

Introduction to Heavy-Ion Physics Part III

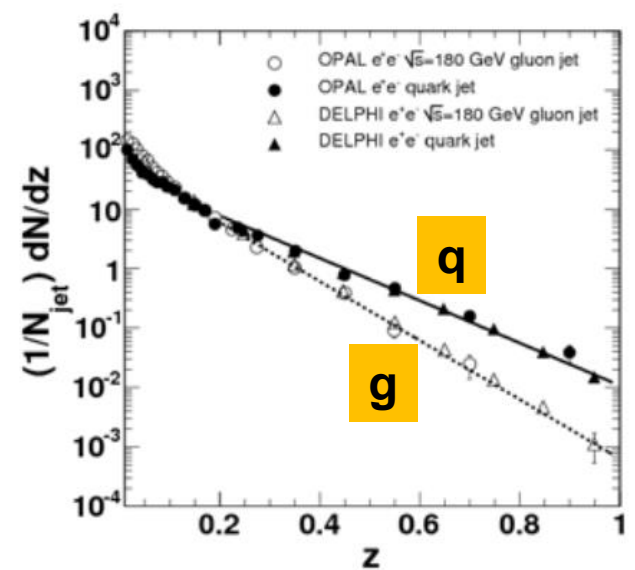
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Summer Student Lectures 2019



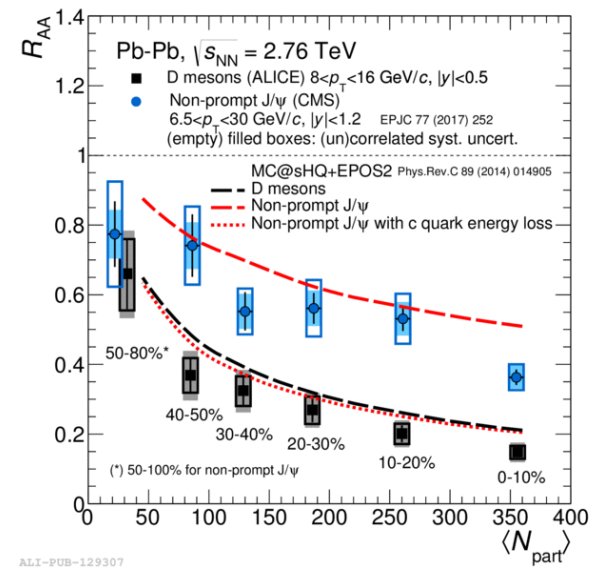
Recap Lecture II

- Energy loss in the medium by elastic and inelastic processes
- Quark-mass dependence expected
 - Fragmentation needs to be considered
 - Harder fragmentation of quark over gluon



- R_{AA} of D and B mesons
 - Analysis complex due to small S/B ratio
 - Mass dependence of energy loss

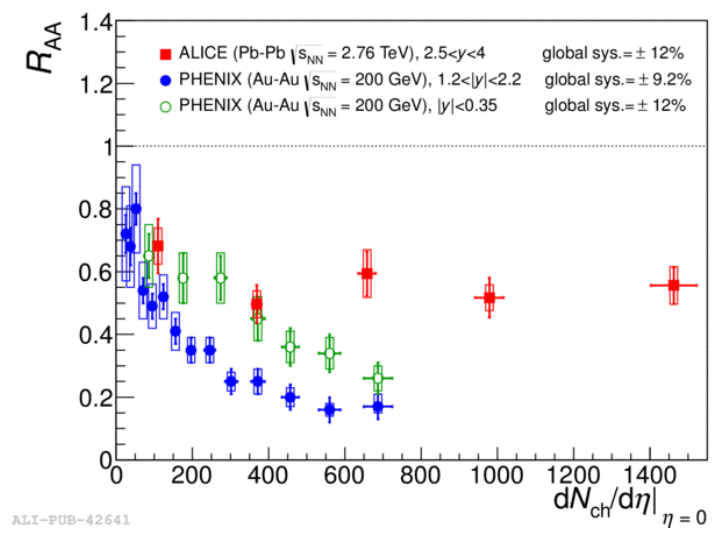
$$R_{AA}^{\pi} \approx R_{AA}^D < R_{AA}^B$$



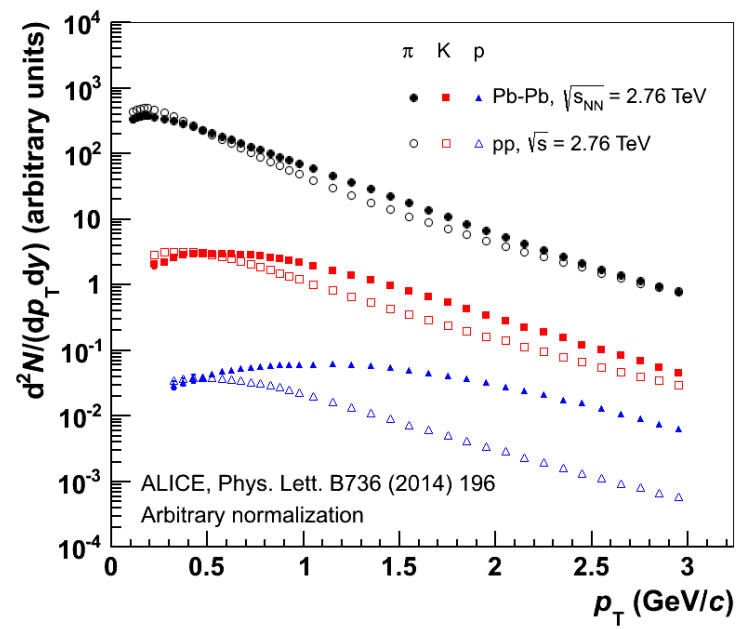


Recap Lecture II

- Quarkonia (c-cbar, b-bar) “melt” due to color screening in the QGP
 - J/ψ suppression
 - Abundance of c at LHC so large that J/ψ regenerate statistically
 - States with lower binding energy are more suppressed
- Hadron yields described by statistical models for $\sqrt{s_{NN}} = 2-2760$ GeV
 - Matter created in HI collisions is in local thermal equilibrium
- Expansion of QGP changes momenta of particles
 - Radial flow (dependent on particle mass)

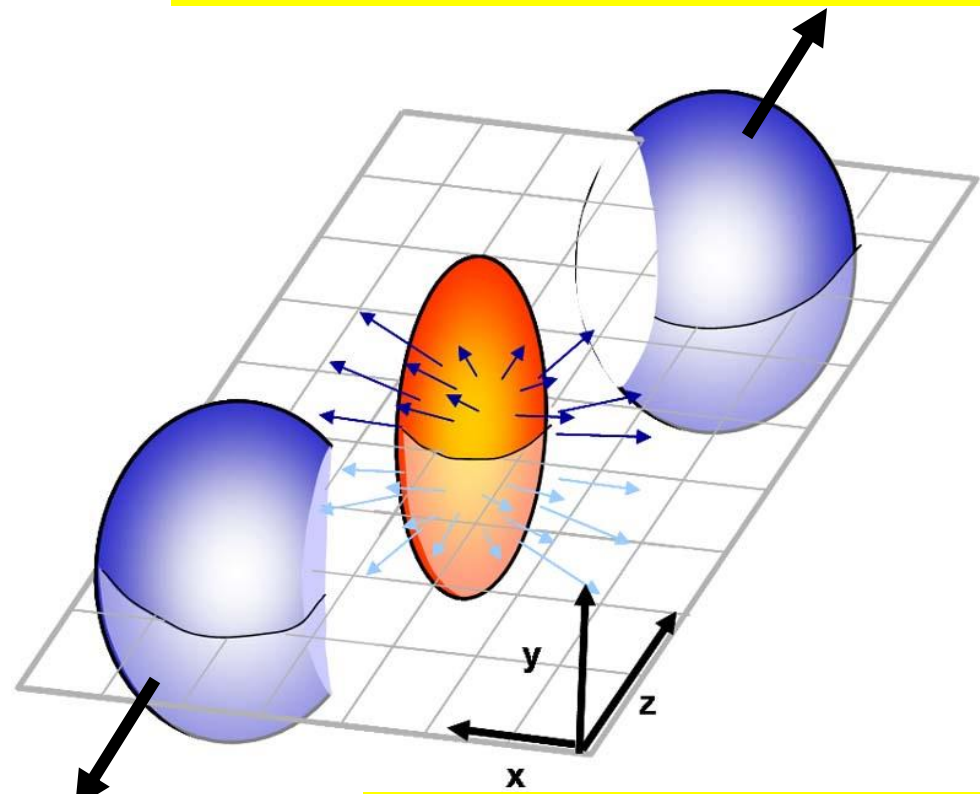


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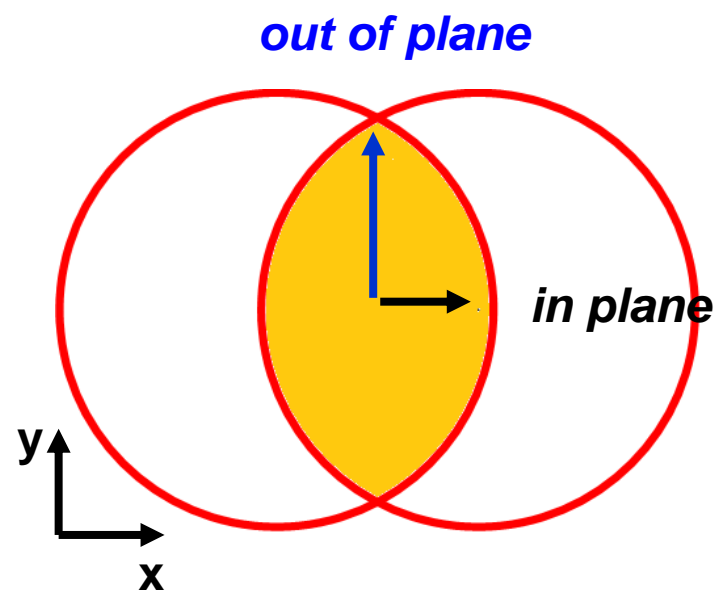


Elliptic Flow

Overlap of colliding nuclei not isotropic in non-central collisions



Defines reaction plane Ψ_{RP}
(spanned by beam axis
and impact parameter vector)



→ Pressure gradients
dependent on direction

$$\text{here: } \frac{dp_x}{dL} > \frac{dp_y}{dL}$$



Elliptic Flow (2)

- Spatial anisotropy (almond shape)

- Quantified by eccentricity ε

$$\varepsilon = \frac{y^2 - x^2}{y^2 + x^2}$$

- Pressure gradient larger in-plane

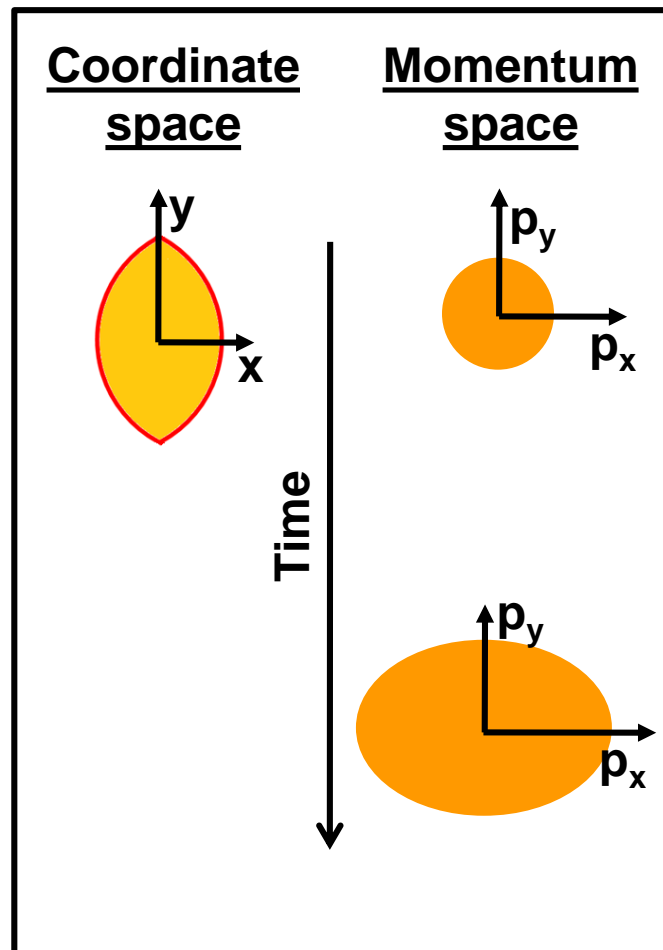
- Pressure pushes partons

- More in in-plane than out-of-plane

- Spatial anisotropy converts into momentum-space anisotropy

- “Faster” particles in-plane

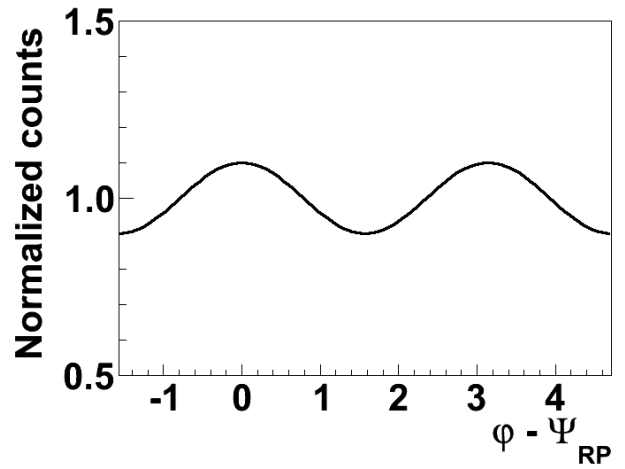
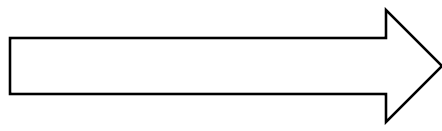
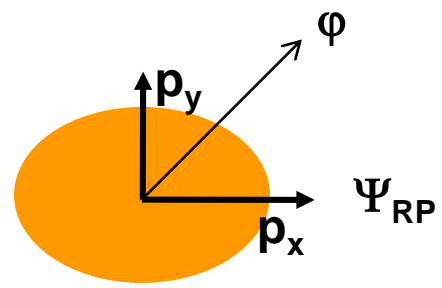
- Measurable in the final state!





