EPFL

Luminosity and Beam-Beam Effects in the Large Hadron Collider (LHC)

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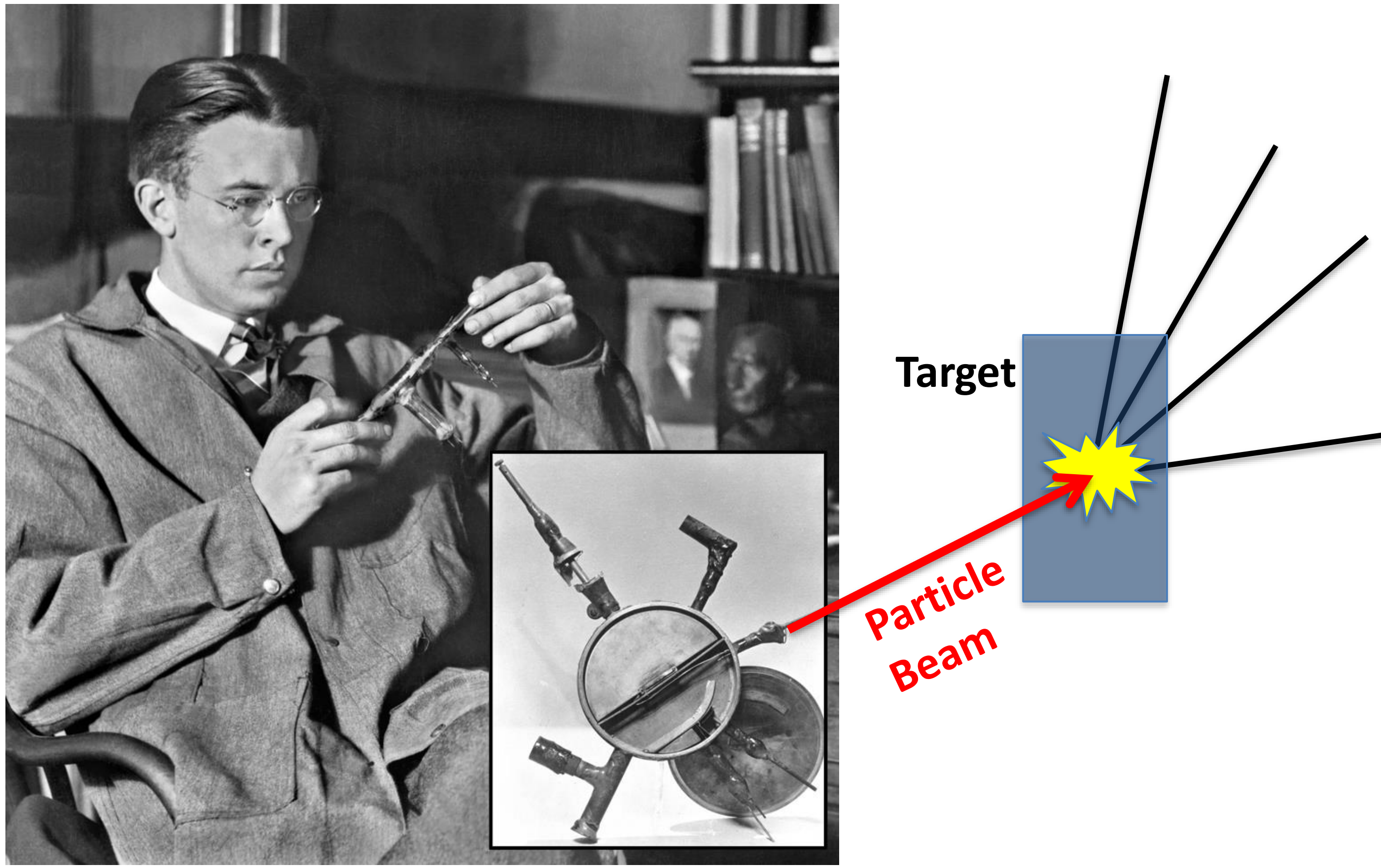
Circular Accelerators: acceleration occurs at every turn!



E. Lawrence 1930

Circular Accelerators: acceleration occurs at every turn!

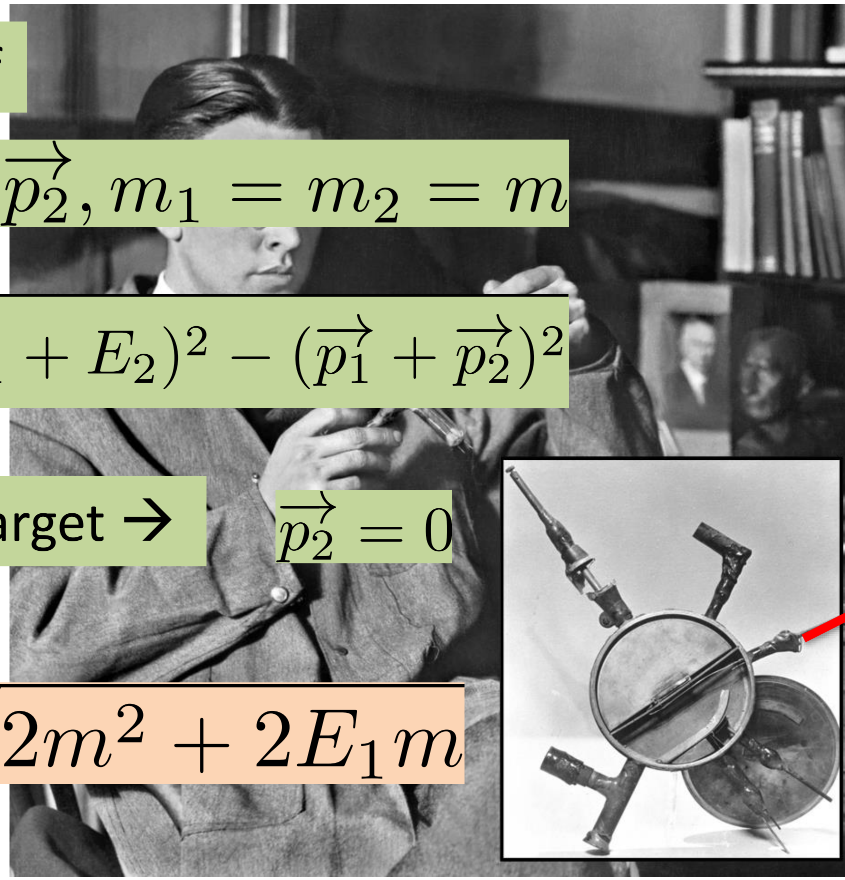
Two Beams of

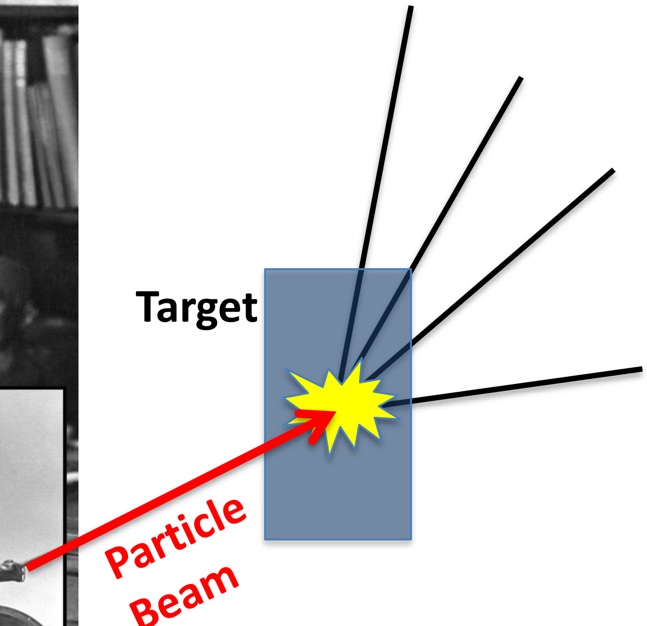
$$E_1, \overrightarrow{p_1}, E_2, \overrightarrow{p_2}, m_1 = m_2 = m$$

$$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\overrightarrow{p_1} + \overrightarrow{p_2})^2}$$

Beam 2 is a Target → $\overrightarrow{p_2} = 0$

$$E_{cm} = \sqrt{2m^2 + 2E_1 m}$$

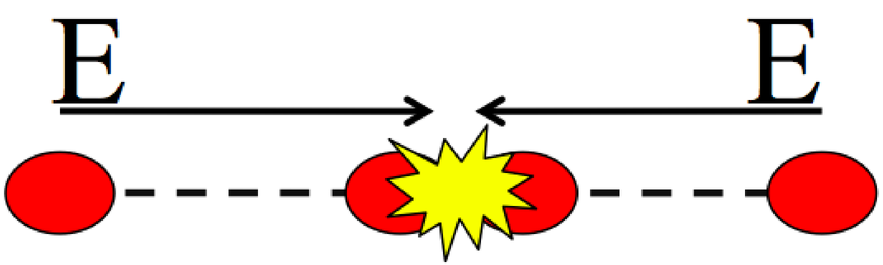




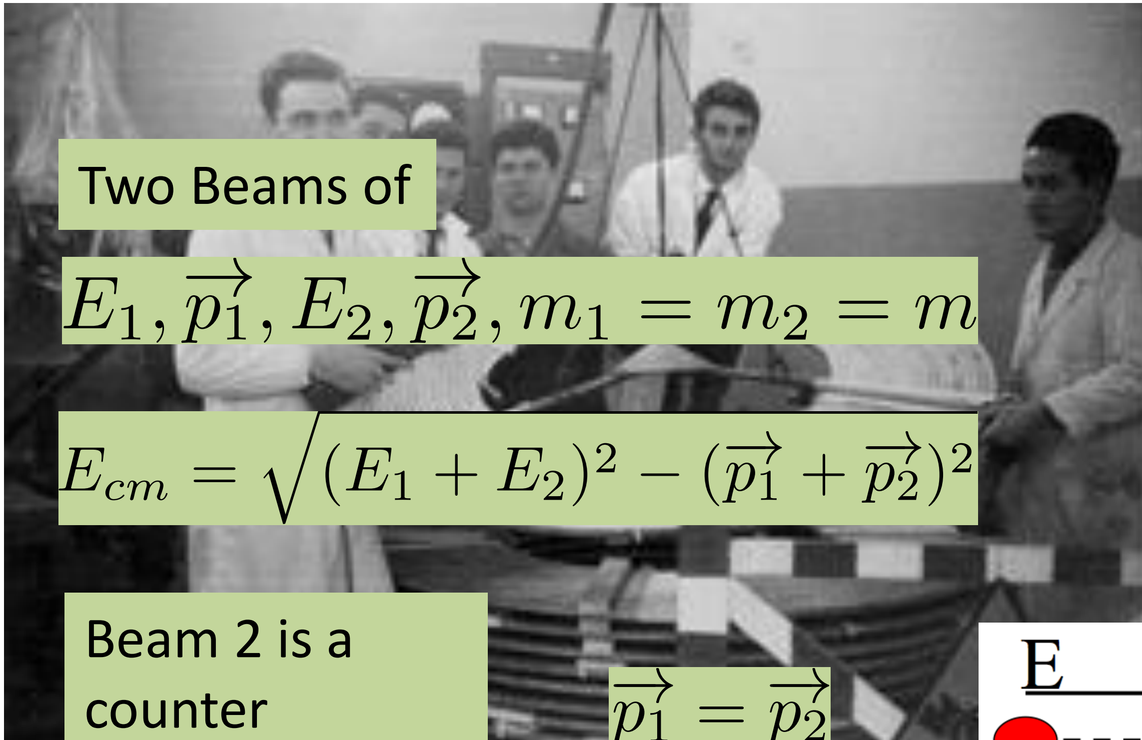
7 TeV proton beam against fix target →115 GeV

Colliders: higher energy

Anello di Accumulazione AdA
B. Touschek 1960



Colliders: higher energy



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$$E_{cm} = \sqrt{(E_1 + E_2)^2 - (\overrightarrow{p_1} + \overrightarrow{p_2})^2}$$

Beam 2 is a counter rotating beam

$$\overrightarrow{p_1} = \overrightarrow{p_2}$$

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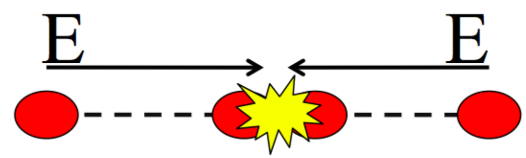
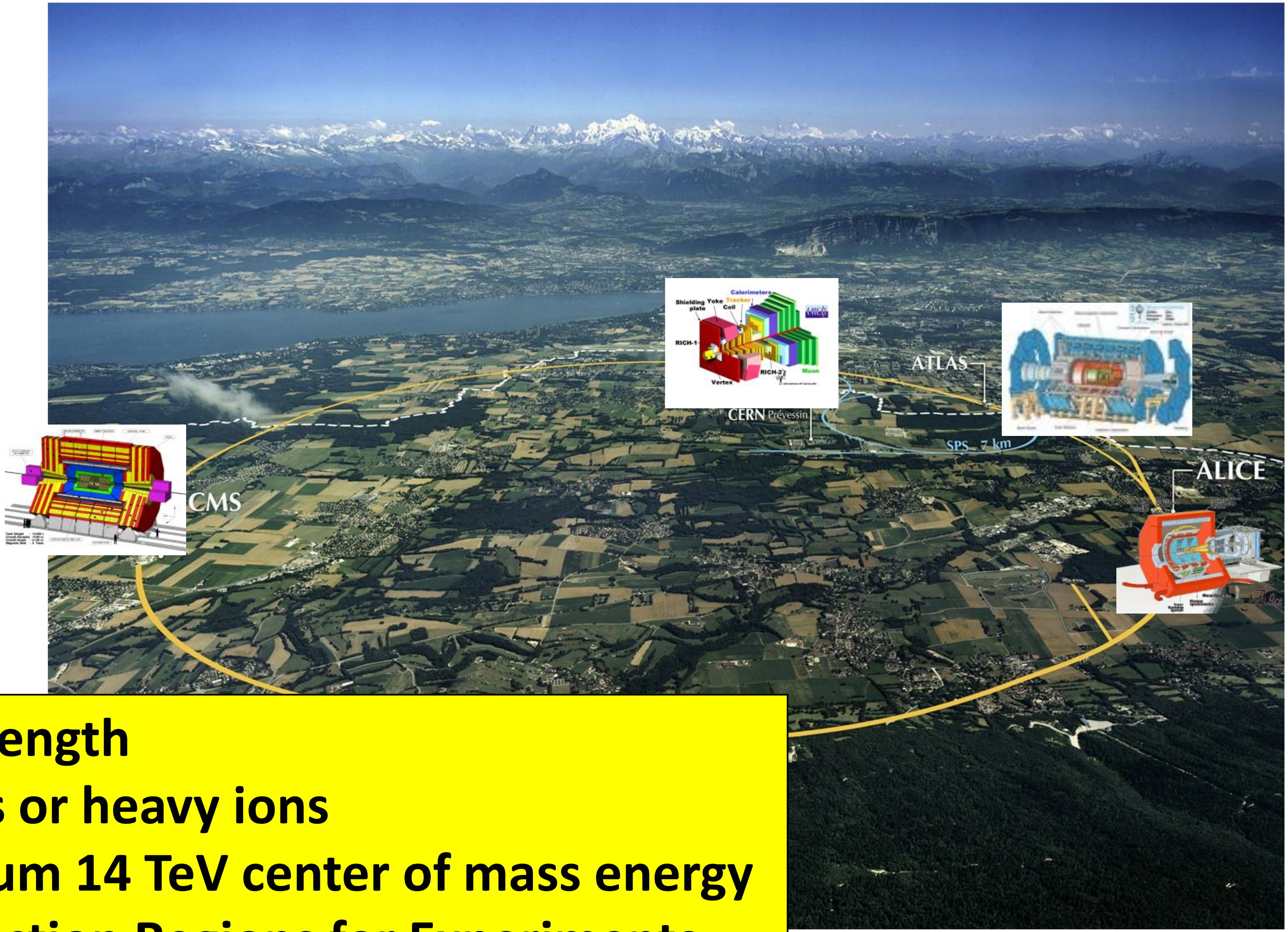


$$E_{cm} = E_1 + E_2$$

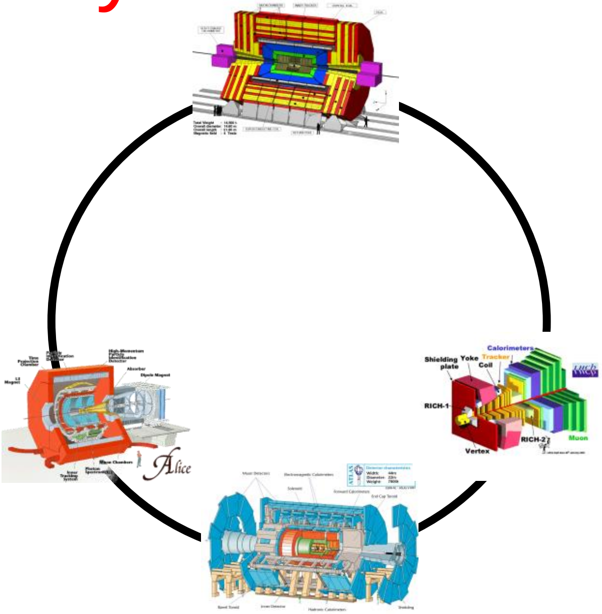
7 TeV proton beam colliding → 14 TeV

The Large Hadron Collider



27 Km length
Protons or heavy ions
Maximum 14 TeV center of mass energy
4 Interaction Regions for Experiments

Circular colliders: Luminosity



Collider Luminosity $\mathcal{L}$

is the proportionality factor between

the cross section σ_{event}

and the number of events per second $\frac{d\mathcal{R}}{dt}$

$$\frac{d\mathcal{R}}{dt} = \mathcal{L} \times \sigma_{event} \qquad units : cm^{-2}s^{-1}$$

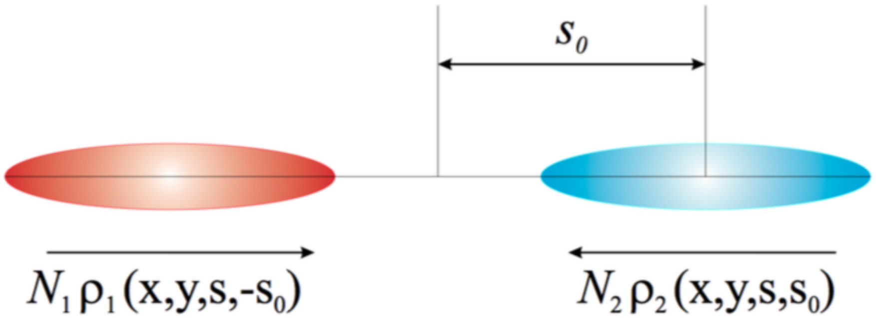
Luminosity is a machine parameter

→Independent of the physical reaction

→Reliable procedure to compute and measure

Luminosity calculation

The overlap integral of two bunches crossing each other head-on is proportional to the luminosity and it is given by:



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,+s_0)\,dx\,dy\,ds\,ds_0$$

$$s_0 = c \cdot t$$

Time variable

$$K = \sqrt{(\vec{v_1} - \vec{v_2})^2 - (\vec{v_1} \times \vec{v_2})^2/c^2}$$

Kinematic Factor

Luminosity formula

$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,+s_0)dxdydsds_0$$

Uncorrelated densities in all planes

→Factorize the distribution density as:

$$\rho_1(x,y,s,-s_0) = \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0)$$

For head-on collisions where

→ "Kinematic Factor" ***K = 2***

To have the luminosity per second

→Needs to multiple by revolution frequency ***f***

In the presence of many bunches ***n_b***

$$\mathcal{L} = 2 \cdot N_1 N_2 \cdot f \cdot n_b \cdot$$

$$\int\int\int\int_{-\infty}^{+\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0) \cdot \rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0)dxdydsds_0$$

Closed solution for Gaussian distributions

Simplest case assumptions:

- Gaussian distributions
- No dispersion at the collision point
- Head-on collision

$$\rho_{i,z}(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\exp(\frac{z^2}{2\sigma_z^2})$$

$$\sigma_{x,y} = \sqrt{\epsilon \cdot \beta^*_{x,y}}$$

K = 2

$$\mathcal{L} = \frac{2N_1N_2fn_b}{(\sqrt{2\pi})^6\sigma_s^2\sigma_x^2\sigma_y^2)}\int\int\int\int e^{-\frac{x^2}{\sigma_x^2}}e^{-\frac{y^2}{\sigma_y^2}}e^{-\frac{s^2}{\sigma_s^2}}e^{-\frac{s_0^2}{\sigma_s^2}}dxdydsds_0$$

Equal Transverse beams “Round” beams

$$\sigma_{1x} = \sigma_{2x}$$
$$\sigma_{1y} = \sigma_{2y}$$

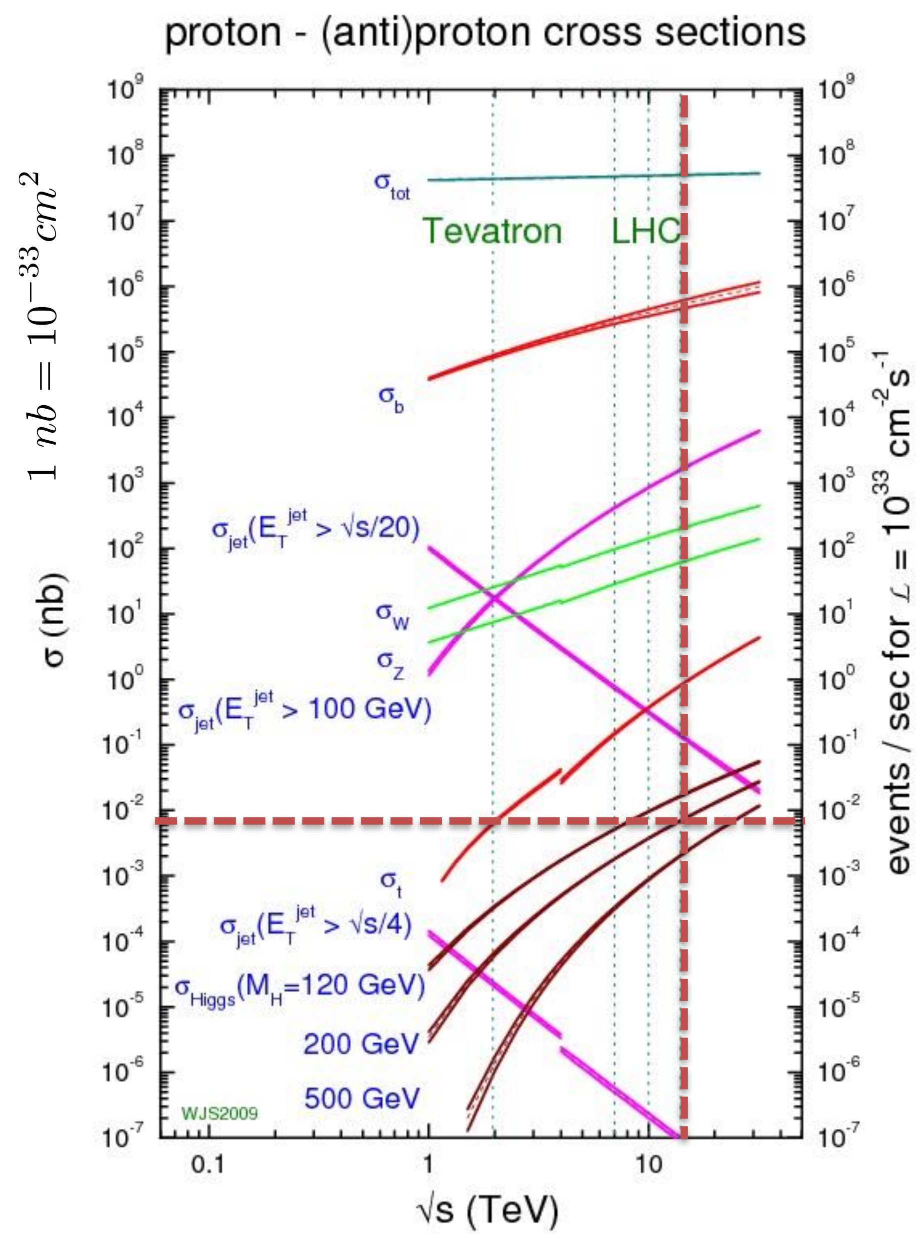
$$\mathcal{L} = \frac{N_1N_2fn_b}{4\pi\sigma_x\sigma_y}$$

Un-Equal Transverse beams “Flat” beams or optics

$$\sigma_{1x} \neq \sigma_{2x}$$
$$\sigma_{1y} \neq \sigma_{2y}$$

$$\mathcal{L} = \frac{N_1N_2fn_b}{2\pi\sqrt{\sigma_{1x}^2+\sigma_{2x}^2}\sqrt{\sigma_{1y}^2+\sigma_{2y}^2}}$$

The LHC design parameters



$$\frac{d\mathcal{R}}{dt} = \mathcal{L} \times \sigma_{event}$$

LHC Design

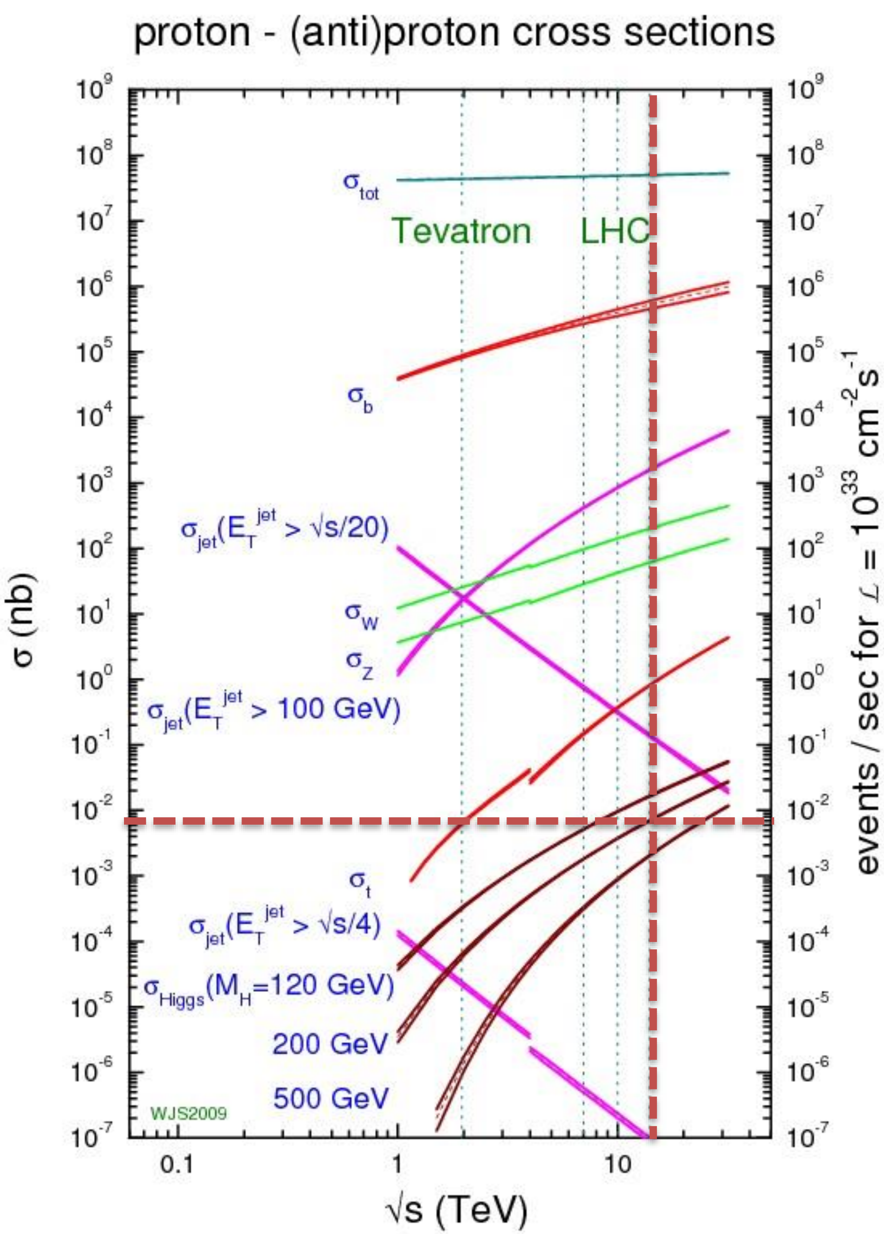
$N_1 = N_2 = 1.15\ 10^{11}$ protons per bunch

