



Making Predictions for Hadron Colliders

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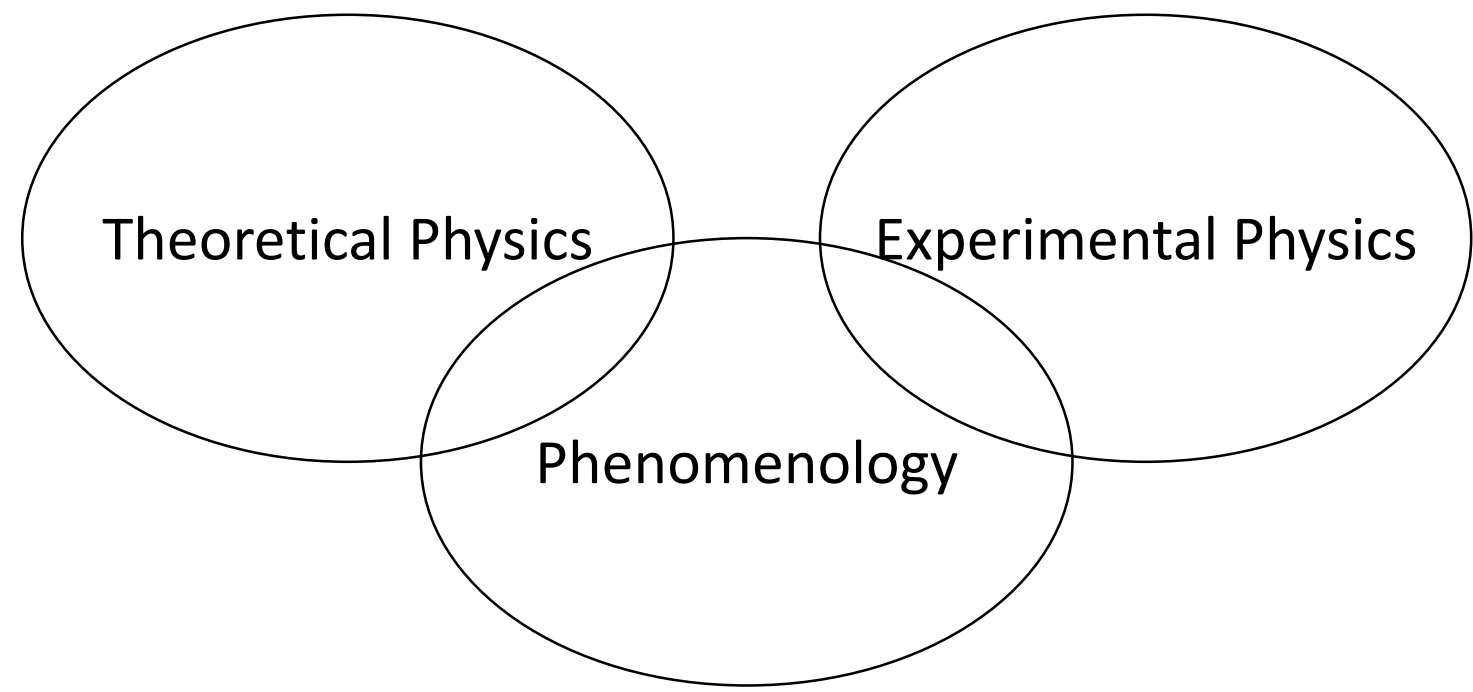


