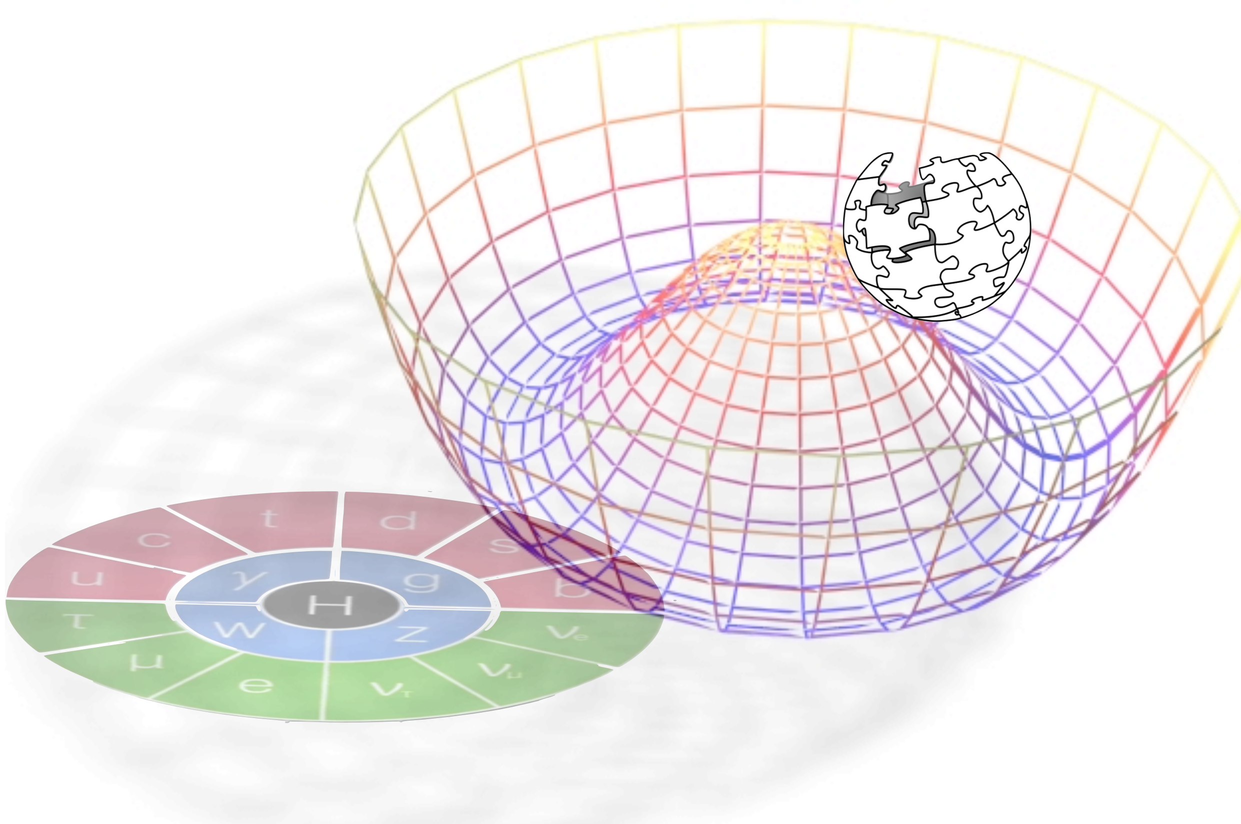
The Standard Model
of particle physics

CERN summer student lectures 2019

Lecture 3/5



Christophe Grojean
DESY (Hamburg)
Humboldt University (Berlin)
(christophe.grojean@desy.de)

Outline

- **Monday**
  - Lagrangians
  - Lorentz symmetry - scalars, fermions, gauge bosons
  - Dimensional analysis: cross-sections and life-time.
- **Tuesday**
  - Dimensional analysis: cross-sections and life-time
  - Nuclear decay, Fermi theory
- **Wednesday**
  - Gauge interactions: U(1) electromagnetism, SU(2) weak interactions, SU(3) QCD
  - Chirality of weak interactions
  - Pion decay
- **Thursday**
  - Spontaneous symmetry breaking and Higgs mechanism
  - Quark and lepton masses
  - Neutrino masses
- **Friday**
  - Running couplings
  - Asymptotic freedom of QCD
  - Anomalies cancelation

Pathology at High Energy

What about weak scattering process, e.g . $e\nu_e \to e\nu_e$?

$$\mathcal{L} = G_F \, J^*_\mu J^\mu \qquad \text{with} \qquad J^\mu = (\bar{n}\gamma^\mu p) + (\bar{e}\gamma^\mu \nu_e) + (\bar{\mu}\gamma^\mu \nu_\mu) + \ldots$$

