

Introduction to Heavy-Ion Physics Part III

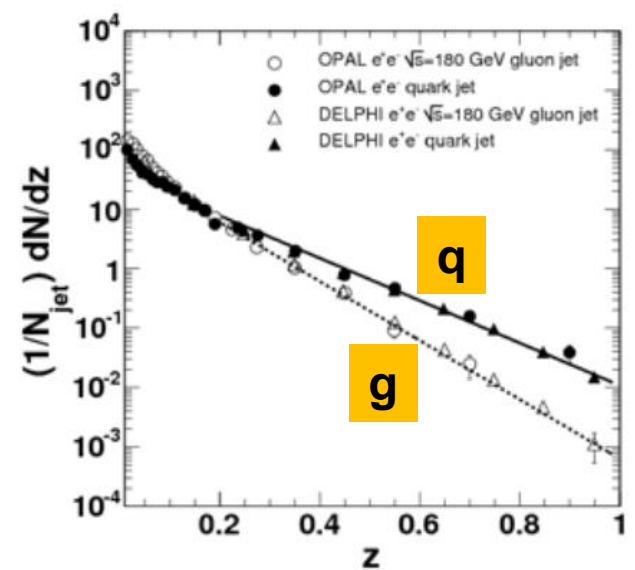
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Francesca Bellini*, CERN

Summer Student Lectures 2019



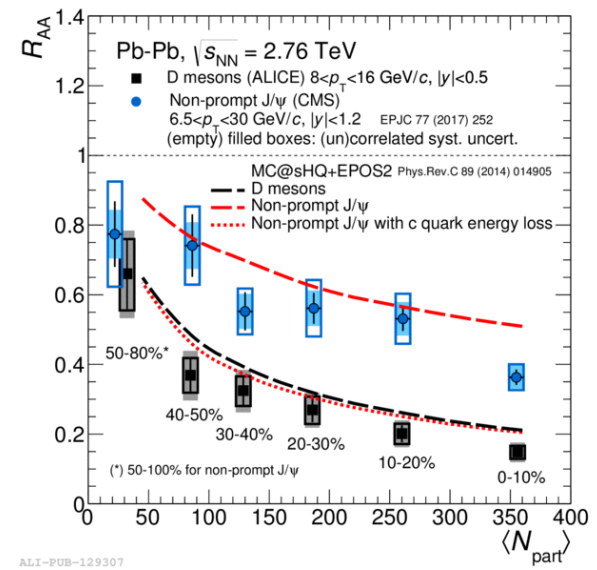
Recap Lecture II

- Energy loss in the medium by elastic and inelastic processes
- Quark-mass dependence expected
 - Fragmentation needs to be considered
 - Harder fragmentation of quark over gluon



- R_{AA} of D and B mesons
 - Analysis complex due to small S/B ratio
 - Mass dependence of energy loss

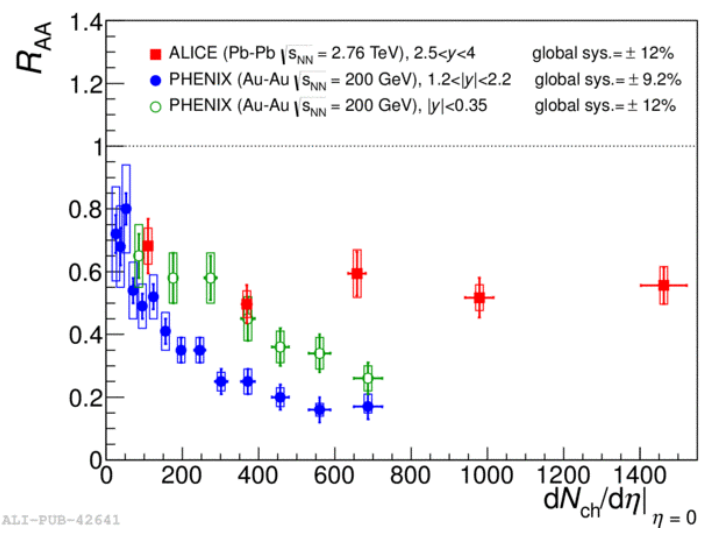
$$R_{AA}^{\pi} \approx R_{AA}^D < R_{AA}^B$$



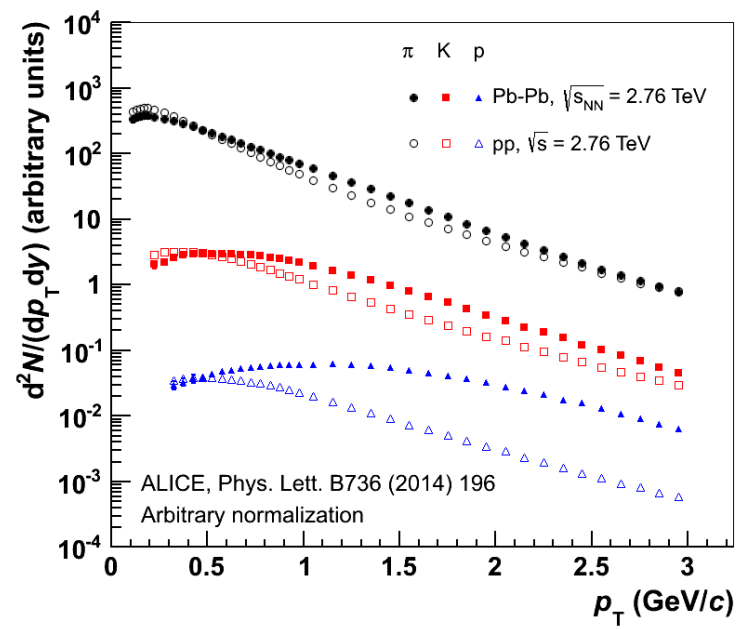


Recap Lecture II

- Quarkonia (c-cbar, b-bar) “melt” due to color screening in the QGP
 - J/ψ suppression
 - Abundance of c at LHC so large that J/ψ regenerate statistically
 - States with lower binding energy are more suppressed
- Hadron yields described by statistical models for $\sqrt{s_{NN}} = 2-2760$ GeV
 - Matter created in HI collisions is in local thermal equilibrium
- Expansion of QGP changes momenta of particles
 - Radial flow (dependent on particle mass)

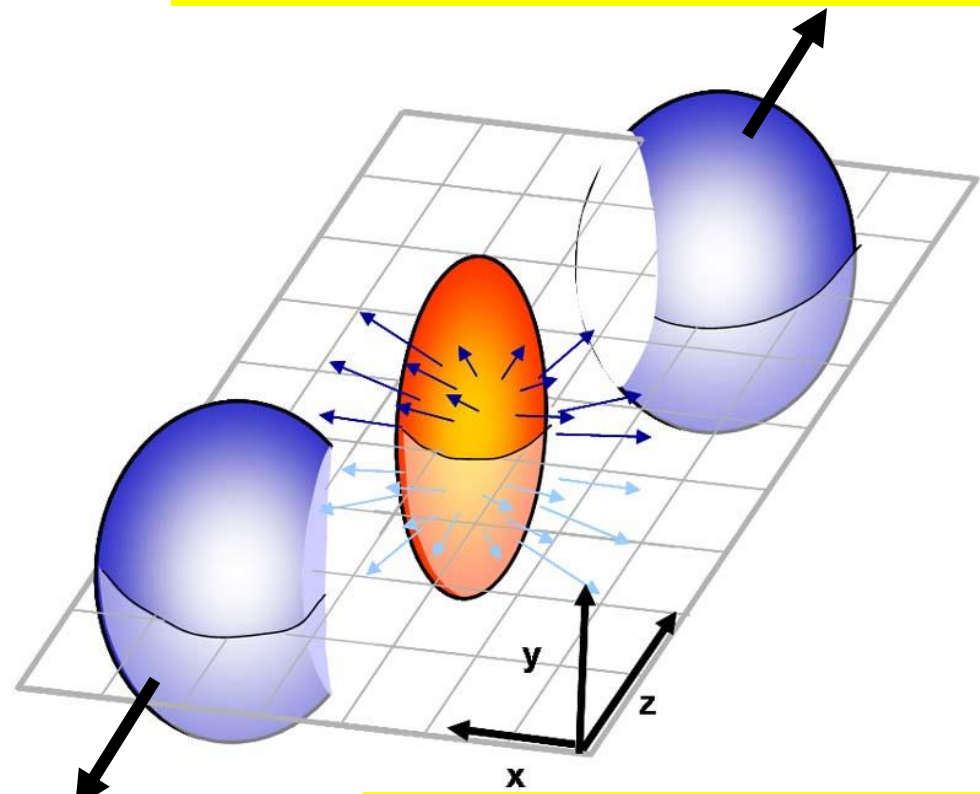


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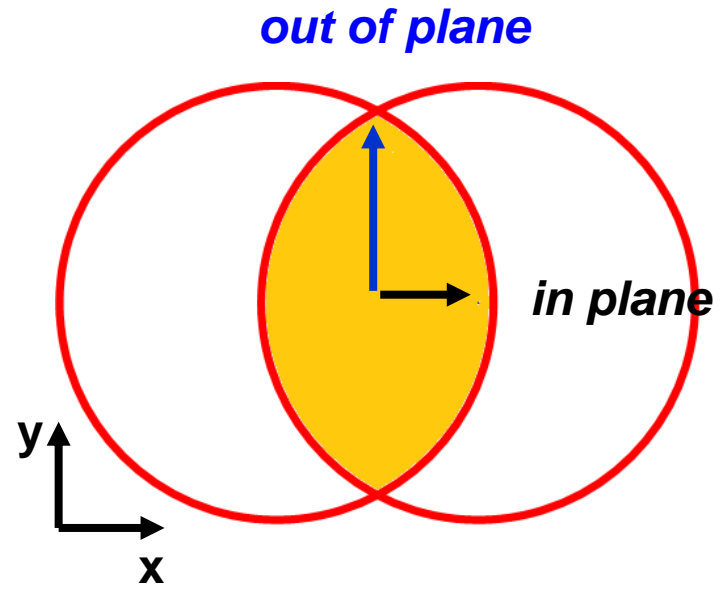


Elliptic Flow

Overlap of colliding nuclei not isotropic in non-central collisions



Defines *reaction plane* Ψ_{RP}
 (spanned by beam axis
 and impact parameter vector)



→ Pressure gradients
 dependent on direction

here: $\frac{dp_x}{dL} > \frac{dp_y}{dL}$



Elliptic Flow (2)

- Spatial anisotropy (almond shape)

- Quantified by eccentricity ε

$$\varepsilon = \frac{y^2 - x^2}{y^2 + x^2}$$

- Pressure gradient larger in-plane

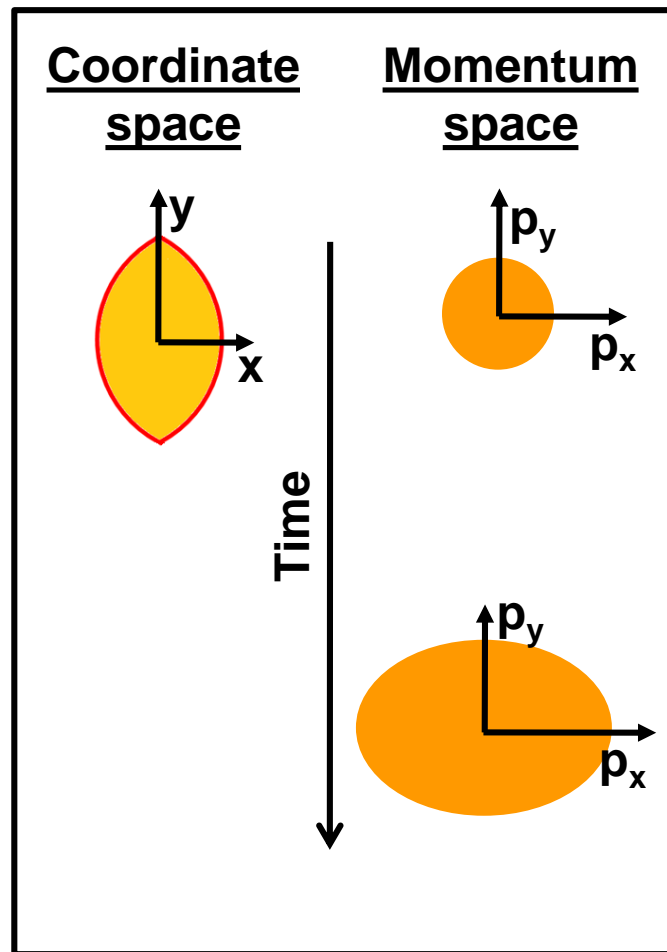
- Pressure pushes partons

- More in in-plane than out-of-plane

- Spatial anisotropy converts into momentum-space anisotropy

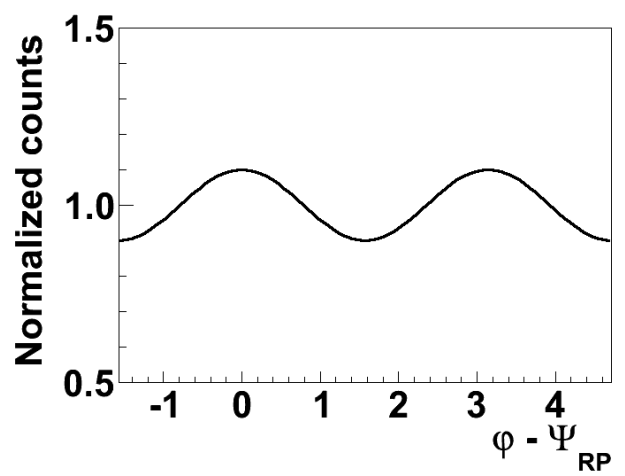
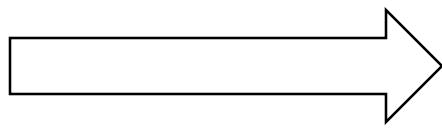
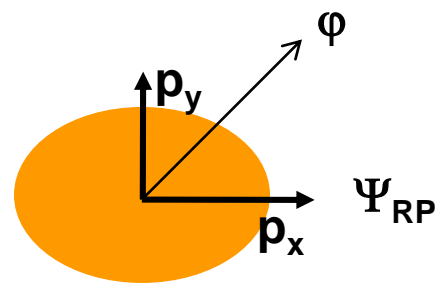
- “Faster” particles in-plane

- Measurable in the final state!