$\sigma_x = \sigma_y = 16.6\ \mu m$

$\beta^* = 55$ cm

$\rightarrow L=10^{34}\ cm^{-2}s^{-1}$

The LHC design parameters



LHC Design

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$\beta^* = 55$ cm

$\rightarrow L = 10^{34}\ cm^{-2}s^{-1}$

LHC Record

$N_1 = N_2 = 1.15\ 10^{11}$ protons per bunch

$\sigma_x = \sigma_y = 9.5\ \mu m$

$\beta^* = 30$ cm

$\rightarrow L = 2 \times 10^{34}\ cm^{-2}s^{-1}$

High Luminosity Upgrade of LHC

$N_1 = N_2 = 2.2\ 10^{11}$ protons per bunch

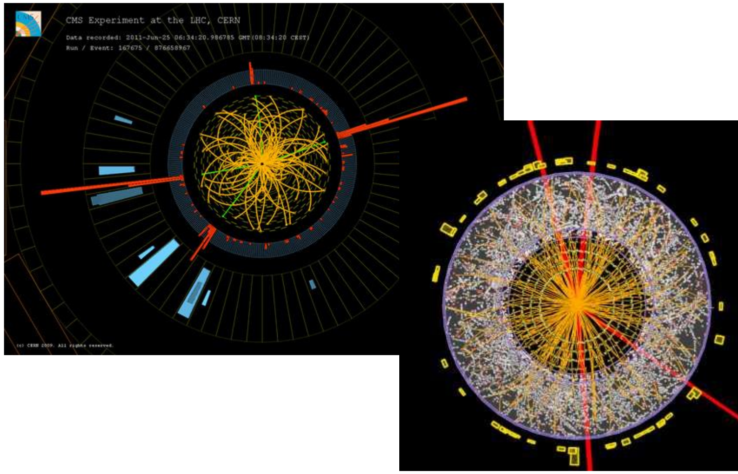
$\sigma_x = \sigma_y = 7.0\ \mu m$

$\beta^* = 64 \rightarrow 15$ cm

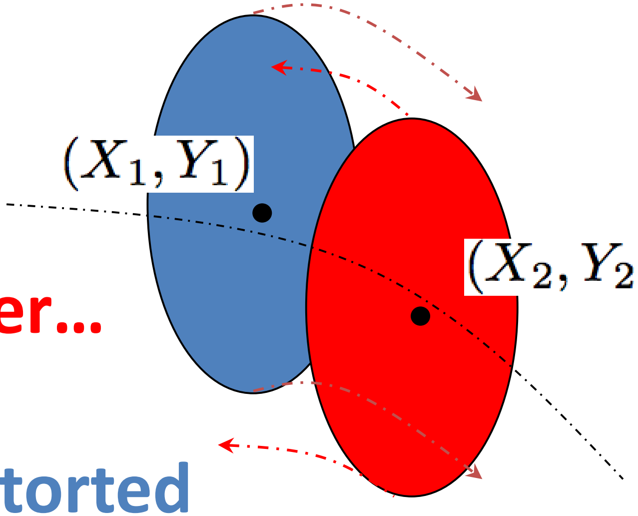
$\rightarrow L = (10\text{-}20) \times 10^{34}\ cm^{-2}s^{-1}$

Different types of collisions

- They occur when two beams get closer and collide



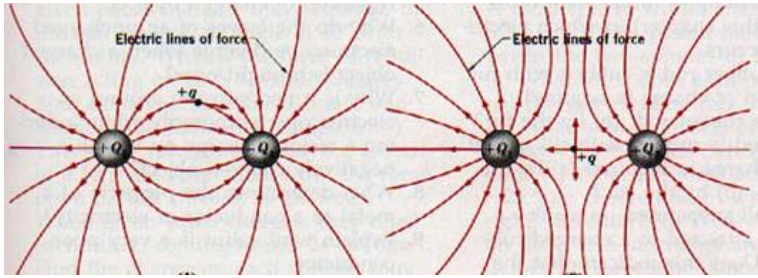
- Two types
  - High energy collisions between two particles (wanted)
  - Distortions of beam by electromagnetic forces (unwanted)



- **Unfortunately: usually both go together…**
- **0.001% (or less) of particles collide**
- **99.999% (or more) of particles are distorted**

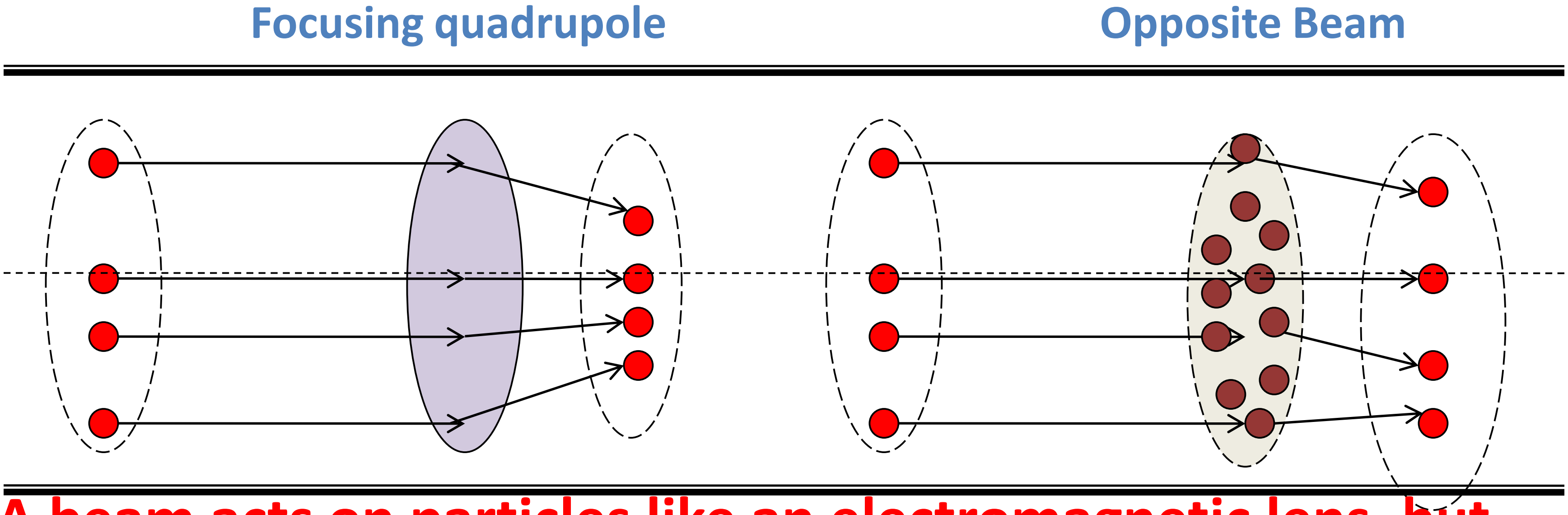
Proton Beams → Electro Magnetic potential

- Beam is a collection of charges
- Beam is an electromagnetic potential for other charges



Electromagnetic force from opposing beam (beam-beam effects)

Single particle motion and whole bunch motion distorted



A beam acts on particles like an electromagnetic lens, but…

Beam-beam Force derivation

General approach in electromagnetic problems **Reference[5]** already applied to beam-beam interactions in **Reference[1,3, 4]**

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x,y,z)$$

Derive potential from Poisson equation for charges with distribution ρ

Solution of Poisson equation

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\int\int\int \frac{\rho(x_0,y_0,z_0)dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}$$

$$\overrightarrow{E} = -\nabla U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

Then compute the fields

$$\overrightarrow{F} = q(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B})$$

From Lorentz force one calculates the force acting on test particle with charge q

Making some assumptions we can simplify the problem and derive analytical formula for the force…

Beam-Beam Force for Round Gaussian distributions

Gaussian distribution for charges
Round beams:
Very relativistic, Force has only radial component :

$$\sigma_x = \sigma_y = \sigma$$

$$\beta \approx 1 \qquad r^2 = x^2 + y^2$$

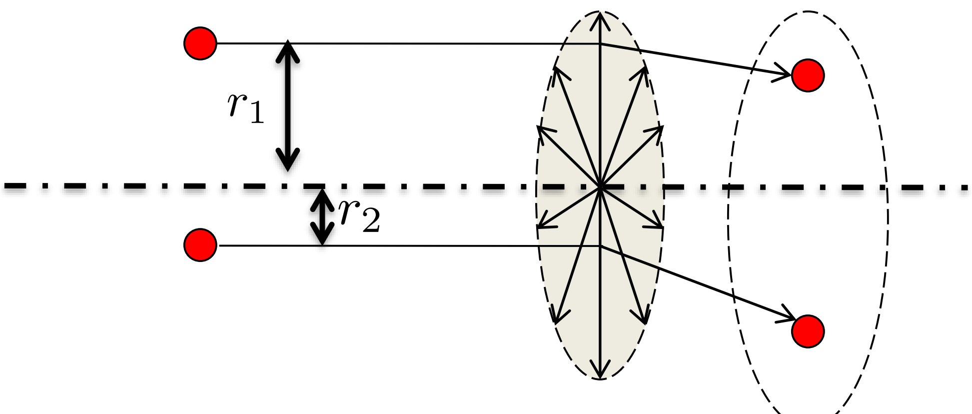
$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Beam-beam Force

$$\Delta r' = \frac{1}{mc\beta\gamma} \int F_r(r,s,t)\ dt$$

$$\Delta r' = -\frac{N_p r_0}{r} \cdot \frac{r}{r^2}[1 - e^{-\frac{r^2}{2\sigma^2}}]$$

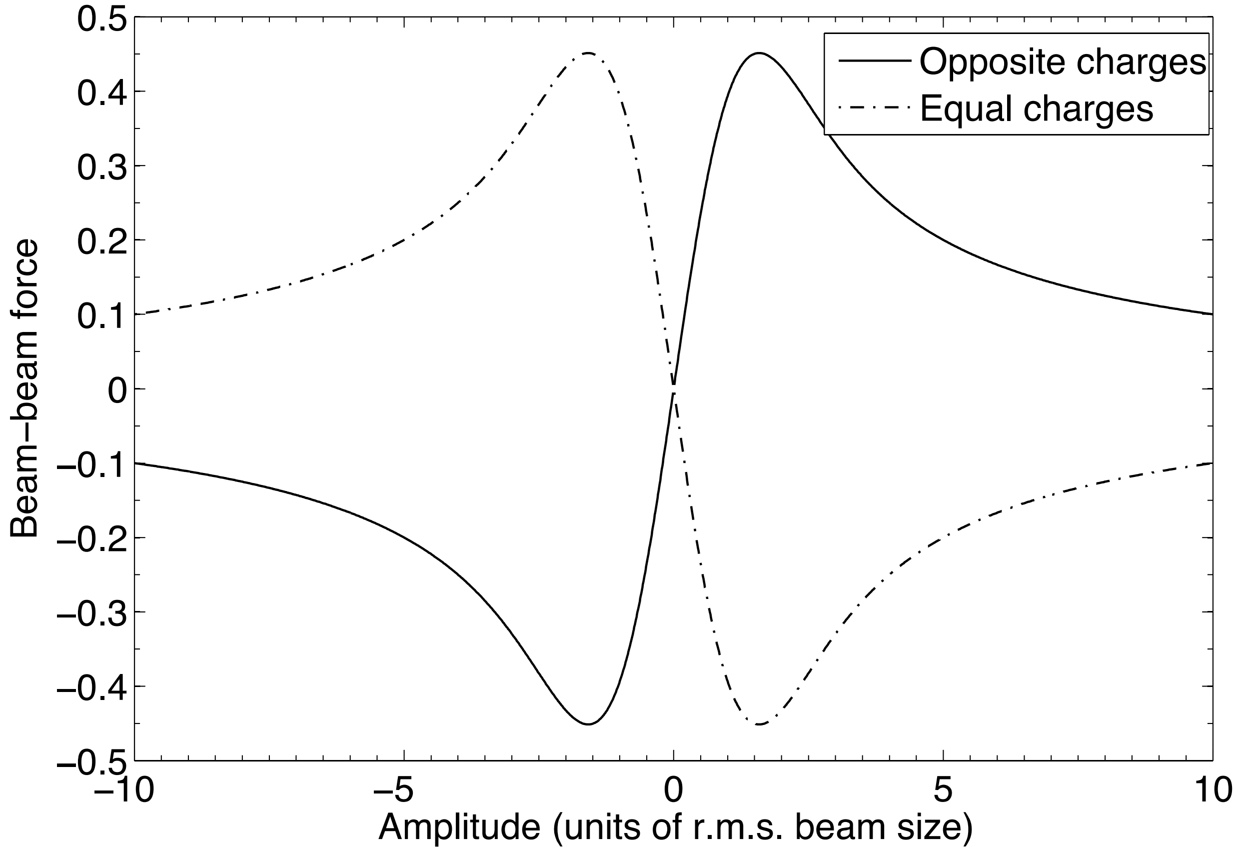
Beam-beam kick obtained integrating the force over the collision (i.e. time of passage)



Only radial component in relativistic case

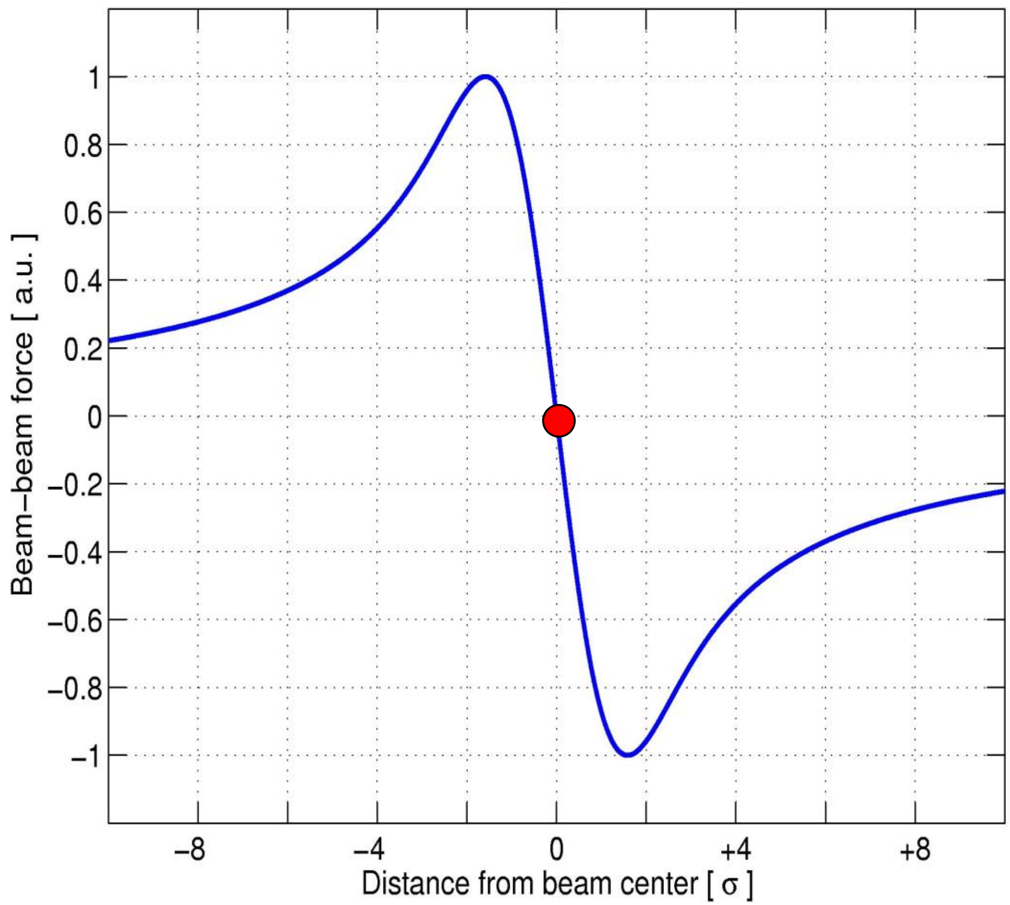
How does this force looks like?

Beam-beam Force



$$F_r(r) = \pm \frac{ne^2(1+\beta_{rel}^2)}{2\pi\epsilon_0}\frac{1}{r}[1-\exp(-\frac{r^2}{2\sigma^2})]$$

Beam-beam Force



$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Why do we care?

Pushing for luminosity means stronger beam-beam effects

$$\mathcal{L} \propto \frac{N_p^2}{\sigma_x \sigma_y} \cdot n_b$$

$$F \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - e^{-\frac{r^2}{2\sigma^2}}\right]$$

Strongest non-linearity in a collider YOU CANNOT AVOID!

Strong non-linear electromagnetic distortion → impact on beam quality (particle losses and emittance blow-up)

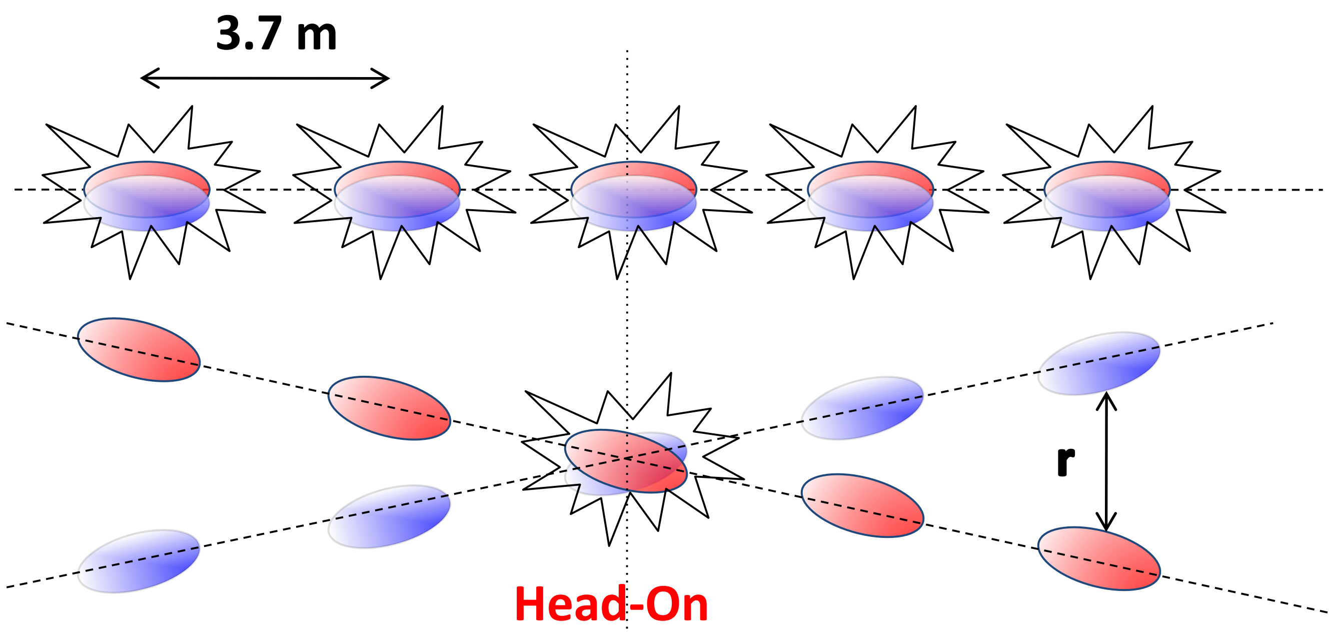
Physics fill lasts for many hours 10h – 24h

Crossing angle operation

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

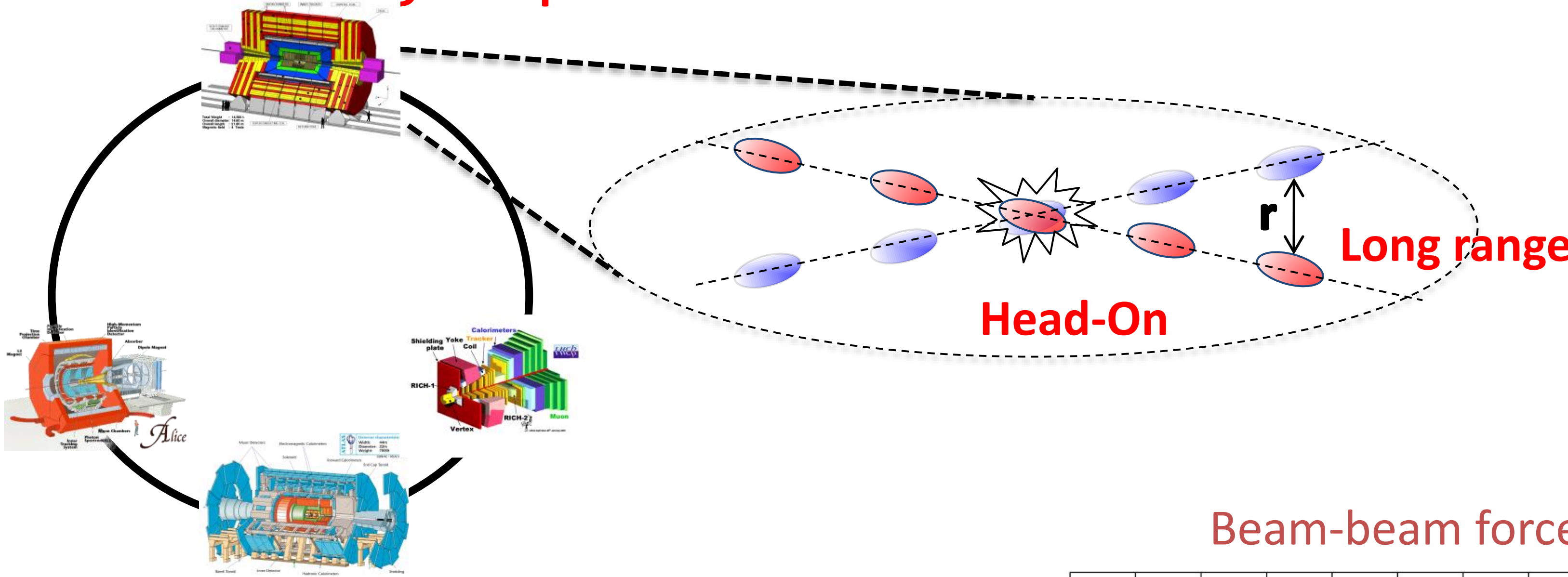
Num. of maximum bunches $n_b = 2808$

Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments



A finite crossing angle has to be applied to avoid multiple collision points

Crossing angle operation and beam-beam interactions



Two type of interactions:

Other beam passing in the center force

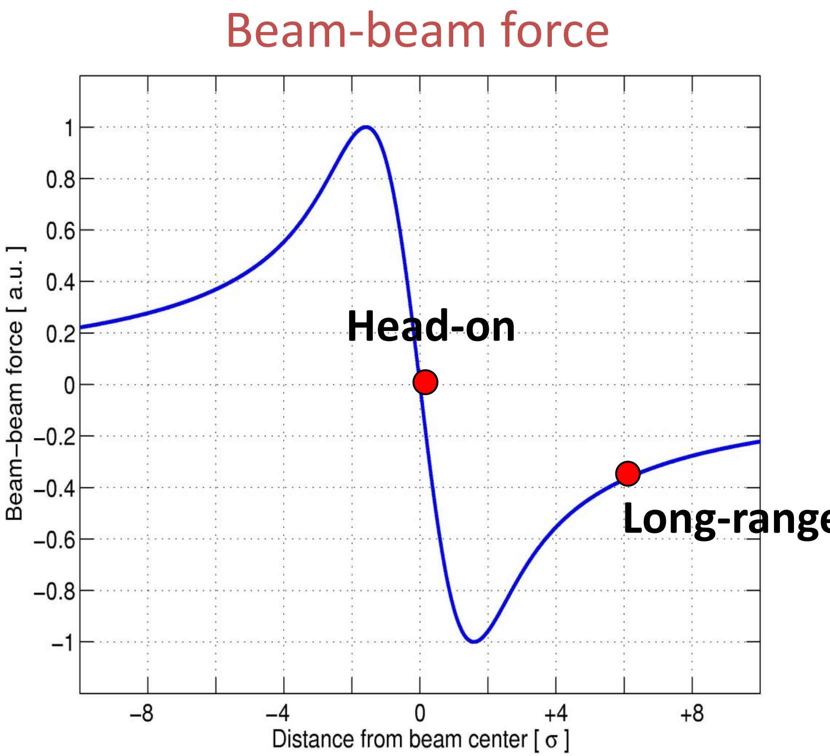
→ HEAD-ON beam-beam interaction

→ **LHC has 4 corresponding to the 4 experiments ATLAS, CMS, Alice, LHCb**

Other beam passing at an offset r

→ LONG-RANGE beam-beam interaction

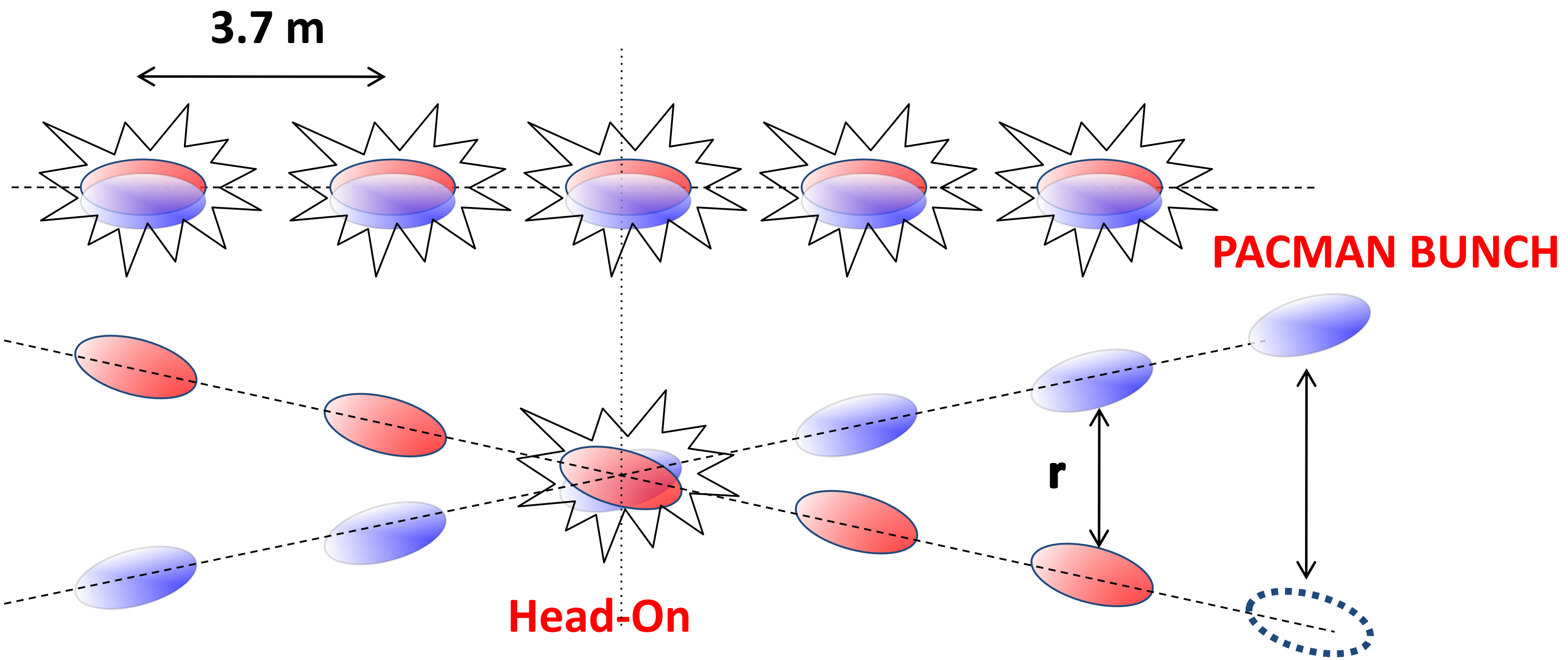
→ **LHC has up to 120 LR interactions**



Multiple bunch Complications

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y}$$

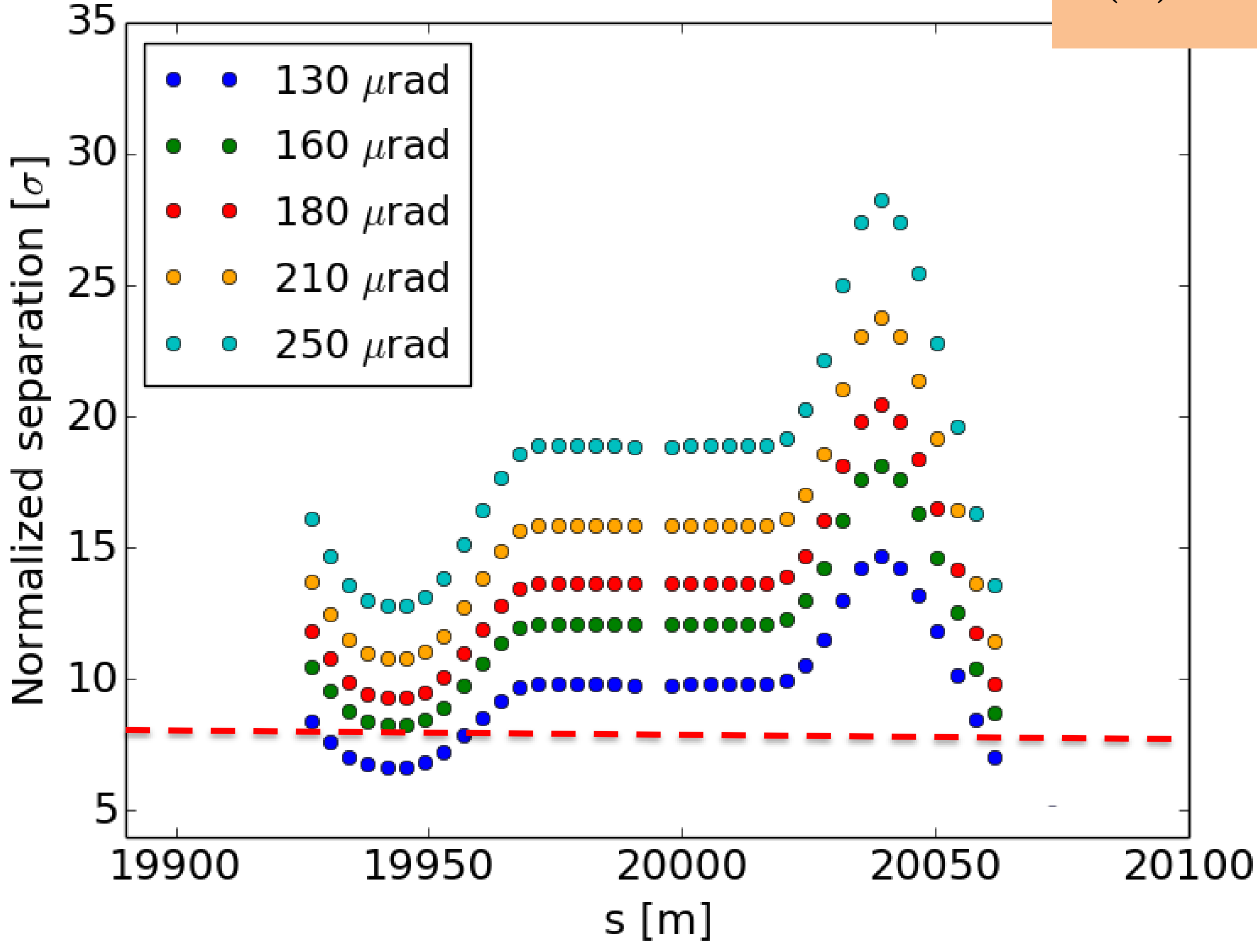
Num. of bunches : $n_b = 2808$



Due to the train structure of the beams → different bunches will experience a different number of interactions!