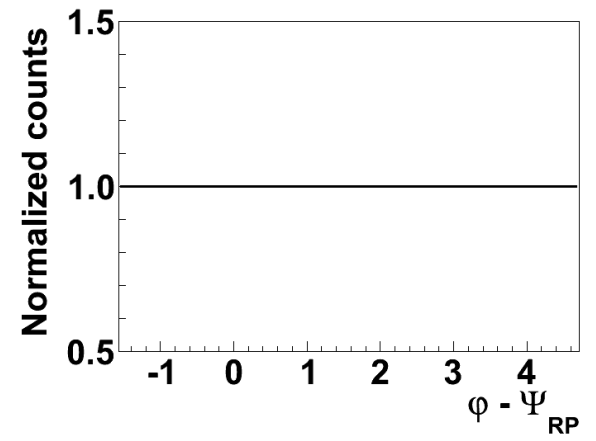
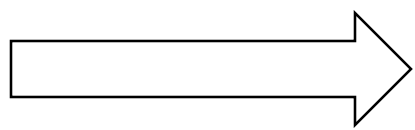
Elliptic Flow (3)

- Particles as a function of $\varphi - \Psi_{RP}$



$$\frac{dN}{d\varphi} = A(1 + 2v_2 \cos 2(\varphi - \Psi_{RP}))$$

- Define $v_2 = \langle \cos 2(\varphi - \Psi_{RP}) \rangle$
 - Second coefficient of Fourier expansion
- Ψ_{RP} common *symmetry plane* (for all particles)
- What if there were no correlations with Ψ_{RP} ?





Measuring Elliptic Flow

$$v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$$

Measure tracks

Measure reaction-plane angle

- Reaction plane angle
 - From the particles themselves

$$Q_x = \sum_i w_i \cos 2\varphi_i \quad Q_y = \sum_i w_i \sin 2\varphi_i \quad \Psi_{RP} = \tan^{-1}(Q_x, Q_y) / 2$$

weight w

- Ψ_{RP} approximates true reaction-plane angle (called *event plane*)
- Calculation of *integrated* $v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$
- $v_2(p_T)$ by considering only particles at given p_T
- Called *event plane method*, denoted $v_2\{EP\}$

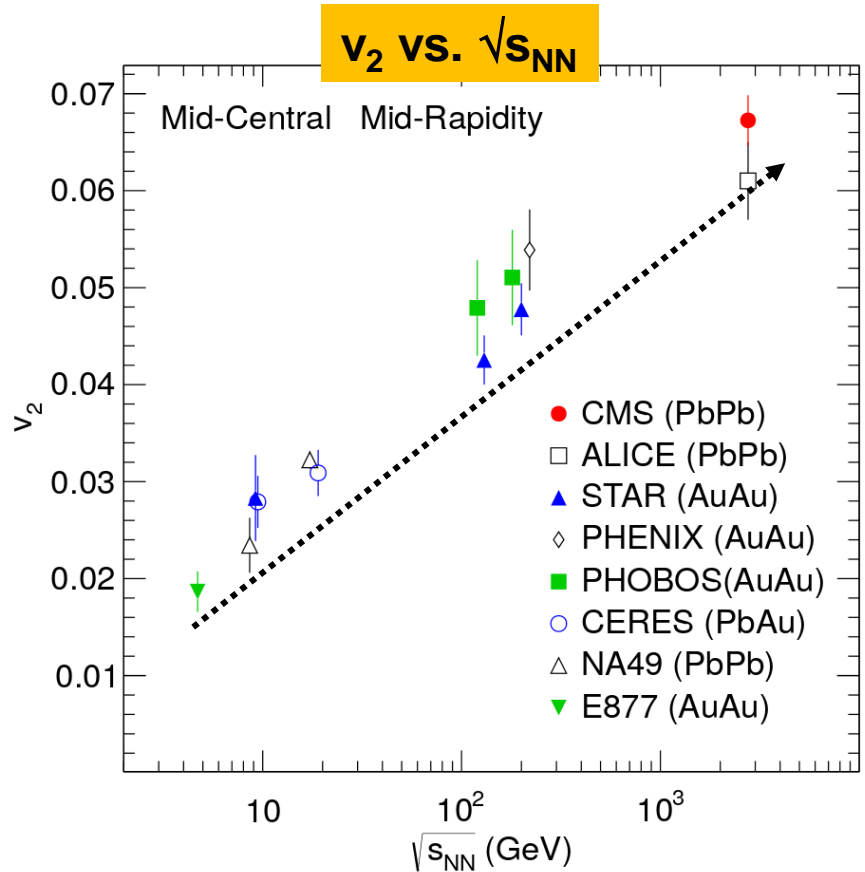
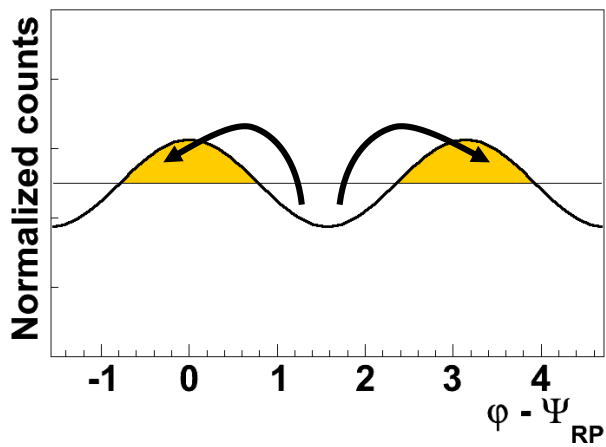


$\sqrt{s_{NN}}$ Dependence

- Increases with $\sqrt{s_{NN}}$
- At LHC $v_2 \sim 0.06$
 - What does that mean?

$$\frac{dN}{d\phi} = A(1 + 2v_2 \cos 2(\phi - \Psi_{RP}))$$

- $2v_2 = 12\%$ of particles “move” from out-of-plane to in-plane

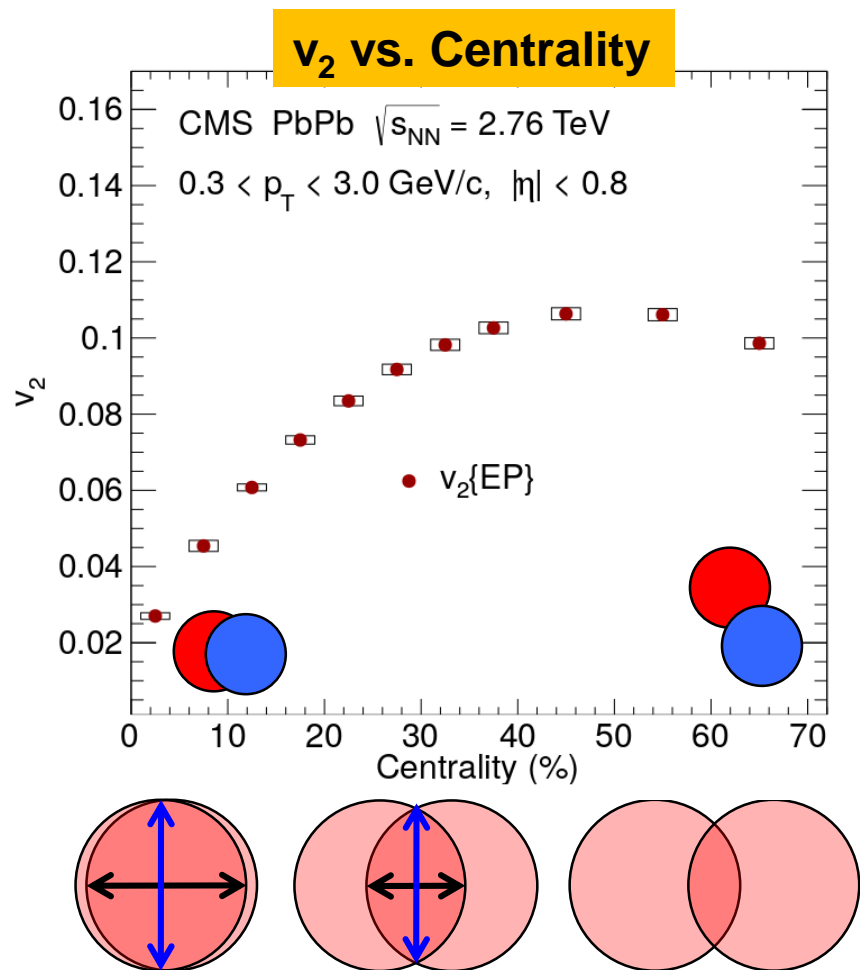


CMS, PRC 87(2013) 014902



Centrality Dependence

- Strong centrality dependence
- v_2 largest for 40-50%
- Spatial anisotropy very small in central collisions
- Largest anisotropy in mid-central collisions
- Small overlap region in peripheral collisions



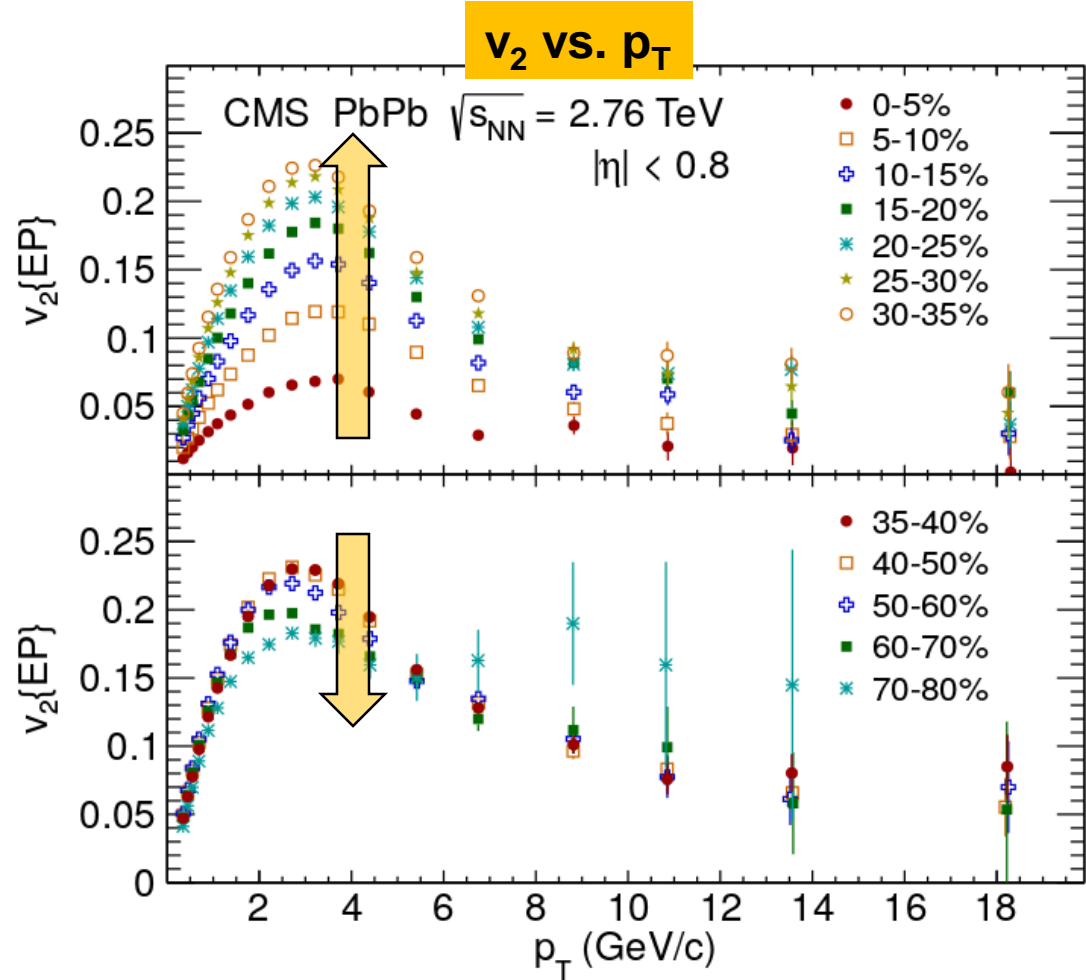
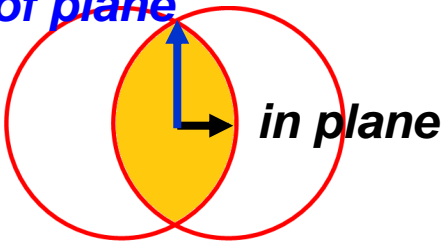
CMS, PRC 87(2013) 014902



p_T Dependence

- Centrality dependence independent of p_T
- Largest v_2 for $p_T \sim 3$ GeV/c
- Low and intermediate p_T , v_2 caused by collective expansion
- Large p_T , v_2 caused by *length-dependent jet quenching*
 - Longer path length out of plane than in plane

out of plane



CMS, PRC 87(2013) 014902



Recap

- Pressure in dense medium affects momenta
- Isotropic expansion effect called *radial flow*
- Overlap of colliding nuclei causes spatial anisotropy
- Converted into momentum-space anisotropy in medium evolution
- Modulation of observed particles
- Quantified by $v_2 = \langle \cos 2 (\varphi - \Psi_{RP}) \rangle$

What other methods exist to measure v_2 ?

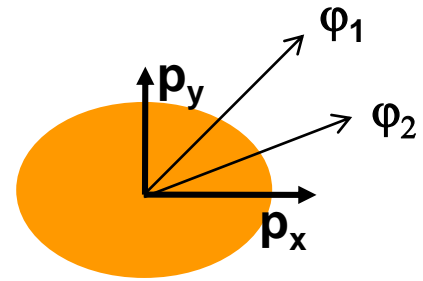
What effect do jet-related particles have on v_2 ?