Making Predictions for Hadron Colliders

1. From Feynman Diagrams to Cross Sections



Phenomenology



Calculating Event Rates

$$N = \mathcal{L} \sigma$$

Number of
events

Integrated
Luminosity

Cross
section

Calculating Cross Sections

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dLIPS$$

Flux factor

$$= 2s = 4E_1E_2$$

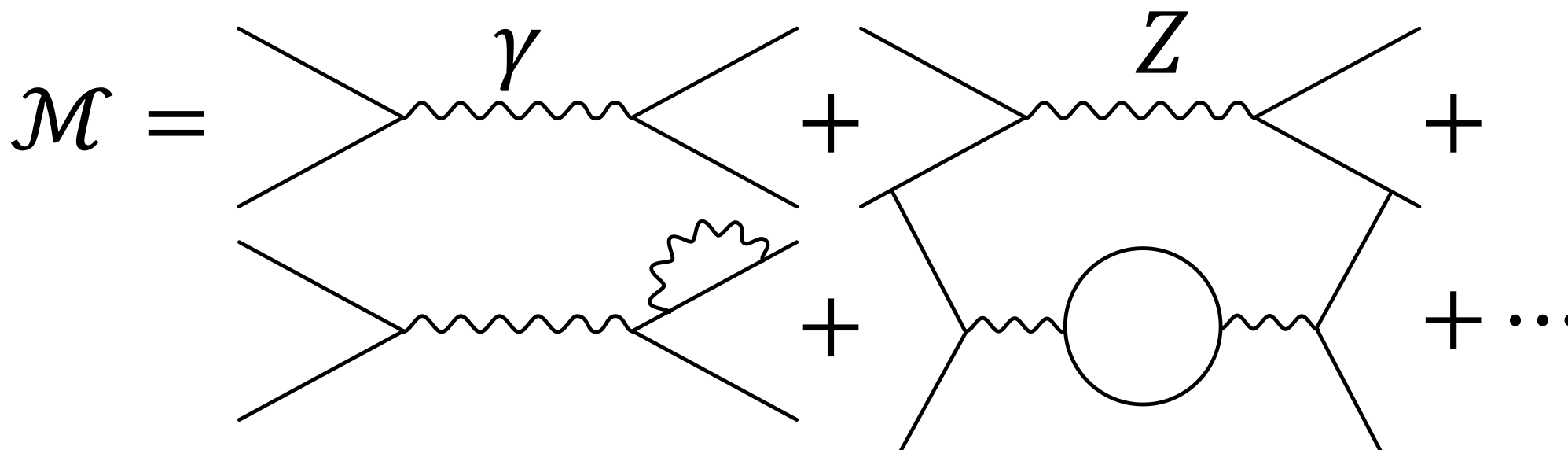
(Quantum
mechanical)
amplitude
squared

Lorentz
Invariant
Phase
Space

$$= \frac{d^4p_i}{(2\pi)^4} (2\pi)\delta(p_i^2 - m_i^2) \frac{d^4p_j}{(2\pi)^4} (2\pi)\delta(p_j^2 - m_j^2) \dots (2\pi)^4 \delta(p_1 + p_2 - p_i - p_j - \dots)$$

Calculating Cross Sections

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dLIPS$$



Feynman Rules

$$\alpha \longrightarrow \longrightarrow \beta \quad \rightarrow \quad \left(\frac{i}{\not{p} - m + i\epsilon} \right)_{\beta\alpha}$$

$$\mu \text{ ~~~~~ } \nu \quad \rightarrow \quad \frac{-i\eta_{\mu\nu}}{p^2 + i\epsilon}$$

$$\begin{array}{c} \beta \\ \nearrow \\ \text{---} \\ \searrow \\ \alpha \end{array} \text{ ~~~~~ } \mu \quad \rightarrow \quad -ie\gamma_{\beta\alpha}^{\mu} (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3).$$

Elementary charge

$$e = \sqrt{4\pi\alpha} \quad \alpha \approx 1/137$$

Tree Diagrams as Leading Order of Expansion in α

$$\mathcal{M} = \begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \end{array} + \begin{array}{c} \text{Diagram 3} \\ \text{Diagram 4} \end{array} + \dots + \mathcal{O}(\alpha^2)$$

