Making Predictions for Hadron Colliders

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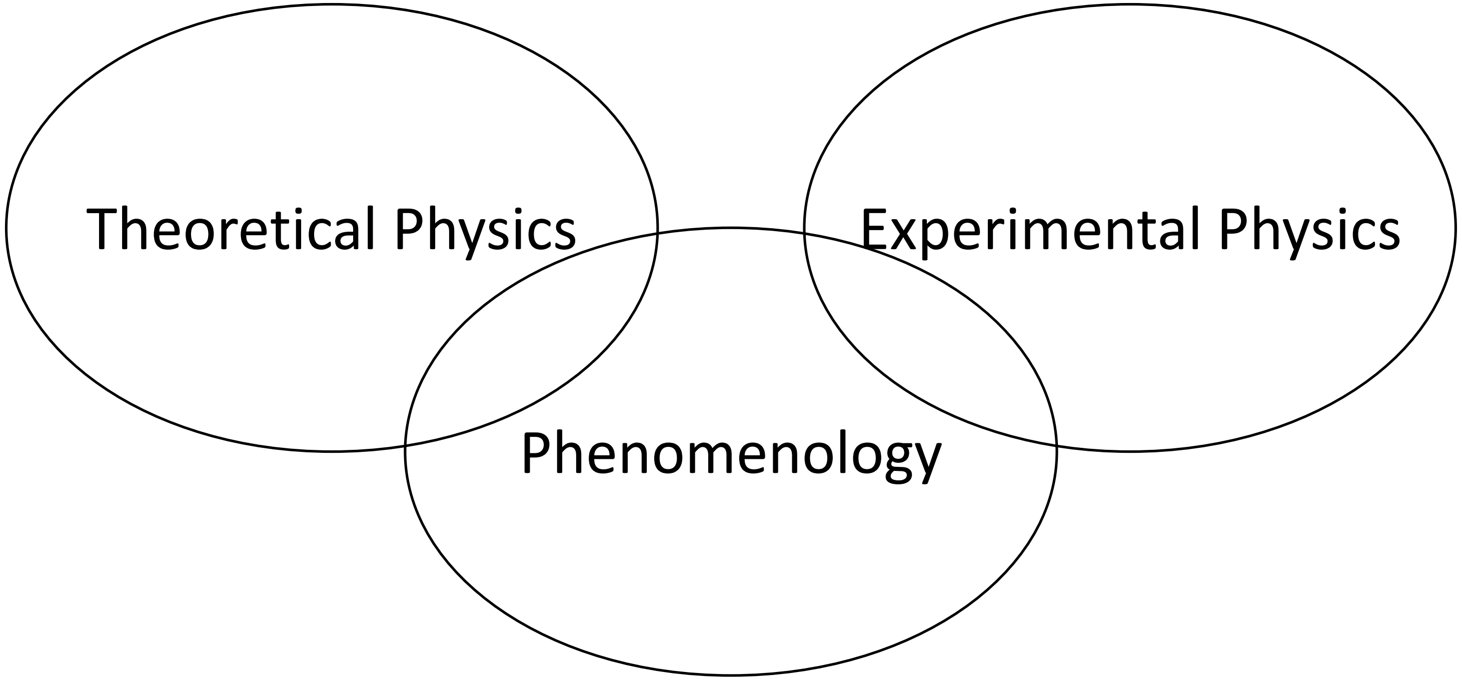
Making Predictions for Hadron Colliders