The same Fermi Lagrangian will thus also contain a term

$$G_F \, (\bar{e}\gamma^\mu \nu_e)(\bar{\nu}_e\gamma^\mu e)$$

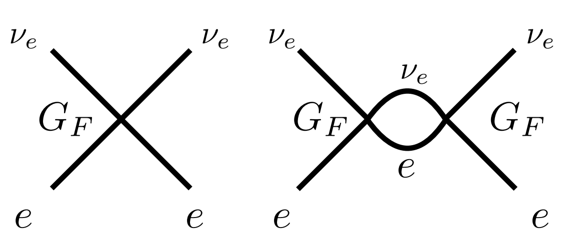
that will generate e-ν_e scattering whose cross-section can be guessed by dimensional arguments

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $\quad [\text{mass}]^{-2\times 2}$ $\quad [\text{mass}]^2$

non conservation of probability
(non-unitary theory)
inconsistent at high energy

It means that at high-energy the quantum corrections to the classical contribution can be sizeable:



$$\sigma \propto G_F^2 E^2 + \frac{1}{16\pi^2} G_F^4 E^6 + \ldots$$

The theory becomes non-perturbative at an energy $E_{\max} = \dfrac{2\sqrt{\pi}}{\sqrt{G_F}} \sim 100\,\text{GeV}–1\,\text{TeV}$

unless new degrees of freedom appears before to change the behaviour of the scattering

U(1) Gauge Symmetry

QED: the phase of an electron is not physical and can be rotated away

$$\phi \to e^{i\theta}\phi$$

The phrase transformation is local, i.e., depends on space-time coordinate, then

$$\partial_\mu \phi \to e^{i\theta}\left(\partial_\mu \phi + i(\partial_\mu \theta)\phi\right)$$

and the kinetic term is no-longer invariant due to the presence of the non-homogenous piece

To make the theory invariant under local transformation, one needs to introduce a **gauge field** that keeps track/memory of how the phase of the electron changes from one point to another. For that, we build a **covariant derivative** that has nice homogeneous transformations

$$D_\mu \phi = \partial_\mu \phi + ieA_\mu \phi \to e^{i\theta} D_\mu \phi \qquad \text{iff} \qquad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu \theta$$

Note that $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \to F_{\mu\nu}$ and the full Lagrangian is invariant under local transformation

Gauge invariance is a dynamical principle: it predicts some interactions among particles

It also explains why the QED interactions are universal (an electron interacts with a photon in the same way on Earth, on the Moon and at the outskirts of the Universe)

SU(N) non-Abelian Gauge Symmetry

We generalise the QED construction by considering general transformation of a N-vector

$$\phi \to U\phi$$

We build a **covariant derivative** that again has nice homogeneous transformations

$$D_\mu\phi = \partial_\mu\phi + igA_\mu\phi \to UD_\mu\phi \qquad \text{iff} \qquad A_\mu \to UA_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

g is the gauge coupling and defines the strength of the interactions

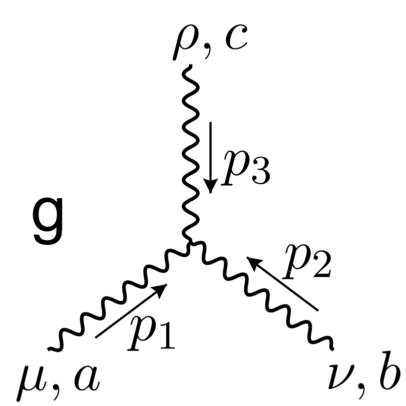
For the field strength to transform homogeneously, one needs to add a non-Abelian piece

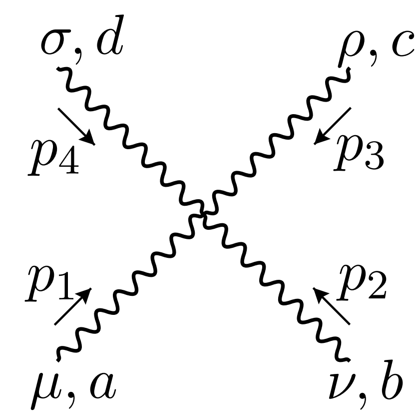
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu] \to UF_{\mu\nu}U^{-1}$$

