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Standard Model | **Homework 2** | **Date: 16.07.2019**

Exercice 1: Particle physics units

We recall that $[\hbar] = M \cdot L^2 \cdot T^{-1}$ and $[c] = L \cdot T^{-1}$.
a) Check the consistency of the classical/quantum correspondence at the dimensional level:

$$E \to i\hbar\frac{\partial}{\partial t} \quad \& \quad p \to i\hbar\frac{\partial}{\partial x}$$

c) Using the Newton constant, $\hbar$ and c, construct a mass scale, a length scale and a time scale. They are defining the Planck scales. Compute the matter density of the universe today (10^{-29} g/cm^3) in Planck units.
d) Using e, m_e and c, construct a length scale. This is the classical radius of the electron. Using e, m_e and $\hbar$, construct a length scale. This is the Bohr radius of the electron.

Exercice 2: Order of magnitude estimates

a) Estimate the energy of the cosmic rays given that the lifetime of a muon is about 1 μs.
b) A 1 cm^3 piece of ice melts in about 40 minutes under the sun. Compute the volume of oil to burn 1 cm-thick ice cap surrounding ths sun at a distance of 150 million kilometres (the effects of the atmosphere will be neglected). Assuming that all the energy radiated by the Sun would be of chemical origin, what would be the maximal lifetime of the Sun? What do you conclude concerning the origin of the energy radiated by the Sun? We recall that burning 1 liter of oil produces about 30 MJ and that 333 kJ are needed to melt 1kg of ice.

Exercice 3: EM action for photons

The photon field strength is defined to be

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,,$$

where the 4-vector A^μ is defined from the scalar and vector potential $(\phi, \vec{A})$.
a) From the classical EM definition of the electric and magnetic fields from the scalar and vector potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial\vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla}\wedge\vec{A}$$

show that

$$\vec{E}_i = -F_{i0} \quad \vec{B}_i = -\frac{1}{2}\epsilon_{ijk}F_{jk}$$

b) Derive the expression of the gauge field Lagrangian density, $-\frac{1}{4\pi}F_{\mu\nu}F^{\mu\nu}$, in terms of the electric and magnetic fields and recognise the usual expression of the energy density stored in

the electromagnetic fields.
c) There is another Lorentz-invariant Lagrangian density that one can construct from the electromagnetic field:

$$\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}.$$

Compute this Lagrangian density in terms of $\vec{E}$ and $\vec{B}$.