Long-Range separations

$$\beta(s) = \beta^* + \frac{s^2}{\beta^*}$$



Multi Bunch operations brings un-wanted interactions left and right of the 4 Experiments

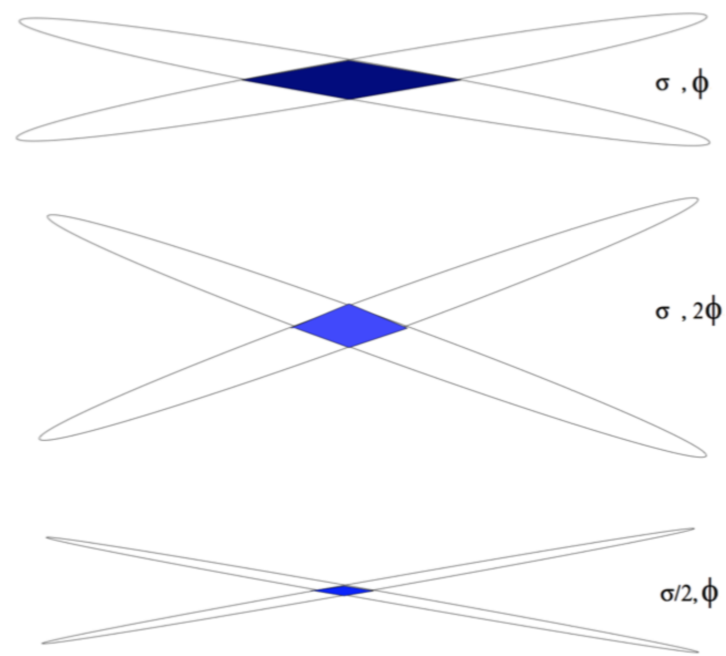
Luminosity Geometric reduction factor

Due to the crossing angle the overlap integral between the two colliding bunches is reduced!

$$\mathcal{L} = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot \mathcal{S}$$

S is the geometric reduction factor

$$\mathcal{S} = \frac{1}{\sqrt{1+(\frac{\sigma_x}{\sigma_s}\tan\frac{\phi}{2})^2}} \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2})^2}}$$



$$\sigma_s >> \sigma_{x,y}$$

Always valid for LHC and HL-LHC
σ_x = 17-7 μm, σ_s = 7.5 cm

$$S \approx \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

LHC design: ϕ = 285 μrad, σ_x = 17 μm, σ_s = 7.5 cm, S=0.84
LHC 2018: ϕ = 320 μrad, σ_x = 9.3 μm, σ_s = 7.5 cm, S=0.61

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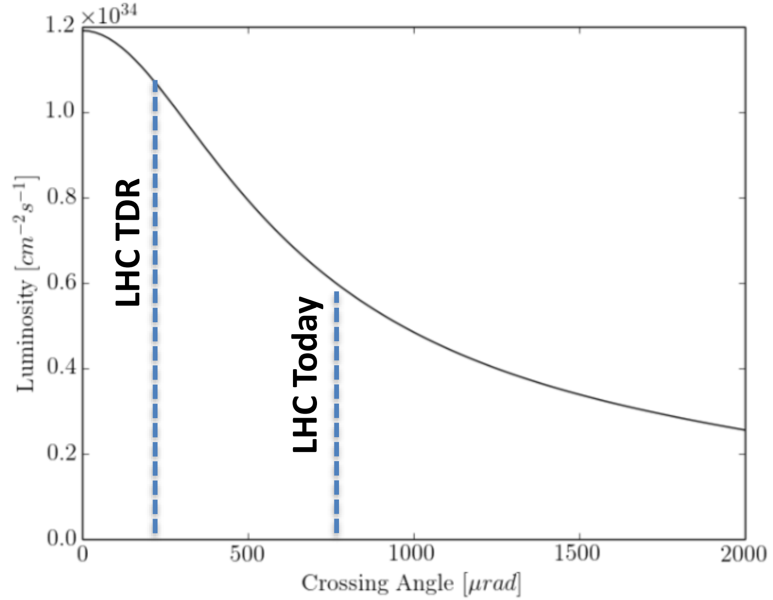

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LHC 2018: ϕ = 320 μrad, σ_x = 9.3 μm, σ_s = 7.5 cm, S=0.61

LHC operates at finite crossing angle

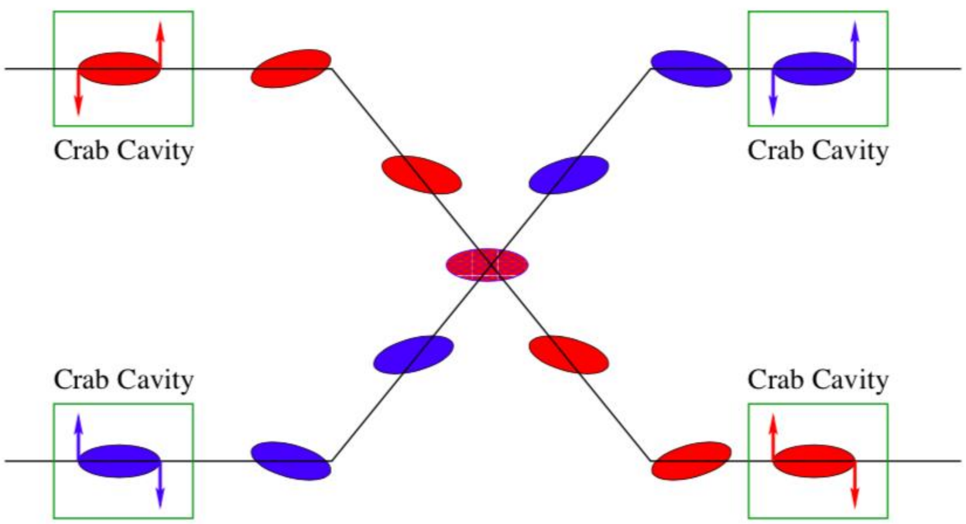
HL-LHC will have bunches of 2.2 10^{11} protons per bunch

ϕ = 590 μrad, σ_x = 9.3 μm, σ_s = 7.5 cm, S=0.26 → 73% of luminosity lost!

$$\sigma_s >> \sigma_{x,y}$$

$$S \approx \frac{1}{\sqrt{1+(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

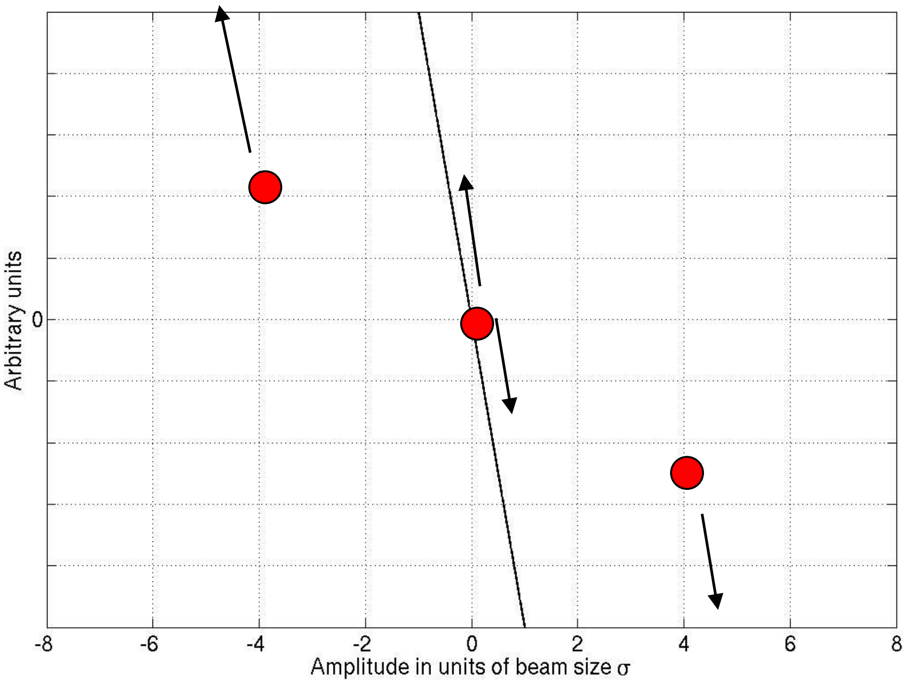
Courtesy of R. Calaga



Crab Cavities used to tilt the bunches longitudinally and compensate for the crossing angle at the collision point! Testing of crab cavities on-going in SPS!

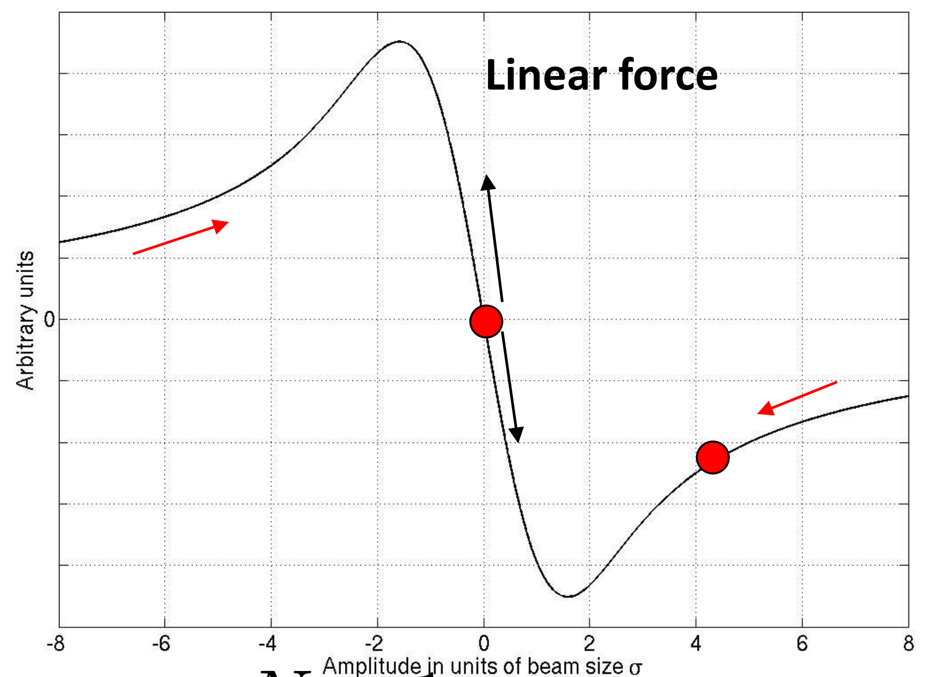
Beam-Beam Force: single particle head-on collision

Lattice defocusing quadrupole



$$F = -k \cdot r$$

Beam-beam force



$$\mathrm{F} \propto \frac{N_p}{\sigma} \cdot \frac{1}{r} \cdot \left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

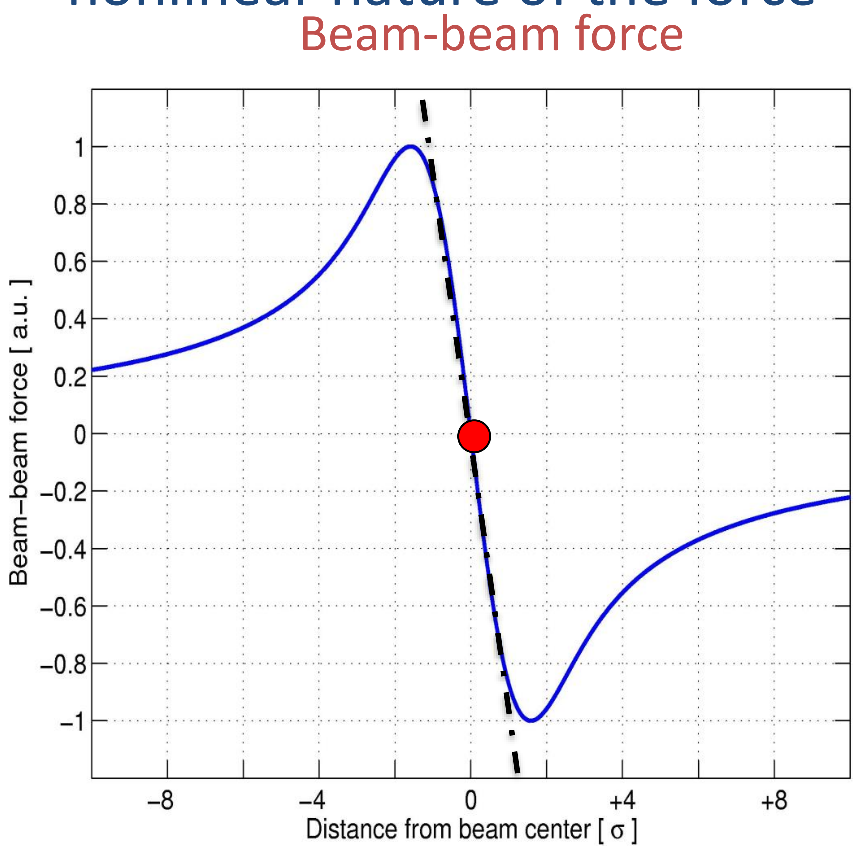
For small amplitudes: linear force

For large amplitude: very non-linear

The beam will act as a strong non-linear electromagnetic lens!

Can we quantify the beam-beam strenght?

Quantifies the strength of the force but does NOT reflect the nonlinear nature of the force



For small amplitudes: linear force

$$F \propto -\xi \cdot r$$

The slope of the force gives you the beam-beam parameter ξ

Beam-Beam Parameter

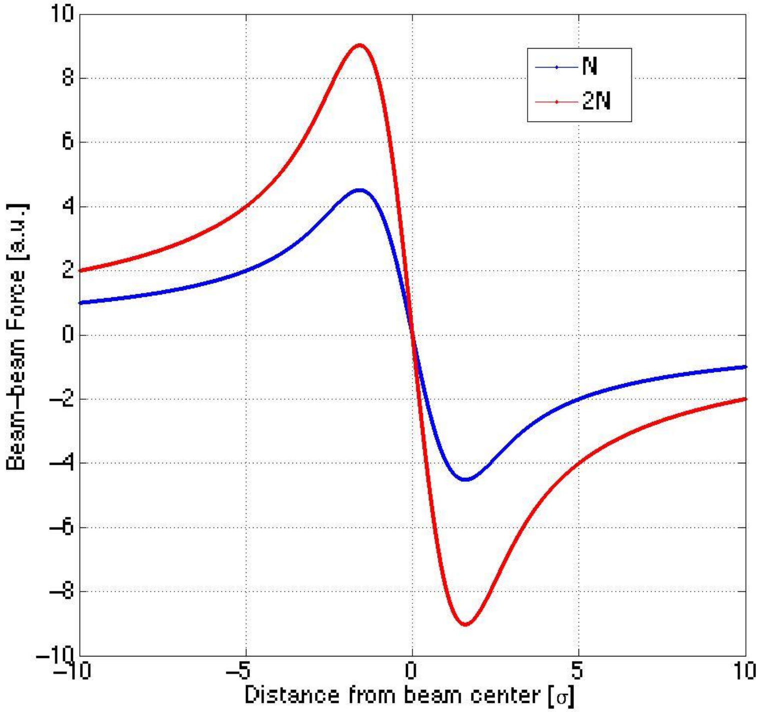
$$\Delta r' = \frac{1}{mc\beta\gamma}\int F_r(r,s,t)\ dt$$

For small amplitudes: linear force

$$\Delta r' = -\frac{N_p r_0}{r}\cdot\frac{r}{r^2}\cdot\left[1 - \mathrm{e}^{-\frac{r^2}{2\sigma^2}}\right]$$

$$\Delta r' = \frac{2N_p r_0}{\gamma}\cdot\frac{1}{r}\cdot\left[1 - \left(1 - \frac{r^2}{2\sigma^2} + \ldots\right)\right]$$

$$\Delta r'|_{r\to 0} = \frac{N r_0 r}{\gamma\sigma^2} = +f\cdot r$$



$$\xi = \frac{\beta^*}{4\pi}\cdot\frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

Beam-Beam parameter:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Examples:

Parameters	LHC TDR	LHC 2012
Intensity $N_{p,e}$/bunch	1.15 10^{11}	1.8 10^{11}
Energy GeV	7000	4000
Beam size H	16.6 μm	16.6 μm
Beam size V	16.6 μm	16.6 μm
$\beta_{x,y}^*$ m	0.55-0.55	0.55-0.55
Crossing angle μrad	290	285
ξ_{bb}	**0.0037**	**0.007**

Beam-Beam parameter:

For round beams:

$$\xi = \frac{\beta^*}{4\pi} \cdot \frac{\delta(\Delta r')}{\delta r} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$

For non-round beams:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)}$$

Examples:

Parameters	LEP (e^+e^-)	LHC(pp)
Intensity $N_{p,e}$/bunch	$4\ 10^{11}$	$1.15\ 10^{11}$
Energy GeV	100	7000
Beam size H	160-200 μm	16.6 μm
Beam size V	2-4 μm	16.6 μm
$\beta_{x,y}^*$ m	1.25-0.05	0.55-0.55
Crossing angle μrad	0	285
ξ_{bb}	**0.07**	**0.0037**

LHC 2012
$1.7\ 10^{11}$
4000
18 μm
18 μm
0.6-0.6
290
0.009

Linear Tune shift due to head-on collision

For small amplitude particles beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

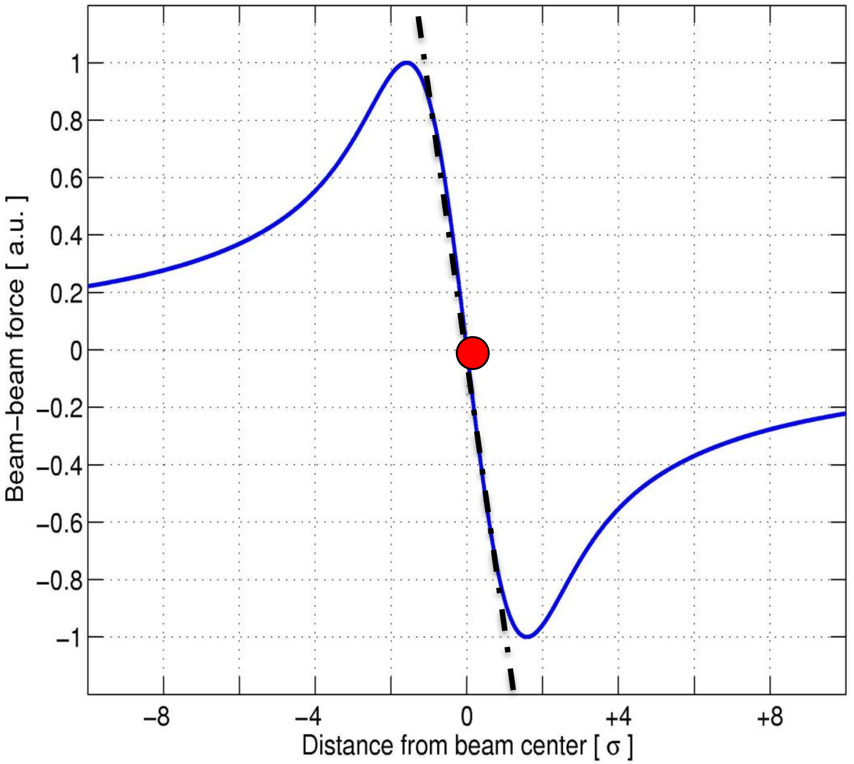
Focal length is given by the beam-beam parameter:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta *}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta *} & 1 \end{pmatrix}$$

Beam-beam force



Equivalent to tune shift

Perturbed one turn matrix

For small amplitudes beam-beam can be approximated as linear force as a quadrupole

$$F \propto -\xi \cdot r$$

Focal length:

$$\frac{1}{f} = \frac{\Delta x'}{x} = \frac{N r_0}{\gamma \sigma^2} = \frac{\xi \cdot 4\pi}{\beta^*}$$

Beam-beam matrix:

$$\begin{pmatrix} 1 & 0 \\ -\frac{\xi \cdot 4\pi}{\beta^*} & 1 \end{pmatrix}$$

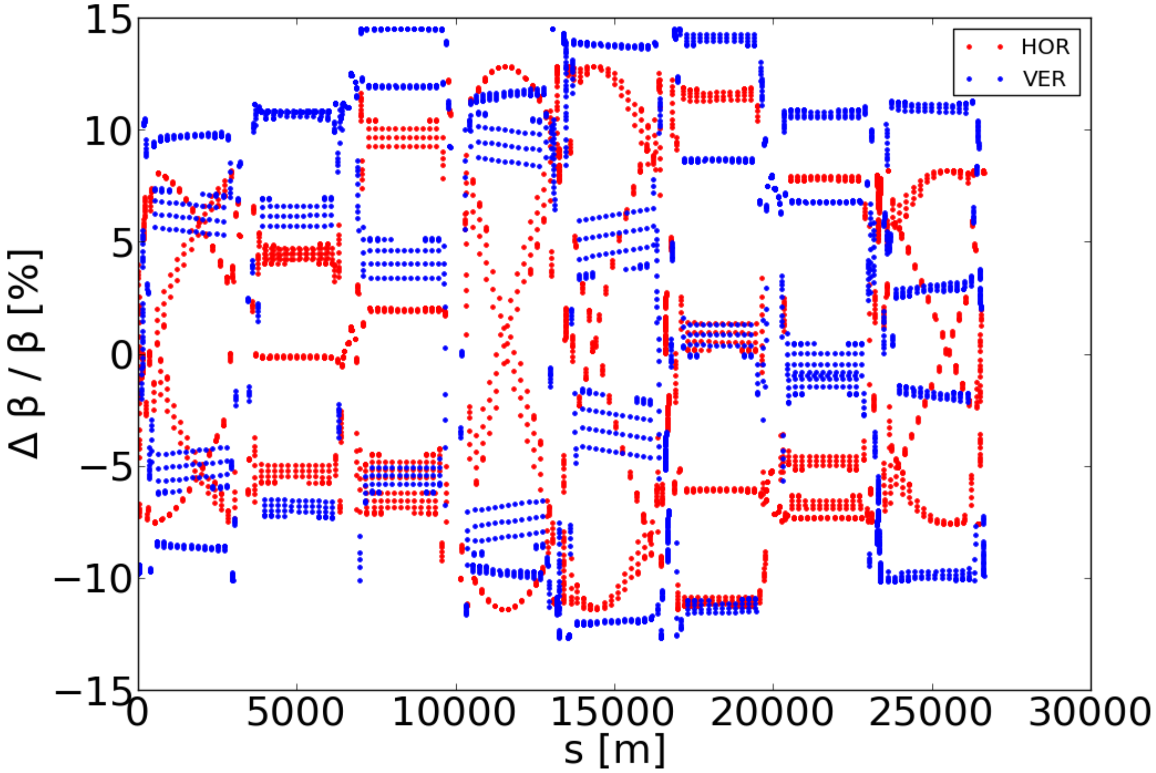
Perturbed one turn matrix with perturbed tune ΔQ and beta function at the IP β*:

$$\begin{pmatrix} cos(2\pi(Q+\Delta Q)) & \beta^* sin(2\pi(Q+\Delta Q)) \\ -\frac{1}{\beta^*} sin(2\pi(Q+\Delta Q)) & cos(2\pi(Q+\Delta Q)) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix} \cdot \begin{pmatrix} cos(2\pi Q) & \beta_0^* sin(2\pi Q) \\ -\frac{1}{\beta_0^*} sin(2\pi Q) & cos(2\pi Q) \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ -\frac{1}{2f} & 1 \end{pmatrix}$$

Dynamic Beta effect

$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q+\Delta Q))} = \frac{1}{\sqrt{1+4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$



$$\xi_{bb} = 0.02$$

Sensitive to:

- Beam-beam parameter: ξ
- Tune : Q
- Configuration (IPS) and optics (phase advance)

LHC case has 5-8 %

HL-LHC 15 %

…or more

Tune shift and and spread

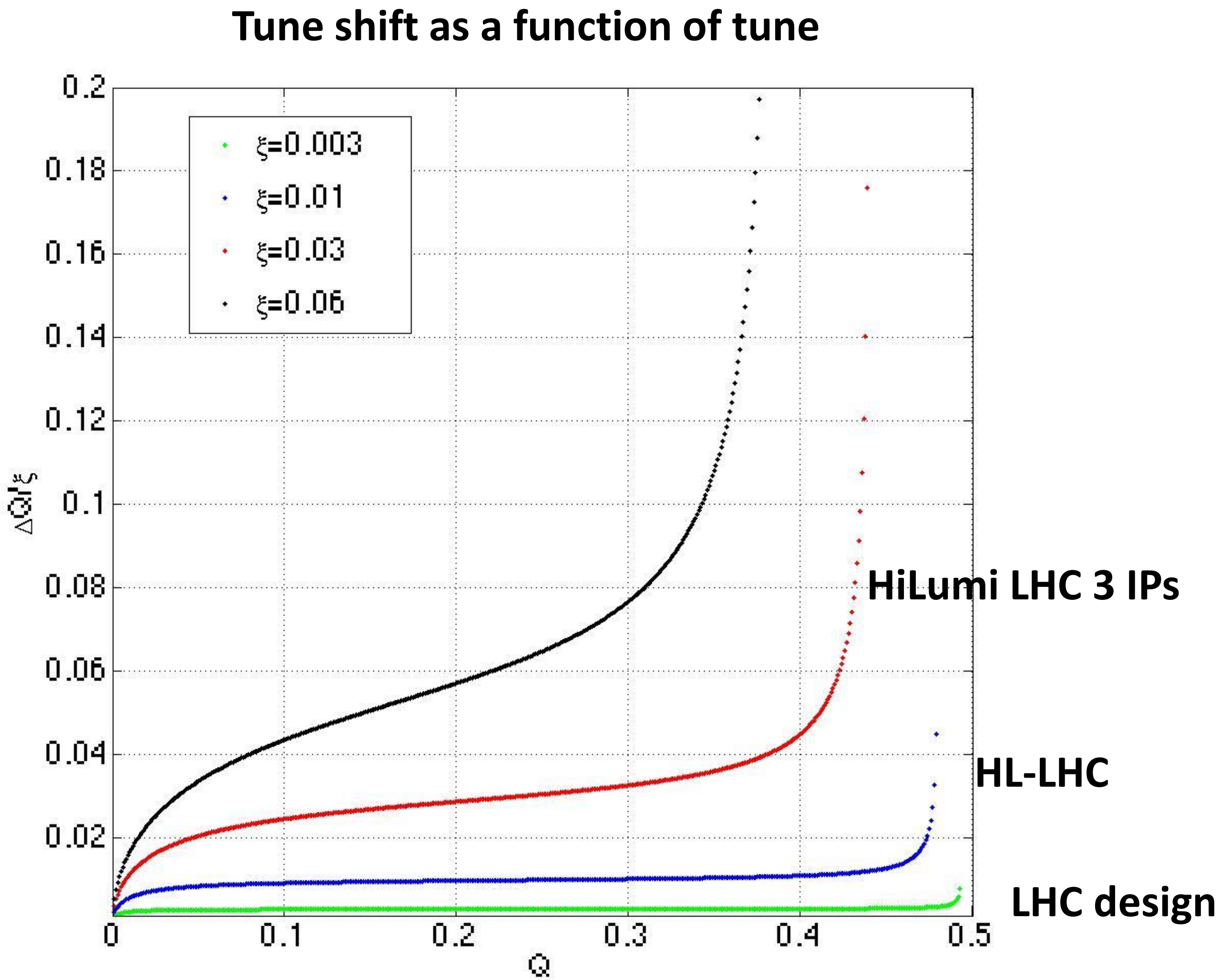
Solving the one turn matrix one can derive the tune shift ΔQ **:**

Tune is changed

$$cos(2\pi(Q + \Delta Q)) = cos(2\pi Q) - \frac{\beta_0^* \cdot 4\pi\xi}{\beta*} sin(2\pi Q)$$

...how does the tune changes?