Contrary to the Abelian case, the gauge fields are now charged and interact with themselves

$$\mathcal{L} \propto F_{\mu\nu}F^{\mu\nu} \supset g\partial AAA + g^2 AAAA$$

g

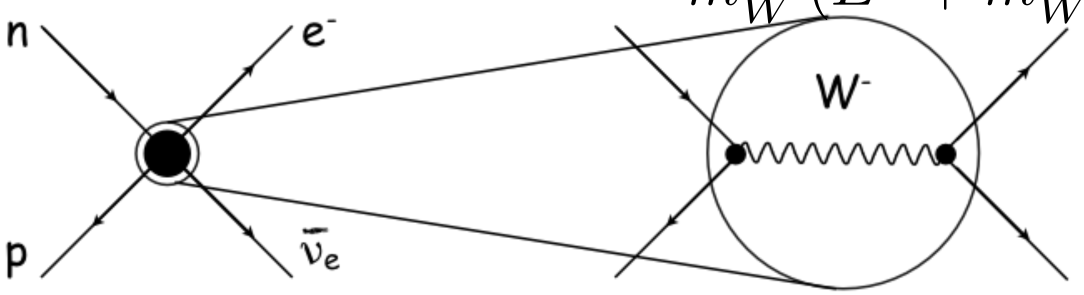




g^2

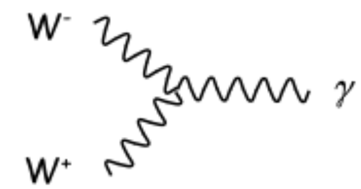
Electroweak Interactions

$\sigma \propto G_F^2 E^2$ $\sigma \propto g^4 \frac{E^2}{m_W^2 (E^2 + m_W^2)}$



$G_F \propto \frac{g^2}{m_W^2}$

charged W ⇛ must couple to photon:



⇛ non-abelian gauge symmetry [Q,T±]=±T±

1. No additional “force”: (Georgi, Glashow ’72) ⇛ extra matter

$SU(2)$

$$[T^a, T^b] = i\epsilon^{abc}T^c$$

$$[T^+, T^-] = Q \qquad [Q, T^\pm] = + \pm T^\pm$$

$$T^\pm = \frac{1}{\sqrt{2}}(T^1 \pm iT^2)$$

$\mathrm{Tr}_{\mathrm{irrep}} T^3 = 0$ ⇛ extra matter $\begin{pmatrix} X_L \\ \nu_L \\ e_L \end{pmatrix}$ $\begin{pmatrix} X_R \\ \nu_R \\ e_R \end{pmatrix}$

$SU(1,1)$

$$[T^+, T^-] = -Q$$

$$[Q, T^\pm] = + \pm T^\pm$$

non-compact

unitary rep. has dim ∞

E_2

2D Euclidean group

$$[T^+, T^-] = 0$$

$$[Q, T^\pm] = + \pm T^\pm$$

only one unitary rep.

of finite dim. = trivial rep.

2. No additional “matter” (SM: Glashow ’61, Weinberg ’67, Salam ’68): SU(2)xU(1)

⇛ extra force

$Q = T^3?$

as Georgi-Glashow

⇛ extra matter

$Q = Y?$

$Q(e_L) = Q(\nu_L)$

$Q = T^3 + Y!$

Gell-Mann ’56, Nishijima-Nakano ’53

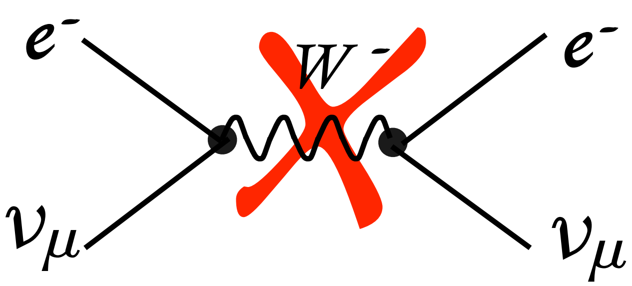
Electroweak Interactions

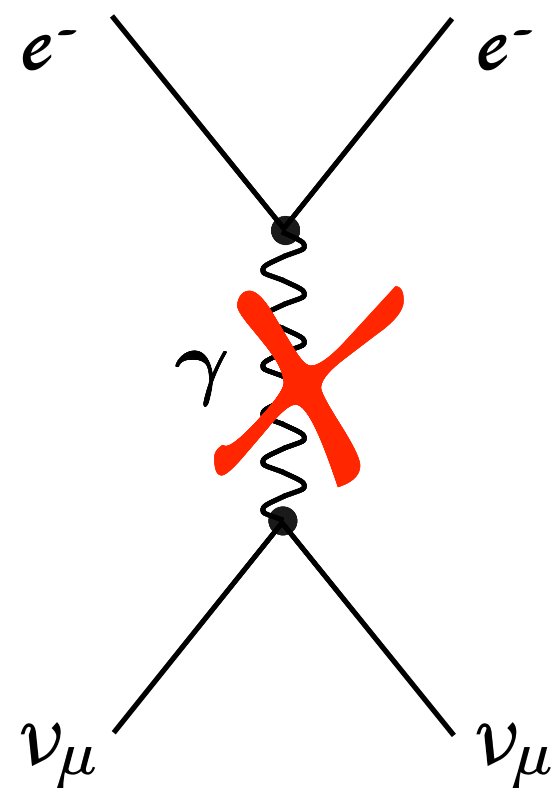
Gargamelle experiment '73 first established the SU(2)xU(1) structure