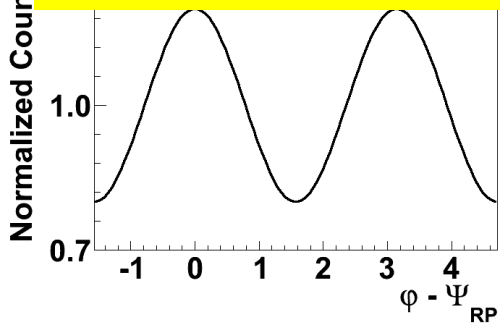
Two-Particle Correlations

- Reaction-plane estimation can be experimentally tricky
- Rewrite $v_2 = \langle \cos 2(\varphi - \Psi_{RP}) \rangle$ as $v_2 = \langle e^{i2(\varphi - \Psi_{RP})} \rangle$
- v_2 can also be measured from 2-particle correlations

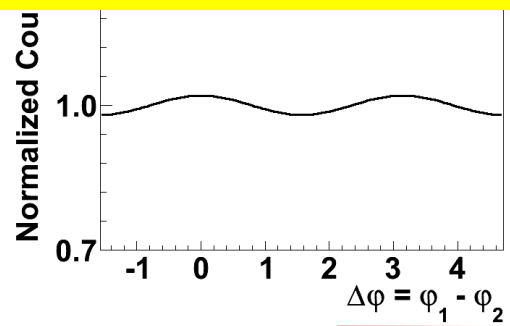
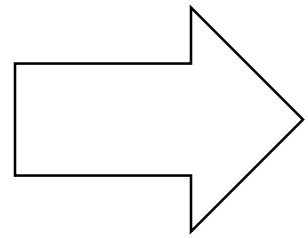
$$\begin{aligned} \langle e^{i2(\varphi_1 - \varphi_2)} \rangle &= \langle e^{i2(\varphi_1 - \Psi_{RP} - (\varphi_2 - \Psi_{RP}))} \rangle = \\ &= \langle e^{i2(\varphi_1 - \Psi_{RP})} \rangle \langle e^{i2(\varphi_2 - \Psi_{RP})} \rangle = v_2^2 \end{aligned}$$



Modulation smaller due to $v_2 \rightarrow (v_2)^2$ but statistical power similar



$$v_2 = \langle e^{i2(\varphi - \Psi_{RP})} \rangle$$



$$v_2^2 = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle$$



Higher-Order Correlations

- Trivial extension to 4-particles (and higher-orders)

$$v_2^4 = \left\langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \right\rangle$$

$$v_2^6 = \left\langle e^{i2(\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5 - \varphi_6)} \right\rangle$$

- NB. sign is arbitrary as long as same amount of positive and negative angles
→ rotational symmetry



Cumulants

- Cumulants extract genuine n-particle correlations
- For 2-particle correlations

$$\langle x_1 x_2 \rangle = \underbrace{\langle x_1 \rangle \langle x_2 \rangle}_{\text{lower order "correlations"}} + \langle x_1 x_2 \rangle_c$$

measured correlation

lower order "correlations"

genuine 2-particle correlations

ϕ dependence only from detector acceptance

- Rewrite (trivially) $\langle x_1 x_2 \rangle_c = \langle x_1 x_2 \rangle - \langle x_1 \rangle \langle x_2 \rangle$
- For 3-particle correlations

$$\langle x_1 x_2 x_3 \rangle = \langle x_1 \rangle \langle x_2 \rangle \langle x_3 \rangle + \langle x_1 x_2 \rangle_c \langle x_3 \rangle + \langle x_1 x_3 \rangle_c \langle x_2 \rangle + \langle x_2 x_3 \rangle_c \langle x_1 \rangle + \underline{\langle x_1 x_2 x_3 \rangle_c}$$

**Higher-order cumulants zero \rightarrow no genuine multi-particle correlation !
No matter what multi particles correlations (i.e. not cumulants) show**



Cumulants for Elliptic Flow

- For uniform detector acceptance, cumulants of 2nd and 4th order:

$$c_2\{2\} = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle = v_2^2 \longleftarrow \text{identical to two-particle correlation}$$

$$c_2\{4\} = \langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle - \underbrace{2 \langle e^{i2(\varphi_1 - \varphi_2)} \rangle^2}_{\text{lower orders are removed}} = -v_2^4$$

lower orders are removed

- $c_2\{4\}$ is genuine 4-particle correlations
 - I.e. if only pairs of particles are correlated $\rightarrow c_2\{4\} = 0$



Flow Methods

- Now we have tons of methods to measure flow
 - Event plane
 - 2-particle and 4-particle correlations, ...
 - 2-particle and 4-particle cumulants, ...



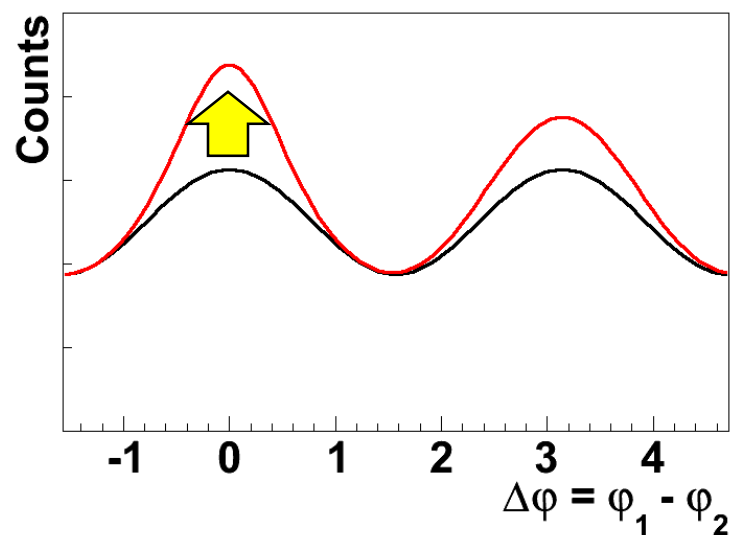
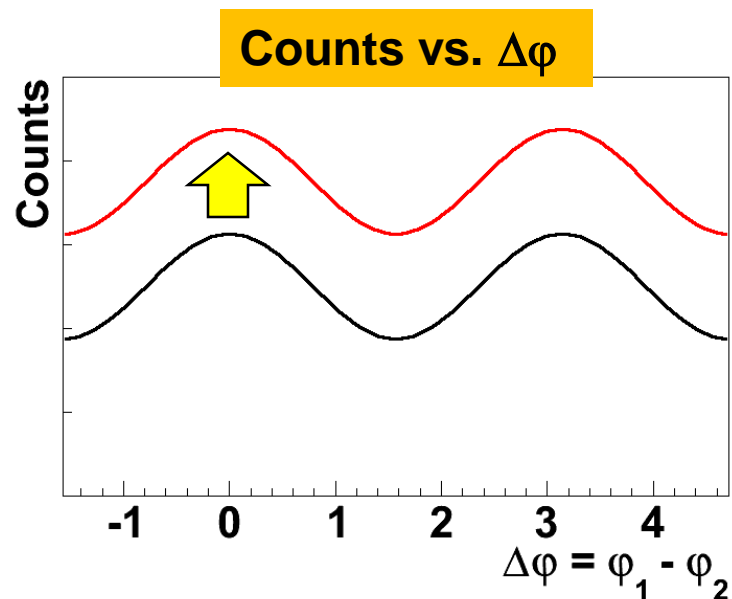
They all estimate v_2 , so what?

Let's have a look, what spoils the flow measurement...



Non-Flow

- Particles are correlated through reaction plane Ψ_{RP}
- Additional isotropically distributed particles
 - Add to baseline, reduce $\cos 2\Delta\phi$ magnitude, but don't distort shape
- Jets
 - Particles which exhibit correlations close in angle (within the same jet) and at $\Delta\phi = \pi$ (back-to-back jet)
 - Distort Ψ_{RP} estimate
 - Distorts shape in 2 particle correlations
- A pure jet-signal results in $v_2 > 0$ (e.g. Pythia)





Non-Flow (2)

- Different effect on different flow methods
- 2-particle correlations / cumulants

$$c_2\{2\} = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle = v_2^2 + \delta_2 \leftarrow \text{non-flow contribution}$$

- 4-particle correlations

$$\langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle = v_2^4 + 4v_2^2\delta_2 + 2\delta_2^2 + \delta_4$$

- 4-particle cumulants

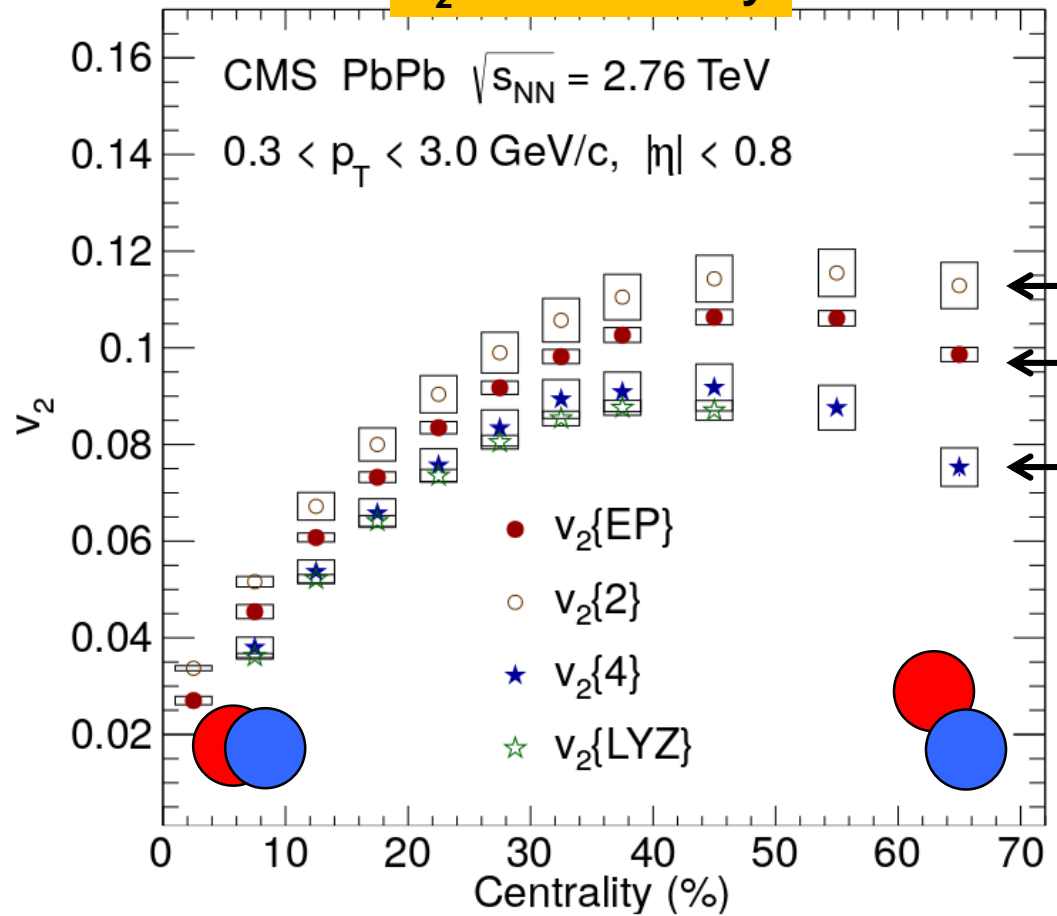
$$\begin{aligned} c_2\{4\} &= \langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle - 2\langle e^{i2(\varphi_1 - \varphi_2)} \rangle = \\ &= v_2^4 + 4v_2^2\delta_2 + 2\delta_2^2 + \delta_4 - 2(v_2^2 + \delta_2) = -v_2^4 + \delta_4 \end{aligned}$$

Second order non-flow dropped out !



Experiment

v₂ vs. Centrality



$$v_2\{2\} = \sqrt{v_2^2 + \delta_2}$$

$$v_2\{EP\}$$

$$v_2\{4\} = \sqrt[4]{v_2^4 - \delta_4}$$

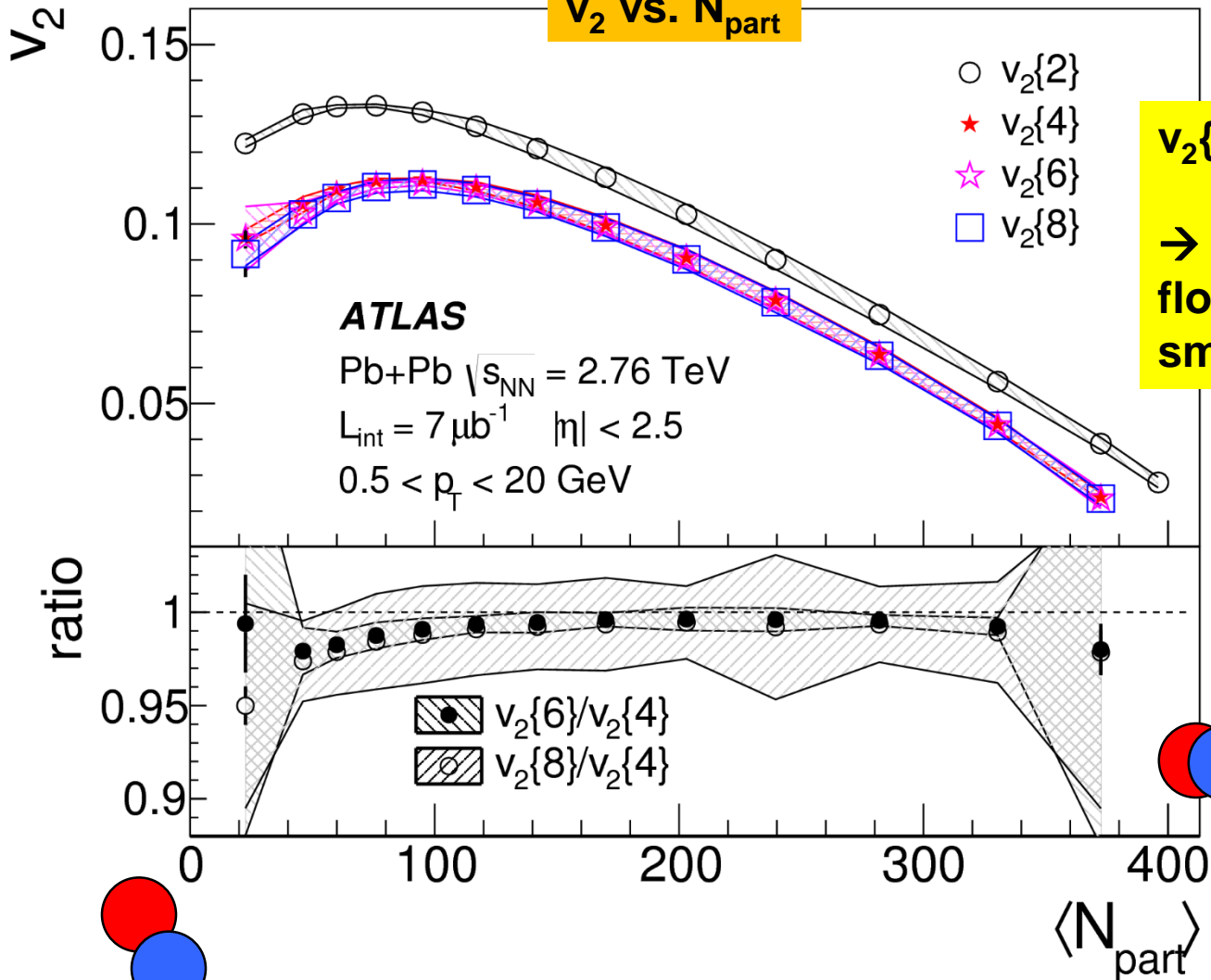
Larger non-flow influence* for v₂{2} than v₂{4}

* neglects fluctuations, see [backup](#)



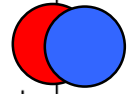
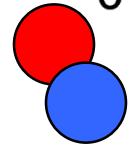
Up to 8 Particles...

