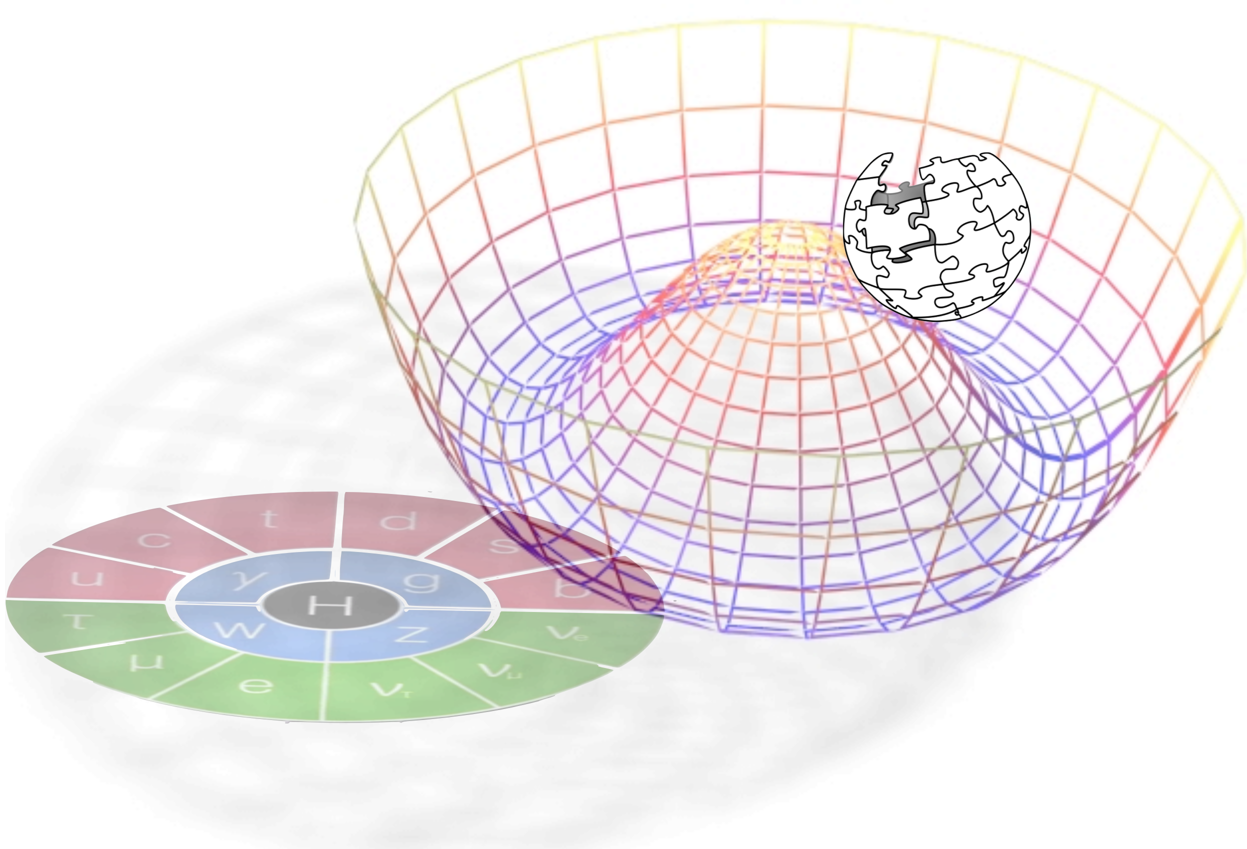
The Standard Model
of particle physics

CERN summer student lectures 2019

Lecture 2/5



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Outline

- **Monday**
  - Lagrangians
  - Lorentz symmetry - scalars, fermions, gauge bosons
  - Dimensional analysis: cross-sections and life-time.
- **Tuesday**
  - Gauge interactions
  - Electromagnetism: U(1)
  - Nuclear decay, Fermi theory and weak interactions: SU(2)
  - ~~Strong interactions: SU(3)~~
- **Wednesday**
  - Chirality of weak interactions
  - Pion decay
- **Thursday**
  - Spontaneous symmetry breaking and Higgs mechanism
  - Quark and lepton masses
  - Neutrino masses
- **Friday**
  - Running couplings
  - Asymptotic freedom of QCD
  - Anomalies cancelation