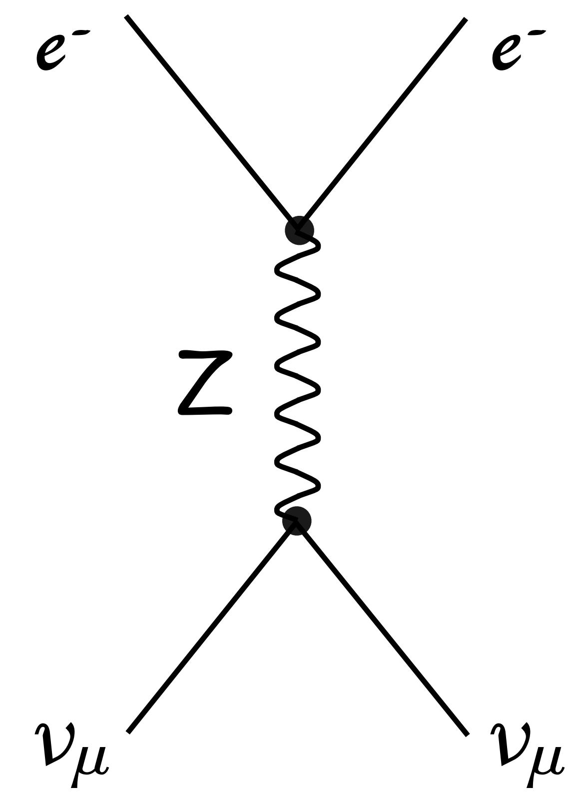
Idea:

rely on a particle that doesn't interact with photon to prove the existence a new neutral current process!

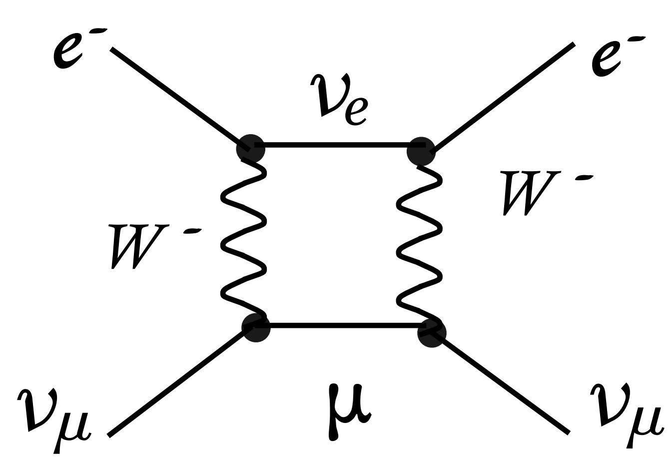
$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$







loop-suppressed contribution from W:



From Gauge Theory to Fermi Theory

We can derive the Fermi current-current contact interactions by "integrating out" the gauge bosons, i.e., by replacing in the Lagrangian the W's by their equation of motion. Here is a simple derivation (a better one should take taking into account the gauge kinetic term and the proper form of the fermionic current that we'll figure out tomorrow, for the moment, take it as a heuristic derivation)

$$\mathcal{L} = -m_W^2 W_\mu^+ W_\nu^- \eta^{\mu\nu} + g W_\mu^+ J_\nu^- \eta^{\mu\nu} + g W_\nu^- J_\nu^+ \eta^{\mu\nu}$$

$$J^{+\mu} = \bar{n}\gamma^\mu p + \bar{e}\gamma^\mu \nu_e + \bar{\mu}\gamma^\mu \nu_\mu + \dots \qquad \text{and} \qquad J^{-\mu} = \left(J^{+\mu}\right)^*$$

The equation of motion for the gauge fields: $\frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \qquad \Rightarrow \qquad W_\mu^- = \frac{g}{m_W^2} J_\mu^-$

Plugging back in the original Lagrangian, we obtain an effective Lagrangian (valid below the mass of the gauge bosons):

$$\mathcal{L} = \frac{g^2}{m_W^2} J_\mu^+ J_\nu^- \eta^{\mu\nu}$$

which is the Fermi current-current interaction. The Fermi constant is given by $G_F = \frac{g^2}{m_W^2}$
(the correct expression involves a different normalisation factor)

In the current-current product, the term $(\bar{n}\gamma^\mu p)(\bar{\nu}_e\gamma^\nu e)\eta_{\mu\nu}$ is responsible for beta decay, while the term $(\bar{\mu}\gamma^\mu \nu_\mu)(\bar{\nu}_e\gamma^\nu e)\eta_{\mu\nu}$ is responsible for muon decay. Both decays are controlled by the same coupling, as indicated by the measurements of the lifetimes of the muon and neutron.