v_2 vs. N_{part}



$v_2\{4\} \sim v_2\{6\} \sim v_2\{8\}$

→ influence of non-flow (and fluctuations) small for ≥ 4 particles





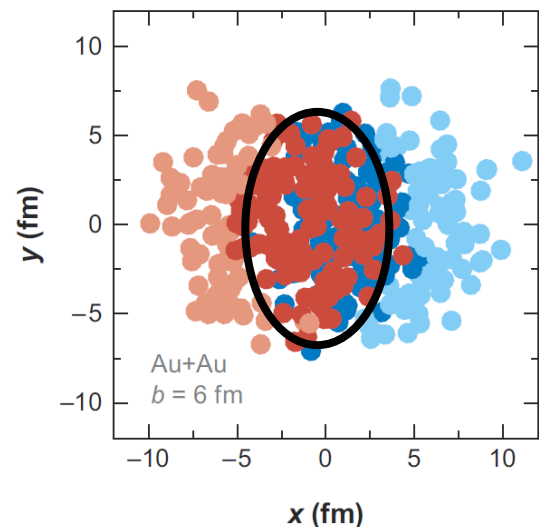
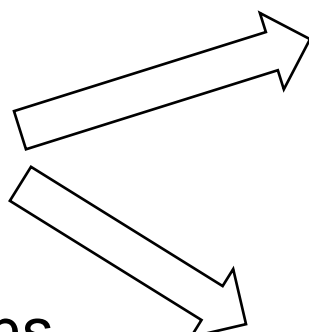
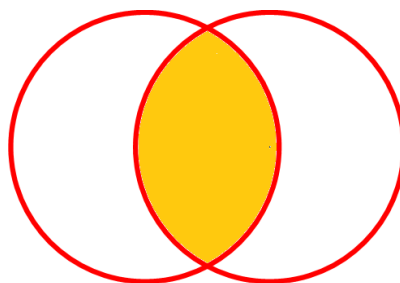
Recap

- Elliptic flow can be measured with different methods
- Cumulants of n^{th} order measure genuine n-particle correlations – not reducible to lower orders
- Mini(jets) and resonances distort the v_2 measurement
- Non-flow influence is different for different methods
 - The higher the order of the cumulant, the smaller the influence

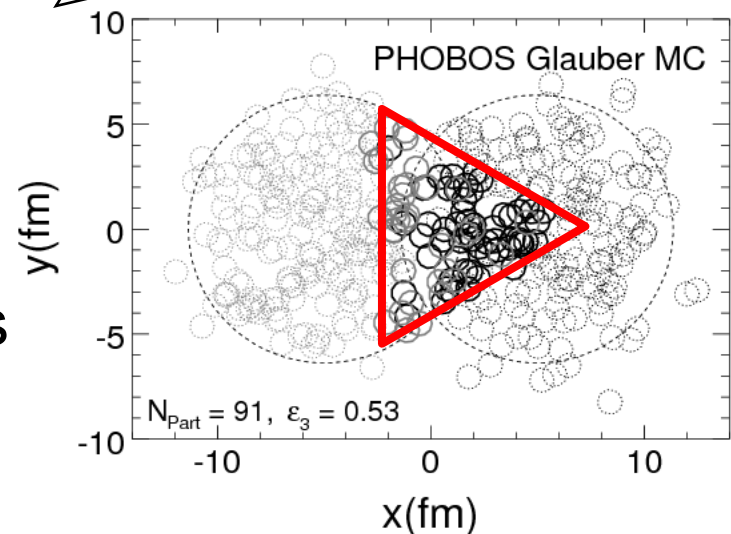
For now we have discussed elliptic flow v_2 – is that all?

Higher-Order Flow

- Geometrical picture
 → 2nd order modulation (v_2)

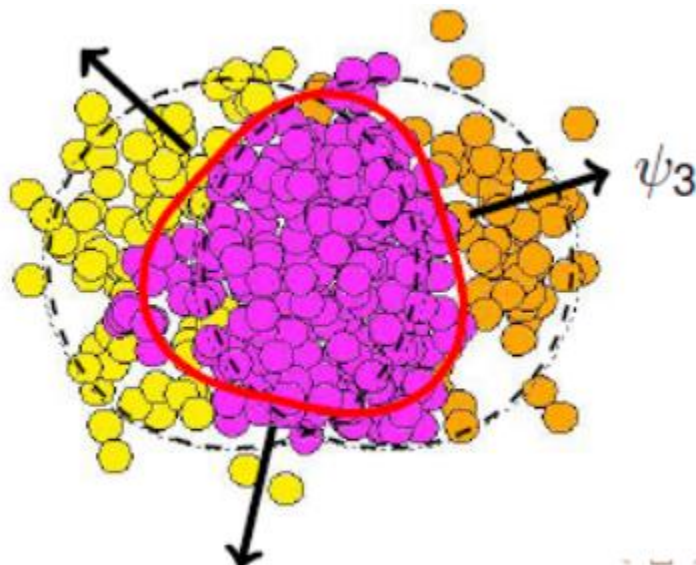


- In practice interacting nucleons need to be considered
 - E.g. estimated with Glauber MC
 - *Initial state density fluctuations*
- These produce all kinds of shapes
 - Elliptic, triangular, quadruple, ...
 - And mixtures of those



nucl-ex/0701025, PRC81 (2010) 054905

Higher-Order Flow (2)



- Reaction plane $\Psi_{RP} \rightarrow n^{\text{th}}$ order participant plane Ψ_n

$$\frac{dN}{d\varphi} = A(1 + 2v_2 \cos 2(\varphi - \Psi_{RP})) \quad \Longrightarrow \quad \frac{dN}{d\varphi} = A\left(1 + 2 \sum_n v_n \cos n(\varphi - \Psi_n)\right)$$

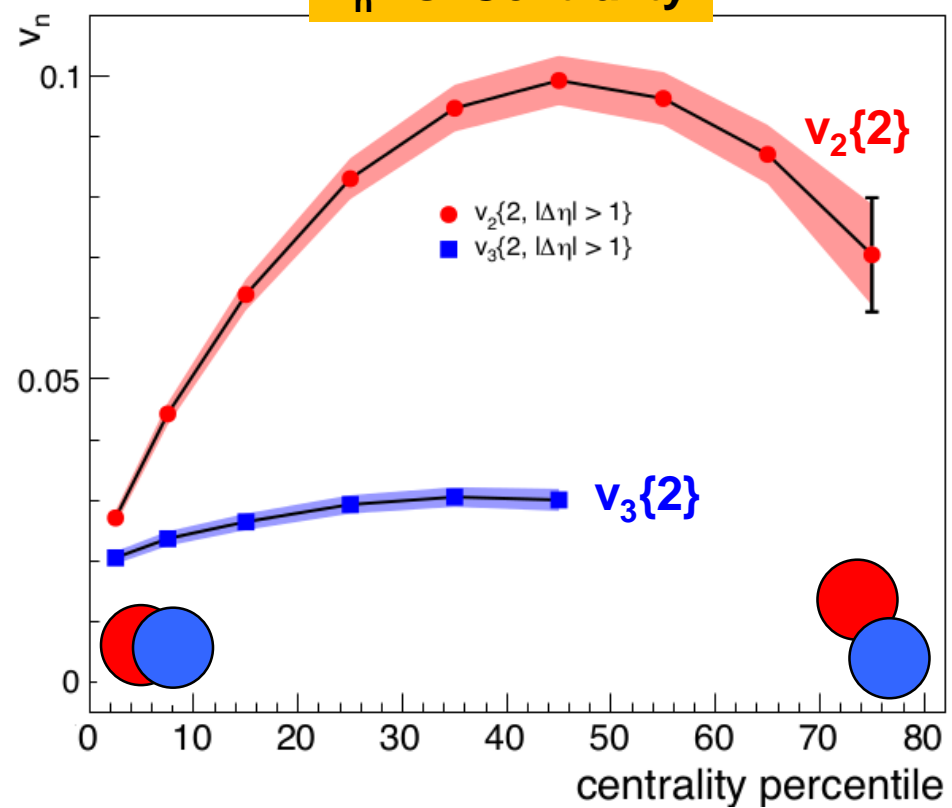
- Formalism can be trivially extended from v_2 to v_n

- E.g. $v_2^2 = \langle e^{i2(\varphi_1 - \varphi_2)} \rangle \longrightarrow v_n^2 = \langle e^{in(\varphi_1 - \varphi_2)} \rangle$ PRC81 (2010) 054905



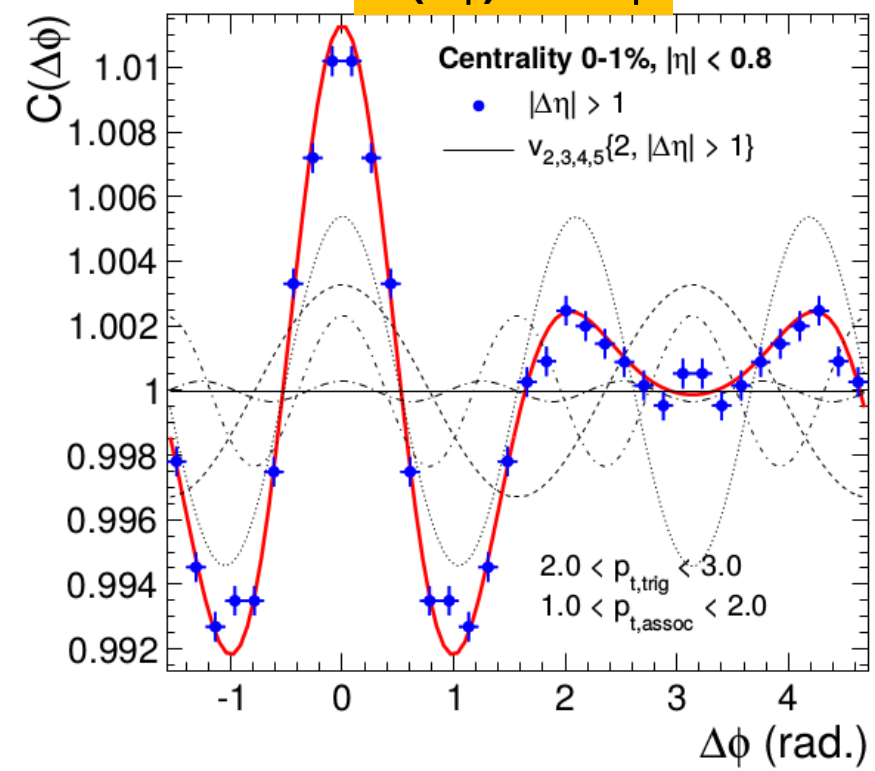
Experiment

v_n vs. Centrality



v_3 sizable
 $v_3 \sim 1/2 v_2$
Weaker centrality dependence

$C(\Delta\phi)$ vs. $\Delta\phi$



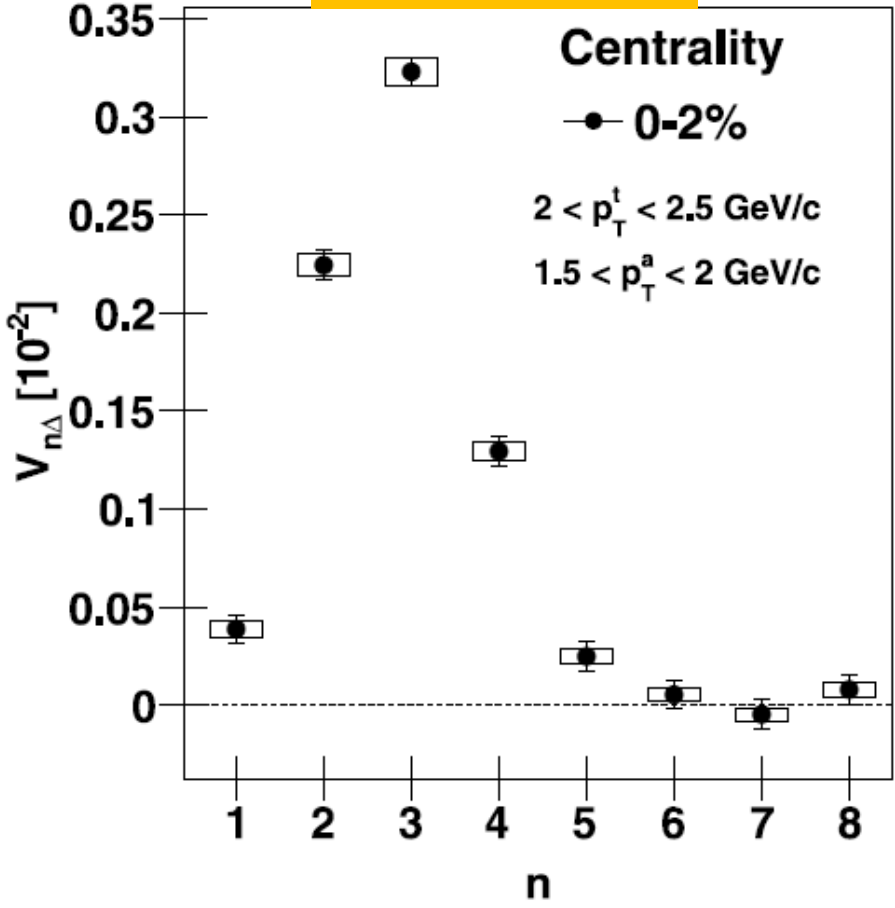
Two-particle correlations can be fully described by $v_2 \dots v_5$

PRL107, 032301 (2011)

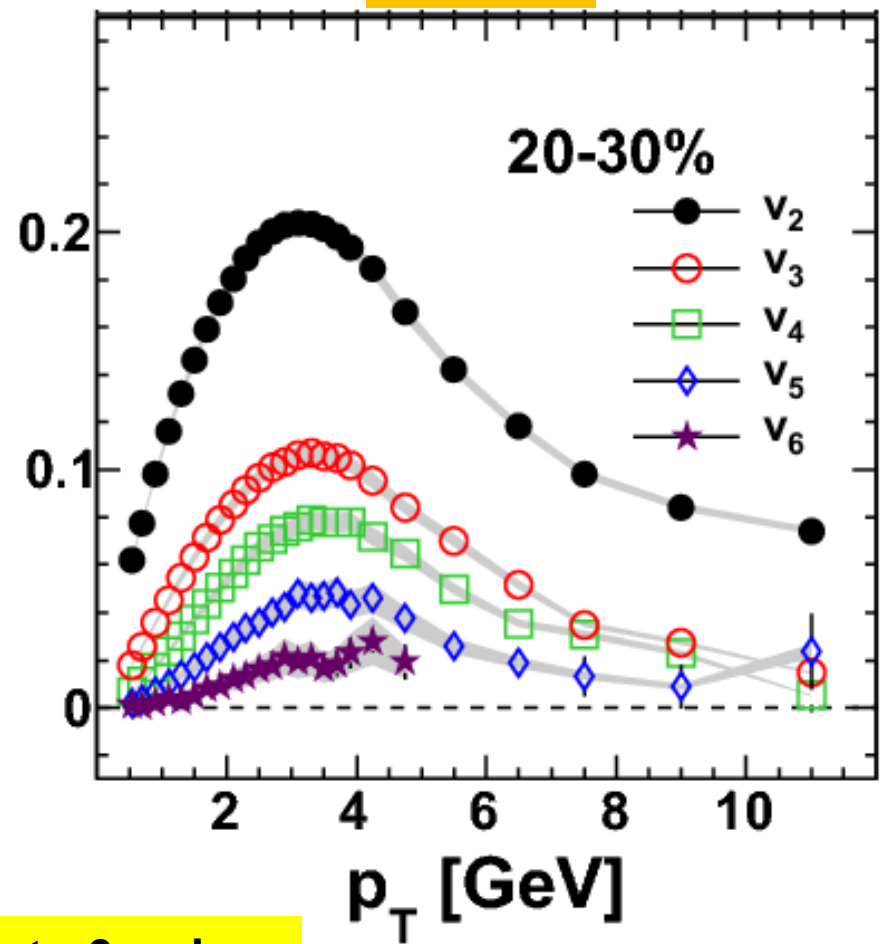


And even higher orders...