Dimensional Analysis

$$[S]_m = 0 \quad \Rightarrow \quad [\mathcal{L}]_m = 4$$

$$S = \int d^4x\, \mathcal{L}$$

Scalar field $\mathcal{L} = \partial_\mu \phi \partial^\mu \phi + \ldots$ $\Rightarrow$ $[\phi]_m = 1$

Spin-1/2 field $\mathcal{L} = \psi^\dagger \gamma^0 \gamma^\mu \partial_\mu \psi$ $\Rightarrow$ $[\psi]_m = 3/2$

Spin-1 field $\mathcal{L} = F_{\mu\nu} F^{\mu\nu} + \ldots \text{ with } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \ldots$ $\Rightarrow$ $[A_\mu]_m = 1$

Particle lifetime of a (decaying) particle: $[\tau]_m = -1$ Width: $[\Gamma = 1/\tau]_m = 1$

Cross-section (“area” of the target): $[\sigma]_m = -2$

Lifetime "Computations"

muon and neutron are unstable particles

$$\mu \to e\nu_\mu\bar{\nu}_e$$

$$n \to p\, e\, \bar{\nu}_e$$

We'll see that the interactions responsible for the decay of muon and neutron are of the form

$$\mathcal{L} = G_F\, \psi^4 \qquad\qquad \Gamma \propto G_F^2 m^5$$

$[\text{mass}]^4$ $\quad$ $[\text{mass}]^{-2}$ $\quad$ $[\text{mass}]^{3/2\times 4}$ $\qquad\qquad$ $[\text{mass}]$

G_F = Fermi constant: $G_F \sim \dfrac{10^{-5}}{m_{\text{proton}}} \sim 10^{-5}\,\text{GeV}^{-2}$

$1 = \hbar c \sim 200\,\text{MeV}\cdot\text{fm}$

E	T	L
1eV	10^{-16}s	10^{-7}m
10^{-16}eV	1s	10^{9}m
10^{-7}eV	10^{-9}s	1m

For the **muon**, the relevant mass scale is the muon mass m_μ=105MeV:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV} \quad \text{i.e.} \quad \tau_\mu \sim 10^{-6}\,\text{s}$$

For the **neutron**, the relevant mass scale is $(m_n\text{-}m_p)\approx 1.29$MeV:

$$\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV} \quad \text{i.e.} \quad \tau_{\text{n}} \sim 10^3\,\text{s}$$

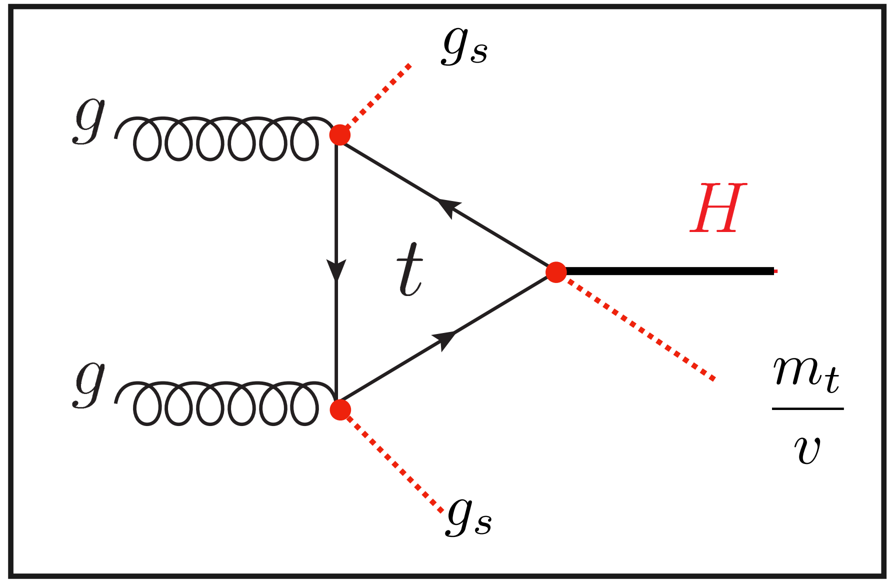
Higgs production “Computation”