1. From Feynman Diagrams to Cross Sections

Phenomenology



Calculating Event Rates

$$N = \mathcal{L}\,\sigma$$

Number of events

Integrated Luminosity

Cross section

Calculating Cross Sections

$$\mathrm{d}\sigma = \frac{1}{F}\,|\mathcal{M}|^2\,\mathrm{d}LIPS$$

Flux factor

$$= 2s = 4E_1E_2$$

(Quantum mechanical) amplitude squared

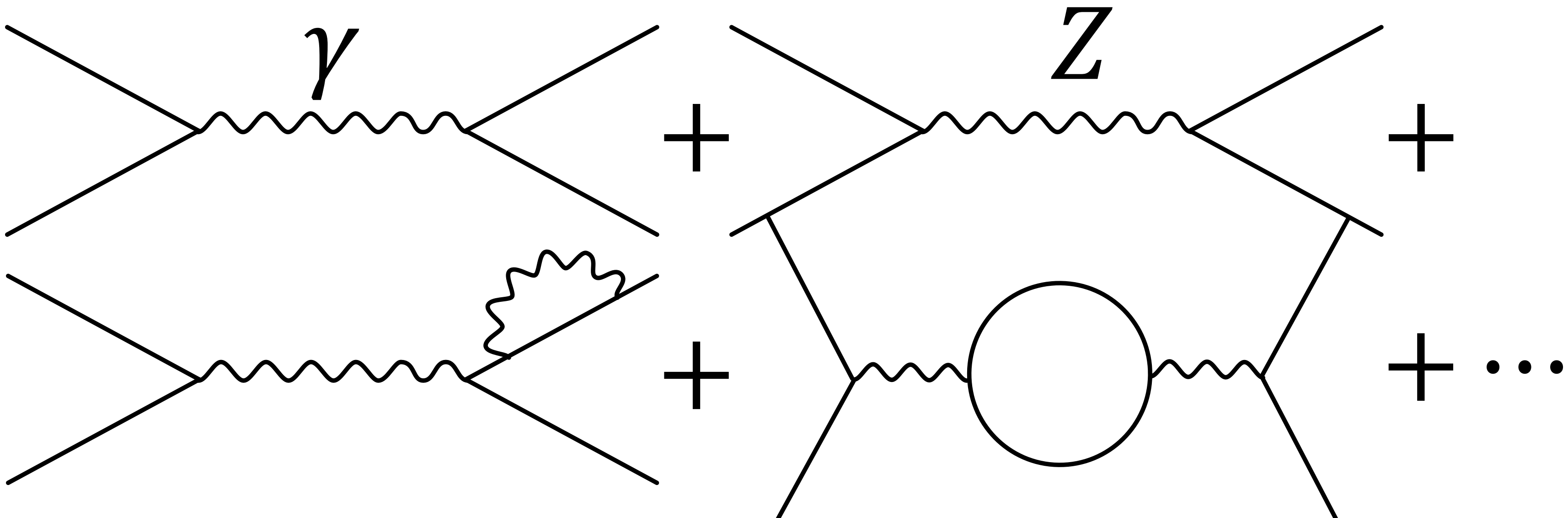
Lorentz Invariant Phase Space

$$= \frac{\mathrm{d}^4p_i}{(2\pi)^4}(2\pi)\delta\left(p_i^2 - m_i^2\right)\,\frac{\mathrm{d}^4p_j}{(2\pi)^4}(2\pi)\delta\left(p_j^2 - m_j^2\right)\ldots\,(2\pi)^4\delta(p_1 + p_2 - p_i - p_j - \ldots)$$