SU(3) QCD

Deep inelastic experiments in the 60's revealed the internal structure of the neutrons and protons
Gell-Mann and others proposed that they are made of "quarks"

Up quark: spin-1/2, Q=2/3
Down quark: spin-1/2, Q=-1/3

SU(2) weak symmetry that changes neutrino into electron also changes up-quark into down-quark

But quarks carry yet another quantum number: "colour"
There 3 possible colours and Nature is colour-blind, i.e, Lagrangian should remain the same when the colours of the quarks are changed, i.e., when we perform a rotation in the colour-space of quarks

$$Q^a \to U^a{}_b Q^b$$

U: 3x3 matrix satisfying $U^\dagger U = 1_3$
such that the quark kinetic term is invariant

SU(3)

hadrons (spin-1/2, #hadronic=1): $p = uud$ $\quad n = udd$

mesons (spin-0, #hadronic=0): $\pi^0 = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$ $\quad \pi^+ = u\bar{d}$ $\quad \pi^- = d\bar{u}$

(Each quark carries a baryon number =1/3)

There are other (heavier) quarks and hence other baryons and mesons

All the interactions of the SM preserve baryon and lepton numbers

$\mu \to e\nu_\mu\bar{\nu}_e$ $\qquad n \to p\, e\, \bar{\nu}_e$ $\qquad \pi^- \to \mu^- \bar{\nu}_\mu$ $\qquad \pi^0 \to \gamma\gamma$ $\qquad p \not\to \pi^0 \bar{e}$

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***

  ➘ Photon γ

  { light, atoms, molecules } — 10^{-5}

- $SU(2)_L$ ***weak interactions***

  ➘ bosons $W^{\pm}, Z^0$

  { β decay

  $$n \xrightarrow{W^{\pm}} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

  } — 10^{-2}

- $SU(3)_c$ ***strong interactions***

  ➘ gluons g^a

  { atomic nuclei

  α decay

  $${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

  } — 1

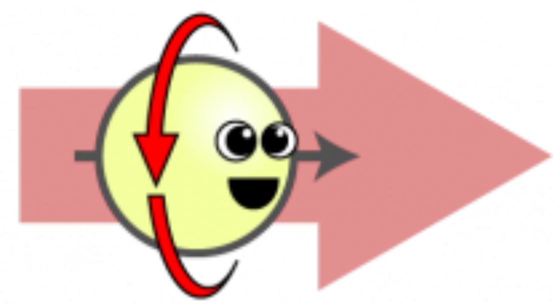
strength ↓

Chirality & Masslessness

Quantum Mechanics 1.0.1

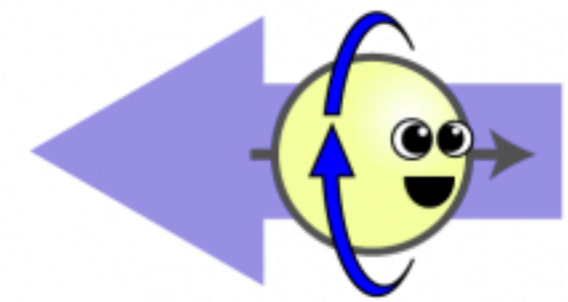
Particle of spin s has 2s+1 polarisation states