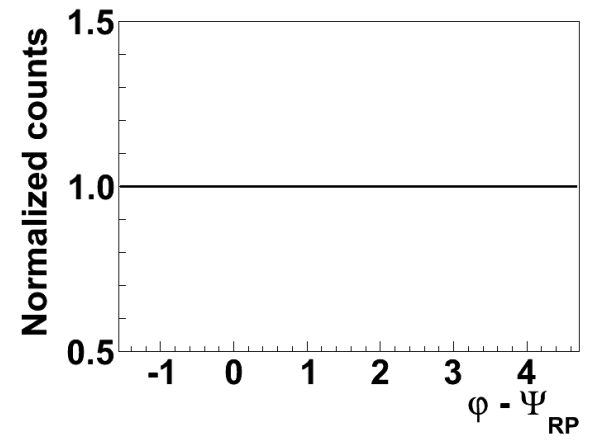
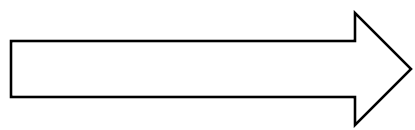
Elliptic Flow (3)

- Particles as a function of $\varphi - \Psi_{RP}$



$$\frac{dN}{d\varphi} = A(1 + 2v_2 \cos 2(\varphi - \Psi_{RP}))$$

- Define $v_2 = \langle \cos 2(\varphi - \Psi_{RP}) \rangle$
 - Second coefficient of Fourier expansion
- Ψ_{RP} common *symmetry plane* (for all particles)
- What if there were no correlations with Ψ_{RP} ?





Measuring Elliptic Flow

$$v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$$

Measure tracks

Measure reaction-plane angle

- Reaction plane angle
 - From the particles themselves

$$Q_x = \sum_i w_i \cos 2\varphi_i \quad Q_y = \sum_i w_i \sin 2\varphi_i \quad \Psi_{RP} = \tan^{-1}(Q_x, Q_y) / 2$$

weight w

- Ψ_{RP} approximates true reaction-plane angle (called *event plane*)
- Calculation of *integrated* $v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$
- $v_2(p_T)$ by considering only particles at given p_T
- Called *event plane method*, denoted $v_2\{EP\}$

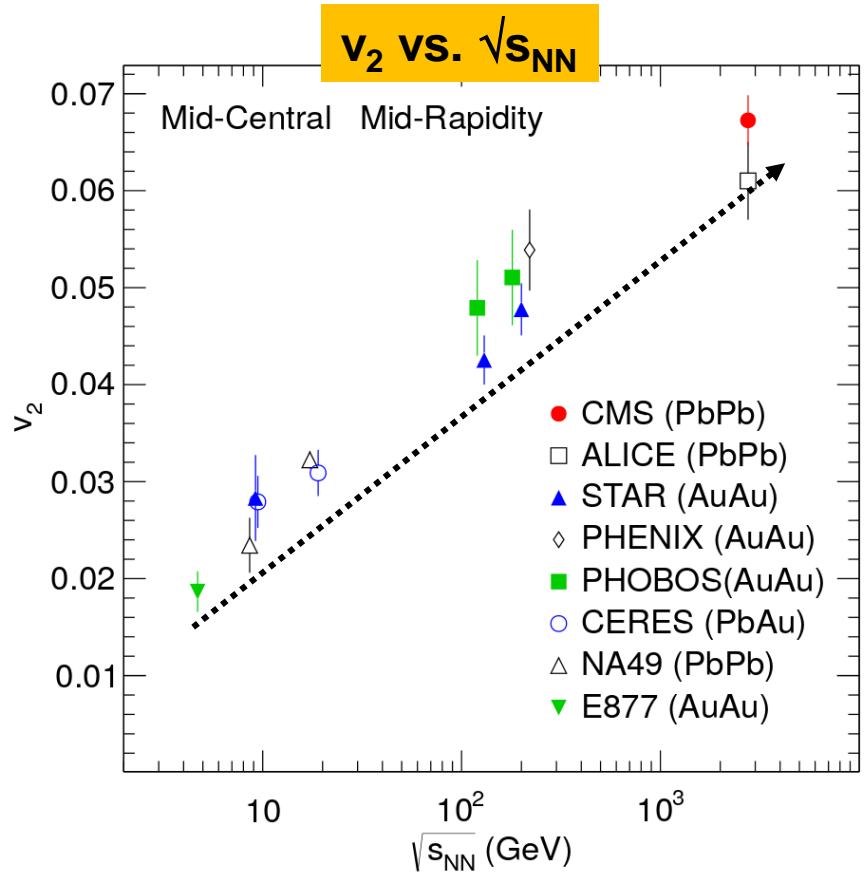
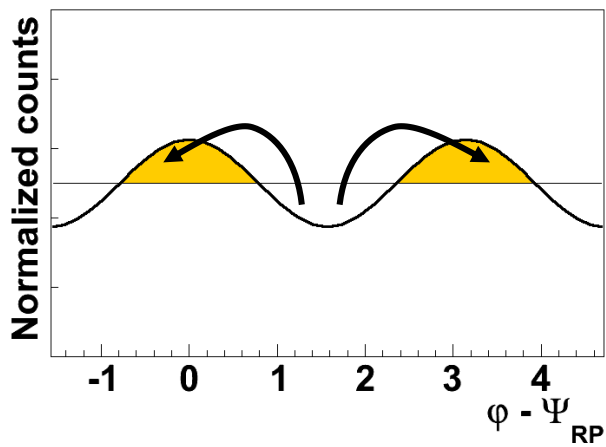


$\sqrt{s_{NN}}$ Dependence

- Increases with $\sqrt{s_{NN}}$
- At LHC $v_2 \sim 0.06$
 - What does that mean?

$$\frac{dN}{d\phi} = A(1 + 2v_2 \cos 2(\phi - \Psi_{RP}))$$

- $2v_2 = 12\%$ of particles “move” from out-of-plane to in-plane

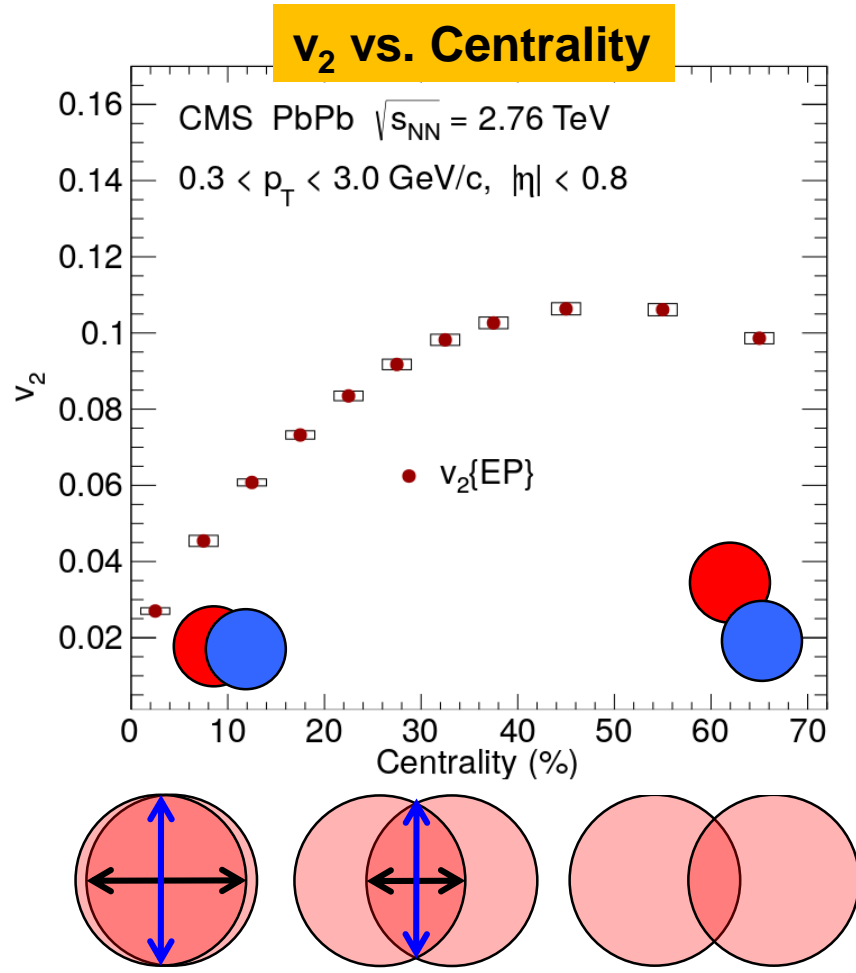


CMS, PRC 87(2013) 014902



Centrality Dependence

- Strong centrality dependence
- v_2 largest for 40-50%
- Spatial anisotropy very small in central collisions
- Largest anisotropy in mid-central collisions
- Small overlap region in peripheral collisions



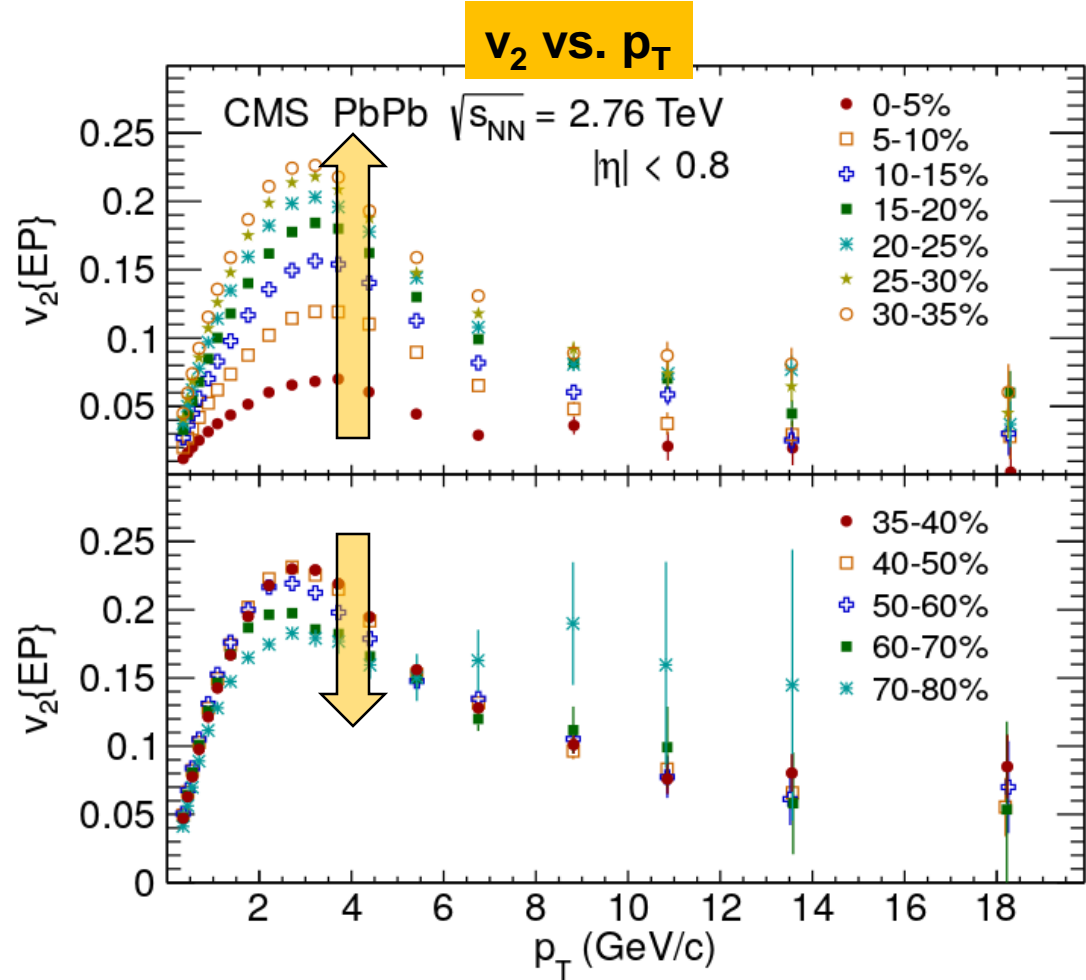
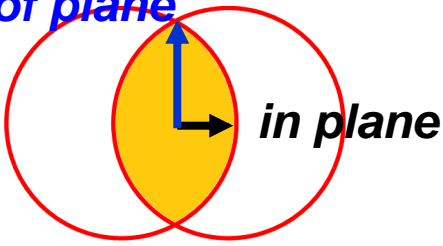
CMS, PRC 87(2013) 014902



p_T Dependence

- Centrality dependence independent of p_T
- Largest v_2 for $p_T \sim 3$ GeV/c
- Low and intermediate p_T , v_2 caused by collective expansion
- Large p_T , v_2 caused by *length-dependent jet quenching*
 - Longer path length out of plane than in plane

out of plane



CMS, PRC 87(2013) 014902



Recap

- Pressure in dense medium affects momenta
- Isotropic expansion effect called *radial flow*
- Overlap of colliding nuclei causes spatial anisotropy
- Converted into momentum-space anisotropy in medium evolution
- Modulation of observed particles
- Quantified by $v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$

What other methods exist to measure v_2 ?

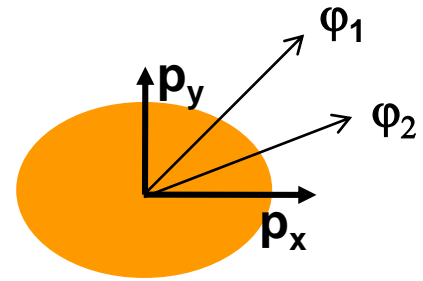
What effect do jet-related particles have on v_2 ?