Calculating Cross Sections

$$\mathrm{d}\sigma = \frac{1}{F}\,|\mathcal{M}|^2\,\mathrm{d}LIPS$$

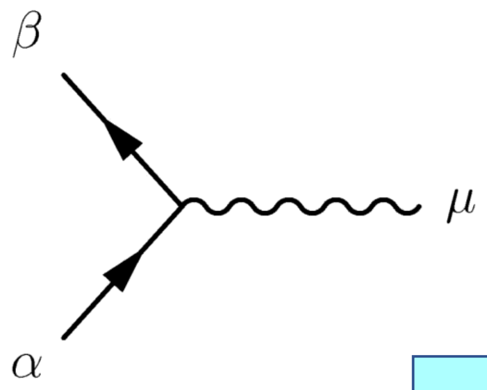
$\mathcal{M} =$



Feynman Rules

$$\rightarrow \left(\frac{i}{\not{p} - m + i\varepsilon}\right)_{\beta\alpha}$$

$$\rightarrow \frac{-i\eta_{\mu\nu}}{p^2 + i\varepsilon}$$



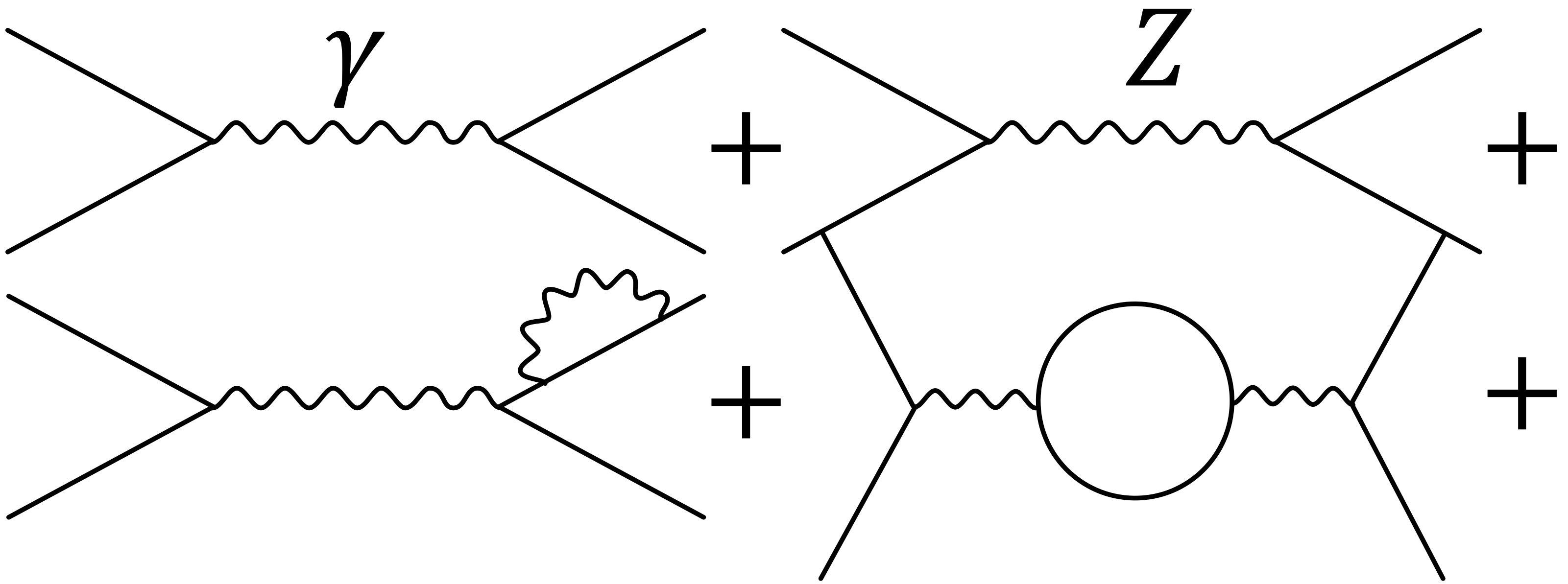
$$\rightarrow -ie\gamma^{\mu}_{\beta\alpha}(2\pi)^4\delta^{(4)}(p_1 + p_2 + p_3).$$

Elementary charge

$$e = \sqrt{4\pi\alpha} \quad \alpha \approx {}^{1}/_{137}$$

Tree Diagrams as Leading Order of Expansion in α

$\mathcal{M} =$



$+ \cdots$

$\mathcal{O}(\alpha)$

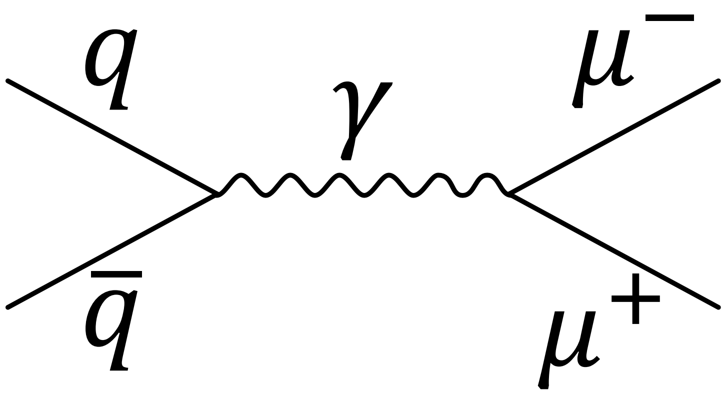
$\mathcal{O}(\alpha^2)$

$\alpha \approx {}^{1}/_{137}$ but $\alpha_s \approx 0.1$

$\Rightarrow$ QCD corrections important

Example: The Drell-Yan process ($pp \to \mu^+\mu^-$)

