Hosing Analysis –
Some HI Growth Rate Calculations

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Formulas for Hosing Growth Rate by C. Schroeder

$$N_{HI} = \frac{3^{3/2}}{4}\left(\mu \hat{k}_\beta^2 k_{pe}^3 \zeta z^2\right)^{\frac{1}{3}} \tag{1}$$

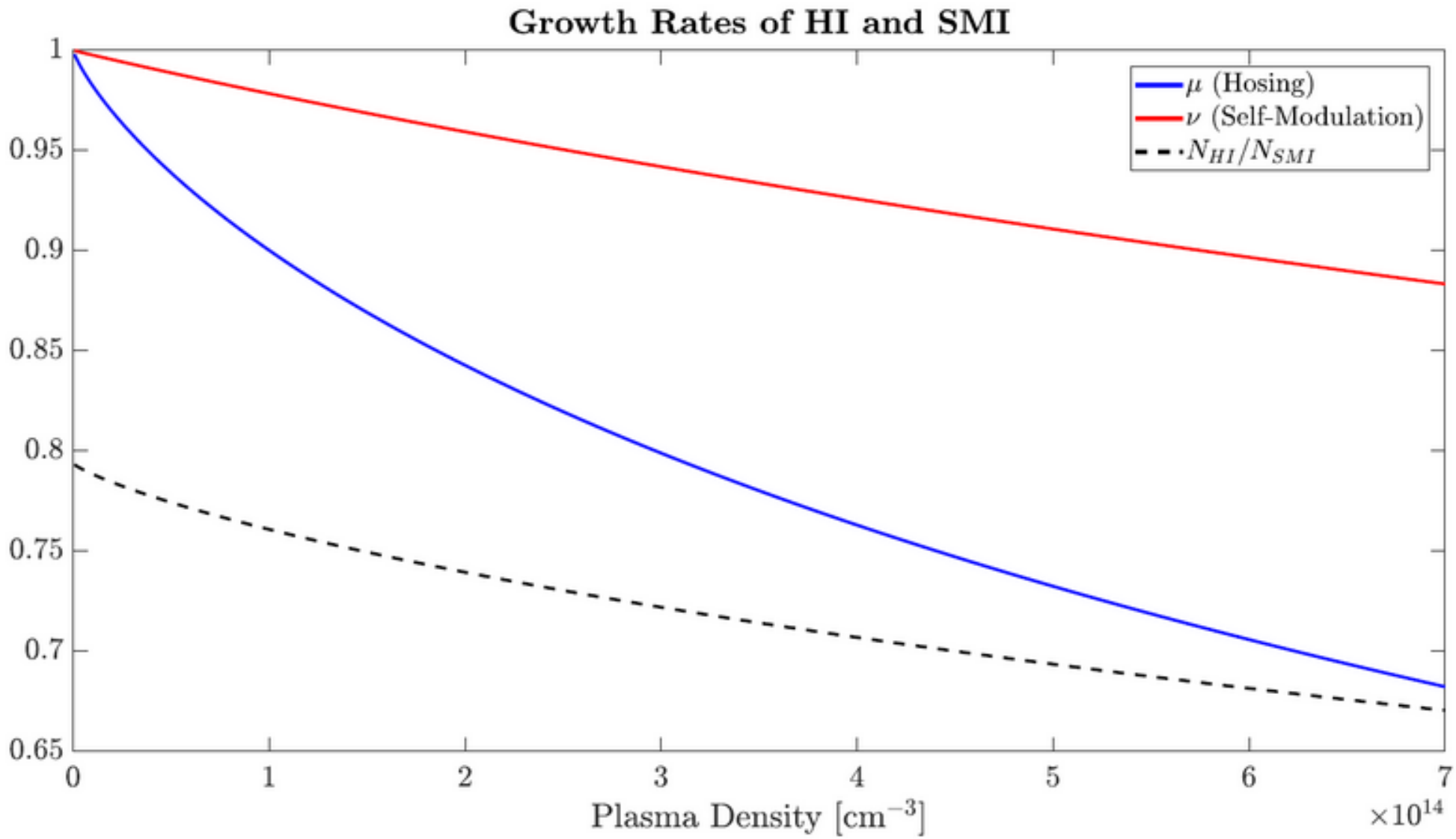
with $\mu = 2I_1(k_{pe}\sigma_r)K_1(k_{pe}\sigma_r)$
and

$$N_{SMI} = \frac{3^{3/2}}{4}\left(2\nu \hat{k}_\beta^2 k_{pe}^3 \zeta z^2\right)^{\frac{1}{3}} \tag{2}$$

with $\nu = 4I_2(k_{pe}r_0)K_2(k_{pe}r_0)$.

$$\frac{N_{HI}}{N_{SMI}} = \left(\frac{\mu}{2\nu}\right)^{\frac{1}{3}} = \left(\frac{1}{4}\cdot\frac{I_1(k_{pe}\sigma_r)K_1(k_{pe}\sigma_r)}{I_2(k_{pe}\sigma_r)K_2(k_{pe}\sigma_r)}\right)^{1/3} \tag{3}$$