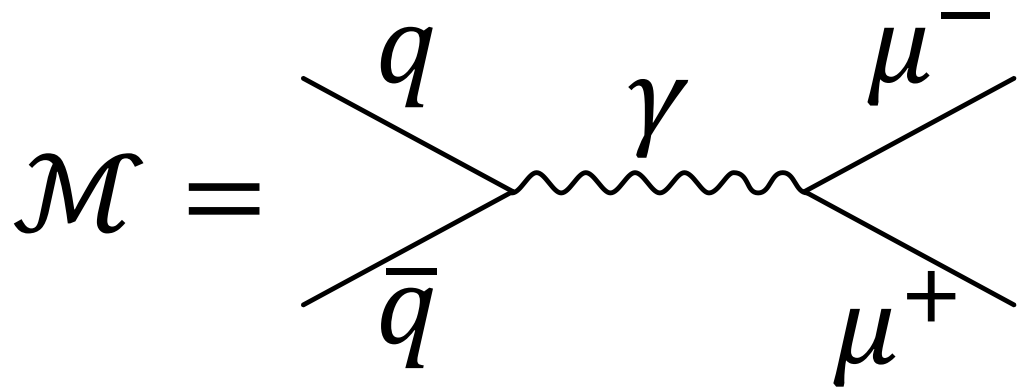
The diagram shows the expansion of the amplitude \mathcal{M} in powers of the coupling constant α . It consists of several terms:

- Two tree-level diagrams (order α):
 - Diagram 1: A wavy line labeled γ connects two vertices, each with two external lines.
 - Diagram 2: A wavy line labeled Z connects two vertices, each with two external lines.
- Two higher-order diagrams (order α^2):
 - Diagram 3: A tree-level diagram with a wavy line and a loop (represented by a circle) attached to it.
 - Diagram 4: A tree-level diagram with a wavy line and a loop (represented by a circle) attached to it.
- Ellipses and $\mathcal{O}(\alpha^2)$ indicate higher-order terms in the expansion.

$$\alpha \approx 1/137 \text{ but } \alpha_s \approx 0.1$$

\Rightarrow QCD corrections important

Example: The Drell-Yan process ($pp \rightarrow \mu^+ \mu^-$)



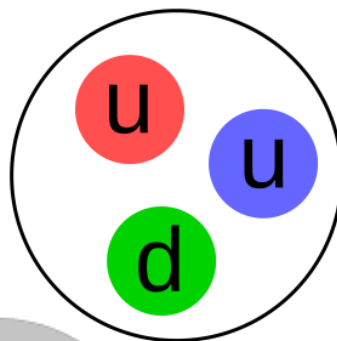
$$\Rightarrow |\mathcal{M}|^2 \propto e_q^2 \alpha^2 \frac{t^2 + u^2}{s^2} \propto e_q^2 \alpha^2 (1 + \cos^2 \theta)$$

$$\Rightarrow \sigma = \frac{4\pi\alpha^2}{9Q^2} e_q^2$$

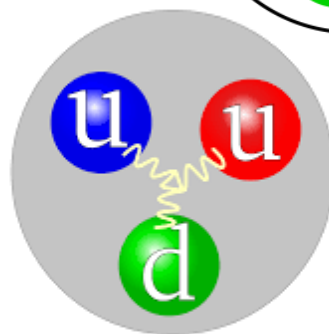
$$(s = Q^2 = (p_q + p_{\bar{q}})^2)$$

Proton structure

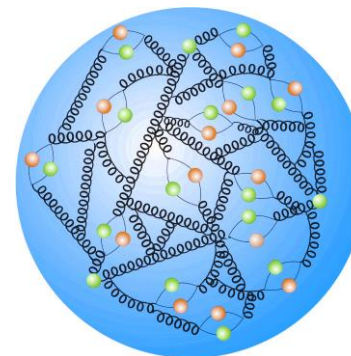
- Proton = uud ?



- Held together by gluons?