Particle spinning anticlockwise wrt its direction of motion



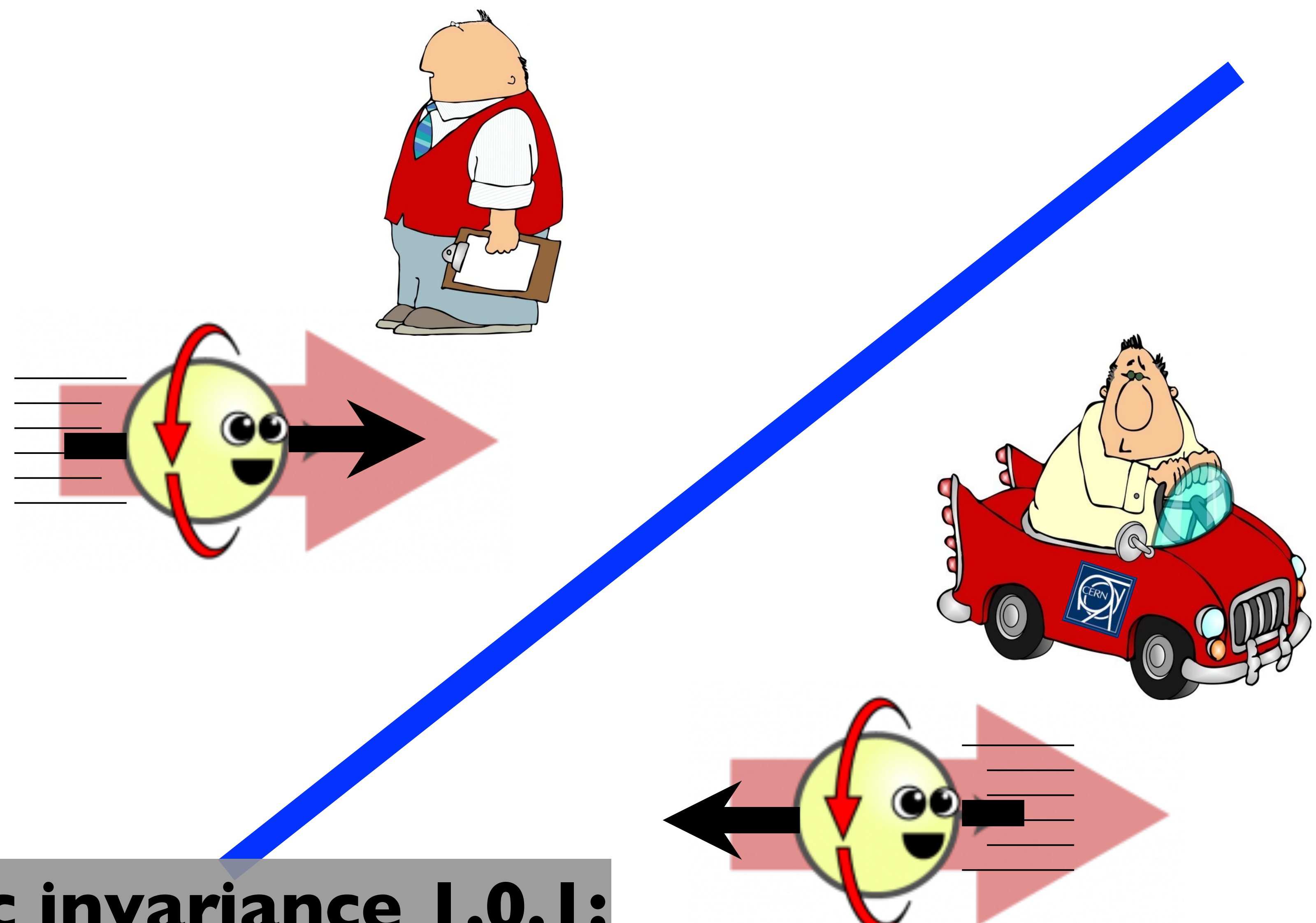
electron has 2 polarisation

Particle spinning clockwise wrt its direction of motion



Picture courtesy to G. Giudice

Chirality & Masslessness



Picture courtesy to G. Giudice

Relativistic invariance 1.0.1:

there must be no distinction for massive particles between particles spinning clockwise or anti-clockwise