$\mathcal{M} =$



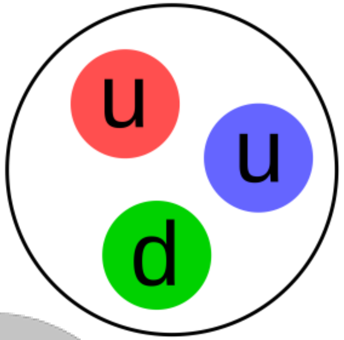
$$\Rightarrow |\mathcal{M}|^2 \propto e_q^2 \alpha^2 \frac{t^2 + u^2}{s^2} \propto e_q^2 \alpha^2 (1 + \cos^2\theta)$$

$$\Rightarrow \sigma = \frac{4\pi\alpha^2}{9Q^2} e_q^2$$

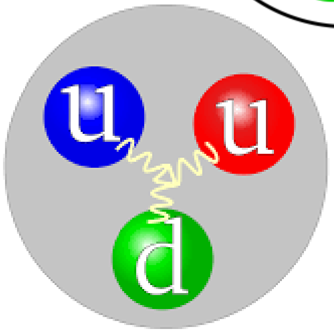
$\left(s = Q^2 = \left(p_q + p_{\bar{q}}\right)^2\right)$

Proton structure

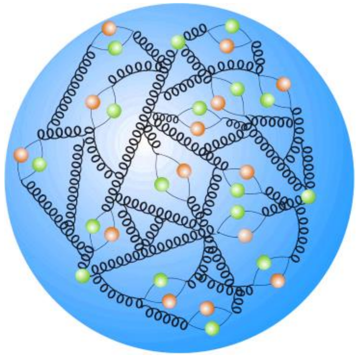
- Proton = uud ?



- Held together by gluons?



- Quantum Field Theory: gluons can create $q\bar{q}$ pairs

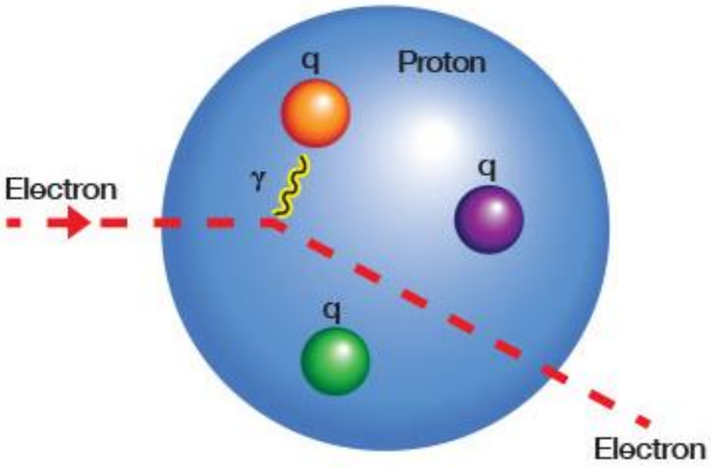


- Proton can interact through any of its partons

Proton structure: parton distribution functions

- How is the proton's energy shared between its parton constituents?
- Measure in deep inelastic electron scattering
- Quantify by *parton distribution function*