- Quantum Field Theory: gluons can create $q\bar{q}$ pairs

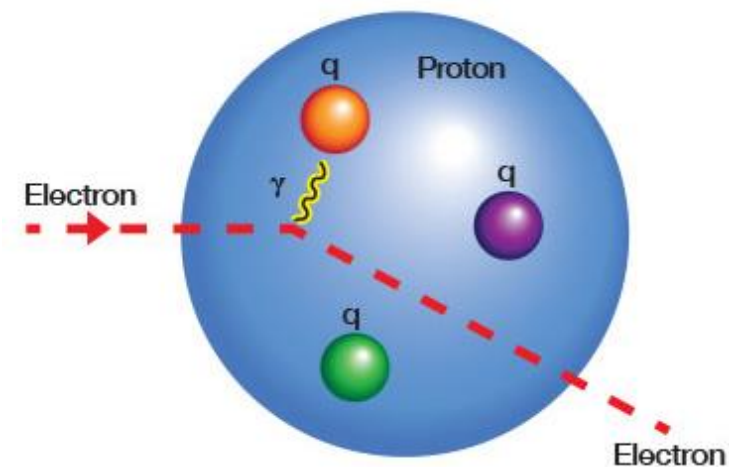


- Proton can interact through any of its partons

Proton structure: parton distribution functions

- How is the proton's energy shared between its parton constituents?
- Measure in deep inelastic electron scattering
- Quantify by *parton distribution function*

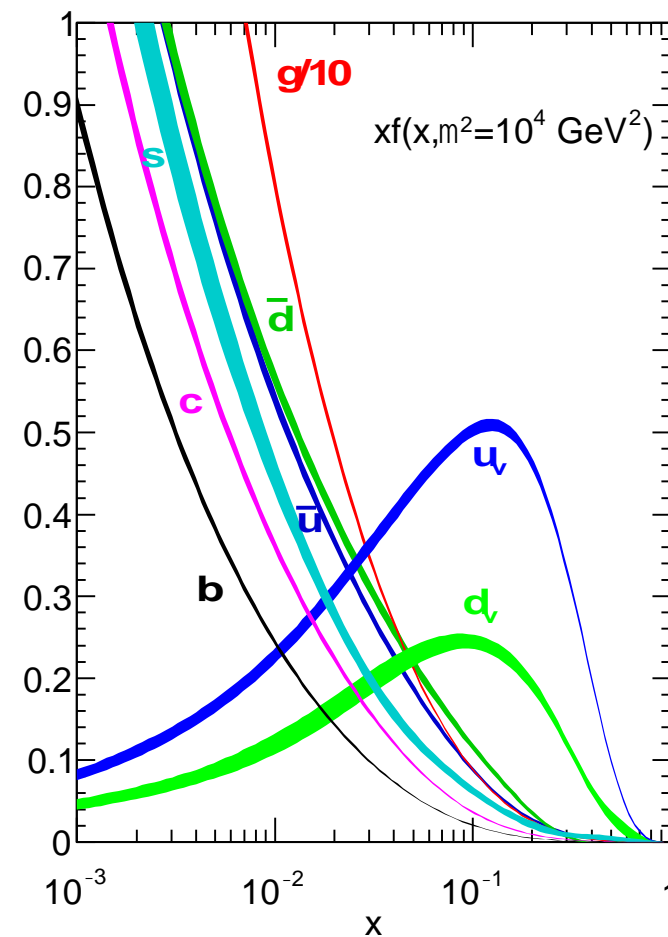
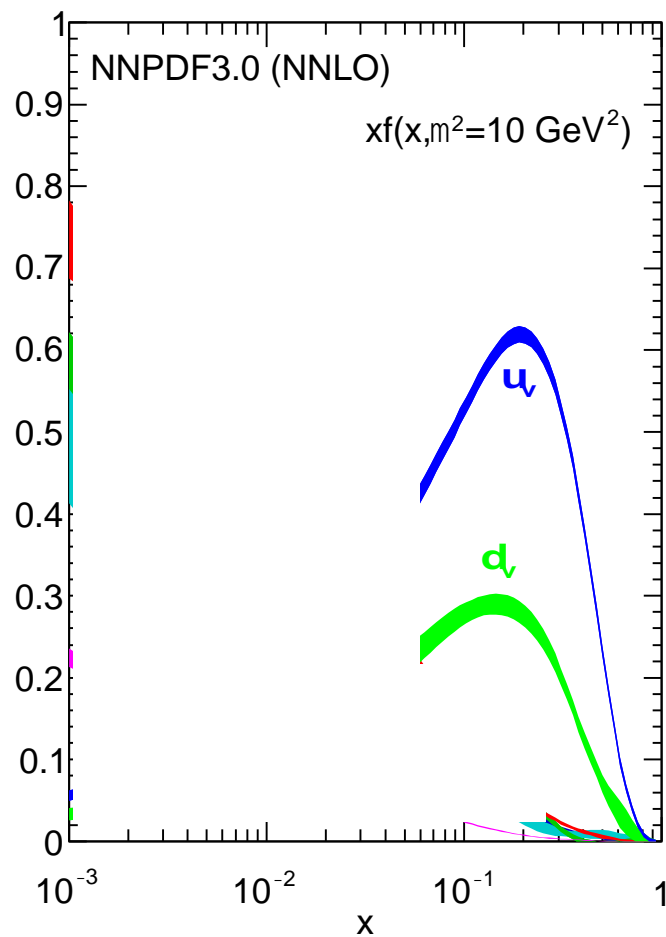
$f_i(x)dx$ = probability that parton of type i is found with fraction of proton's momentum between x and $x + dx$