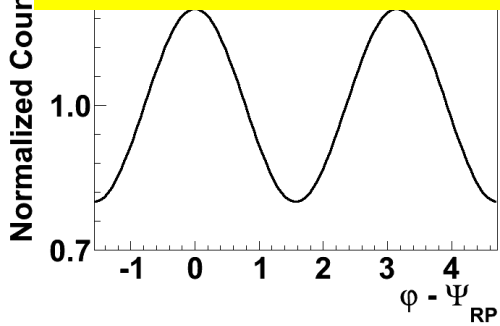
Two-Particle Correlations

- Reaction-plane estimation can be experimentally tricky
- Rewrite $v_2 = \langle \cos 2(\varphi - \Psi_{RP}) \rangle$ as $v_2 = \langle e^{i2(\varphi - \Psi_{RP})} \rangle$
- v_2 can also be measured from 2-particle correlations

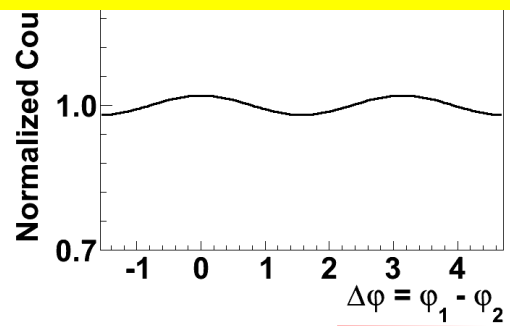
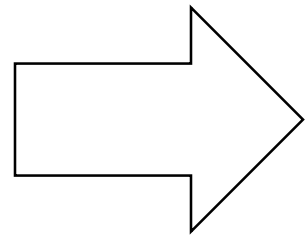
$$\begin{aligned} \langle e^{i2(\varphi_1 - \varphi_2)} \rangle &= \langle e^{i2(\varphi_1 - \Psi_{RP} - (\varphi_2 - \Psi_{RP}))} \rangle = \\ &= \langle e^{i2(\varphi_1 - \Psi_{RP})} \rangle \langle e^{i2(\varphi_2 - \Psi_{RP})} \rangle = v_2^2 \end{aligned}$$



Modulation smaller due to $v_2 \rightarrow (v_2)^2$ but statistical power similar



$$v_2 = \langle e^{i2(\varphi - \Psi_{RP})} \rangle$$



$$v_2^2 = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle$$



Higher-Order Correlations

- Trivial extension to 4-particles (and higher-orders)

$$v_2^4 = \left\langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \right\rangle$$

$$v_2^6 = \left\langle e^{i2(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6)} \right\rangle$$

- NB. sign is arbitrary as long as same amount of positive and negative angles
→ rotational symmetry



Cumulants

- Cumulants extract genuine n-particle correlations
- For 2-particle correlations

$$\langle x_1 x_2 \rangle = \underbrace{\langle x_1 \rangle \langle x_2 \rangle}_{\text{lower order "correlations"}} + \langle x_1 x_2 \rangle_c$$

measured correlation

lower order "correlations"

genuine 2-particle correlations

ϕ dependence only from detector acceptance

- Rewrite (trivially) $\langle x_1 x_2 \rangle_c = \langle x_1 x_2 \rangle - \langle x_1 \rangle \langle x_2 \rangle$

- For 3-particle correlations

$$\langle x_1 x_2 x_3 \rangle = \langle x_1 \rangle \langle x_2 \rangle \langle x_3 \rangle + \langle x_1 x_2 \rangle_c \langle x_3 \rangle + \langle x_1 x_3 \rangle_c \langle x_2 \rangle + \langle x_2 x_3 \rangle_c \langle x_1 \rangle + \underline{\langle x_1 x_2 x_3 \rangle_c}$$

**Higher-order cumulants zero \rightarrow no genuine multi-particle correlation !
 No matter what multi particles correlations (i.e. not cumulants) show**



Cumulants for Elliptic Flow

- For uniform detector acceptance, cumulants of 2nd and 4th order:

$$c_2\{2\} = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle = v_2^2 \longleftarrow \text{identical to two-particle correlation}$$

$$c_2\{4\} = \langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle - \underbrace{2 \langle e^{i2(\varphi_1 - \varphi_2)} \rangle^2}_{\text{lower orders are removed}} = -v_2^4$$

lower orders are removed

- $c_2\{4\}$ is genuine 4-particle correlations
 - I.e. if only pairs of particles are correlated $\rightarrow c_2\{4\} = 0$



Flow Methods

- Now we have tons of methods to measure flow
 - Event plane
 - 2-particle and 4-particle correlations, ...
 - 2-particle and 4-particle cumulants, ...



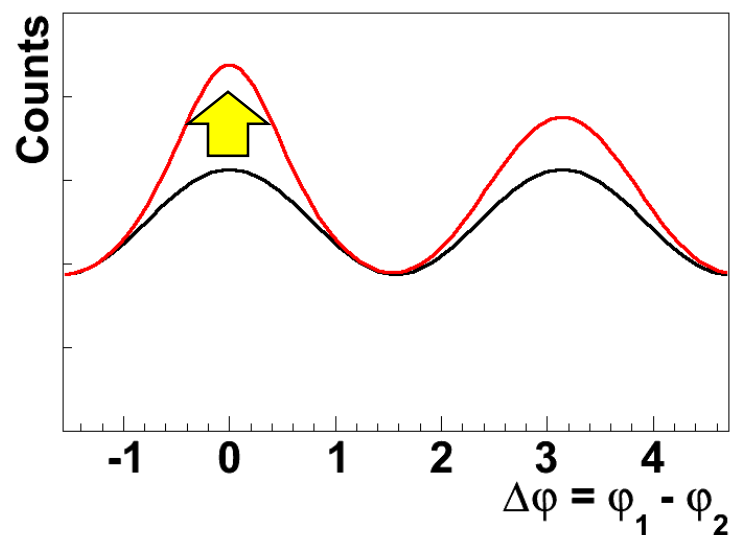
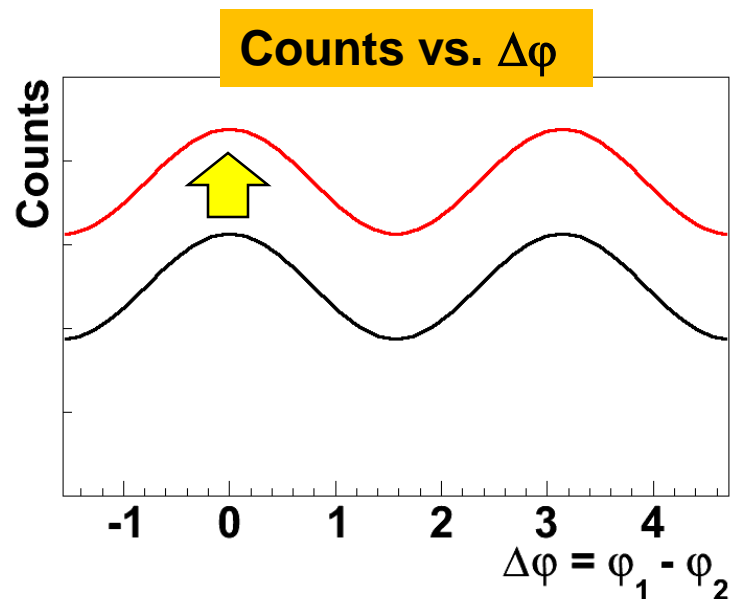
They all estimate v_2 , so what?

Let's have a look, what spoils the flow measurement...



Non-Flow

- Particles are correlated through reaction plane Ψ_{RP}
- Additional isotropically distributed particles
 - Add to baseline, reduce $\cos 2\Delta\phi$ magnitude, but don't distort shape
- Jets
 - Particles which exhibit correlations close in angle (within the same jet) and at $\Delta\phi = \pi$ (back-to-back jet)
 - Distort Ψ_{RP} estimate
 - Distorts shape in 2 particle correlations
- A pure jet-signal results in $v_2 > 0$ (e.g. Pythia)





Non-Flow (2)

- Different effect on different flow methods
- 2-particle correlations / cumulants

$$c_2\{2\} = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle = v_2^2 + \delta_2 \leftarrow \text{non-flow contribution}$$

- 4-particle correlations

$$\langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle = v_2^4 + 4v_2^2\delta_2 + 2\delta_2^2 + \delta_4$$

- 4-particle cumulants

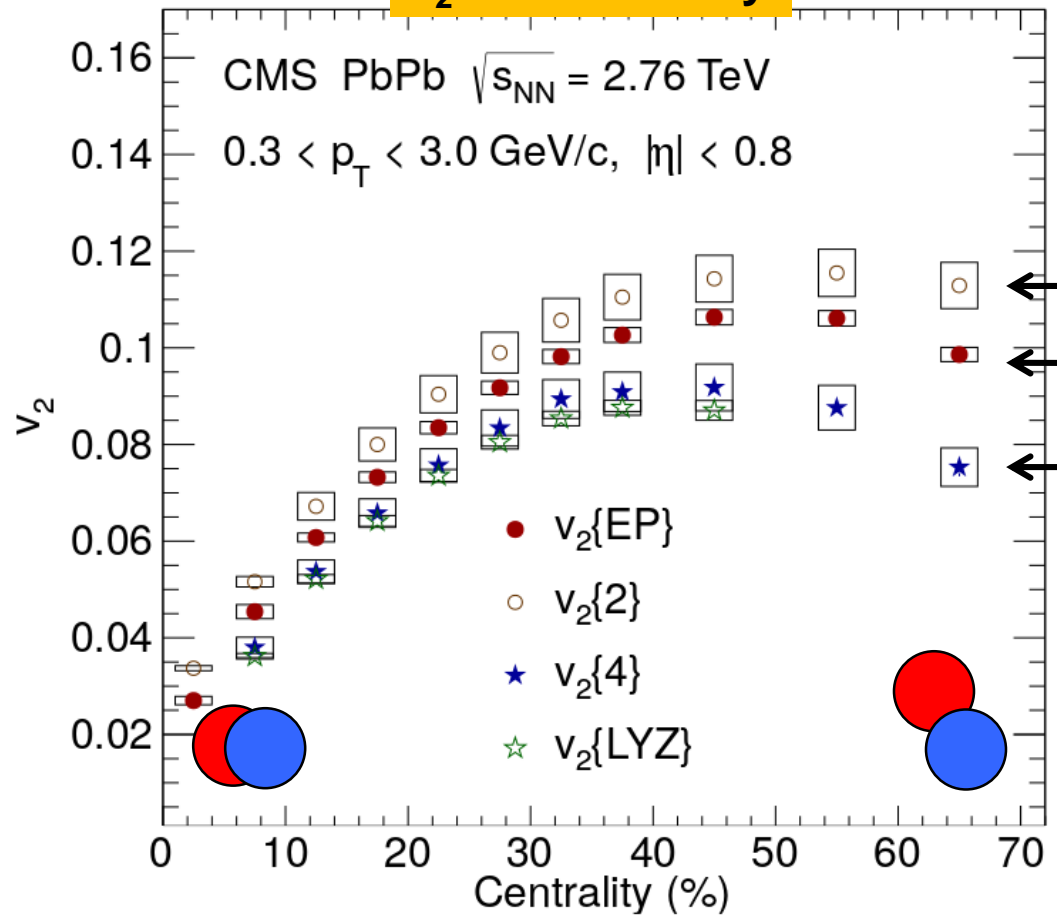
$$\begin{aligned} c_2\{4\} &= \langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle - 2\langle e^{i2(\varphi_1 - \varphi_2)} \rangle = \\ &= v_2^4 + 4v_2^2\delta_2 + 2\delta_2^2 + \delta_4 - 2(v_2^2 + \delta_2) = -v_2^4 + \delta_4 \end{aligned}$$

Second order non-flow dropped out !



Experiment

v₂ vs. Centrality



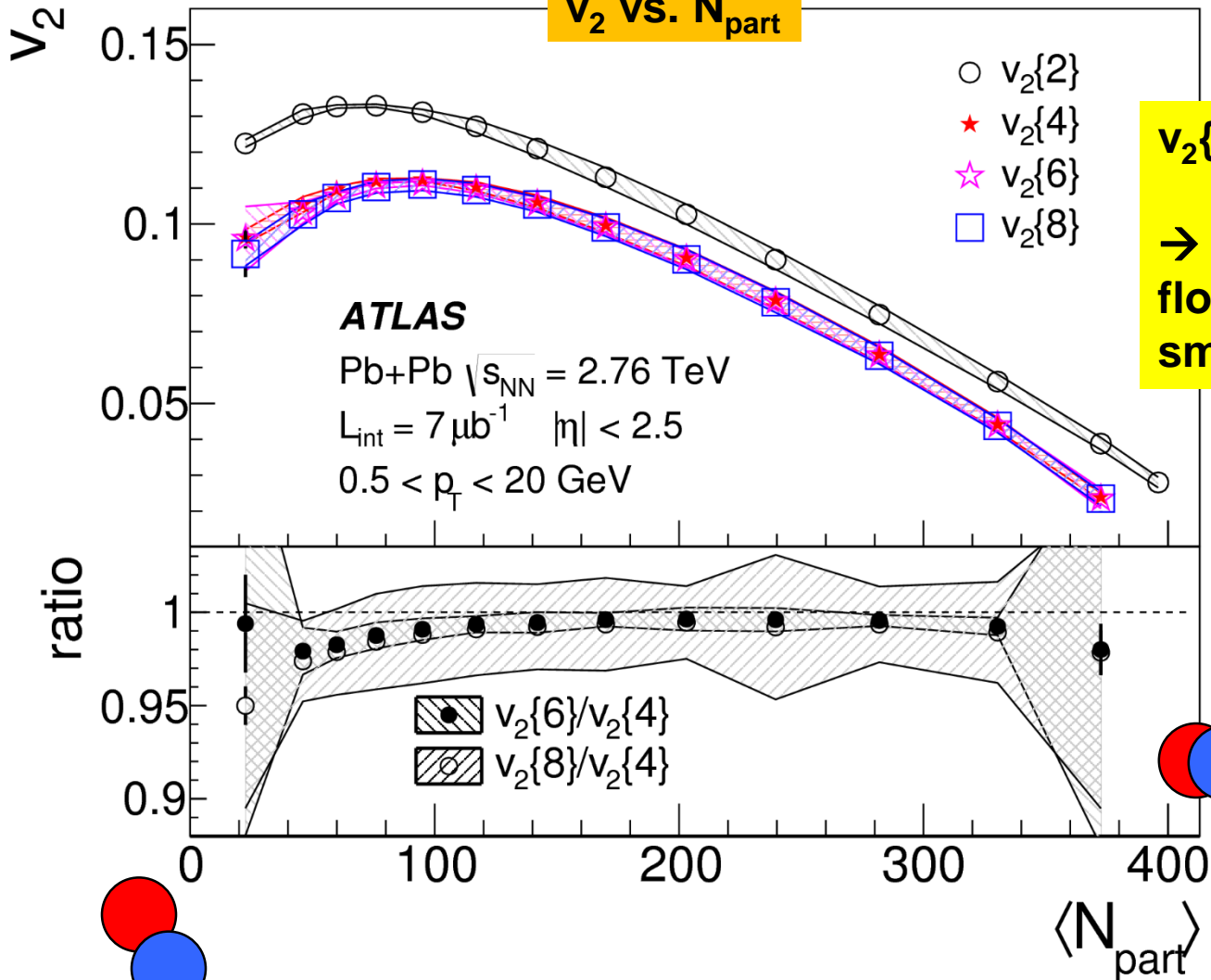
**Larger non-flow influence*
for $v_2\{2\}$ than $v_2\{4\}$**

* neglects fluctuations, see [backup](#)

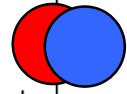
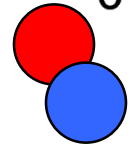


Up to 8 Particles...

v_2 vs. N_{part}



$v_2\{4\} \sim v_2\{6\} \sim v_2\{8\}$
→ influence of non-flow (and fluctuations) small for ≥ 4 particles