Tune shift due to beam-beam interactions

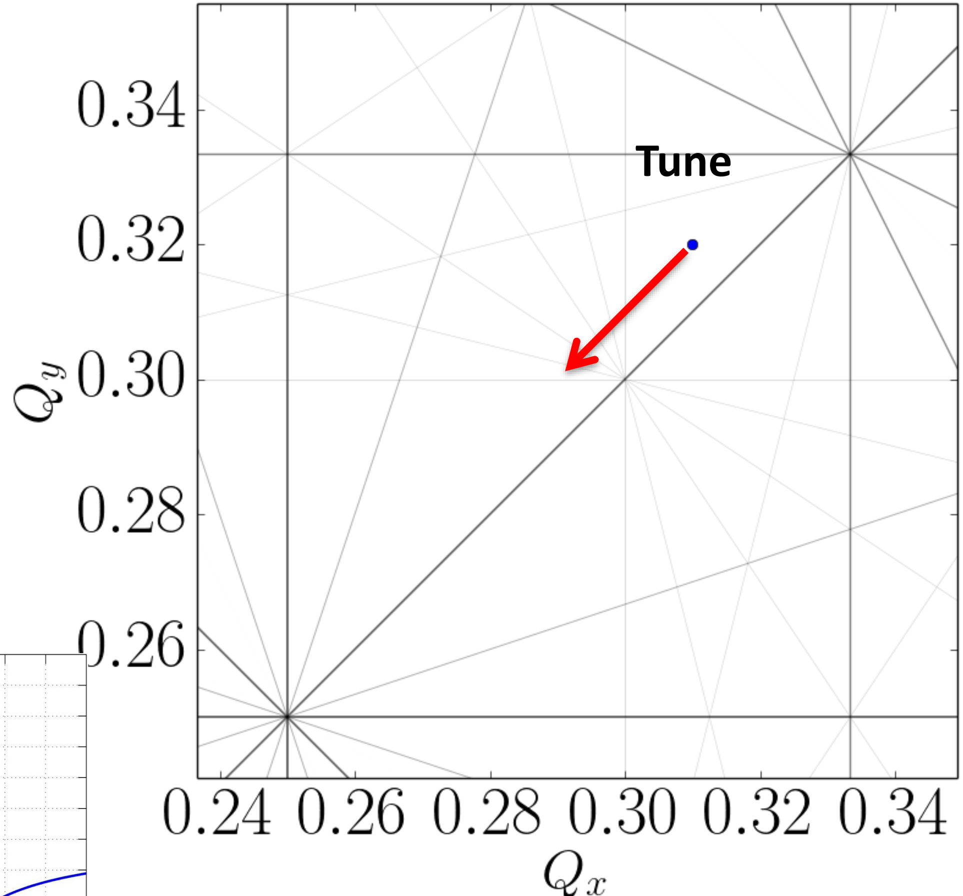


Larger ξ → Strongest variation with Q

Effects of multiple Interaction Points does not add linearly (phase advance between IP..)

Linear head-on Tune shift

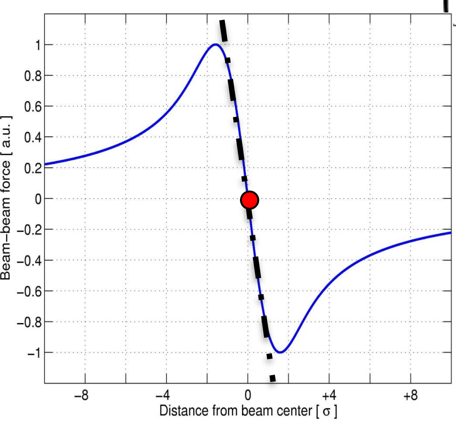
Tune shift in 2 dimensional case equally charged beams and tunes far from integer and half



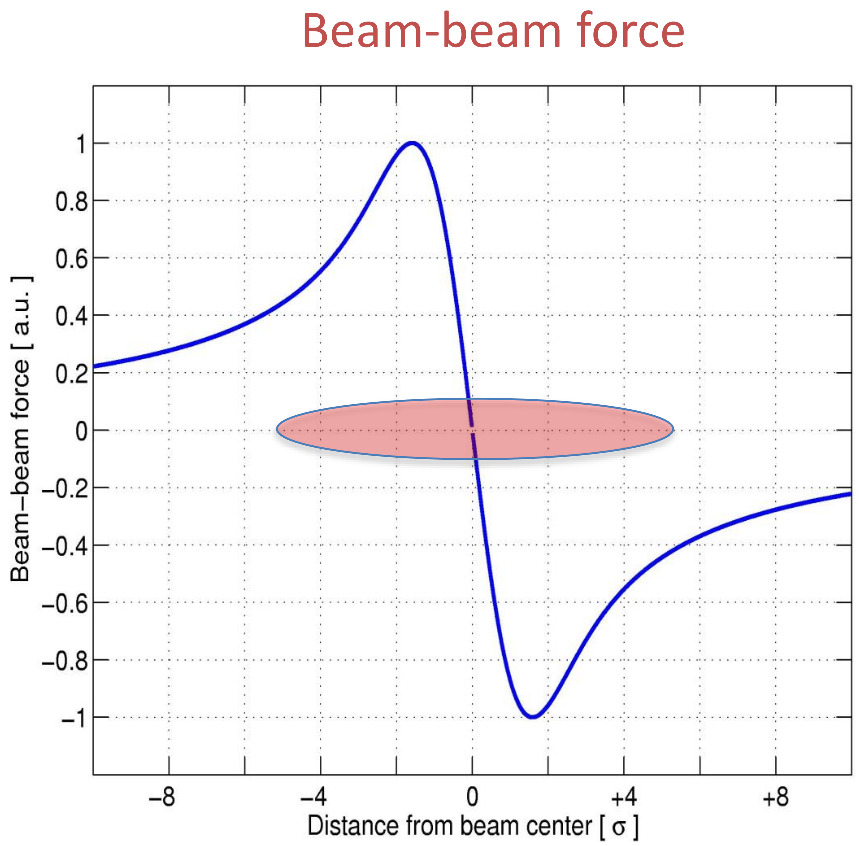
$$\xi_{bb} = 0.02$$

Zero amplitude particle will fill an extra defocusing term

$$\Delta Q \approx \xi_{bb}$$



A beam is a collection of particles

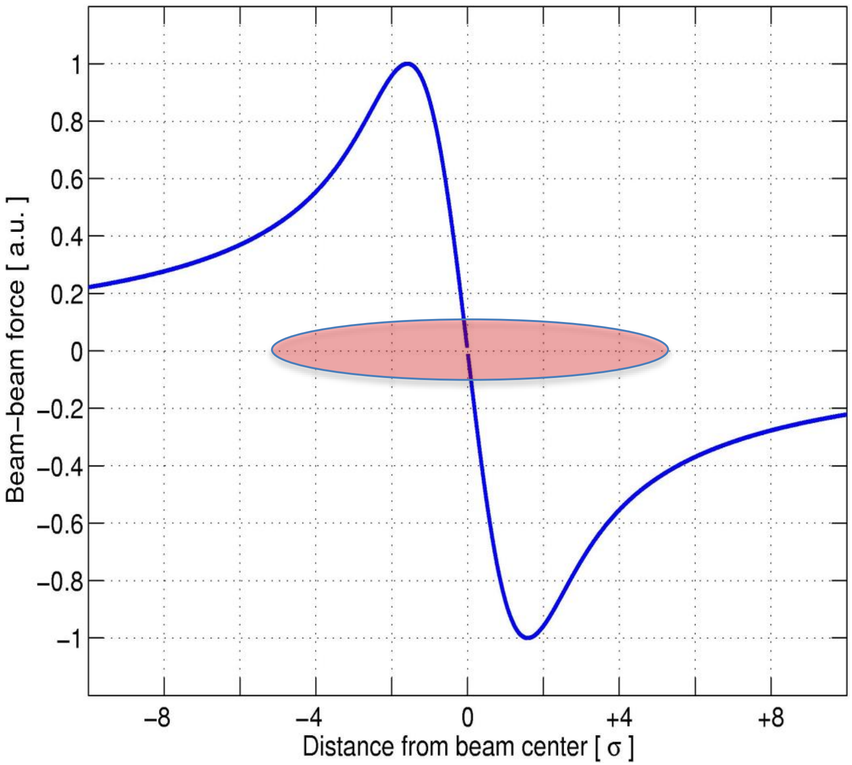


Beam 2 passing in the center of force produce by Beam 1
Particles of Beam 2 will experience different ranges of the beam-beam forces

Tune shift as a function of amplitude
(detuning with amplitude or tune spread)

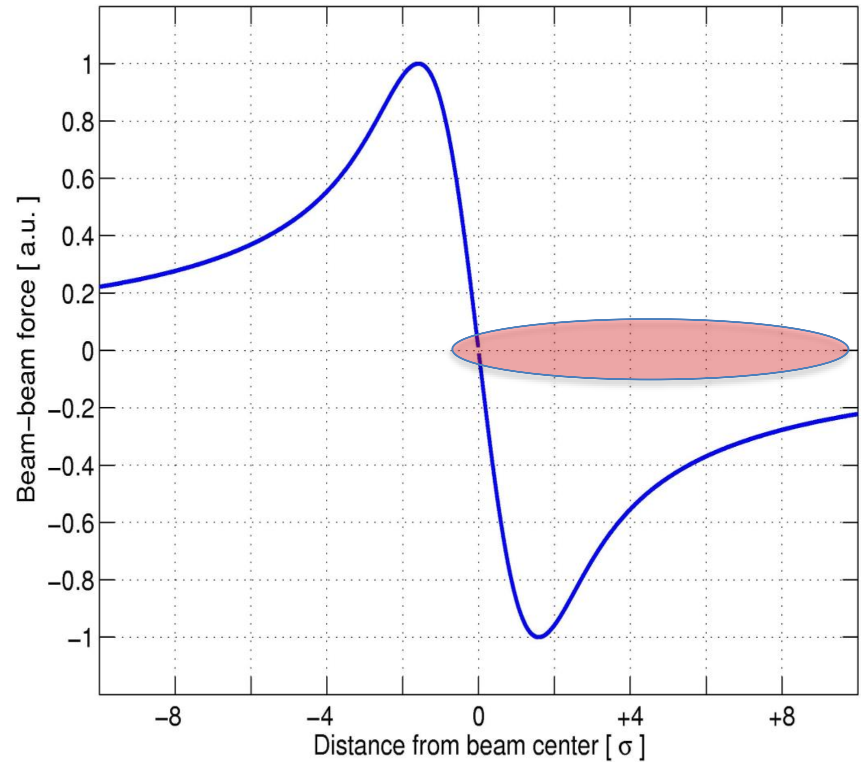
A beam will experience all the force range

Beam-beam force



Second beam passing in the center
HEAD-ON beam-beam interaction

Beam-beam force



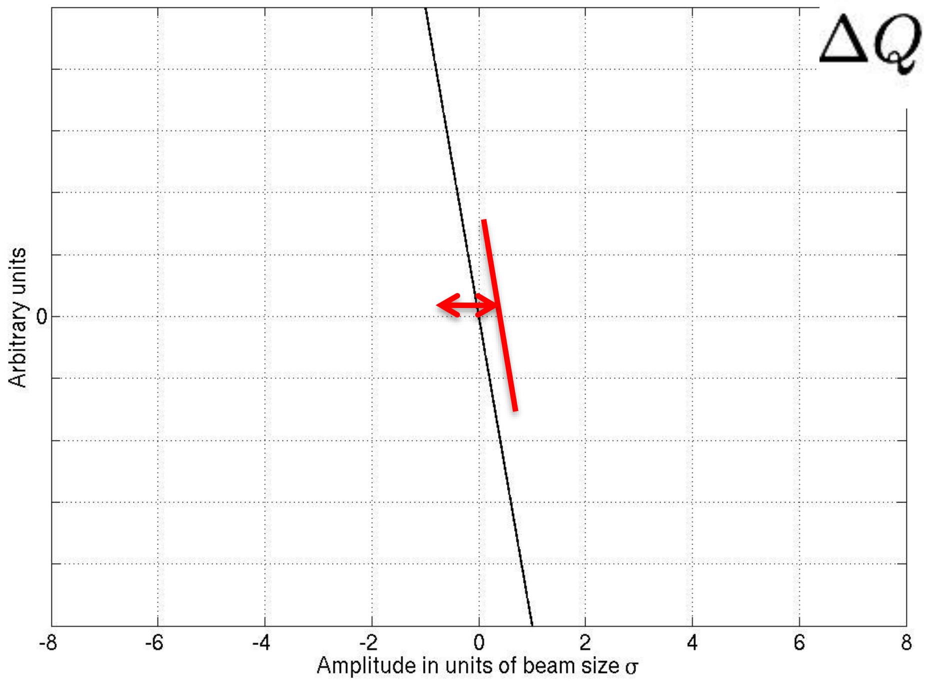
Second beam displaced offset
LONG-RANGE beam-beam interaction

Different particles will see different force

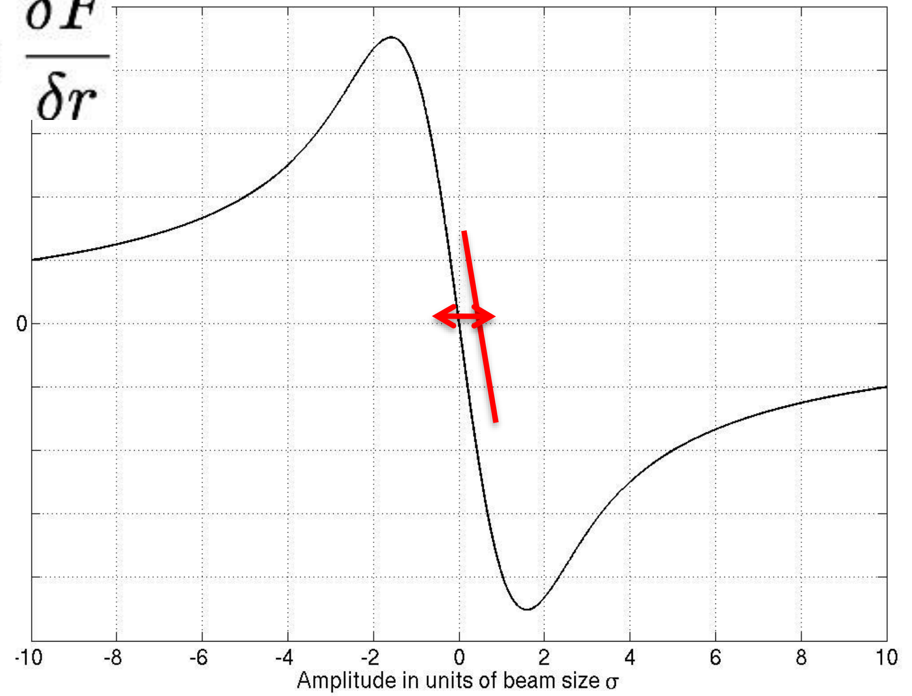
Detuning with Amplitude for head-on

Instantaneous tune shift of test particle when it crosses the other beam is related to the derivative of the force with respect to the amplitude

$$\Delta Q \propto \frac{\delta F}{\delta r}$$



$$\Delta Q_{quad} = const$$



$$\Delta Q_{bb} \approx const$$

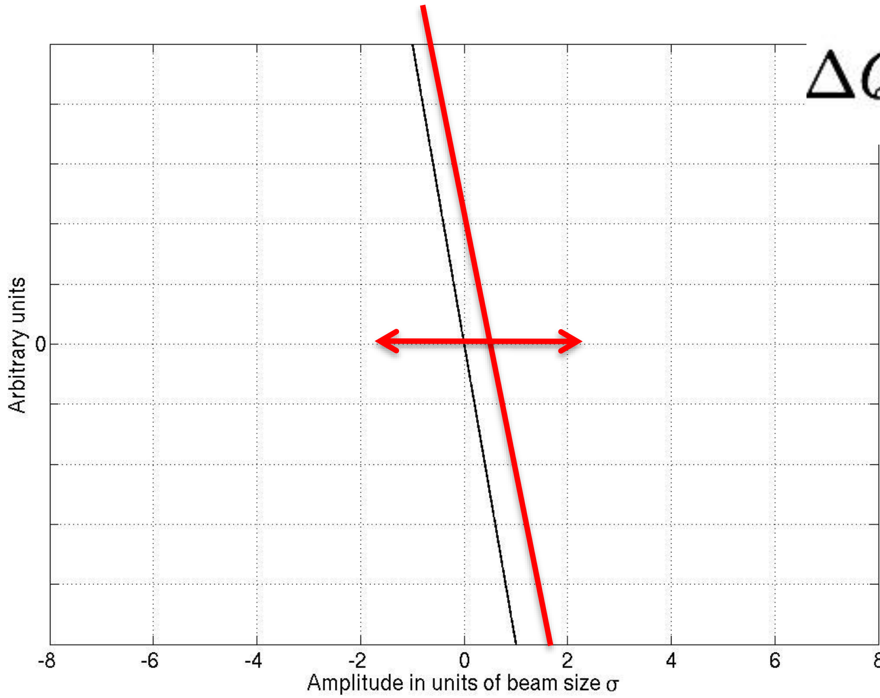
For small amplitude test particle linear tune shift

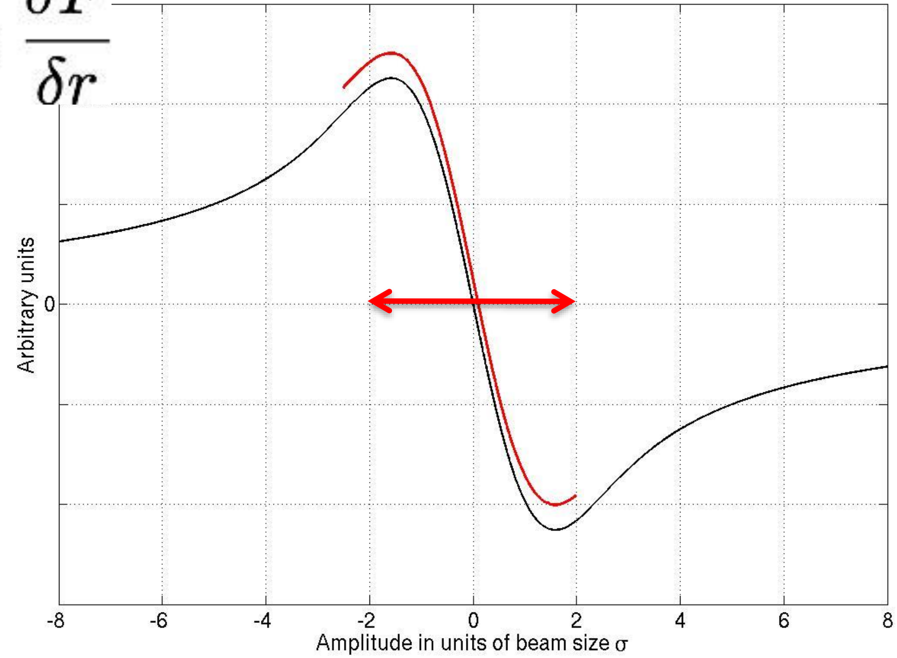
$$\lim_{r \to 0} \Delta Q(r) = -\frac{N r_0 \beta^*}{4\pi\gamma\sigma^2} = \xi$$

Detuning with Amplitude for head-on

Beam with many particles this results in a tune spread

$$\Delta Q \propto \frac{\delta F}{\delta r}$$





$$\Delta Q_{quad} = const \qquad\qquad \Delta Q_{bb} \neq const$$

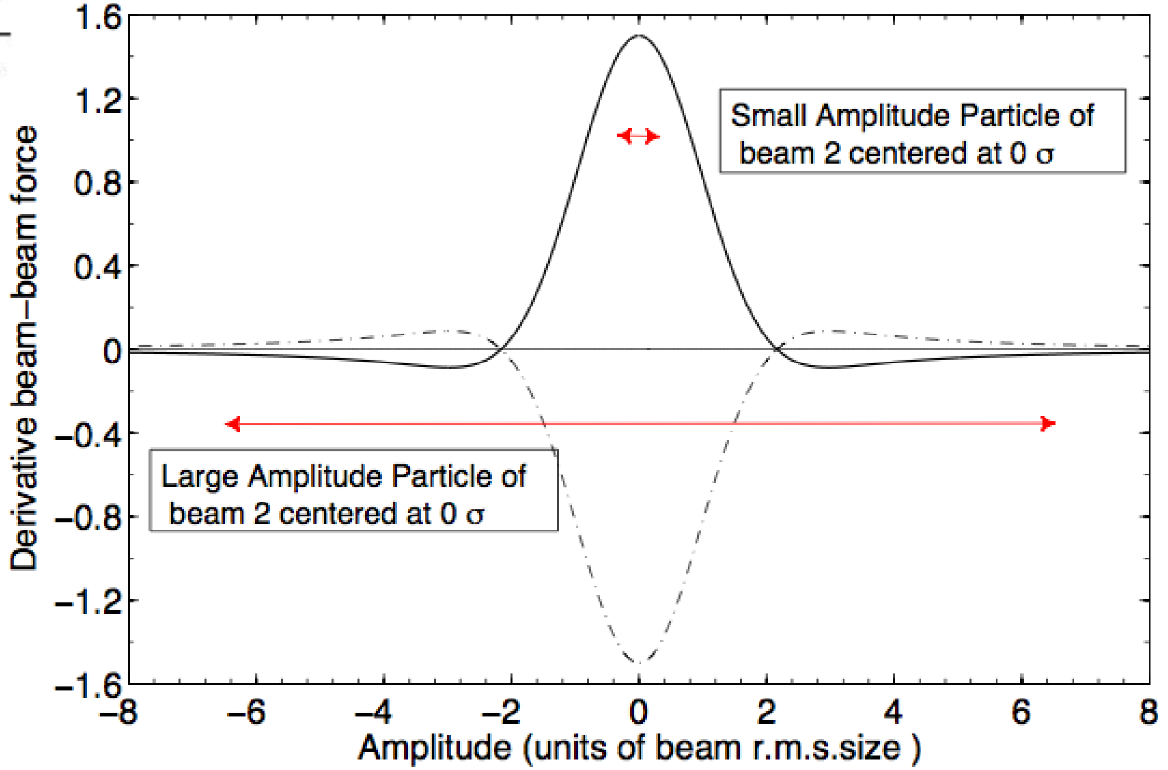
$$\Delta Q(x) = \frac{N r_0 \beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{x}{2})^2} \cdot (\exp -(\frac{x}{2})^2 I_0(\frac{x}{2})^2 - 1)$$

Mathematical derivation in Ref [3] using Hamiltonian formalism and in Ref [4] using Lie Algebra