At the LHC, the dominant Higgs production mode is gluon fusion

strong coupling constant $g_s \sim 1$

$g_s\, G_\mu \bar{t}\gamma^\mu t$

[mass]^0 [mass]^1 [mass]$^{3/2\text{x}2}$



Higgs coupling proportional to the mass

$g_{ttH}\, \bar{t}tH$

[mass]^0 [mass]$^{3/2\text{x}2}$ [mass]^1

v=246 GeV

$$\sigma = \frac{1}{8\pi}\,\frac{1}{16\pi^2}\,g_s^4\,\frac{m_t^2}{v^2}\,\frac{1}{m_t^2}$$

flux loop couplings dimensionally $[\sigma]_m$=-2

i.e. $\sigma \sim 10^{-25}\,\mathrm{eV}^{-2} \sim 10^{-39}\,\mathrm{m}^2 = 10\,\mathrm{pb}$

$$1\,\mathrm{barn} = 10^{-28}\,\mathrm{m}^2$$

How many Higgs bosons produced at LHC? $\sigma \times \int dt\,\mathcal{L} = 10\,\mathrm{pb} \times 100\,\mathrm{fb}^{-1} \sim 10^6$

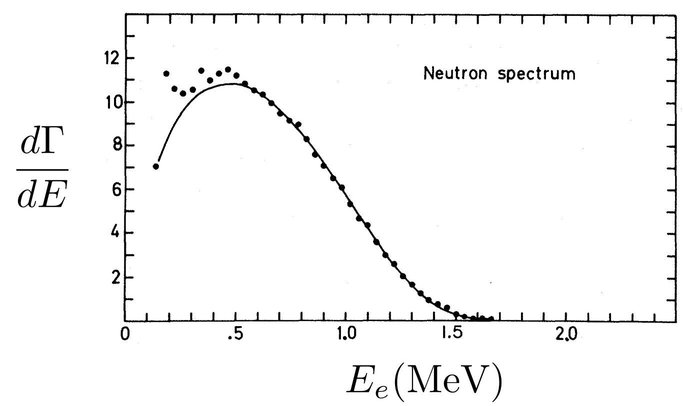
Beta decay

$${}^{40}_{19}\mathrm{K} \to {}^{40}_{20}\mathrm{Ca}^+ + e^- \qquad {}^{64}_{29}\mathrm{Cu} \to {}^{64}_{30}\mathrm{Zn}^+ + e^- \qquad {}^{3}_{1}\mathrm{H} \to {}^{3}_{2}\mathrm{He}^+ + e^-$$

- Two body decays: A→B+C

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} c^2 \qquad p = \frac{\sqrt{\lambda(m_A, m_B, m_C)}}{2m_A} c$$

$$\lambda(m_A, m_B, m_C) = (m_A + m_B + m_C)(m_A + m_B - m_C)(m_A - m_B + m_C)(m_A - m_B - m_C)$$



fixed energy of daughter particles (pure SR kinematics, independent of the dynamics)

$\Rightarrow$ non-conservation of energy?

Pauli '30: ∃ neutrino, very light since end-point of spectrum is close to 2-body decay limit

ν first observed in '53 by Cowan and Reines

- N-body decays: A→B$_1$+B$_2$+...+B$_N$

$$E_{B_1}^{\min} = m_{B_1} c^2 \qquad E_{B_1}^{\max} = \frac{m_A^2 + m_{B_1}^2 - (m_{B_2} + \ldots + m_{B_N})^2}{2m_A} c^2$$

How are neutrinos produced?

$\pi \to \mu\nu$ (more about pion decay later later) $\qquad \mu \to e\bar{\nu}_e\nu_\mu$

need 2 neutrino flavors and flavor conservation since $\mu \not\to e\gamma$

electron energy from decay of muon at rest: $m_e c^2 \approx 511\ \mathrm{keV} \leq E_e \leq 53\ \mathrm{MeV} \approx \dfrac{m_\mu^2 + m_e^2}{2m_\mu} c^2$

Lederman, Schwartz, Steinberger '62: $p\,\bar{\nu}_\mu \to n\,\mu^+$ but $p\,\bar{\nu}_\mu \not\to n\,e^+$

Fermi theory '33

(paper rejected by Nature: declared too speculative !)

$$\mathcal{L} = G_{\mathcal{F}}(\bar{n}p)(\bar{\nu}_e e)$$

exp: $G_F = 1.166\times10^{-5}\ \mathrm{GeV}^{-2}$

We'll see later that the structure is a bit more complicated

Universality of Weak Interactions

How can we be sure that muon and neutron decays proceed via the same interactions?

