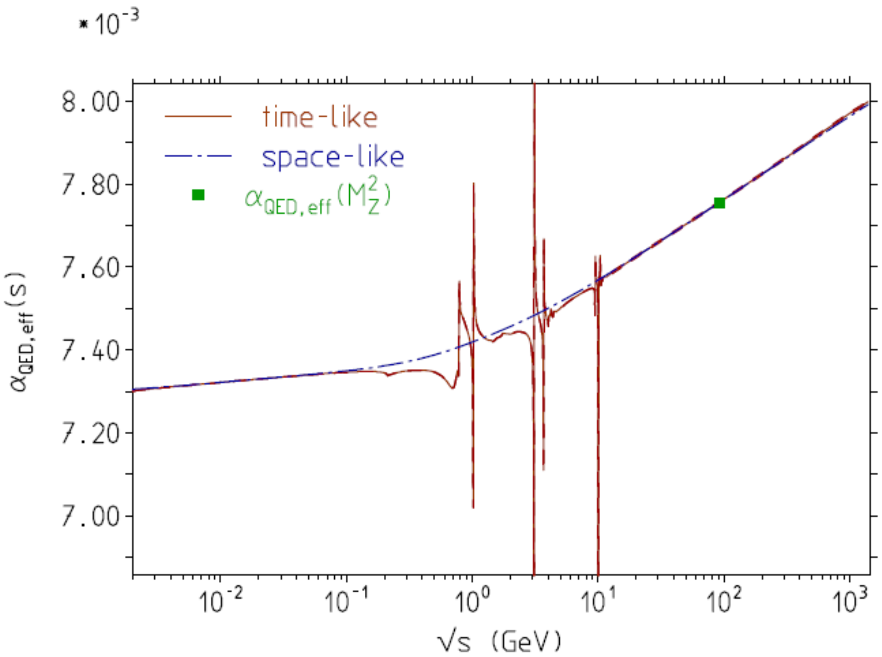
$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02761 ± 0.00011

±0.00036 on $\Delta\alpha_{had}^{(5)}(M_Z^2)$ -> ±0.00013 on $\sin^2\theta_{eff}$

$\alpha_{QED}(s)$ and $\sin^2\theta_W(s)$

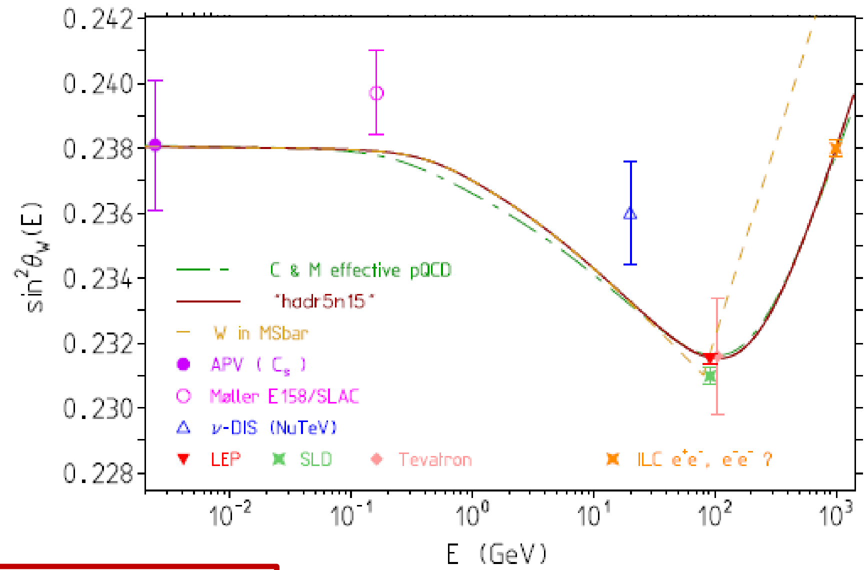
$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

F. Jegerlehner
FCCee WS 2019



$$\sin^2\Theta_i\ \cos^2\Theta_i\ = \frac{\pi\,\alpha}{\sqrt{2}\,G_\mu\,M_Z^2}\frac{1}{1-\Delta r_i}$$

$$\Delta r_i = \Delta r_i(\alpha,\ G_\mu,\ M_Z,\ m_H,\ m_{f\neq t},\ m_t)$$



$$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.027738 \pm 0.000158\ [0.027523 \pm 0.000119]$$

Effective weak mixing angle @ LHC

Here convoluted with line-shape and cosθ* distribution of MC events.

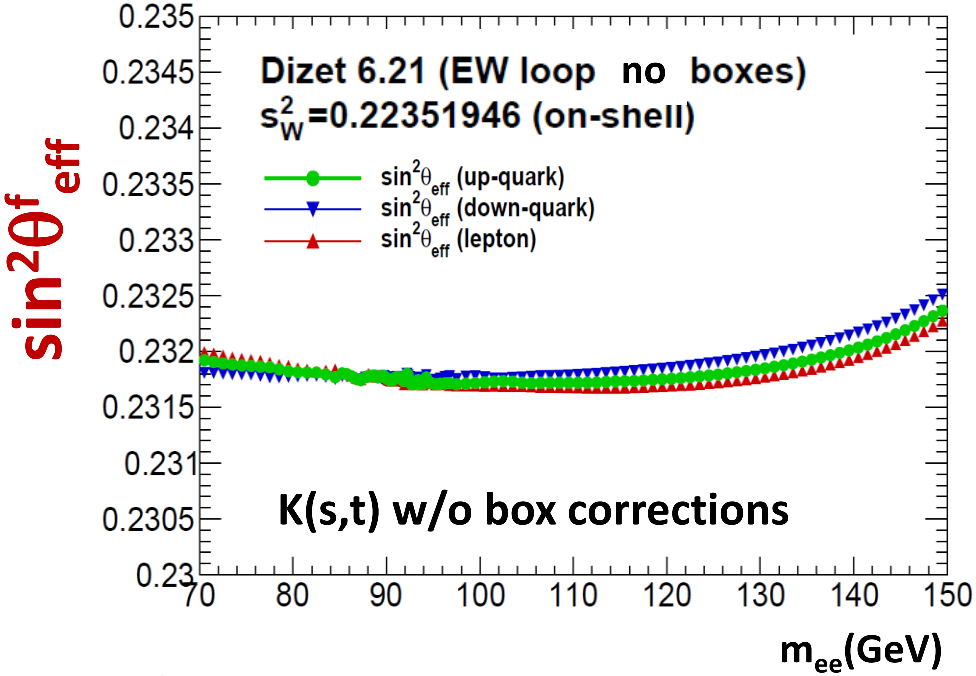
$$\sin^2\theta_W^{eff} = Re(K_Z^f)s_W^2 + I_f^2$$