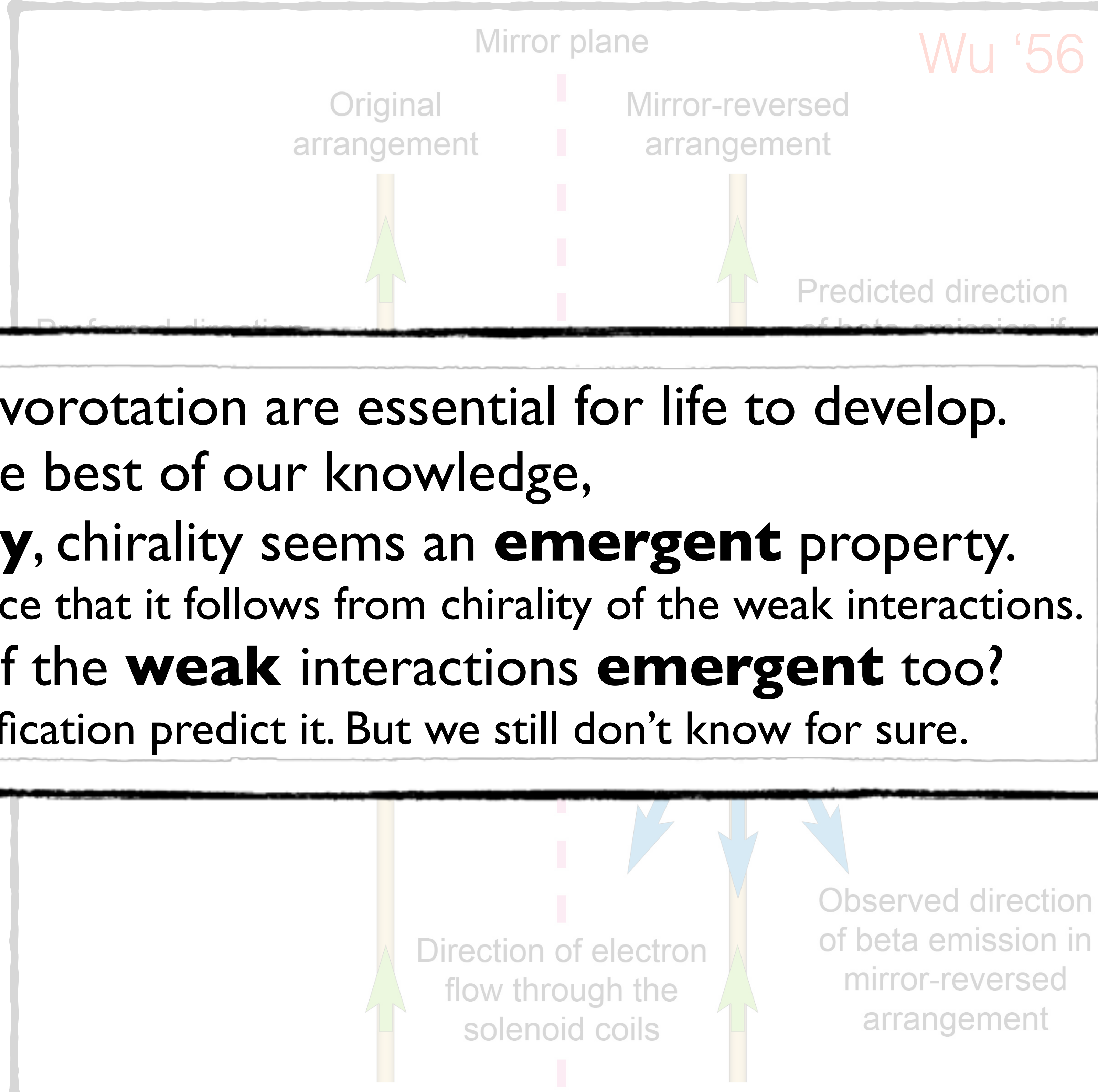
[chirality operator doesn't commute with the Hamiltonian]

Theorem ⇒ **If your theory sees a difference between e_L and e_R, either your theory is wrong or m_e=0**

Chirality of SM & Mass problem

Weak interaction
(force responsible for
neutron decay)
is chiral!

[e_L and e_R are fundamentally



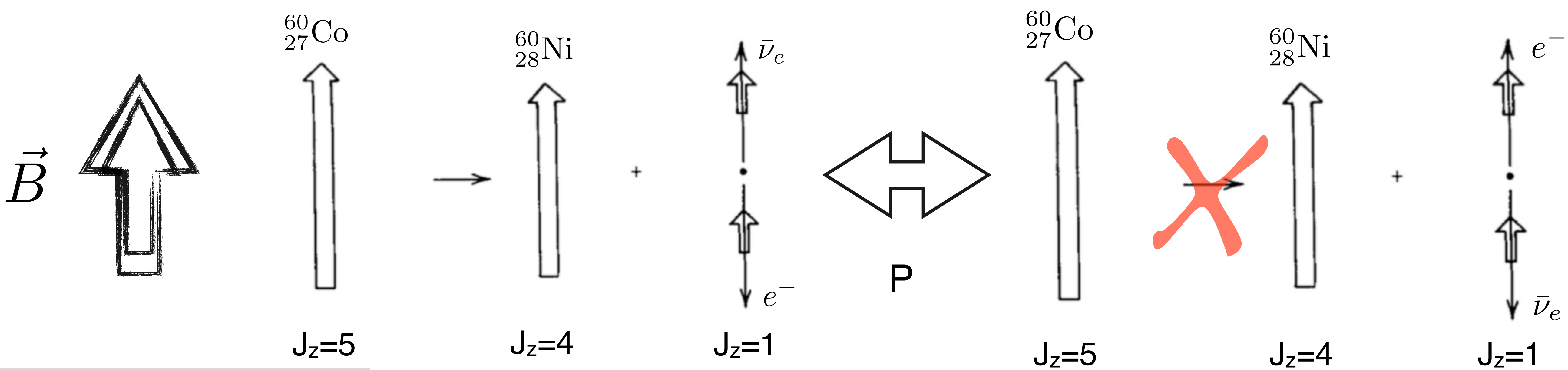
Dextrorotation and Levorotation are essential for life to develop.
To the best of our knowledge,
in **molecular biology**, chirality seems an **emergent** property.
At least, there is no clear evidence that it follows from chirality of the weak interactions.
Are the chiral nature of the **weak** interactions **emergent** too?
Some models of grand unification predict it. But we still don't know for sure.