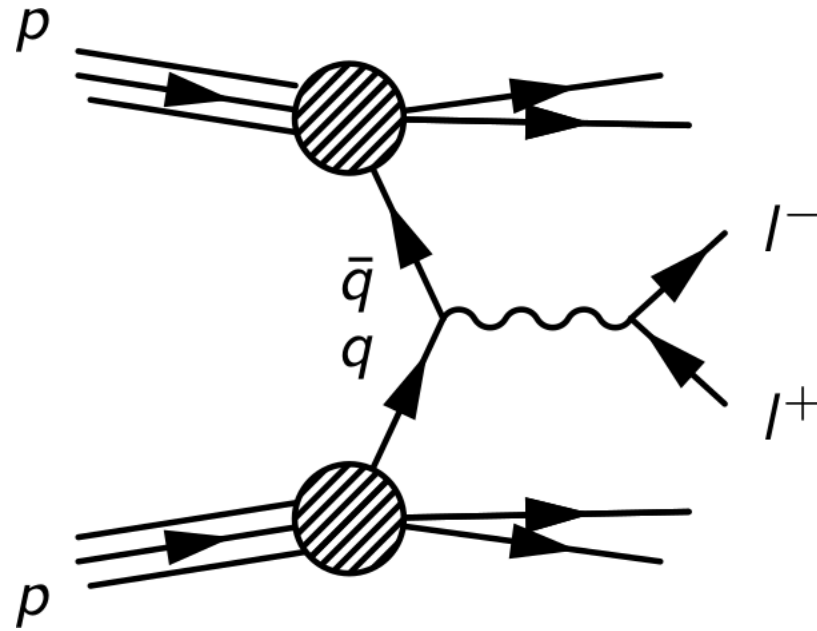
- But how long do those quantum fluctuations live?

⇒ PDFs depend on the momentum scale of the probe $f_i(x, Q^2)dx$

Proton structure: parton distribution functions



The Drell-Yan process ($pp \rightarrow \mu^+ \mu^-$)



$$\frac{d\sigma}{dQ^2} = \sum_q \int dx_1 f_q(x_1, Q^2) dx_2 f_{\bar{q}}(x_2, Q^2) \frac{4\pi\alpha^2}{9Q^2} e_q^2 \delta(x_1 x_2 s - Q^2)$$

Loop Diagrams as Higher Order Corrections

$$\mathcal{M} = \text{tree diagram } \mathcal{O}(\alpha) + \text{loop diagram } \mathcal{O}(\alpha\alpha_s) + \dots$$

$$|\mathcal{M}|^2 = |\mathcal{M}_0|^2 + 2\Re(\mathcal{M}_0^* \mathcal{M}_1) + |\mathcal{M}_1|^2 + \dots$$