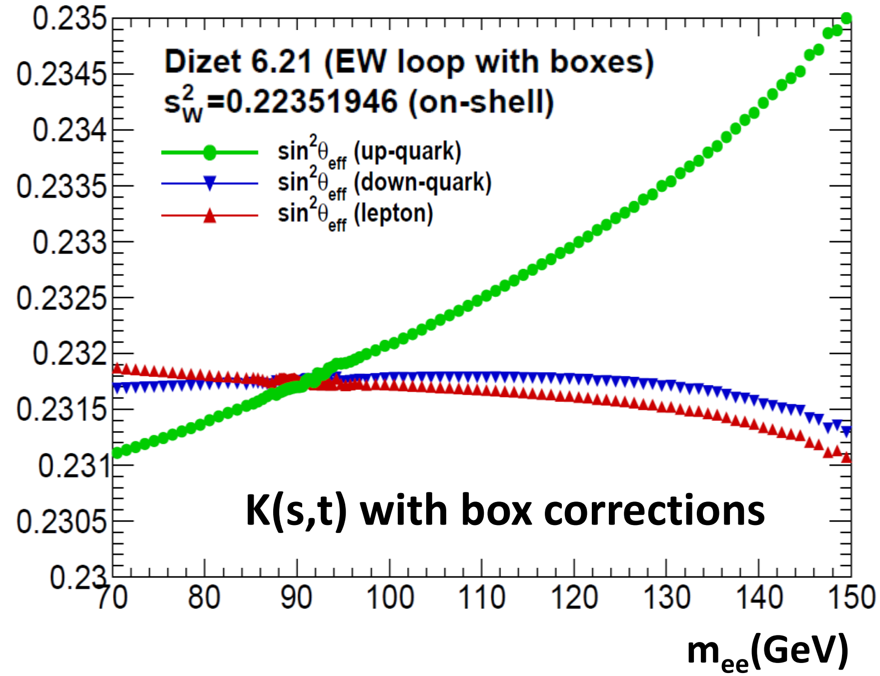
↑(s,t)

$$I_f^2 = \alpha^2(s)\frac{35}{18}[1 - \frac{8}{3}Re(K_Z^f)s_W^2]$$ = ~ 10^{-4}

$$s_W^2 = 1 - M_W^2/M_Z^2$$

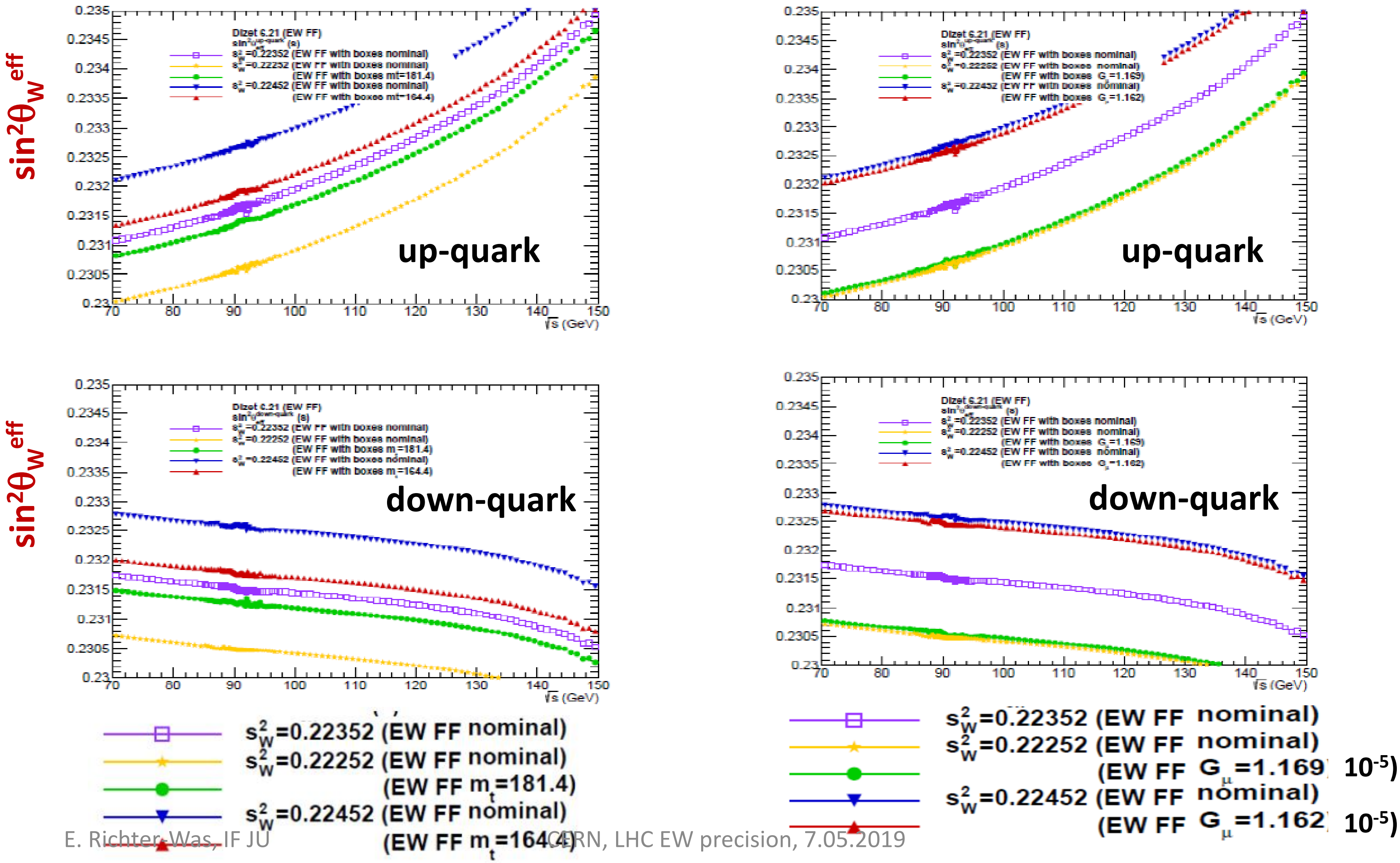
$\sin^2\theta_W^{eff}$ defines ratio of couplings $g_Z^f = \frac{v_f}{a_f} = 1 - 4|q_f|(K_Z^f s_W^2 + I_f^2)$