But since we know it is not true, we

need a new phenomena to generate mass: Higgs mechanism

SM is a Chiral Theory

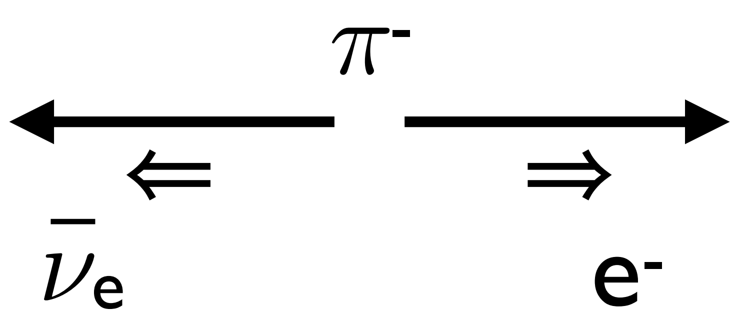
Weak interactions maximally violates P



$^{60}_{27}\text{Co} \to {}^{60}_{28}\text{Ni} + e^- + \bar{\nu}_e$

only LH e⁻ produced

TH: Yang&Lee '56. EXP: Wu '57



Conservation of momentum and spin imposes to have a RH e⁻

Weak decays proceed only w/ LH e⁻
So the amplitude is prop. to m_e

$$\mathcal{L}_{\text{Dirac}} = \bar{\psi}_L \gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R \gamma^\mu \partial_\mu \psi_R + m \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right)$$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{m_e^2}{m_\mu^2} \sim 2 \times 10^{-5} \sim 10^{-4}_{\text{obs}}$$

↑ Extra phase-space factor

Technical Details for Advanced Students

Chirality

Chirality matrix

$$\boxed{\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3}$$

A few remarkable properties

$$\left(\gamma^5\right)^2 = 1_4$$

$$\{\gamma^5, \gamma^\mu\} = 0$$

$$\gamma^{5\dagger} = \gamma^5 = -\gamma^0\gamma^5\gamma^0$$

Chiral/Weyl spinor

A **chiral/Weyl** spinor is an eigenvector of the chirality matrix $\psi_{L,R} = \pm\gamma^5\psi_{L,R}$

From the Lorentz-transformation law of a spinor, it is obvious that the chirality condition is frame-independent

A Dirac spinor can also be written as a sum of two chiral spinors

$$\psi = \frac{1}{2}\left(1_4 + \gamma^5\right)\psi + \frac{1}{2}\left(1_4 - \gamma^5\right)\psi \equiv \psi_L + \psi_R$$

Charge conjugation

In general, ψ and ψ^* do not transform in the same way under Lorentz transformations and the naive reality condition $\psi = \psi^*$ is frame dependent

But it is possible to find a matrix C, called charge conjugation matrix, such that

$$\psi \quad \text{and} \quad \psi_C = C\,\psi^*$$

transform in the same way under Lorentz transformations

The matrix C needs to satisfy $C\gamma^* = -\gamma^\mu C$

In the Dirac and Weyl representations, $C = i\gamma^2$

In the Majorana representation, $C = 1_4$

Basic properties of the charge conjugation matrix: $C^2 = 1_4,\ C^\dagger = C,\ C^* = C$

The charge conjugated spinor, ψ_C, satisfies the same Dirac equation as ψ, with the same mass but opposite electric charge (when the spinor is minimally coupled to a U(1) gauge field)

A **Majorana** spinor satisfies the (Lorentz invariant!) condition $\psi = \psi_C$

Note that in 4D, a spinor cannot be simultaneously chiral and Majorana

Dirac and Majorana Masses

By construction, the following two mass terms in the Lagrangian are Lorentz-invariant

Dirac mass: $\mathcal{L}_{\rm Dirac} = m\bar{\psi}\psi$ (conserves fermion number)

Majorana mass: $\mathcal{L}_{\rm Majorana} = m\bar{\psi}_C\,\psi$ (changes fermion number by 2)

These two mass terms have different a chirality structure

$$\mathcal{L}_{\rm Dirac} = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right)$$

$$\mathcal{L}_{\rm Majorana} = m\left(\bar{\psi}_{L_C}\psi_L + \bar{\psi}_{R_C}\psi_R\right)$$

A chiral fermion can have a Majorana mass
A Dirac mass requires spinors of opposite chirality

Whether or not a Dirac or a Majorana mass can be included in the Lagrangian depends on transformation laws of the spinors under the gauge transformations

Within the SM (with the Higgs field), a Dirac mass can written for the charged leptons and the quarks while a Majorana mass can be written for the neutrinos.