Head-on detuning with amplitude

1-D plot of detuning with amplitude for opposite and equally charged beams

$$\Delta Q \propto \frac{\delta F}{\delta r}$$

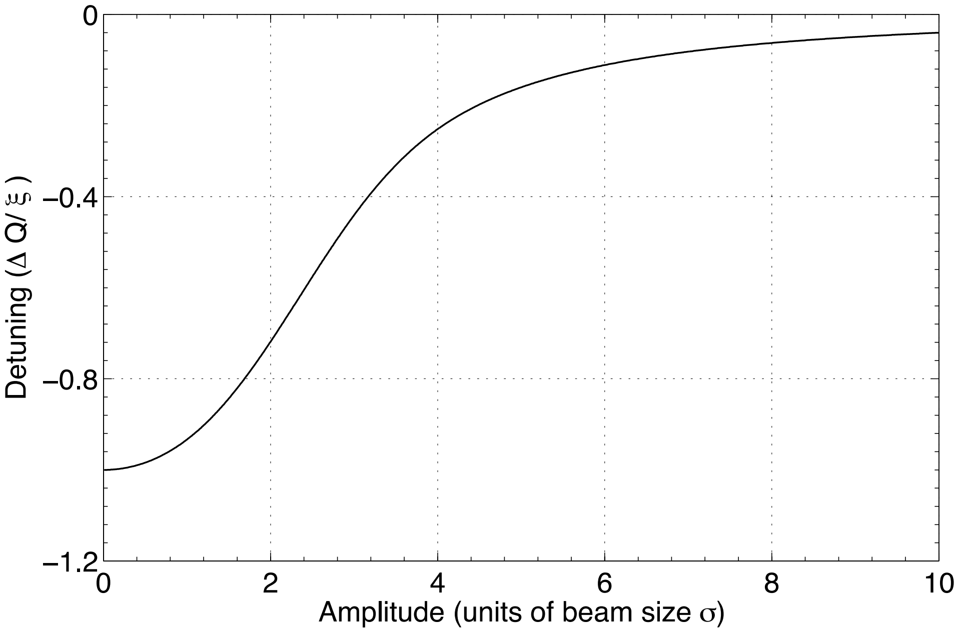


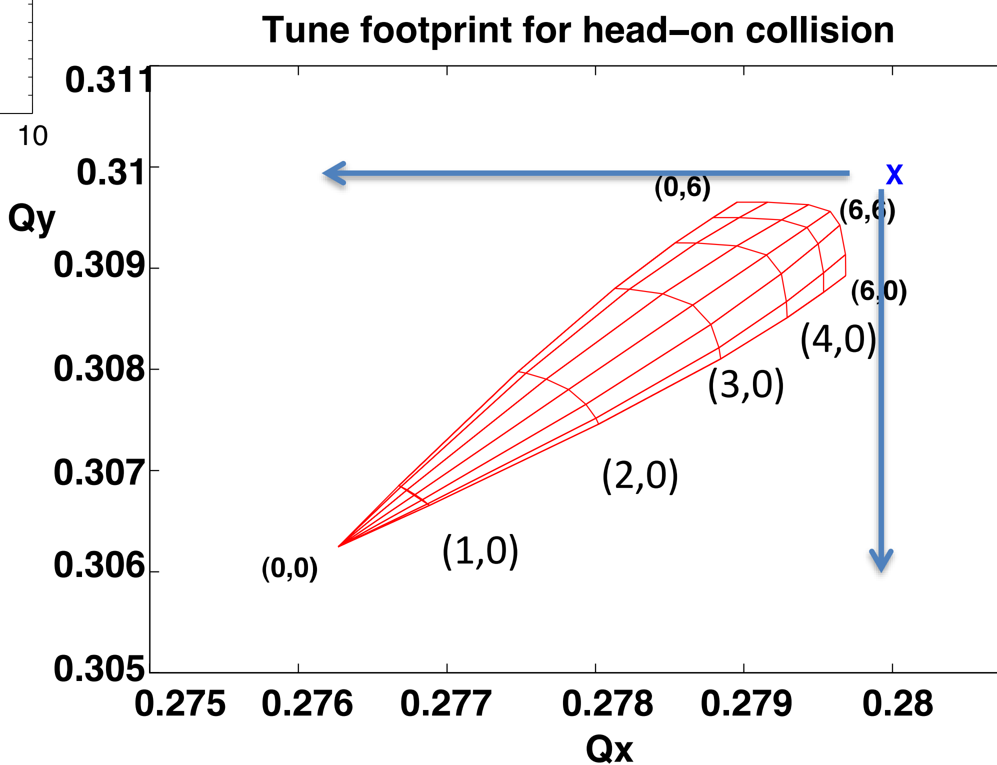
Maximum tune shift for small amplitude particles
Zero tune shift for very large amplitude particles

And in the other plane? THE SAME DERIVATION

Head-on detuning with amplitude and footprints

1-D plot of detuning with amplitude



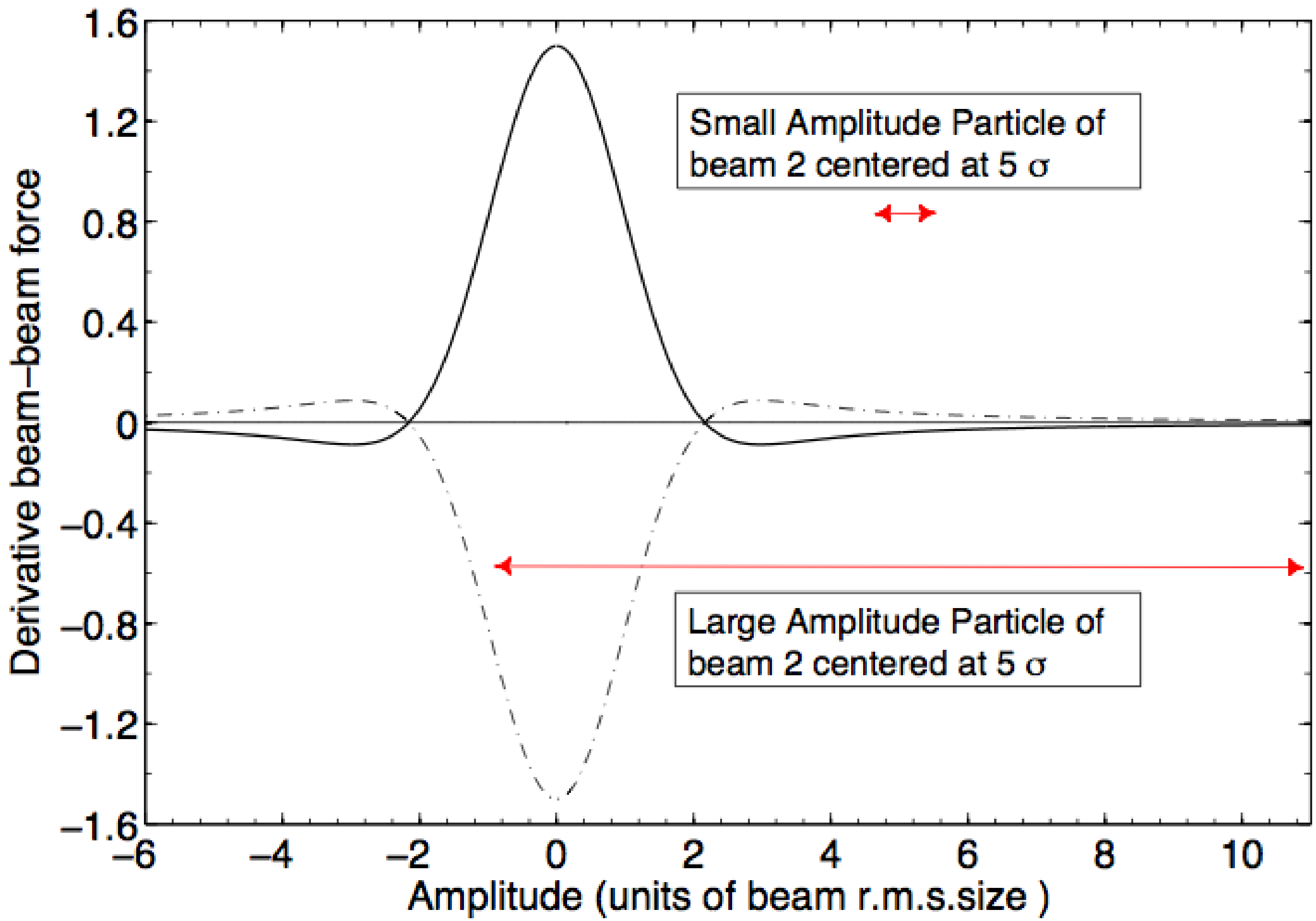


FOOTPRINT

2-D mapping of the detuning with amplitude of particles

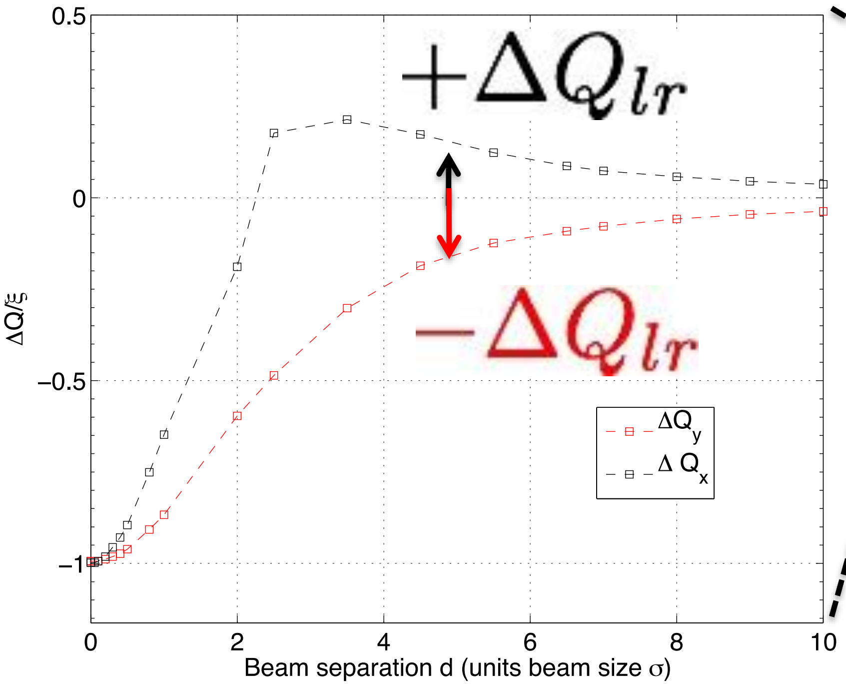
Long Range detuning with amplitude

1-D plot of detuning with amplitude for opposite and equally charged beams



Maximum tune shift for large amplitude particles
Smaller tune shift detuning for zero amplitude particles and opposite sign

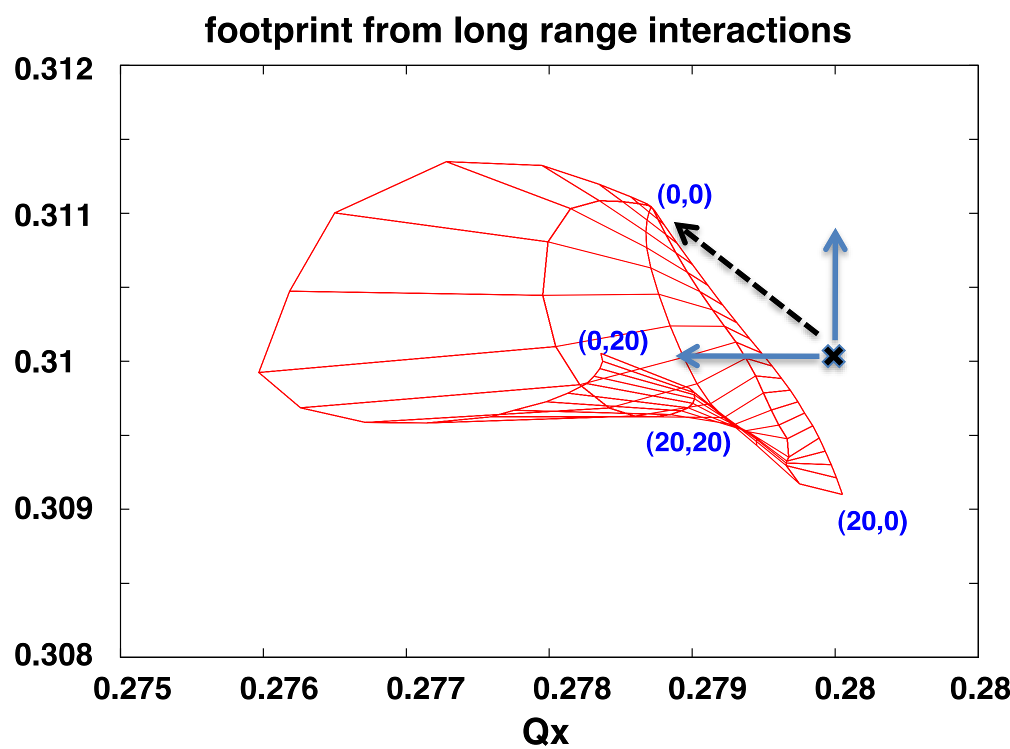
2-D Long Range detuning with amplitude



Tune shift as a function of separation in horizontal plane

In the **horizontal** plane **long range tune shift**

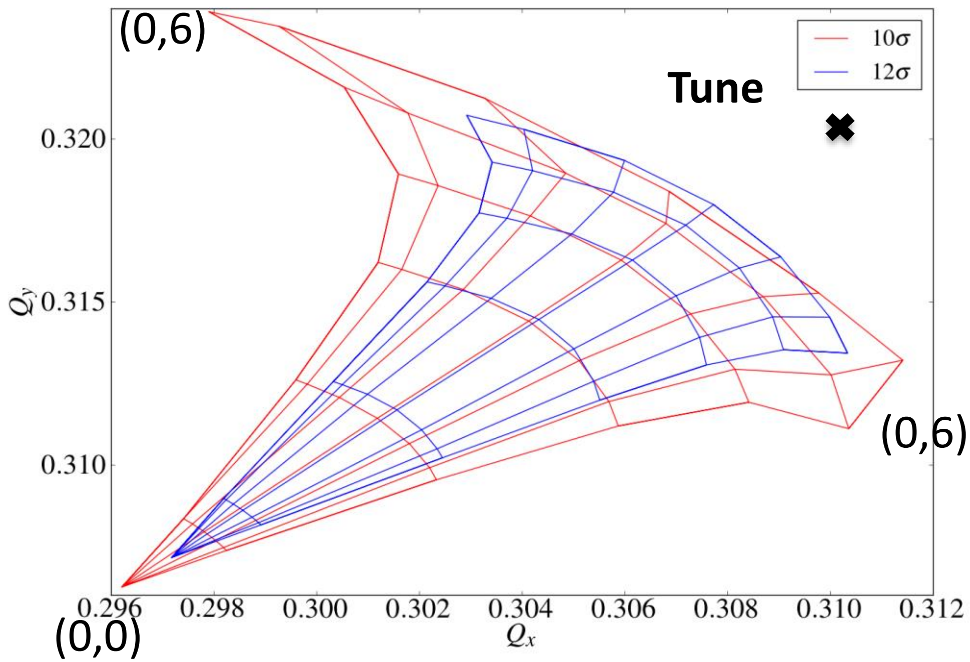
In the **vertical plane opposite sign!**



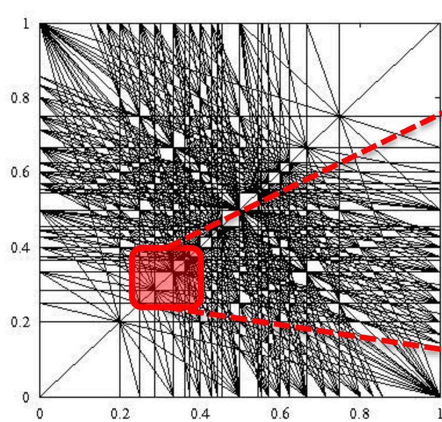
Long range tune shift scaling for distances $d > 6\sigma$

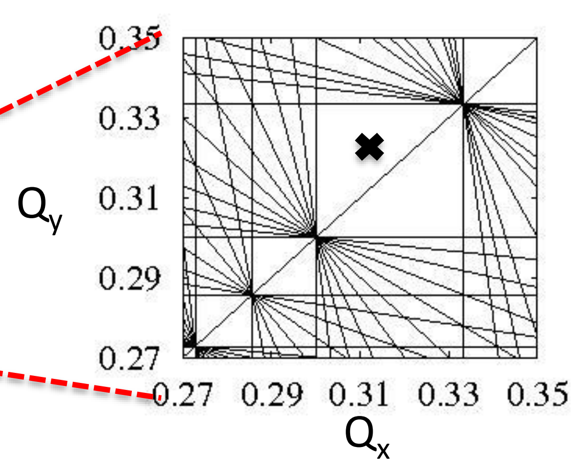
$$\Delta Q_{lr} \propto -\frac{N}{d^2}$$

Beam-beam tune shift and spread



Higher Luminosity → increases this area
We need to keep it small to avoid resonances and preserve the long term stability of particles

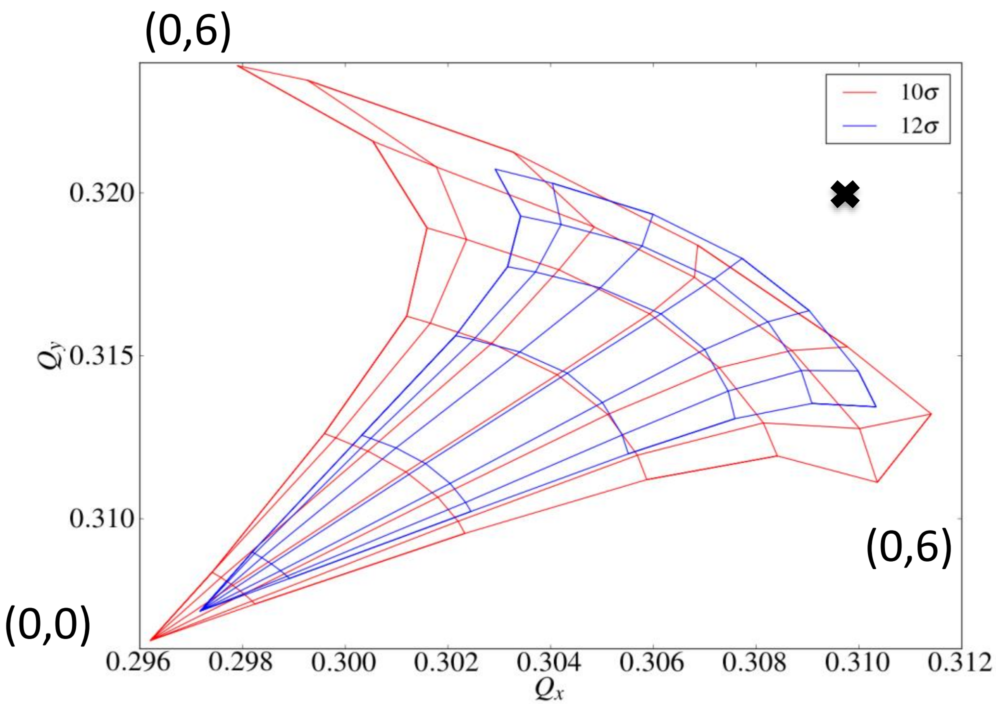




The footprint from beam-beam sits in the tune diagram

Beam-beam tune shift and tune spread

Head-on and Long range interactions detuning with amplitude



Footprints depend on:

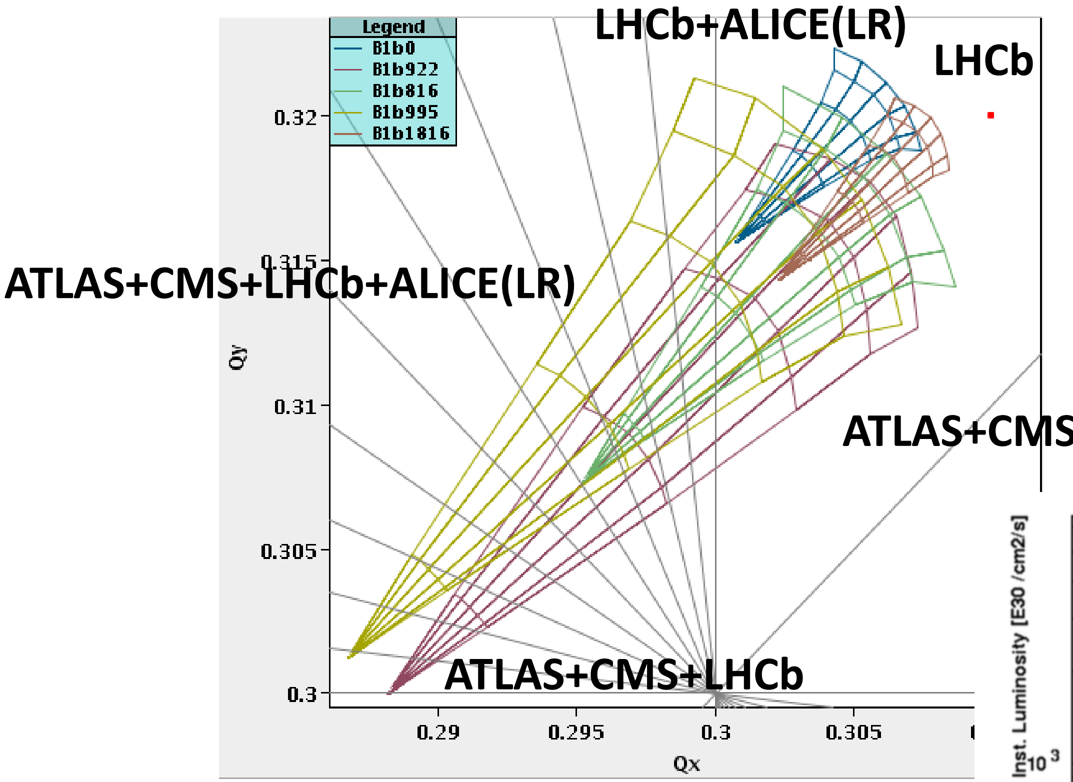
- number of interactions (124 per turn)
- Type (Head-on and long-range)
- Separation
- Plane of interaction

Very complicated depending on collision scheme

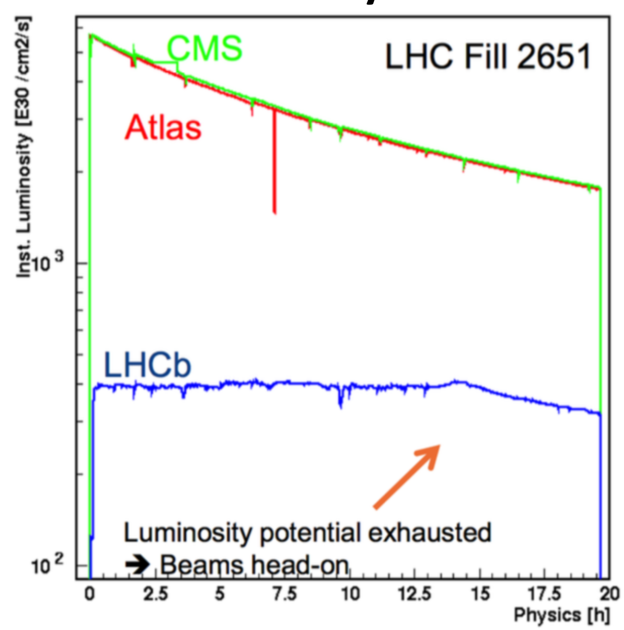
Strongest non-linearity in a collider

LHC Footprints and multiple experiments

LHC 2012



Luminosity Evolution



…operationally it is even more complicated!

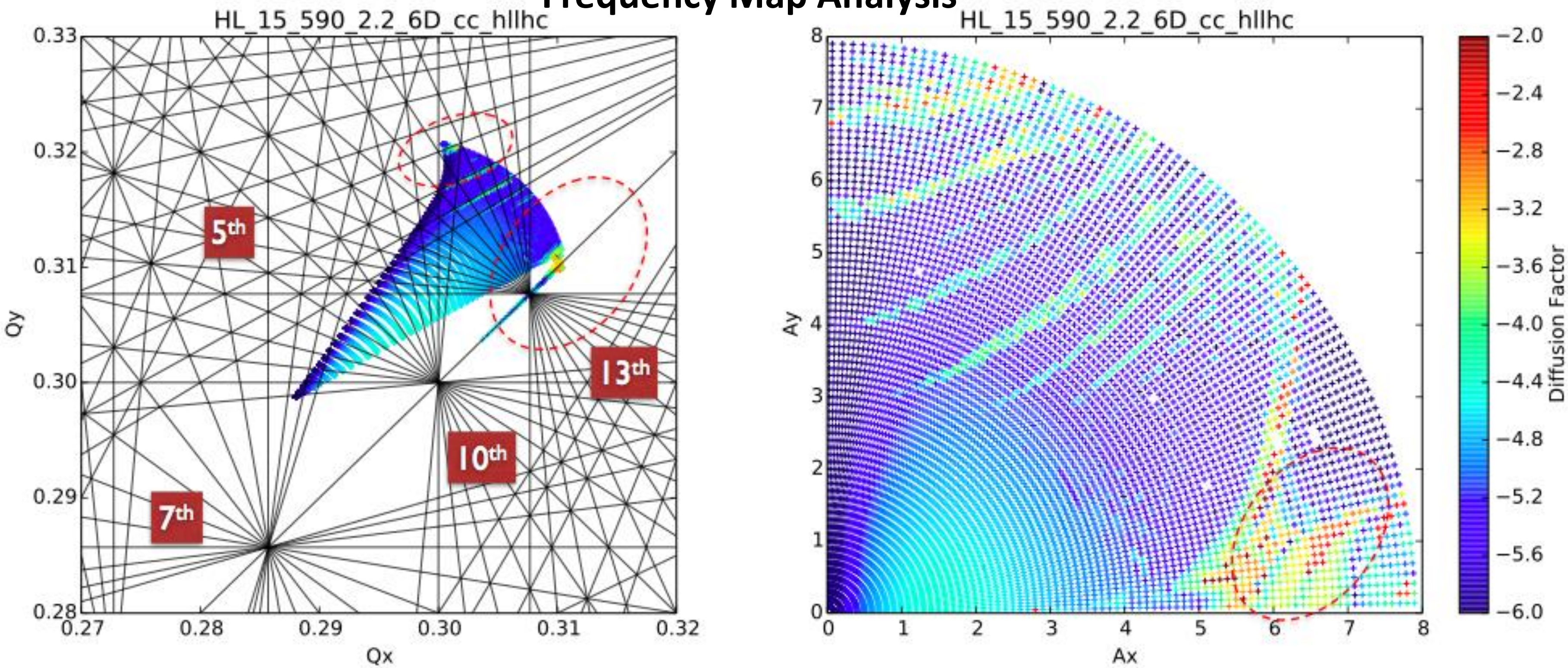
…different intensities, emittances…

Dynamical Aperture and Particle Losses

Dynamic Aperture: area in amplitude space with stable motion

Stable area of particles depends on beam intensity and crossing angle

Frequency Map Analysis

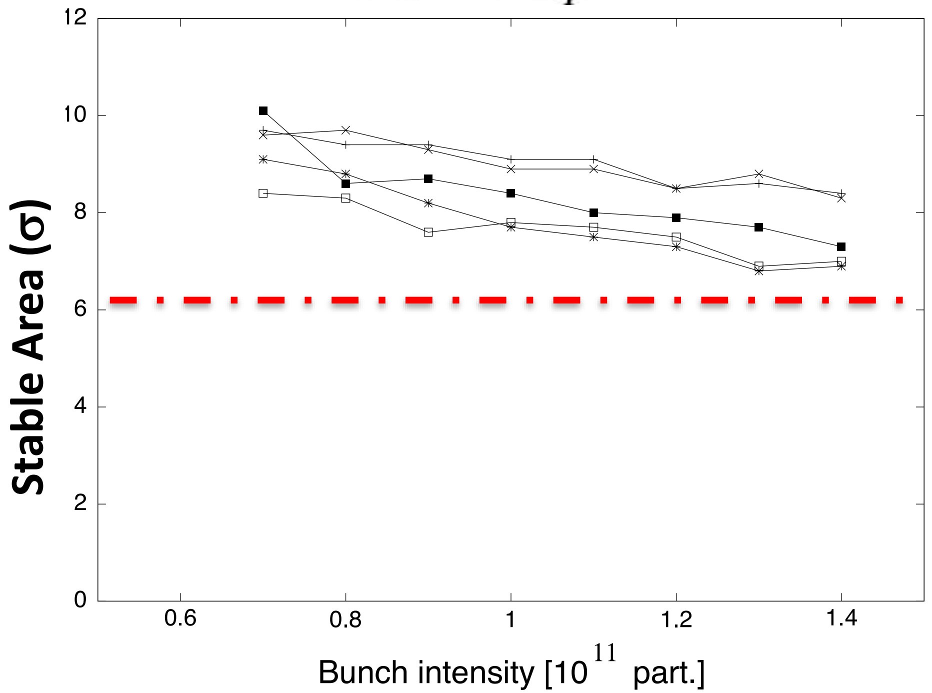


Stable area depends on beam-beam interactions therefore the choice of running parameters (crossing angles, β^*, intensity) is the result of careful study of different effects!