Effect on $\sin^2\theta_W^{eff}$ from recalculating form-factors

Variation in m_t or G_μ corresponding to ± 100 10^{-5} variation in s^2_W



How to scan s^2_W around its nominal value

We want to stay consistent within SM QED/EW field theory

optME=1 : change $s^2_W + \delta_{s2w}$ as if due to new V-coupling X-particle
no SM parameters modifications

$$v_\ell = (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 + \delta_{S2W}) \cdot K_\ell(s,t))/\Delta$$
$$v_f = (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 + \delta_{S2W}) \cdot K_f(s,t))/\Delta$$

$$\begin{aligned} vv_{\ell f} = \; & \frac{1}{v_\ell \cdot v_f}[(2 \cdot T_3^\ell)(2 \cdot T_3^f) \\ & -4 \cdot q_\ell \cdot (s_W^2 + \delta_{S2W}) \cdot K_f(s,t)(2 \cdot T_3^\ell) \\ & -4 \cdot q_f \cdot (s_W^2 + \delta_{S2W}) \cdot K_\ell(s,t)(2 \cdot T_3^f) \\ & +(4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{\ell f}(s,t) \\ & +2 \cdot (4 \cdot q_\ell))(4 \cdot q_f \cdot) \cdot s_W^2 \cdot \delta_{S2W} K_{\ell f}(s,t)] \frac{1}{\Delta^2} \end{aligned}$$

optME=2 : change s^2_W as if due to changes in the SM input parameters,
like m_t or G_μ .
Followed (option) by form-factors recalculation.

optME=3 : add δ_V directly in the vertex, almost equivalent to optME=1 for
$s^2_W + \delta_V / K_l$ and $s^2_W + \delta_V / K_f$

$$v_\ell = \quad (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot (s_W^2 \cdot K_\ell(s,t) + \delta_V))/\Delta$$
$$v_f = \quad (2 \cdot T_3^f - 4 \cdot q_f \cdot (s_W^2 \cdot K_f(s,t) + \delta_V))/\Delta$$

$$\begin{aligned} vv_{\ell f} = \; & \frac{1}{v_\ell \cdot v_f}[(2 \cdot T_3^\ell)(2 \cdot T_3^f) \\ & -4 \cdot q_\ell \cdot (s_W^2 + \cdot K_f(s,t) + \delta_V)(2 \cdot T_3^\ell) \\ & -4 \cdot q_f \cdot (s_W^2 \cdot K_\ell(s,t) + \delta_V)(2 \cdot T_3^f) \\ & +(4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2) K_{\ell f}(s,t) \\ & +2 \cdot (4 \cdot q_\ell))(4 \cdot q_f \cdot) \cdot s_W^2 \cdot K_{\ell f}(s,t) \cdot \delta_V] \frac{1}{\Delta^2} \end{aligned}$$

How to scan s^2_W

April 2018

Around Z-pole slope of $A_4(s^2_W)$ or $A_{fb}(s^2_W)$ depends on how variation is done in practice.