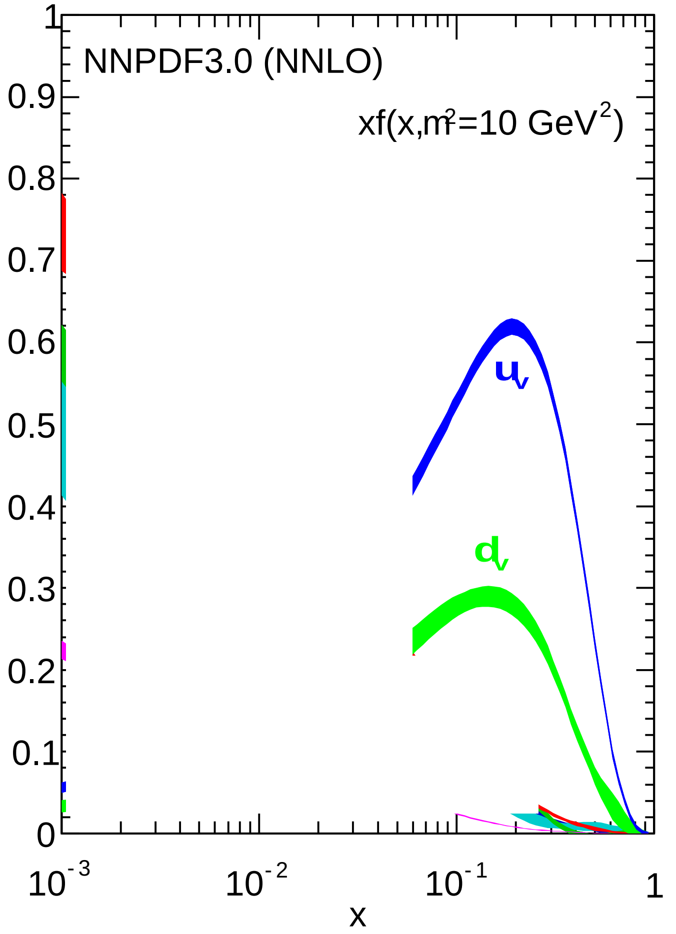
  $f_i(x)\mathrm{d}x$ = probability that parton of type i is found with fraction of proton's momentum between x and $x + \mathrm{d}x$

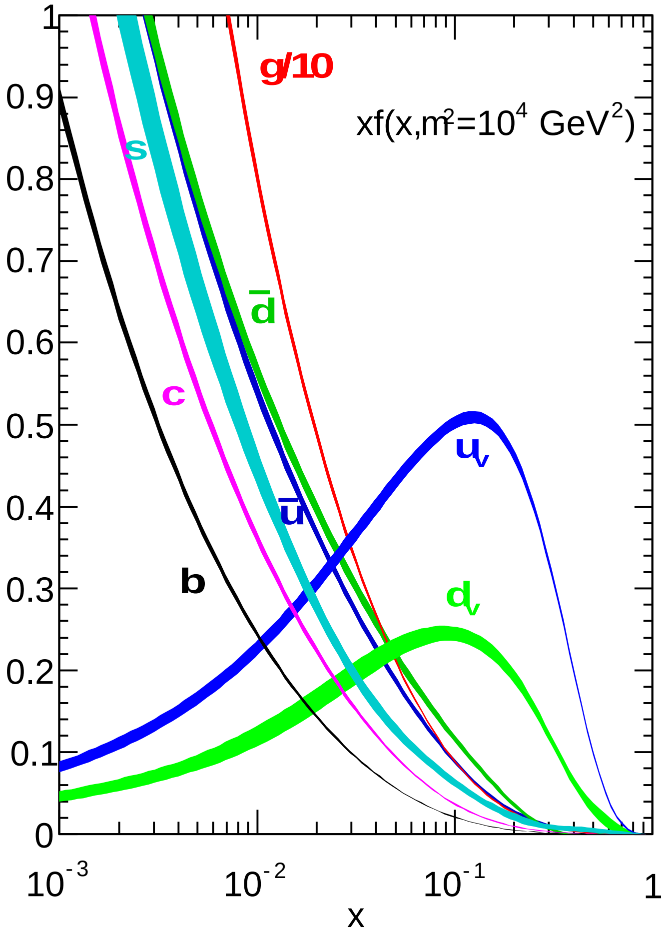


- But how long do those quantum fluctuations live?