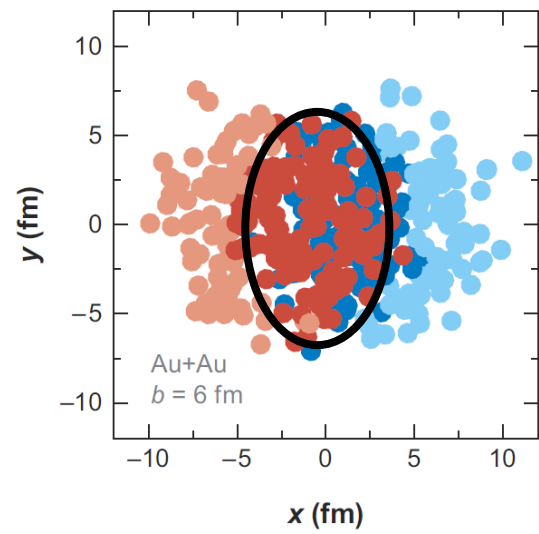
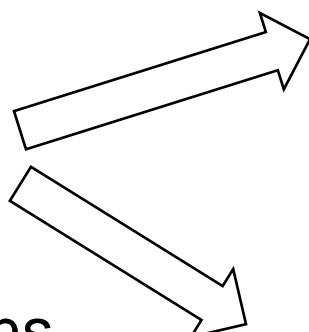
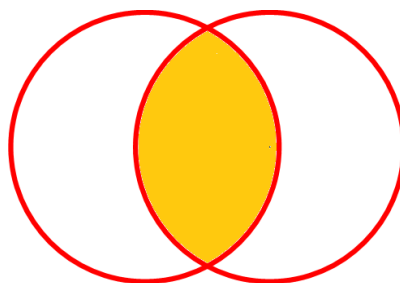
Recap

- Elliptic flow can be measured with different methods
- Cumulants of n^{th} order measure genuine n-particle correlations – not reducible to lower orders
- Mini(jets) and resonances distort the v_2 measurement
- Non-flow influence is different for different methods
 - The higher the order of the cumulant, the smaller the influence

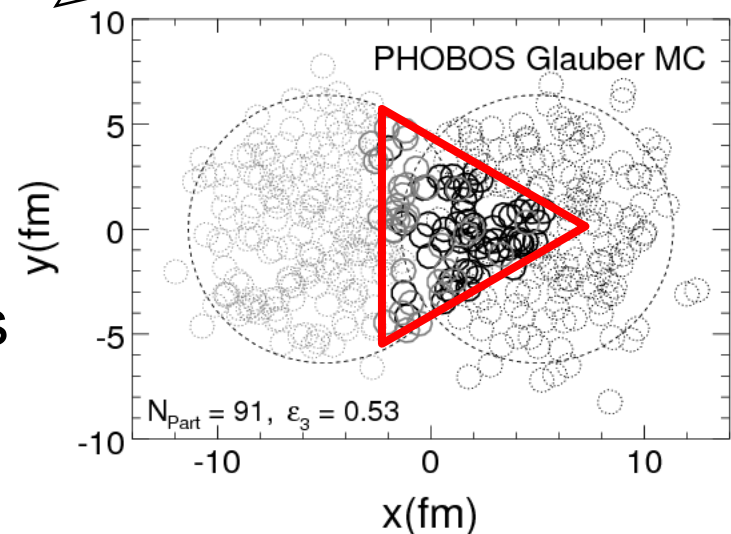
For now we have discussed elliptic flow v_2 – is that all?

Higher-Order Flow

- Geometrical picture
 → 2nd order modulation (v_2)

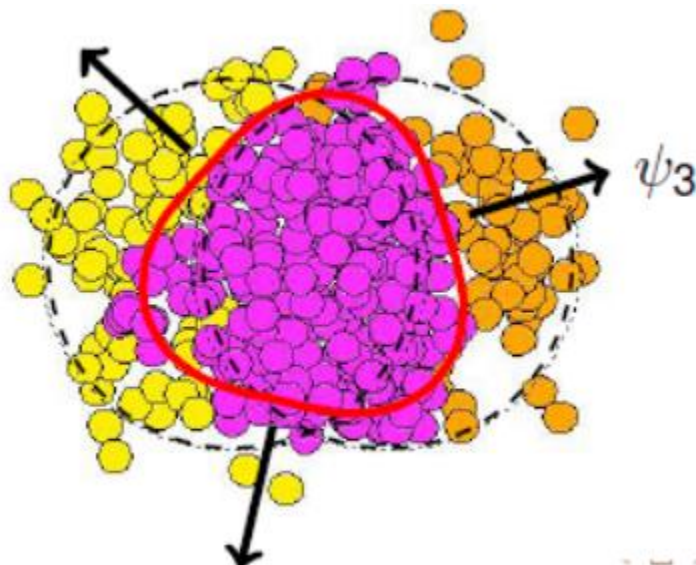


- In practice interacting nucleons need to be considered
 - E.g. estimated with Glauber MC
 - *Initial state density fluctuations*
- These produce all kinds of shapes
 - Elliptic, triangular, quadruple, ...
 - And mixtures of those



nucl-ex/0701025, PRC81 (2010) 054905

Higher-Order Flow (2)



- Reaction plane $\Psi_{RP} \rightarrow n^{\text{th}}$ order participant plane Ψ_n

$$\frac{dN}{d\varphi} = A(1 + 2v_2 \cos 2(\varphi - \Psi_{RP})) \quad \Longrightarrow \quad \frac{dN}{d\varphi} = A\left(1 + 2 \sum_n v_n \cos n(\varphi - \Psi_n)\right)$$

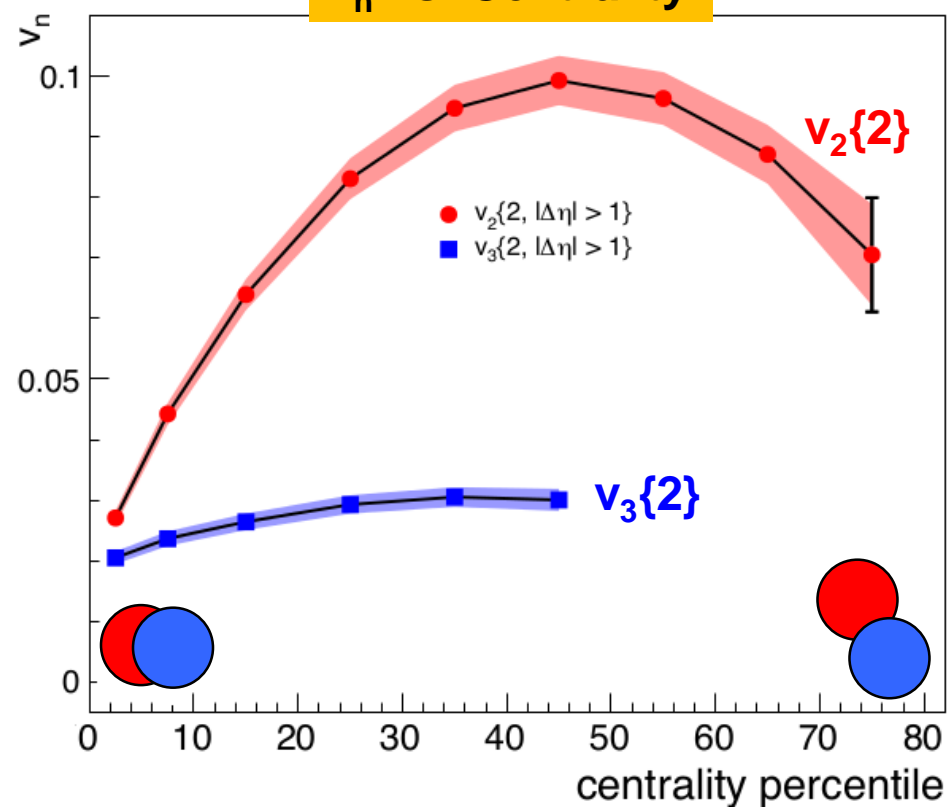
- Formalism can be trivially extended from v_2 to v_n

- E.g. $v_2^2 = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle \longrightarrow v_n^2 = \langle e^{in(\varphi_1 - \varphi_2)} \rangle$ PRC81 (2010) 054905



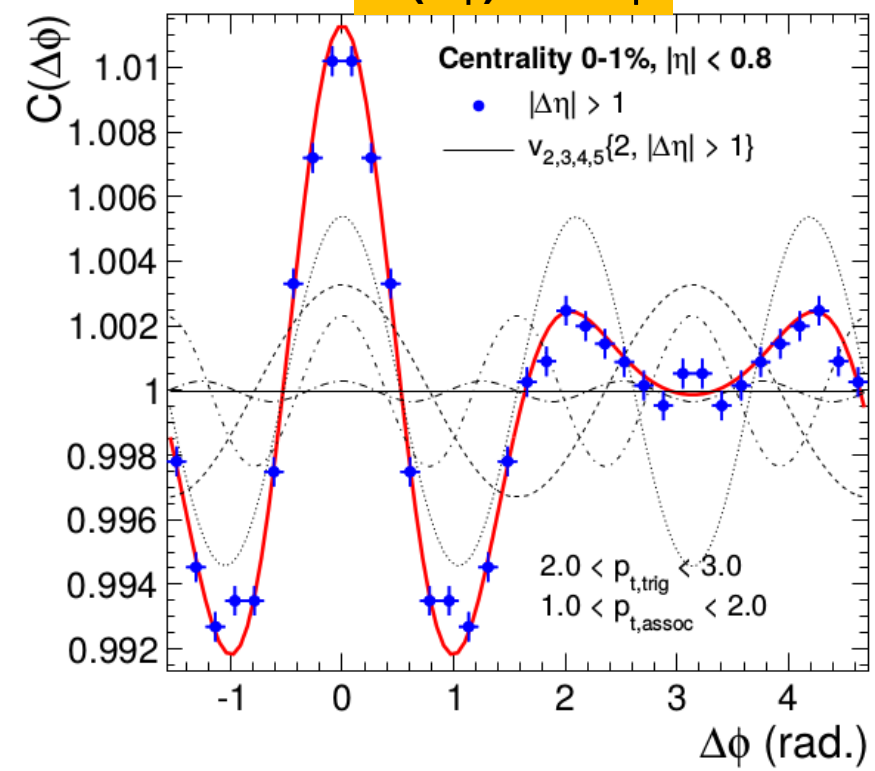
Experiment

v_n vs. Centrality



v_3 sizable
 $v_3 \sim \frac{1}{2} v_2$
Weaker centrality dependence

$C(\Delta\phi)$ vs. $\Delta\phi$



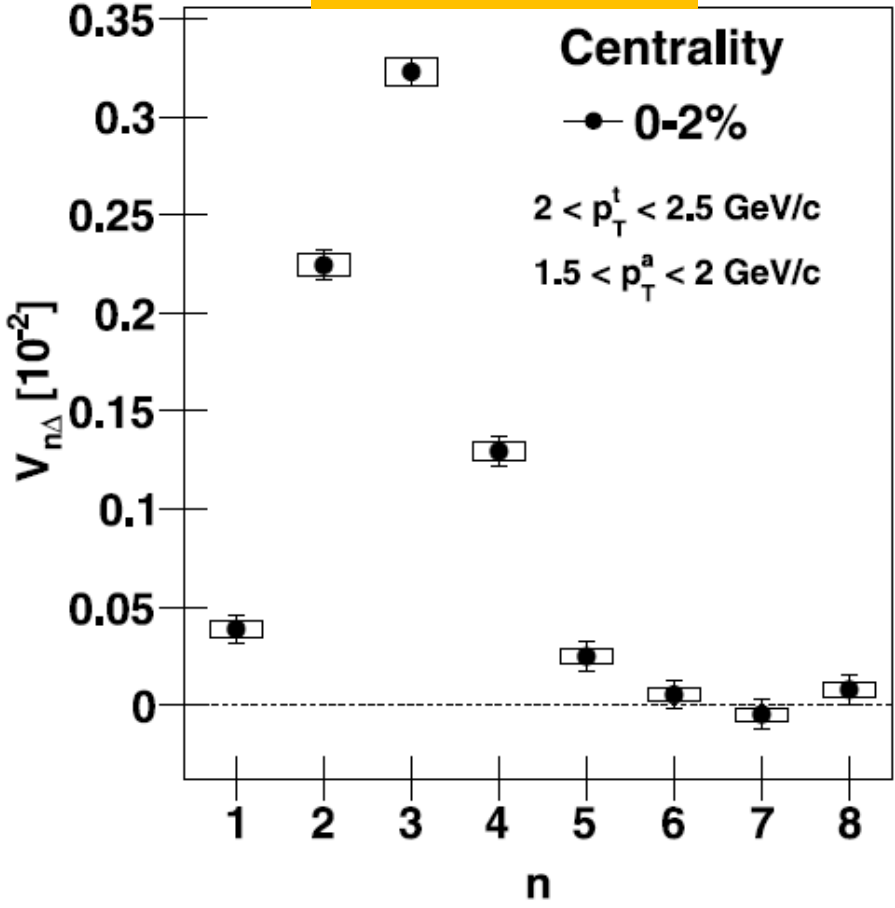
Two-particle correlations can be fully described by $v_2 \dots v_5$

PRL107, 032301 (2011)

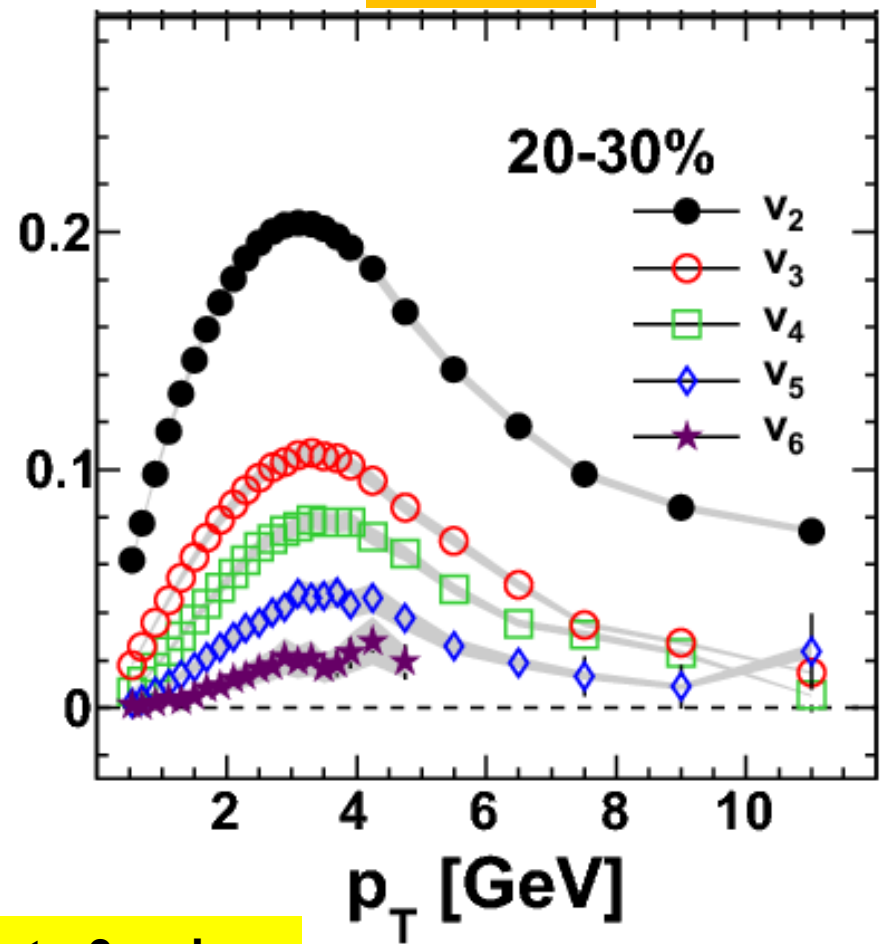


And even higher orders...