Sensitivity:

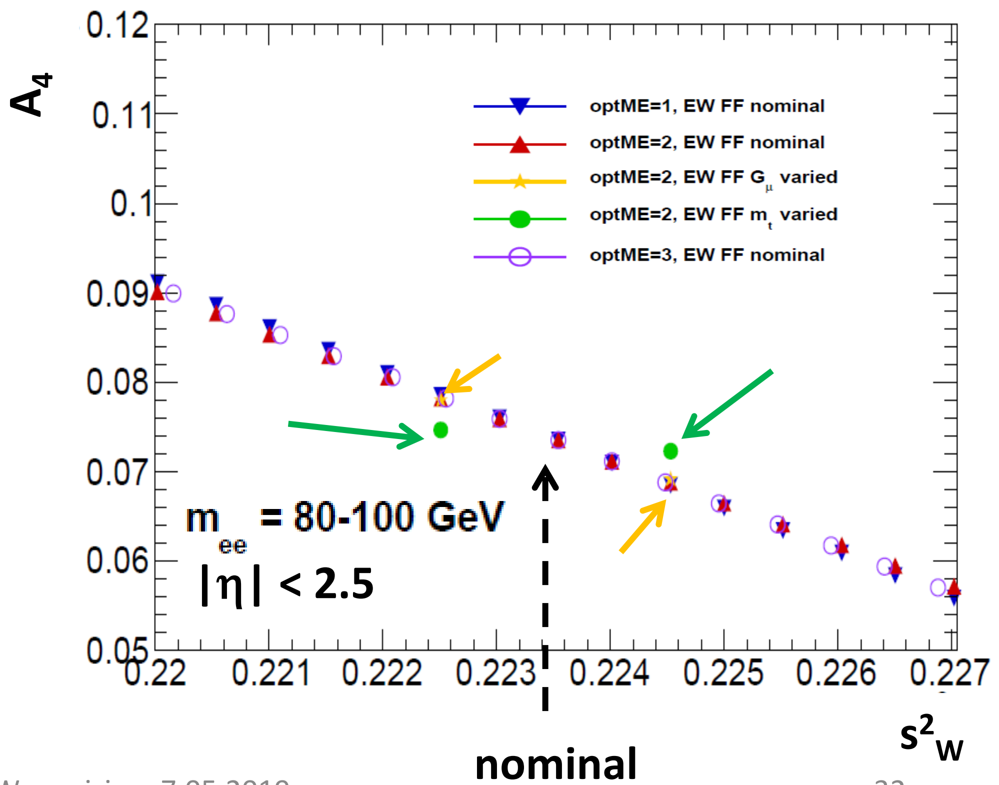
1% relative uncertainty on A_{fb} or A_4 corresponds to $20 * 10^{-5}$ uncertaint. on s^2_W (except case of m_t variation).

Sensitivity in case of m_t variation much weaker due to compensating corrections from recalculated FF

Recalculating FF makes procedure formally gauge-invariant, but requires m_t or G_μ unreasonably far from measured values.

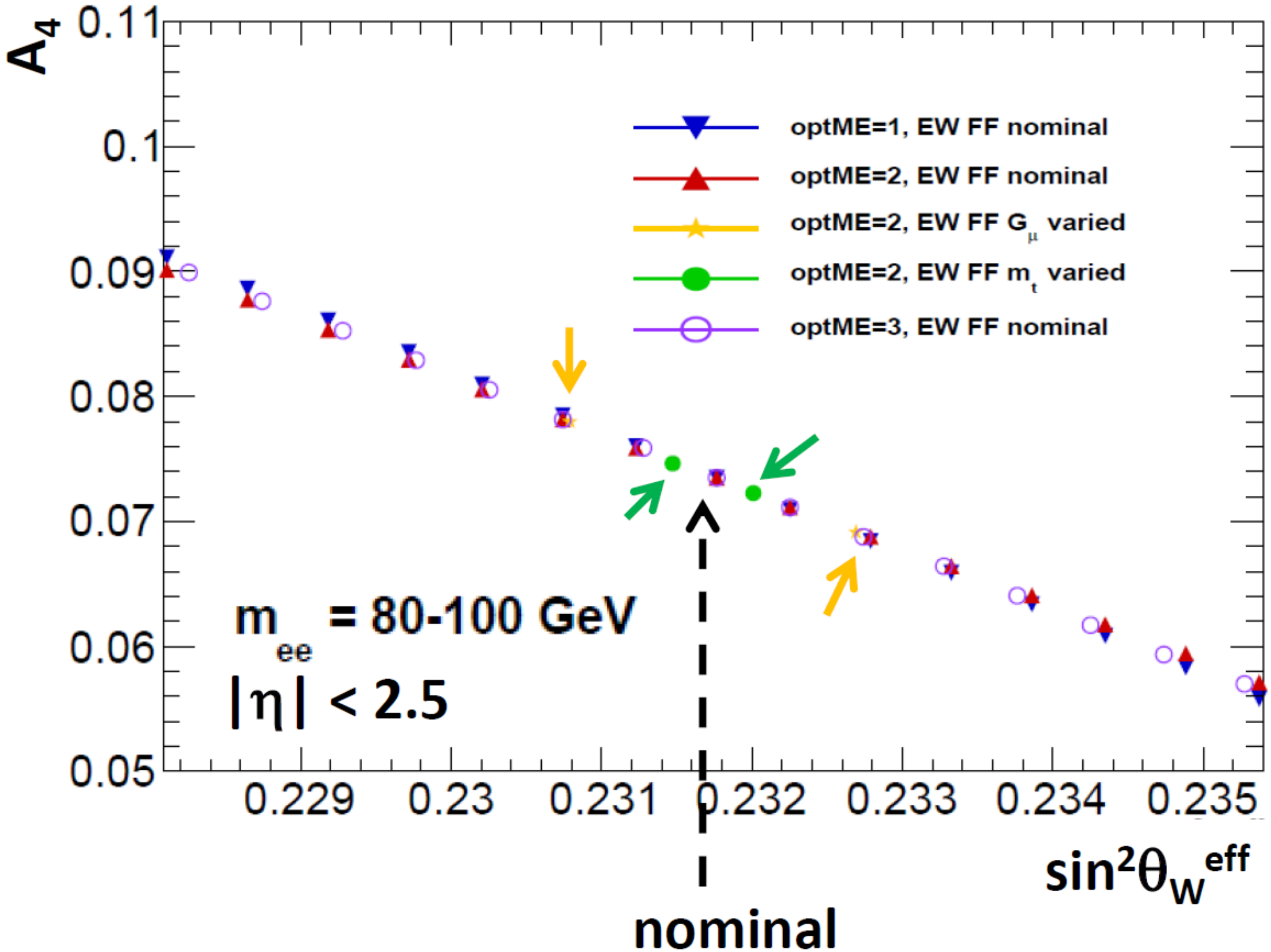
Scan was performed using per event weight wt^{EW} introduced earlier.

