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Standard Model | **Homework 4** | **Date: 18.07.2019**

Exercice 1: Goldstone equivalence theorem

a) Draw the Feynman diagram for the decay of the top quark, $t \to bW$.
b) Neglecting the bottom mass, calculate, in the unitary gauge, the decay rates for $t \to bW_T^+$ and $t \to bW_L^+$. Reproduce the latter result using the equivalence theorem, i.e. by computing the $t \to b\phi^+$, where ϕ^+ is the would-be Goldstone boson.
c) Draw the Feynman diagram for the Higgs decay into a pair of W gauge bosons.
d) In the heavy Higgs mass limit, compute, in the unitary gauge, the decay widths of $h \to W_T^+W_T^-$ and $h \to W_L^+W_L^-$. Reproduce the latter result using the equivalence theorem, i.e. by computing the $h \to \phi^+\phi^-$, where $\phi^\pm$ are the would-be Goldstone bosons.

Exercice 2: Higgs decays

In the lecture we have seen that in the Standard Model the Higgs boson couples to fermion proportionally to their mass, i.e., the interaction Lagrangian writes $\mathcal{L} = -\frac{\sqrt{2}m_f}{v} h\bar{f}f$.
a) Write down the Feynman rule for the vertex between a Higgs boson and fermion and its anti-fermion.
b) Compute the amplitude associated to the decay $h \to b\bar{b}$.
c) Compute $\langle|M|^2\rangle$. *Help:* You can use the completeness relation $\sum_{spin}(\bar{u}(p)v(q))(\bar{u}(p)v(q))^* = \mathrm{Tr}((\gamma^\mu p_\mu + m_f)(\gamma^\nu q_\nu - m_f))$ and remenber that $\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4\eta^{\mu\nu}$.
d) Compute the partial decay width $\Gamma(h \to b\bar{b})$. *Help:* We recall that the partial decay width for a two body decay is obtained from the amplitude as $\Gamma = \frac{p\langle|M|^2\rangle}{8\pi m_h^2}$ where p is the modulus of the 3D momentum of the two daughter particles in the rest-frame of the decaying particle (momentum that you'll compute using energy conservation). And you'll use the numerical input: $v \approx 246\,\mathrm{GeV}$, $m_h \approx 125\,\mathrm{GeV}$ and $m_b \approx 4\,\mathrm{GeV}$.
e) Given that $BR(h \to b\bar{b}) \approx 57\%$, compute the total decay width of the Higgs boson.
f) If the Higgs boson were a pseudo-scalar instead of a scalar field, how would you write the interaction Lagrangian?
g) In the pseudo-scalar case, compute the Higgs decay width. *Help:* You can use the completeness relation $\sum_{spin}(\bar{u}(p)\gamma^5 v(q))(\bar{u}(p)\gamma^5 v(q))^* = \mathrm{Tr}((\gamma^\mu p_\mu + m_f)\gamma^5(\gamma^\nu q_\mu - m_f)\gamma^5)$ and remember that $\gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$ and $(\gamma^5)^2 = 1_4$.