$V_{n\Delta} = (v_n)^2$ vs. n



v_n vs. p_T



Significant up to 6 orders

ATLAS-CONF-2011-074
PLB708 (2012) 249



Recap

- Geometry of overlapping nuclei \rightarrow elliptic flow
- Initial-state density fluctuations lead to different 'shapes' of overlap region \rightarrow flow at higher orders
- Flow measured up to 6th order

What does a medium need for collective effects?

What can we learn from these results?



Hydrodynamics

- Calculating space-time evolution of QGP from first principles (QCD Lagrangian) is too complex (non-abelian, strong coupling, many-body system, ...)
- Expanding medium can be described macroscopically with hydrodynamical models
 - Conservation of energy-momentum $\partial_\mu T^{\mu\nu} = 0$
 - Conservation of charges, mainly baryon number $\partial_\mu N_i^\mu = 0$
 - Local thermodynamical equilibrium $N_i^\mu = nu^\mu$
- Needed input
 - Initial conditions
 - Equation of State (EoS), from lattice QCD
 - Relativistic fluid dynamics
 - Perfect or dissipative (\rightarrow transport coefficients)

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$



Hydrodynamics (2)

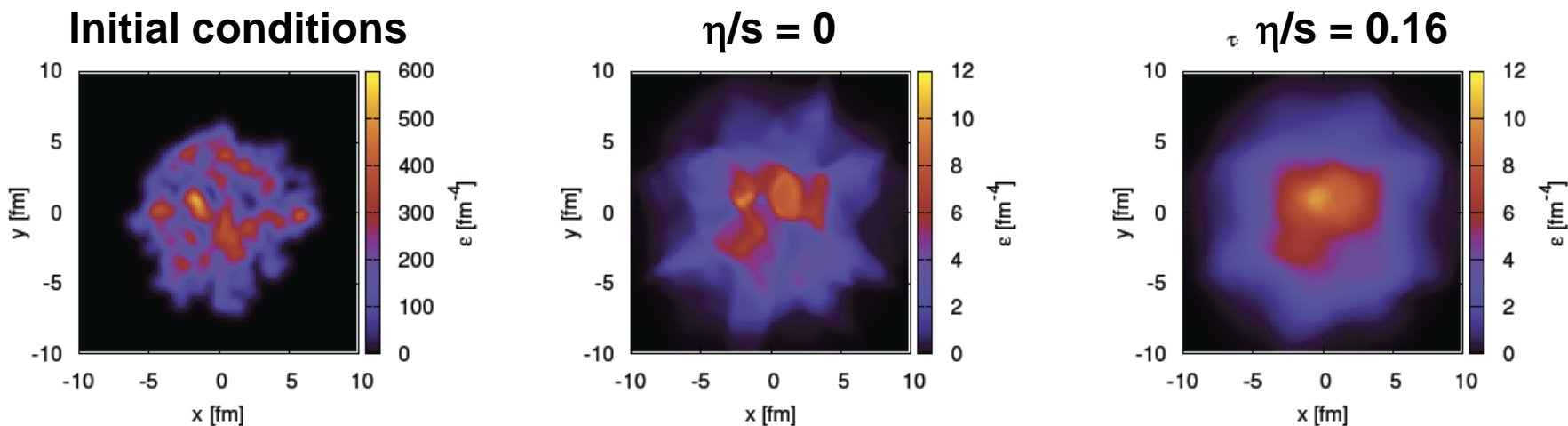
- Once dynamics well described, hydrodynamic “output” can be used in other calculations: jet quenching, J/ψ melting, etc.
- Flow observables:
Initial-state anisotropies \rightarrow final-state anisotropies
 - Translate from initial-state eccentricity ε_n to final-state flow v_n
- Deduce conclusions on initial conditions, EoS and transport coefficients by data comparison

Shear Viscosity

- Shear viscosity washes out initial-state anisotropies
 - Expressed as η/s (shear viscosity over entropy)
 - Ideal hydrodynamics : $\eta/s = 0$ \longrightarrow
 - Viscous hydrodynamics : $\eta/s > 0$
 - Large influence on higher-order flow

not to confuse with ideal (free streaming) gas \rightarrow no interactions

Density in collision region (x vs. y)

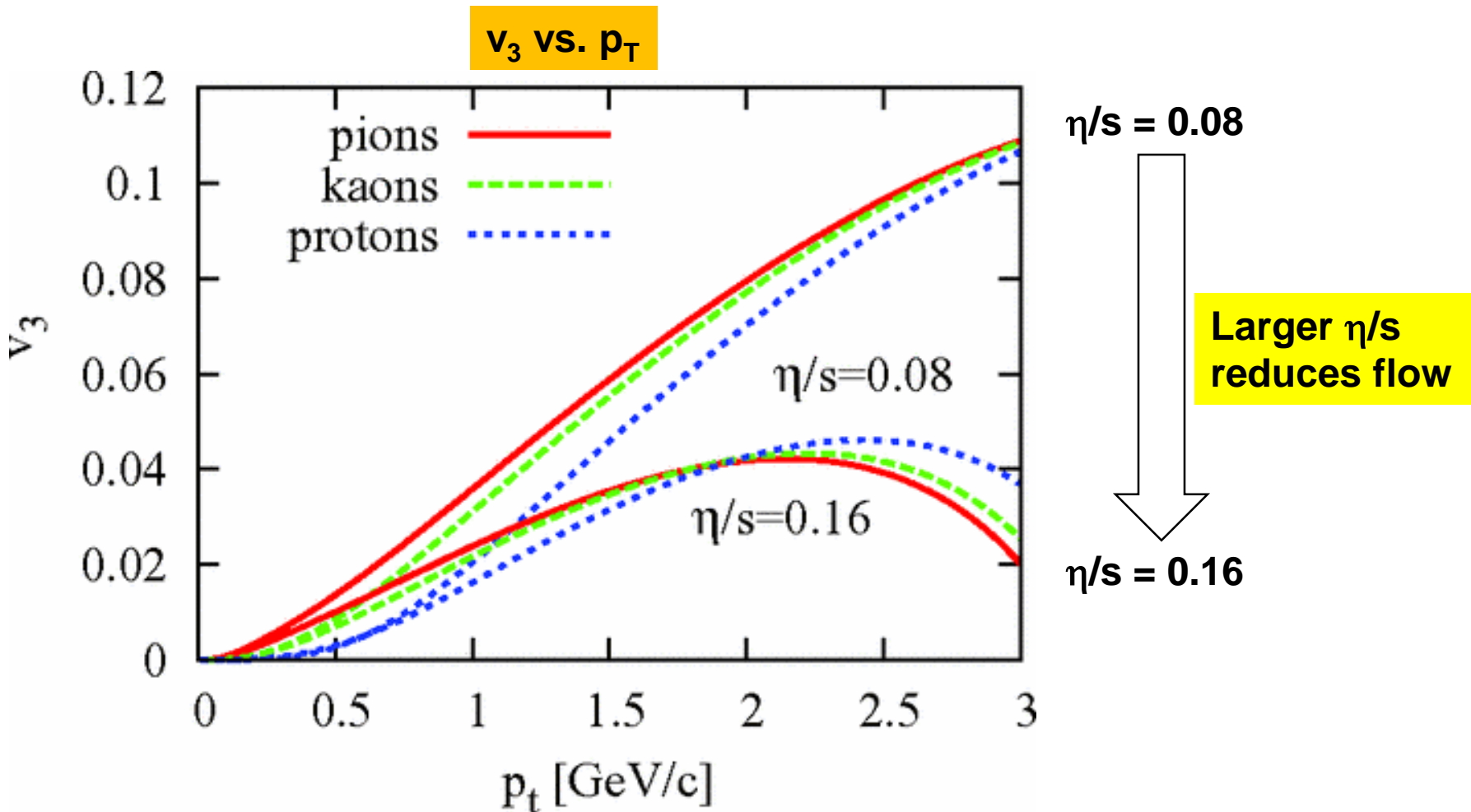


Water: $\eta/s \sim 30$ | Olive oil $\eta/s \sim 240$

MUSIC, Sangyong Jeon



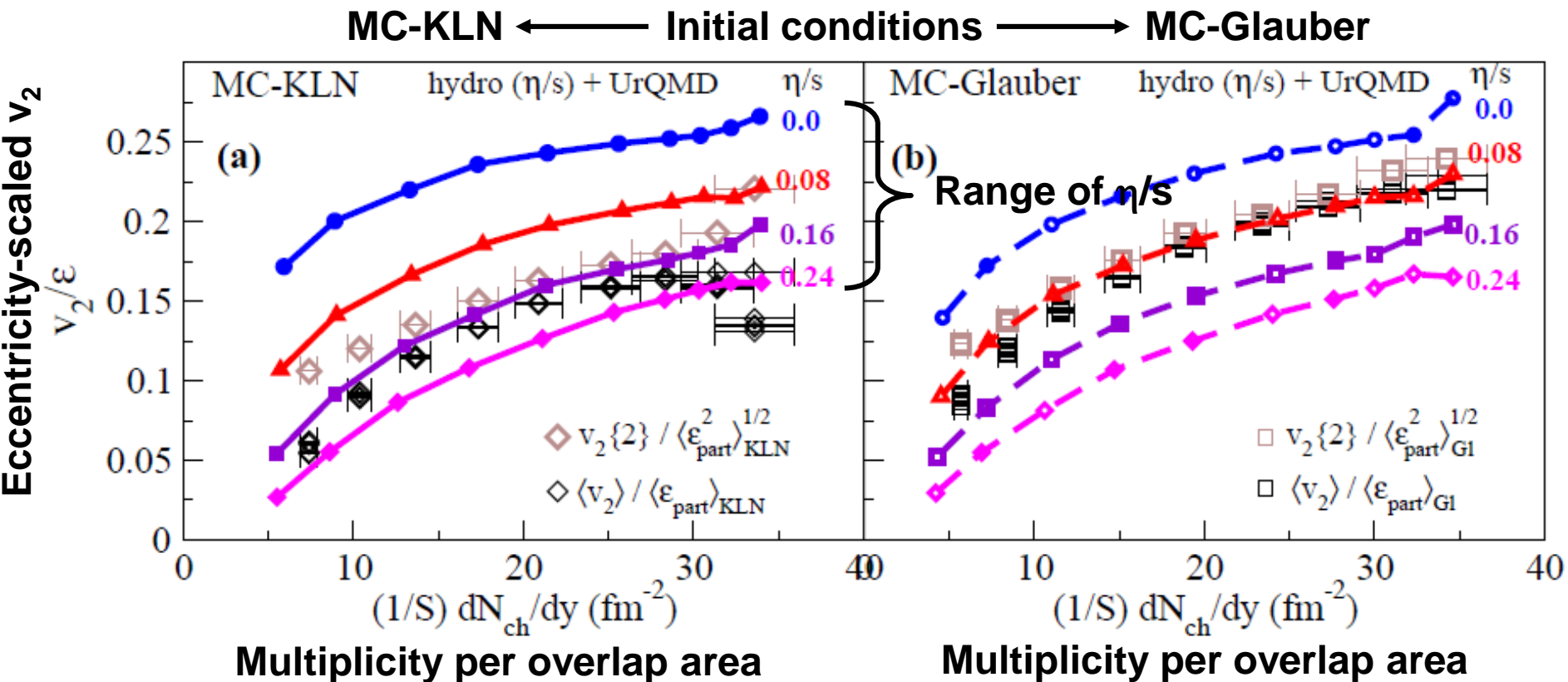
Example: Shear Viscosity



Shear viscosity hampers the build-up of flow !

PRC 82, 034913 (2010)

Hydro vs. Data



MC-KLN with $\eta/s = 0.16$ or MC-Glauber with $\eta/s = 0.08$

Water: $\eta/s \sim 30$ | Olive oil $\eta/s \sim 240$

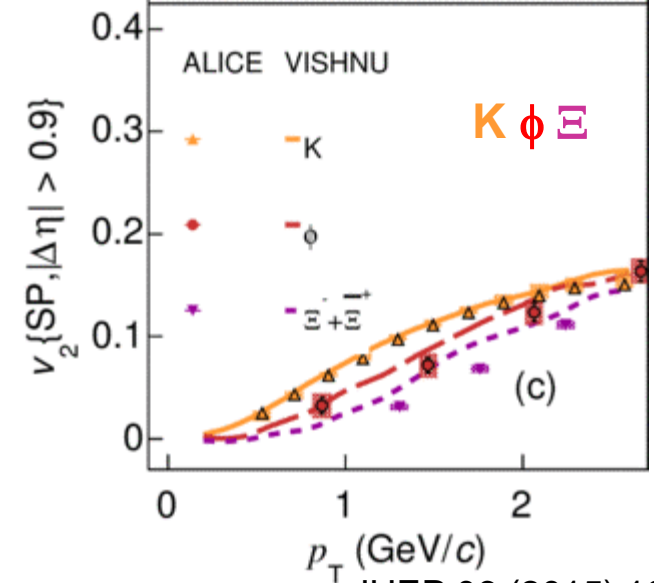
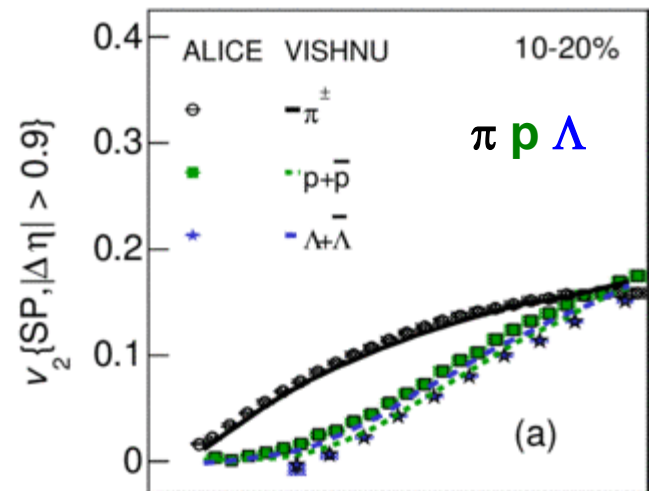
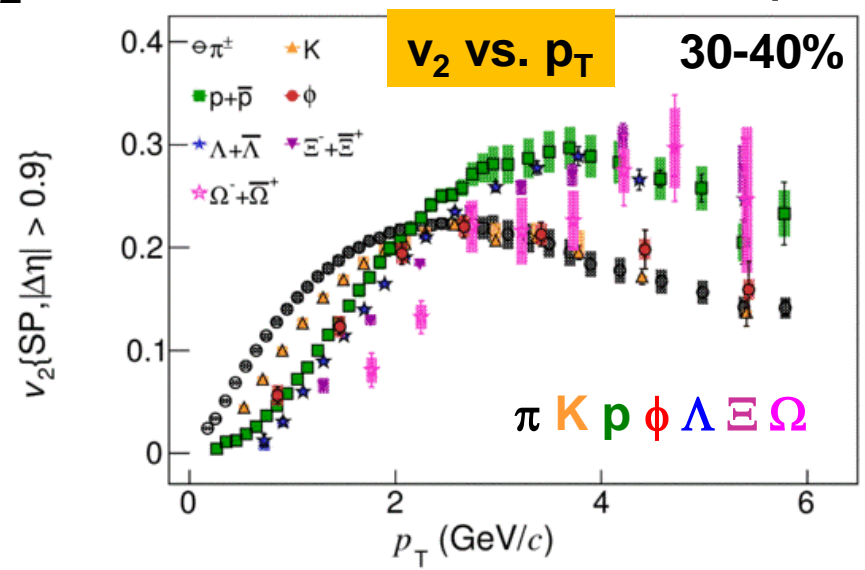
Annu.Rev.Nucl.Part.Sci. 63 (2013) 123 (Data: STAR 200 GeV)