@ 8 TeV	A_4	$\frac{\Delta A_4/A_4 = 0.01}{\Delta s^2_W}$
optME=2	0.0735	0.00017
optME=2, G_μ varied		0.00019
optME=2, m_t varied		0.00070
optME=3		0.00017
optME=1		0.00016



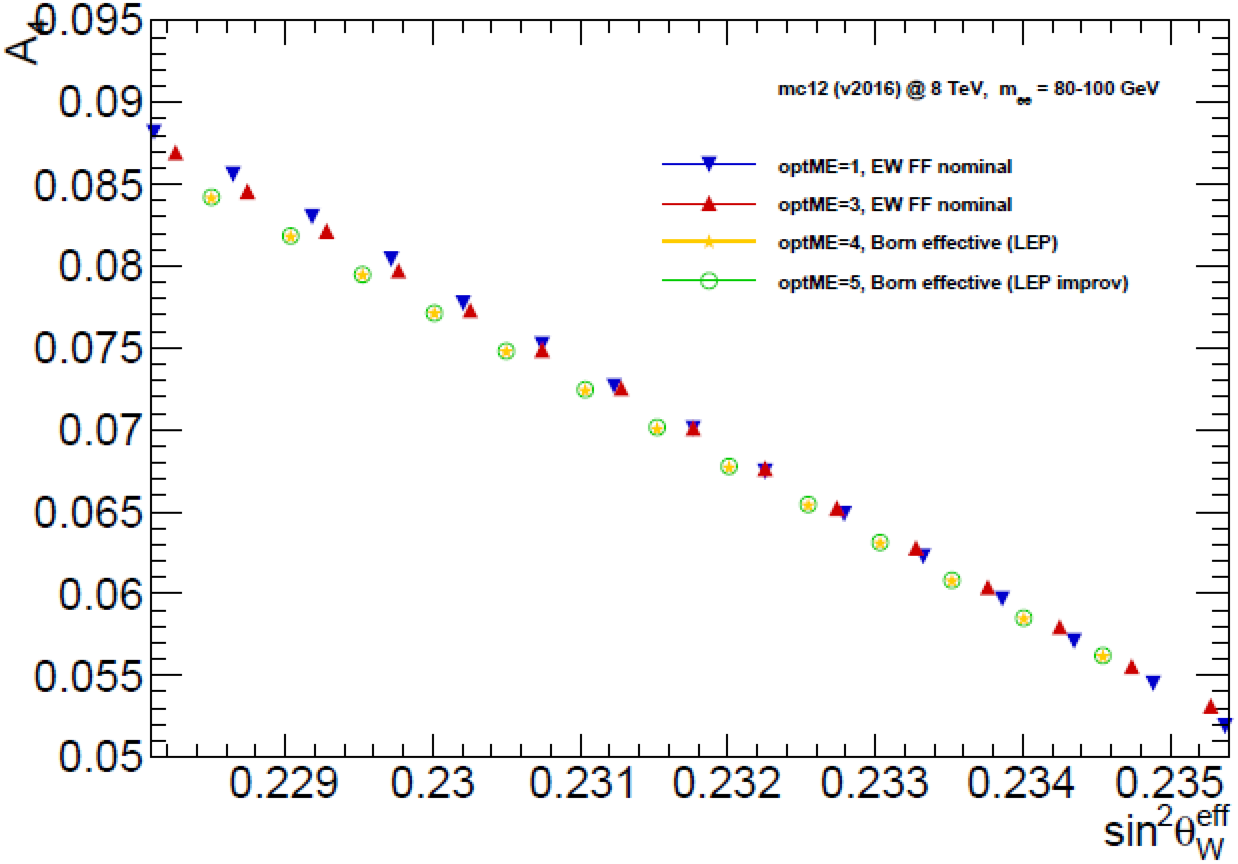
How to scan $\sin^2\theta_{eff}$

April 2018



How to scan $\sin^2\theta_{eff}$

April 2018



<u>Scan with EW LO:</u>

optME = 4,5 use Born EW LO amplitude, parametrised with $\sin^2\theta_{eff}$ and $\alpha(M_Z)$

Shift by 20 10-5 in predicted $\sin^2\theta_{eff}$ corresponding to the same A_{fb} or A4.