$V_{n\Delta} = (v_n)^2$ vs. n



v_n vs. p_T



Significant up to 6 orders

ATLAS-CONF-2011-074
PLB708 (2012) 249



Recap

- Geometry of overlapping nuclei \rightarrow elliptic flow
- Initial-state density fluctuations lead to different 'shapes' of overlap region \rightarrow flow at higher orders
- Flow measured up to 6th order

What does a medium need for collective effects?

What can we learn from these results?



Hydrodynamics

- Calculating space-time evolution of QGP from first principles (QCD Lagrangian) is too complex (non-abelian, strong coupling, many-body system, ...)
- Expanding medium can be described macroscopically with hydrodynamical models
 - Conservation of energy-momentum $\partial_\mu T^{\mu\nu} = 0$
 - Conservation of charges, mainly baryon number $\partial_\mu N_i^\mu = 0$
 - Local thermodynamical equilibrium $N_i^\mu = nu^\mu$
- Needed input
 - Initial conditions
 - Equation of State (EoS), from lattice QCD
 - Relativistic fluid dynamics
 - Perfect or dissipative (\rightarrow transport coefficients)

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}$$



Hydrodynamics (2)

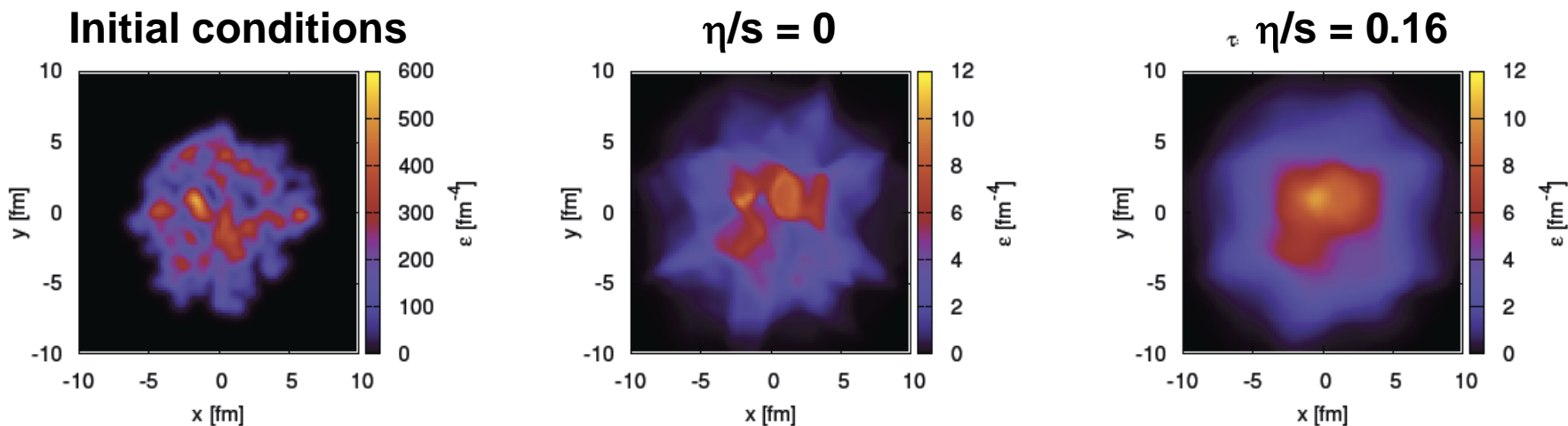
- Once dynamics well described, hydrodynamic “output” can be used in other calculations: jet quenching, J/ψ melting, etc.
- Flow observables:
Initial-state anisotropies \rightarrow final-state anisotropies
 - Translate from initial-state eccentricity ε_n to final-state flow v_n
- Deduce conclusions on initial conditions, EoS and transport coefficients by data comparison

Shear Viscosity

- Shear viscosity washes out initial-state anisotropies
 - Expressed as η/s (shear viscosity over entropy)
 - Ideal hydrodynamics : $\eta/s = 0$ \longrightarrow
 - Viscous hydrodynamics : $\eta/s > 0$
 - Large influence on higher-order flow

not to confuse with ideal (free streaming) gas \rightarrow no interactions

Density in collision region (x vs. y)

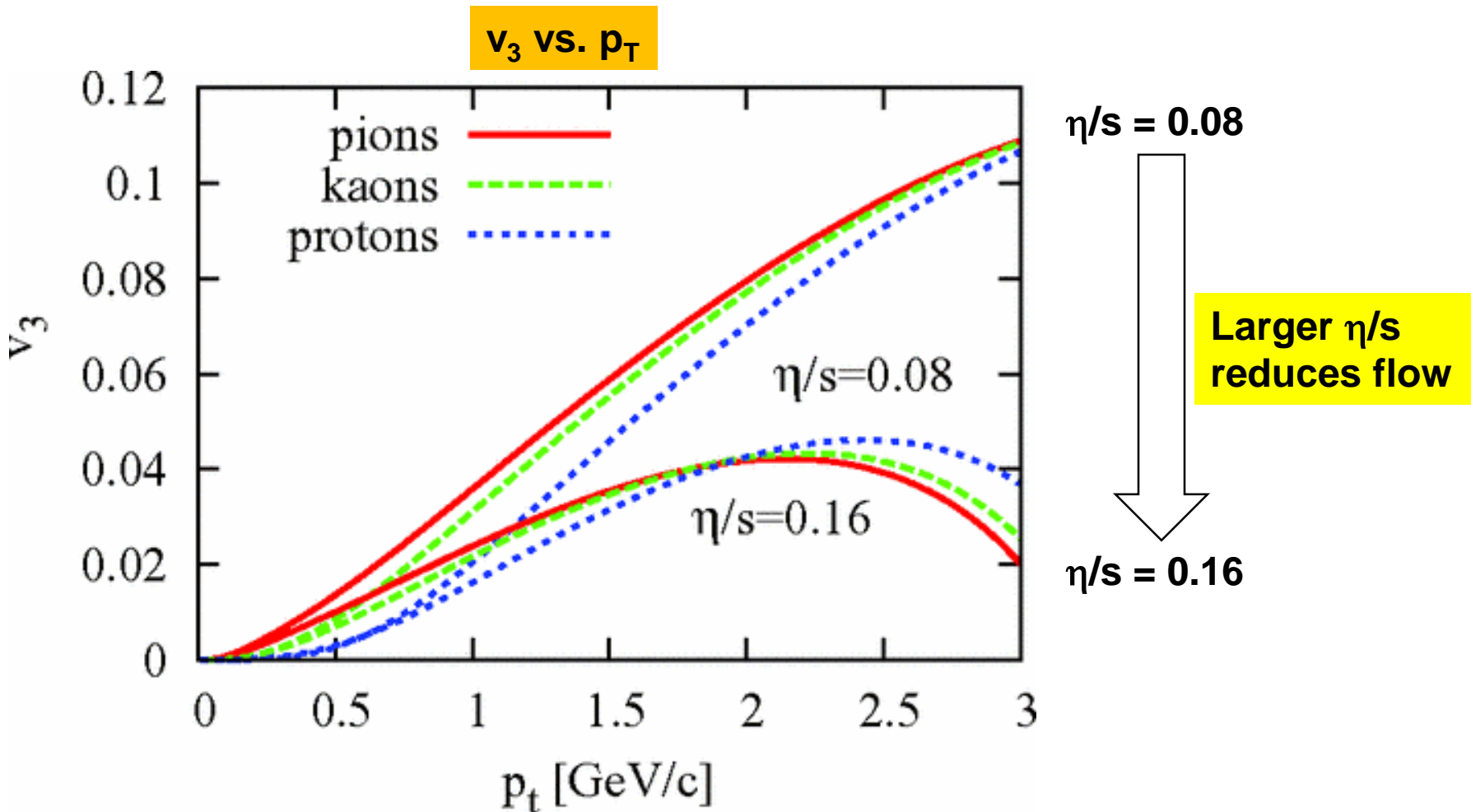


Water: $\eta/s \sim 30$ | Olive oil $\eta/s \sim 240$

MUSIC, Sangyong Jeon



Example: Shear Viscosity

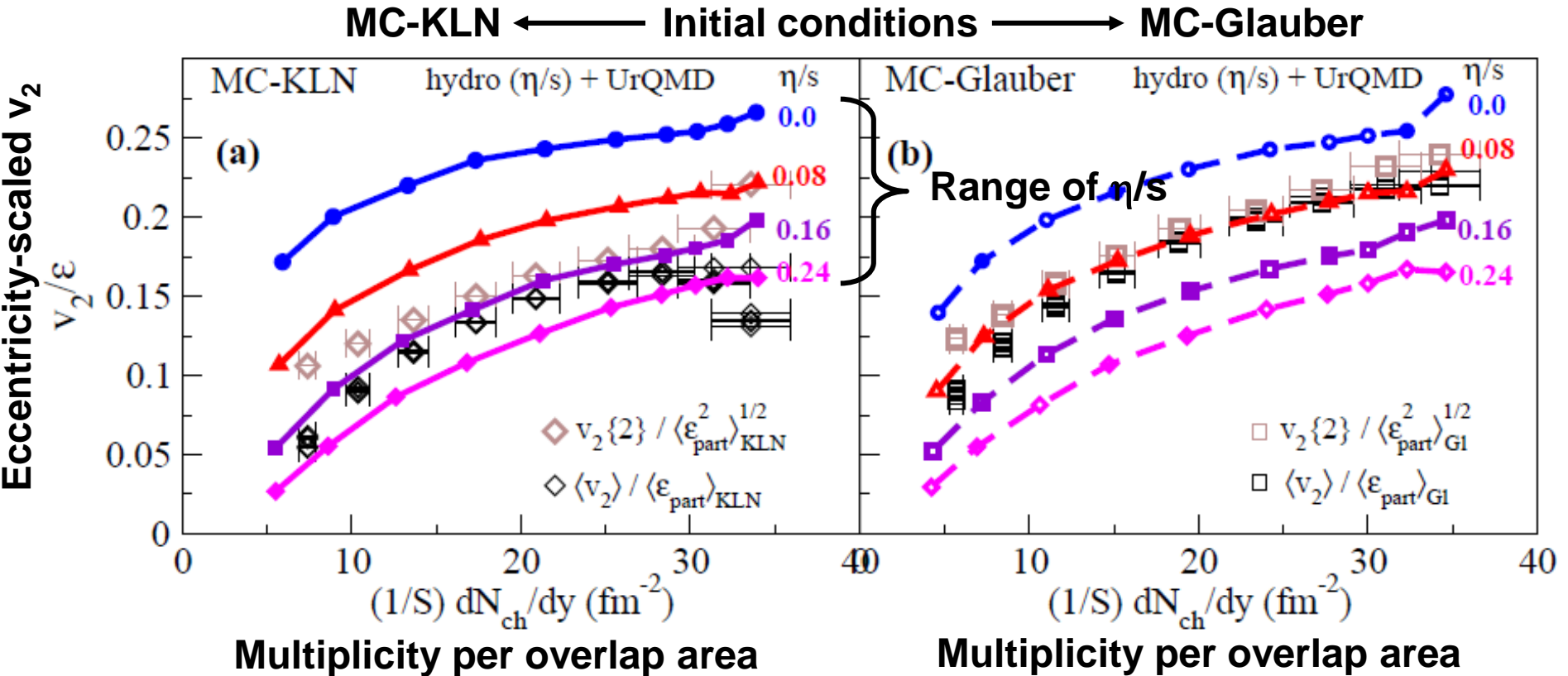


Shear viscosity hampers the build-up of flow !

PRC 82, 034913 (2010)



Hydro vs. Data



MC-KLN with $\eta/s = 0.16$ or MC-Glauber with $\eta/s = 0.08$

Water: $\eta/s \sim 30$ | Olive oil $\eta/s \sim 240$

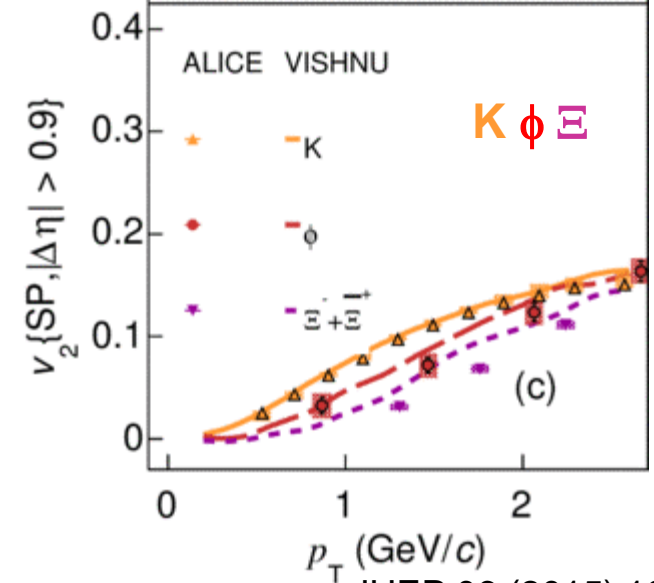
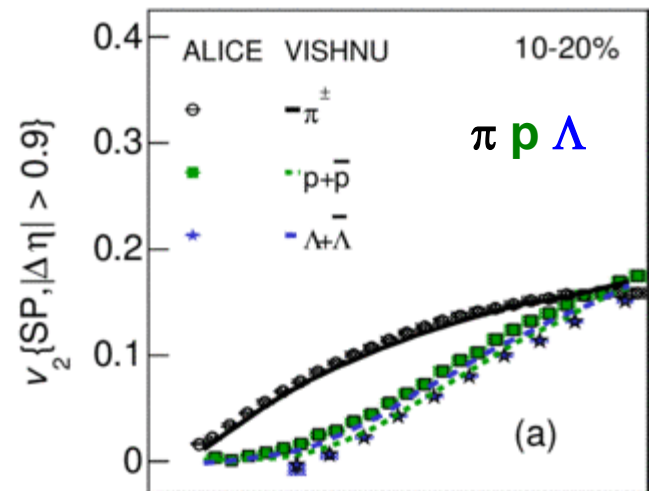
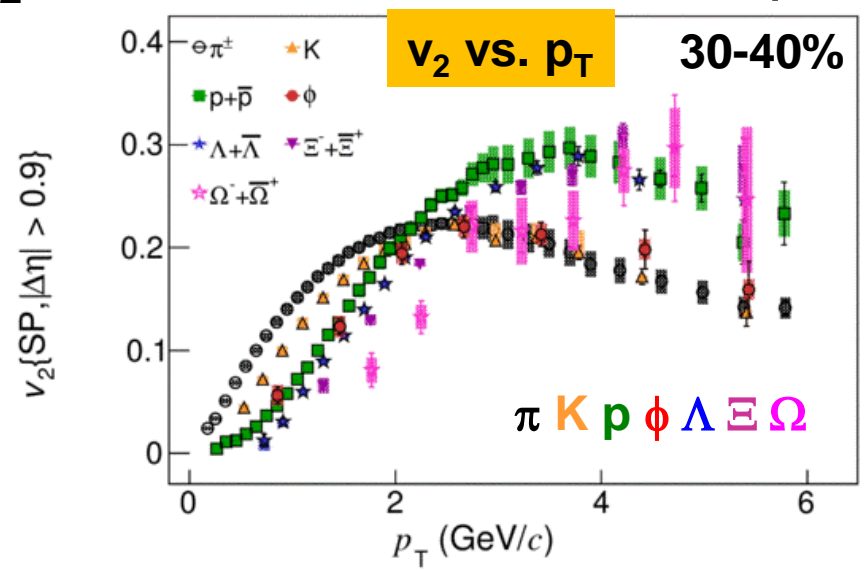
Annu.Rev.Nucl.Part.Sci. 63 (2013) 123 (Data: STAR 200 GeV)