$\tau_\mu \approx 10^{-6}$s vs. $\tau_{neutron} \approx 900$s

By analogy with electromagnetism, one can see the Fermi force as a current-current interaction

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The cross-terms generate both neutron decay and muon decay
The life-times of the neutron and muon tell us that the relative factor between the electron and the muon is the current is of order one, i.e., the weak force has the same strength for electron and muon

What about $\pi^\pm$ decay $\tau_\pi \approx 10^{-8}$s?

Why $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \underset{\text{Exp}}{\sim} 10^{-4}$? And not $\dfrac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim \dfrac{(m_\pi - m_e)^5}{(m_\pi - m_\mu)^5} \underset{\text{Th}}{\sim} 500$?

Does it mean that our way to compute decay rate is wrong?
Is pion decay mediated by another interaction?
Is the weak interaction non universal, i.e. is the value of G_F processus dependent?

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F\, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F\,(\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e\gamma^\mu e)$$

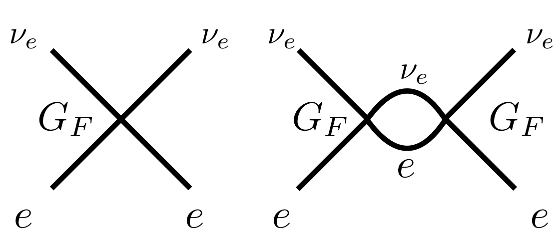
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $\quad[\text{mass}]^{-2\times 2}$ $\quad[\text{mass}]^{2}$

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that at high-energy the quantum corrections to the classical contribution can be sizeable:



$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots$$

The theory becomes non-perturbative at an energy $E_{\max} = \dfrac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\mathrm{GeV}\text{–}1\,\mathrm{TeV}$

unless new degrees of freedom appears before to change the behaviour of the scattering

U(1) Gauge Symmetry

QED: the phase of an electron is not physical and can be rotated away

$$\phi \to e^{i\theta}\phi$$

The phrase transformation is local, i.e., depends on space-time coordinate, then

$$\partial_\mu \phi \to e^{i\theta}\left(\partial_\mu \phi + i(\partial_\mu \theta)\phi\right)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under local transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another. For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \to e^{i\theta} D_\mu \phi \qquad \text{iff} \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$$

Note that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \to F_{\mu\nu}$ and the full Lagrangian is invariant under local transformation

Gauge invariance is a dynamical principle: it predicts some interactions among particles

It also explains why the QED interactions are universal (an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \to U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu\phi = \partial_\mu\phi + igA_\mu\phi \to UD_\mu\phi \qquad \text{iff} \qquad A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

g is the gauge coupling and defines the strength of the interactions

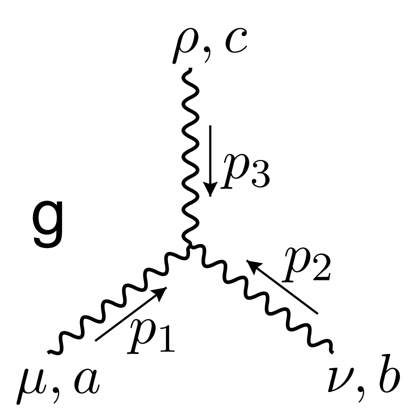
For the field strength to transform homogeneously, one needs to add a non-Abelian piece