EW schemes: LEP and LHC paradigms

LEP

- **EW scheme: „on-mass-schell" regularisation; input: ($\alpha(0)$, G_F, M_Z), the most precisely known quantities:**

  $\Delta\alpha/\alpha$ ~ 3.6 10-9; $\Delta G_F/G_F$ ~8.6 10-6; $\Delta M_Z/M_Z$ ~ 2.4 10-5
- **TH precision @Z-pole: 4 MeV on m_W; 5 10-5 on $\sin^2\theta_{eff}$**
- **Parametric precision on $\sin^2\theta_{eff}$: 4 10-5 (dominated by $\Delta\alpha_{had}$)**
- **Used and develloped still today: GFitter, FCCee preparation.**

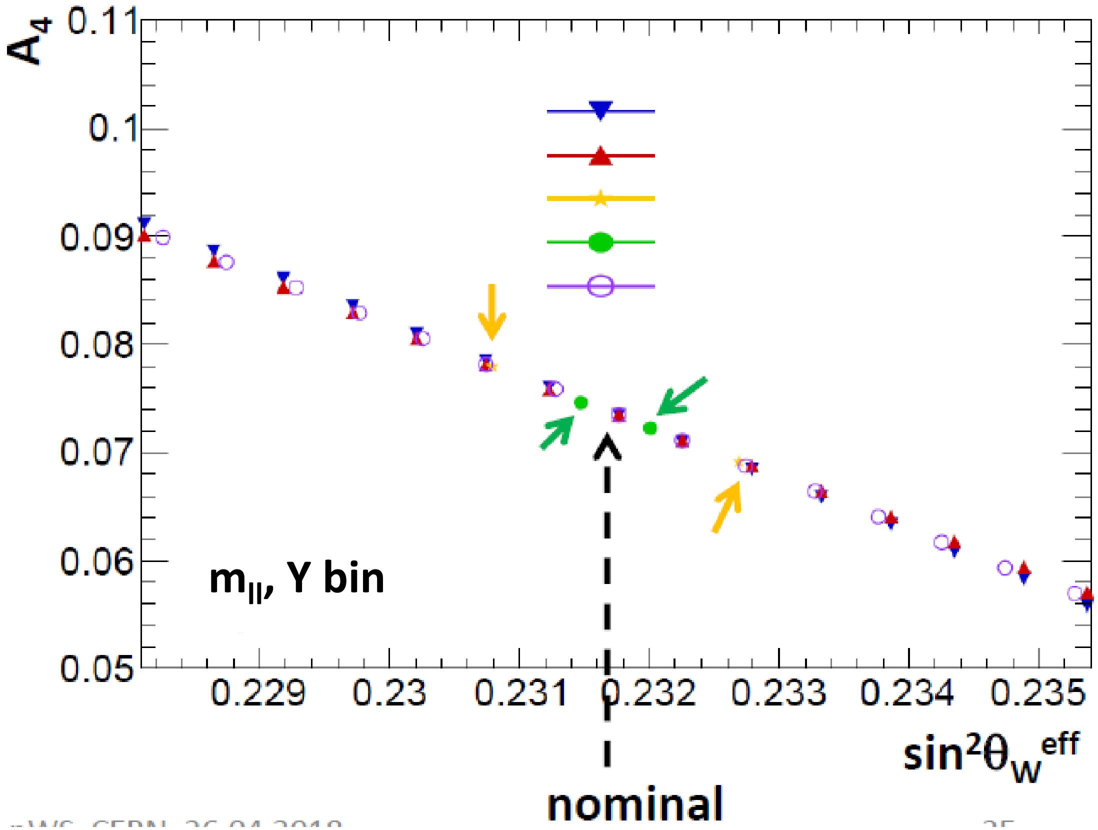
LHC

- **EW scheme: „pole-mass" regularisation; input: (G_F, M_Z, M_W)**

  $\Delta G_F/G_F$ ~8.6 10-6; $\Delta M_Z/M_Z$ ~ 2.4 10-5; $\Delta M_W/M_W$ ~ 1.9 10-4
- **Relations to EWPO's: not established.**
- **EW corrections complete at EW NLO, only some at EW NLO+HO**
- **TH precision @Z-pole: not established.**
- **Parametric precision: ±15 MeV on M_W -> 30 10-5 on $\sin^2\theta_W$**

How to scan s^2w: outlook

- Similarly here: compare different options, on vertical scale either A_{FB} or A_4. Expose, how other SM parameters need to vary in you scheme.
- We need it with granularity 5 10-5, in a range ±100 10-5 around central value, in different mass and Y bins.



EW LO schemes: running/fixed width

Running and fixed Z-boson width in the propagator: taking into account photon-loop corrections to Γ_Z

- **Fixed width** $$\chi_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}.$$

- **Running width (LEP legacy)**

$$\chi'_Z(s) = \frac{1}{s - M_Z^2 + i \cdot \Gamma_Z \cdot s/M_Z}$$

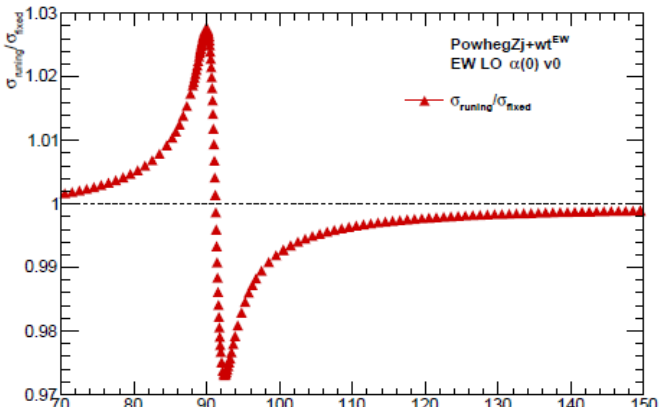
$$\begin{aligned}
\chi'_Z(s) &= \frac{1}{s(1 + i \cdot \Gamma_Z/M_Z) - M_Z^2} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{s(1 + \Gamma_Z^2/M_Z^2) - M_Z^2(1 - i \cdot \Gamma_Z/M_Z)} \\
&= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} \frac{1}{s - \frac{M_Z^2}{1+\Gamma_Z^2/M_Z^2} + i \cdot \frac{\Gamma_Z M_Z}{1+\Gamma_Z^2/M_Z^2}} \\
&= N_Z \frac{1}{s - {M'_Z}^2 + i\Gamma'_Z M'_Z} \\
M'_Z &= \frac{M_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}} \\
\Gamma'_Z &= \frac{\Gamma_Z}{\sqrt{1 + \Gamma_Z^2/M_Z^2}} \\
N_Z &= \frac{(1 - i \cdot \Gamma_Z/M_Z)}{(1 + \Gamma_Z^2/M_Z^2)} = \frac{(1 - i \cdot \Gamma'_Z/M'_Z)}{(1 + {\Gamma'_Z}^2/{M'_Z}^2)}
\end{aligned}$$

Both equivalent if redefined parameters m_Z, Γ_Z, N_Z (normalization). Change in the normalisation can (?) be absorbed into G_μ redefinition. In case of „pole” convention (last slide) it was absorbed into α.