Dynamical Aperture and Particle Losses

Beam-beam linear dependency with Intensity

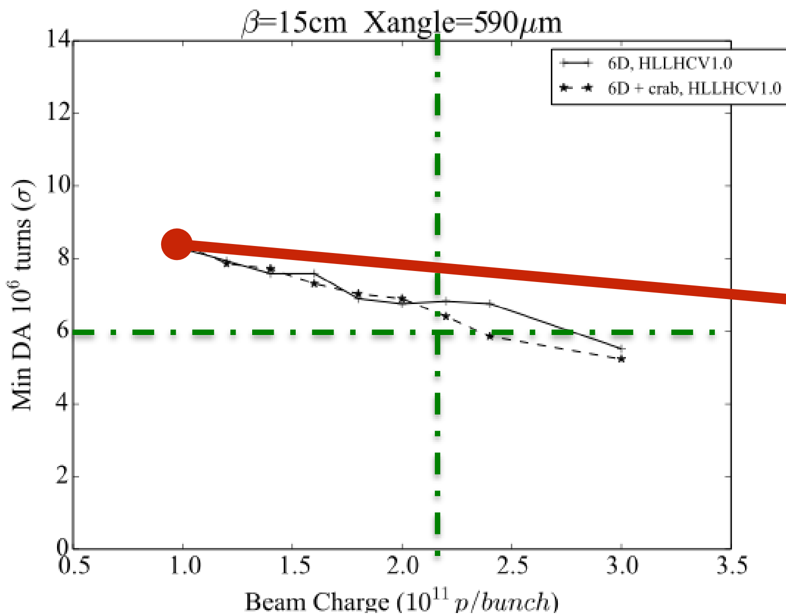
$$F_{bb} \propto N_p$$

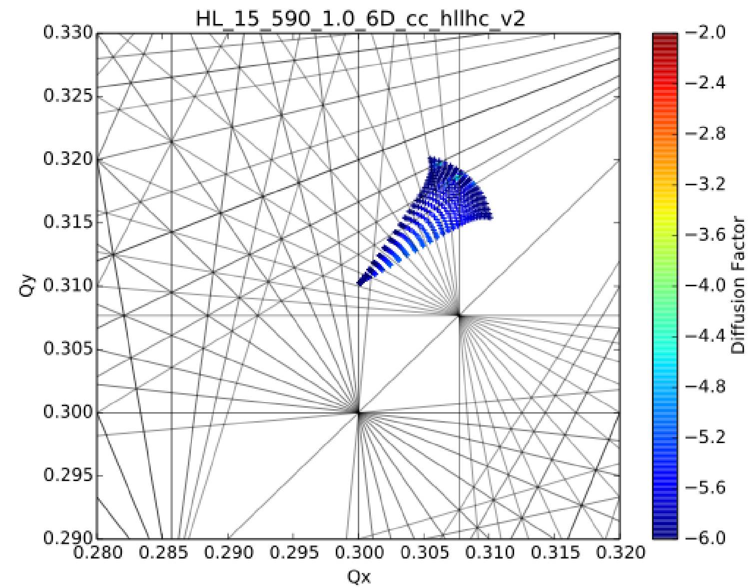


Our goal: keep dynamical aperture above 6 σ → all particles up to 6 σ amplitude not lost over long tracking time (10^6 turns in simulation) equivalent to 1 minute of collider

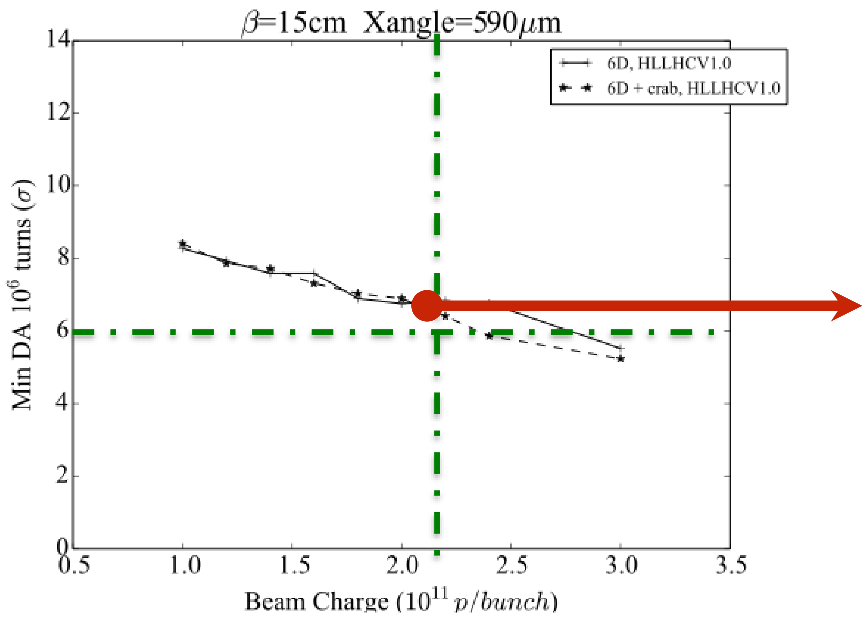
Example collider collision time : 24 hours

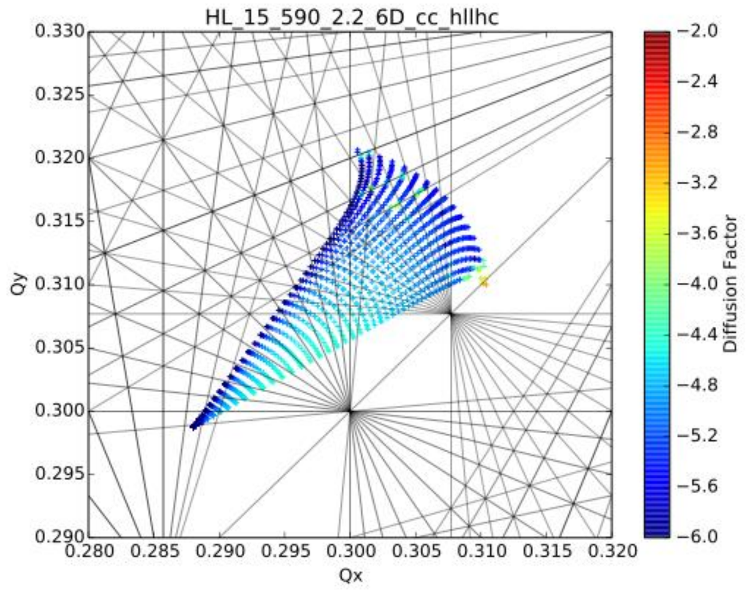
Round optics15 cm, 590μrad: intensity scan



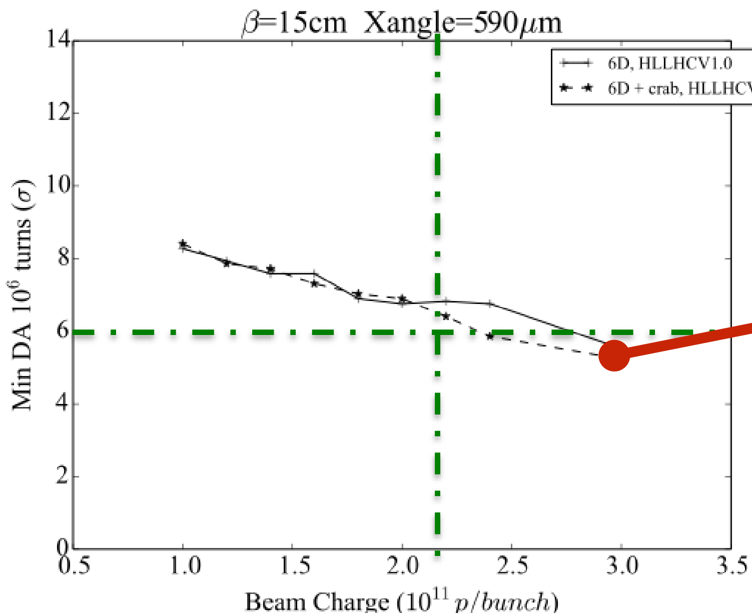


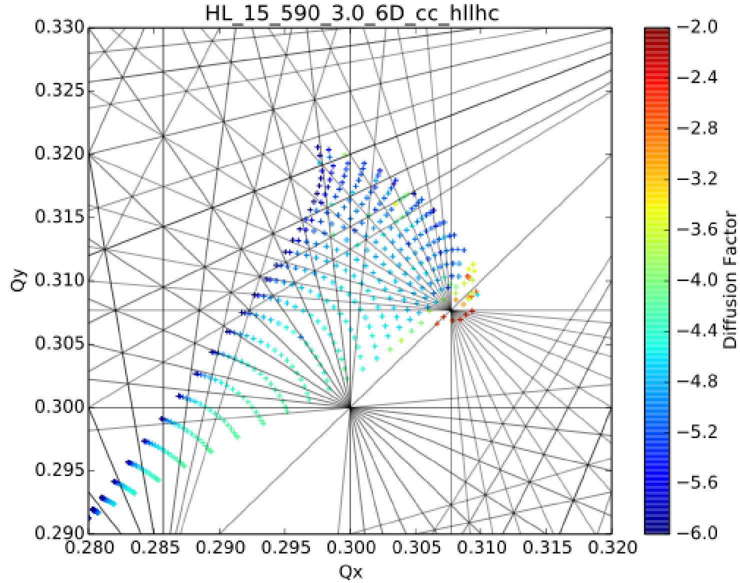
Round optics15 cm, 590µrad: intensity scan





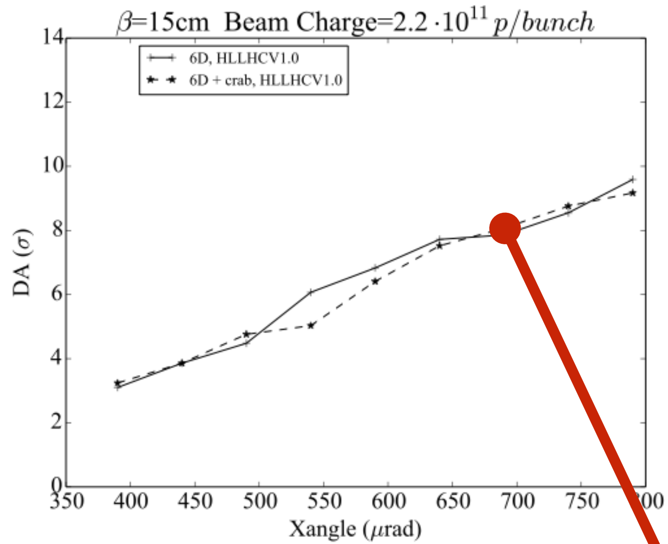
Round optics15 cm, 590μrad: intensity scan



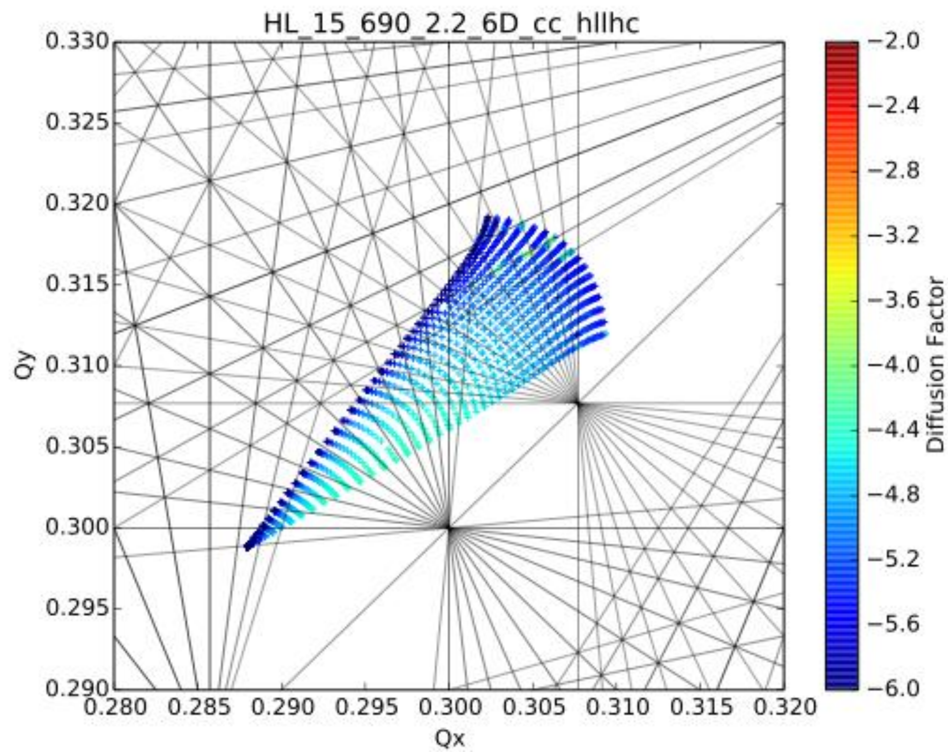


AT high intensity the beam-beam force gets too strong and makes particles unstable and eventually are lost

Round 15cm, 2.2E11, 690µrad

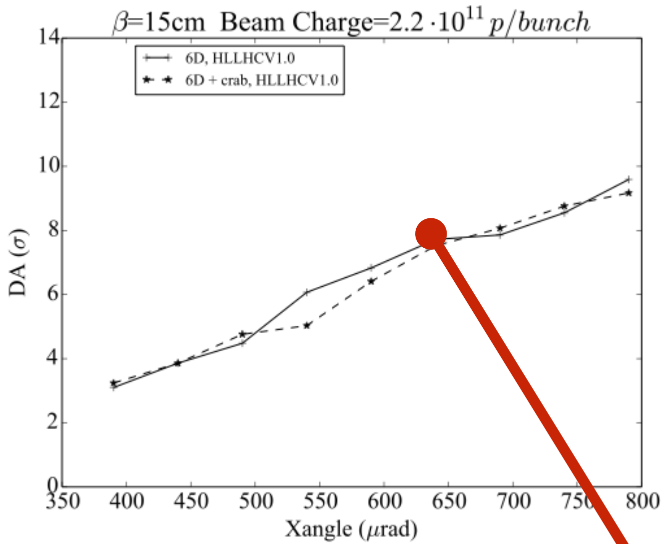


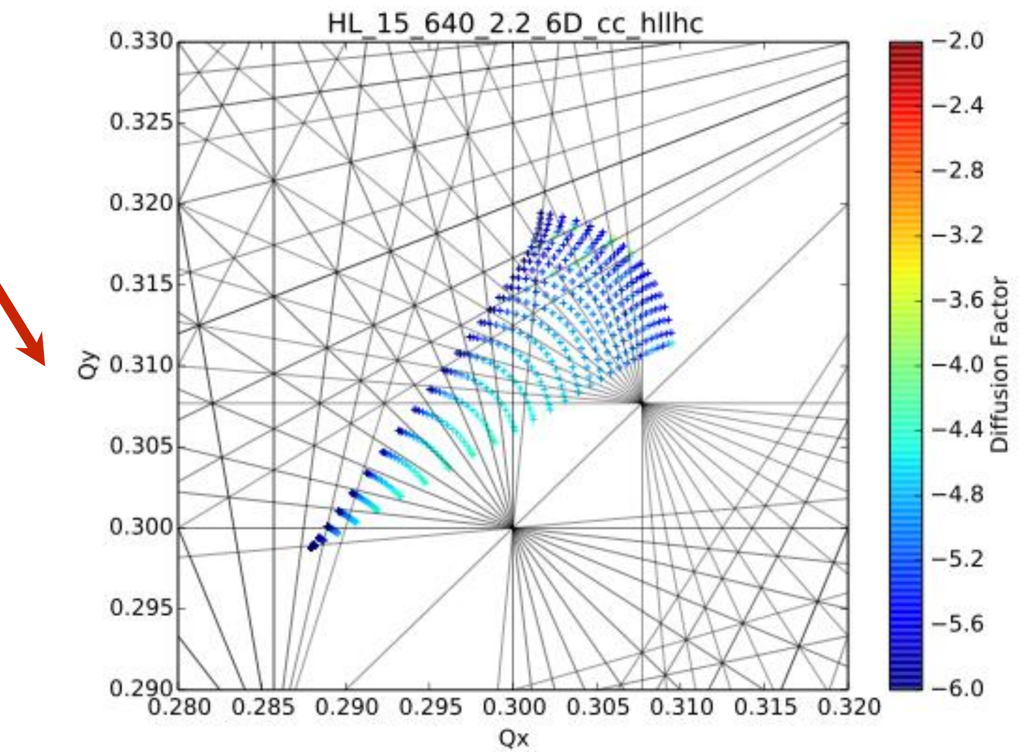
$$d_{lr} \propto \sqrt{\frac{\beta^* \alpha^2 \gamma}{\epsilon_n}}$$



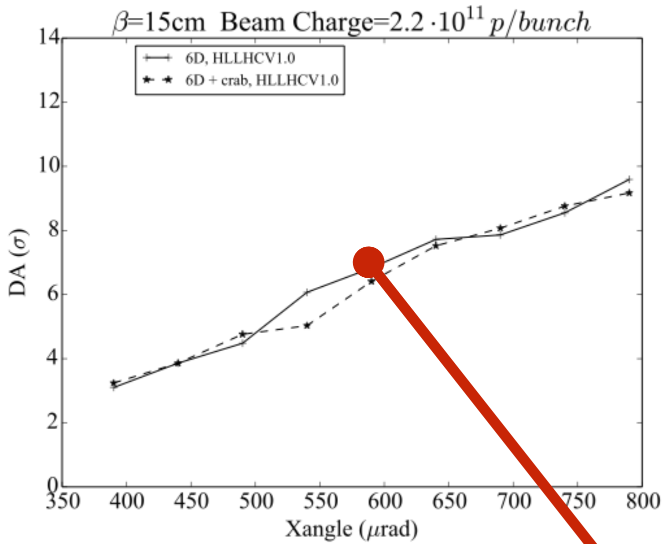
Smaller beam-beam separation at parasitic long-range encounters stronger non linearities → smaller dynamical aperture