$\mathcal{O}(\alpha^2)$ $\mathcal{O}(\alpha^2\alpha_s)$

- Quantum mechanics: sum over unobserved quantum numbers = integrate over gluon momenta

Loop Diagrams as Higher Order Corrections

$$\mathcal{M} = \text{tree diagram } \mathcal{O}(\alpha) + \text{loop diagram } \mathcal{O}(\alpha\alpha_s) + \dots$$

The diagram shows the expansion of the amplitude \mathcal{M} . The first term is a tree-level diagram with a wavy gluon line between two quark vertices, labeled $\mathcal{O}(\alpha)$. The second term is a one-loop diagram with a gluon loop on the left side of the wavy gluon line, labeled $\mathcal{O}(\alpha\alpha_s)$. The series continues with an ellipsis.

- Gluon momentum integral is divergent! (= *minus* infinity)
- Divergence comes from:
 - Momentum = 0
 - Momentum = parallel to quark or antiquark

Gluon Emission as Higher Order Correction

$$\mathcal{M} = \text{[Diagram 1]} + \text{[Diagram 2]} \quad \mathcal{O}(\alpha\sqrt{\alpha_s})$$

The diagram shows the sum of two Feynman diagrams for the process $q\bar{q} \rightarrow \mu^+\mu^-g$. The first diagram shows a quark and antiquark annihilating into a virtual photon, which then decays into a muon and antimuon, with a gluon emitted from the quark line. The second diagram is identical but the gluon is emitted from the antiquark line. The overall amplitude is of order $\mathcal{O}(\alpha\sqrt{\alpha_s})$.