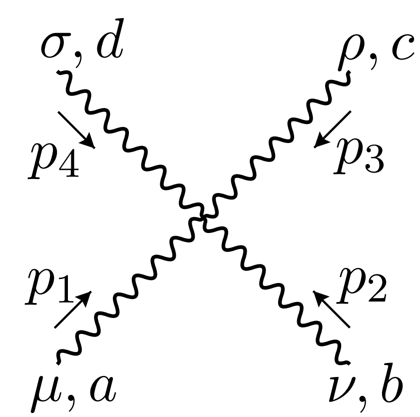
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L} \propto F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

g

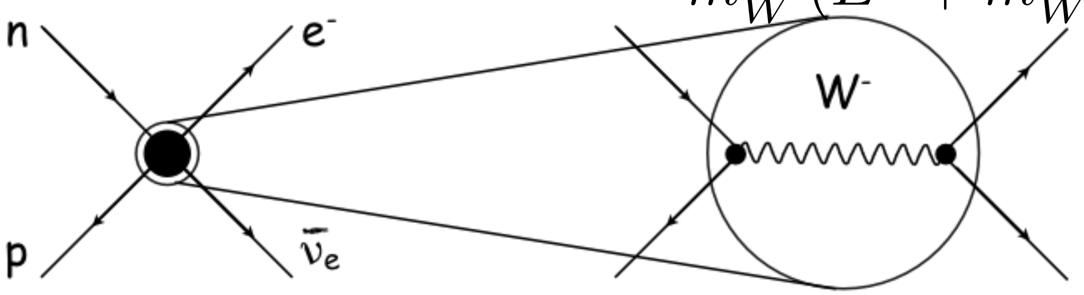




g^2

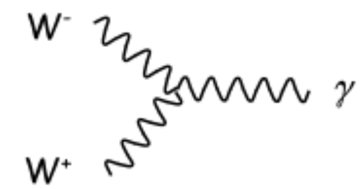
Electroweak Interactions

$\sigma \propto G_F^2 E^2$ $\qquad$ $\sigma \propto g^4 \dfrac{E^2}{m_W^2\,(E^2+m_W^2)}$



$G_F \propto \dfrac{g^2}{m_W^2}$

charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry [Q,T±]=±T±

1. No additional "force": (Georgi, Glashow '72) ⇛ extra matter

$SU(2)$

$$[T^a,T^b]=i\epsilon^{abc}T^c$$

$$[T^+,T^-]=Q \qquad [Q,T^\pm]=+\pm T^\pm$$

$$T^\pm=\frac{1}{\sqrt{2}}(T^1\pm iT^2)$$

$\mathrm{Tr}_{\mathrm{irrep}}T^3=0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix}$ $\begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+,T^-]=-Q$$

$$[Q,T^\pm]=+\pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+,T^-]=0$$

$$[Q,T^\pm]=+\pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional "matter" (SM: Glashow '61, Weinberg '67, Salam '68): SU(2)xU(1)