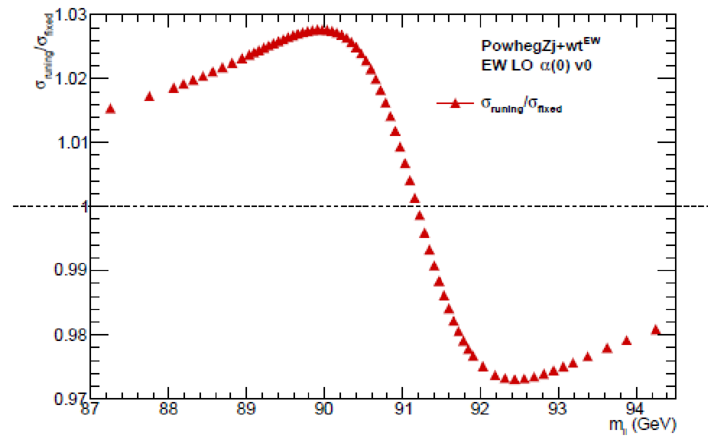
EW LO schemes: running/fixed width

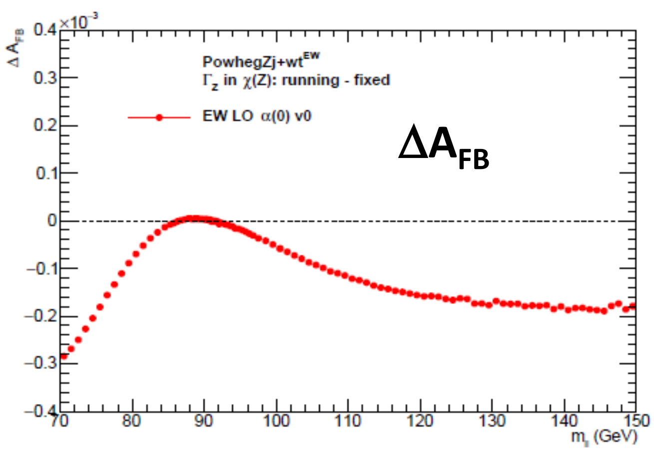
Nowdays MC's are using fixed-width propagators with on-shell $M_Z = 91.1876$.
How does it affect predictions, if running-> fixed without reparametrizing?
ΔA_{FB} (m_{ll}=80-100 GeV) = 0.0005 (thanks to D. Walker for pointing it out)

Lineshape: ratio

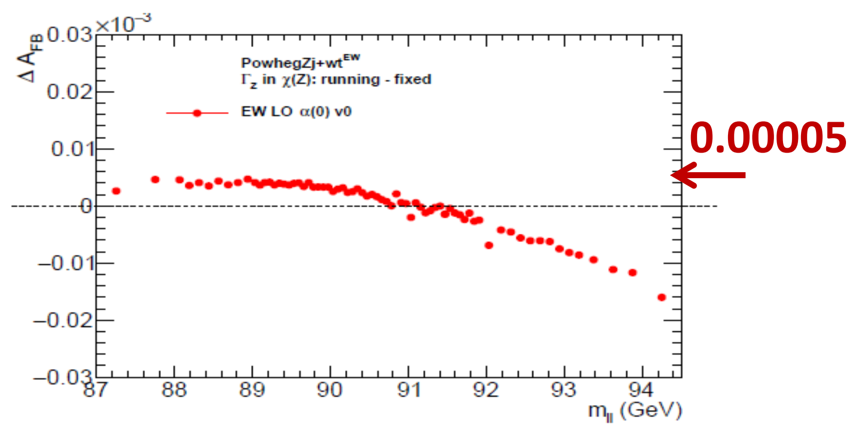


zoom





zoom



Dictionary

EW LO Born (LO = lowest order):
tree-level vertex and propagator of the Z/γ^* bosons, setting of SM EW parameters defines the EW scheme.

EW effective Born:
tree-level vertex and propagator of the Z/γ^* bosons, EW couplings: $\alpha(m_Z)$, $\sin^2\theta_W(m_Z)$, m_Z, set of best measured values.

EW Improved Born Approximation (IBA):
tree-level vertex and propagator of the Z/γ^* bosons, EW couplings and propagators multiplied by form-factors dependent on the scattering angle of the lepton (choice of frame) and virtuality of Z/γ^*.