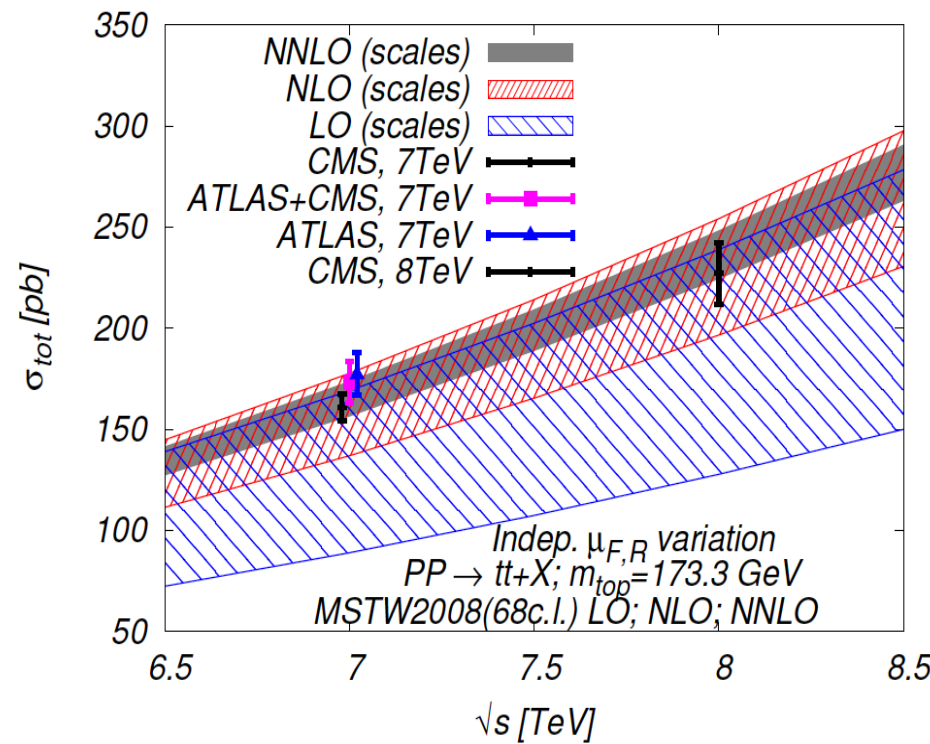
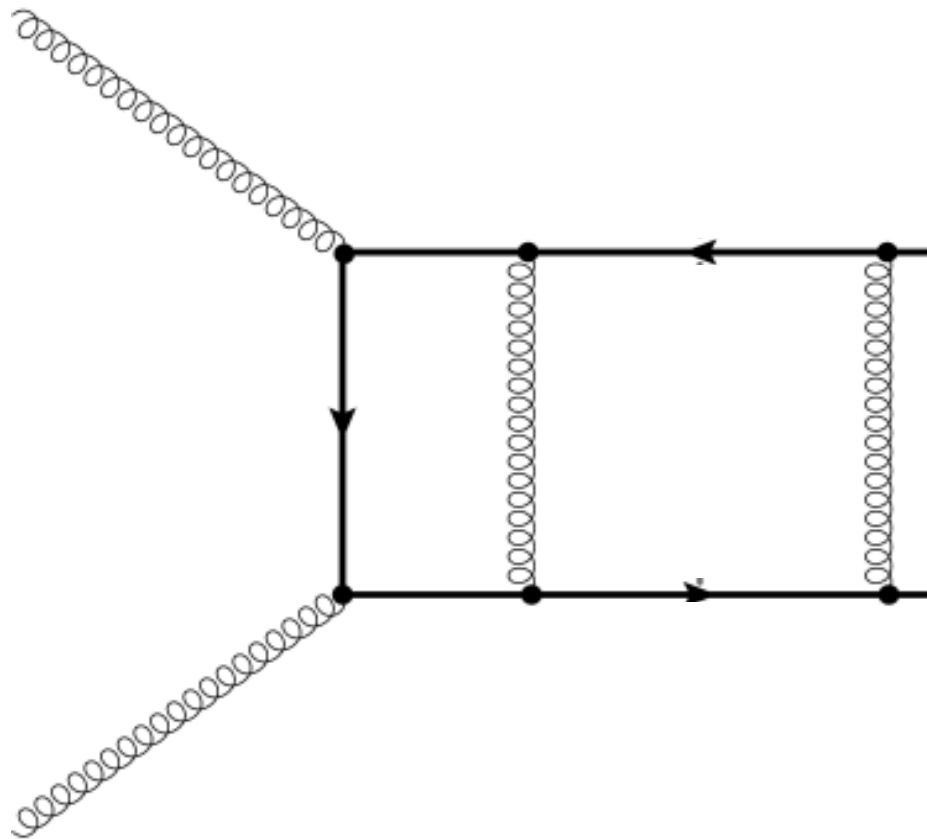
- Gluon emission describes a different process ($q\bar{q} \rightarrow \mu^+\mu^-g$)
- But if we are only interested in the total cross section for Drell-Yan pairs, must integrate over gluon momenta
- Divergent from momentum = 0 or parallel to quark or antiquark
- Cancels loop divergence

Next-to-Leading Order (NLO) cross section

- $\sigma_{NLO} = \sigma_{tree} + \sigma_{loop} + \sigma_{emission}$
- σ_{loop} and $\sigma_{emission}$ each divergent
 - must regularize and expose singularities of each
 - *Subtraction algorithms*
- Fully automated,
 - e.g. in Madgraph/aMC@NLO, MCFM, Sherpa, Herwig ...

State of the Art – NNLO Calculations

e.g. $pp \rightarrow t\bar{t}$



From Feynman Diagrams to Cross Sections

- Major part of phenomenology = calculating cross sections
- LO = write down all tree diagrams, integrate phase space numerically
- Convolute with parton distribution functions (fitted to data)
- NLO = one-loop diagrams, one-emission processes
 - Extract singularities from integrals, integrate analytically
 - Integrate remainders numerically
- NNLO = two-loop diagrams, one-emission at one-loop, and two emissions
- But LHC events contain *hundreds* of additional particles...