Hydro vs. Data (2)

v_2 vs. p_T

- v_2 measured for 7 different species



- Strong species dependence
 - Different masses and quark content
- Stringent test for hydro
 - Very good agreement with VISHNU (hydro + hadronic cascade model (UrQMD), initial conditions MC-KLN, $\eta/s \sim 0.16$)

JHEP 06 (2015) 190

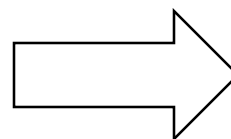


Summary

Collective Flow & Hydrodynamics

- Quark-gluon plasma expands rapidly (up to $\sim 0.65c$)
- Spatial anisotropy of collision region causes anisotropic flow quantified as Fourier coefficients v_n
 - Measured up to 6th order
 - Initial-state fluctuations influence v_n
- Well described by viscous hydrodynamics with a very low shear viscosity ($\eta/s \sim 0.08 - 0.16$) “perfect liquid”

Hydrodynamical models describe collective flow



Matter created in HI collisions is in local thermal equilibrium



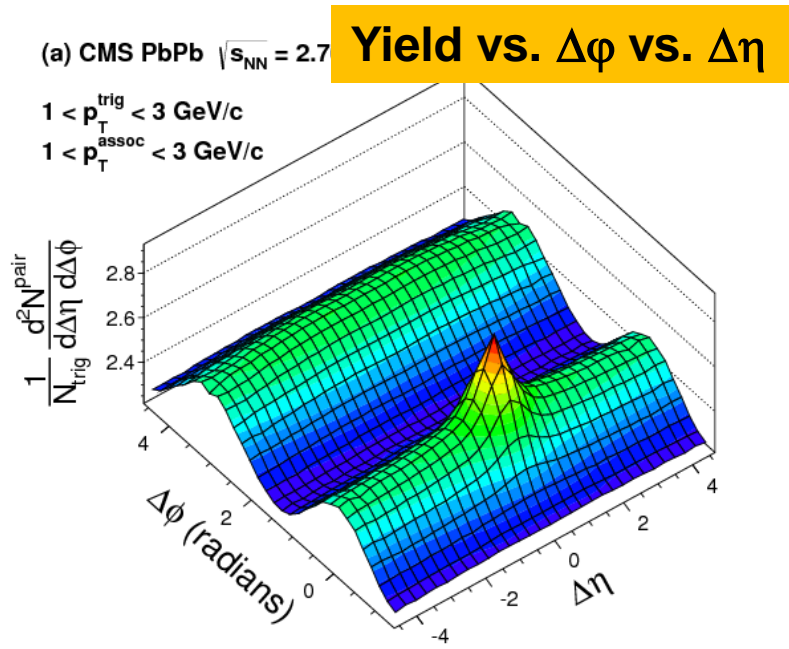
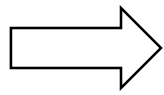
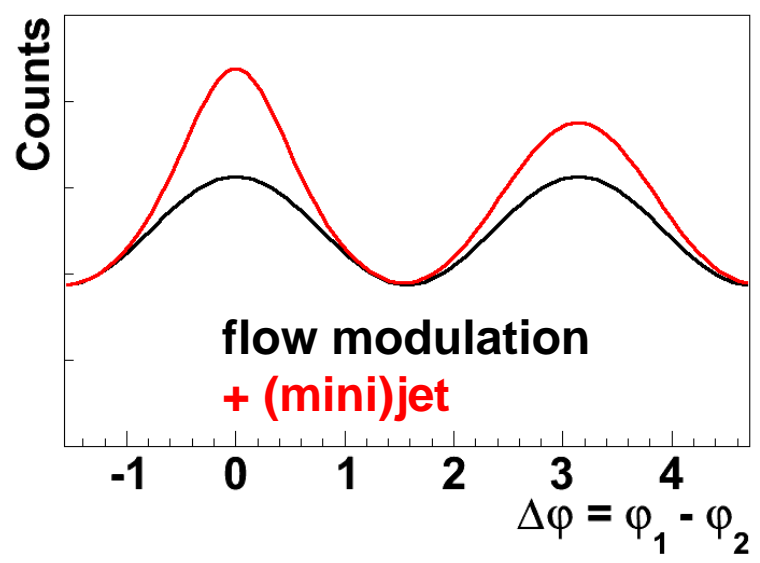
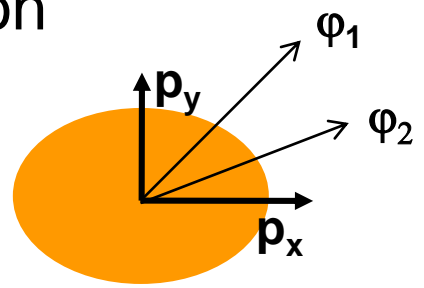
Collectivity in Small Systems

Some surprises...



Recap Two-Particle Correlations

- For v_n measurement, we discussed contribution from flow and non-flow ((mini)jets)
- This can also be looked at in two dimensions
 - Azimuth $\Delta\phi$ and pseudorapidity $\Delta\eta$

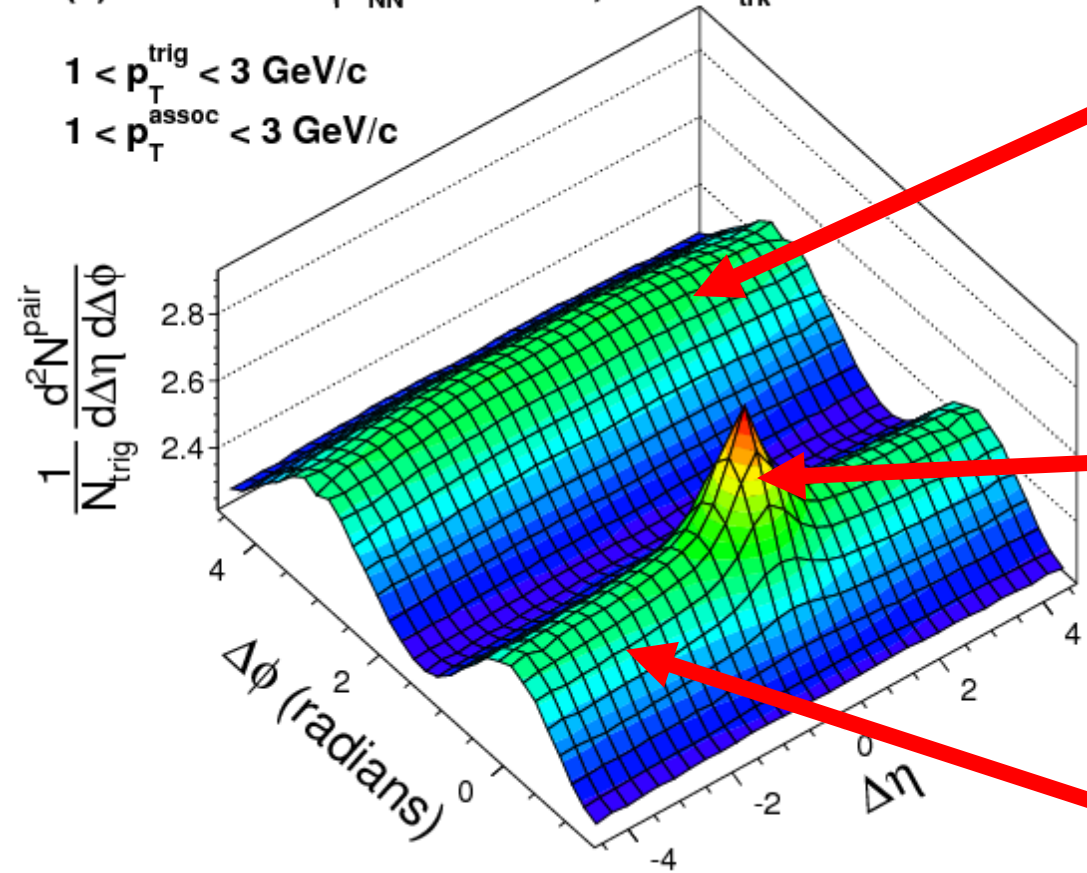




Typical Two-Particle Correlation

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{trk}^{offline} < 260$

$1 < p_T^{trig} < 3$ GeV/c
 $1 < p_T^{assoc} < 3$ GeV/c



Away-side jet + flow
($\Delta\phi \sim \pi$, elongated in $\Delta\eta$)

Near-side jet + resonances, ...
($\Delta\phi \sim 0$, $\Delta\eta \sim 0$)

Near-side flow ridge
($\Delta\phi \sim 0$, elongated in $\Delta\eta$)

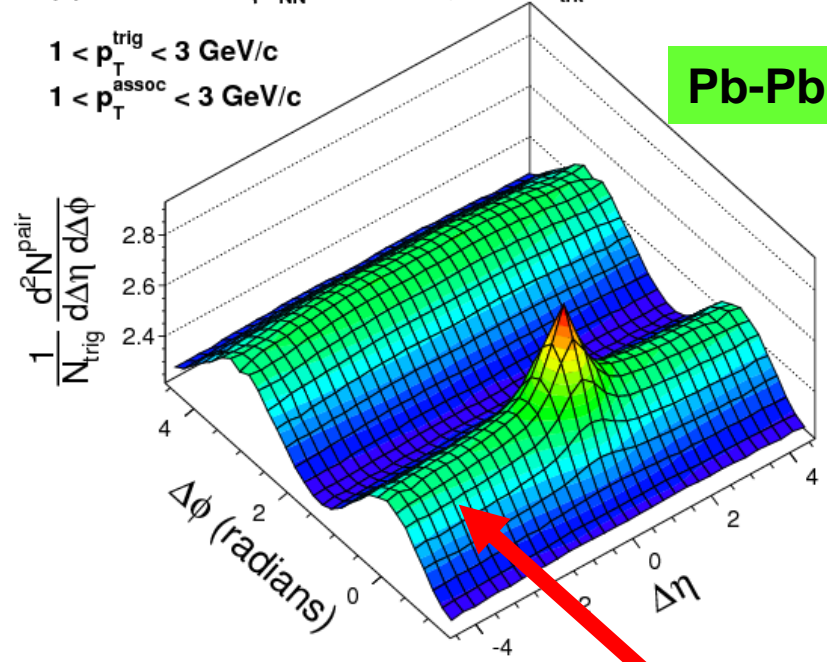


Pb-Pb vs. pp

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{trk}^{offline} < 260$

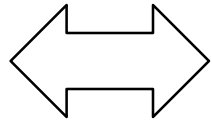
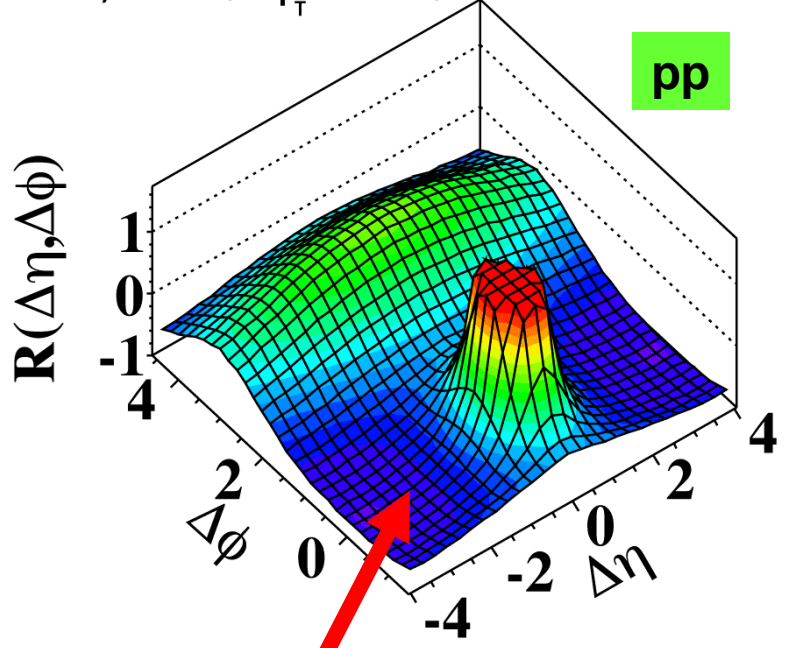
$1 < p_T^{trig} < 3$ GeV/c
 $1 < p_T^{assoc} < 3$ GeV/c

Pb-Pb



CMS 2010, $\sqrt{s} = 7$ TeV
MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$

pp



Near-side ridge (flow) only in Pb-Pb

at least everyone thought so for a long time...