⇛ extra force

$Q=T^3?$

as Georgi-Glashow

⇛ extra matter

$Q=Y?$

$Q(e_L)=Q(\nu_L)$

$Q=T^3+Y!$

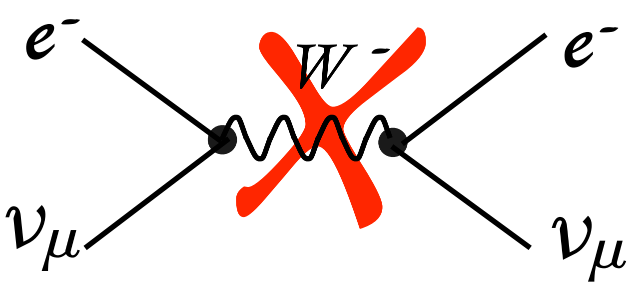
Gell-Mann '56, Nishijima-Nakano '53

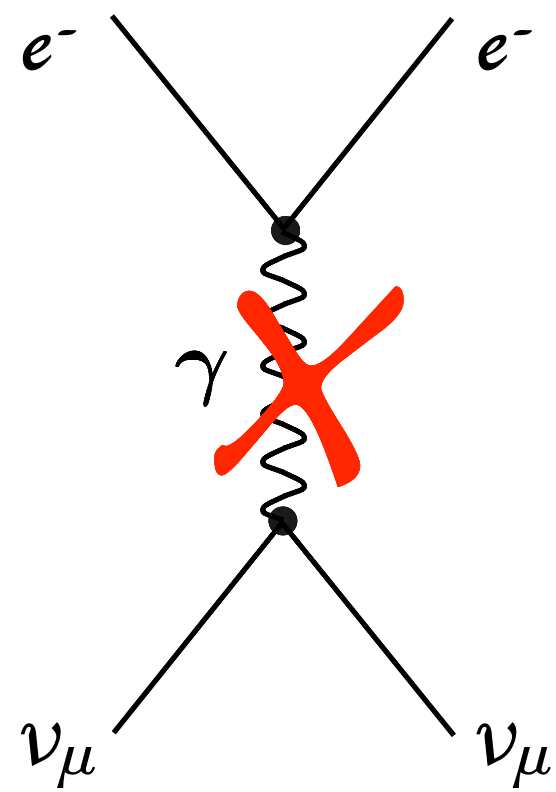
Electroweak Interactions

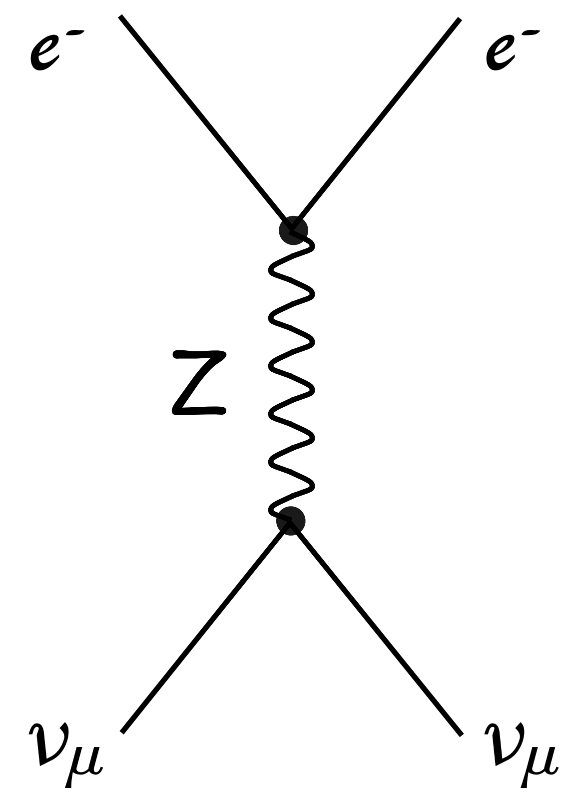
Gargamelle experiment '73 first established the SU(2)xU(1) structure

Idea:
rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

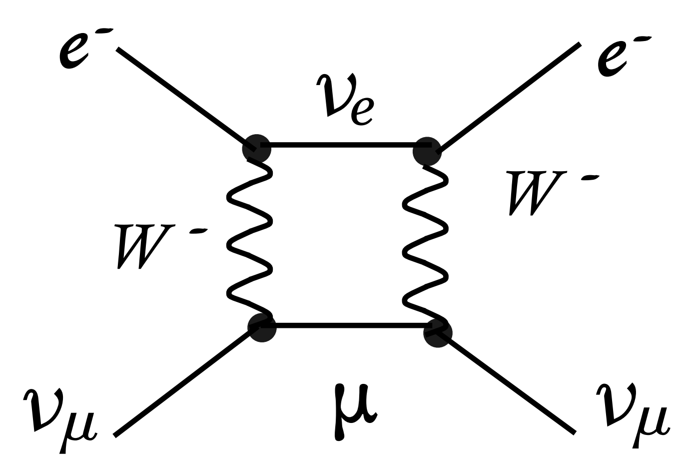
$$\nu_\mu \, e^- \rightarrow \nu_\mu \, e^-$$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation (a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \ldots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\dfrac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \quad \Rightarrow \quad W_\mu^- = \dfrac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an effective Lagrangian (valid below the mass of the gauge bosons):

$$\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}$$

which is the Fermi current-current interaction. The Fermi constant is given by $G_F = \dfrac{g^2}{m_W^2}$

(the correct expression involves a different normalisation factor)

In the current-current product, the term $(\bar{n}\gamma^\mu p)(\bar{\nu}_e\gamma^\nu e)\eta_{\mu\nu}$ is responsible for beta decay, while the term $(\bar{\mu}\gamma^\mu \nu_\mu)(\bar{\nu}_e\gamma^\nu e)\eta_{\mu\nu}$ is responsible for muon decay. Both decays are controlled by the same coupling, as indicated by the measurements of the lifetimes of the muon and neutron.