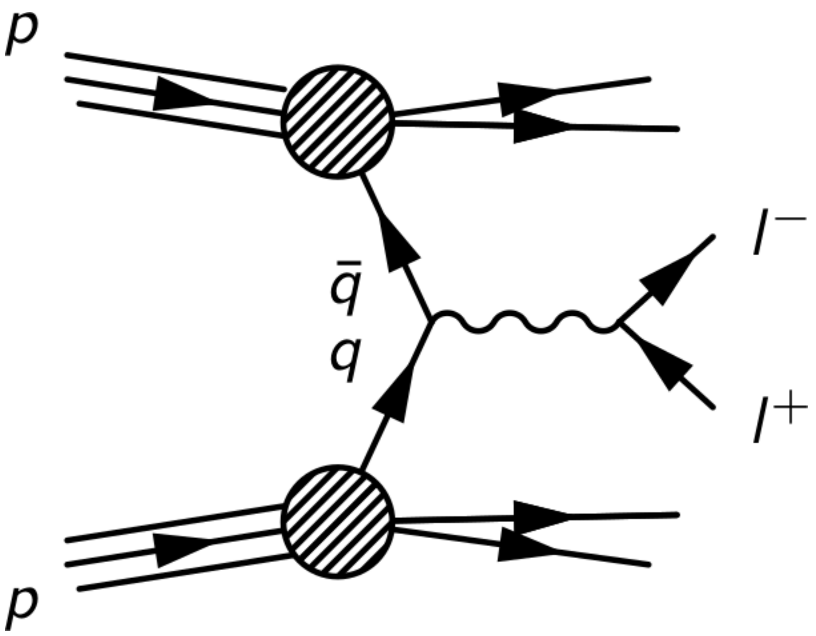
⇒ PDFs depend on the momentum scale of the probe $f_i(x, Q^2)\mathrm{d}x$

Proton structure: parton distribution functions



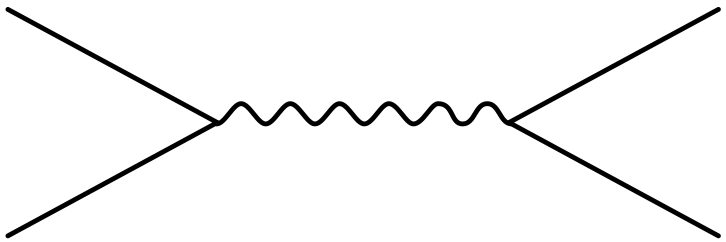
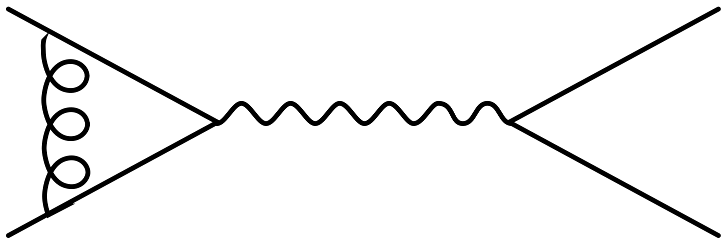


The Drell-Yan process ($pp \to \mu^+\mu^-$)



$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \sum_q \int \mathrm{d}x_1\, f_q(x_1, Q^2)\, \mathrm{d}x_2\, f_{\bar{q}}(x_2, Q^2)\, \frac{4\pi\alpha^2}{9Q^2}\, e_q^2\, \delta(x_1 x_2 s - Q^2)$$

Loop Diagrams as Higher Order Corrections

$\mathcal{M} =$  $+$  $+\cdots$

$\mathcal{O}(\alpha)$ $\qquad$ $\mathcal{O}(\alpha\alpha_s)$

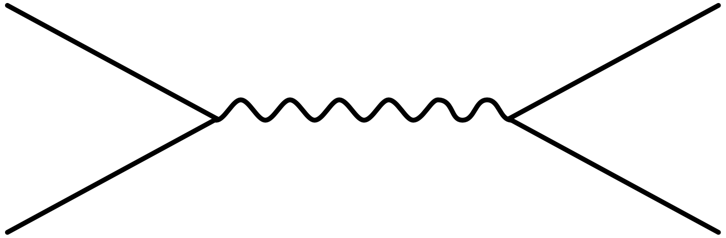
$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 + 2\Re(\mathcal{M}_0^*\mathcal{M}_1) + |\mathcal{M}_1|^2 + \cdots$$

$\mathcal{O}(\alpha^2)$ $\qquad$ $\mathcal{O}(\alpha^2\alpha_s)$

- Quantum mechanics: sum over unobserved quantum numbers
  = integrate over gluon momenta

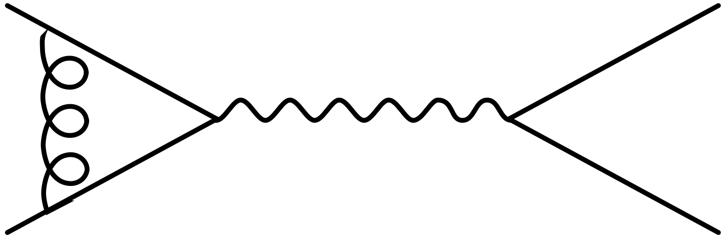
Loop Diagrams as Higher Order Corrections