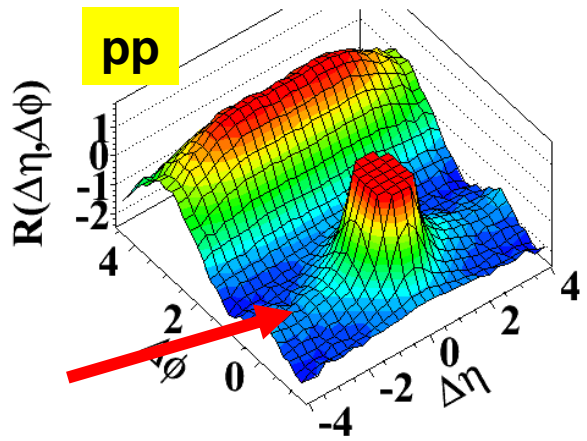
here: $\eta = \eta_{lab}$

Near-Side Ridge

0.005% of MB

- ...observed in very high-multiplicity pp collisions
 - 0.005% events with highest multiplicity

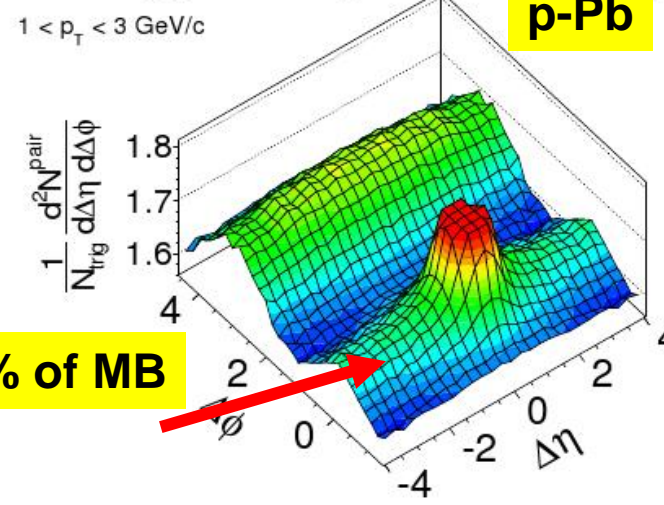
(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



CMS, JHEP09(2010)091

- ...observed in high multiplicity p-Pb collisions
 - ~40% events with highest multiplicity
 - Surprisingly large magnitude

$N_{\text{offline, trk}} > 110$
CMS pPb $\sqrt{s_{NN}} = 5.02 \text{ TeV}$, $N_{\text{trk}}^{\text{offline}} \geq 110$



3.1% of MB

CMS, PLB718 (2013) 795



The Double Ridge

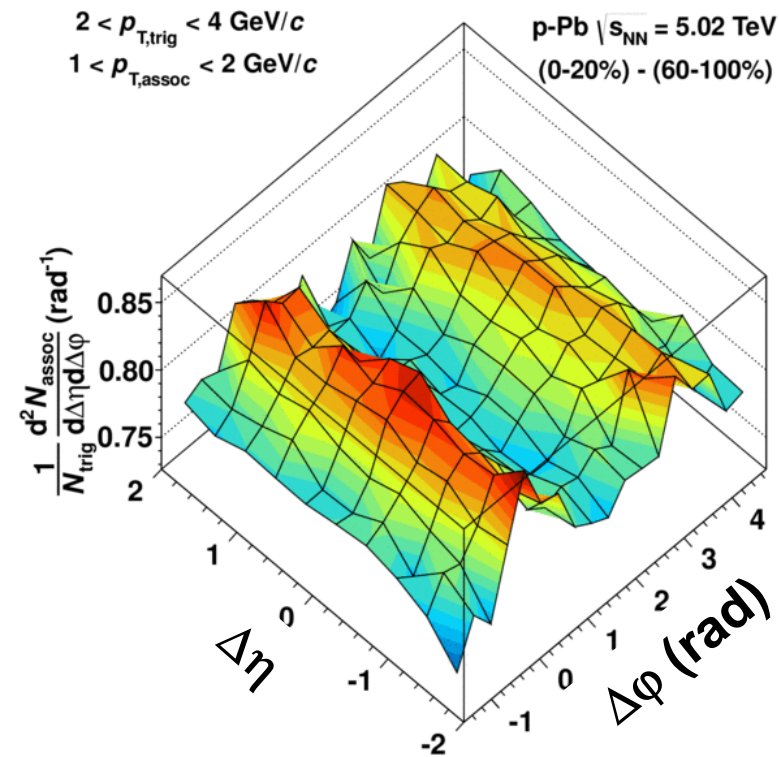
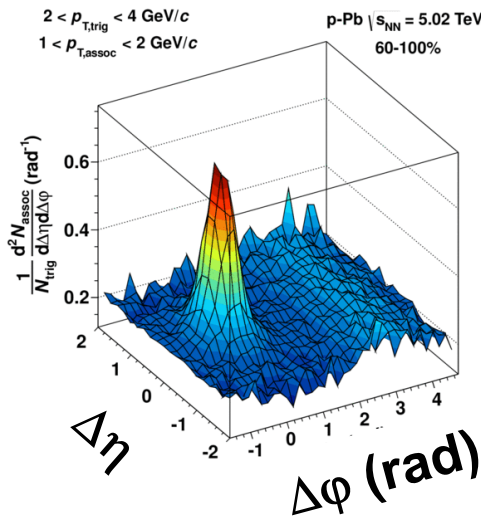
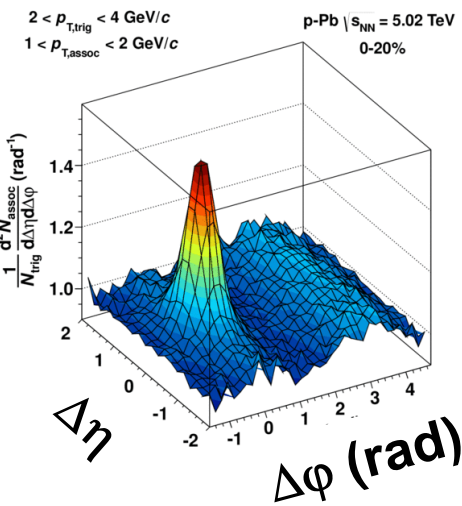
- Subtraction procedure to “isolate” ridge contribution from jet correlations
 - No ridge seen in 60-100% and similar to pp

0-20%

60-100%

$2 < p_{T,\text{trig}} < 4 \text{ GeV}/c$
 $1 < p_{T,\text{assoc}} < 2 \text{ GeV}/c$

p-Pb | $\sqrt{s_{\text{NN}}} = 5.02 \text{ TeV}$
 (0-20%) - (60-100%)



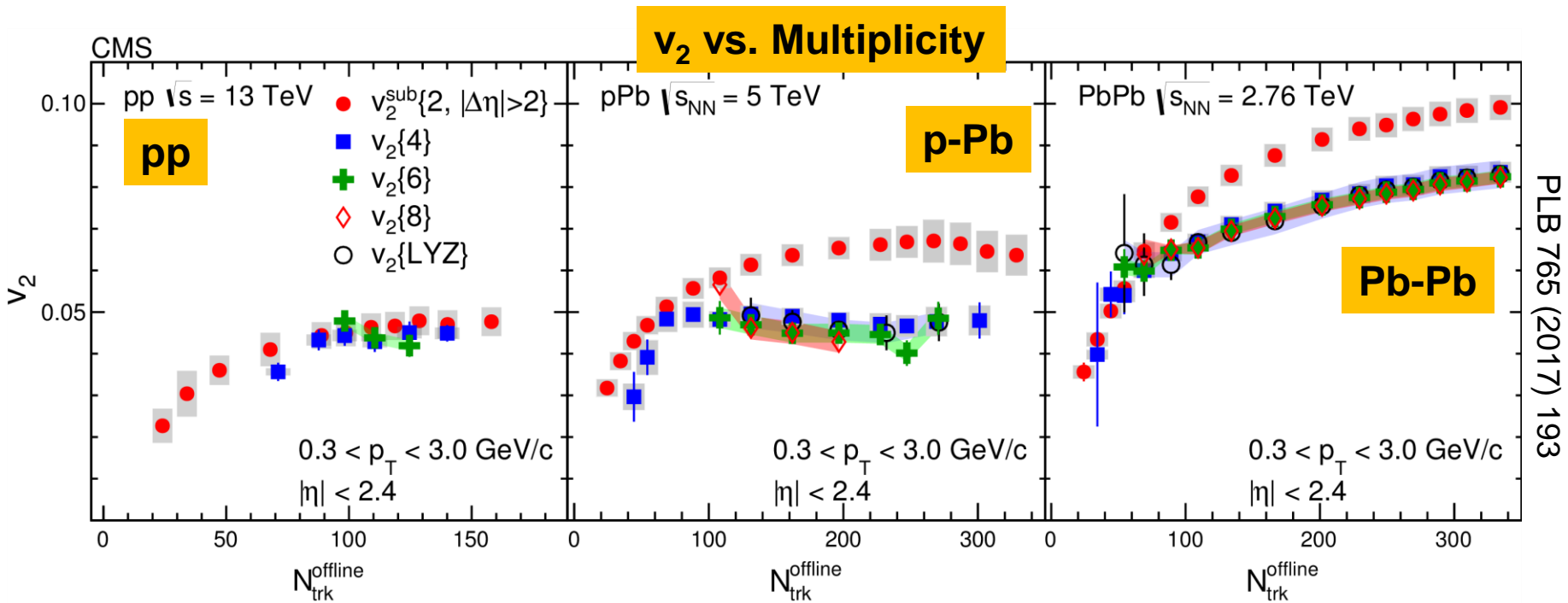
Two ridges !

ALICE, PLB719 (2013) 29



Today's Understanding

- Various “HI observables” in p-Pb and high-multiplicity pp
 - V_2, V_3, \dots
 - Multi-particle correlation $v_2\{4\} = v_2\{6\} = v_2\{8\}$
 - Mass ordering of particle species E.g. $v_2\{p\} < v_2\{\pi\}$ for $p_T < 2$ GeV/c

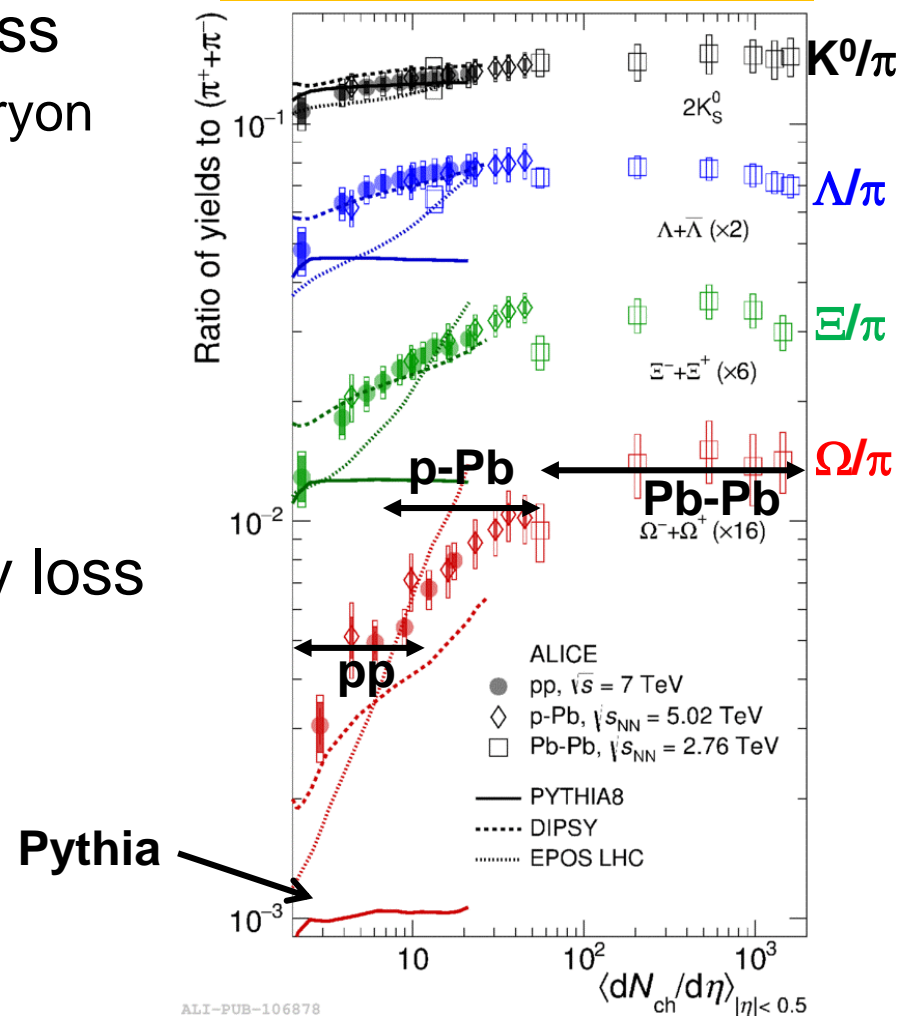




Today's Understanding (2)

- Particle ratios and strangeness
 - Smooth increase of strange baryon production
 - From pp, over p-Pb to Pb-Pb
 - Multiplicity dependence not reproduced by MC generators
- But: No sign of parton energy loss

Strange / π ratio vs. N_{ch}




Nature Phys. 13 (2017) 535



Summary Collectivity in Small Systems

- Typical Pb-Pb collision effects observed in pp and p-Pb
- Paradigm shift in interpretation of small systems
- Many hints that (mini) QGP is created in high-multiplicity p-Pb collisions (and pp collisions?)

For LHC	pp	p-Pb	Pb-Pb
Size collision region (fm ²)	2	12	150
Volume at freeze-out (fm ³)	25	160	5000
Energy density (GeV/fm ³)	?	3 (?)	10

- Debate on influence of the initial state effect as opposed to a collective approach (rescattering) 

Topic of ongoing exciting research – Stay tuned... or even better: join in!



What Next?

- Observations challenge **two paradigms** at once
 - For how small systems does the HI “standard model” remain valid?
 - Can the standard tools for pp physics remain standard?