Hydro vs. Data (2)

v_2 vs. p_T

- v_2 measured for 7 different species



- Strong species dependence
 - Different masses and quark content
- Stringent test for hydro
 - Very good agreement with VISHNU (hydro + hadronic cascade model (UrQMD), initial conditions MC-KLN, $\eta/s \sim 0.16$)

JHEP 06 (2015) 190

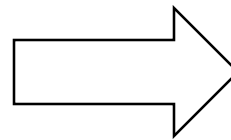


Summary

Collective Flow & Hydrodynamics

- Quark-gluon plasma expands rapidly (up to $\sim 0.65c$)
- Spatial anisotropy of collision region causes anisotropic flow quantified as Fourier coefficients v_n
 - Measured up to 6th order
 - Initial-state fluctuations influence v_n
- Well described by viscous hydrodynamics with a very low shear viscosity ($\eta/s \sim 0.08 - 0.16$) “perfect liquid”

Hydrodynamical models describe collective flow



Matter created in HI collisions is in local thermal equilibrium



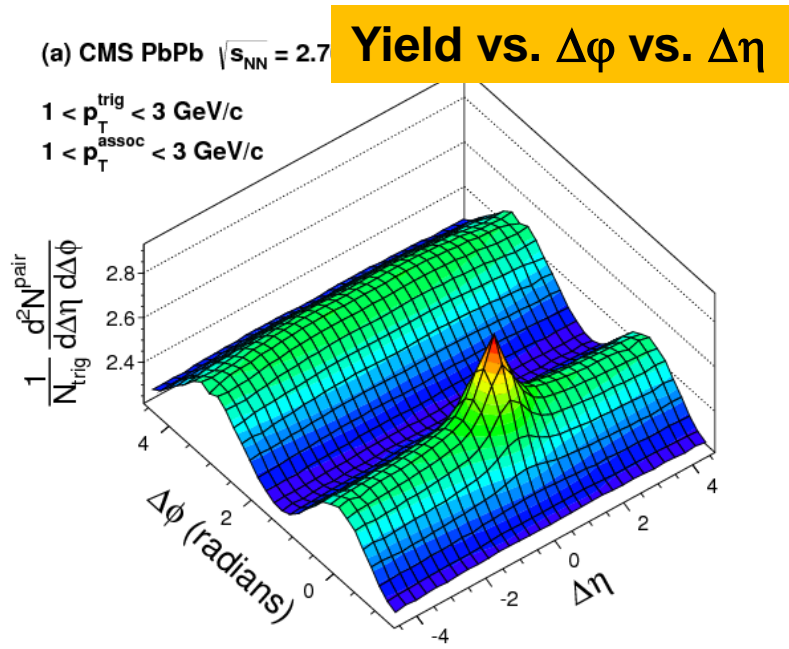
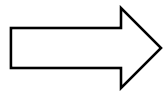
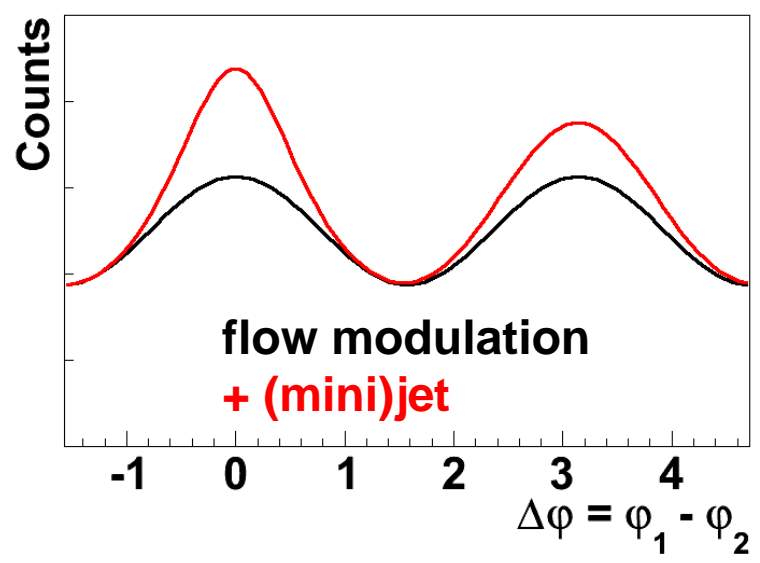
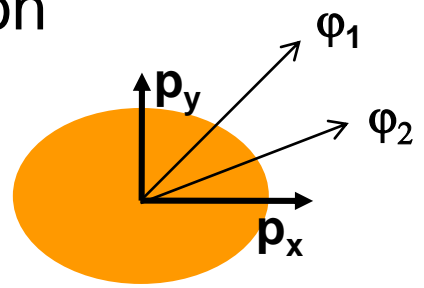
Collectivity in Small Systems

Some surprises...



Recap Two-Particle Correlations

- For v_n measurement, we discussed contribution from flow and non-flow ((mini)jets)
- This can also be looked at in two dimensions
 - Azimuth $\Delta\phi$ and pseudorapidity $\Delta\eta$

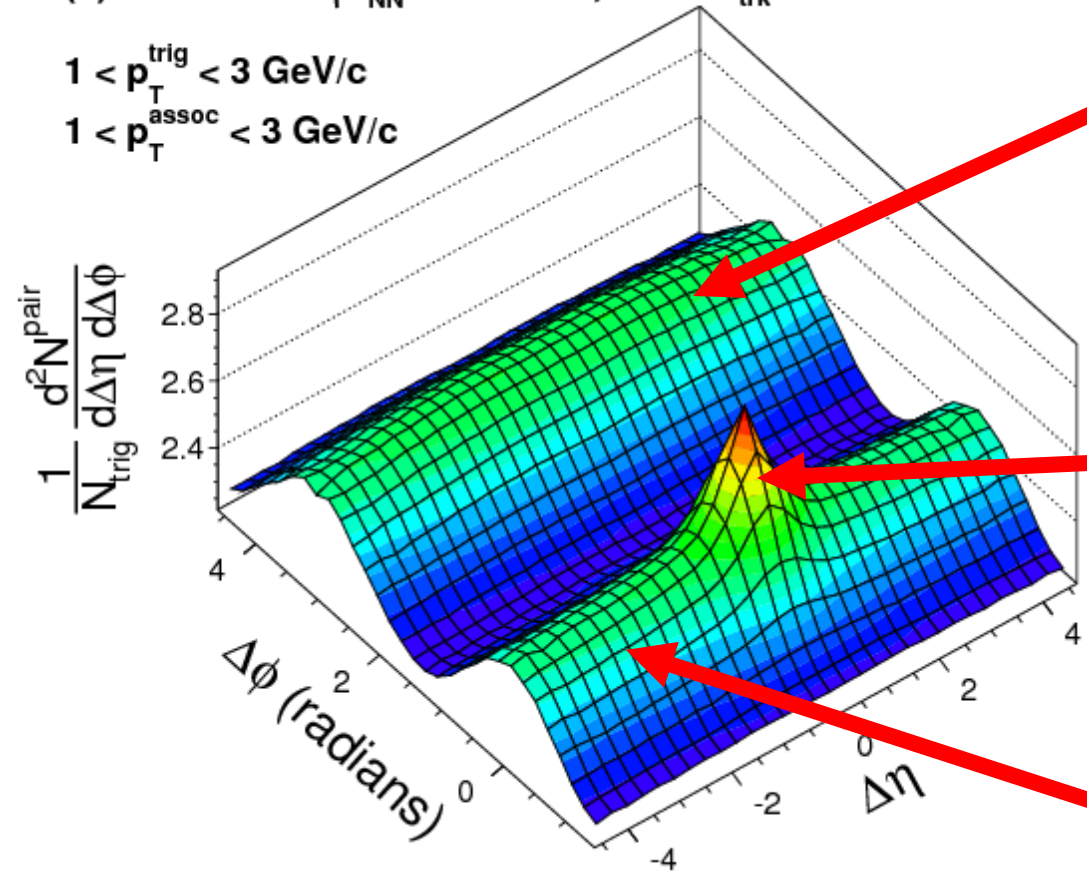




Typical Two-Particle Correlation

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{trk}^{offline} < 260$

$1 < p_T^{trig} < 3$ GeV/c
 $1 < p_T^{assoc} < 3$ GeV/c



Away-side jet + flow
($\Delta\phi \sim \pi$, elongated in $\Delta\eta$)

Near-side jet + resonances, ...
($\Delta\phi \sim 0$, $\Delta\eta \sim 0$)

Near-side flow ridge
($\Delta\phi \sim 0$, elongated in $\Delta\eta$)

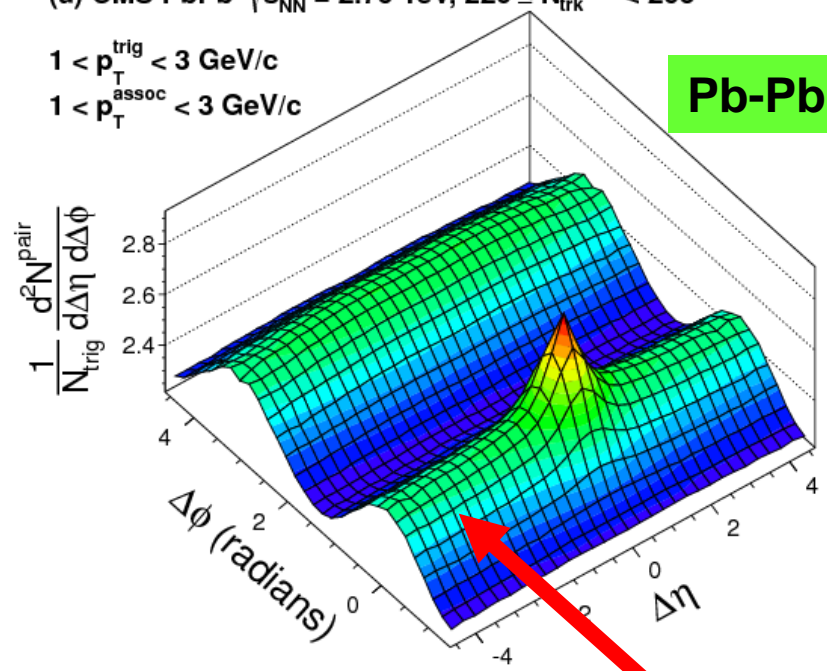


Pb-Pb vs. pp

(a) CMS PbPb $\sqrt{s_{NN}} = 2.76$ TeV, $220 \leq N_{trk}^{offline} < 260$

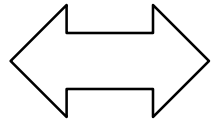
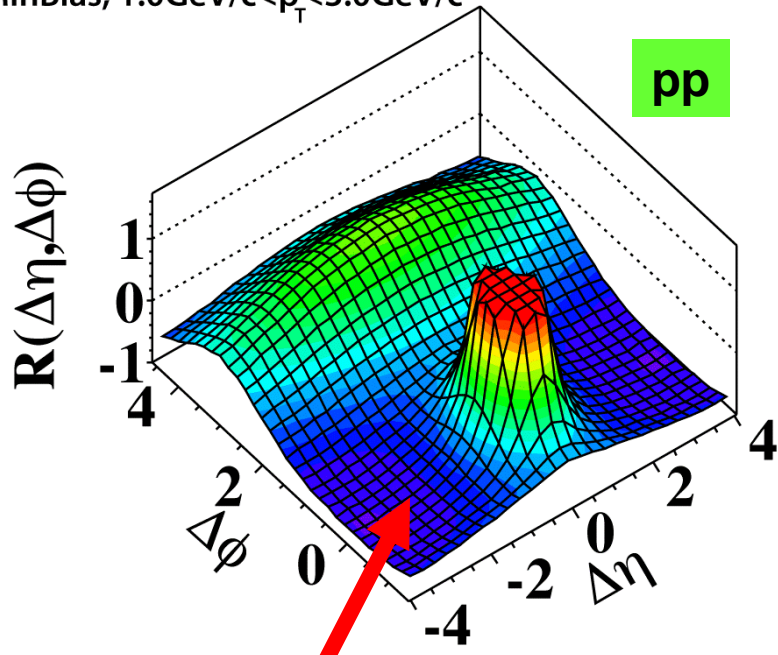
$1 < p_T^{trig} < 3$ GeV/c
 $1 < p_T^{assoc} < 3$ GeV/c

Pb-Pb



CMS 2010, $\sqrt{s} = 7$ TeV
MinBias, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$

pp



Near-side ridge (flow) only in Pb-Pb

at least everyone thought so for a long time...