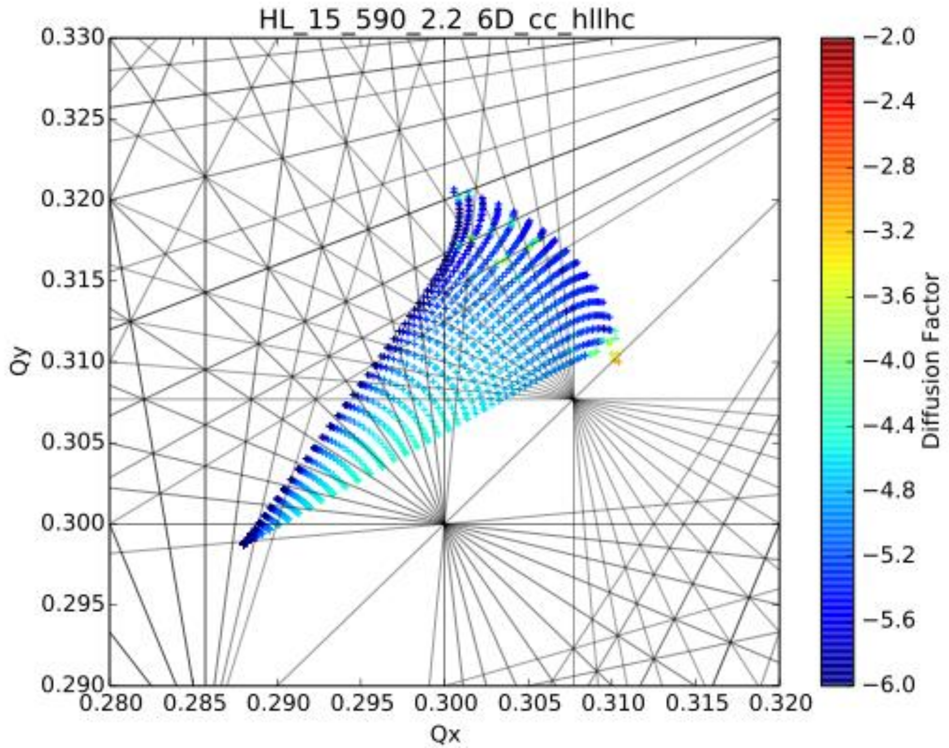
Round 15cm, 2.2E11, 650μrad



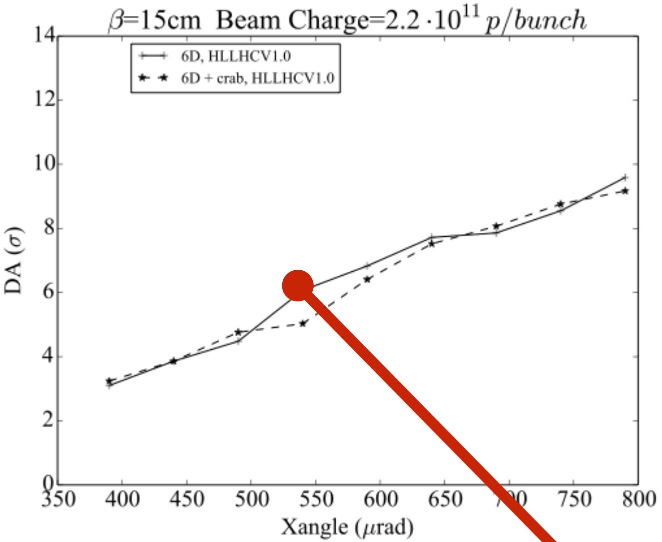


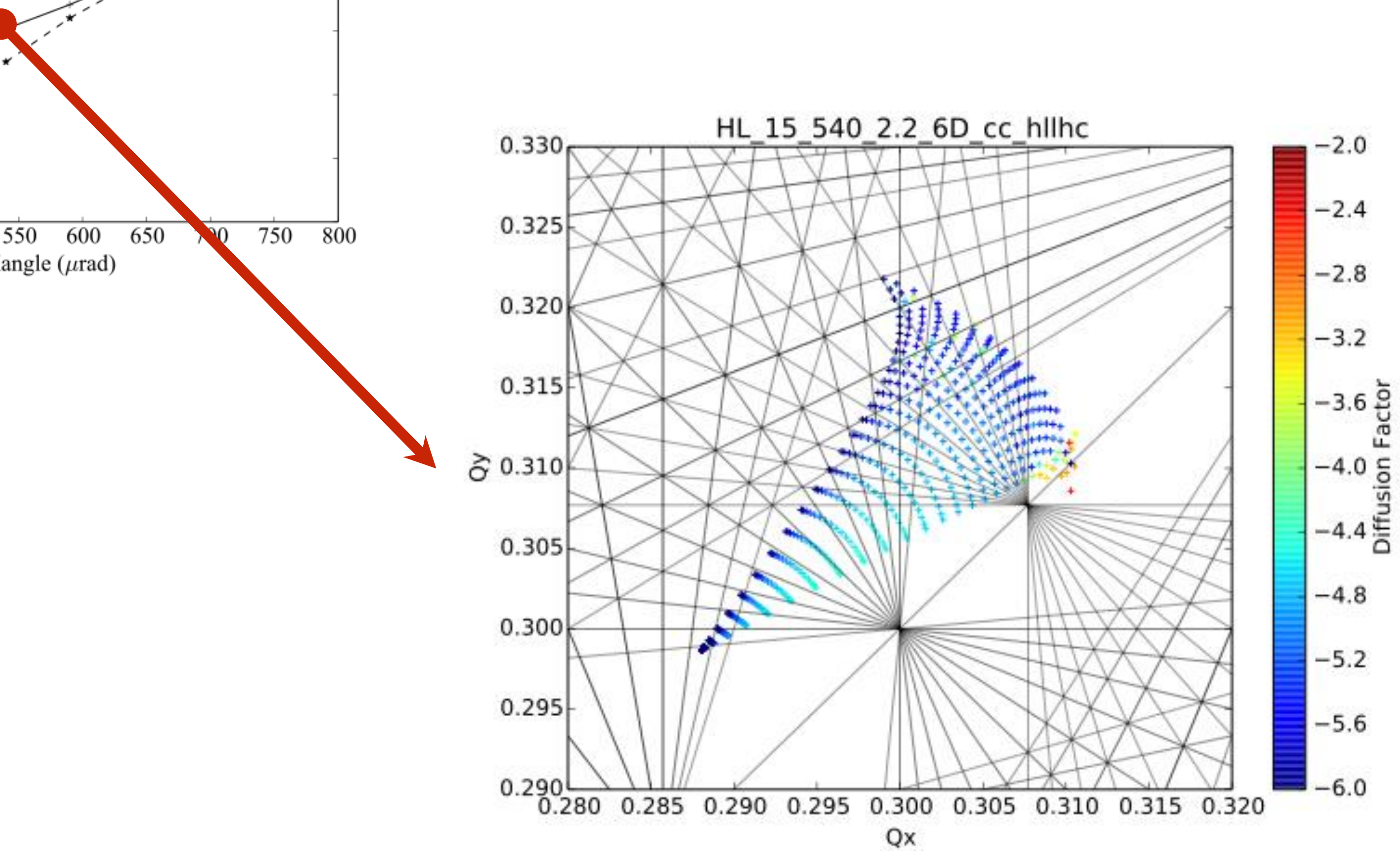
Round 15cm, 2.2E11, 590μrad



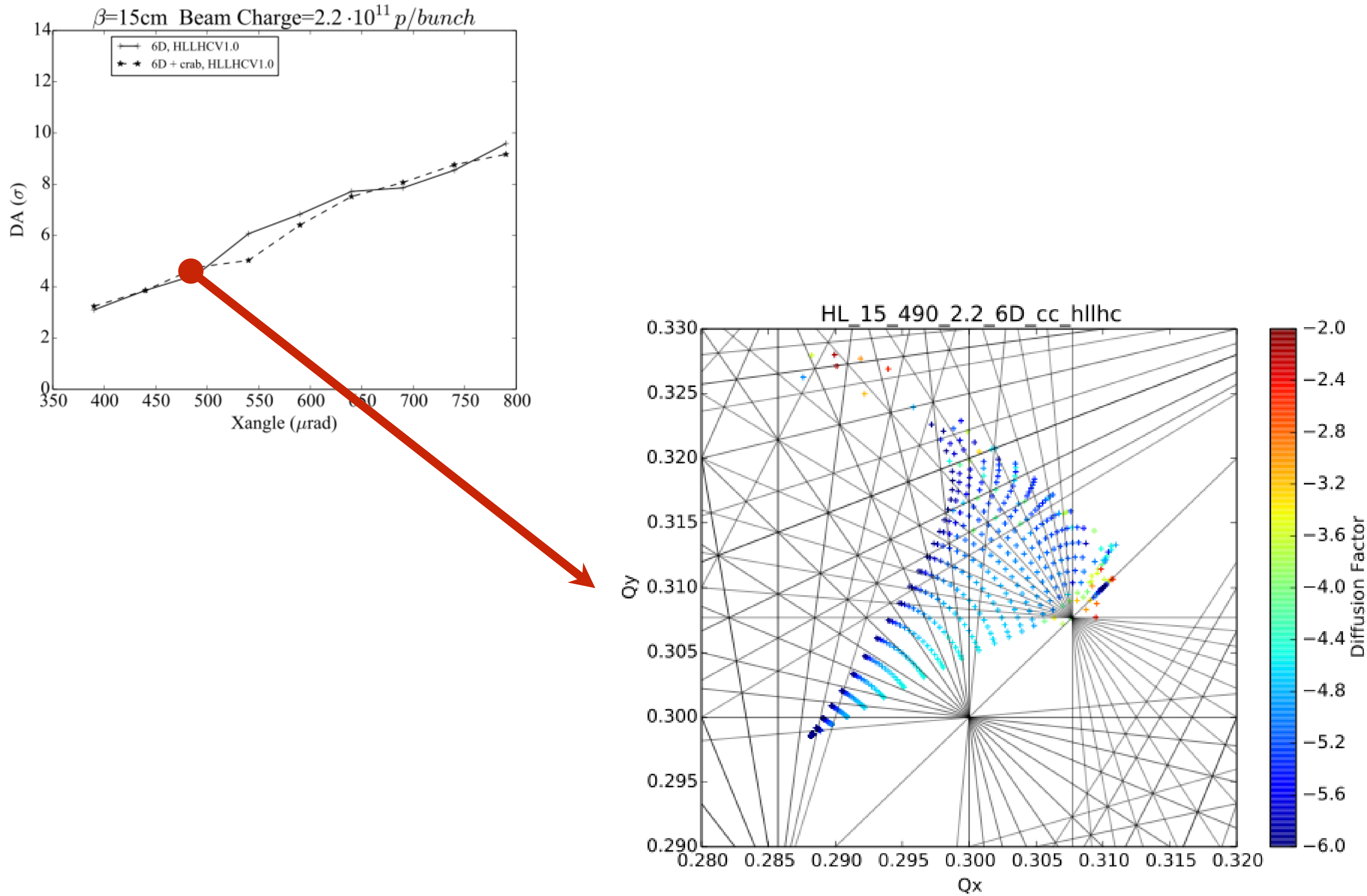


Round 15cm, 2.2E11, 540μrad

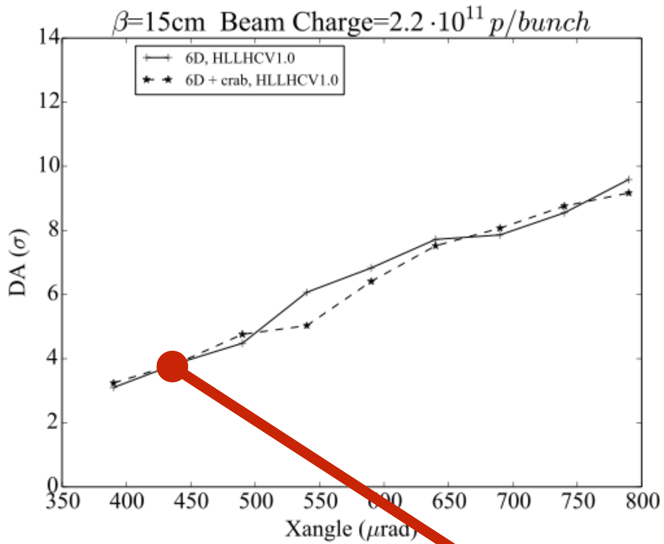


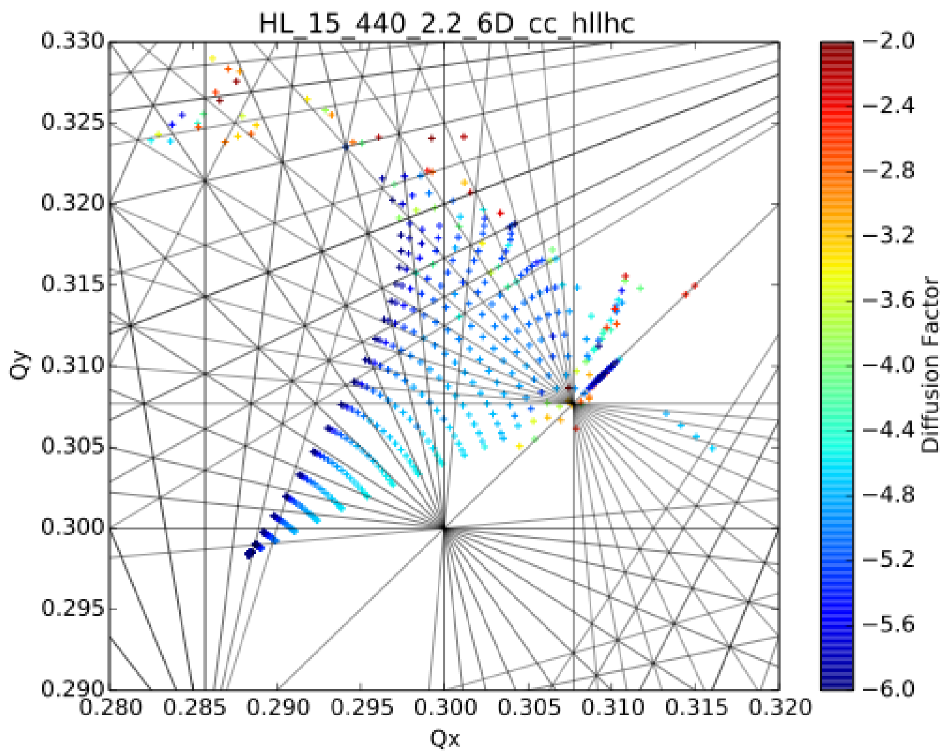


Round 15cm, 2.2E11, 490μrad

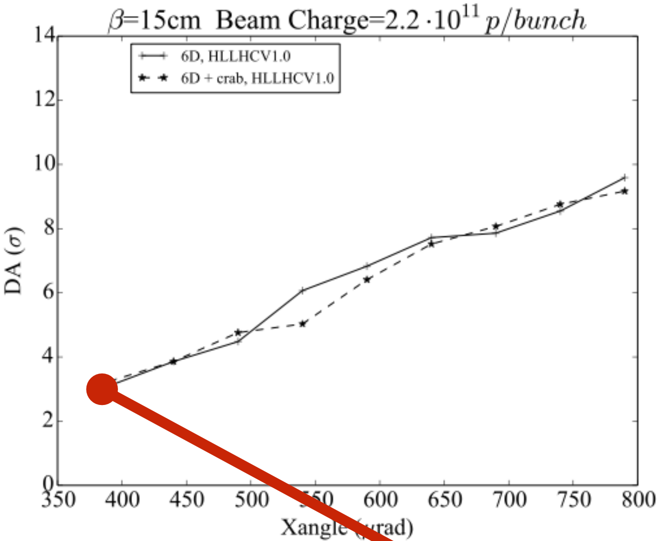


Round 15cm, 2.2E11, 440μrad

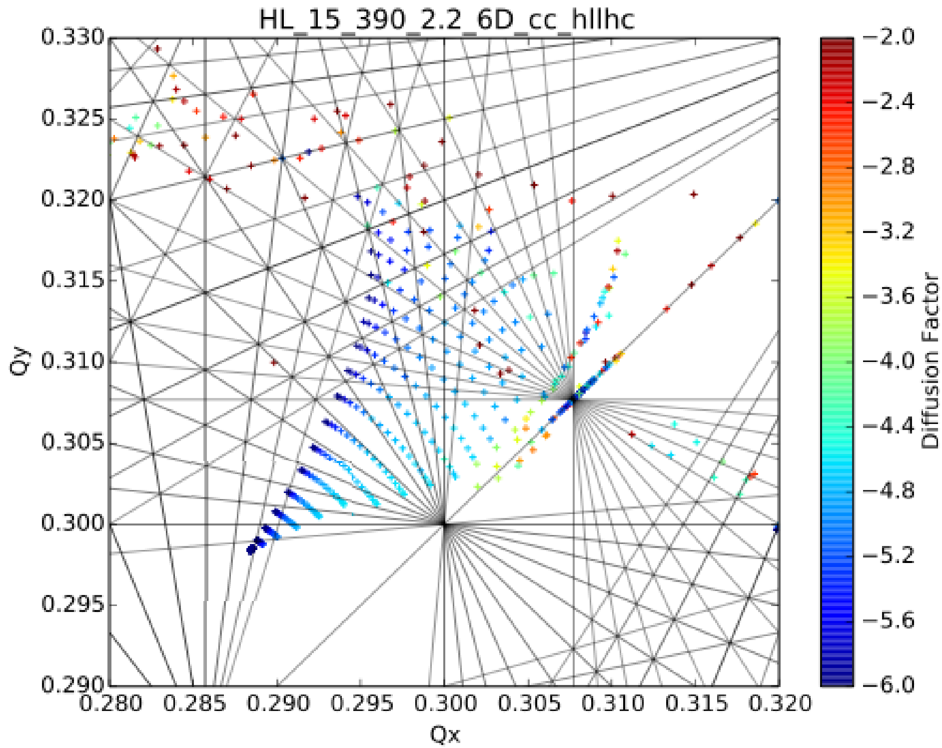




Round 15cm, 2.2E11, 390μrad



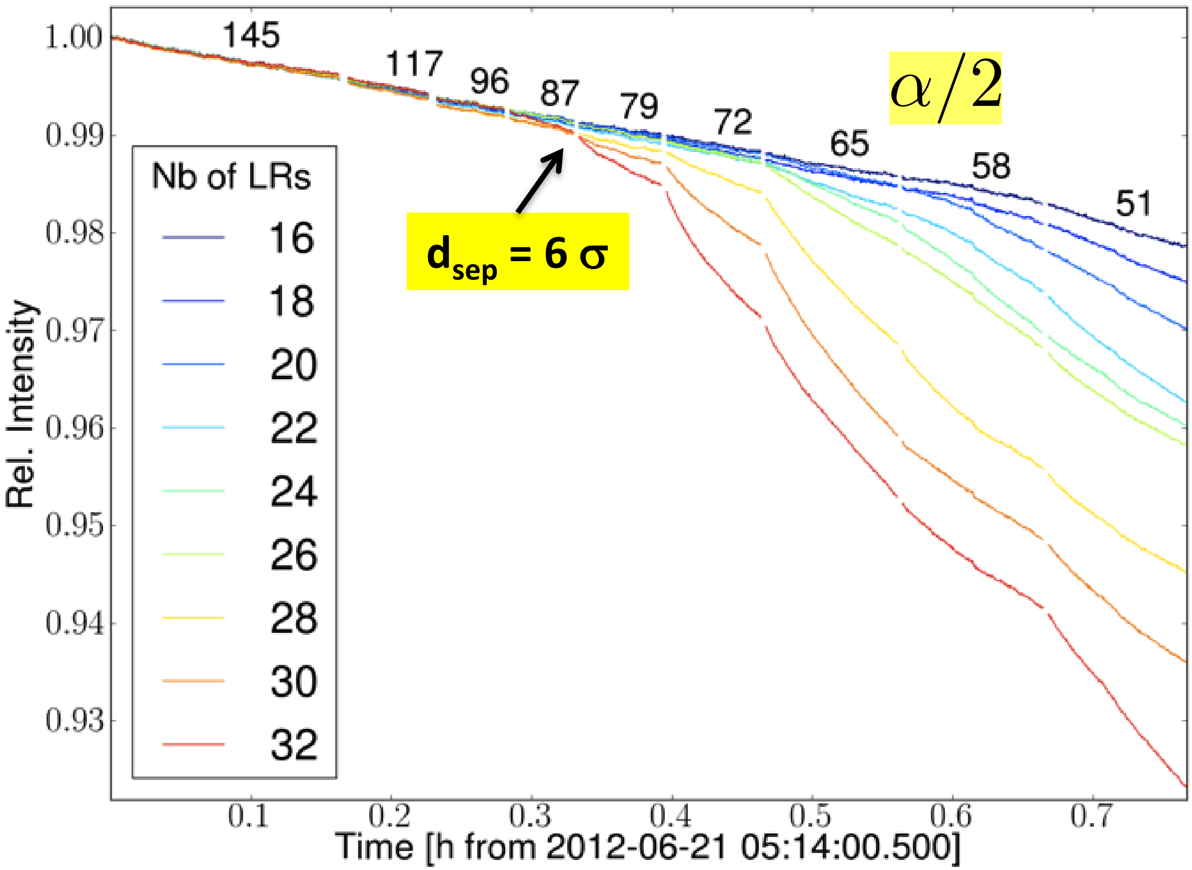
Crossing angle changes the separation and the strength of BB-LR that strongly affects the tails. 0σ particle are almost not affected.



At small separation particles gets unstable and eventually lost

How does it look like in the LHC?

Relative intensity decay 2012 experiment



Beam-Beam separation at first LR

$$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$$

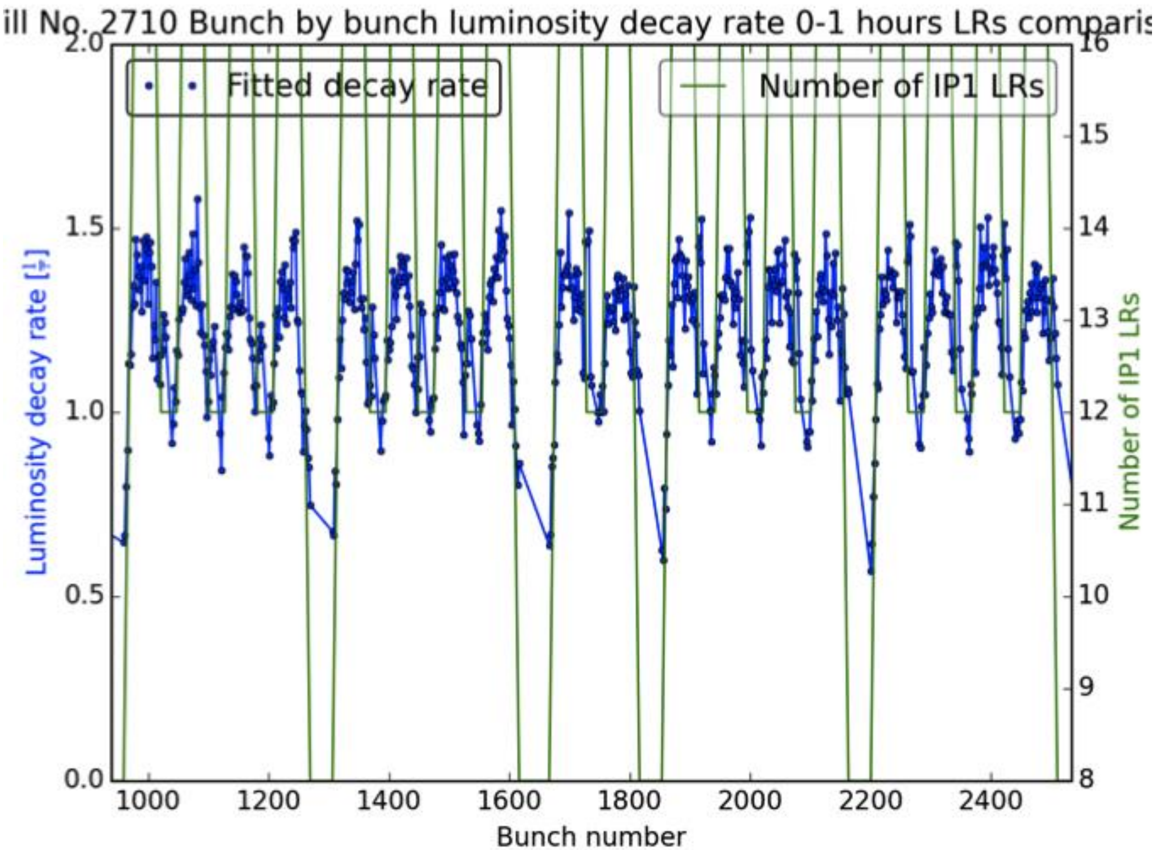
Small crossing angle = small separation

If separation of long range too small particles become unstable and are lost proportionally to the number of long range encounters

Particle losses follow number of Long range interactions

Do we see the particle losses?

Regular Physics Fill of 2012 RUN LHC



Beam-Beam separation at first LR

$$d_{sep} = \alpha \cdot \sqrt{\frac{\gamma \cdot \beta^*}{\epsilon}}$$

Small crossing angle = small separation

Luminosity decays following the long range numbers… higher number of long range interactions larger losses

Particle losses follow number of Long range interactions
Machine protection implication and beam lifetimes gets worse…

Best peformance of collider always a trade off between beam-beam and luminosity

Long-range Beam-Beam effects: orbit

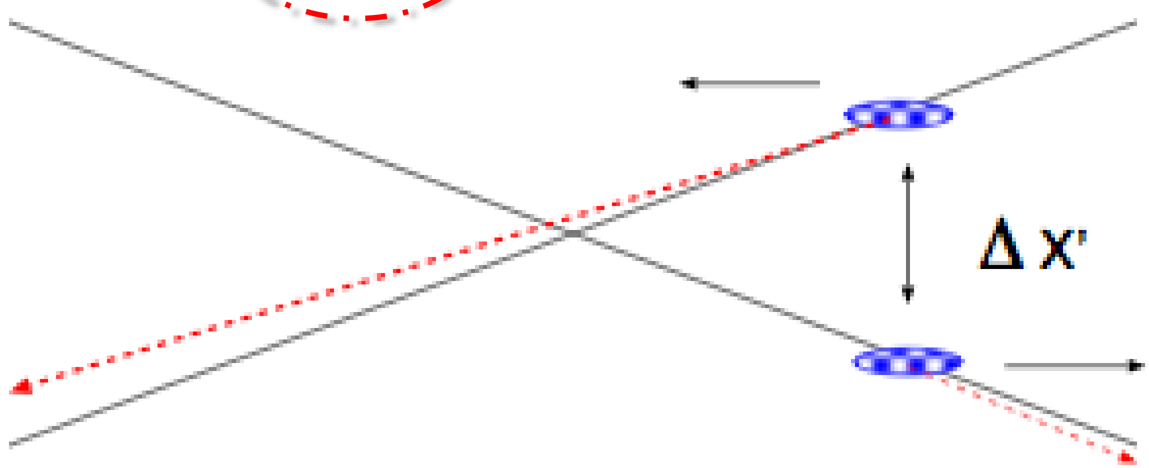
Long Range Beam-beam interactions lead to several effects…

Long range angular kick $$\Delta x'(x+d,y,r) = -\frac{2Nr_0}{\gamma}\frac{(x+d)}{r^2}[1-\exp(-\frac{r^2}{2\sigma^2})$$

For well separated beams $$d \gg \sigma$$

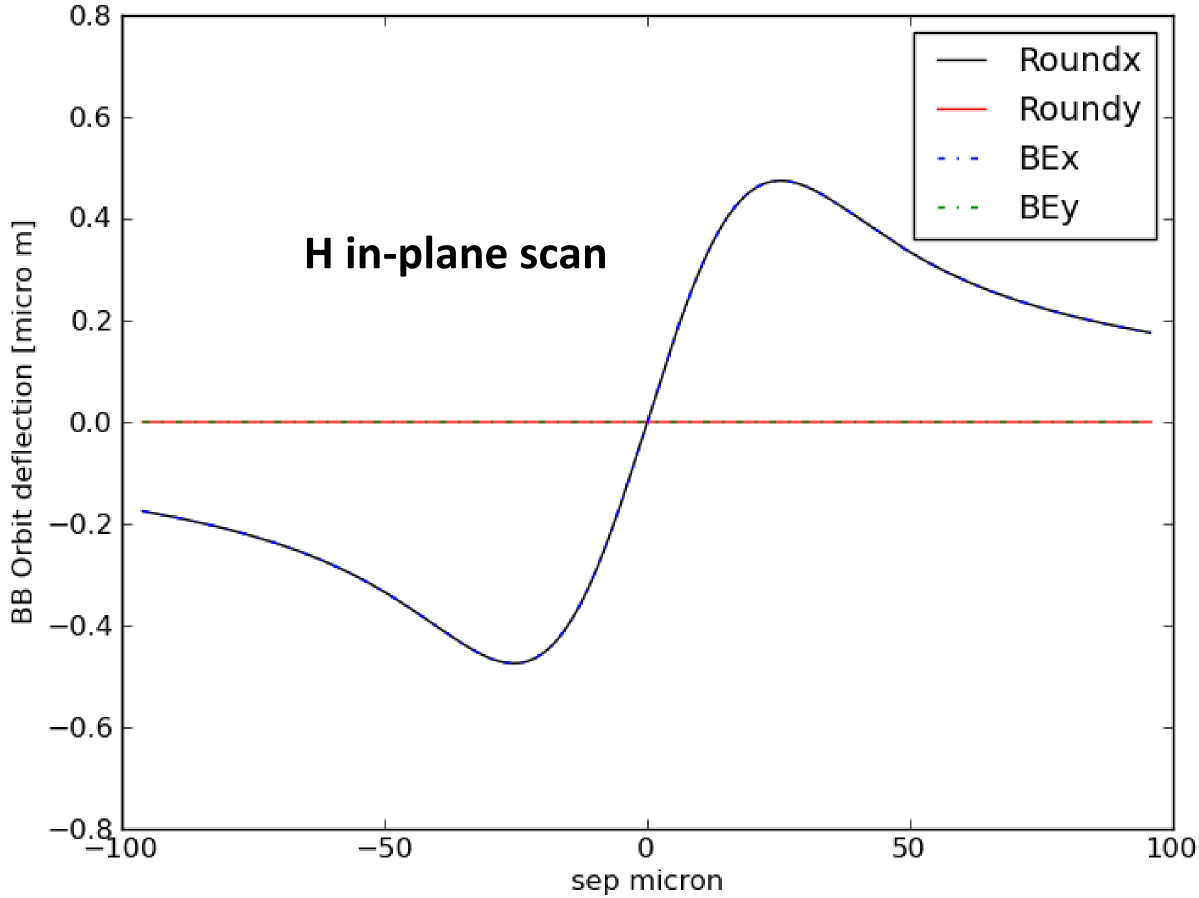
The force has several components at first order we have an amplitude independent contribution: ORBIT KICK

$$\Delta x' = \frac{const}{d}[1 - \frac{x}{d} + O(\frac{x^2}{d^2}) + \ldots]$$



In simple case (1 interaction) one can compute it analytically

Orbit effect as a function of separation



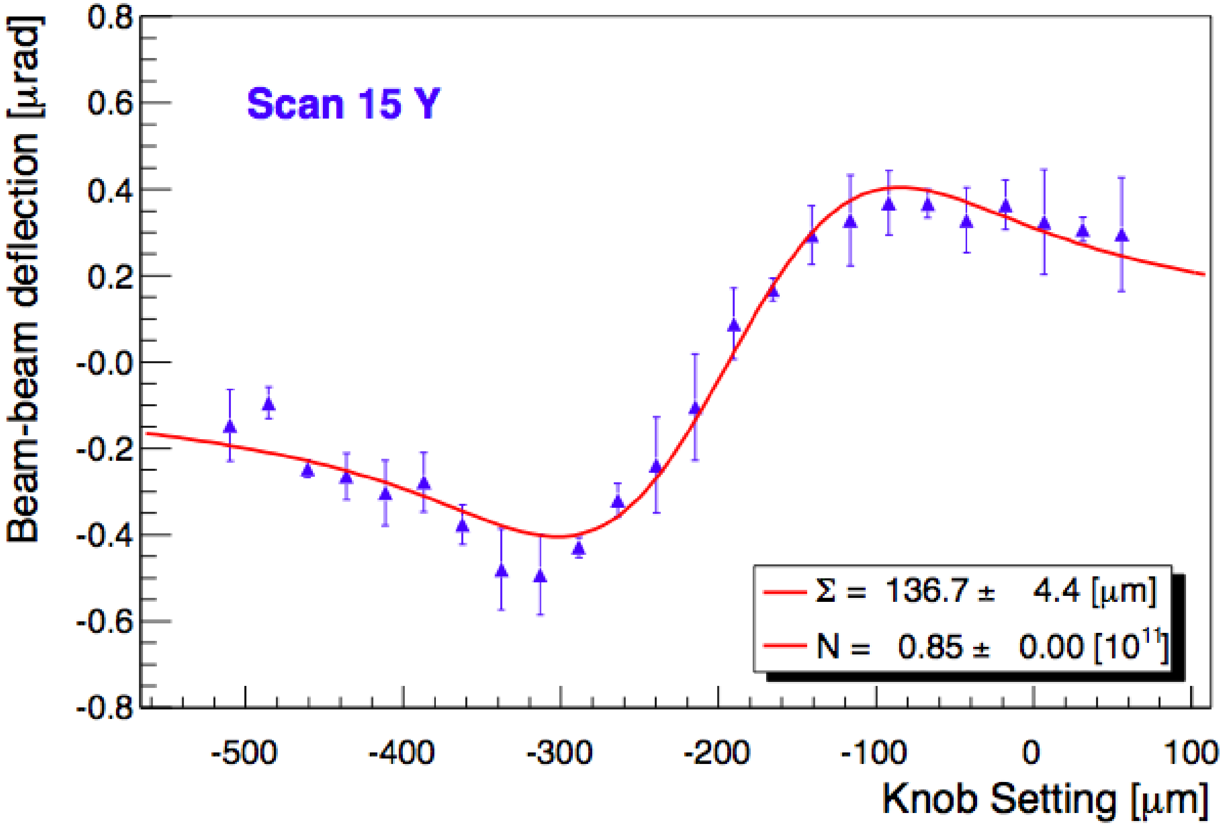
Angular Deflections:

$$\theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma)$$

Closed Orbit effect:

$$Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})}$$

Orbit effect as a function of separation



Angular Deflections: $$\theta_y + i\theta_x = \frac{2r_p}{\gamma} N_p F_0(x, y, \Sigma)$$

Closed Orbit effect: $$Orb_{x,y} = \theta_{x,y} \cdot \beta_{x,y} \cdot \frac{1}{2\tan(\pi \cdot Q_{x,y})}$$

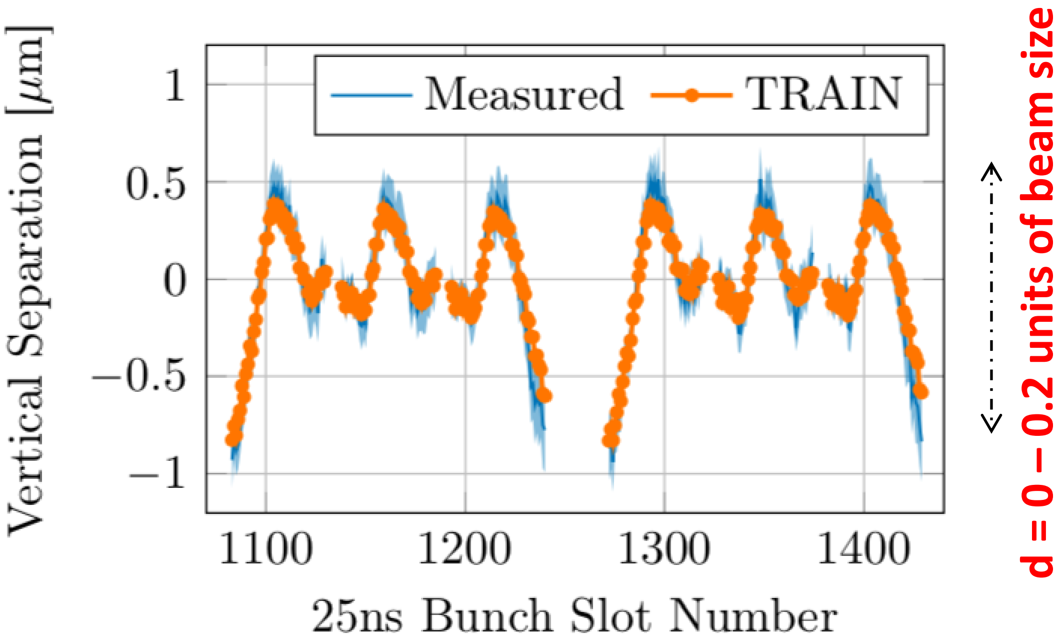
Orbit can be corrected but we should remember PACMAN effects

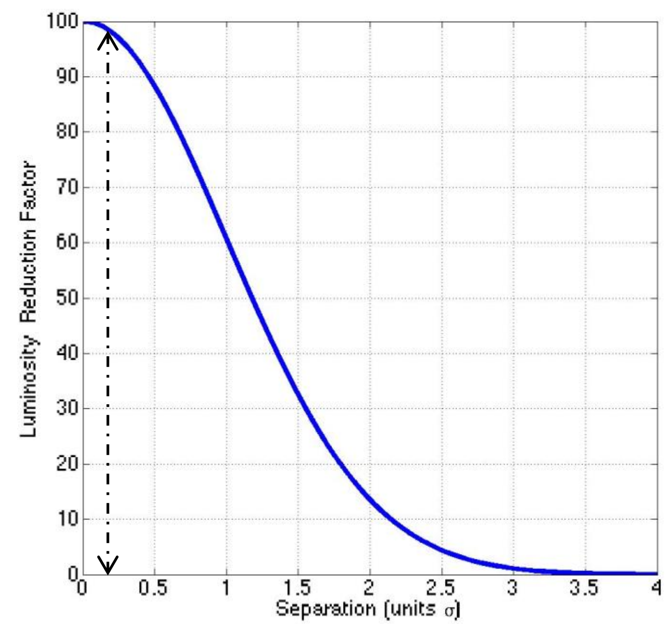
LHC orbit effects

Many long range interactions could become important effect!
Holes in bunch structure leads to PACMAN effects this cannot be corrected!

$$L = L_0 \cdot e^{-\frac{d^2}{4\sigma_x^2}}$$

Self consistent evaluation





1-2% Luminosity loss due to beam-beam orbit effects

…not covered here…

- *Beam-Beam compensation schemes*
- *Landau damping and beam-beam*
- *Beam-Beam coherent effects*
- *Asymmetric beams effects*
- *Noise on colliding beams*
- *Van der Meer scans*
- *Leveling luminosity*
- *….*

Thank you!