$\mathcal{M} =$



$\mathcal{O}(\alpha)$

$+$

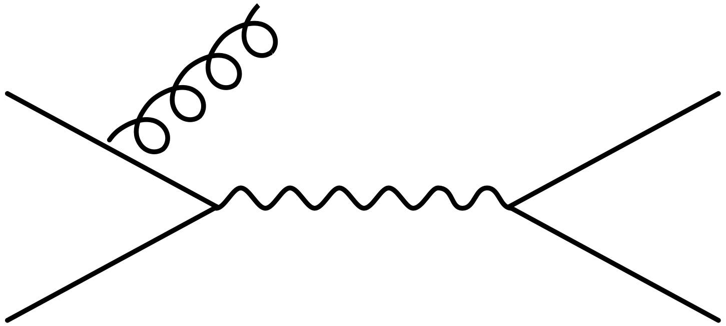
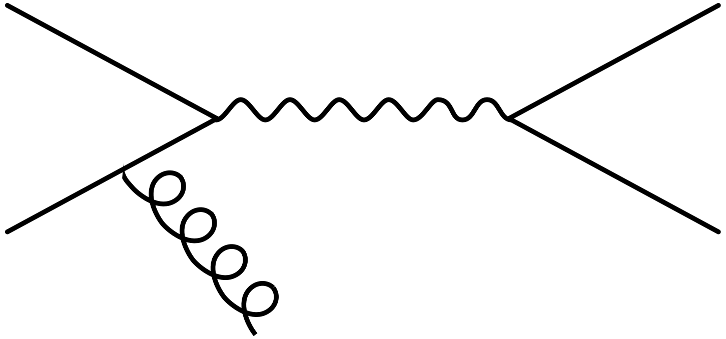


$\mathcal{O}(\alpha\alpha_s)$

$+ \cdots$

- Gluon momentum integral is divergent! (= *minus* infinity)
- Divergence comes from:
  - Momentum = 0
  - Momentum = parallel to quark or antiquark

Gluon Emission as Higher Order Correction

$\mathcal{M} =$  $+$  $\mathcal{O}(\alpha\sqrt{\alpha_s})$

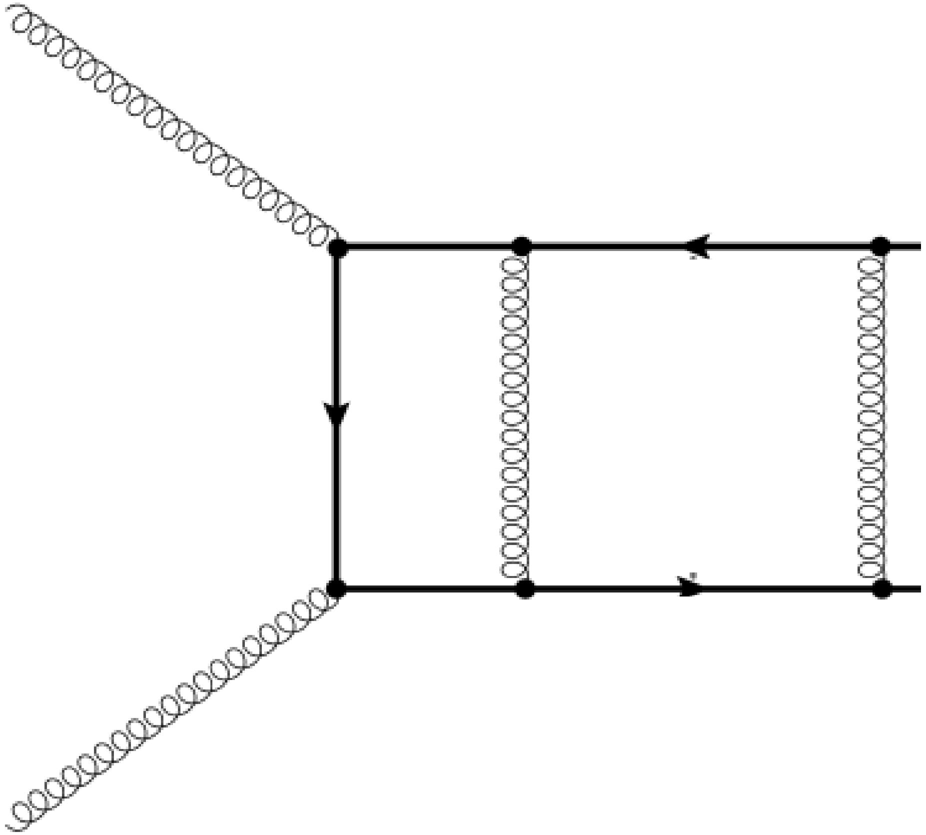
- Gluon emission describes a different process $(q\bar{q} \to \mu^+\mu^- g)$
- But if we are only interested in the total cross section for Drell-Yan pairs, must integrate over gluon momenta
- Divergent from momentum = 0 or parallel to quark or antiquark
- Cancels loop divergence