QED/EW corrections: D. Bardin et al. arXiv:9908433
separate set of „QED corrections” and „genuine EW + lineshape corrections”

Tree level Standard Model relations:

$$G_F = \frac{\pi\alpha}{\sqrt{2} m_W^2 \sin^2\theta_W^{tree}},$$

$$\rho_0 = \frac{m_W^2}{m_Z^2 \cos^2\theta_W^{tree}} = 1$$

Powheg_ew: EW LO, NLO, NLO+HO

Status:
September 2018

F. Piccinini et al.

	EW order	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	m_{ee} = 70 - 120 GeV
A_{FB} $\alpha(0)$ v0	LO	0.06691361	0.06392369	0.06253754
A_{FB} $\alpha(0)$ v1	LO	0.04653886	0.04343789	0.04212883
A_{FB} G_μ	LO	0.04653886	0.04343789	0.04212883
A_{FB} $\alpha(0)$ v1	NLO	0.03004289	0.02690785	0.02569858
A_{FB} G_μ	NLO	0.02905841	0.02592168	0.02471918
A_{FB} $\alpha(0)$ v1	NLO+HO	0.03083234	0.02770533	0.02649700
A_{FB} G_μ	NLO+HO	0.03090286	0.02777783	0.02656851
ΔA_{FB} $\alpha(0)$ v1	NLO-LO	-0.0164959	-0.0165300	-0.0164302
ΔA_{FB} G_μ	NLO-LO	-0.0174805	-0.0175162	-0.0174096
ΔA_{FB} $\alpha(0)$ v1	NLO+HO-LO	-0.0157065	-0.0157326	-0.0156318
ΔA_{FB} G_μ	NLO+HO-LO	-0.0156360	-0.0156596	-0.0155603

A_{FB}

ΔA_{FB} (NLO – LO)

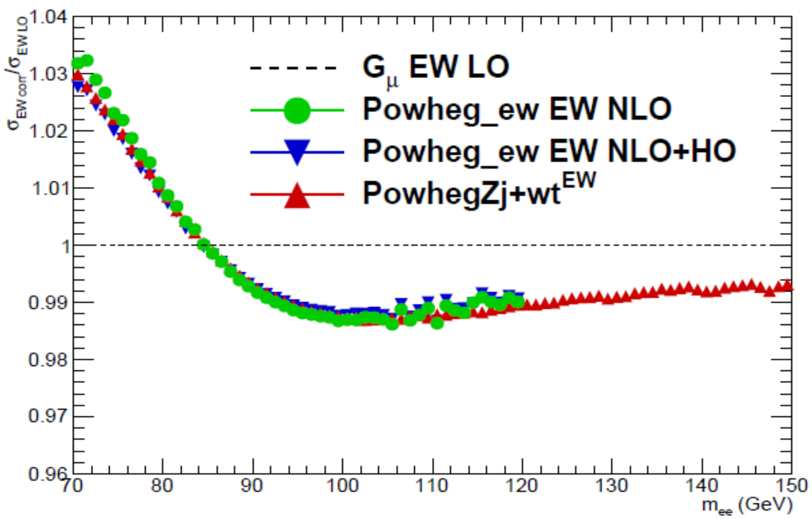
ΔA_{FB} (NLO+HO – LO)

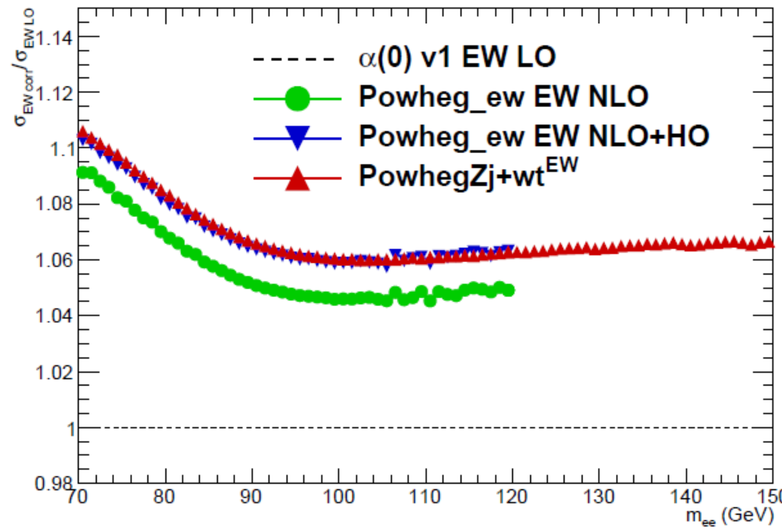
ΔA_{FB}	EW order	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	m_{ee} = 70 - 120 GeV
$\alpha(0)$ v1 - $\alpha(0)$ v0	LO	-0.020375	-0.020486	-0.020487
G_μ - $\alpha(0)$ v0	LO	-0.020375	-0.020486	-0.0204871
G_μ - $\alpha(0)$ v1	LO	0.0	0.0	0.0
G_μ - $\alpha(0)$ v1	NLO	-0.00098	-0.00098	-0.00098
G_μ - $\alpha(0)$ v1	NLO + HO	-0.00007	-0.00007	-0.00007

ΔA_{FB} between EW schemes at LO, NLO, NLO+HO

Better than 0.0001 agreement on A_{FB} at NLO+HO between two EW schemes !

Cross-section: EW LO, NLO, NLO+HO





Powheg_ew

	EW order	m_{ee} = 89 - 93 GeV
$\alpha(0)$ v1	NLO/LO	1.050350
G_μ	NLO/LO	0.991230
$\alpha(0)$ v1	NLO+HO/LO	1.063247
G_μ	NLO+HO/LO	0.991038

	EW order	m_{ee} = 89 - 93 GeV
Powheg_ew	NLO+HO/LO	
$\alpha(0)$ v1		1.06325
G_μ		0.99104
PowhegZj+wt^{EW}	NLO+HO/LO	
$\alpha(0)$ v0		0.96452
$\alpha(0)$ v1		1.06506
G_μ		0.99167

In G_μ scheme, NLO corrections < 1%
HO corrections < 0.02%