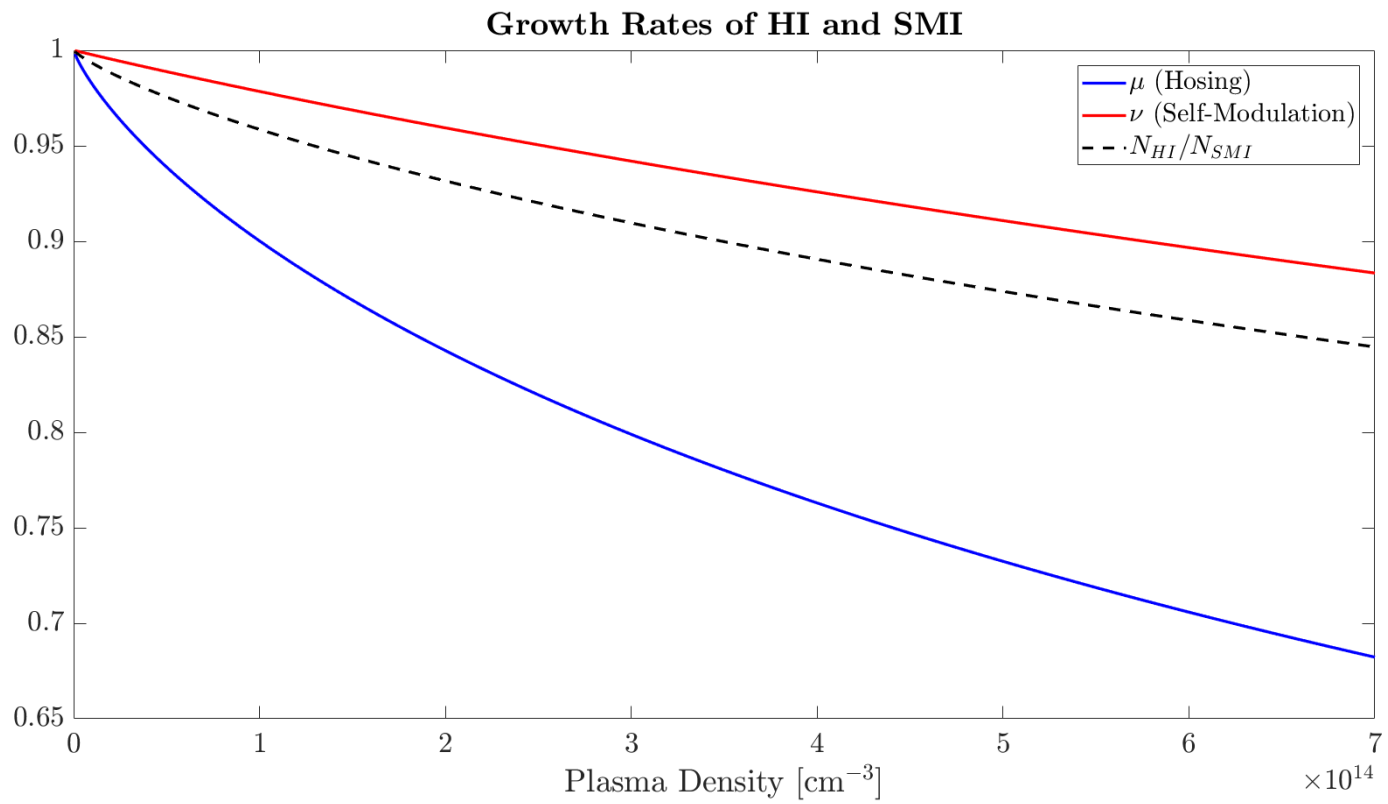
- C. Schroeder et al., Growth and Phase Velocity of Self-Modulated Beam-Driven Plasma Waves. Phys. Rev. Lett.,107:145002, Sep 2011
- C. Schroeder et al., Coupled beam hose and self-modulation instabilities in overdense plasma. Physical Review E , 86(2):026402, 2012.


Growth Rates of HI and SMI
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0
1
2
3
4
5
6
7
Plasma Density [cm^{-3}]
$\times 10^{14}$
μ (Hosing)
ν (Self-Modulation)
N_{HI}/N_{SMI}

Taking SMI-growth formula from Pukhov*:

$$\Gamma = \frac{3\sqrt{3}}{4}\omega_p\left(\frac{n_b m}{2n_e m_p \gamma_b}\frac{\xi}{ct}\right)^{1/3}$$

--> $$\frac{N_{HI}}{N_{SMI}} = \left(\frac{\mu}{\nu}\right)^{\frac{1}{3}} = \left(\frac{I_1(k_{pe}\sigma_r)K_1(k_{pe}\sigma_r)}{I_2(k_{pe}\sigma_r)K_2(k_{pe}\sigma_r)}\right)^{1/3}$$



*A. Pukhov et al., Phase Velocity and Particle Injection in a Self-Modulated Proton-Driven Plasma Wakefield Accelerator, PRL 107 (2011), DOI: 10.1103/PhysRevLett.107.145003.