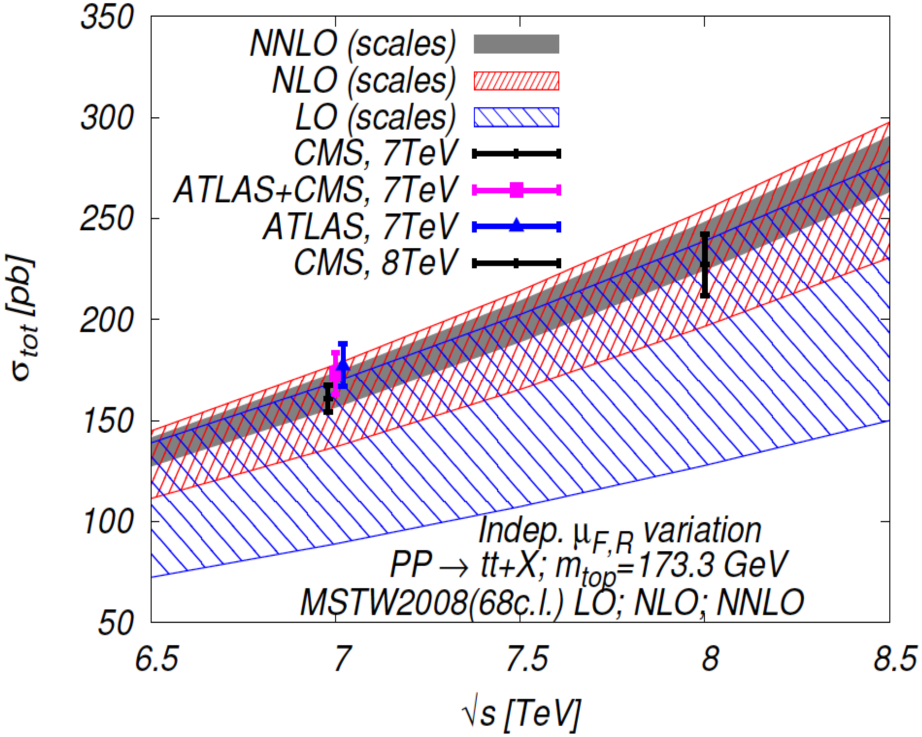
Next-to-Leading Order (NLO) cross section

- $\sigma_{NLO} = \sigma_{tree} + \sigma_{loop} + \sigma_{emission}$
- σ_{loop} and $\sigma_{emission}$ each divergent
  - must regularize and expose singularities of each
  - *Subtraction algorithms*
- Fully automated,
  - e.g. in Madgraph/aMC@NLO, MCFM, Sherpa, Herwig …

State of the Art – NNLO Calculations

e.g. $pp \to t\bar{t}$





From Feynman Diagrams to Cross Sections

- Major part of phenomenology = calculating cross sections
- LO = write down all tree diagrams, integrate phase space numerically
- Convolute with parton distribution functions (fitted to data)
- NLO = one-loop diagrams, one-emission processes
  - Extract singularities from integrals, integrate analytically
  - Integrate remainders numerically
- NNLO = two-loop diagrams, one-emission at one-loop, and two emissions
- But LHC events contain *hundreds* of additional particles…