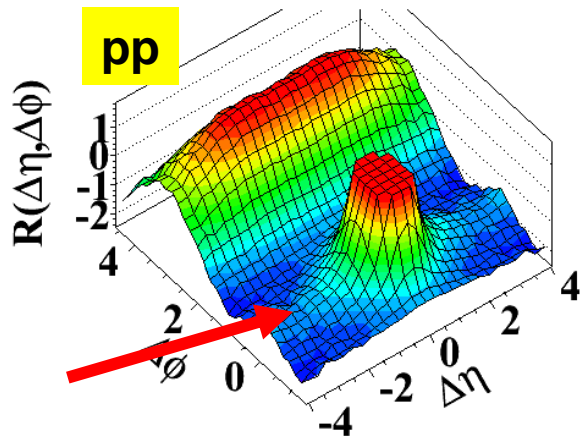
here: $\eta = \eta_{lab}$

Near-Side Ridge

0.005% of MB

- ...observed in very high-multiplicity pp collisions
 - 0.005% events with highest multiplicity

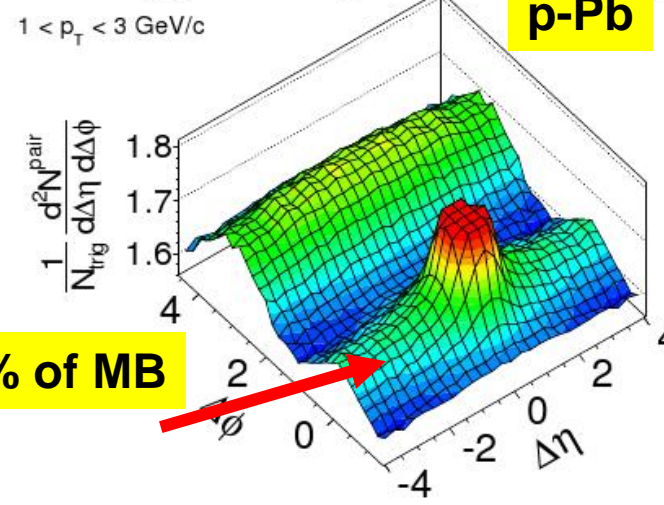
(d) CMS $N \geq 110$, $1.0 \text{ GeV}/c < p_T < 3.0 \text{ GeV}/c$



CMS, JHEP09(2010)091

- ...observed in high multiplicity p-Pb collisions
 - ~40% events with highest multiplicity
 - Surprisingly large magnitude

$N_{\text{offline, trk}} > 110$
CMS pPb $\sqrt{s_{NN}} = 5.02 \text{ TeV}$, $N_{\text{trk}}^{\text{offline}} \geq 110$



3.1% of MB

CMS, PLB718 (2013) 795



The Double Ridge

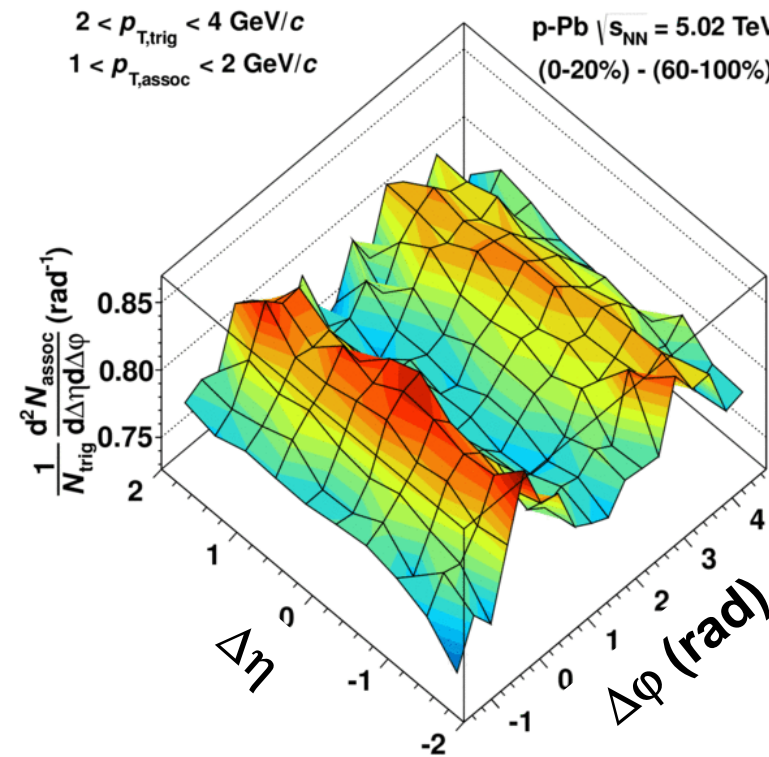
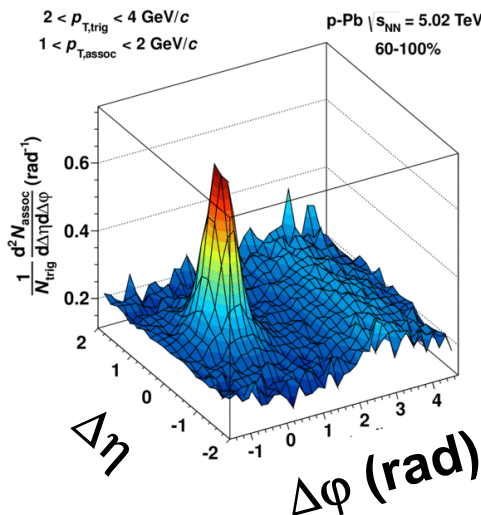
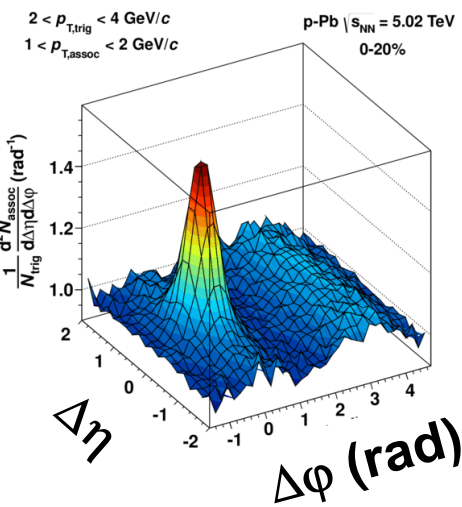
- Subtraction procedure to “isolate” ridge contribution from jet correlations
 - No ridge seen in 60-100% and similar to pp

0-20%

60-100%

$2 < p_{T,\text{trig}} < 4 \text{ GeV}/c$
 $1 < p_{T,\text{assoc}} < 2 \text{ GeV}/c$

p-Pb | $s_{\text{NN}} = 5.02 \text{ TeV}$
 (0-20%) - (60-100%)



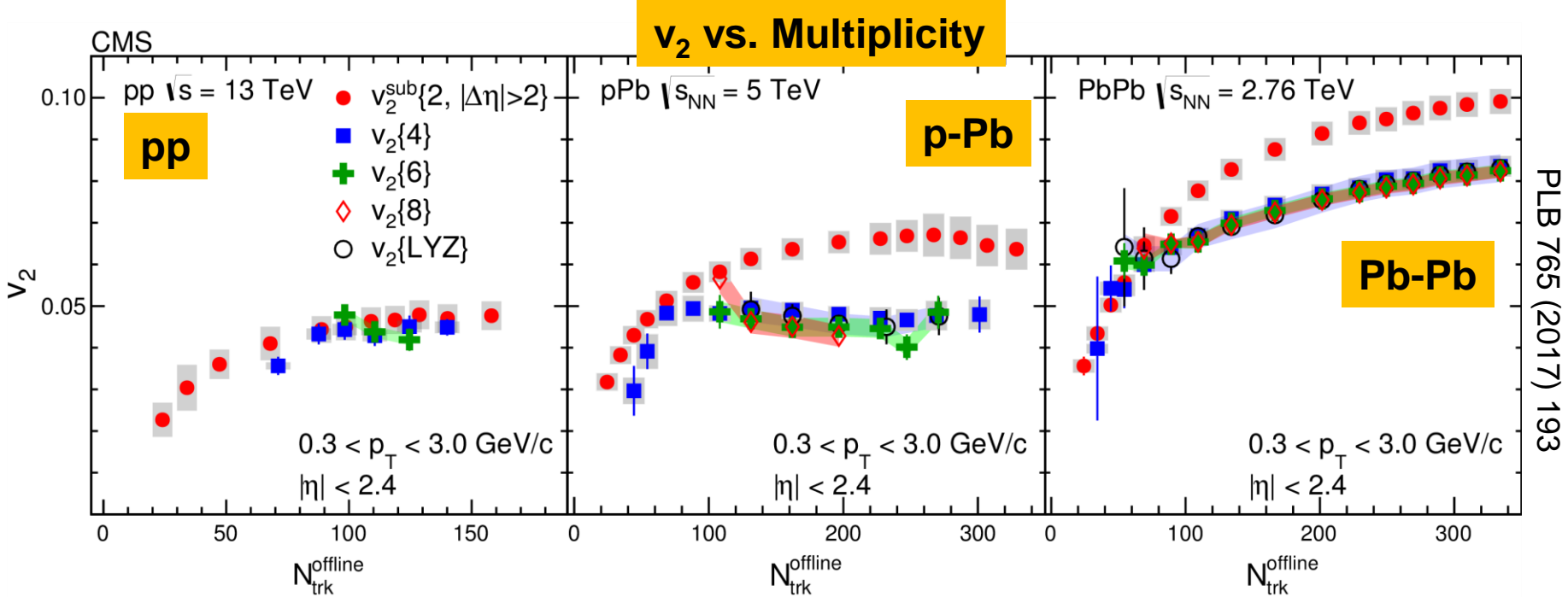
Two ridges !

ALICE, PLB719 (2013) 29



Today's Understanding

- Various “HI observables” in p-Pb and high-multiplicity pp
 - V_2, V_3, \dots
 - Multi-particle correlation $v_2\{4\} = v_2\{6\} = v_2\{8\}$
 - Mass ordering of particle species E.g. $v_2\{p\} < v_2\{\pi\}$ for $p_T < 2$ GeV/c

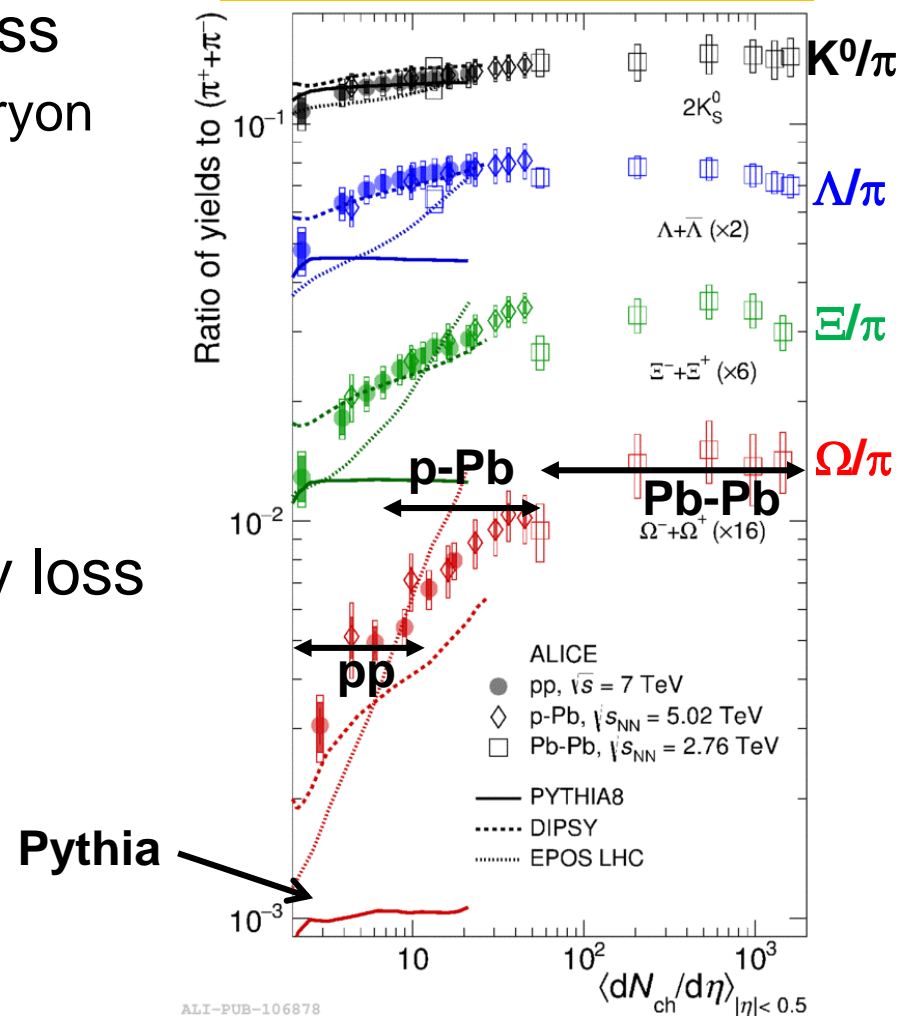




Today's Understanding (2)

- Particle ratios and strangeness
 - Smooth increase of strange baryon production
 - From pp, over p-Pb to Pb-Pb
 - Multiplicity dependence not reproduced by MC generators
- But: No sign of parton energy loss

Strange / π ratio vs. N_{ch}




Nature Phys. 13 (2017) 535



Summary Collectivity in Small Systems

- Typical Pb-Pb collision effects observed in pp and p-Pb
- Paradigm shift in interpretation of small systems
- Many hints that (mini) QGP is created in high-multiplicity p-Pb collisions (and pp collisions?)

For LHC	pp	p-Pb	Pb-Pb
Size collision region (fm ²)	2	12	150
Volume at freeze-out (fm ³)	25	160	5000
Energy density (GeV/fm ³)	?	3 (?)	10

- Debate on influence of the initial state effect as opposed to a collective approach (rescattering) 

Topic of ongoing exciting research – Stay tuned... or even better: join in!



What Next?

- Observations challenge **two paradigms** at once
 - For how small systems does the HI “standard model” remain valid?
 - Can the standard tools for pp physics remain standard?