Run 1 + 2 (2009-2018)

- Discovery of heavy-ion like phenomena in small systems
- Characterization of multi-particle correlations and strangeness enhancement

Non-flow-free correlation measurements
→ nature of higher-order correlations

Energy-loss signals
→ role of final-state interactions

Run
3 + 4

2021-2029

Strangeness enhancement
→ insight into baryon production

Thermal radiation
→ isotropization / equilibration

Chance to find unified description of underlying dynamics across system size



Summary Medium Evolution

$dN_{ch}/d\eta \sim 1600$ particles

Large pressure
↓
collective flow

Dense medium
↓
Energy loss,
Quarkonia melting

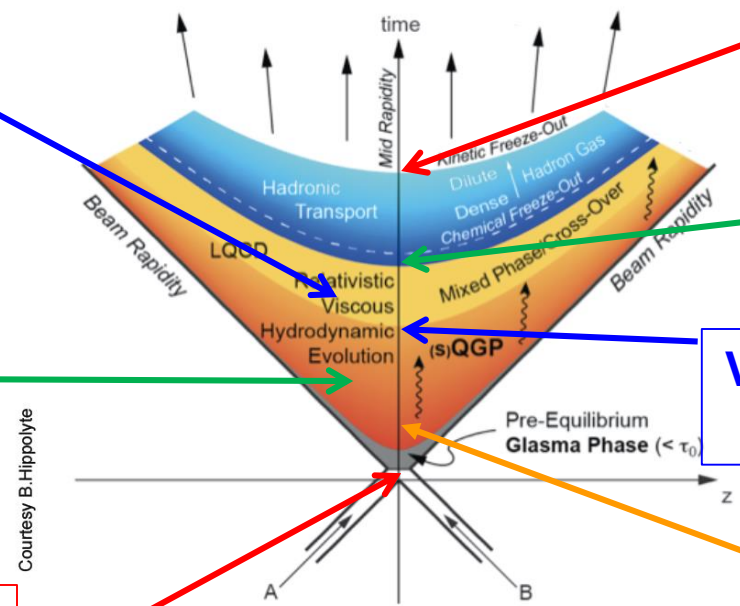
Density fluctuations
↓
spatial anisotropies

Kinetical
freeze-out
~ 90 MeV

Chemical
freeze-out
~ 155 MeV

Viscous hydrodynamics
 $\eta/s \sim 0.08 - 0.16$

Initial
temperature*
~ 300 MeV



Values for central $\sqrt{s_{NN}} = 2.76$ TeV collisions (LHC)
* from direct photons (not discussed)



Take-Home Messages

- Dense colored strongly coupling medium is produced in heavy-ion collisions (the *Quark-Gluon Plasma*)
 - Particle production is strongly suppressed
- Created matter is in local thermal equilibrium
 - Particle production described by statistical models
 - Expansion described by viscous hydrodynamics “perfect liquid”
- Recent discoveries and observations in p-Pb collisions hint at collective “QGP-like” effects in small systems
 - Universal description across system size?

**Thank you for
your attention**

Many thanks for useful discussions and inspiring previous lectures to Federico Antinori, Davide Caffarri, Leticia Cunqueiro, Andrea Dainese, Michele Floris, Alexander Kalweit, Andreas Morsch, Raimond Snellings, Alberica Toia



Backup



Fluctuations

- Initial-state density fluctuations cause higher-order flow
- For a given order
 - Value is not the same event by event
 - Usually we look at averages

$$\langle e^{in(\phi_1 - \phi_2)} \rangle = v_n^2 \text{ means actually } \left\langle \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle_{\text{tracks}} \right\rangle_{\text{events}} = \langle \langle 2 \rangle \rangle = \langle v_n^2 \rangle$$

$$\langle \langle 4 \rangle \rangle = -\langle v_n^4 \rangle \text{ etc.}$$

non-flow not shown for simplicity
 for $\sigma_{v_n} \ll \langle v_n \rangle$

- However we look for $\langle v_n \rangle$
- $\langle v_n \rangle^k = \langle v_n^k \rangle$ without fluctuations



Deviates with fluctuations

$$v_n \{2\} = \langle v_n^2 \rangle^{1/2} \approx \langle v_n \rangle + \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle} \quad v_n \{4\} = \langle v_n^4 \rangle^{1/4} \approx \langle v_n \rangle - \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

← fluctuation
← average



Fluctuations (2)

- v_2 distribution is broad
- Influence of fluctuations significant
- Estimate of fluctuations

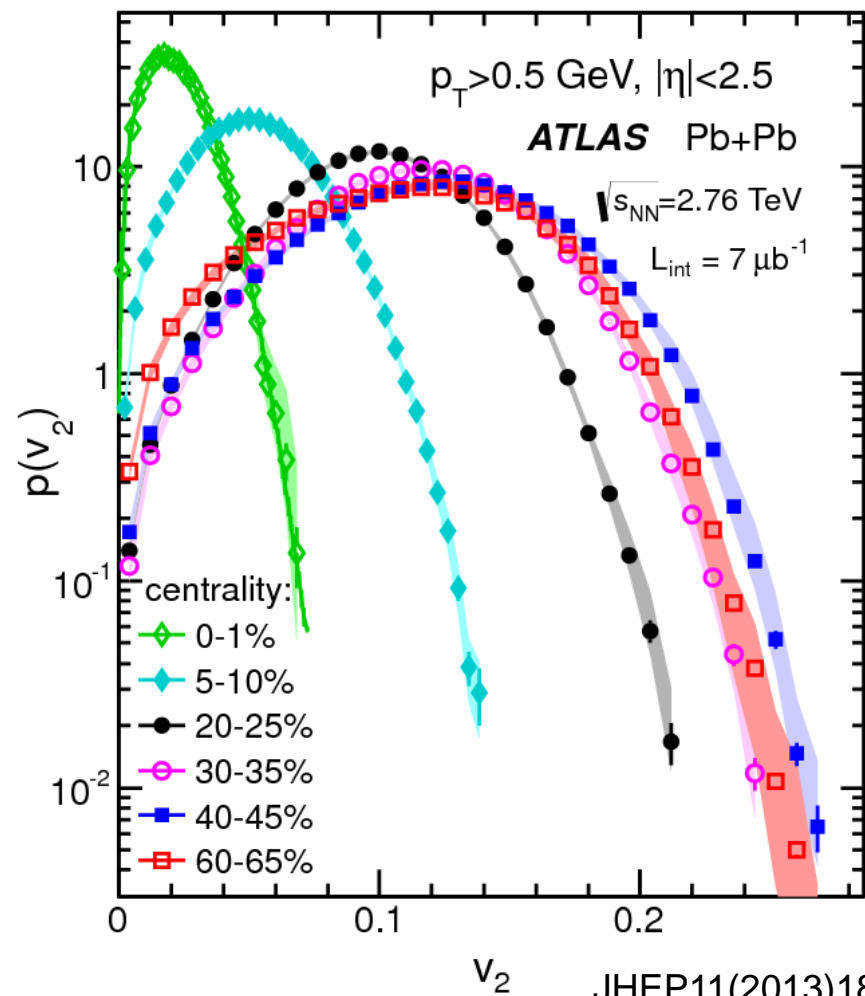
$$v_n \{2\} \approx \langle v_n \rangle + \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

$$v_n \{4\} \approx \langle v_n \rangle - \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

$$\frac{\sigma_{v_n}}{\langle v_n \rangle} \approx \sqrt{\frac{v_n^2 \{2\} - v_n^2 \{4\}}{v_n^2 \{2\} + v_n^2 \{4\}}}$$

for $\sigma_{v_n} \ll \langle v_n \rangle$

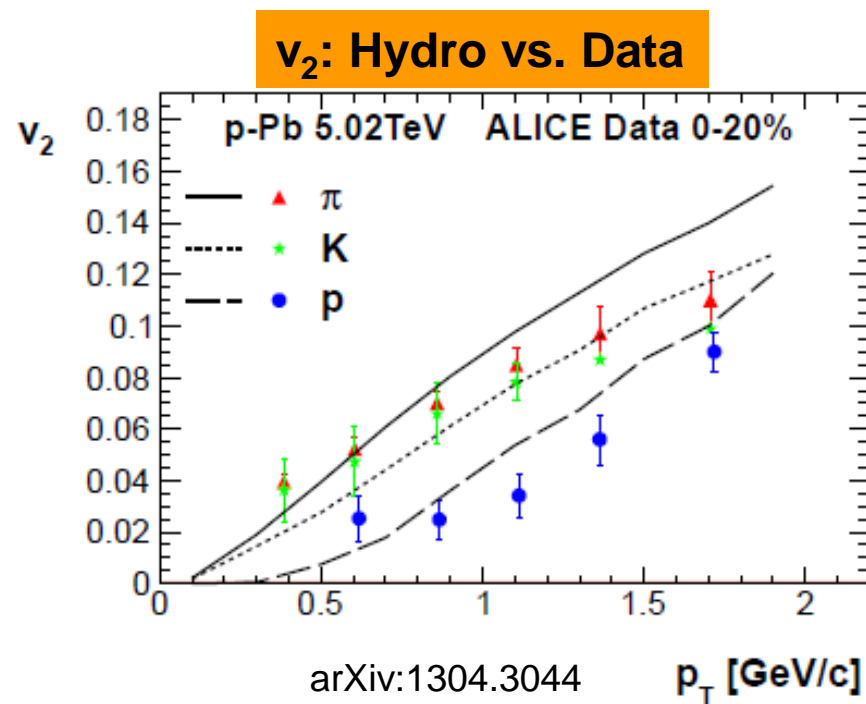
P(v_2) vs v_2





Interpretation Hydro?

- Observed effects associated to hydrodynamical evolution in Pb-Pb collisions
- Hydrodynamics in p-Pb collisions?
 - Number of interactions?
 - Sufficient time for constituents to see each other?
- Hydrodynamics in p-Pb collisions reproduces measurements
 - Assuming 0.2-0.6 fm/c for beginning of hydro evolution

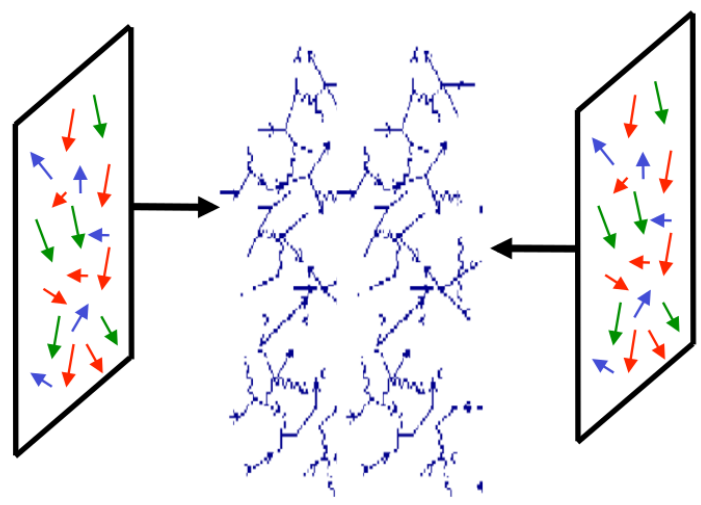




Interpretation

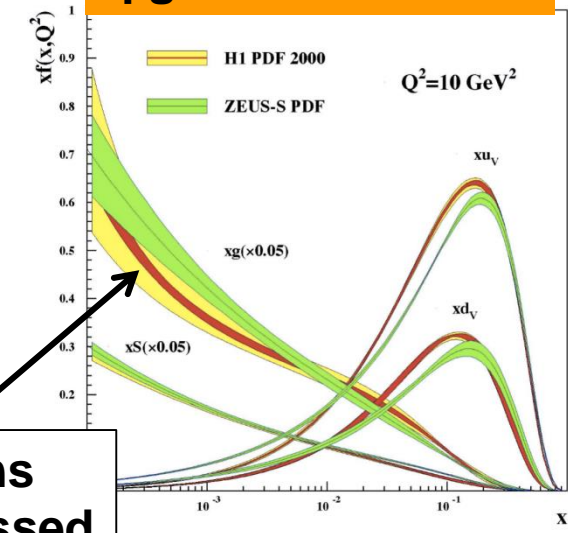
Initial-state effect?

- At low x , gluon density rises
- In nucleus density increases by $A^{1/3} \sim 6$
→ saturation
- Model of *Color Glass Condensate*

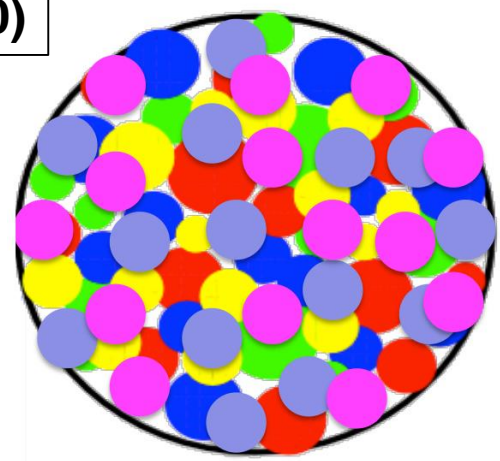


Color: gluon color charge
Glass: solid on short time scale, liquid on large time scales
Condensate: high density

q/g densities vs. x



Gluons
 (suppressed
 by factor 20)

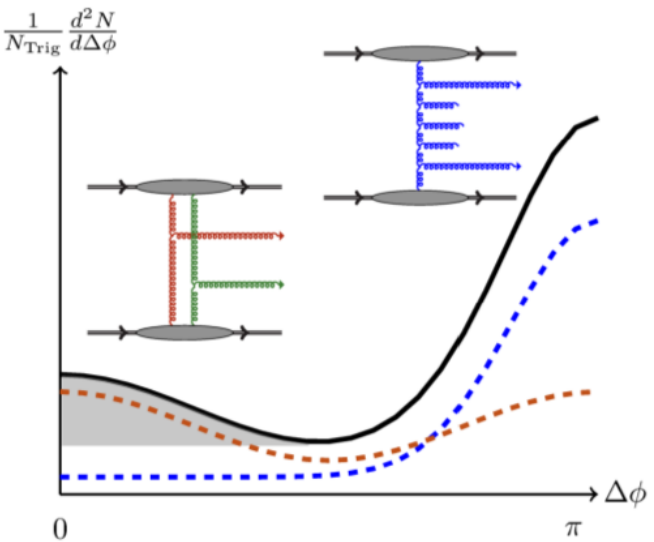




Interpretation (2)

Initial-state effect?

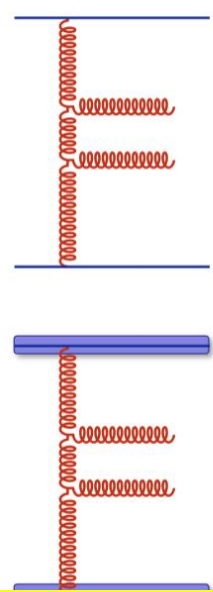
- Saturation enhances certain graphs by orders of α_S
 - Glasma graph enhanced by twice the order of magnitude than jet graph



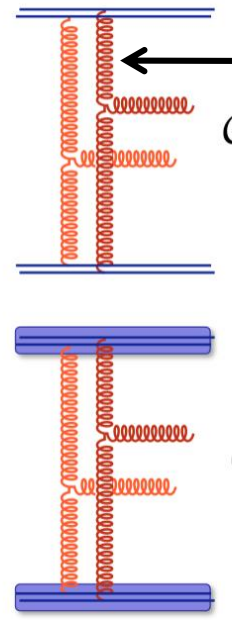
Low color density

High color density

"jet" graph



"glasma" graph



Within these models, ridge can be calculated quantitatively
Then there are lots of other qualitative ideas...

PRD 87, 094034 (2013)