Questions?

References:

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[9] Y. Alexahin, "On the Landau damping and decoherence of transverse dipole oscillations in colliding beams ", Part. Acc. 59, 43 (1996).

[10] R. Assmann et al., "Results of long-range beam-beam studies - scaling with beam separation and intensity "

...much more on the LHC Beam-beam webpage:

http://lhc-beam-beam.web.cern.ch/lhc-beam-beam/

From the potential of charge beam to the Beam-beam Force

We need to calculate the field E and B of opposing beam

In rest frame only electrostatic field: $\vec{E} \neq 0, \vec{B} = 0$

We can derive the electrostatic field

In the lab frame the electric and magnetic fields can be obtained:

$$E_{\parallel} = E'_{\parallel}, \quad E_{\perp} = \gamma \cdot E'_{\perp} \quad \text{with :} \quad \vec{B} = \vec{\beta} \times \vec{E}/c$$

Lorentz force gives: $\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$

Ultra-relativistic case $F_r = qE_{\perp}(1 + \beta^2)$

Beam-Beam Effect is mainly a TRANSVERSE EFFECT

Beam-beam potential and force

General approach in electromagnetic problems **Reference[5]**
already applied to beam-beam interactions in **Reference[1,3, 4]**

We need to calculate the field E and B of opposing beam

In rest frame only electrostatic field: $\vec{E} \neq 0, \vec{B} = 0$

$$\Delta U = -\frac{1}{\epsilon_0}\rho(x, y, z)$$

Scalar Potential can be derived from Poisson equation which relates the potential to the charge density ρ

$$\overrightarrow{E} = -\nabla U(x, y, z, \sigma_x, \sigma_y, \sigma_z)$$

Then compute the Electric Field from Gauss Law

Then back to the Lab frame we can compute the force

Lorentz force gives: $\vec{F} = q(\vec{E} + \vec{\beta} \times \vec{B})$

Beam-beam potential

In the case of Gaussian Beam density distribution we can factorize the density distribution

$$\rho(x_0, y_0, z_0) = \rho(x_0)\cdot\rho(y_0)\cdot\rho(z_0)$$

$$\rho(x_0, y_0, z_0) = \frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3} e^{\left(-\frac{x_0^2}{2\sigma_x^2}-\frac{y_0^2}{2\sigma_y^2}-\frac{z_0^2}{2\sigma_z^2}\right)}$$

N is the number of particles in bunch

The poison equation can be formally solved using the Green's function G(x,y,z,x0,y0,z0) method [25]

Solution of Poisson equation

$$U(x,y,z) = \frac{1}{\epsilon_0}\int G(x,y,z,x_0,y_0,z_0)\cdot\rho(x_0,y_0,z_0)dx_0dy_0dz_0$$

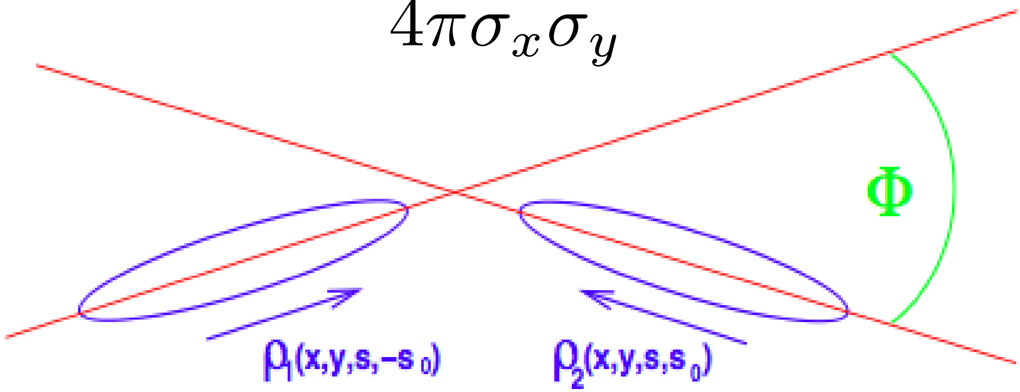
The potential get's the form:

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3}\int\int\int\frac{e^{\left(-\frac{x_0^2}{2\sigma_x^2}-\frac{y_0^2}{2\sigma_y^2}-\frac{z_0^2}{2\sigma_z^2}\right)}dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}$$

This is difficult to solve but following [29] we can solve the diffusion equation.

Crossing angle effect

$$L = \frac{N_1 N_2 f n_b}{4\pi\sigma_x\sigma_y} \cdot S$$



S is the geometric reduction factor
For small crossing angle

$$\sigma_s >> \sigma_{x,y}$$

$$S \approx \frac{1}{\sqrt{1 + (\frac{\sigma_s}{\sigma_x}\frac{\phi}{2})^2}}$$

Examples: LHC (7 TeV): ϕ = 285 μrad, σ_x = 17 μm, σ_s = 7.5 cm, **S=0.84**
HL-LHC (7 TeV) ϕ=590 μrad, σ_x = 7 μm, σ_s = 7.5 cm, **S=0.3**

70% loss of luminosity if not compensated

S. Kheifets proposal

From the diffusion equation:

$$\Delta V - A^2 \cdot \frac{\delta V}{\delta t} = -\frac{1}{\epsilon_0}\rho(x,y,z) \qquad (\text{for } \ t \geq 0)$$

We obtain the potential U by going to the limit of A → 0 $\quad U = \lim_{A \to 0} V$

Solving the diffusion equation instead of Poisson gives a Green's function of the form:

$$G(x,y,z,t,x_0,y_0,z_0) = \frac{A^3}{(2\sqrt{\pi t})^3} \cdot e^{-A^2/4t \cdot ((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}$$

We can then compute the potential

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

$$\frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3\epsilon_0}\int_0^t d\tau \int\int\int e^{\left(-\frac{x_0^2}{2\sigma_x^2}-\frac{y_0^2}{2\sigma_y^2}-\frac{z_0^2}{2\sigma_z^2}\right)} \frac{A^3 \cdot e^{-A^2/4\tau((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}{(2\sqrt{\pi\tau})^3} dx_0 dy_0 dz_0$$

S. Kheifets proposal

From Poisson Equation:

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3}\int\int\int\frac{e^{\left(-\frac{x_0^2}{2\sigma_x^2}-\frac{y_0^2}{2\sigma_y^2}-\frac{z_0^2}{2\sigma_z^2}\right)}dx_0dy_0dz_0}{\sqrt{(x-x_0)^2+(y-y_0)^2+(z-z_0)^2}}$$

From Diffusion equation:

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z)$$

$$\frac{Ne}{\sigma_x\sigma_y\sigma_z\sqrt{2\pi}^3\epsilon_0}\int_0^t d\tau\int\int\int e^{\left(-\frac{x_0^2}{2\sigma_x^2}-\frac{y_0^2}{2\sigma_y^2}-\frac{z_0^2}{2\sigma_z^2}\right)}\frac{A^3\cdot e^{-A^2/4\tau((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)}}{(2\sqrt{\pi\tau})^3}dx_0dy_0dz_0$$

This allows to avoid the denominator in the integral and to collect the exponential which can be integrated

The potential of charge beam: 2D case

Changing the independent variable τ to $q = 4\tau/A^2$ and using the three integrations:

$$\int_{-\infty}^{\infty} e^{-(au^2+2bu+c)}du \;=\; \sqrt{\frac{\pi}{a}}e^{\left(\frac{b^2-ac}{a}\right)} \qquad (for: u = x_0, y_0, z_0)$$

Our potential assumes the form of:

$$U(x,y,z,\sigma_x,\sigma_y,\sigma_z) = \frac{1}{4\pi\epsilon_0}\frac{Ne}{\sqrt{\pi}}\int_0^{\infty}\frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q}-\frac{y^2}{2\sigma_y^2+q}-\frac{z^2}{2\sigma_z^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)(2\sigma_z^2+q)}}dq$$

Since we are interest in the transverse fields, in a two dimensional case

$$\rho(\mathrm{x,y}) = \rho(\mathrm{x}) \cdot \rho(\mathrm{y})$$

$$\rho_u(u) = \frac{1}{\sigma_u\sqrt{2\pi}}\exp\left(-\frac{u^2}{2\sigma_u^2}\right) \text{ where } \; u = x, y$$

2 dimensional problem

The two dimensional potential is then given by:

$$U(x,y,\sigma_x,\sigma_y) = \frac{ne}{4\pi\epsilon_0}\int_0^\infty \frac{\exp(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q})}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}}dq$$

n is the line density of particles in the beam
e is the elementary charge
ε Is the permittivity of free space

From the potential we can derive the field

$$\vec{E} = -\nabla U(x,y,\sigma_x,\sigma_y)$$

Radial Force

In cylindrical coordinates

$$r^2 = x^2 + y^2$$

$$E_r = -\frac{ne}{4\pi\epsilon_0} \cdot \frac{\delta}{\delta r} \int_0^\infty \frac{\exp(-\frac{r^2}{(2\sigma^2+q)})}{(2\sigma^2+q)} dq$$

Radial component

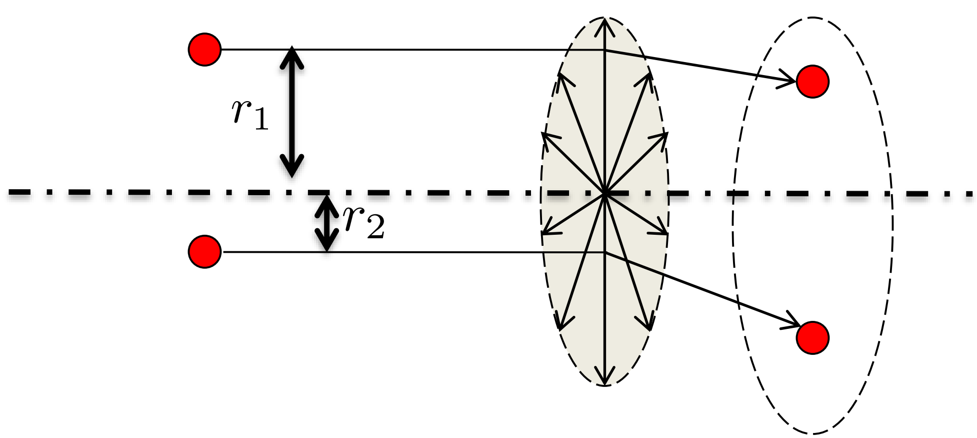
$$B_\Phi = -\frac{ne\beta c\mu_0}{4\pi} \cdot \frac{\delta}{\delta r} \int_0^\infty \frac{\exp(-\frac{r^2}{(2\sigma^2+q)})}{(2\sigma^2+q)} dq$$

Azimuthal component

From Lorentz Force

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

Force has a radial component



Beam-Beam Force: round beams

For the case of q=-e opposite charges

In cylindrical Coordinates $r^2 = x^2 + y^2$

$$F_r(r) = \frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{1}{r} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

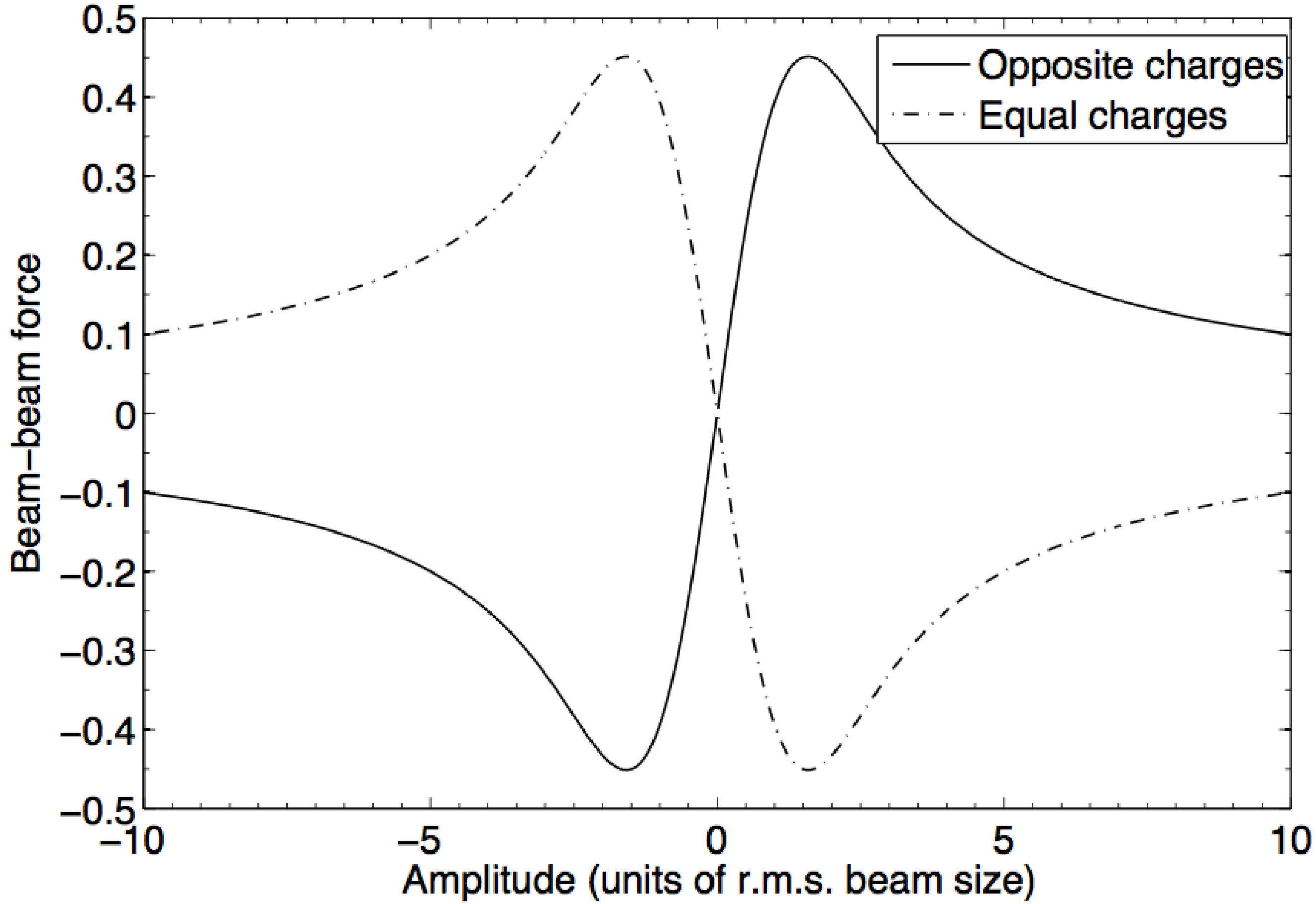
In Cartesian Coordinates:

$$F_x(r) = \frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{x}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

$$F_y(r) = \frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{y}{r^2} \cdot \left[1 - \exp(-\frac{r^2}{2\sigma^2})\right]$$

Beam-Beam Force

If we normalize the separations in units of the beam transverse rms size:



$$F_r(r) = \pm \frac{ne^2(1+\beta_{rel}^2)}{2\pi\epsilon_0}\frac{1}{r}[1-\exp(-\frac{r^2}{2\sigma^2})]$$

Why do we care?

- Tune shift has opposite sign in plane of separation
- Break the symmetry between the planes, much more resonances are excited
- Mostly affect particles at large amplitude
- Cause effects on closed orbit, tune shift, chromaticity…
- PACMAN effects complicates the picture

Dynamic beta effect and beating

- The beam-beam collision at the experiment changes also the optics of the machine
- This leads to changes in the phase $\Delta\mu$ and to an "optical error" $\Delta\beta^*$
- Source of force at the position s, and the effect at position s_0 in perturbation theory is given by:

$$\Delta\beta(s_0) = -\frac{\beta(s_0)}{2sin(2\pi Q)}\int_{s_1}^{s_1+C}\beta(s)\Delta k(s)cos\left[2(\mu(s)-\mu(s_0))-2\pi Q\right]ds$$

If our case if optics changes → beam-beam force changes → optics changes → beam-beam force changes ...

Self-consistent calculation is required to evaluate the effect

Dynamic Beta effect

In a simple case with one beam-beam interaction and seen as a perturbation
And taking the effect at the source of the error ($s=s_0$)

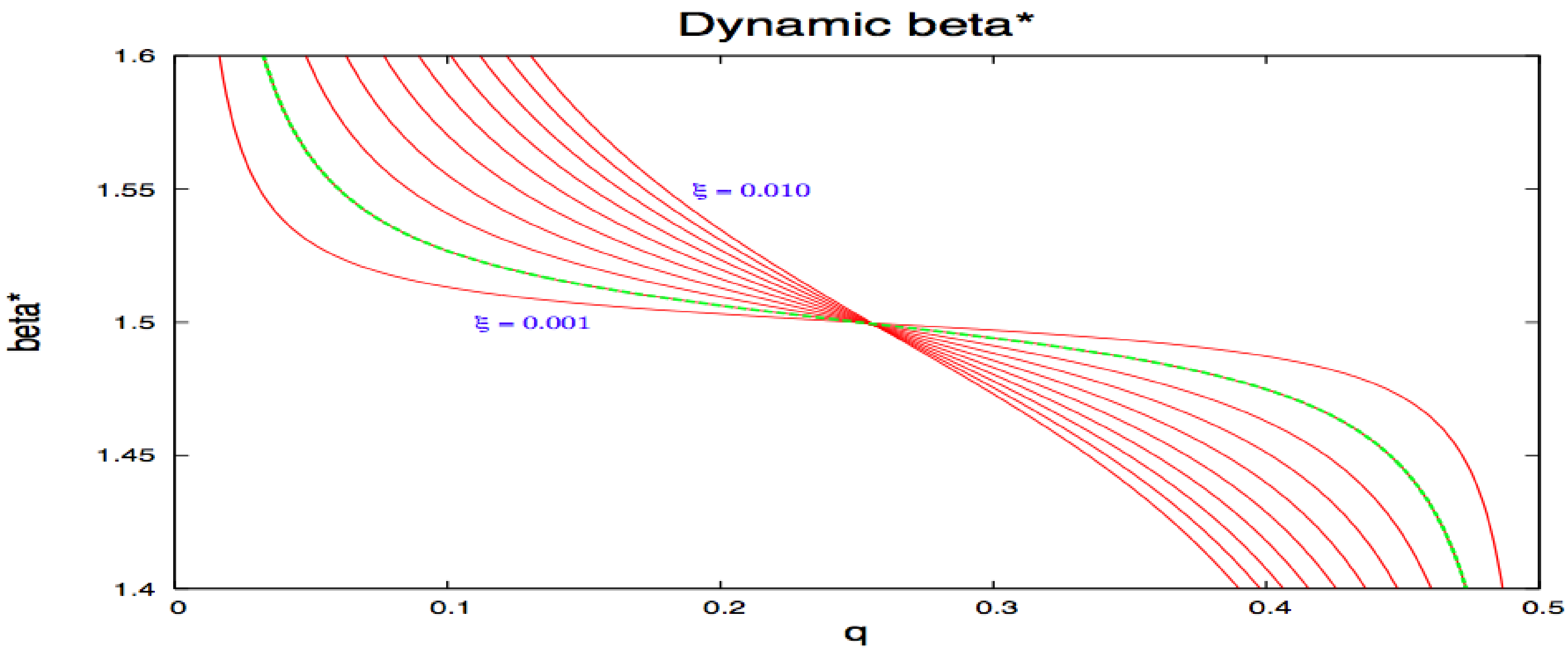
$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q+\Delta Q))} = \frac{1}{\sqrt{1+4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$

Beam-beam interaction leads to optical distortion at <ins>interaction point itself</ins> **Dynamic beta**

Beam-beam interaction leads to optical distortion at <ins>all other interaction points</ins> **Dynamic beating**

Expression above not valid during scan or several interaction points → needs optics code for calculation

Dynamic Beta effect single Interaction point



$$\frac{\beta^*}{\beta_0^*} = \frac{sin(2\pi Q)}{sin(2\pi(Q + \Delta Q))} = \frac{1}{\sqrt{1 + 4\pi\xi cot(2\pi Q) - 4\pi^2\xi^2}}$$

Sensitive to:

- **Beam-beam parameter: ξ**
- **Tune : Q**
- **Configuration (IPS) and optics (phase advance)**

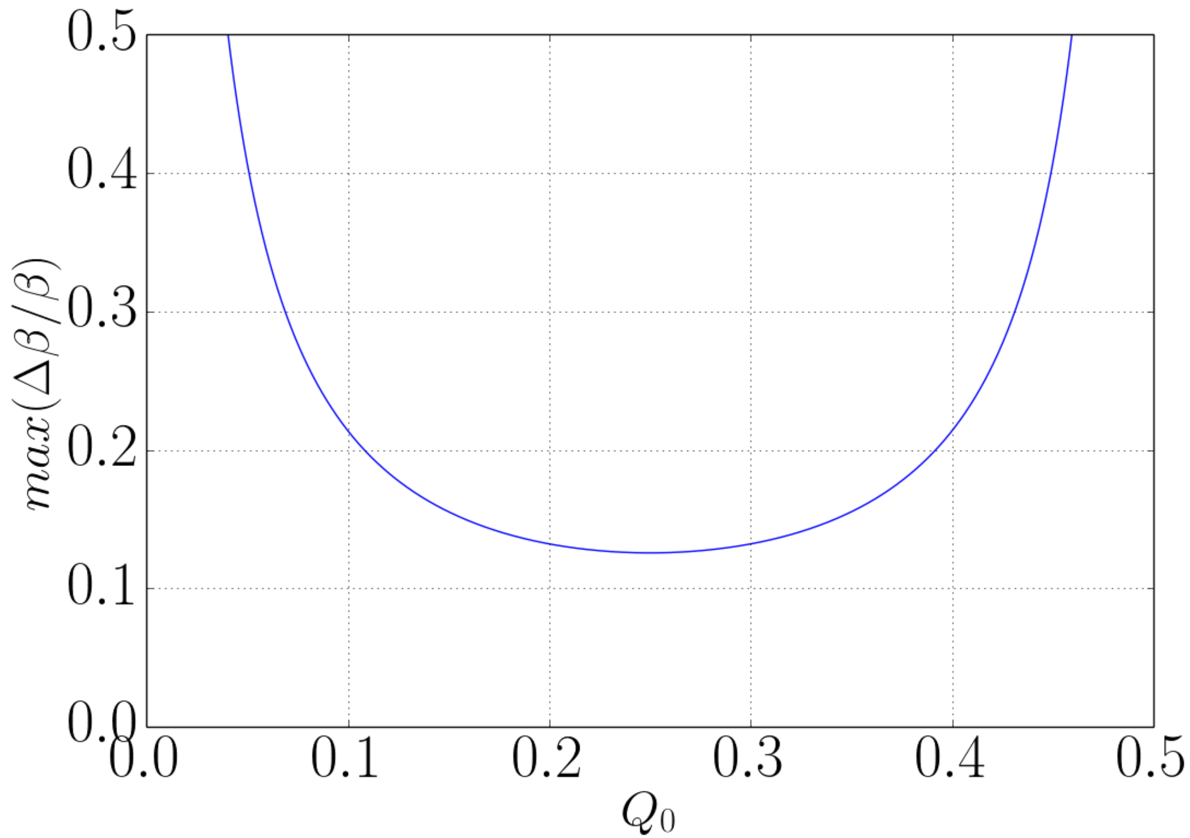
LHC case has 1-2 %
HL-LHC 3-6 %
…or more

Dynamic beta-beating due to beam-beam effects

Maximum beta change as a function of unperturbed tune

$$\max\left(\frac{\Delta\beta}{\beta}\right) = \frac{2\pi\xi}{sin(2\pi Q_0)}$$

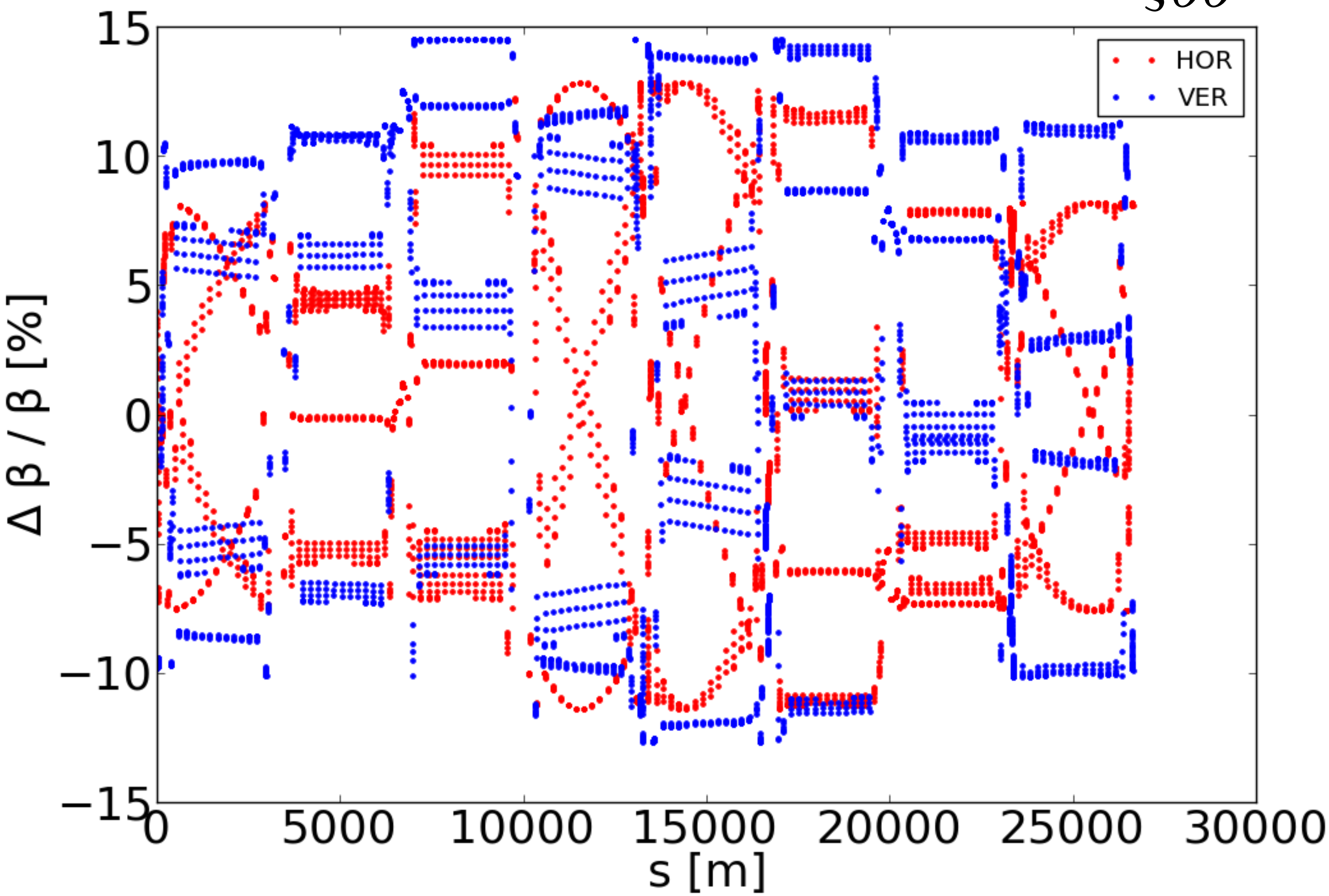
$$\xi_{bb} = 0.02$$



Maximum beating as a function of tune

Dynamic beta-beating due to beam-beam effects

$$\xi_{bb} = 0.02$$



From optics codes beating along the accelerator

How will cleaning efficiency and machine protection deal with such beating?