Run 1 + 2 (2009-2018)

- Discovery of heavy-ion like phenomena in small systems
- Characterization of multi-particle correlations and strangeness enhancement

Non-flow-free correlation measurements
→ nature of higher-order correlations

Energy-loss signals
→ role of final-state interactions

Run
3 + 4

2021-2029

Strangeness enhancement
→ insight into baryon production

Thermal radiation
→ isotropization / equilibration

Chance to find unified description of underlying dynamics across system size



Summary Medium Evolution

$$dN_{ch}/d\eta \sim 1600 \text{ particles}$$

Large pressure
↓
collective flow

Dense medium
↓
Energy loss,
Quarkonia melting

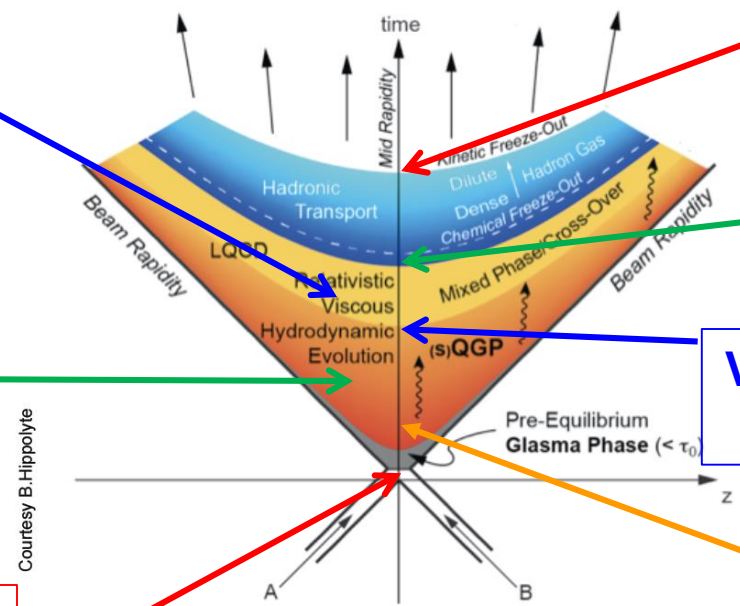
Density fluctuations
↓
spatial anisotropies

Kinetical
freeze-out
~ 90 MeV

Chemical
freeze-out
~ 155 MeV

Viscous hydrodynamics
 $\eta/s \sim 0.08 - 0.16$

Initial
temperature*
~ 300 MeV



Values for central $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ collisions (LHC)
* from direct photons (not discussed)



Take-Home Messages

- Dense colored strongly coupling medium is produced in heavy-ion collisions (the *Quark-Gluon Plasma*)
 - Particle production is strongly suppressed
- Created matter is in local thermal equilibrium
 - Particle production described by statistical models
 - Expansion described by viscous hydrodynamics “perfect liquid”
- Recent discoveries and observations in p-Pb collisions hint at collective “QGP-like” effects in small systems
 - Universal description across system size?

**Thank you for
your attention**

Many thanks for useful discussions and inspiring previous lectures to Federico Antinori, Davide Caffarri, Leticia Cunqueiro, Andrea Dainese, Michele Floris, Alexander Kalweit, Andreas Morsch, Raimond Snellings, Alberica Toia



Backup



Fluctuations

- Initial-state density fluctuations cause higher-order flow
- For a given order
 - Value is not the same event by event
 - Usually we look at averages

$$\langle e^{in(\phi_1 - \phi_2)} \rangle = v_n^2 \text{ means actually } \left\langle \left\langle e^{in(\phi_1 - \phi_2)} \right\rangle_{\text{tracks}} \right\rangle_{\text{events}} = \langle \langle 2 \rangle \rangle = \langle v_n^2 \rangle$$

$$\langle \langle 4 \rangle \rangle = -\langle v_n^4 \rangle \text{ etc.}$$

**non-flow not shown
for simplicity**

for $\sigma_{v_n} \ll \langle v_n \rangle$

- However we look for $\langle v_n \rangle$
- $\langle v_n \rangle^k = \langle v_n^k \rangle$ without fluctuations



Deviates with fluctuations

$$v_n \{2\} = \langle v_n^2 \rangle^{1/2} \approx \langle v_n \rangle + \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle} \quad v_n \{4\} = \langle v_n^4 \rangle^{1/4} \approx \langle v_n \rangle - \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

← fluctuation
← average



Fluctuations (2)

- v_2 distribution is broad
- Influence of fluctuations significant
- Estimate of fluctuations

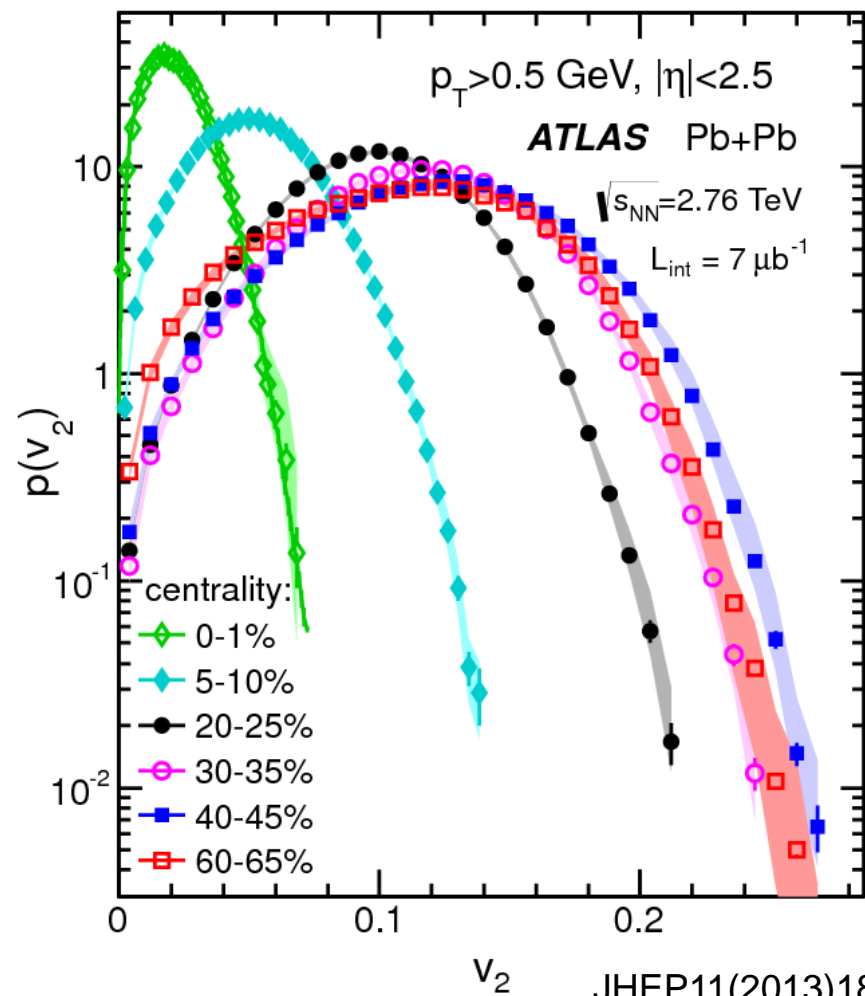
$$v_n \{2\} \approx \langle v_n \rangle + \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

$$v_n \{4\} \approx \langle v_n \rangle - \frac{1}{2} \frac{\sigma_{v_n}^2}{\langle v_n \rangle}$$

$$\frac{\sigma_{v_n}}{\langle v_n \rangle} \approx \sqrt{\frac{v_n^2 \{2\} - v_n^2 \{4\}}{v_n^2 \{2\} + v_n^2 \{4\}}}$$

for $\sigma_{v_n} \ll \langle v_n \rangle$

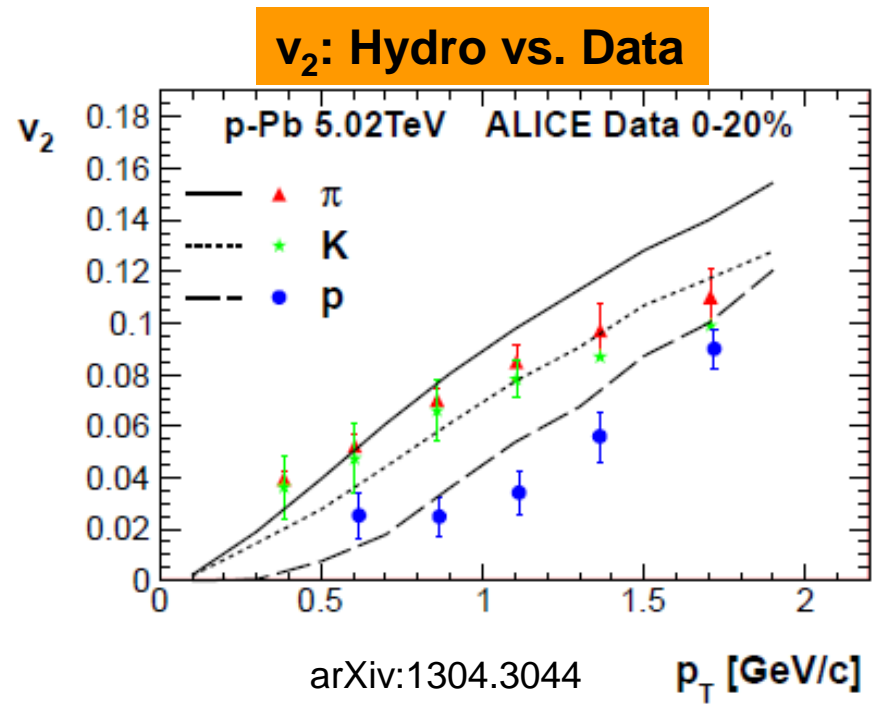
P(v_2) vs v_2





Interpretation Hydro?

- Observed effects associated to hydrodynamical evolution in Pb-Pb collisions
- Hydrodynamics in p-Pb collisions?
 - Number of interactions?
 - Sufficient time for constituents to see each other?
- Hydrodynamics in p-Pb collisions reproduces measurements
 - Assuming 0.2-0.6 fm/c for beginning of hydro evolution

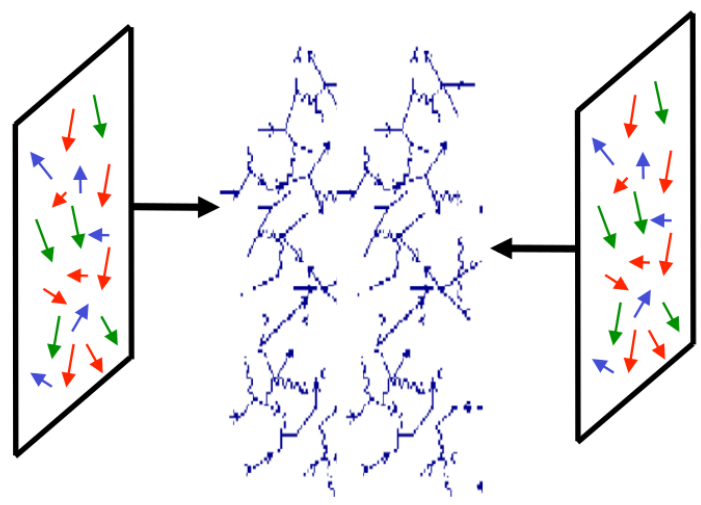




Interpretation

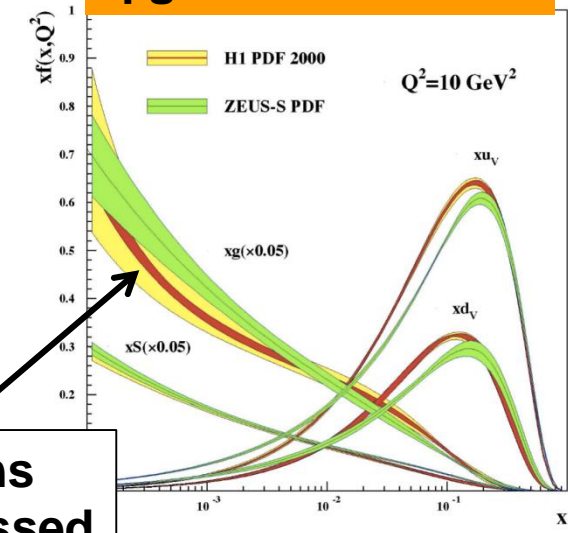
Initial-state effect?

- At low x , gluon density rises
- In nucleus density increases by $A^{1/3} \sim 6$
→ saturation
- Model of *Color Glass Condensate*

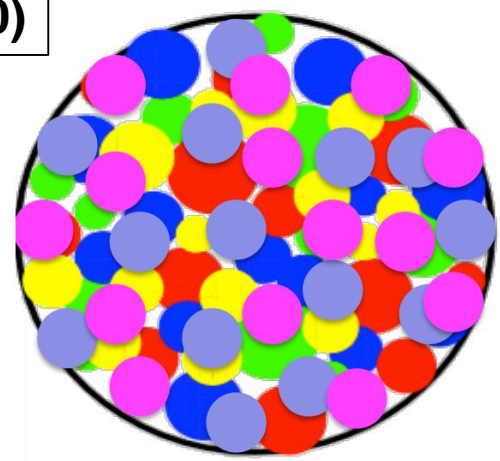


Color: gluon color charge
Glass: solid on short time scale, liquid on large time scales
Condensate: high density

q/g densities vs. x



Gluons
 (suppressed
 by factor 20)

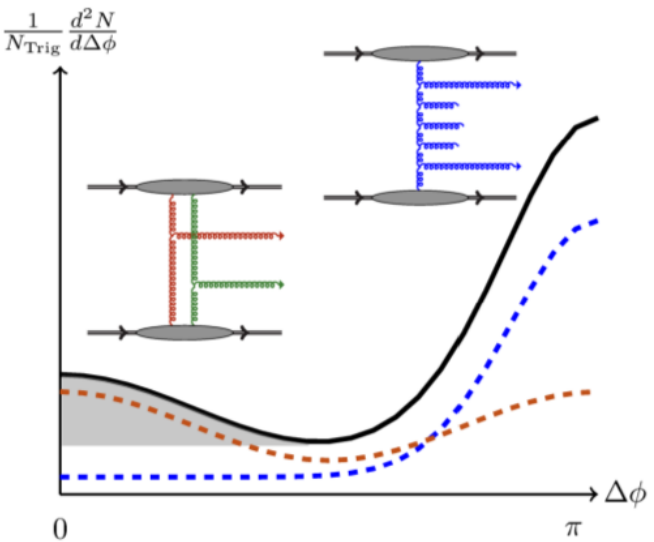




Interpretation (2)

Initial-state effect?

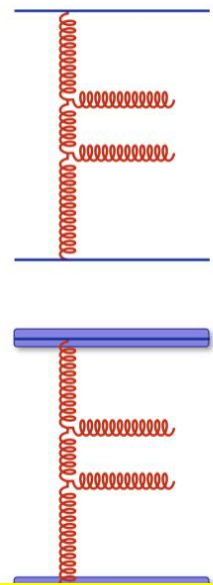
- Saturation enhances certain graphs by orders of α_S
 - Glasma graph enhanced by twice the order of magnitude than jet graph



Low color density

High color density

"jet" graph

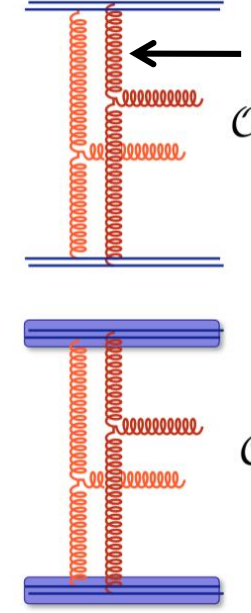


$$\mathcal{O}(g^8)$$

$$\mathcal{O}(g^0)$$

$\times 10^8$

"glasma" graph



$$\mathcal{O}(g^{12})$$

$$\mathcal{O}(1/g^4)$$

$\times 10^{16}$

Within these models, ridge can be calculated quantitatively

Then there are lots of other qualitative ideas...

PRD 87, 094034 (2013)